



MADE EASY

India's Best Institute for IES, GATE & PSUs

ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-3 : Analog and Digital Communication Systems [All topics]

Signals and Systems-1 + Microprocessors and Microcontroller [Part Syllabus]

Network Theory-2 + Control Systems-2 [Part Syllabus]

Name :

Roll No :

Test Centres

Delhi ☐ Bhopal ☐ Jaipur ☐
Pune ☐ Kolkata ☐ Bhubaneswar ☐ Hyderabad ☒

Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	

Signature of Evaluator

Cross Checked by



MADE EASY

India's Best Institute for IES, GATE & PSUs

ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-3 : Analog and Digital Communication Systems [All topics]

Signals and Systems-1 + Microprocessors and Microcontroller [Part Syllabus]

Network Theory-2 + Control Systems-2 [Part Syllabus]

Name : T. Manohar Reddy

Roll No : 23 MT HY

E	C	A	M	T	H	Y				A	0	0	2
---	---	---	---	---	---	---	--	--	--	---	---	---	---

Test Centres

Delhi ☐ Bhopal ☐ Jaipur ☐
Pune ☐ Kolkata ☐ Bhubaneswar ☐ Hyderabad ☒

Student's Signature

T. Manohar

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	

Signature of Evaluator

Cross Checked by

Corp. office : 44 - A/1, Kalu Sarai, New Delhi-110016

Ph: 9021300500 | Web: www.madeeasy.in



Scanned with OKEN Scanner

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

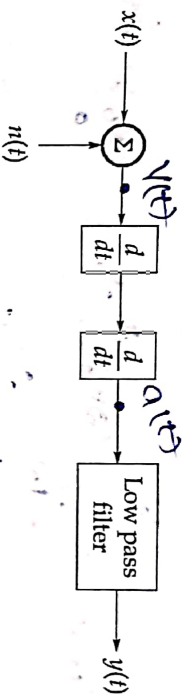
1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Analog and Digital Communication Systems

Q.1 (a)

Consider the system shown in figure. The signal $x(t)$ is defined by:

$$x(t) = A \cos 2\pi f_c t$$



The low pass filter has unity gain in the passband and bandwidth W , where $f_c < W$. The noise $n(t)$ is white with two sided power spectral density $\frac{1}{2} N_0$. Determine the signal to noise ratio at the output $y(t)$.

[12 marks]

$$V(t) = A \cos 2\pi f_c t + n(t)$$

transfer function of sy Low pass filter be $H(\omega)$

$$\therefore \frac{d}{dt} \leftrightarrow j\omega$$

$$\therefore y(j\omega) \leftrightarrow (j\omega)^2 H(j\omega) \cdot V(j\omega)$$

output Signal due to $x(t) = X(j\omega) (j\omega)^2 \cdot H(j\omega)$

$$\text{at } \omega = 0 \quad \frac{d^2}{dt^2} (x(t)) = \frac{d^2}{dt^2} A \cos 2\pi f_c t$$

$$\frac{d^2}{dt^2} A \cos 2\pi f_c t = -A 2\pi f_c \sin 2\pi f_c t$$

Given $f_c \ll W$

$$\text{at } \omega = 0 \quad = -A (\sin f_c)^2 \cos 2\pi f_c t$$

$$\text{Signal power} = \frac{(A (\sin f_c)^2)^2}{2} = \frac{A^2 (4\pi^2 f_c^2)^2}{2}$$

$$= \frac{A^2 (4\pi^2)^2 f_c^4}{2}$$

output PSD due to noise = $|H(\omega)|^2 \times \frac{N_0}{2}$

$$= (2\pi f_c)^4 \times -10 < \omega < \omega_1$$

$$\text{Output Noise} = \int_{-\omega_1}^{\omega_1} (2\pi f_c)^4 \frac{N_0}{2} = \frac{N_0 W (2\pi f_c)^4}{2}$$

$$\text{SNR at output} = \frac{A_c^2 (4\pi^2)^2 f_c^4}{2 N_0 W (2\pi f_c)^4} = \frac{A_c^2}{2 N_0 W} \times \frac{1}{4\pi^2}$$

$$(\text{SNR})_{\text{OP}} = \frac{A_c^2}{2 N_0 W \pi^2}$$

$$(\text{SNR})_{\text{OP}} = \frac{A_c^2}{2 N_0 W}$$

Q.1(b)

Consider a continuous input signal whose amplitude V lies in the range $[-V_{\max}, V_{\max}]$. This is applied to a uniform quantizer of mid-rise type where the step size is given by Δ and L denotes the number of representation levels. Let σ_Q^2 represent the variance of the quantization error and ' n ' represent the number of bits per sample. Show that $\sigma_Q^2 = \frac{1}{3} V_{\max}^2 \cdot 2^{-2n}$ and that the output signal to noise ratio of a uniform quantizer is

$$(\text{SNR}_0) = \frac{3P}{V_{\max}^2} \cdot 2^{2n} \text{ where } P \text{ is signal power}$$

$$\text{Step Size } \Delta = \frac{V_{\max} - (-V_{\max})}{L} = \frac{2V_{\max}}{L} \quad [12 \text{ marks}]$$

Part:- 1

for uniform quantizer, quantization noise power is $\frac{\Delta^2}{12}$

$$E(\sigma_e^2) = \frac{\Delta^2}{12} = \frac{\left(\frac{2V_{\max}}{L}\right)^2}{12} = \frac{4V_{\max}^2}{12L^2}$$

$$= \frac{V_{\max}^2}{3L^2}$$

assuming the mean of quantizer is zero
variance $\sigma_a^2 = E(\sigma_e^2) - [E(\sigma_e)]^2$

$$\sigma_{e}^2 = E(e^2)$$

$$[\because E[e] = 0]$$

$$= \frac{V_{\max}^2}{3L^2}$$

We know that No. of Level $L = 2^n$

$$\sigma_e^2 = \frac{V_{\max}^2}{3 \cdot 2^{2n}} = \frac{V_{\max}^2 \cdot 2^{-2n}}{3}$$

Part 2:-

Signal Power at output = P

$$(SNR)_o = \frac{\text{Signal power}}{\text{Noise power}}$$

at output of quantizer the noise is due to quantization error.

So we can assume noise power as quantization noise power

$$(SNR)_o = \frac{P}{\frac{V_{\max}^2 \cdot 2^{-2n}}{3}}$$

$$(SNR)_o = \frac{3P \cdot 2^{2n}}{V_{\max}^2}$$

Q.1 (c)

The random process $X(t)$ is defined by

$$X(t) = X \cos 2\pi f_0 t + Y \sin 2\pi f_0 t$$

where X and Y are two zero mean independent Gaussian random variable each with variance σ^2 .

(i) Find $m_X(t)$.(ii) Find $R_X(t+\tau, t)$. Is $X(t)$ stationary? Is it cyclostationary?

[12 marks]

$$\text{Given } X(t) = X \cos 2\pi f_0 t + Y \sin 2\pi f_0 t$$

$$(i) m_X(t) = E[X(t)]$$

$$= E[X \cos 2\pi f_0 t] + E[Y \sin 2\pi f_0 t]$$

$$= E[X] E(\cos 2\pi f_0 t) + E[Y] E(\sin 2\pi f_0 t)$$

$$E(X) = 0, E(Y) = 0 \Rightarrow \text{given}$$

$$m_X(t) = (0) E(\cos 2\pi f_0 t) + (0) E(\sin 2\pi f_0 t)$$

$$m_X(t) = 0$$

$$(ii) R_X(t+\tau, t) = E[X(t+\tau) X(t)]$$

$$= E[X \cos 2\pi f_0 (t+\tau) + Y \sin 2\pi f_0 (t+\tau)] [X \cos 2\pi f_0 t + Y \sin 2\pi f_0 t]$$

$$= E[X^2 \cos 2\pi f_0 (t+\tau) \cos 2\pi f_0 t + XY \cos 2\pi f_0 (t+\tau) \sin 2\pi f_0 t + XY \sin 2\pi f_0 (t+\tau) \cos 2\pi f_0 t + Y^2 \sin 2\pi f_0 (t+\tau) \sin 2\pi f_0 t]$$

$$= E[X^2 \cos 2\pi f_0 (t+\tau) \cos 2\pi f_0 t + X^2 \cos 2\pi f_0 (t+\tau) \cos 2\pi f_0 t + XY \cos 2\pi f_0 (t+\tau) \sin 2\pi f_0 t + XY \sin 2\pi f_0 (t+\tau) \cos 2\pi f_0 t + Y^2 (\cos 2\pi f_0 (t+\tau) \cos 2\pi f_0 t - \cos 2\pi f_0 t \cos 2\pi f_0 t)]$$

Given X and Y are independent $\therefore E(XY) = 0$

Above equation can be written as

$$R_x(t+n, t) = E(x^2) E(\cos \pi f_s (2t+n)) + E(x^2) E(\cos 2\pi f_s t) + E(y^2) E(\cos 2\pi f_s (2t+n)) - E(y^2) E(\cos 2\pi f_s t)$$

$$= \sqrt{2} \left[E(\cos \pi f_s (2t+n)) + \cos \pi f_s t \right] + \sqrt{2} \left[E(\cos \pi f_s (2t+n)) - \cos \pi f_s t \right]$$

$\therefore R_x(t+n, t)$ depends upon time
Auto Correlation $R_x(t+n, t)$ is not Stationary

Q.1 (d) A PCM system uses a uniform quantizer followed by a 8-bit binary encoder. The bit rate of the system is equal to 60 Mbps.

- What is the maximum message bandwidth for which the system operates satisfactory?
- Determine signal to quantization noise ratio for uniform distributed sample of message signal having uniform quantization level.

[12 marks]

(i) $n = 8$, Bit rate $R_b = 60 \text{ Mbps}$

We know that $R_b = n f_s$

$$60 \times 10^6 = 8 \times f_s$$

$$f_s = 7.5 \text{ MHz}$$

A sampling f_s satisfies Nyquist Sampling

$$f_s \geq 2 f_m$$

$$f_m \leq \frac{f_s}{2} = \frac{7.5}{2}$$

$$f_m \leq 3.75 \text{ MHz}$$

\therefore Maximum Message Bandwidth = 3.75 MHz

(ii)

for uniformly - quantized encoder and assuming input as sinusoidal

$$S_{\text{SNR}} = (8 + 6n) \text{ dB}$$

$$S_{\text{SNR}} = 49.8 \text{ dB}$$

Q.1 (e) What are the capture effect and threshold effect in an FM system? List two different methods used for FM threshold improvement.

[12 marks]

Capture effect:-

When at the input of FM receiver, we receive two different signals of same frequency then receiver doesn't understand which signal to receive between two signals. There is a tendency for the receiver to accept the signal which has higher magnitude and it doesn't accept low magnitude signal compared to both.

This phenomenon of accepting higher amplitude signal from different signals

having Same frequency is known as
Capture effect

Threshold effect:-

When the Signal to noise ratio of
input decreases then output Signal to noise
ratio also decreases proportionally, but when
SNR at input decreases below certain
value called threshold then output SNR
decreases rapidly and this is known as
Threshold effect

Methods to ~~improve~~ decrease threshold effect

- ① Use Pre emphasis Circuit at transmitter
and deemphasis Circuit at receiver side
- ② by using feedback Circuits in demodulator
Circuit like Phase locked loop and
frequency locked loop.

Q.2 (a)

A communication channel has a bandwidth of 100 kHz. This channel is to be used for transmission of an analog source $m(t)$, where $|m(t)| < 1$, whose bandwidth is 4 kHz. The power content of the message signal is 0.1 W.

- (i) Find the ratio of the output SNR of an FM system that utilizes the whole bandwidth, to the output SNR of a conventional AM system with a modulation index of $\mu = 0.85$. What is this ratio in dB?
- (ii) Show that if an FM system and a PM system are employed and these systems have same output signal to noise ratio, we have

$$\frac{BW_{PM}}{BW_{FM}} = \frac{\sqrt{3}\beta_f + 1}{\beta_f + 1} \quad (\beta_f = \text{Modulation index of FM})$$

[10 + 10 marks]

Q.2(b)

An analog signal having 5 kHz bandwidth is sampled at twice the Nyquist rate and each sample is quantized into one of 256 equally likely levels. Assume the samples to be statistically independent.

- (i) Calculate the information rate of the source.
- (ii) Can the output of the source be transmitted without error over an AWGN channel with a bandwidth of 10 kHz and $\left(\frac{S}{N}\right)$ ratio of 40 dB?
- (iii) Find the $\left(\frac{S}{N}\right)$ ratio so that the output of this source is transmitted without error over an AWGN channel with a bandwidth of 10 kHz.
- (iv) Find the bandwidth requirement for an AWGN channel for an error free transmission of the output of this source if $\left(\frac{S}{N}\right)$ ratio is 40 dB.

[20 marks]

Q.2 (c)

- (i) The two sided power spectral density of the channel noise is 1×10^{-11} W/Hz and the carrier used in the transmitter is $15 \cos(2\pi f_c t)$ mV. Binary data (equiprobable bits) with a rate of 0.5 Mbps is transmitted through an AWGN channel using different modulation schemes. In each case of different modulation schemes, the signal are received by their respective correlator receiver with exact phase synchronisation and with optimum threshold detection. Find the average symbol error probability for modulation schemes BASK, BFSK and BPSK.
- (ii) For a minimum hamming distance of "5",
1. How many errors can be detected?
 2. How many errors can be detected and corrected?

[14 + 6 marks]

Find the value of x if $\int_0^x \frac{1}{\sqrt{1-t^2}} dt = \frac{\pi}{4}$

Ans. We have $\int_0^x \frac{1}{\sqrt{1-t^2}} dt = \frac{\pi}{4}$

$$\frac{1}{\sqrt{1-t^2}} = \frac{1}{\sqrt{1-\left(\frac{t}{1}\right)^2}} \quad \text{Let } t = \sin \theta$$

$$\frac{1}{\sqrt{1-t^2}} = \frac{1}{\sqrt{1-\sin^2 \theta}} = \frac{1}{\cos \theta}$$

$$\therefore \int_0^x \frac{1}{\sqrt{1-t^2}} dt = \int_0^{\frac{\pi}{2}} \frac{1}{\cos \theta} d\theta = \frac{\pi}{4}$$

$$\therefore \frac{\pi}{2} = \frac{\pi}{4} \quad \text{Hence } x = \frac{1}{2}$$

$$\therefore \int_0^x \frac{1}{\sqrt{1-t^2}} dt = \frac{\pi}{4} \quad \text{Hence } x = \frac{1}{2}$$

Q.3 (a)

A Gaussian signal pulse given by,

$$x(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t^2/2\sigma^2)}$$

is applied to the input of matched filter and the noise on the channel is a white noise with power density spectrum of $\frac{N_0}{2} = 10^{-20}$ Watt/Hz, then calculate the maximum

signal to noise ratio $\left(\frac{S}{N}\right)_{\max}$ in dB achieved by this filter with $\sigma = 1$.

[20 marks]

When $x(t)$ is given as input then SNR at output of matched filter is $\frac{2E}{N_0}$

E = energy of input $x(t)$

$$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma^2 2\pi} \left(e^{-\frac{t^2}{2\sigma^2}} \right)^2 dt$$

$$= \frac{1}{2\pi \sigma^2} \int_{-\infty}^{\infty} e^{-\frac{t^2}{\sigma^2}} dt$$

$$\text{Put } \frac{t}{\sigma} = x \Rightarrow \frac{2t}{\sigma} dx = dt$$

$$\text{Given } \sigma^2 = 1 \Rightarrow \sigma = 1$$

$$E_{x(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-x^2} dx$$

Example: Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{c} = 3\hat{i} + 4\hat{j} + 5\hat{k}$.
 Find the value of $\vec{a} \cdot (\vec{b} \times \vec{c})$.

Sol: We know that $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Ans: 1

$$= \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = 2(10-9) - 3(5-9) + 4(5-6) = 2(1) - 3(-4) + 4(-1) = 2 + 12 - 4 = 10$$

$$= 2(10-9) - 3(5-9) + 4(5-6) = 2(1) - 3(-4) + 4(-1) = 2 + 12 - 4 = 10$$

$$= 2(10-9) - 3(5-9) + 4(5-6) = 2(1) - 3(-4) + 4(-1) = 2 + 12 - 4 = 10$$

$$= 2(10-9) - 3(5-9) + 4(5-6) = 2(1) - 3(-4) + 4(-1) = 2 + 12 - 4 = 10$$

$$= 2(10-9) - 3(5-9) + 4(5-6) = 2(1) - 3(-4) + 4(-1) = 2 + 12 - 4 = 10$$

$$= 2(10-9) - 3(5-9) + 4(5-6) = 2(1) - 3(-4) + 4(-1) = 2 + 12 - 4 = 10$$

Q.3 (b)

For each of the following processes, find the power spectral density.

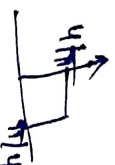
- (i) $X(t) = A \cos(2\pi f_0 t + \theta)$, where A is a constant and θ is a random variable uniformly distributed on $\left[0, \frac{\pi}{4}\right]$.

- (ii) $X(t) = x + y$, where x and y are independent, x is uniformly distributed on $[-1, 1]$ and y is uniformly distributed on $[0, 1]$.

[10 + 10 marks]

$$(i) \quad x(t) = A \cos(2\pi f_0 t + \theta)$$

$$R_x(\tau) = E[x(t)x(t+\tau)]$$



$$= E[A^2 \cos(2\pi f_0 t + \theta) \cos(2\pi f_0 (t + \tau) + \theta)]$$

$$= E[A^2 \cos(2\pi f_0 t + \theta) \cos(2\pi f_0 t + 2\pi f_0 \tau + \theta)]$$

$$= E[A^2] E[\cos(2\pi f_0 t + \theta) \cos(2\pi f_0 t + 2\pi f_0 \tau + \theta)] + E[\cos(2\pi f_0 t + \theta) \cos(2\pi f_0 t + 2\pi f_0 \tau + \theta)]$$

$$= E[A^2] E[\cos(2\pi f_0 \tau)]$$

$$= A^2 \int_0^{\pi/4} \cos 2\pi f_0 \tau \times \frac{1}{\pi} d\tau =$$

$$2/5 = \frac{14x}{100}$$

$$\Rightarrow \frac{2}{5} \times 100 = \frac{14x}{100} \times 100$$

$$\frac{2}{5} \times 100 = 14x$$

$$(x+1) \times 100 = 14x \times 100$$

$$[100(x+1)] \times 100 = 14x \times 100$$

$$[100x + 100] \times 100 = 14x \times 100$$

$$\left(\frac{100}{100}\right) \times 100 + \left(\frac{100}{100}\right) \times 100 = 14x \times 100$$

$$\left(\frac{100}{100}\right) \times 100 + \left(\frac{100}{100}\right) \times 100 = 14x \times 100$$

∴ (100) × 100 = 14x × 100

∴ 100 × 100 = 14x × 100

$$1 = \frac{14x}{100} \times 100$$

$$1 = 14x$$

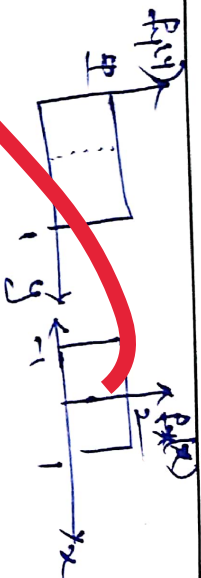
$$(100 \times 100) \div 14$$

$$(100 \times 100) \div 14 = 714.28$$

$$[100 \times 100] \div 14$$

(ii)

$$X = x + y$$



$$E(y) = \int_0^1 x \, dx = 0.5$$

$$E(x) = \int_{-1}^1 y \, dy = 0$$

$$E\left(\frac{x^2}{2}\right) = \int_0^1 x^2 \, dx = \frac{1}{3}$$

$$E\left(\frac{y^2}{3}\right) = \int_{-1}^1 y^2 \, dy = \frac{y^3}{3} \Big|_{-1}^1 = \frac{1+1}{3} = \frac{2}{3}$$

$$R_X(t) = E(X(t) X(t))$$

$$= E((x+y)(x+y))$$

$$= E\left[x^2 + 2xy + y^2\right]$$

$$= E(x^2) + E(2xy) + E(y^2)$$

$$= E\left(\frac{x^2}{2}\right) + 2E(xy) + E\left(\frac{y^2}{3}\right)$$

given x and y are independent $\therefore E(xy) = 0$

$$R_X(t) = \frac{1}{3} + \frac{2}{3} = 1$$

$$R_X(t) = 1$$

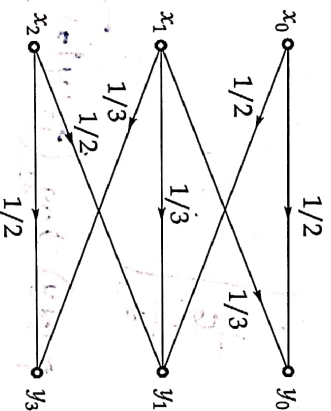
$$S_X(\omega) = F.T. R_X(t)$$

$$\leftarrow 2\pi \delta(\omega)$$

$$S_X(\omega) = 2\pi \delta(\omega)$$

Q.3 (c)

Consider the discrete memoryless channel shown below:



If the input probabilities are $P(x_0) = P(x_2) = \frac{1}{4}$ and $P(x_1) = P(x_3) = \frac{1}{2}$, then determine the mutual information $I(X; Y)$.

[20 marks]

$$I(X; Y) = H(Y) - H(Y|X)$$

$$P(Y|X) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

$$P(X) = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \end{bmatrix}$$

$$P(Y) = P(X)P(Y|X) = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \end{bmatrix}$$

$$= \left[\frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4} \quad 0 \right]$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P(x_0) = \frac{7}{24}, \quad P(y_1) = \frac{5}{12}, \quad P(y_2) = \frac{1}{24}$$

$$H(y) = -[P(y_0) \log_2 P(y_0) + P(y_1) \log_2 P(y_1) + P(y_2) \log_2 P(y_2)]$$

$$= -\left[2 \left(\frac{7}{24} \log_2 \left(\frac{7}{24}\right) + \frac{5}{12} \log_2 \frac{5}{12}\right)\right]$$

$$= 1.563 \text{ bits/symbols}$$

$$P(x,y) = [P(x)]_A [P(y)_B]$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & 0 \\ 0 & \frac{1}{6} & \frac{1}{8} \\ 0 & 0 & \frac{1}{8} \end{bmatrix}$$

$$H(x,y) = -\left[\sum_{i=1}^{r-1} \sum_{j=1}^{s-1} P(x,y) \log_2 P(x,y)\right]$$

$$= -\left[\frac{1}{8} \log_2 \left(\frac{1}{8}\right) + \frac{1}{8} \log_2 \left(\frac{1}{8}\right) + \frac{3}{8} \log_2 \left(\frac{1}{8}\right) + \frac{2}{8} \log_2 \left(\frac{1}{8}\right)\right]$$

$$= -\left[\frac{1}{2} \log_2 \left(\frac{1}{2}\right) + \frac{3}{8} \log_2 \left(\frac{1}{8}\right)\right] = 1.29 \text{ bits/sym}$$

$$I(x,y) = H(y) - H(y|x)$$

$$= 1.563 - 1.29$$

$$I(x,y) = 0.273 \text{ bits/symbols}$$

Q4 (a)

An AM signal has the form

$$u(t) = [20 + 2 \cos 3000\pi t + 10 \cos 6000\pi t] \cos 2\pi f_c t$$

where $f_c = 10^5$ Hz.

- Sketch the (voltage) spectrum of $u(t)$.
- Determine the power in each of the frequency components.
- Determine the modulation index.
- Determine the power in the sidebands, the total power, and the ratio of the sidebands power to the total power.

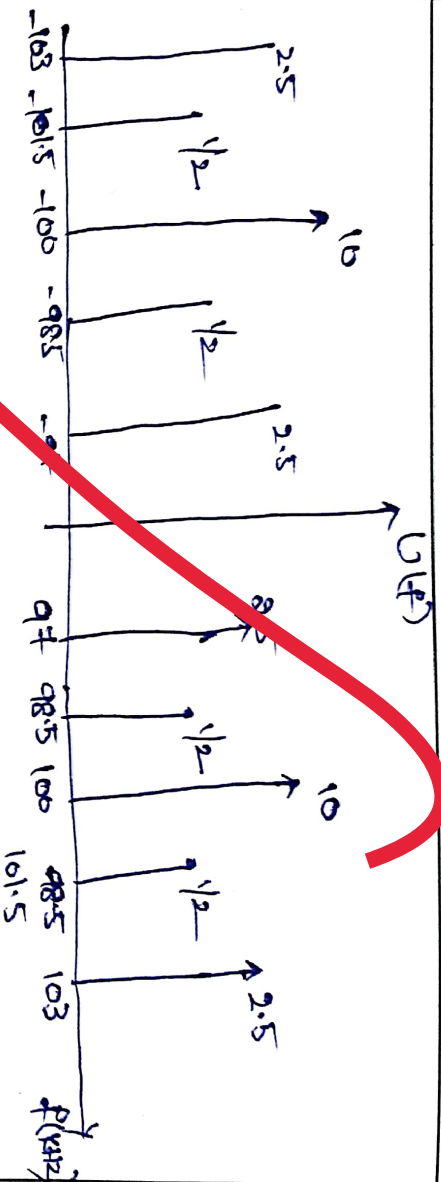
Given $u(t) = [20 + 2 \cos 3000\pi t + 10 \cos 6000\pi t] \cos 2\pi f_c t$ [5 × 4 marks]

$$u(t) = 20 \cos 2\pi f_c t + 2 \cos 2\pi f_c t \cos 3000\pi t + 10 \cos 2\pi f_c t \cos 6000\pi t$$

$$= 20 \cos 2\pi f_c t + 2 \cos \pi (101500)t + \cos (2\pi 98500)t + 5 \cos (2\pi 103000t) + 5 \cos (2\pi 97000t)$$

$$U(f) = 20 \left[\delta(f - f_c) + \delta(f - f_c) + \frac{1}{2} \delta(f - 101500) + \frac{1}{2} \delta(f - 101500) + \frac{1}{2} \delta(f - 98500) + \frac{1}{2} \delta(f - 98500) + \frac{1}{2} \delta(f - 103000) + \frac{1}{2} \delta(f - 103000) + \frac{1}{2} \delta(f - 97000) + \frac{1}{2} \delta(f - 97000) \right]$$

(i)



(ii)

Power in Carrier Component i.e. $P_c = 10^2 \text{ W} \Rightarrow \frac{10^2}{2} = 50 \text{ W}$

Power in $f = 97 \text{ kHz}$ Component $= \frac{10^2}{2} = 12.5 \text{ Watts}$

Power in $f = 98.5 \text{ kHz}$ Component $= \frac{1^2}{2} = 0.5 \text{ Watts}$

Power in $f = 101.5 \text{ kHz}$ Component $= \frac{2^2}{2} = 0.5 \text{ Watts}$

Power in $f = 103 \text{ kHz}$ Component $= \frac{5^2}{2} = 12.5 \text{ Watts}$

In the Spectrum Power will be divided equally at negative and positive frequencies

i.e. Power $P_c = 50 \text{ Watts}$, $P_{f=97 \text{ kHz}} = 0.5 \text{ Watts}$

$P_{f=98.5 \text{ kHz}} = 6.25 \text{ Watts}$

$P_{f=101.5 \text{ kHz}} = 0.25 \text{ Watts}$

$P_{f=103 \text{ kHz}} = 6.25 \text{ Watts}$

(iii)

$$u(t) = [20 + 2 \cos 3000\pi t + 10 \cos 6000\pi t] \cos 97\pi t$$

$$= 20 [1 + 0.1 \cos 300\pi t + 0.5 \cos 600\pi t] \cos 97\pi t$$

Comparing it with Standard multiplex AM

Signal

$$u(t) = A_c [1 + \mu_1 \cos \omega_1 t + \mu_2 \cos \omega_2 t] \cos \omega_c t$$

$$A_c = 20; \mu_1 = 0.1; \mu_2 = 0.5$$

$$\text{total Modulation index } \mu_T = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.1^2 + 0.5^2}$$

$$\mu_T = 0.5099$$

$$\text{Power in Side bands} = 12.5 + 0.5 + 0.5 + 12.5$$

$$= 26 \text{ Watts}$$

$$\text{Power in Carrier} = 200 \text{ Watts}$$

$$\text{Total Power} = 200 + 26$$

$$= 226 \text{ Watts}$$

$$\text{Ratio of Side band power to total power} = \frac{26}{226}$$

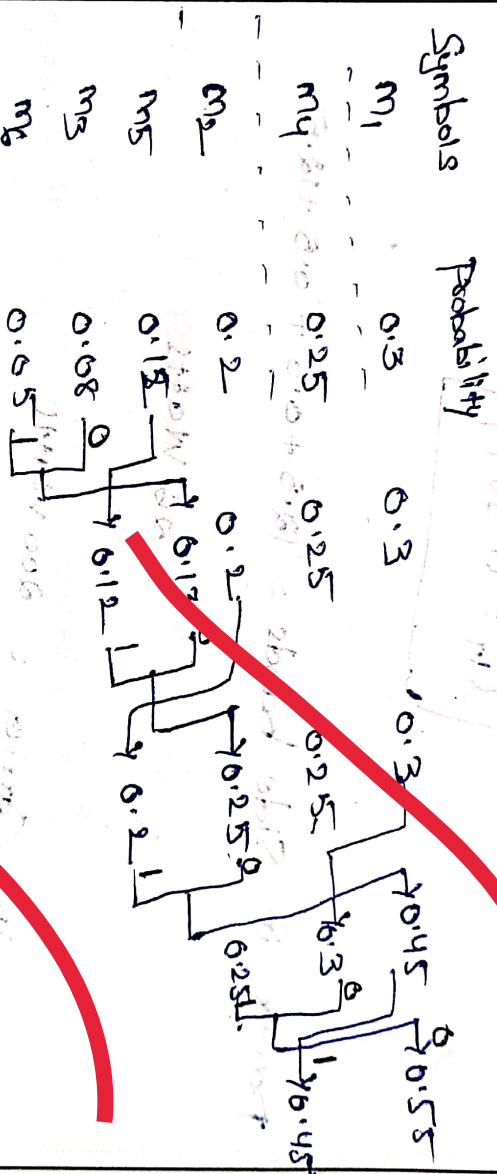
$$= 0.13$$

Q.4(b)

- (i) A message source generates six message symbols m_1, m_2, \dots, m_6 with probabilities 0.3, 0.2, 0.08, 0.25, 0.12, 0.05 respectively. Give Huffman code for these symbols. Determine the efficiency and redundancy of the code.
- (ii) For an AM modulator with carrier frequency $f_c = 200$ kHz and a maximum modulating signal frequency $f_{m(\max)} = 6$ kHz, determine,
1. Frequency limits for the upper and lower sidebands.
 2. Bandwidth
 3. Upper and lower side frequencies produced when the modulating signal is a single frequency 2 kHz tone.

[10 + 10 marks]

(i)



Codes

length

 $m_1 \Rightarrow 00$

2

Average length $L = 2.38$ bits $m_2 \Rightarrow 11$

2

 $L = 0.3 \times 2 + 0.2 \times 2 + 4 \times 0.08$ $m_3 \Rightarrow 1000$

4

 $L = 0.05 \times 4 + 3 \times 0.12 + 0.05 \times 4$ $m_4 \Rightarrow 01$

2

 $L = 2.38$ bits $m_5 \Rightarrow 101$

3

 $m_6 \Rightarrow 1001$

4

$$H(x) = - \sum_{i=1}^n P(x_i) \log_2 P(x_i)$$

$$= - [0.3 \log_2(0.3) + 0.2 \log_2(0.2) + 0.08 \log_2(0.08) + 0.25 \log_2(0.25) + 0.12 \log_2(0.12) + 0.05 \log_2(0.05)]$$

$$= 2.36$$

$$\text{Efficiency, } \eta = \frac{\frac{P_{av}}{P_{dc}}}{1 + \frac{P_{av}}{P_{dc}}} = \frac{2.36}{2.38} = 0.991$$

$$\eta = 99.1\%$$

$$\text{Redundancy} = 1 - \eta = 0.008$$

(ii) Given $f_c = 800 \text{ kHz}$, $f_m \text{ max} = 6 \text{ kHz}$

$$\begin{aligned} \text{(i) Upper Side band} &= f_c + f_m \\ &= 800 + 6 \\ &= 806 \text{ kHz} \end{aligned}$$

$$\text{limits of upper Side band} \Rightarrow 800 \text{ kHz} - 806 \text{ kHz}$$

$$\begin{aligned} \text{Lower Side band} &= f_c - f_m \\ &= 800 - 6 = 794 \text{ kHz} \end{aligned}$$

$$\text{limits of lower Side band} \Rightarrow 794 \text{ kHz} - 800 \text{ kHz}$$

$$\text{a) Bandwidth of AM} = 2 f_m \text{ max}$$

$$(BW)_{AM} = 12 \text{ kHz}$$

$$\text{3) if } f_m = 2 \text{ kHz}$$

$$\text{Upper Side band} = f_c + f_m = 800 + 2$$

$$f_{us} = 802 \text{ kHz}$$

Lower Side band $f_{LSB} = f_c - f_m$

$$= 800 - 2$$

$$f_{LSB} = 198 \text{ kHz}$$

Q.4 (c)

A single-tone modulating signal $m(t) = A_m \cos(2\pi f_m t)$ is used to generate the VSB signal

$$S(t) = \frac{1}{2} a A_m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_m A_c (1-a) \cos[2\pi(f_c - f_m)t]$$

where 'a' is a constant, less than unity, representing the attenuation of the upper side frequency.

- Find the quadrature component of the VSB signal $S(t)$.
- The VSB signal, plus the carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced in recovering the message signal.
- What is the value of constant 'a' for which this distortion reaches its worst possible condition?

(i)

$$S(t) = \frac{1}{2} a A_m A_c \cos(2\pi(f_c + f_m)t) + \frac{1}{2} A_m A_c (1-a) \cos(2\pi(f_c - f_m)t)$$

[20 marks]

$$\frac{1}{2} A_m A_c (1-a) \cos(2\pi(f_c - f_m)t)$$

$$= \frac{1}{2} a A_m A_c \cos(2\pi(f_c + f_m)t) - \frac{1}{2} a A_m A_c \cos(2\pi(f_c - f_m)t)$$

$$+ \frac{1}{2} A_m A_c \cos(2\pi(f_c - f_m)t)$$

(11)

$$0 = C_1 y^2 + C_2 y^2 + C_3 y^2 + C_4 y^2$$

$$0 = C_1 y^2 + C_2 y^2 + C_3 y^2 + C_4 y^2$$

$$13 + 28 = [(-1 + 20 + 1 - 2)] y^2$$

$$\frac{2}{4+2} + \frac{1}{1+2} = \frac{13+28}{(-1+20+1-2)} = C_1 y^2$$

$$1 = \frac{13+28}{(-1+20+1-2)} = [(-1+20+1-2)] y^2$$

$$0 = \frac{13+28}{14+2} = [(-1+20+1-2)] y^2$$

$$\frac{2}{4+2} + \frac{1}{1+2} = C_1 y^2$$

$$\left\{ C_1 y^2 + C_2 y^2 + C_3 y^2 + C_4 y^2 \right\}$$

**Section B : Signals and Systems-1 + Microprocessors and Microcontroller-1
+ Network Theory-2 + Control Systems-2**

Q.5 (a)

Consider a system described by the differential equation $\ddot{y}(t) + 2\dot{y}(t) + 3y(t) = x(t)$ with $x(t) = 3e^{-4t}$, $y(0) = 3$ and $\dot{y}(0) = 4$. Find its Z.I.R and Z.S.R.

[12 marks]

for Z.I.R ; $x(t) = 0$

$$\ddot{y}(t) + 2\dot{y}(t) + 3y(t) = 0$$

apply unilateral Laplace transform,

$$s^2 Y(s) - sy(0) - \dot{y}(0) + 2(sY(s) - y(0)) + 3Y(s) = 0$$

$$Y(s) [s^2 + 3s + 2] = 3s + 4$$

$$Y(s) = \frac{3s+4}{s^2+3s+2} = \frac{3s+4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \left. (s+1)Y(s) \right|_{s=-1} = \frac{3(-1)+4}{(-1+2)} = 1$$

$$B = \left. (s+2)Y(s) \right|_{s=-2} = \frac{3(-2)+4}{-2+1} = 2$$

$$Y(s) = \frac{1}{s+1} + \frac{2}{s+2}$$

$$y(t) = e^{-t} + 2e^{-2t}$$

For ZSR, $y(0)=0$, $\dot{y}(s)=4$, $y(s)=\frac{3}{s+4}$

$$\left[\frac{3}{(s+2)(s+4)} \right] y(s) = \frac{3}{s+4}$$

$$A = y(s)(s+1) = \frac{3}{(s+3)(s+4)} = -1$$

$$y(s) = \frac{3}{(s+4)(s+1)}$$

$$B = y(s)(s+3) = -1.5$$

$$y(s) = \frac{-1}{s+4} + \frac{1.5}{s+1}$$

$$C = y(s)(s+1) = 0.5$$

$$y(t) = -e^{-4t} + 1.5e^{-t} + 0.5e^{-t}$$

Q.5 (b)

Describe the following instructions of 8086:

- (i) LDS R_n, M
- (ii) AAM
- (iii) DAS
- (iv) CLI

[12 marks]

i) LDS R_n, M

Used to load DS Register and
other provided Register from the memory

$$[DS] \leftarrow [M] \quad \& \quad [R_n] \leftarrow [M]$$

(ii)

AAM

used to adjust ASCII codes after
multiplication

AAM: ASCII adjust for multiplication

(iii)

DAS:-

Decimal after

Subtraction

This instruction is used to adjust

decimal after

Subtraction

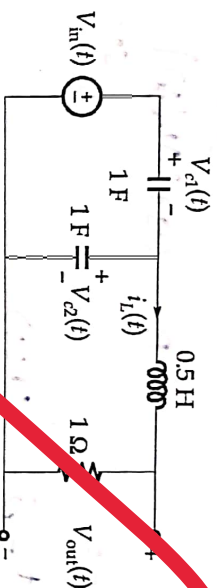
CLI:- Clear Interrupt

Used to clear the interrupt enable
flag to 0, i.e. disable INTR interrupt

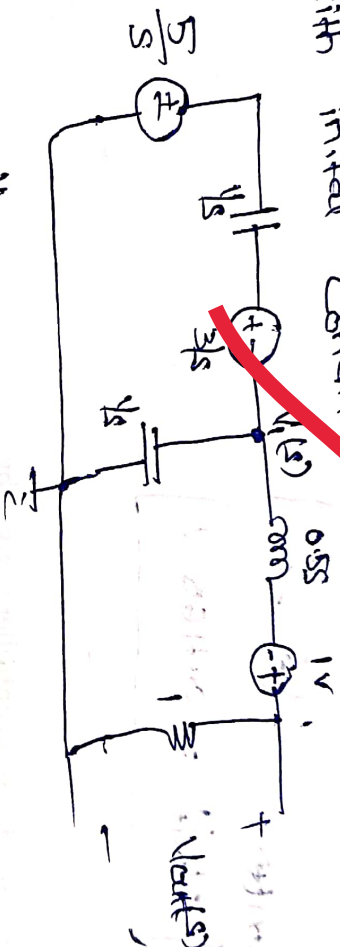
$[R0] \rightarrow [R2] \rightarrow [R0]$

Q.5 (c)

Consider the circuit below in which $V_{in}(t) = 5u(t)$ V, $V_{c1}(0^-) = 3$ V, $V_{c2}(0^-) = 0$ V and $i_L(0^-) = 2$ A. Find $V_{out}(t)$ and also obtain V_{out} at $t = 1$ sec.



Replacing the circuit in Laplace domain [12 marks]
With initial conditions

KVL at V_1

$$\frac{V_1 - \frac{5}{s} + \frac{3}{s}}{1/s} + \frac{V_1}{1/s} + \frac{V_1 + 1}{1 + 0.5s} = 0$$

$$s(V_1 - \frac{3}{s}) + V_1 s + \frac{V_1 + 1}{0.5s + 1} = 0$$

$$2sV_1 - 2 + \frac{V_1}{0.5s + 1} + \frac{1}{0.5s + 1} = 0$$

$$V_1 \left(2s + \frac{2}{s+2} \right) = 2 - \frac{2}{s+2}$$

$$V_1 \frac{(2s^2 + 4s + 2)}{s+2} = \frac{2s+4}{s+2}$$

$$V_1 = \frac{2s+2}{2s^2+4s+2} = \frac{s+1}{s^2+2s+1} = \frac{1}{s+1}$$

$$V_{out}(s) = \frac{1}{1+0.5s} \times V_1 = \frac{2}{s+2} \times \frac{1}{s+1}$$

$$N_{out}(s) = \frac{A}{s+2} + \frac{B}{s+1}$$

$$A = (s+2)N_{out}(s) \Big|_{s=-2} = -2$$

$$= \frac{2}{s+1} - \frac{2}{s+2}$$

$$B = (s+1)N_{out}(s) \Big|_{s=-1} = \frac{2}{2-1} = 2$$

$$N_{out}(t) = 2[e^{-t} - e^{-2t}] u(t)$$

$$\text{at } t=1$$

$$N_{out}(t) = 2[e^{-1} - e^{-2}]$$

$$N_{out}(t) = 0.4705$$

Q.5(d)

The transfer function of a controller is given by,

$$G_c(s) = \frac{s}{10s+4}$$

If this controller is realised using an operational amplifier, then find the other parameters of the controller assuming the capacitor value of 25 μF .

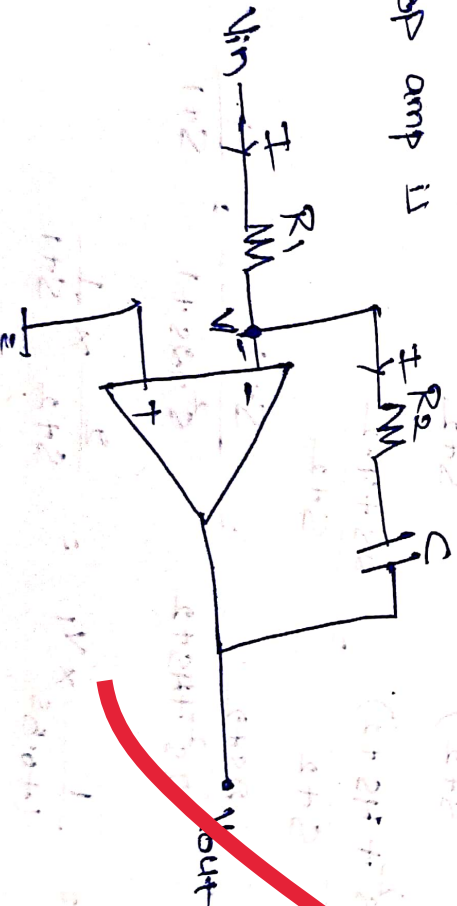
[12 marks]

$$G_c(s) = \frac{10s+4}{s} = 10 + \frac{4}{s} = K_P + \frac{K_I}{s} \rightarrow (1)$$

Given Controller is a P-I Controller

the transfer function of PI Controller using

op amp is



by applying Virtual Short $V^- = V^+ = 0$

V_{in} at V^-

$$\frac{V_{in}}{R_1} = \frac{0 - V_{out}}{R_2 + \frac{1}{sC}}$$

$$\frac{V_{in}}{R_1} = \frac{-V_{out}(sC)}{sCR_2 + 1}$$

We can neglect negative sign

$$\frac{V_{out}}{V_{in}} = \frac{sCR_2 + 1}{R_1 sC}$$

$$= \frac{R_2}{R_1} + \frac{1}{sR_1 C} \rightarrow (2)$$

Comparing (1) & (2)

$$K_P = \frac{R_2}{R_1} = 10$$

$$\frac{1}{R_1 C} = K_F = 4$$

$$R_2 = 10R_1$$

$$R_1 C = \frac{1}{4}$$

$$R_2 = 10 \times 10$$

$$R_2 = 100k\Omega$$

$$R_1 = \frac{1}{4 \times 85 \times 10^6}$$

$$R_1 = 10k\Omega$$

Write a 8085 program to find 2's complement of the number stored in memory location 9000 H, and store the result in memory location 9001 H. Also give the flow chart of the program and calculate execution time of program if operating frequency is 5 MHz.

[12 marks]

1000

[illegible]

$\frac{11.20}{2.29} = 4.89$
 4.89

(16)
F

6
30
20

7

30
20

Completed

$$\frac{1}{10} = \frac{1}{10} = \frac{1}{10}$$

100

11

1000

$\beta = 10^{-3}$

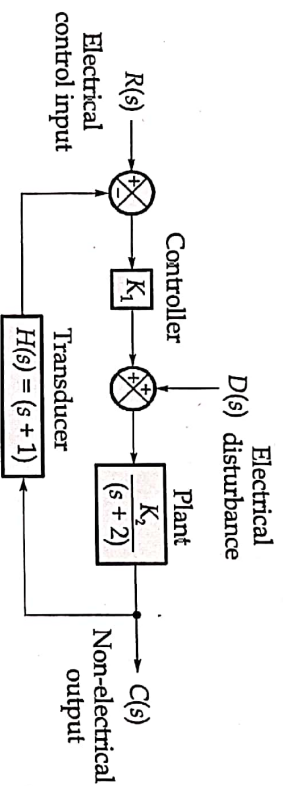
Q.6 (a)

- (i) Explain all the basic machine cycles of 8085 microprocessor and differentiate between instruction cycle (IC) and machine cycle (MC).
- (ii) Draw the timing diagram of OUT instruction for 8085 microprocessor.

[10 + 10 marks]

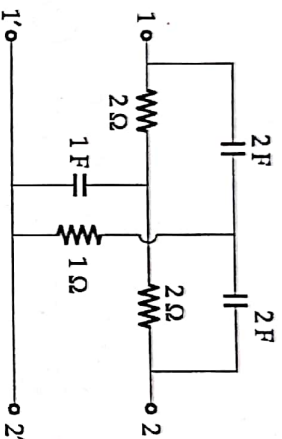
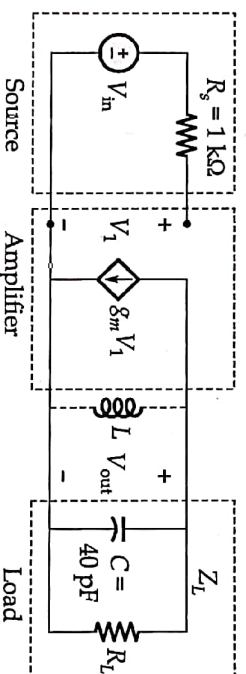
Q.6 (b)

For the system shown in the figure below, both the electrical control input and the disturbance are unit step signals. Find the sensitivity of the steady-state error for changes in K_1 and in K_2 individually, when $K_1 = 100$ and $K_2 = 0.10$.



[20 marks]

Q.6 (c)

(i) Determine the Y parameters of given network.(ii) Below given figure displays an amplifier model containing a VCCS with $g_m = 2 \text{ mS}$ (milli-Siemens) and $R_L = 20 \text{ k}\Omega$. The applied sinusoidal voltage $V_{in}(j\omega)$ has a magnitude of 0.1 V at 10 MHz . The load is modeled by the parallel combination of R_L and the 40-pF capacitor. The capacitance accounts for such real-world phenomena as wiring capacitance, the device input capacitance, and other embedded capacitances. This capacitance cannot be removed from the circuit and often has deleterious effects on the amplifier performance.

1. With the load connected directly as shown (without L), find the magnitude of the output voltage.
2. If an inductance L is connected across the load to tune out the effect of the capacitance, find the value of L and the resulting $|V_{out}|$. What is the impact on the amplifier gain?

[8 + 12 marks]

- Q.7 (a) (i) Explain the addressing modes of 8086 with one example each.
- (ii) Obtain the physical address and effective address for different addressing modes of 8086 with the contents of register as given below:
- Offset = 1000 H; [AX] = 5000 H; [BX] = 2000 H; [SI] = 3000 H; [BP] = 5000 H;
[DI] = 4000 H; [SP] = 6000 H, [DS] = 7000 H
1. Register indirect addressing mode (assuming DI).
 2. Based addressing mode (assuming BX)
 3. Based index addressing mode (assuming DX).
 4. Based index with displacement addressing mode (assuming BX).

[14 + 6 marks]

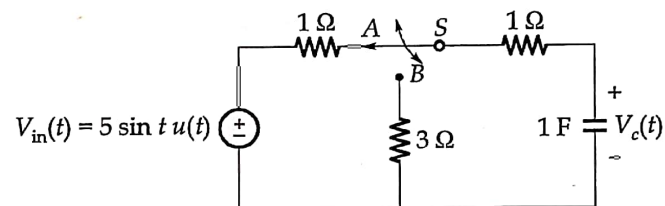
Q.7 (b) A system is described by the following state and output equations:

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t); \quad \frac{dx_2(t)}{dt} = -2x_2(t) + u(t); \quad y(t) = x_1(t)$$

If $u(t)$ is the input and $y(t)$ is the output, then find the system transfer function and state transition matrix of the above system.

[20 marks]

- Q.7 (c) In the circuit given below $V_{in}(t) = 5 \sin t u(t)$ V and $V_c(0^-) = 0$. The switch is initially in position A. The switch 'S' moves from position 'A' to position 'B' at $t = 1$ s and from position 'B' to position 'A' at $t = 2$ s, where it remains for all subsequent time. Find $V_c(t)$ for $t \geq 0$.



[20 marks]

Q.8 (a)

Determine the unilateral Laplace transform of the signals given below. Specify the property used, if any, in each step.

(i) $x(t) = [u(t-1) + u(-t-4)] * e^{-2t}u(t-1)$

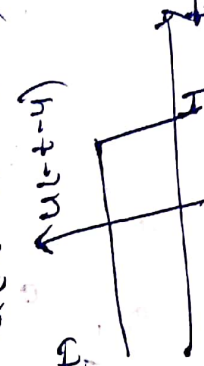
(ii) $x(t) = t \cdot \frac{d}{dt} \left[e^{-t} \cdot \cos t u(t) + e^{-(t+1)} u(-(t+1)) \right]$

[10 + 10 marks]

(i) $x(t) = [u(t-1) + u(-t-4)] * e^{-2t}u(t-1)$

Let $x_1(t) = u(t-1) + u(-t-4)$

$u(t-1-4) = u(t-5)$



unilateral laplace transform of $x_1(t)$

$$x_1(s) = \int_0^\infty x_1(t) e^{-st} dt$$

$$= \int_0^\infty u(t-1) dt + \int_0^\infty u(-t-4) dt$$

$$x_1(s) = \frac{e^{-s}}{s} + \frac{1}{s} [1 - e^{-4s}]$$

$$x_2(t) = e^{-2t} u(t-1) \quad ; \quad e^{-2t} u(t) \leftrightarrow \frac{1}{s+2}$$

$$e^{-2(t-1)} u(t-1) \leftrightarrow \frac{e^{-s}}{s+2}$$

$$X_2(s) = e^{-2} \frac{e^{-s}}{s+2}$$

$$x(t) = x_1(t) * x_2(t)$$

by convolution property

$$X(s) = X_1(s) \cdot X_2(s) \\ = \frac{1}{s} \left[e^{-s} + 1 - e^{-1s} \right] \times \frac{e^{-2} e^{-s}}{s+2}$$

$$X(s) = \frac{e^{-2} \left[e^{-s} + e^{-2s} + e^{-3s} \right]}{s(s+2)}$$

$$ii) x(t) = t \frac{d}{dt} \left[e^{-t} \cos t u(t) + e^{-(t+1)} u(-(t+1)) \right]$$

$$x_1(t) = e^{-(t+1)} u(-(t+1))$$

$$u(-(t+1))$$

$$X_1(s) = \int_0^1 e^{-(t+1)} e^{-st} dt$$

$$= e^{-1} \int_0^1 e^{-(s+1)t} dt = e^{-1} \left[\frac{1 - e^{-(s+1)}}{s+1} \right]$$

$$= e^{-1} = \frac{1}{s+1} \left[e^{-1} + e^{-2} + e^{-3} \right]$$

$$e^{-t} u(t) \cos t \Leftrightarrow \frac{s}{(s+1)^2 + 1}$$

$$\frac{d}{dt} \left[e^{-t} \cos t u(t) + e^{-(t+1)} u(-(t+1)) \right]$$

$$= s \left[\frac{s}{(s+1)^2 + 1} + \frac{e^{-1}}{s+1} + \frac{e^{-1-s} e^{-2}}{s+1} \right]$$

from differentiation in frequency domain
 Property $f(x(t)) \leftrightarrow -\frac{d}{ds} X(s)$

$$X(s) = \left[\frac{-\frac{d}{ds} \left[\frac{s^2}{(s+1)^2 + 1} \right] + \frac{s(e^{-1} \cdot e^{-s \cdot 2})}{s+1} \right]$$

Q.8(b) A series circuit consists of a 300Ω non-inductive resistor, a $7.95 \mu\text{F}$ capacitor and a 2.06 H inductor of negligible resistance.

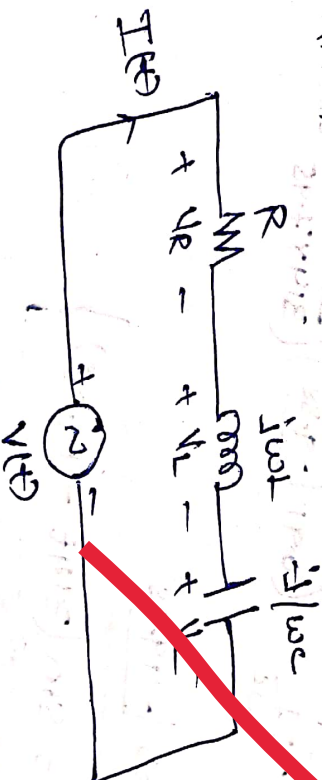
If the supply voltage is

$$v(t) = 250\sqrt{2} \sin(314t + 30^\circ) \text{ V, calculate}$$

- the circuit current,
- the voltage drop across each component in the circuit,
- the power consumed in the circuit.

(i)

Given circuit can be drawn as



$$\omega = 314 \text{ rad/sec}$$

$$Z = R + j(\omega L - \frac{1}{\omega C})$$

$$= 300 + j(314 \times 2.06 - \frac{10^6}{314 \times 7.95})$$

$$= 300 + j246.24 = 388.11 \angle 39.38^\circ$$

$$I(t) = \frac{V(t)}{Z} = \frac{V(t)}{388.11 \angle 39.38^\circ} = \frac{250\sqrt{2} \angle 36^\circ}{388.11 \angle 39.38^\circ}$$

$$= 0.91 \angle -9.38^\circ$$

$$\therefore I(t) = 0.91 \sin(314t - 9.38^\circ) \text{ amp}$$

(ii) $V_R = I(t) \cdot R = 0.91 \sin(314t - 9.38^\circ) \times 360$

$$= 273 \sin(314t - 9.38^\circ) \text{ Volts}$$

$$V_R = 273 \angle -9.38^\circ \text{ V}$$

$$V_L = I(t) \cdot j\omega L = (0.91 \angle -9.38^\circ) (314 \times 2.06 \angle 90^\circ)$$

$$= 285.74 \angle +80.62^\circ \text{ Volts}$$

$$V_L(t) = 285.74 \sin(314t + 80.62^\circ)$$

$$V_C = I(t) \cdot \frac{-j}{\omega C} = (0.91 \angle -9.38^\circ) \left(\frac{-j10^6}{314 \times 7.95} \angle -90^\circ \right)$$

$$= 364.53 \angle -99.38^\circ$$

$$V_C(t) = 364.53 \sin(314t - 99.38^\circ)$$

(iii)

Power consumed $S = V(t) I^*(t)$

$$= \frac{250\sqrt{2}}{\sqrt{2}} \angle 30^\circ \times \frac{0.91}{\sqrt{2}} \angle 9.38^\circ$$

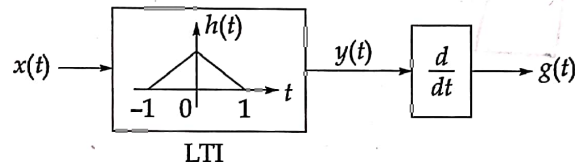
$$= 160.86 \angle 39.38^\circ \text{ VA}$$

$$\text{Total power } S = (124.33 + j102.05) \text{ VA}$$

$$\text{Active power } P = 124.33 \text{ Watts}$$

$$\text{Reactive power } Q = 102.05 \text{ VAR}$$

Q.8 (c) (i) Consider an LTI system has the impulse response $h(t)$ shown in figure below:



If the input $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$, then sketch output $g(t)$.

(ii) A voltage waveform $V(t)$ has a period $T = 2$ second, its Fourier series coefficient values are:

$$C_0 = 1, C_1 = 2j, C_2 = 2j$$

Obtain the value of $V(t)$ at $t = 0$.

[10 + 10 marks]

(i)

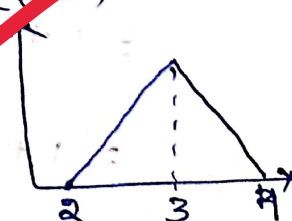
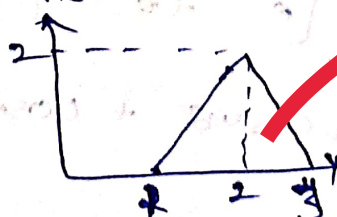
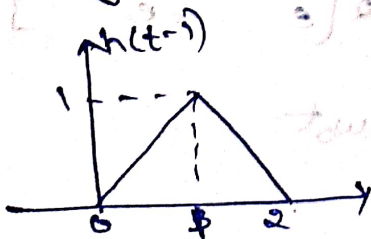
$$y(t) = h(t) * x(t)$$

$$= h(t) * (\delta(t-1) + \delta(t-2) + \delta(t-3))$$

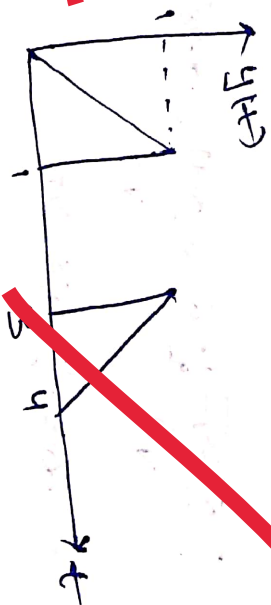
$$= h(t) * \delta(t-1) + h(t) * \delta(t-2) + h(t) * \delta(t-3)$$

$$\therefore x(t) * \delta(t-t_0) = x(t-t_0)$$

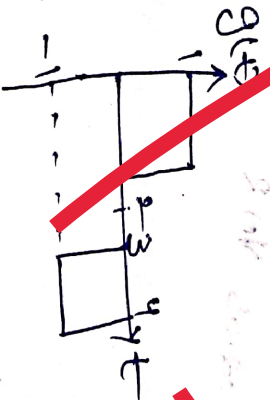
$$\therefore y(t) = h(t-1) + h(t-2) + h(t-3)$$



$y(t)$ can be drawn as



$$g(t) = \frac{d}{dt} y(t)$$



(ii)

Given $T = 2\text{sec}$

$$C_0 = 1, C_1 = 2j, C_2 = 2$$

for odd signals C_n will be zero, but C_0

Assuming $V(t)$ is even signal

$$\text{i.e. } C_k = C_{-k}$$

$$C_{-2} = 2, C_{-1} = 2j$$

$V(t)$ can be written as

$$\begin{aligned} V(t) &= 1 + 2je^{-j\omega t} + 2je^{j\omega t} + 2e^{-j2\omega t} + 2e^{j2\omega t} \\ &= 1 + 2j[e^{-j\omega t} + e^{j\omega t}] + 2[e^{-j2\omega t} + e^{j2\omega t}] \\ &= 1 + 2j \cos \omega t + 2 \cos 2\omega t \end{aligned}$$

$$V(t) = 1 + j \cos \omega t + \cos 2\omega t$$

$$\text{at } t = 0$$

$$V(0) = 1 + j \cos(0) + \cos(0)$$

$$V(0) = (2 + j)$$

0000