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Detailed Solutions

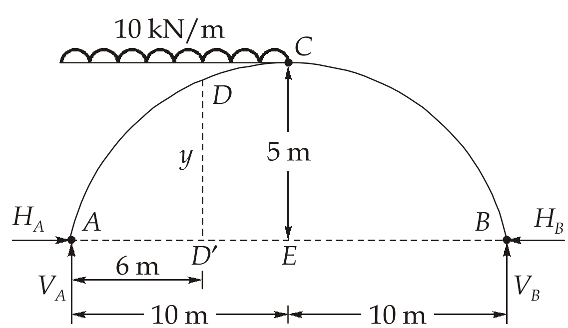
ESE-2023 Mains Test Series

Civil Engineering Test No : 6

Section A : Structural Analysis + CPM PERT

Q.1 (a) Solution:

Let H_A and H_B be horizontal reactions and V_A and V_B be vertical reactions at A and B respectively.



Now, $\Sigma F_y = 0$

$\Rightarrow \quad V_A + V_B - 10 \times 10 = 0$ ...(i)

$\Sigma F_x = 0$

$\Rightarrow \quad H_A - H_B = 0$

$\Rightarrow \quad H_A = H_B$ ...(ii)

$\Sigma M_B = 0$

$\Rightarrow \quad V_A \times 20 - 10 \times 10 \times 15 = 0$ ...(iii)

$\Rightarrow \quad V_A = \dfrac{1500}{20} = 75 \text{ kN}$

From (i),

$\Rightarrow \quad V_B = 100 - 75 = 25 \text{ kN}$

$\Sigma M_C = 0$

$\Rightarrow \quad V_A \times 10 - 10 \times 10 \times 5 - H_A \times 5 = 0$

$\Rightarrow \quad 75 \times 10 - 500 = 5 H_A$

$\Rightarrow \quad H_A = 50 \text{ kN}$

So, $\quad H_B = 50 \text{ kN}$

To calculate y :

Using the property of circle,

$(2R - 5) \times 5 = 10 \times 10$

$\Rightarrow \quad 2R = 25$

$\Rightarrow \quad R = 12.5 \text{ m}$

In ΔOFG

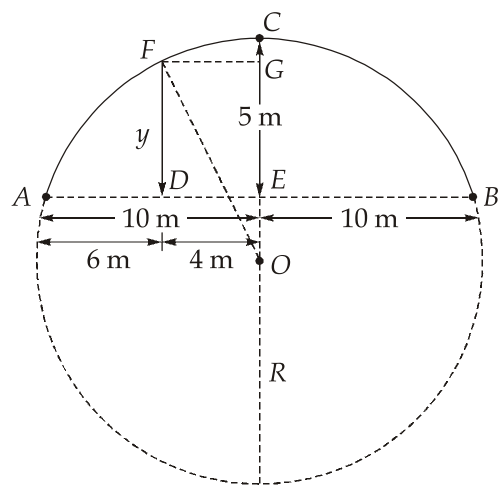
$DF = GE = y$

Also, $\quad OE = 12.5 - 5 = 7.5 \text{ m}$

Also, $\quad OF^2 = FG^2 + OG^2$

$\Rightarrow \quad (12.5)^2 = 4^2 + (7.5 + y)^2$

$\Rightarrow \quad y = 4.34 \text{ m}$



Now, $\quad (\text{B.M})_D = V_A \times 6 - H_A \times 4.34 - 10 \times \frac{6^2}{2}$

$= 75 \times 6 - 50 \times 4.34 - 5 \times 36$

$= 53 \text{ kN-m}$

Q.1 (b) Solution:

The figure shown below represents the original network with earliest occurrence time (T_E) and latest occurrence time (T_L).

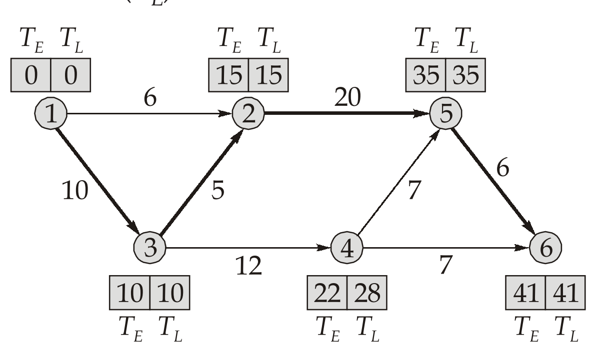


Fig. Original network

The critical path, shown by bold lines is along 1 - 3 - 2 - 5 - 6.

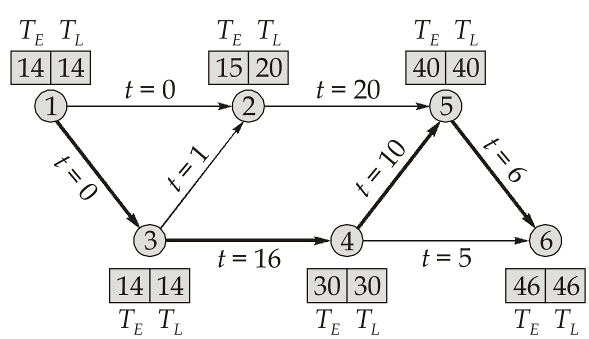
Table given below shows the details of execution of the various activities at the end of 14 days.

Table: Activities to be executed after 14 days

Activity	Whether completed		Additional time required for activities in progress	Completion time required for activities yet to begin (days)
	Yes/No	If Yes, time taken (days)		
1 - 2	Yes	6	-	-
1 - 3	Yes	10	-	-
3 - 2	No	-	1	-
3 - 4	No	-	16	-
2 - 5	No	-	-	20
4 - 5	No	-	-	10
4 - 6	No	-	-	5
5 - 6	No	-	-	6

For those activities which have already been completed, completion time 't' is taken as zero, since they require zero time after 14th day.

The updated network is shown below,

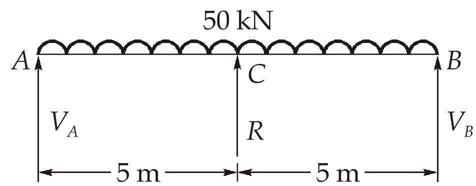


Thus, the critical path of updated network has now changed, it is now along activities 1 - 3 - 4 - 5 - 6 shown by dark lines.

According to updated network, the project will take a total time of 46 days instead of 41 days which was originally planned. Thus, the remaining duration of project = 46 – 41 = 5 days.

Q.1 (c) Solution:

As the 50 kN load is distributed uniformly, so load will act as a uniformly distributed load of 5 kN/m



Let the load on prop be R,

So, $$V_A = V_B = \frac{5\times 10 - R}{2} = \frac{50-R}{2}$$

[∵ Due to symmetry]

Now, strain energy stored by beam, $$U = 2\int_0^5 \frac{M_x^2 dx}{2EI}$$

where $$M_x = V_A x - \frac{wx^2}{2} = \left(\frac{50-R}{2}\right)x - 5\times\frac{x^2}{2}$$

So, $$U = \frac{2\int_0^5 \left(\left(\frac{50-R}{2}\right)x - \frac{5x^2}{2}\right)^2 dx}{2EI}$$

$$= \frac{1}{EI}\int_0^5 \left(\left(\frac{50-R}{2}\right)x - \frac{5}{2}x^2\right)^2 dx \quad \text{...(i)}$$

Now, Deflection at C, $$\Delta_C = \frac{\partial U}{\partial R} = -KR \quad \text{...(ii)}$$

[∵ Negative sign indicates that deflection is opposite to direction of load]

$$\frac{\partial}{\partial R}\left(\frac{1}{EI}\int_0^5 \left(\left(\frac{50-R}{2}\right)x - \frac{5}{2}x^2\right)^2\right)dx = -KR$$

$$\Rightarrow \frac{1}{EI}\int_0^5 2\left(\left(\frac{50-R}{2}\right)x - \frac{5}{2}x^2\right)\left(\frac{-x}{2}\right)dx = -KR$$

$$\Rightarrow \frac{1}{EI}\left[-\left(\frac{50-R}{2}\right)\frac{x^3}{3} + \frac{5x^4}{8}\right]_0^5 = -KR$$

$$\Rightarrow \qquad \frac{1}{EI}\left[-\left(\frac{50-R}{2}\right)\times\frac{5^3}{3}+\frac{5\times5^4}{8}\right] = -KR$$

$$\Rightarrow \qquad \frac{1}{EI}\left[\frac{-6250}{6}+\frac{125R}{6}+\frac{3125}{8}\right] = -KR$$

$$\Rightarrow \qquad \frac{125R}{6}-\frac{15625}{24} = -KREI$$

$$\Rightarrow \qquad \left(\frac{125}{6}+KEI\right)R = \frac{15625}{24}$$

$$\Rightarrow \qquad R = \frac{15625}{24\times\left(\frac{125+6EIK}{6}\right)} = \frac{15625}{500+24EIK}$$

Q.1 (d) (i) Solution:

1. **Percentage rate contract:**
   - In this form of contract, the department draws up 'item rate tender' i.e. bill of quantities with rate, amount and total amount.
   - The contractors are required to offer to carry the work as per with the rates shown in the specific price schedule or some percentage above or below the rates indicated in the schedule of work attached with the tender.

   Merits:
   - The ranking amongst the contractors is easily known just on the opening of the tender.
   - The benefit due to increased quantity with a beneficial rate can not be availed by the contractor.

   Demerits:
   - By negotiating among the contractors, two or more may quote same rate in order to get a part of work at a high rate. There may be difficulty to divide the work at equal amount among the contractors.

2. **Cost plus percentage rate contract:**
   - In this type of contract, a contractor agrees to take the work of construction on the actual cost of work plus on agreed percentage in addition, for his services.
   - It is generally adopted when the labour and material cost are liable to fluctuate heavily in market.

Merits:

- The work can be completed in the shortest possible time.
- Suitable for works when there is uncertainty and fluctuations in the market rates of labour and materials.

Demerits:

- The contractor's only aim is to make the cost of the project as high as possible in order to seek greater margin of profit.
- A proper control over purchase of materials and of labour shall have to be exercised by the department or owner.

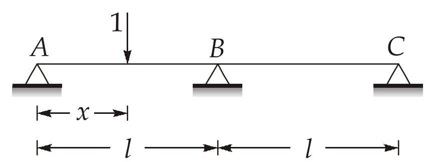
Q.1 (d) (ii) Solution:

1. **Earnest Money Deposit (EMD):**
    - It is the amount to be deposited by all the bidders when they submit their tender/bid.
    - The EMD amount varies from 1% to 3% of the tender value.
    - Once the contract is finalized, the EMD remitted by all the unsuccessful bidders are returned back.
    - In the event of the successful bidders with drawing his offer, or refusing to take up the work, then the EMD retained will be fortified.
2. **Security Deposit:**
    - It is the amount to be deposited by the successful bidders after the contract is finalized.
    - Normally, it is about 2.5% of the total value of the contract.
    - This amount is collected as the safety measure so that the contractor fulfills all the terms and conditions of the contract and completes the work in accordance with the terms of the contract.
    - The deposit amount will be refunded to the contractor on the completion of the work and after observing the work for defects if any till the 'defect liability period'. (Normally around six months to one year).

Q.1 (e) Solution:

Muller Breslau's principle: "The influence line diagram for any stress function in a structure is represented by its deflected shape obtained by removing the restraint offered by stress function (S.F, B.M and reaction etc) and introducing a directly generalized unit displacement in the positive direction of that stress function."

When unit load is in AB



Fixed end moments:

$$M_{FAB} = \frac{-x(l-x)^2}{l^2}\ ; \quad M_{FBC} = 0$$

$$M_{FBA} = \frac{x^2(l-x)}{l^2}\ ; \quad M_{FCB} = 0$$

Now, **distribution factor,**

Joint	Member	Stiffness	Total stiffness	D.F.
B	BA	$\frac{3EI}{l}$	$\frac{6EI}{l}$	$\frac{1}{2}$
	BC	$\frac{3EI}{l}$		$\frac{1}{2}$

Moment distribution table,

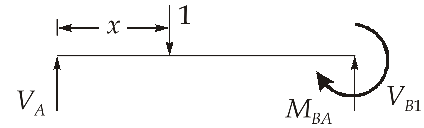
Joint	A		B	C
		0.5	0.5	
D.F.	$\frac{-x(l-x)^2}{l^2}$	$\frac{-x^2(l-x)}{l^2}$		
End correction	$\frac{x(l-x)^2}{l^2} \rightarrow$	$\frac{x(l-x)^2}{2l^2}$		
Corrected F.E.M.	0	$\frac{x(l^2-x^2)}{2l^2}$		
B.M.		$\frac{-x(l^2-x^2)}{4l^2}$	$\frac{-x(l^2-x^2)}{4l^2}$	
Final end moment	0	$\frac{x(l^2-x^2)}{4l^2}$	$\frac{-x(l^2-x^2)}{4l^2}$	0

Support reactions

Now, $\Sigma M_B = 0$

$\Rightarrow \quad V_A \times l - (l-x) + M_{BA} = 0$

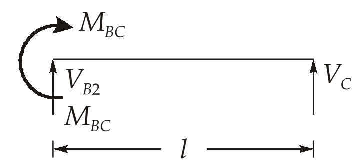
$$V_A = \frac{-M_{BA} + (l - x)}{l}$$



$$= \frac{\frac{-x(l^2 - x^2)}{4l^2} + (l - x)}{l} = \frac{(l - x)(4l^2 - lx - x^2)}{4l^3}$$

$$\Sigma M_B = 0$$

$$\Rightarrow \quad -V_C \times l + M_{BC} = 0$$



$$\Rightarrow \quad V_C = \frac{M_{BC}}{l} = \frac{-x(l^2 - x^2)}{4l^3}$$

$$\therefore \quad V_B = 1 - (V_A + V_C)$$

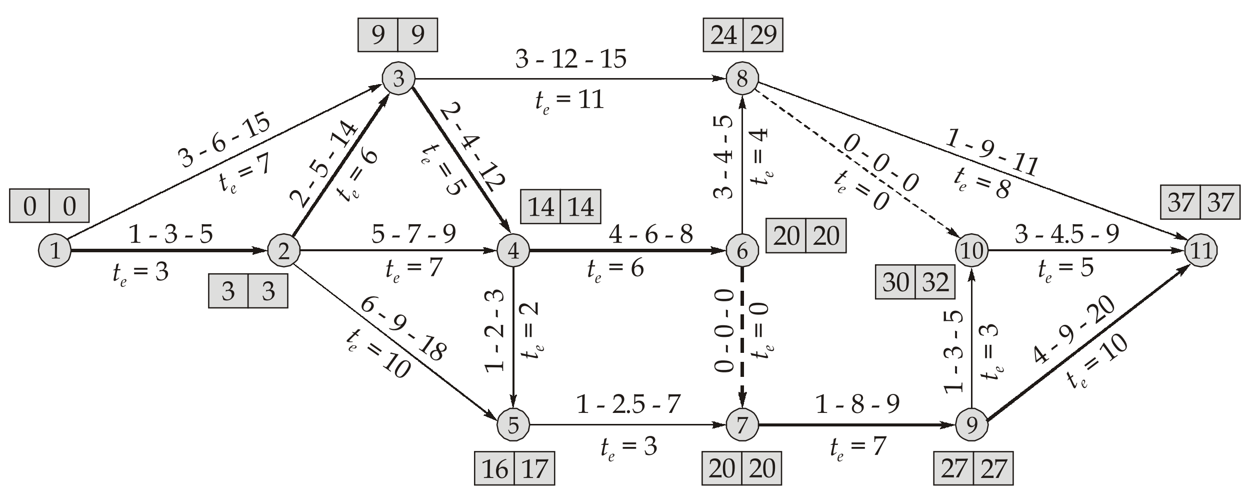
$$= 1 - \left(\frac{(l - x)(4l^2 - lx - x^2)}{4l^3} - \frac{x(l^2 - x^2)}{4l^3}\right)$$

$$= 1 - \frac{(l - x)^2(2l + x)}{2l^3}$$

When the load is in BC, V_A and V_C will interchange their values but V_B will remain same. However, when load is in BC, x in expression of 'reactions' will be from end 'C'.

Q.2 (a) Solution:

(i) PERT network:



(ii) Expected duration of each activity,

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

Standard deviation of each activity,

$$\sigma_t = \frac{t_p - t_0}{6}$$

Variance of each activity, $(\sigma_t)^2 = \left(\frac{t_p - t_0}{6}\right)^2$

Activity (i – j)	Three time estimates (weeks)			t_e (weeks)	σ_t (weeks)	$(\sigma_t)^2$ (weeks)^2	Remarks
	t_0	t_m	t_p				
1 - 2	1	3	5	3	2/3	4/9	Critical
1 - 3	3	6	15	7	2	4	-
2 - 3	2	5	14	6	2	4	Critical
2 - 4	5	7	9	7	2/3	4/9	-
2 - 5	6	9	18	10	2	4	-
3 - 8	3	12	15	11	2	4	-
3 - 4	2	4	12	5	5/3	25/9	Critical
4 - 5	1	2	3	2	1/3	1/9	-
4 - 6	4	6	8	6	2/3	4/9	Critical
5 - 7	1	2.5	7	3	1	1	-
6 - 7	0	0	0	0	0	0	Critical
6 - 8	3	4	5	4	1/3	1/9	-
7 - 9	1	8	9	7	4/3	16/9	Critical
8 - 10	0	0	0	0	0	0	-
8 - 11	1	9	11	8	5/3	25/9	-
9 - 10	1	3	5	3	2/3	4/9	-
9 - 11	4	9	20	10	8/3	64/9	Critical
10 - 11	3	4.5	9	5	1	1	-

Critical path is shown in bold lines and the same is:

1 - 2 - 3 - 4 - 6 - 7 - 9 - 11

(iii) Expected project length = 37 days

(iv) Variance of the project = Sum of variance of critical activities lying on critical path

$$= \frac{4}{9} + 4 + \frac{25}{9} + \frac{4}{9} + 0 + \frac{16}{9} + \frac{64}{9} = 16.556 \text{ weeks}$$

Standard deviation of project = $\sqrt{16.556}$ = 4.069 weeks

(v) 1. $\because \quad Z = \frac{x-u}{\sigma} = \frac{37-37}{\sigma} = 0$

For Z = 0, Probability = 50%

2. For $x = 37 + 4 = 41$ weeks

$$\therefore \quad Z = \frac{x-u}{\sigma} = \frac{41-37}{4.069} = \frac{4}{4.069} = 0.983$$

$$\therefore \quad \text{Probability } (Z = 0.983) = 81.59 + \frac{(81.59-84.13)}{(1-0.9)}(0.983-0.9) = 83.6982\%$$

(vi) For 95% probability, $Z = 1.65$

$$\therefore \quad 1.65 = \frac{x-37}{4.069}$$

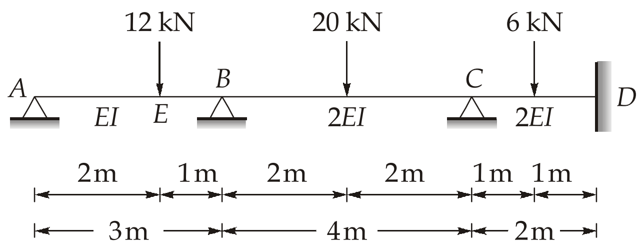
$$\Rightarrow \quad x = 43.714 \text{ weeks}$$

Q.2 (b) (i) Solution:

The points of comparison between force and displacement methods are:

Force Methods	Displacement Methods
1. In these methods, forces (i.e. SF, BM, reactions) are taken as unknowns and equations are expressed in terms of these unknown forces.	1. In these methods, displacements (i.e. Δ, θ) are taken as unknowns and equations are expressed in terms of these unknown forces.
2. Additional compatibility equations are required. Hence, this method is also called compatibility method.	2. Additional equilibrium equations are written to find displacement components. Hence this method is also called equilibrium method
3. In the compatibility equations, flexibilities of members appear, hence this method is also called flexibility method.	3. In this method stiffnesses of members appear, hence this method is also called stiffness method.
4. When degree of kinematic indeterminacy (D_k) is greater than degree of static indeterminacy (D_s), then the force methods are better.	4. When degree of kinematic indeterminacy (D_k) is less than degree of static indeterminacy (D_s), then displacement methods are better.
5. Examples of force methods are virtual work method, Clapeyron's three moment theorem, Castigliano's theorem, strain energy method, etc.	5. Examples of displacement methods are moment distribution method, slope deflection method, Kani's method etc.

Q.2 (b) (ii) Solution:



1. Fixed end moments:

$$M_{FAB} = \frac{-12\times 2\times 1}{3^2} = -2.67 \text{ kNm}$$

$$M_{FBA} = \frac{12\times 1\times 4}{3^2} = 5.33 \text{ kNm}$$

$$M_{FBC} = \frac{-20\times 4}{8} = -10 \text{ kNm}$$

$$M_{FCB} = \frac{20\times 4}{8} = 10 \text{ kNm}$$

$$M_{FCD} = \frac{-6\times 2}{8} = -1.5 \text{ kNm}$$

$$M_{FDC} = \frac{6\times 2}{8} = 1.5 \text{ kNm}$$

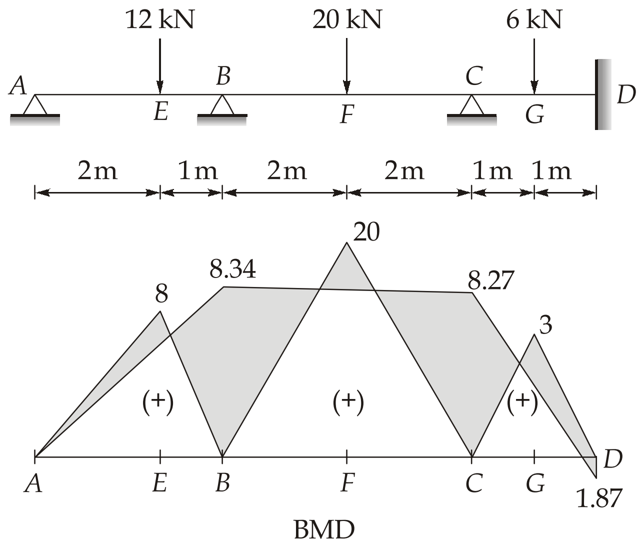
2. Distribution factors:

Joint	Member	Stiffness	Total stiffness	D.F.
B	*BA*	$\frac{3EI}{3}$	3*EI*	0.33
	BC	$\frac{8EI}{4}$		0.67
C	*CB*	$\frac{8EI}{4}$	6*EI*	0.33
	CD	$\frac{8EI}{2}$		0.67

Moment distribution table:

Joint	A	B		C		D
Distribution factor		0.33	0.67	0.33	0.67	
Fixed end moment	–2.67	5.33	–10	10	–1.5	1.5
End Correction	2.67 →	1.34				
Corrected F.E.M.	0	6.67	–10	10	–1.5	1.5
Balancing moment		1.11	2.22	–2.83	–5.67	
Carry over moment			–1.415	1.11		–2.835
Balancing moment		0.47	0.945	–0.37	–0.74	
Carry over moment			–0.185	0.47		–0.37
Balancing moment		0.062	0.123	–0.16	–0.31	
Carry over moment			–0.08	0.06		–0.15
Balancing moment		0.027	0.053	–0.02	–0.04	
Carry over moment			–0.01	0.026		–0.02
	0	8.34	–8.34	8.27	–8.27	–1.87

B.M.D.



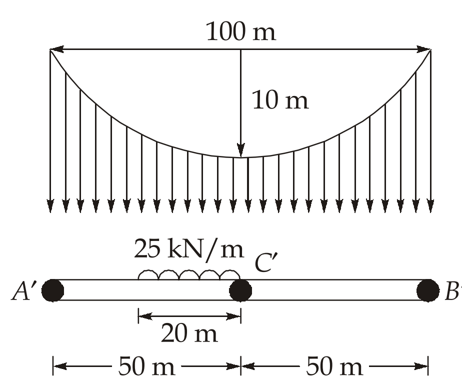
(BM values in kNm)

Q.2 (c) (i) Solution:

Analysis of live load:

Live load on girder will be transferred to cables in the form of uniformaly distributed load. Let the intensity of uniformly distributed load acting upwards on girder be w_l kN/m.

Let, the reactions developed at A' and B' due to w_l and live load be V_A' and V_B' respectively



Now, $\Sigma M_B' = 0$

$\Rightarrow V_A' \times 100 + w_l \times 100 \times 50 - 25 \times 20 \times 60 = 0$

$\Rightarrow \quad 100V_A' + 5000w_l - 30000 = 0$ ...(i)

Also, $\Sigma M_C' = 0$

$\Rightarrow V_A' \times 50 + w_l \times 50 \times 25 - 25 \times 20 \times 10 = 0$

$\Rightarrow \quad 50V_A' + 1200w_l - 5000 = 0$ ...(ii)

Solving (i) and (ii), we get

$$V'_A = -100 \text{ kN}$$

$$w_l = 8 \text{ kN/m}$$

Also, $V'_A + V'_B + w_l \times 100 = 25 \times 20$

$\Rightarrow$ $-100 + V'_B + 8 \times 100 = 500$

$\Rightarrow$ $V'_B = -200 \text{ kN}$

This uniformly distributed load 'w_l' is acting downwards on cable throughout length.

So, vertical reaction at A, $V_{A1} = \frac{w_l l}{2} = \frac{8 \times 100}{2} = 400 \text{ kN}$

Horizontal reaction at A, $H_{A1} = \frac{w_l l^2}{8h} = \frac{8 \times 100^2}{8 \times 10} = 1000 \text{ kN}$

Analysis for dead load:

As the dead load of 15 kN/m is acting on whole span of girder, it will be transferred as uniformly distributed load of 15 kN/m to cable,

So, Vertical reaction at A, $V_{A2} = \frac{w_d l}{2} = \frac{15 \times 100}{2} = 750 \text{ kN}$

(where w_d is uniformly distributed load transferred to cable due to dead load on girder)

Horizontal reaction at A, $H_{A2} = \frac{w_d l^2}{8h} = \frac{15 \times 100^2}{8 \times 10} = 1875 \text{ kN}$

Therefore, total vertical reaction at A, $V_A = V_{A1} + V_{A2}$

$= 400 + 750 = 1150 \text{ kN}$

Total horizontal reaction at A, $H_A = H_{A1} + H_{A2}$

$= 1000 + 1875 = 2875 \text{ kN}$

So, maximum tension in cable $= \sqrt{H_A^2 + V_A^2} = \sqrt{1150^2 + 2875^2} = 3096.47 \text{ kN}$

Q.2 (c) (ii) Solution:

Length of wire, $L = l + \frac{8}{3}\frac{h^2}{l}$ (where h is central dip)

$$= 30 + \frac{8}{3} \times \frac{(0.5)^2}{30} = 30.022 \text{ m}$$

So, Weight of cable, $W = 30.022 \times \frac{\pi}{4} \times (0.02)^2 \times 78.5 = 0.74 \text{ kN}$

Now, Horizontal thrust, $H = \frac{Wl}{8h} = \frac{0.74 \times 30}{8 \times 0.5} = 5.55$ kN

Vertical reaction, $V = \frac{W}{2} = \frac{0.74}{2} = 0.37$ kN

So, $T_{max} = \sqrt{H^2 + V^2} = \sqrt{5.55^2 + 0.37^2} = 5.56$ kN

Hence, maximum stress in wire $= \frac{5.56}{\frac{\pi}{4} \times (0.02)^2} = 17698.03$ kN/m^2 = 17.7 N/mm^2

Now, $\delta h = \frac{3}{16}\frac{l}{y_c}\delta L = \frac{3}{16}\frac{l^2 \alpha \Delta t}{h}$ $\quad [\because \delta l \approx l\alpha\Delta T]$

$= \frac{3}{16} \times \frac{30^2}{0.5} \times 1.2 \times 10^{-5} \times \Delta t = 4.05 \times 10^{-3}\ \Delta t$

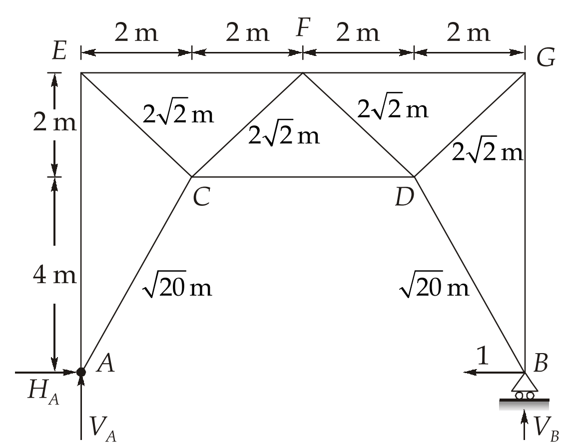
Also, $\frac{\delta f}{f} = \frac{-\delta h}{h}$

$\Rightarrow \left(\frac{20 - 17.7}{17.7}\right) = \frac{4.05 \times 10^{-3} \times \Delta t}{0.5}$

$\Rightarrow \Delta t = 16.04° \text{ C}$

Hence, the fall in temperature required to raise the stress in wire to 20 N/mm^2 is 16.04° C.

Q.3 (a) Solution:



In figure,

Number of members, $m = 11$

Number of external reactions, $r_e = 4$

Number of joints, $j = 7$

So, degree of static indeterminacy,

$D_s = m + r_e - 2j$

$= 11 + 4 - 2 \times 7$

$= 1$

Hence, the frame is statically indeterminate by one degree and indeterminacy is external because number of external reactions are more than three. So, pin joint at 'B' is replaced by a roller and a unit load is applied at B in horizontal direction. Let, the reactions developed at A be V_A and H_A and at joint B be V_B. Forces in members of frame due to unit load is represented by K_i.

Now, $\Sigma F_x = 0$

$\Rightarrow$ $H_A - 1 = 0$

$\Rightarrow$ $H_A = 1$

$\Sigma F_y = 0$

$\Rightarrow$ $V_A + V_B = 0$ ...(i)

$\Sigma M_B = 0$

$\Rightarrow$ $V_A \times 8 = 0$

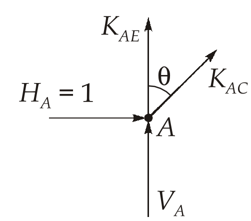
$\Rightarrow$ $V_A = 0$

From equation (i),

$V_B = 0$

***K* system of forces.**

- **Joint *A*:**



$$\sin\theta = \frac{2}{\sqrt{20}}$$

$$\cos\theta = \frac{4}{\sqrt{20}}$$

Now, $\Sigma F_x = 0$

$\Rightarrow$ $H_A + K_{AC}\sin\theta = 0$

$\Rightarrow$ $K_{AC} = \frac{-1}{\sin\theta} = \frac{-1\sqrt{20}}{2} = -\sqrt{5}$

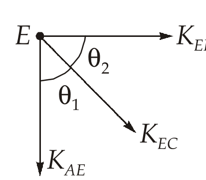
$\Sigma F_y = 0$

$\Rightarrow$ $K_{AE} + K_{AC}\cos\theta + V_A = 0$

$\Rightarrow$ $K_{AE} = -K_{AC}\cos\theta = -\left(-\sqrt{5}\right)\times\frac{4}{\sqrt{20}} = 2$

- **Joint *E*:**

In figure,



$$\sin\theta_1 = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos\theta_1 = \frac{1}{\sqrt{2}}$$

$$\sin\theta_2 = \frac{1}{\sqrt{2}}$$

$$\cos\theta_2 = \frac{1}{\sqrt{2}}$$

Now, $\Sigma F_y = 0$

$\Rightarrow$ $K_{AE} + K_{EC}\cos\theta_1 = 0$

$\Rightarrow$ $2 + K_{EC} \times \frac{1}{\sqrt{2}} = 0$

$\Rightarrow$ $K_{EC} = -2\sqrt{2}$

$\Sigma F_x = 0$

$\Rightarrow$ $K_{EF} + K_{EC}\cos\theta_2 = 0$

$\Rightarrow$ $K_{EF} = -K_{EC}\cos\theta_2$

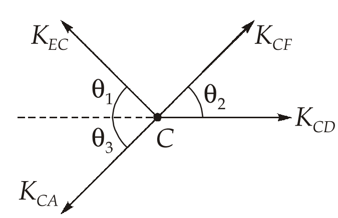
$$= -\left(-2\sqrt{2}\right) \times \frac{1}{\sqrt{2}} = 2$$

- **Consider joint C:**

$$\sin\theta_1 = \frac{1}{\sqrt{2}}, \qquad \sin\theta_2 = \frac{1}{\sqrt{2}}$$

$$\sin\theta_3 = \frac{4}{\sqrt{20}}$$

$$\cos\theta_1 = \frac{1}{\sqrt{2}}$$



$$\cos\theta_2 = \frac{1}{\sqrt{2}}$$

$$\cos\theta_3 = \frac{2}{\sqrt{20}}$$

Now, $\Sigma F_x = 0$

$$K_{EC}\cos\theta_1 + K_{CA}\cos\theta_3 = K_{CF}\cos\theta_2 + K_{CD}$$

$\Rightarrow$ $-2\sqrt{2} \times \frac{1}{\sqrt{2}} + \left(-\sqrt{5}\right) \times \frac{2}{\sqrt{20}} = \frac{K_{CF}}{\sqrt{2}} + K_{CD}$

$\Rightarrow$ $-3 = \frac{K_{CF}}{\sqrt{2}} + K_{CD}$ ...(i)

$\Sigma F_y = 0$

$$\Rightarrow \quad K_{CA}\sin\theta_3 = K_{EC}\sin\theta_1 + K_{CF}\sin\theta_2$$

$$\Rightarrow \quad -\sqrt{5}\times\frac{4}{\sqrt{20}} = \frac{K_{EC}}{\sqrt{2}} + \frac{K_{CF}}{\sqrt{2}}$$

$$\Rightarrow \quad -2\sqrt{2} = K_{EC} + K_{CF} \qquad \text{...(ii)}$$

$$\Rightarrow \quad K_{CF} = -2\sqrt{2} - K_{EC} = -2\sqrt{2} - \left(-2\sqrt{2}\right) = 0$$

Putting value of K_{CF} in (i),

$$K_{CD} = -3 - \frac{K_{CF}}{\sqrt{2}} = -3$$

As the frame is symmetrical, therefore

$$K_{BD} = K_{AC} = -\sqrt{5}$$

$$K_{BG} = K_{AE} = 2$$

$$K_{GF} = K_{EF} = 2$$

$$K_{DF} = K_{CF} = 2$$

$$K_{GD} = K_{CE} = -2\sqrt{2}$$

Now, horizontal movement of roller end 'B' due to temperature rise = $\Sigma l\alpha\Delta t k_i$

Also, horizontal movement of roller end 'B' due to unit load at B,

$$\delta = \frac{\Sigma K^2 l}{AE}$$

So, horizontal movement of roller end 'B' due to horizontal thrust 'H' at end B

$$= \frac{H\Sigma K^2 l}{AE}$$

$$\text{So, total movement of joint } B = \Sigma l\alpha\Delta t K_i + \frac{H\Sigma K^2 l}{AE}$$

But as the end B is hinged and thus,

$$\Sigma l\alpha\Delta t K_i + \frac{H\Sigma K^2 l}{AE} = 0$$

$$\Rightarrow \quad H = \frac{-\Sigma l\alpha\Delta t K_i}{\dfrac{\Sigma K^2 l}{AE}}$$

There values are calculated in table below,

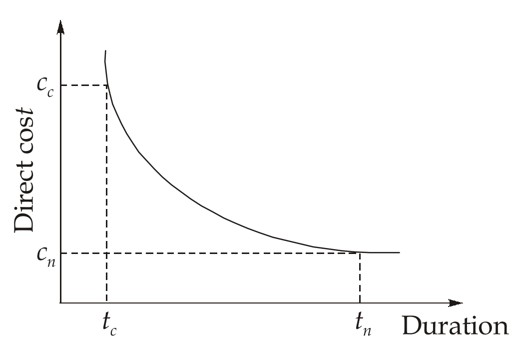
Member	l (in m)	$l\alpha\Delta t$	K	K^2l	$l\alpha\Delta tK_i$
AC	$2\sqrt{5}$	1.34×10^{-3}	$-\sqrt{5}$	22.36	-3×10^{-3}
BD	$2\sqrt{5}$	1.34×10^{-3}	$-\sqrt{5}$	22.36	-3×10^{-3}
AE	6	1.8×10^{-3}	2	24	3.6×10^{-3}
BG	6	1.8×10^{-3}	2	24	3.6×10^{-3}
EF	4	1.2×10^{-3}	2	16	2.4×10^{-3}
GF	4	1.2×10^{-3}	2	16	2.4×10^{-3}
EC	$2\sqrt{2}$	0.85×10^{-3}	$-2\sqrt{2}$	22.62	-2.4×10^{-3}
GD	$2\sqrt{2}$	0.85×10^{-3}	$-2\sqrt{2}$	22.62	-2.4×10^{-3}
CF	$2\sqrt{2}$	0.85×10^{-3}	0	0	0
DF	$2\sqrt{2}$	0.85×10^{-3}	0	0	0
CD	4	1.2×10^{-3}	-3	36 $\Sigma = 205.96$	-3.6×10^{-3} -2.4×10^{-3}

Now,
$$H = \frac{\dfrac{-(-2.4\times10^{-3})}{205.96}}{30\times2\times10^{5}\times100} = 6991.65\text{ N}$$
$$= 6.99\text{ kN} \approx 7\text{ kN (say)}$$

Q.3 (b) (i) Solution:

Direct cost:

- Direct project costs are those expenditures which are directly chargeable to and can be identified specifically with the activities of the project.
- These include labour cost, material cost, equipment cost etc.
- The direct cost curve falls with increase in duration.

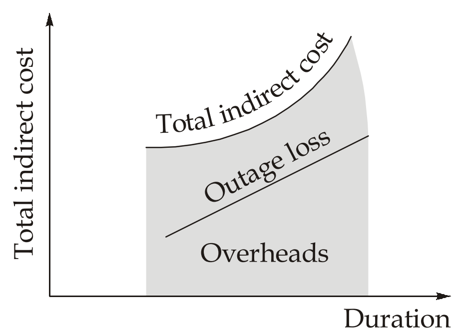


Indirect cost:

- Those expenditures which can not be clearly allocated to the individual activities of a project but are assessed as a whole.
- The indirect cost includes all the expenditures related to administrative and establishment charges, overhead, supervision, loss of revenue, penalty etc.
- Indirect cost increases with increase in duration.

Outage loss:

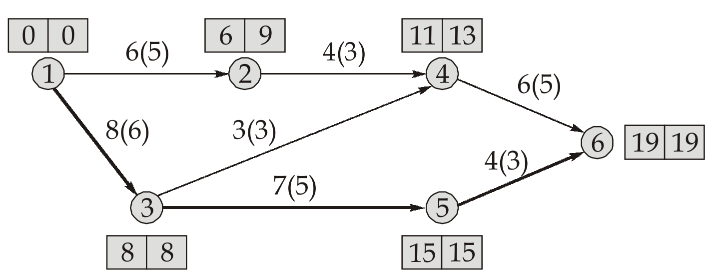
- When there is a loss in profits, due to inability to meet demand or due to some penalty due to delay, a corresponding cost increase must be added to the cost of overhead. Such a loss is called as outage loss.



Q.3 (b) (ii) Solution:

Stage-1 :

The network diagram is shown in figure below.



The critical path of the network is 1 - 3 - 5 - 6 and corresponding project duration is 19 days.

$$\because \qquad \text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

The cost slope of each activity is computed and shown below:

Activity	Normal time (Days)	Normal cost (Rs.)	Crash time (Days)	Crash cost (Rs.)	Cost slope (Rs./day)
1 – 2	6	1100	5	1300	200
1 – 3	8	2200	6	2400	100
2 – 4	4	1600	3	1800	200
3 – 4	3	1400	3	1400	Nil
3 – 5	7	1300	5	1500	100
4 – 6	6	2100	5	2600	500
5 – 6	4	1600	3	1800	200

$$\text{Total cost of project} = 1100 + 2200 + 1600 + 1400 + 1300 + 2100 + 1600 + (19 \times 350)$$

$$= 11300 + 6650 = \text{Rs. } 17950$$

The duration along non-critical paths are as under:

1 - 2 - 4 - 6 : 16 days

1 - 3 - 4 - 6 : 17 days

Stage-2 :

Crash the activities along the critical path in such a way that the duration of the critical path is reduced to non-critical path.

∴ Crash the critical path by 2 days.

Since, cost slope of activities 1 - 3 and 3 - 5 are equal but less than cost slope of activity 5 - 6.

∴ Crash activity 1 - 3 by 2 days and hence, duration of project is 17 days

$$\therefore \quad \text{Total cost} = \text{Rs. } 17950 + (2 \times 100 - 2 \times 350)$$

$$= \text{Rs. } 17450$$

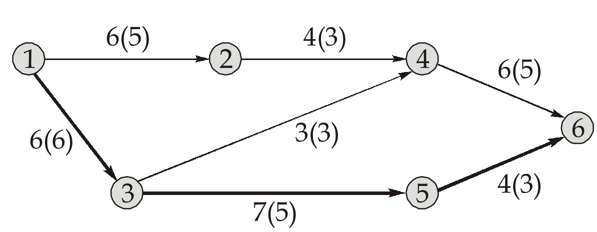


Fig. Crashed network

Stage-3 :

Now,

Critical path : 1 - 3 - 5 - 6 : 17 days

Non-critical path : 1 - 2 - 4 - 6 : 16 days

Non-critical path : 1 - 3 - 4 - 6 : 15 days

Since, activity 3- 5 has minimum cost slope.

∴ Crash activity 3 - 5 by 1 day and duration of project becomes 16 days.

∴ Total cost = Rs. 17450 + (1 × 100 – 1 × 350)

= Rs. 17200

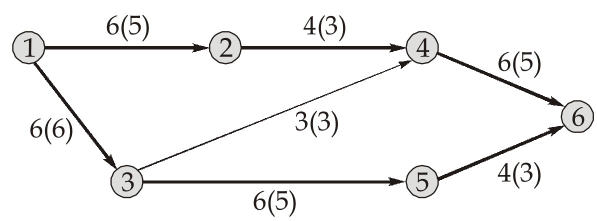


Fig. Crashed network

Stage-4 :

Critical path : 1 - 3 - 5 - 6 : 16 days

Critical path : 1 - 2 - 4 - 6 : 16 days

Noncritical path : 1 - 3 - 4 - 6 : 15 days

Both the critical paths need to be crashed.

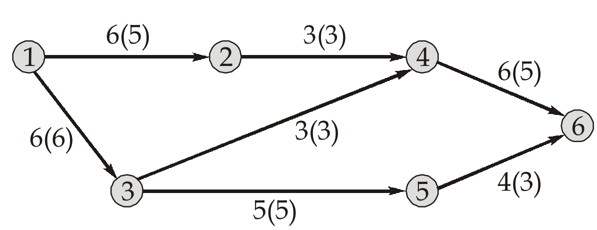
Since, activity 3 - 5 from path 1 - 3 - 5 - 6 and activity 2 - 4 from path 1 - 2 - 4 - 6 has minimum cost slope.

So, activity 3 - 5 and activity 2 - 4 to be crashed each by 1 day.

∴ Duration of project = 15 days

Total cost = Rs. 17200 + (100 + 200 – 1 × 350)

= Rs. 17150



Stage-5 :

Critical path : 1 - 3 - 5 - 6 : 15 days

Critical path : 1 - 2 - 4 - 6 : 15 days

Critical path : 1 - 3 - 4 - 6 : 15 days

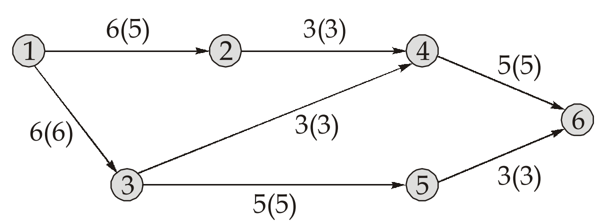
Now, activity 4 - 6 and 5 - 6 meets at common node, these are to be crashed simultaneously.

∴ Both the activities 4 - 6 and 5 - 6 can be crashed by one day.

∴ Duration of project = 14 days

∴ Total cost = Rs. 17150 + (500 + 200 – 350)

= Rs. 17500



Hence,

Minimum cost of project = Rs. 17150

Corresponding duration of project = 15 days

Q.3 (c) (i) Solution:

Dragline:

- A dragline is used to excavate the earth and load into hauling units such as trucks or tractor pulled wagons or to deposit it into dams/embankments.
- Since, the basic character of the machine is dragging the bucket against the material to be dug, it is called as dragline.

Advantages of dragline over power shovel are as follows:

- A dragline usually does not have to go into a pit or hole for excavating the earth. It may operate on natural firm ground.
- When the excavated earth is to be deposited on nearby banks or dams, it is better to use dragline with a long boom enough to disposal of the earth in one operation, eliminating the need of hauling units.
- A dragline is excellent for excavating trenches without shoring.

Q.3 (c) (ii) Solution:

Dozers:

- Dozers are very efficient excavating tools for short hauling distance of upto (say) 100 m.
- It is a tractor power unit that has blade attached to the machine's front.
- It has no set volumetric capacity. The amount of material the dozer moves is dependent on the quantity that will remain in front of the blade during the push.

Comparison between wheel mounted dozer and crawler mounted dozer are as follows:

Wheel Mounted Dozer	Crawler Mounted Dozer
• Able to operate on firm soil and paved roads.	• Able to operate on a variety of soils.
• Not able to travel over very soft, loose or muddy soil.	• Able to travel over loose or muddy soil.
• Elimination of hauling units for transporting the dozer.	• Hauling units required for transporting the dozer.
• Higher travel speed.	• Lesser travel speed.
• Greater output, especially where considerable travailing is involved.	• Greater flotation because of low pressure under the tyres.

Q.3 (c) (iii) Solution:

Maximum possible rimpull prior to slippage of driving tyres

$$= 0.50 \times 17450 \text{ kg} = 8725 \text{ kg}$$

Total weight of unit $= 28550 \text{ kg} = 28.55$ tonnes

Rolling resistance of the haul road $= 50$ kg/tonnes of gross weight

$$= 50 \times 28.55 = 1427.5 \text{ kg}$$

$\therefore$ Available rimpull to negotiate the slope

$$= 8725 - 1427.5$$

$$= 7297.5 \text{ kg}$$

The pull required per 1 tonne of the gross weight per 1% slope = 10 kg

$\therefore$ The pull required for 28.55 tonnes per 1% slope = 28.55 × 10 = 285.50 kg

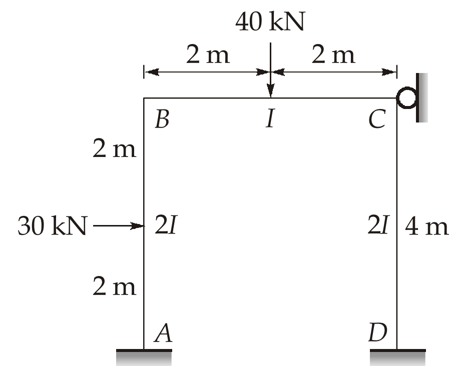
Since, 285.50 kg pull required per 1% slope

$$\therefore 7297.5 \text{ kg pull required for } \frac{7297.5}{285.5} \times 1\% = 25.56\% \text{ slope.}$$

Hence, greatest slope up which the tractor may move without slipping = 25.56%.

Q.4 (a) (i) Solution:

Fixed end moments:



$$M_{FAB} = \frac{-30 \times 4}{8} = -15 \text{ kNm}$$

$$M_{FBA} = \frac{30 \times 4}{8} = 15 \text{ kNm}$$

$$M_{FBC} = \frac{-40 \times 4}{8} = -20 \text{ kNm}$$

$$M_{FCB} = \frac{40 \times 4}{8} = 20 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

Slope deflection equations:

$$M_{AB} = -15 + \frac{2E(2I)}{4}(\theta_B) = -15 + EI\theta_B \quad \text{...(i)}$$

$$M_{BA} = 15 + \frac{2E(2I)}{4}(2\theta_B) = 15 + 2EI\theta_B \quad \text{...(ii)}$$

$$M_{BC} = -20 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$= -20 + EI\theta_B + 0.5EI\theta_C \quad \text{...(iii)}$$

$$M_{CB} = 20 + \frac{2EI}{4}(2\theta_C + \theta_B)$$

$$= 20 + EI\theta_C + 0.5EI\theta_B \quad \text{...(iv)}$$

$$M_{CD} = \frac{2E(2I)}{4}(2\theta_C) = 2IE\theta_C \quad \text{...(v)}$$

$$M_{DC} = \frac{2E(2I)}{4}(\theta_C) = EI\theta_C \quad \text{...(vi)}$$

Joint equilibrium condition at joint *B*

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow 15 + 2EI\theta_B - 20 + EI\theta_B + 0.5EI\theta_C = 0$$

$$\Rightarrow \quad 3EI\theta_B + 0.5EI\theta_C = 5 \quad \text{...(vii)}$$

Joint equilibrium condition joint *C*

$$M_{CB} + M_{CD} = 0$$

$$\Rightarrow \quad 20 + EI\theta_C + 0.5EI\theta_B + 2EI\theta_C = 0$$

$$\Rightarrow \quad 0.5EI\theta_B + 3EI\theta_C = -20 \qquad \text{...(viii)}$$

Solving equations (vii) and (viii), we get

$$EI\theta_B = 2.86, \quad EI\theta_C = -7.14$$

So,

$$M_{AB} = -15 + 2.86 = -12.14 \text{ kNm}$$

$$M_{BA} = 15 + 2 \times 2.86 = 20.72 \text{ kNm}$$

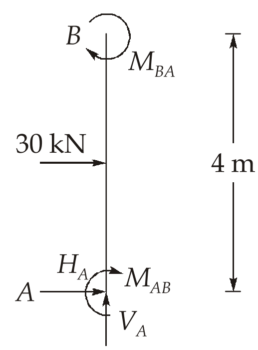
$$M_{BC} = -20 + 2.86 + 0.5 \times (-7.14) = -20.72 \text{ kNm}$$

$$M_{CB} = 20 + (-7.14) + 0.5 \times (2.86) = 14.28 \text{ kNm}$$

$$M_{CD} = 2 \times (-7.14) = -14.28 \text{ kNm}$$

$$M_{DC} = -7.14 \text{ kNm}$$

Reactions at supports:



F.B.D of *AB*

$$\Sigma M_B = 0$$

$$\Rightarrow \quad -H_A \times 4 - 30 \times 2 + M_{AB} + M_{BA} = 0$$

$$\Rightarrow \quad -H_A \times 4 - 60 + (-12.14) + 20.72 = 0$$

$$\Rightarrow \quad H_A = -12.855 \text{ kN}$$

So, $H_A = 12.855\ (\leftarrow)$

$$\Sigma M_C = 0$$

$$\Rightarrow \quad H_D \times 4 + M_{DC} + M_{CD} = 0$$

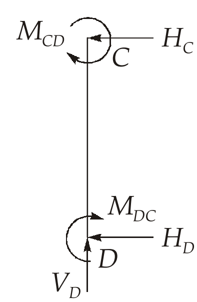
$$\Rightarrow \quad H_D \times 4 - 14.28 - 7.14 = 0$$

$$\Rightarrow \quad H_D = 5.355 \text{ kN } (\leftarrow)$$

Also, $H_A + H_D + H_C - 30 = 0$

$$\Rightarrow \quad 12.855 + 5.355 + H_C - 30 = 0$$

$$\Rightarrow \quad H_C = 11.79 \text{ kN } (\leftarrow)$$



F.B.D of *CD*

Also, $\Sigma F_y = 0$

$$\Rightarrow \quad V_A + V_D = 0 \qquad \text{...(ix)}$$

$$\Sigma M_D = 0$$

$$\Rightarrow V_A \times 4 + 30 \times 2 - 40 \times 2 - H_C \times 4 + M_{AB} + M_{DC} = 0$$

$$\Rightarrow V_A \times 4 + 60 - 80 - 11.79 \times 4 - 12.14 - 7.14 = 0$$

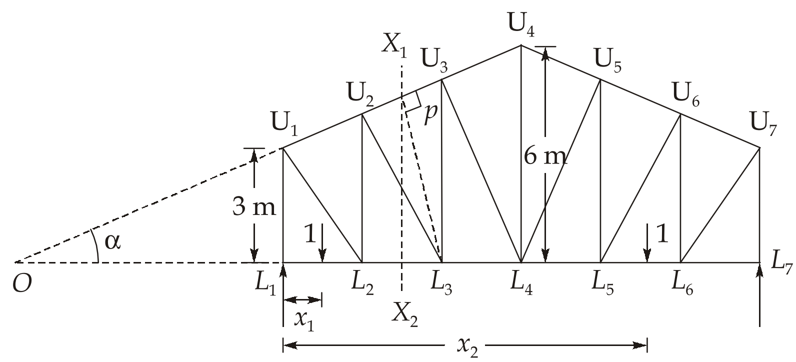
$$\Rightarrow \quad V_A = 21.61 \text{ kN}$$

So, $V_D = 40 - V_A$

$= 40 - 21.61 = 18.39$ kN

Q.4 (a) (ii) Solution:

ILD for U_2U_3



6 panels @ 4 m each

In figure, $\tan\alpha = \dfrac{U_1L_1}{OL_1} = \dfrac{U_4L_4}{OL_4}$

$$= \frac{3}{OL_1} = \frac{6}{OL_1 + 12}$$

$\Rightarrow$ $OL_1 = 12$ m

So, $\tan\alpha = \dfrac{3}{12} = \dfrac{1}{4}$

$\therefore$ $\sin\alpha = \dfrac{1}{\sqrt{17}}$

and, $\cos\alpha = \dfrac{4}{\sqrt{17}}$

Cut a section $X_1 - X_2$ through U_2U_3 , L_2L_3 and U_2L_3 and let, perpendicular distance of U_2U_3 from L_3 be p

Then, $p = OL_3 \sin\alpha = 20 \times \dfrac{1}{\sqrt{17}} = \dfrac{20}{\sqrt{17}}$

- When unit load is in L_1L_2

  Let V_{L1} and V_{L7} are vertical reactions developed at L_1 and L_7 respectively

  Now, $V_{L1} = \dfrac{24 - x_1}{24}$ where $0 < x_1 < 4$

So, considering right portion of section $X_1 - X_2$ and taking moments about L_3, we get

$$-F_{U_2U_3} \times p - V_7 \times 16 = 0$$

$$\Rightarrow \quad F_{U_2U_3} = \frac{-V_7}{p} \times 16 = \frac{-x \times 16 \times \sqrt{17}}{24 \times 20} = \frac{-x\sqrt{17}}{30}$$

So, when load is at L_1, $\quad x_1 = 0 \Rightarrow F_{U_2U_3} = \frac{-0 \times \sqrt{17}}{30} = 0$

When load is at L_2, $\quad x_1 = 4 \Rightarrow F_{U_2U_3} = \frac{-2\sqrt{17}}{15}$

- When unit load is in L_3L_7

  Let load is at 'x_2' distance from L_1

So, $$V_{L1} = \frac{24 - x_2}{24}$$

$$V_{L7} = \frac{x_2}{24}$$

Now, considering left portion of section $X_1 - X_1$ and taking moments about L_3,

We get, $\quad V_{L1} \times 8 + F_{U_2U_3} \times p = 0$

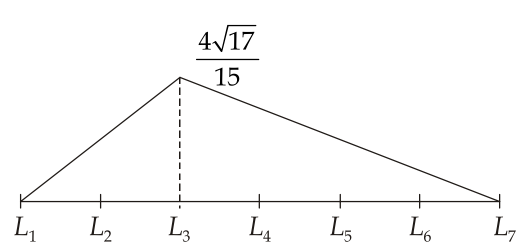
$$\Rightarrow \quad F_{U_2U_3} = \frac{-V_{L1} \times 8}{p} = \frac{-(24 - x_2)}{24 \times 20} \times 8 \times \sqrt{17}$$

$$= \frac{-(24 - x_2)\sqrt{17}}{60}$$

When load is at L_3, $\quad x_2 = 8 \Rightarrow F_{U_2U_3} = \frac{-4\sqrt{17}}{15}$

When load is at L_7, $\quad x_2 = 24 \Rightarrow F_{U_2U_3} = 0$

So, it can be said that when load is from L_1 to L_3, compressive force in U_2U_3 increases and when load is from L_3 to L_7, compressive force decreases. Its i.l.d is shown below:



I.L.D. of force U_2U_3

Q.4 (b) (i) Solution:

$$\text{Moment at } B,\ M_B = \frac{-wx^2}{2}$$

$$\text{Moment at } D,\ M_D = \frac{-wx^2}{2}$$

Applying the theorem of three moments to span BC and CD

$$M_B \times l + 2M_C(l+l) + M_D \times l = -6\left[\frac{\frac{2}{3}\times\frac{wl^2}{3}\times l\times\frac{l}{2}}{l} + \frac{\frac{2}{3}\times\frac{wl^2}{7}\times l\times\frac{l}{2}}{l}\right]$$

$$\Rightarrow \quad \frac{-wx^2}{2}\times l + 4M_C l - \frac{wx^2}{2}\times l = \frac{-wl^2}{2}$$

$$\Rightarrow \quad 4M_C = \frac{-wl^2}{2} + wx^2$$

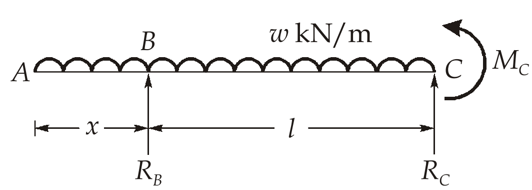
$$\Rightarrow \quad M_C = \frac{-wl^2}{8} + \frac{wx^2}{4}$$

Now, total load on beam = $2w\ (x + l)$

As all the vertical reactions are equal and thus,

$$R_B = R_C = R_D = \frac{2w(x+l)}{3}$$

Now, cut the beam at C and consider left portion of beam



Taking moments about C,

$$R_B \times l - M_C - \frac{w(l+x)^2}{2} = 0$$

$$\Rightarrow \quad R_B = \frac{w(l+x)^2}{2l} + \frac{M_c}{l} = \frac{w(x+l)^2}{2l} + \left(\frac{\frac{-wl^2}{8}+\frac{wx^2}{4}}{l}\right)$$

But, $$R_B = \frac{2w(l+x)}{3}$$

So, $$\frac{2w(l+x)}{3} = \frac{w(l+x)^2}{2l} + \left(\frac{\frac{-wl^2}{8} + \frac{wx^2}{4}}{l}\right)$$

$$\Rightarrow \quad 8 \times \frac{2}{3}(x+l) \times l = 4(l + x)^2 - l^2 + 2x^2$$

$$\Rightarrow \quad 16l^2 + 16l\,x = 12(l^2 + 2lx + x^2) - 3l^2 + 6x^2$$

$$\Rightarrow \quad 18x^2 + 8l\,x - 7l^2 = 0$$

$$\therefore \quad x = \frac{-8l \pm \sqrt{64l^2 + 4 \times 7 \times 18l^2}}{2 \times 18} = \frac{-8l \pm 2l\sqrt{16+126}}{2 \times 18}$$

$$= \frac{-8l \pm 2l \times 11.916}{36} = 0.4398l$$

Q.4 (b) (ii) Solution:

Moment of inertia of column, $$I = \frac{230 \times 300^3}{12} = 5.175 \times 10^8 \text{ mm}^4$$

So, flexural rigidity, $$EI = 22 \times 10^3 \times 5.175 \times 10^8$$
$$= 1.1385 \times 10^{13} \text{ N-mm}^2$$

Now, equivalent lateral stiffness of column is,

$$K = \frac{2 \times 12EI}{L_1^3} + \frac{3EI}{L_2^3}$$

$$= \frac{2 \times 12 \times 1.138 \times 10^{13}}{(5000)^3} + \frac{3 \times 1.138 \times 10^{13}}{(3000)^3}$$

$$= 3449.40 \text{ N/mm}$$

1. Natural frequency, $$w_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3449.4 \times 10^3}{\left(\frac{500 \times 10^3}{9.81}\right)}} = 8.23 \text{ rad/s}$$

Now, damped natural frequency,

$$w_D = w_n\sqrt{1-\xi^2} = 8.23\sqrt{1-0.1^2} = 8.19 \text{ rad/s}$$

2. Amplitude, $A = \sqrt{y_0^2 + \left(\frac{\dot{y}_0 + w_n \xi y_0}{w_D}\right)^2}$

$$= \sqrt{22^2 + \left(\frac{12 + 8.23 \times 0.1 \times 22}{8.19}\right)^2} = 22.30 \text{ mm}$$

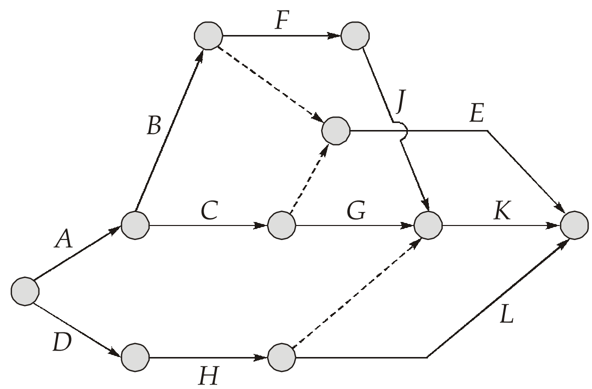
3. Displacement, $y = e^{-w_n \xi t}\left[y_0 \cos w_D t + \left(\frac{\dot{y}_0 + w_n \xi y_0}{w_D}\right)\sin w_D t\right]$

$$= e^{-8.23\times0.1t}\left[22\cos 8.19t + \left(\frac{12 + 8.23 \times 0.1 \times 22}{8.19}\right)\sin 8.19t\right]$$

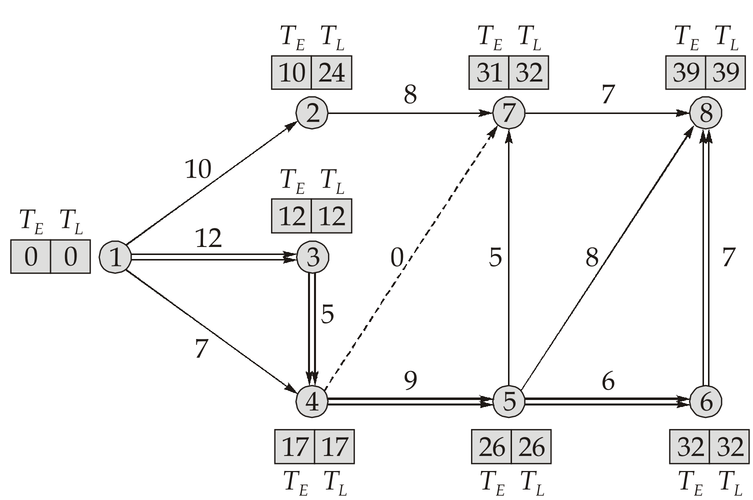
$$= e^{-0.823t}\left[22\cos 8.19t + 3.68\sin 8.19t\right]$$

Q.4 (c) Solution:

(i)



(ii)



The critical path is 1 - 3 - 4 - 5 - 6 - 8

$$\text{Project duration} = 39 \text{ days}$$

$$EST = T_E^i$$

$$EFT = EST + t^{ij}$$

$$LST = LFT - t^{ij}$$

$$LFT = T_L^j$$

Total float, $$F_T = T_L^j - T_E^i - t^{ij} = LFT - EFT$$

Free float, $$F_F = F_T - S_j = \left(T_L^j - T_E^j\right)$$

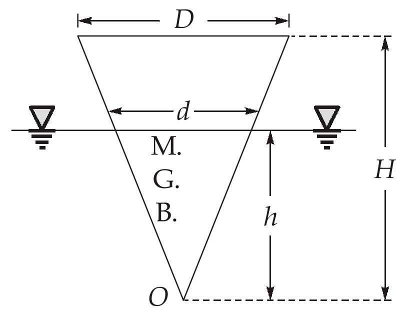
On incorporating the above formulas, early start, early finish, late start, late finish time of all the activities are shown in table below:

Activity	Tail Event		Head Event		Duration (t^{ij})	*EST*	*EFT*	*LST*	*LFT*	F_T	$F_F = F_T - S_j$
	T_E^i	T_L^i	T_E^j	T_L^j							
1 - 2	0	0	10	24	10	0	10	14	24	14	0
1 - 3	0	0	12	12	12	0	12	0	12	0	0
1 - 4	0	0	17	17	7	0	7	10	17	10	10
2 - 7	10	24	31	32	8	10	18	24	32	14	13
3 - 4	12	12	17	17	5	12	17	12	17	0	0
4 - 5	17	17	26	26	9	17	26	17	26	0	0
4 - 7	17	17	31	32	0	17	17	32	32	15	14
5 - 6	26	26	32	32	6	26	32	26	32	0	0
5 - 7	26	26	31	32	5	26	31	27	32	1	0
5 - 8	26	26	39	39	8	26	34	31	39	5	5
6 - 8	32	32	39	39	7	32	39	32	39	0	0
7 - 8	31	32	39	39	7	31	38	32	39	1	1

Section B : Flow of fluids, hydraulic machines and hydro power-1 + Design of concrete and Masonry Structures-2

Q.5 (a) Solution:

The situation is as shown in figure below.



$$\text{Weight of cone} = \text{Weight of volume of water displaced}$$

$$\Rightarrow \quad \frac{1}{12}\pi D^2 H(S\gamma_w) = \frac{1}{12}\pi d^2 h \gamma_w \qquad (\gamma_w = \text{specific weight of water})$$

$$\Rightarrow \quad D^2H(S) = d^2h \qquad \text{...(i)}$$

Also,
$$\frac{D}{H} = \frac{d}{h}$$

$$\Rightarrow \quad d = \frac{D}{H}\cdot h \qquad \text{...(ii)}$$

Putting value of d in (i),

$$\Rightarrow \quad D^2H(S) = \frac{D^2}{H^2}\cdot h^3$$

$$\Rightarrow \quad h^3 = H^3S \quad \Rightarrow h = S^{1/3}H$$

Height of centre of buoyancy, $OB = \frac{3}{4}h$

Height of centre of gravity of cone,

$$OG = \frac{3}{4}H$$

If M is the metacentre, then,

$$BM = \frac{I_{min}}{V_{displaced}} = \frac{\frac{\pi}{64}d^4}{\frac{\pi}{12}d^2h} = \frac{3}{16}\frac{d^2}{h}$$

$$= \frac{3}{16h}\cdot\left(\frac{D}{H}h\right)^2 = \frac{3}{16}\frac{D^2h}{H^2}$$

For stable equilibrium,

$$GM > 0 \text{ or } OM > OG$$

$$\Rightarrow \quad OB + BM > OG$$

$$\Rightarrow \quad \frac{3}{4}h + \frac{3}{16}\frac{D^2h}{H^2} > \frac{3}{4}H$$

$$\Rightarrow \quad \frac{3h}{4}\left(1+\frac{D^2}{4H^2}\right) > \frac{3}{4}H$$

$$\Rightarrow \quad S^{1/3}H\left(1+\frac{D^2}{4H^2}\right) > H$$

$$\Rightarrow \quad \frac{D^2}{4H^2} > S^{-1/3}-1$$

$$\Rightarrow \quad \frac{4H^2}{D^2} < \frac{1}{S^{-1/3}-1}$$

$$\Rightarrow \quad \frac{H}{D} < \frac{1}{2}\sqrt{\frac{S^{1/3}}{1-S^{1/3}}}$$

So, Maximum value of $\frac{H}{D} = \frac{1}{2}\sqrt{\frac{S^{1/3}}{1-S^{1/3}}}$

Q.5 (b) Solution:

1. Size of footing:

Load from column = 330 kN

Weight of footing + backfill = 0.1 × 330 = 33 kN

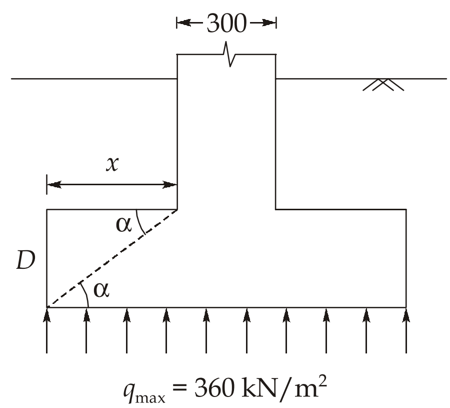
Total load = 363 kN

Area of footing required = $\frac{363}{360} = 1 \text{ m}^2$

Size of footing = 1 m

Provide 1.0 × 1.0 m size of footing.

2. Depth of footing:



$$\tan\alpha \le 0.9\sqrt{\frac{100 q_{max}}{f_{ck}}+1} = 0.9\sqrt{\frac{100\times360}{10^3\times20}+1}$$

$$\tan\alpha \le 1.506$$

Depth of footing, $D \ge x\tan\alpha$

$$x = \frac{1-0.3}{2} = 0.35 \text{ m}$$

∴ $D \ge 0.35 \times 1.506 = 0.5271 \text{ m} \approx 0.53 \text{ m (say)}$

∴ Provide footing depth of 530 mm

Hence provide a concrete block of 1000 × 1000 × 530 mm

$$A_{st,\,min} = \frac{0.12}{100}\times1000\times530 = 636 \text{ mm}^2$$

∴ Provide 6 - 12 ϕ bars (A_{st} = 678 mm²)

3. Check for bearing:

$$f_b = \frac{1.5\times330\times10^3}{300\times300}$$

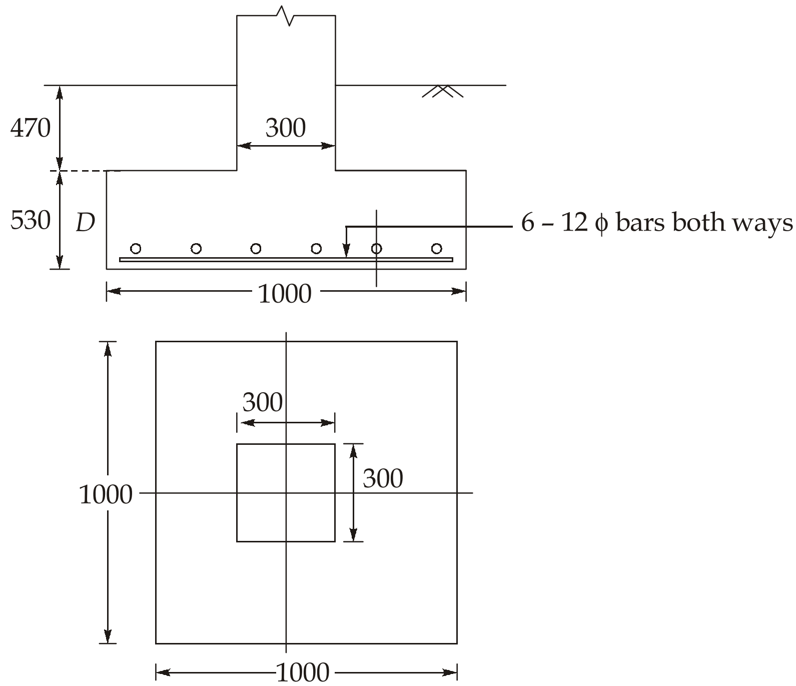
$$= 5.5 \text{ N/mm}^2 < (0.45 f_{ck} = 9 \text{ N/mm}^2) \qquad \therefore \text{Safe}$$

Assuming unit weight of concrete and soil as 24 kN/m³ and 18 kN/m³ respectively,

Actual gross soil pressure,

$$q_{max} = \frac{330}{1.0\times1.0} + (24\times0.53) + (18\times0.47)$$

$$= 351.2 \text{ kN/m}^2 < 360 \text{ kN/m}^2 \qquad \therefore \text{Safe}$$

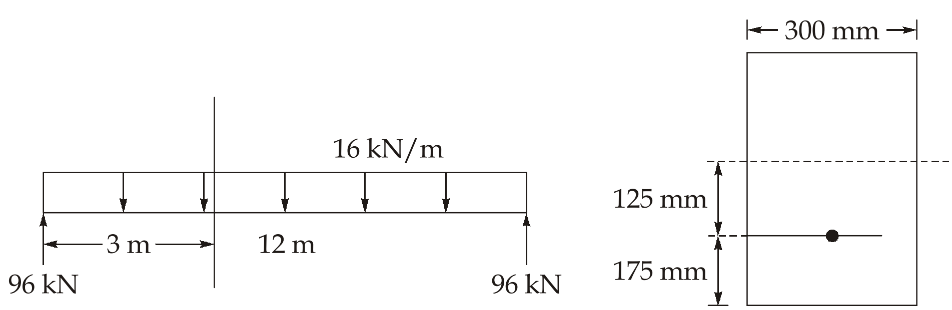


(All dimensions in mm)

Q.5 (c) Solution:

$$P = 800 \times 2000 \times 10^{-3} = 1600 \text{ kN}$$

$$e = 300 - 175 = 125 \text{ mm}$$



$$M \text{ @ } (x = 3 \text{ m}) = 96 \times 3 - \frac{16 \times 3 \times 3}{2} = 216 \text{ kN-m}$$

$$A = 300 \times 600 = 18 \times 10^4 \text{ mm}^4$$

$$Z = \frac{300 \times 600^2}{6} = 18 \times 10^6 \text{ mm}^3$$

(i) Stress concept method:

$$\frac{P}{A} = \frac{1600 \times 10^3}{18 \times 10^4} = +8.89 \text{ N/mm}^2$$

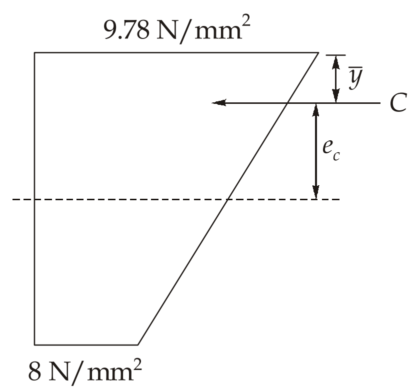
$$\frac{Pe}{Z} = \frac{1600 \times 10^3 \times 125}{18 \times 10^6} = \mp 11.11 \text{ N/mm}^2$$

$$\frac{M}{Z} = \frac{216 \times 10^6}{18 \times 10^6} = \pm 12 \text{ N/mm}^2$$

$$f_{top} = 8.89 - 11.11 + 12 = 9.78 \text{ N/mm}^2$$

$$f_{bottom} = 8.89 + 11.11 - 12 = 8 \text{ N/mm}^2$$

(ii) Strength concept method:



$$C = \left(\frac{9.78 + 8}{2}\right) \times 600 \times 300 \approx 1600 \times 10^3 \text{ N} = 1600 \text{ kN}$$

$$\bar{y} = \left(\frac{9.78 + (2 \times 8)}{9.78 + 8}\right) \times \frac{600}{3} = 290 \text{ mm}$$

$$e_c = 300 - 290 = 10 \text{ mm}$$

$$\frac{C}{A} = \frac{1600 \times 10^3}{18 \times 10^4} = +8.89 \text{ N/mm}^2$$

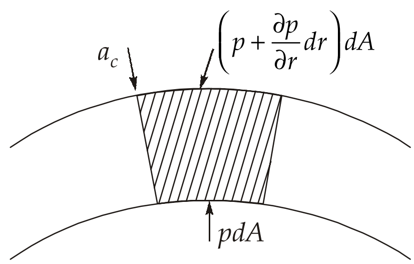
$$\frac{Ce_c}{Z} = \frac{1600 \times 10^3 \times 10}{18 \times 10^6} = \pm 0.89 \text{ N/mm}^2$$

$$f_{top} = 8.89 + 0.89 = 9.78 \text{ N/mm}^2$$

$$f_{bottom} = 8.89 - 0.89 = 8 \text{ N/mm}^2$$

Q.5 (d) Solution:

Let us assume a differential fluid element at a distance *r* from the centre, having a thickness of *dr* and mass *dm*.



Force analysis:

$$\left(p+\frac{\partial p}{\partial r}dr\right)dA - pdA = dma_c \quad \text{...(i)}$$

where a_c is centripetal acceleration

$$\therefore \quad \frac{\partial p}{\partial r}drdA = \rho dAdr\omega^2 r \quad \text{...(ii)}$$

[$\because d_m = \rho\, dAdr,\ ac = \omega^2 r$]

$$\Rightarrow \quad \frac{\partial p}{\partial r} = \rho\omega^2 r \quad \text{...(iii)}$$

As we know, $p = f(r, z)$

$$\Rightarrow \quad dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$$

$$\Rightarrow \quad dp = \rho\omega^2 rdr - \rho gdz \quad \text{...(iv)} \left\{\frac{\partial p}{\partial r} = \rho\omega^2 r,\ \frac{\partial p}{\partial z} = -\rho g\right\}$$

Equation of isobars in forced vortex motion,

So from (iv), $dp = 0$ [$\because$ At isobars pressure change is zero]

$$\Rightarrow \quad \omega^2 rdr = gdz$$

Since, ω is constant in forced vortex motion

$$\therefore \quad \omega^2 \int_{r=0}^{r=r} r.dr = g\int_{z=0}^{z=h} dz$$

$$\Rightarrow \quad \frac{\omega^2 r^2}{2} = gh$$

$$\Rightarrow \quad h = \frac{w^2 r^2}{2g} \quad \text{...(v)}$$

$$\therefore \quad h \propto r^2$$

Take a differential element of width '*dr*' at a distance '*r*' from centre. The height '*h*' of *t*

Volume of paraboloid:

$$dV = \pi r^2 dh$$

From (v),
$$r^2 = 2g\frac{h}{\omega^2}$$

$\therefore$
$$\int dV = \frac{2\pi g}{\omega^2}\int_0^H h.dh$$

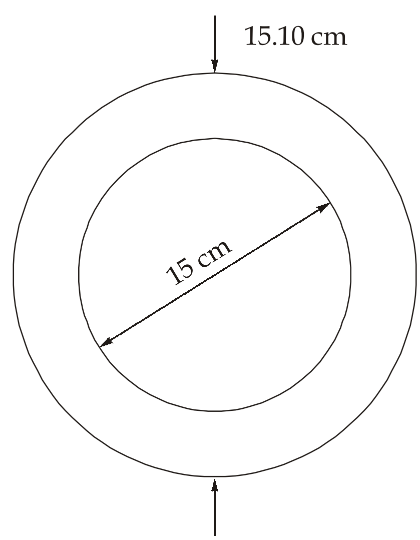
$\Rightarrow$
$$V = \frac{2\pi g}{\omega^2}\left[\frac{h^2}{2}\right]_0^H = \frac{\pi H^2 g}{\omega^2}$$

$$= \pi\frac{\omega^2 R^2}{2g}H\frac{g}{\omega^2} \qquad \left[H = \frac{\omega^2 R^2}{2g}\right]$$

$\therefore$
$$V = \frac{\pi R^2 H}{2}$$

$\therefore$ Volume of paraboloid $= \frac{1}{2}\times$ Vol. circumscribing cylinder Hence proved.

Q.5 (e) Solution:



Given data: Torque = 12 Nm, $r = \frac{15}{2} = 7.5\,\text{cm}$

N = 100 rpm, l = 25 cm

Given linear velocity profile within the thin oil film and thus viscous shear stress is,

$$\tau = \frac{\mu du}{dy} = \mu \times \frac{u}{t}$$

Viscous resistance or viscous force

$$= \text{Shear stress} \times \text{Area}$$

$$= \mu \times \frac{u}{t} \times (2\pi r l)$$

Viscous torque = Viscous force × Torque arm

The torque arm equals the radius of the cylinder which is rotating.

∴ Viscous torque, $$T = \mu \times \frac{u}{t} \times (2\pi r l) \times r \quad \text{...(i)}$$

$$t = \frac{15.1 - 15}{2} = 0.05\,\text{cm} = 0.0005\,\text{m}$$

$$r = \frac{15}{2} = 7.5\,\text{cm} = 0.075\,\text{m}$$

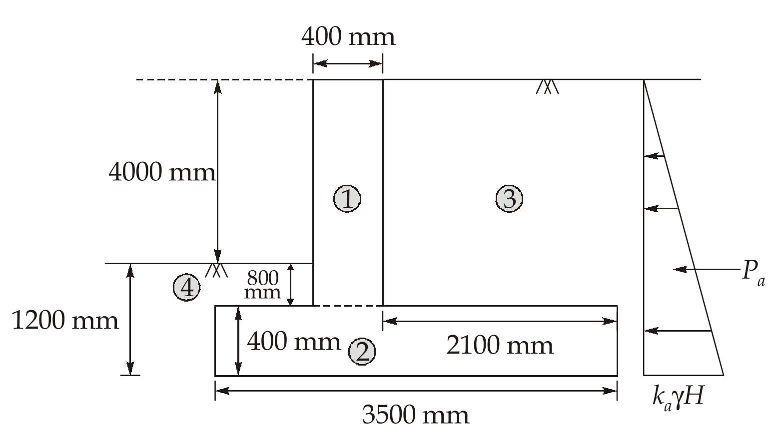
$$u = \frac{2\pi r N}{60} = \frac{2 \times \pi \times 0.075 \times 100}{60} = 0.785\,\text{m/s}$$

Using eq. (i) $$12 = \mu \times \frac{0.785}{0.0005} \times (2\pi \times 0.075 \times 0.25) \times 0.075$$

⇒ $\mu = 0.865\ \text{Ns/m}^2$

⇒ $\mu = 8.65$ poise

Q.6 (a) Solution:



1. Earth pressure

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

$$K_a\gamma H = \frac{1}{3}\times 18\times(4+1.2) = 31.2 \text{ kN/m}^2$$

$$P_a = \frac{1}{2}\times 5.2\times 31.2 = 81.12 \text{ kN}$$

2. Load calculation: (For 1 m length of retaining wall)

Loads	Distance from toe	Moment about toe
RCC $W_1 = 0.4 \times 4.8 \times 25 = 48$ kN $W_2 = 0.4 \times 3.5 \times 1 \times 25 = 35$ kN	 1 + 0.4/2 = 1.2 m 3.5/2 = 1.75 m	 57.6 kN – m 61.25 kN – m
Soil $W_3 = 2.1 \times 4.8 \times 1 \times 18 = 181.44$ kN $W_4 = 1 \times 0.8 \times 1 \times 18 = 14.4$ kN	 1 + 0.4+ 2.1/2 = 2.45 m 1/2 = 0.5 m	 444.53 kN – m 7.2 kN – m
Total $\Sigma W = 278.84$ kN		570.58 kN – m

3. Check against overturning:

$$\text{Overturning moment} = P_a \times \frac{4}{3} = 81.12 \times \frac{5.2}{3} = 140.61 \text{ kN-m}$$

$$\text{Balancing moment} = 570.58 \text{ kN-m}$$

$$\text{FOS against overturning} = \frac{0.9\times 570.58}{140.61} = 3.65 > 1.40 \qquad \therefore \text{Safe}$$

4. Check against sliding:

$$K_p = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 30°}{1-\sin 30°} = 3$$

$$P_p = \frac{1}{2}\times K_p\gamma H^2 = \frac{1}{2}\times 3\times 18\times 1.2^2 = 38.88 \text{ kN}$$

$$\text{Sliding force} = P_a = 81.12 \text{ kN}$$

$$\begin{aligned}\text{Balancing force} &= P_p + \mu(\Sigma W)\\ &= 38.88 + 0.52 \times 278.84\\ &= 183.88 \text{ kN}\end{aligned}$$

$$\text{FOS against sliding} = \frac{0.9 \times 183.88}{81.12} = 2.04 > 1.4 \qquad \therefore \text{Safe}$$

5. Soil pressure:

$$\bar{x} = \frac{\Sigma M_{net}}{\Sigma V} = \frac{M_R - M_O}{\Sigma V}$$

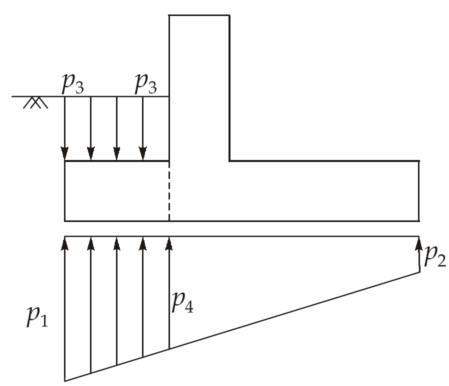
$$= \frac{570.58 - 140.61}{278.84} = 1.542 \text{ m}$$

$$e = \frac{B}{2} - \bar{x} = \frac{3.5}{2} - 1.542 = 0.208 \text{ m}$$

$$p_1 = \frac{\Sigma V}{B}\left[1 + \frac{6e}{B}\right] = \frac{278.84}{3.5}\left[1 + \frac{6 \times 0.208}{3.5}\right] = 108 \text{ kN/m}^2$$

$$p_2 = \frac{\Sigma V}{B}\left[1 - \frac{6e}{B}\right] = \frac{278.84}{3.5}\left[1 - \frac{6 \times 0.208}{3.5}\right] = 51.26 \text{ kN/m}^2$$

6. Net design soil pressure for toe:



$$p_3 = 0.8 \times 18 + 0.4 \times 25 = 24.4 \text{ kN/m}^2$$

$$p_4 = 52 + \left(\frac{108 - 52}{3.5}\right) \times 2.5 = 92 \text{ kN/m}^2$$

$$\text{Area of steel, } A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right) Bd$$

$$= \frac{0.5 \times 30}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 59 \times 10^6}{30 \times 1000 \times 340^2}}\right) \times 1000 \times 340$$

$$= 490.66 \text{ mm}^2$$

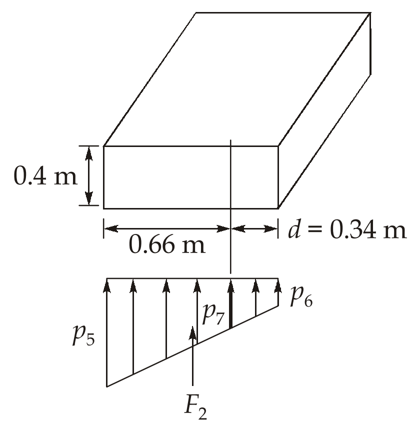
$$A_{st} \text{ minimum} = \frac{0.12}{100} \times 1000 \times 400 = 480 \text{ mm}^2 < 490.66 \text{ mm}^2$$

$$\text{Spacing of 10 mm dia. bars} = \frac{1000 \times \frac{\pi}{4} \times (10)^2}{490.66} = 160.07 \text{ mm} \approx 160 \text{ mm c/c (say)}$$

∴ Provide 10 mm ϕ @ 160 mm c/c

Check for shear:

Critical section is at distance 'd' from face



$$p_7 = 67.6 + \left(\frac{83.6 - 67.6}{1}\right) \times 0.34 = 73 \text{ kN/m}^2$$

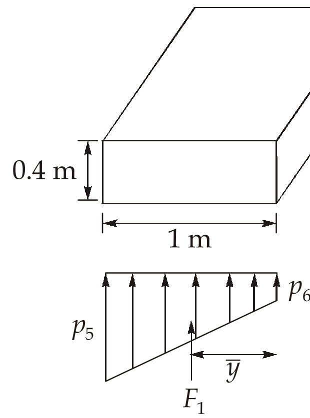
$$F_2 = \left(\frac{73 + 83.6}{2}\right) \times 0.66 \times 1 = 51.68 \text{ kN}$$

$$V_u = 1.5 \times 51.68 = 77.52 \text{ kN}$$

$$\tau_v = \frac{77.52 \times 10^3}{1000 \times 340} = 0.228 \text{ N/mm}^2 < \tau_{min} (= 0.29 \text{ N/mm}^2)$$

∴ Safe

7. Design of toe:



$$p_5 = p_1 - p_3 = 108 - 24.4 = 83.6 \text{ kN/m}^2$$

$$p_6 = p_4 - p_3 = 92 - 24.4 = 67.6 \text{ kN/m}^2$$

$$F_1 = \frac{83.6 + 67.6}{2} \times 1 \times 1 = 75.6 \text{ kN}$$

$$\bar{y} = \left(\frac{67.6 + 2 \times 83.6}{67.6 + 83.6}\right) \times \frac{1}{3} = 0.52 \text{ m}$$

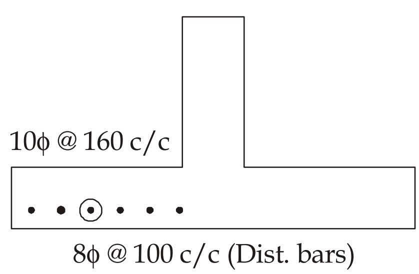
$$\text{Factored B.M} = 1.5 \times 75.6 \times 0.52$$

$$= 58.968 \text{ kN-m} \approx 59 \text{ kN-m (say)}$$

Depth of toe slab, $d = \sqrt{\frac{M_u}{Q \cdot B}} = \sqrt{\frac{59 \times 10^6}{0.138 \times 30 \times 1000}} = 119.4 \text{ mm}$

Depth available = 400 – 60 = 340 mm > 119.4 mm (OK)

(Assuming an effective cover of 60 mm)



Q.6 (b) (i) Solution:

Given: μ = 1.5 poise = 0.15 Pa.s, ρ = 0.85 × 1000 = 850 kg/m^3

1. Wall shear stress $\tau_0 = \frac{R}{2}\left(\frac{\gamma \cdot h_f}{L}\right)$

$$= \frac{0.15}{2}\left(\frac{850 \times 9.81 \times 20}{3000}\right) = 4.17 \text{ Pa}$$

2. Shear stress 'τ' at, r = 10 cm

$$\tau = \frac{\tau_0}{R} \cdot r = \frac{4.17}{0.15} \times 0.1 = 2.78 \text{ Pa}$$

3. If the flow is laminar, then

$$h_f = \frac{32\mu VL}{\rho g d^2}$$

$$\Rightarrow \quad V = \frac{h_f}{L} \times \frac{\rho g \cdot d^2}{32\mu} = \frac{20}{3000} \times \frac{850 \times 9.81 \times 0.3^2}{32 \times 0.15} = 1.04 \text{ m/s}$$

$$R_e = \frac{\rho VD}{\mu} = \frac{850 \times 1.04 \times 0.3}{0.15} = 1768 < 2000$$

Thus the flow is laminar.

$\therefore$ Friction factor, $f = \frac{64}{R_e} = \frac{64}{1768} = 0.0362$

Q.6 (b) (ii) Solution:

$$V = f_n(\rho, \mu, D, H, g)$$

Number of variables involved = 6

Number of basic dimensions involved = 3

Hence number of dimensionless terms = 3

Select ρ , g , H as the repeating variables

First term π_1 :

$$\pi_1 = \rho^a g^b H^c V$$

$$\Rightarrow \quad M^0L^0T^0 = [ML^{-3}]^a [LT^{-2}]^b [L]^c [LT^{-1}]$$

Equating powers of M, L and T, we get

$$a = 0$$

$$-3a + b + c + 1 = 0$$

$$-2b - 1 = 0$$

Solving, $a = 0, b = \frac{-1}{2},$ $\quad c = \frac{-1}{2}$

$$\therefore \quad \pi_1 = \frac{V}{\sqrt{gH}}$$

Second term π_2 :

$$\pi_2 = \rho^a g^b H^c \mu$$

$$\Rightarrow \quad M^0L^0T^0 = [ML^{-3}]^a\,[LT^{-2}]^b\,[L]^c\,[ML^{-1}T^{-1}]$$

Equating powers of M, L and T, we get

$$a + 1 = 0$$

$$-3a + b + c - 1 = 0$$

$$-2b - 1 = 0$$

Solving, $a = -1, b = \frac{-1}{2}, c = \frac{-3}{2},$

$$\therefore \quad \pi_2 = \frac{\mu}{\rho g^{1/2} H^{3/2}}$$

Third term π_3 :

$$\pi_3 = \rho^a g^b H^c D$$

$$\Rightarrow \quad M^0L^0T^0 = [ML^{-3}]^a\,[LT^{-2}]^b\,[L]^c\,[L]$$

Equating powers of M, L and T, we get

$$a = 0$$

$$-3a + b + c + 1 = 0$$

$$-2b = 0$$

Solving, $a = 0, b = 0, c = -1,$

$$\therefore \quad \pi_3 = \frac{D}{H}$$

Thus, from above three π terms, we get

$$\frac{V}{\sqrt{gH}} = f_n\left[\left(\frac{\mu}{\rho g^{1/2} H^{3/2}}\right), \left(\frac{D}{H}\right)\right]$$

Also, $$\pi_2 = \frac{\mu}{\rho g^{1/2} H^{3/2}} \times \frac{V}{V}$$

$$\Rightarrow \quad \pi_2 = \frac{\mu}{\rho V H} \times \frac{V}{\sqrt{gH}}$$

$$\Rightarrow \qquad \pi_2 = \frac{\mu}{\rho VH} \times \pi_1$$

Thus,
$$\frac{V}{\sqrt{gH}} = \phi\left[\left(\frac{\mu}{\rho VH}\right),\left(\frac{D}{H}\right)\right]$$

$$\Rightarrow \qquad V = \sqrt{2gH}\,\phi_1\left[\left(\frac{\mu}{\rho VH}\right),\left(\frac{D}{H}\right)\right]$$

Q.6 (c) Solution:

1. **Effective depth:**

Let, effective depth of slab, $d = \frac{4500}{20 \times 1.3} = 173.077 \text{ mm} \approx 180 \text{ mm (say)}$

$\therefore$ Overall depth, $D = 180 + 20 = 200$ mm

2. **Load calculation:**

Dead load of slab = $25 \times 0.2 = 5$ kN/m^2

Floor finish = 0.5 kN/m^2

Live load = 2 kN/m^2

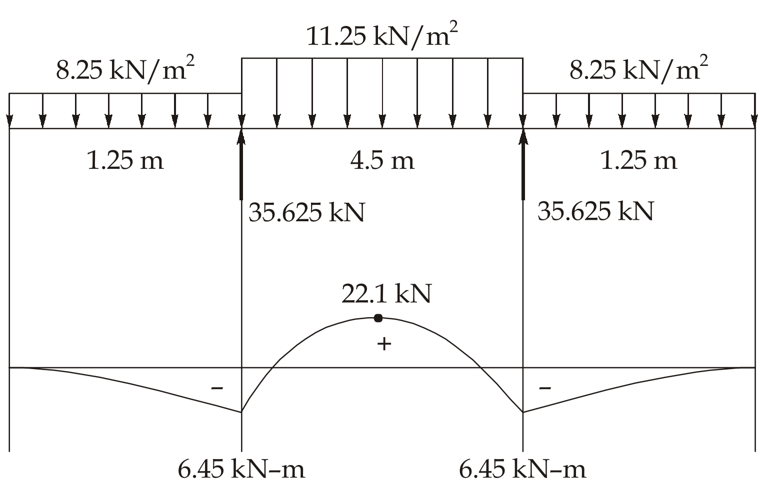
Factored dead load = $1.5(5 + 0.5) = 8.25$ kN/m^2

Factored total load = $1.5(5 + 0.5 + 2) = 11.25$ kN/m^2

3. **Design of span between the supports:**

The maximum positive bending moment for this span will occur when the live load covers only this span. Hence the loading on this span will be 11.25 kN/m^2, while the loading on the cantilever parts will be 8.25 kN/m^2.

Consider 1 m wide strip of slab,



Maximum positive BM at the centre,

$$M_u = \left(35.625 \times \frac{4.5}{2}\right) - \left(11.25 \times 2.25 \times \frac{2.25}{2}\right) - (8.25 \times 1.25 \times 2.875)$$

$$= 22.03125 \text{kN-m} \approx 22.1 \text{ kN-m (say)}$$

Depth of slab required:

$$d = \sqrt{\frac{M_u}{QB}} = \sqrt{\frac{22.1 \times 10^6}{0.138 \times 20 \times 1000}} = 89.48 \text{ mm}$$

Effective depth provided = 180 mm > 89.48 mm $\quad \therefore$ OK

Calculation of steel:

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right) B d$$

$$= \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 22.1 \times 10^6}{20 \times 1000 \times 180^2}}\right) \times 1000 \times 180 = 354.74 \text{ mm}^2$$

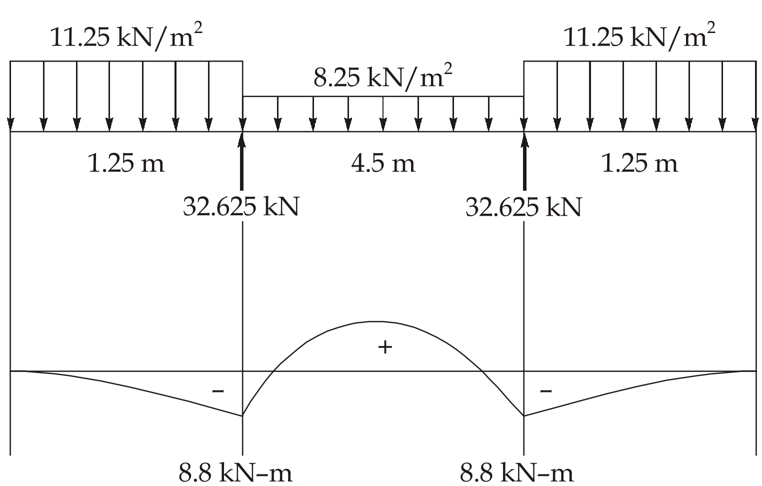
Spacing of 10 mm dia bars

$$= \frac{1000 \times \frac{\pi}{4} \times 10^2}{354.74} = 221.4 \text{ mm} \approx 220 \text{ mm c/c (say)}$$

$\therefore$ Provide 10 mm ϕ bars @ 220 mm c/c

4. Design of contilever part of the slab:

This is subjected to maximum hogging moment when the total load i.e. 11.25 kN/m^2 is applied on the cantilever part and the load between the supports is the dead load.



$$\text{Maximum logging moment} = 11.25 \times 1.25 \times \frac{1.25}{2} = 8.8 \text{ kN-m}$$

$$\text{Depth required, } d = \sqrt{\frac{8.8 \times 10^6}{0.138 \times 20 \times 1000}} = 56.5 \text{ mm}$$

Effective depth provided = 180 mm > 56.5 mm ∴ OK

Calculation of steel:

$$A_{st} = \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 8.8 \times 10^6}{20 \times 1000 \times 180^2}}\right) \times 1000 \times 180$$

$$= 137.66 \text{ mm}^2$$

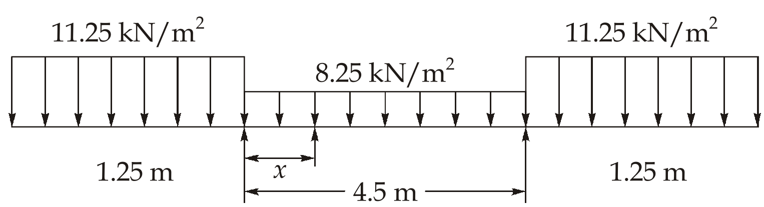
Minimum percentage of steel to be provided = 0.12%

$$\therefore \quad A_{st} = \frac{0.12}{100} \times 1000 \times 200 = 240 \text{ mm}^2$$

$$\text{Spacing of 8 mm } \phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 8^2}{240} = 209 \text{ mm} \approx 200 \text{ mm (say)}$$

∴ Provide 8 mm ϕ bars @ 200 mm c/c

Point of centra flexure:



$$M_x = (32.625 \times x) - 11.25 \times 1.25 \times \left(x + \frac{1.25}{2}\right) - 8.25 \times x \times \frac{x}{2}$$

$$\Rightarrow \quad M_x = 32.625\,x - 14.0625\,x - 8.79 - 4.125x^2$$

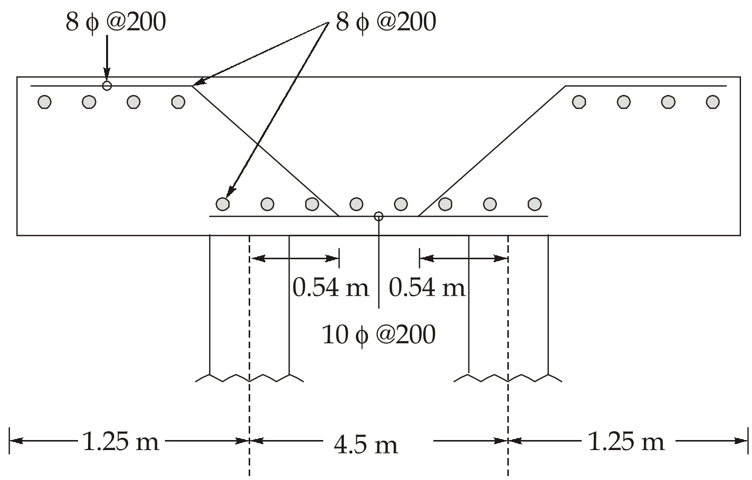
$$M_x = 0$$

$$\Rightarrow -4.125x^2 + 18.5625x - 8.79 = 0$$

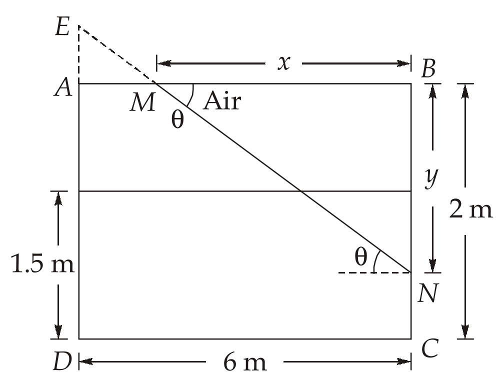
$$\therefore \quad x = 0.54 \text{ m}$$

$$\text{Also, distribution steel} = \frac{0.12}{100} \times 1000 \times 200 = 240 \text{ mm}^2$$

∴ Provide 8 mm ϕ bars @ 200 c/c



Q.7 (a) (i) Solution:



Given, $a_x = 2.5 \text{ m/s}^2$

Water surface slope, $\tan\theta = \dfrac{a_x}{g}$

$\Rightarrow \quad \tan\theta = \dfrac{2.5}{9.81} \Rightarrow \theta = 14.297 \approx 14.3°$

Volume of air in the tank $= 0.5 \times 6 \times 3 = 9 \text{ m}^3$

As there is no spill of water, volume of air will remain the same.

Let MN be the new water surface at an inclination of θ to the horizontal.

If $MB = x$ and $BN = y$, then

$\therefore \quad \dfrac{1}{2} \times x \times y \times 3 = 9$

$\Rightarrow \quad xy = 6$

Also, $\tan\theta = \dfrac{y}{x}$

Substituting,

$$x\,(x \tan 14.3°) = 6$$

$\Rightarrow$ $$x = 4.852 \text{ m}$$

and $$y = 1.237 \text{ m}$$

Now, in ΔEAM

$$AM = 6 - x = 6 - 4.852 = 1.148 \text{ m}$$

$$AE = AM \tan 14.3° = 1.148 \tan 14.3° = 0.293 \text{ m}$$

The pressure profile on the top is represented by the triangle EAM extending over the width. Pressure force on the top,

$$F_{top} = \frac{1}{2} \times AM \times AE \times \text{width} \times \gamma$$

$$= \frac{1}{2} \times 1.148 \times 0.293 \times 3 \times 9.81 = 4.95 \text{ kN}$$

This force acts vertically upwards at $\frac{1}{3}$(AM) = 0.383 m from A at the mid-width of the section.

Q.7 (a) (ii) Solution:

1. We know, $$u = \frac{-\partial\phi}{\partial x} = \frac{-\partial\psi}{\partial y} \quad ...(i)$$

and $$v = \frac{-\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x} \quad ...(ii)$$

where u and v are the velocities along X and Y direction respectively and ϕ and ψ are the equipotential and stream function respectively.

From (i),

$$\frac{-\partial}{\partial x}(x^2 - y^2 + y) = \frac{-\partial\psi}{\partial y}$$

$\Rightarrow$ $$2x = \frac{\partial\psi}{\partial y}$$

$\Rightarrow$ $$\psi = 2xy + f(x) + C \quad \text{(where } C \text{ is a constant)}$$

Now, from equation (ii),

$$\frac{-\partial}{\partial y}(x^2 - y^2 + y) = \frac{\partial}{\partial x}(2xy + f(x) + c)$$

$$\Rightarrow \quad -(-2y + 1) = (2y + f'(x))$$

$$\Rightarrow \quad f'(x) = -1$$

Integrating,

$$f(x) = -x + D, \qquad \text{(where } D \text{ is a constant)}$$

Substituting in ψ,

$$\therefore \quad \psi = 2xy + (-x + D) + C$$

$$= 2xy - x + C' \qquad \text{(where } C' \text{ is a constant)}$$

2. The flow rate between the streamline passing through points (1, 1) and (1, 2) is given by,

$$Q = \psi_{(1,\,2)} - \psi_{(1,\,1)}$$

$$\Rightarrow \quad Q = (2xy - x + C')_{(1,\,2)} - (2xy - x + C')_{(1,\,1)}$$

$$= (2(1)(2) - (1) + C') - (2(1)(1) - (1) + C')$$

$$= 3 + C' - 1 - C'$$

$$= 2 \text{ units}$$

Q.7 (b) Solution:

1. **Size of footing:**

Load from column = 1500 kN

Self weight of footing = 0.1 × 1500 = 150 kN

Total load = 1650 kN

$$\text{Area of square footing} = \frac{1650}{150} = 11 \text{ m}^2$$

$$\text{Size of the footing} = \sqrt{11} = 3.317 \text{ m} \approx 3.4 \text{ m (say)}$$

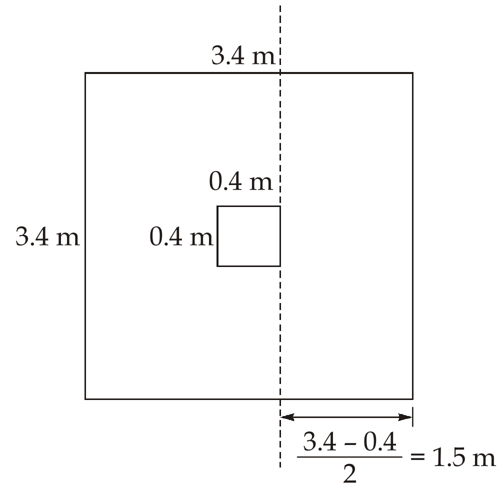
∴ Provide a square footing of size 3.4 m × 3.4 m

2. **Net design soil pressure:**

$$w_0 = \frac{P}{A} = \frac{1500}{3.4 \times 3.4} = 129.76 \text{ kN/m}^2 < \text{SBC} (= 150 \text{ kN/m}^2)$$

Factored soil pressure, $w_4 = 1.5 \times 129.76 = 194.64 \text{ kN/m}^2$

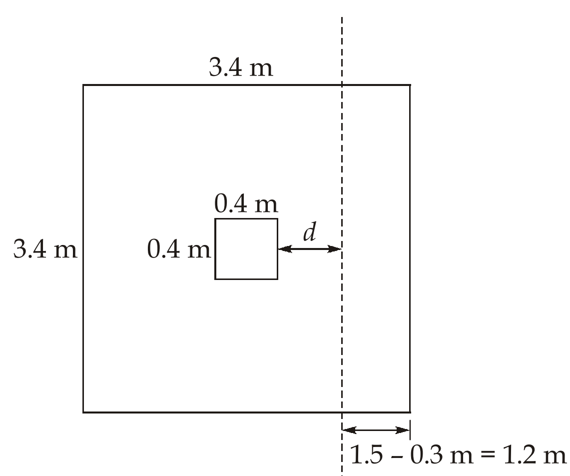
3. Check for bending moment: (For 1 m width)



$$M_{u_x} = 194.64 \times 1.5 \times \frac{1.5}{2} = 218.97 \text{ kN-m}$$

$$\text{Effective depth required, } d = \sqrt{\frac{218.97 \times 10^6}{0.138 \times 20 \times 1000}} = 281.67 \text{ mm} \approx 300 \text{ mm (say)}$$

4. Check for one way shear:



Maximum S.F. $V_u = 194.64 \times 1.2 = 233.568$ kN

$$\tau_v = \frac{V_u}{Bd} = \frac{233.568 \times 10^3}{1000 \times 300} = 0.7786 \text{ N/mm}^2$$

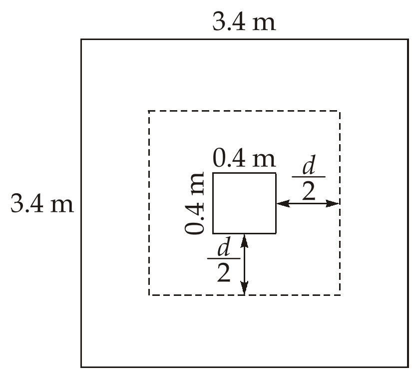
But, τ_1 (minimum for M20) $= 0.28$ N/mm^2

$$\therefore \quad 0.28 \geq \frac{233.56 \times 10^3}{1000 \times d}$$

$$\Rightarrow \quad d \geq 834.17 \text{ mm}$$

$$\therefore \text{ Provide,} \quad d = 840 \text{ mm}$$

5. Check for punching shear:



Net punching shear force,

$$P_{u\,net} = 1.5 \times 1500 - 194.64\,(0.4 + 0.84)^2$$

$$= 1950.72 \text{ kN}$$

Punching shear stress, $$\tau_{vp} = \frac{1950.72 \times 10^3}{4(400 + 840) \times 840} = 0.47 \text{ N/mm}^2$$

Permissible punching shear stress

$$= K_\beta \times 0.25 \times \sqrt{f_{ck}}$$

$$K_\beta = 0.5 + \beta_c = 0.5 + 1 = 1.5 \leq 1.0 \qquad \therefore K_\beta = 1.0$$

∴ Permissible punching shear stress

$$= 1 \times 0.25 \times \sqrt{20}$$

$$= 1.118 \text{ N/mm}^2 > \tau_{vp} \qquad \therefore \text{OK}$$

6. Area of steel:

$$d = 840 \text{ mm},$$

Let, effective cover $= 60$ mm

$$\therefore \quad D = 840 + 60 = 900 \text{ mm}$$

$$\text{Area of steel (for 1 m width)} = \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 218.97 \times 10^6}{20 \times 1000 \times 840^2}}\right) \times 1000 \times 840$$

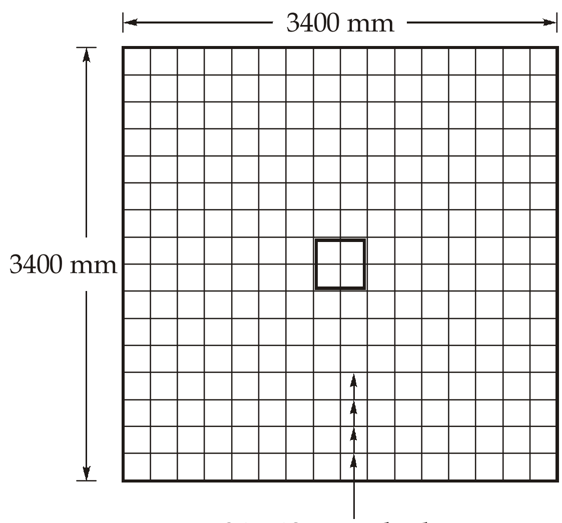
$$= 735.74 \text{ mm}^2 \; < A_{st\,min}$$

$$A_{st\,min} = \frac{0.12}{100} \times 1000 \times 900 = 1080 \text{ mm}^2$$

$\therefore$ Provide A_{st} = 1080 mm^2

$$\therefore \text{ Spacing of 12 mm } \phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{1080} = 104.72 \text{ mm} \approx 100 \text{ mm (say)}$$

$$\text{Total no. of bars required for 3.4 m width} = \frac{3400}{100} = 34$$



Q.7 (c) Solution:

Given, E_C = 38 kN/mm^2

$$I_C = \frac{150 \times 300^3}{12} = 3.375 \times 10^8 \text{ mm}^4$$

$$E_C I_C = 128.25 \times 10^8 \text{ kN-mm}^2$$

Self-weight of beam = 24 × 0.15 × 0.3 = 1.08 kN/m

$$\text{Deflection due to self-weight} = \frac{5wl^4}{384 E_C I_C} = \left[\frac{5 \times 1.08 \times (8)^4}{384 \times 128.25 \times 10^8 \times 10^{-6}}\right] \times 10^3$$

= 4.4912 mm (downwards)

$$\text{Deflection due to initial prestress} = \frac{-5P(e_1 + e_2)L^2}{48 E_C I_C} + \frac{P e_1 L^2}{8 E_C I_C}$$

where, e_1 = 25 mm and e_2 = 75 mm

$$= \frac{-5\times 350\times 10^3(25+75)\times 8000^2}{48\times 128.25\times 10^{11}}$$

$$+\frac{350\times 10^3\times 25\times 8000^2}{8\times 128.25\times 10^{11}}$$

$$= (-)18.1936 + 5.4581$$

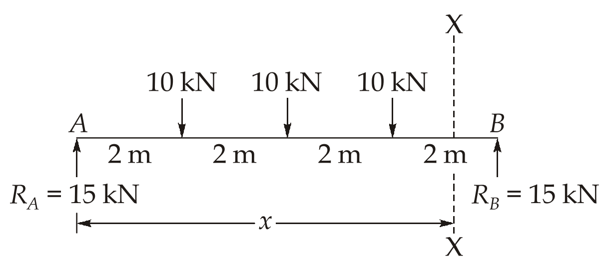
$$= -12.7355 \text{ mm (upwards)}$$

(i) Instantaneous deflection due to prestress + self-weight

$$= -12.7355 + 4.4912$$

$$= -8.2443 \text{ (upwards)}$$

(ii) Deflection due to live load:



$$M_x = 15x - 10(x-2) - 10(x-4) - 10(x-6)$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = 15x - 10(x-2) - 10(x-4) - 10(x-6)$$

$$\therefore \quad EI\frac{dy}{dx^2} = \frac{15x^2}{2} - \frac{10(x-2)^2}{2} - \frac{10(x-4)^2}{2} - \frac{10(x-6)^2}{2} + C_1$$

and

$$EIy = \frac{15x^3}{6} - \frac{10(x-2)^3}{6} - \frac{10(x-4)^3}{6} - \frac{10(x-6)^3}{6} + C_1x + C_2$$

At $x = 0$, $y = 0 \Rightarrow C_2 = 0$

At $x = 8$ m, $y = 0$

$$\therefore \quad 0 = \frac{15\times 8^3}{6} - \frac{10\times 6^3}{6} - \frac{10\times 4^3}{6} - \frac{10\times 2^3}{6} + C_1 8$$

$$\Rightarrow \quad C_1 = \left[\frac{-15\times 8^3}{6} + \frac{10\times 6^3}{6} + \frac{10\times 4^3}{6} + \frac{10\times 2^3}{6}\right]\times\frac{1}{8}$$

$\Rightarrow \quad C_1 = -100$

$$\therefore \quad EI\,y = \frac{15x^3}{6} - \frac{10(x-2)^3}{6} - \frac{10(x-4)^3}{6} - \frac{10(x-6)^3}{6} - 100x$$

At $x = 4$ m,

$$EI\,y = \frac{15\times 4^3}{6} - \frac{10\times 2^3}{6} - 100\times 4$$

$$\Rightarrow \quad y = \frac{-760}{3\times EI}$$

$$\Rightarrow \quad y = \left(\frac{-760}{3\times 128.25\times 10^8\times 10^{-6}}\right)\times 10^3 = -19.7531 \text{ mm}$$

'–ve' sign in double integration method indicates downward deflection.

∴ Deflection due to live load = 19.753 mm (downwards)

The long term deflection is obtained from the expression,

$$a_f = a_{i1}(1+\phi) - a_{i_p}\left[\left(1-\frac{L_p}{P_i}\right)+\left(1-\frac{L_p}{2P_i}\right)\phi\right]$$

(In this expression, the negative sign refers to the deflection in the upward direction)

where, a_{i1} = initial deflection due to transverse loads.

a_{ip} = initial deflection due to prestress.

L_p = loss of prestressing force = $(P_i - P_t)$

P_i = initial prestress

P_t = prestress after a time, t

ϕ = creep coefficient

Now,

$$a_f = (19.7531 + 4.4912)(1+1.8) - 12.7355\left[(1-0.2)+\left(1-\frac{0.2}{2}\right)1.8\right]$$

$= (24.2443)\,(2.8) - (12.7355 \times 2.42)$

$= 37.0641$ mm (downward)

Note: A much simplified but an approximate procedure is suggested by *Lin* for computing long-term deflection,

$$a_f = \left(a_{i1} - a_{ip}\times\frac{P_t}{P_i}\right)(1+\phi)$$

$= [24.2443 - 12.7355 \times 0.8]\,(1 + 1.8)$

$= 39.3565$ mm (downward)

Q.8 (a) Solution:

(i) Velocity in suction pipe, $V_1 = \frac{Q}{\frac{\pi}{4}D_1^2} = \frac{25\times10^{-3}}{\frac{\pi}{4}(0.15)^2} = 1.415\text{ m/s}$

Velocity in delivery pipe, $V_2 = \frac{Q}{\frac{\pi}{4}D_2^2} = \frac{25\times10^{-3}}{\frac{\pi}{4}(0.12)^2} = 2.21\text{ m/s}$

Head loss in suction pipe, $h_{f_1} = \frac{8fl_1Q^2}{\pi^2 g D_1^5} = \frac{8\times0.02\times30\times(0.025)^2}{\pi^2\times9.81\times(0.15)^5} = 0.408\text{ m}$

Inlet loss, $h_{l_1} = \frac{0.5V_1^2}{2g} = \frac{0.5(1.415)^2}{2\times9.81} = 0.051\text{ m}$

Head loss in delivery pipe, $h_{f_2} = \frac{8fl_2Q^2}{\pi^2 g D_2^5} = \frac{8\times0.02\times200\times(0.025)^2}{\pi^2\times9.81\times(0.12)^5} = 8.301\text{ m}$

Exit loss, $h_{l_2} = \frac{V_2^2}{2g} = \frac{(2.21)^2}{2\times9.81} = 0.249\text{ m}$

Now, applying Bernoulli's equation between A and C,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A + H_P = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C + h_{f_1} + h_{l_1} + h_{f_2} + h_{l_2}$$

$$\Rightarrow \quad 95 + H_p = 110 + 0.408 + 0.051 + 8.301 + 0.249$$

Head delivered by the pump,

$$H_p = 24.01\text{ m}$$

Power delivered, $P = \rho Q g H_p$

$= 10^3 \times 0.025 \times 9.81 \times 24.01$

$= 5888.45\text{ W} = 5.89\text{ kW}$

(ii) Discharge when the pump delivers a power of 6.5 kW

$$P_{\text{delivered}} = 6.5\text{ kW}$$

Head delivered by the pump, $H_p = \frac{P_{\text{delivered}}}{\rho Q g}$

Applying Bernoulli's equation between A and C,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A + H_p = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C + h_{f_1} + h_{l_1} + h_{f_2} + h_{l_2}$$

$$\Rightarrow \quad 95 + H_p = 110 + h_{f_1} + h_{l_1} + h_{f_2} + h_{l_2}$$

$$\Rightarrow \quad 95 + \frac{P_{\text{delivered}}}{\rho Q g} = 110 + \frac{8 f l_1 Q^2}{\pi^2 g D_1^5} + \frac{0.5}{2g}\left(\frac{Q}{\frac{\pi}{4} D_1^2}\right)^2 + \frac{8 f l_2 Q^2}{\pi^2 g D_2^5} + \frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4} D_2^2}\right)^2$$

$$\Rightarrow \quad 95 + \frac{6.5}{Q(9.81)} = 110 + \frac{8(0.02)(30)Q^2}{\pi^2 g (0.15)^5} + \frac{4Q^2}{\pi^2 g (0.15)^4} + \frac{8(0.02)(200)Q^2}{\pi^2 g (0.12)^5} + \frac{8Q^2}{\pi^2 g (0.12)^4}$$

$$\Rightarrow \quad \frac{6.5}{Q} = 15 \times 9.81 + \frac{4Q^2}{\pi^2 (0.15)^4}\left(\frac{2 \times 0.02 \times 30}{0.15} + 1\right) + \frac{8Q^2}{\pi^2 (0.12)^4}\left(\frac{0.02 \times 200}{0.12} + 1\right)$$

On solving,

$$Q = 0.02643 \text{ m}^3/\text{s} = 26.43 \text{ L/s}$$

(iii) Pressure at the suction side of the pump,

Applying Bernoulli's equation between A and P

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_S}{\rho g} + \frac{V_1^2}{2g} + Z_P + h_{l_1} + h_{f_1}$$

$$\Rightarrow \quad 10 + 95 = \frac{P_S}{\rho g} + \frac{(1.415)^2}{2 \times 9.81} + 103 + 0.051 + 0.408$$

$$\Rightarrow \quad \frac{P_S}{\rho g} = 1.439 \text{ m}$$

$$\Rightarrow \quad P_S = 1.439 \times 9.81 = 14.12 \text{ kPa (abs)}$$

Q.8 (b) (i) Solution:

At the centre of the plate, $x = 1.5$ m, $u = 2$ m/s

$$\text{Re}_x = \frac{ux}{\nu} = \frac{2 \times 1.5}{10^{-4}} = 3 \times 10^4 < (5 \times 10^5)$$

Hence the boundary layer is laminar,

$$\because \quad \frac{\delta}{x} = \frac{5}{\sqrt{\text{Re}_x}}$$

$$\Rightarrow \quad \delta = \frac{5 \times 1.5}{\sqrt{3 \times 10^4}} = 0.0433 \text{ m} = 4.33 \text{ cm}$$

Local friction coefficient, $C_{fx} = \dfrac{0.664}{\sqrt{\text{Re}_x}}$

Shear stress, $\tau_0 = \dfrac{C_f \rho U^2}{2} = \dfrac{0.664}{\sqrt{3 \times 10^4}} \times \dfrac{1}{2} \times 900 \times 2^2 = 6.9 \text{ N/m}^2$

At the trailing edge,

$$x = L = 3 \text{ m}$$

$$\text{Re}_L = \frac{uL}{\nu} = \frac{2 \times 3}{10^{-4}} = 6 \times 10^4 = (< 5 \times 10^5)$$

Hence the boundary layer is laminar,

$$\because \quad \frac{\delta_L}{L} = \frac{5}{\sqrt{\text{Re}_L}}$$

$$\Rightarrow \quad \delta_L = \frac{5 \times 3}{\sqrt{6 \times 10^4}} = 0.0612 = 6.12 \text{ cm}$$

$$C_{f_L} = \frac{0.664}{\sqrt{\text{Re}_L}}$$

Shear stress, $\tau_{0_L} = C_{f_L} \dfrac{\rho U^2}{2} = \dfrac{0.664}{\sqrt{6 \times 10^4}} \times \dfrac{1}{2} \times 900 \times 2^2 = 4.88 \text{ N/m}^2$

Power required:

Total force on plate, $F = \dfrac{1}{2} C_D \rho A U^2$

$$C_D = \frac{1.328}{\sqrt{\text{Re}_L}}$$

$$F = \frac{1}{2} \times \frac{1.328}{\sqrt{6 \times 10^4}} \times 900 \times (2 \times 2 \times 3) \times 2^2 = 117.105 \text{ N}$$

Power required, $P = F \times U$

$$= 117.105 \times 2 = 234.21 \text{ W}$$

Q.8 (b) (ii) Solution:

If suffixes 1 and 2 refer to the inlet and the throat respectively, then applying Bernoulli's equation between 1 and 2, we get

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

where, H_L is the head loss between 1 and 2,

Also, $$\left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = \Delta h$$

$$\Rightarrow \quad \Delta h - H_L = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Rightarrow \quad \Delta h - H_L = \frac{1}{2g}\left(\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right)$$

$$\Rightarrow \quad Q^2 = \frac{2g(\Delta h - H_L)A_1^2 A_2^2}{A_1^2 - A_2^2}$$

$$\Rightarrow \quad Q = \frac{\sqrt{2g(\Delta h - H_L)}A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \qquad \text{...(i)}$$

But we know, $$Q = \frac{C_d A_1 A_2 \sqrt{2g(\Delta h)}}{\sqrt{A_1^2 - A_2^2}} \qquad \text{...(ii)}$$

Comparing equations (i) and (ii),

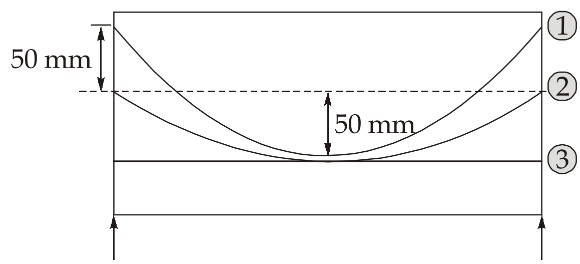
$$\therefore \quad C_d\sqrt{\Delta h} = \sqrt{\Delta h - H_L}$$

$$\Rightarrow \quad C_d^2(\Delta h) = \Delta h - H_L$$

$$\Rightarrow \quad H_L = \Delta h(1 - C_d^2)$$

Hence proved.

Q.8 (c) Solution:



Force in each cable,

$$P_1 = P_2 = P_3 = 1200 \times 200 \text{ N} = 240 \text{ kN}$$

$$\text{Area of beam cross-section} = 100 \times 300 = 3 \times 10^4 \text{ mm}^4$$

$$w_d + w_L = 2 \text{ kN/m}$$

$$\text{Moment due to load} = \frac{wl^2}{8} = \frac{2 \times 10^2}{8} = 25 \text{ kN-m}$$

$$I = \frac{100 \times 300^3}{12} = 225 \times 10^6 \text{ mm}^4$$

Cable no. 3 → Loss of stress = 0

Cable no. 2 → Loss of stress due to P_3 only

When cable 3 is tensioned and anchored, stress at the level of cable 2 is calculated as below:

At support : (at the level of cable 2 due to cable 3)

$$f_{c_1} = \frac{240 \times 10^3}{3 \times 10^4} + \frac{(240 \times 10^3) \times 50}{225 \times 10^6} \times 0 = 8 \text{ N/mm}^2$$

At mid span:

$$f_{c_2} = \frac{240 \times 10^3}{3 \times 10^4} + \frac{(240 \times 10^3) \times 50}{225 \times 10^6} \times 50 - \frac{25 \times 10^6}{225 \times 10^6} \times 50$$

$$= 5.11 \text{ N/mm}^2$$

$$f_{c_{avg}} = f_{c_1} + \frac{2}{3}(f_{c_2} - f_{c_1}) = 8 + \frac{2}{3}(5.11 - 8)$$

$$= 6.07 \text{ N/mm}^2$$

$$\text{Loss in cable-2} = m f_{c_{avg}} = 6 \times 6.07 = 36.42 \text{ N/mm}^2$$

Cable no. 1 → Loss of stress due to P_2 and P_3

At support: (At the level of cable 1 due to cables 2 and 3)

$$f_{c_1} = \frac{(240+240)\times 10^3}{3\times 10^4} - \frac{240\times 10^3 \times 50}{225\times 10^6}\times 50$$

$$= 13.33 \text{ N/mm}^2$$

At mid span:

$$f_{c_2} = \frac{(240+240)\times 10^3}{3\times 10^4} + \frac{240\times 10^3 \times 50}{225\times 10^6}\times 50$$

$$+\frac{240\times 10^3 \times 50}{225\times 10^6}\times 50 - \frac{25\times 10^6}{225\times 10^6}\times 50$$

$$= 15.78 \text{ N/mm}^2$$

$$f_{c_{avg}} = f_{c_1} + \frac{2}{3}(f_{c_2} - f_{c_1}) = 13.33 + \frac{2}{3}(15.78 - 13.33)$$

$$= 14.96 \text{ N/mm}^2$$

Loss of stress in cable-1 = $m f_{c_{avg}}$ = 6 × 14.96 = 89.76 N/mm^2

∴ Loss of stress due to elastic shorting in cable-1 = 89.76 N/mm^2

Loss of stress due to elastic shorting in cable-2 = 36.44 N/mm^2

Loss of stress due to elastic shorting in cable-3 = 0

Loss of stress due to creep of concrete:

Loss in cable-1 = $\phi m f_{c_{avg}} = 1.6 \times 89.76 = 143.616 \text{ N/mm}^2$

Loss in cable-2 = 1.6 × 36.42 = 58.27 N/mm^2

Loss in cable-3 = 0

Due to friction:

$$y = \frac{4e}{L^2}\times x(L-x)$$

∴ $$\frac{dy}{dx} = \frac{4e}{L^2}(L-2x)$$

Slope at (x = 0), $$\frac{dy}{dx} = \frac{4e}{L}$$

Slope at (x = L), $$\frac{dy}{dx} = \frac{-4e}{L}$$

Cumulative angle between tangents, α

$$= \frac{4e}{L} - \left(\frac{-4e}{L}\right) = \frac{8e}{L}$$

For cable-1,

$$\alpha = \frac{8e}{L} = \frac{8 \times 100}{10 \times 10^3} = 0.08 \text{ radians}$$

$$\text{Loss of stress} = P_0(\mu\alpha + kx)$$

$$= 1200[0.35 \times 0.08 + 0.0015 \times 10]$$

$$= 51.6 \text{ N/mm}^2$$

For cable-2,

$$\alpha = \frac{8e}{L} = \frac{8 \times 50}{10 \times 10^3} = 0.04 \text{ radians}$$

$$\text{Loss of stress} = P_0(\mu\alpha + kx)$$

$$= 1200[0.35 \times 0.04 + 0.0015 \times 10]$$

$$= 34.8 \text{ N/mm}^2$$

For cable-3,

$$\alpha = 0$$

$$\text{Loss of stress} = 1200[0 + 0.0015 \times 10] = 18 \text{ N/mm}^2$$

Total loss of stress in cable-1,

$$89.76 + 143.616 + 51.6 = 284.976 \text{ N/mm}^2$$

Total loss of stress in cable-2,

$$36.42 + 58.27 + 34.8 = 129.49 \text{ N/mm}^2$$

Total loss of stress in cable-3,

$$0 + 0 + 18 = 18 \text{ N/mm}^2$$

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