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ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-3 : Power Systems + Systems and Signal Processing-1 + Microprocessors-1 + Electrical Circuits-2 + Control Systems-2

Name : ....

Roll No :

Test Centres | **Student's Signature**

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	36
Q.2	
Q.3	
Q.4	49
Section-B	
Q.5	55
Q.6	
Q.7	51
Q.8	43
Total Marks Obtained	234

Signature of Evaluator: Saurabh kumar ....

Cross Checked by ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Power Systems

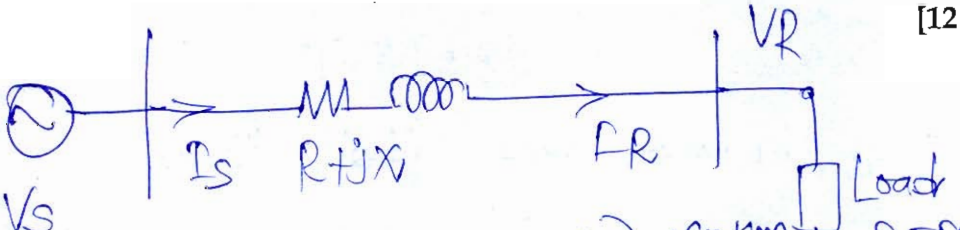
Q.1 (a) A 66 kV, 60 km long, transmission line delivers a load of 25 MW at 0.8 lagging power factor. If the line have series resistance and inductance of 0.08 Ω/km and 1.25 mH/km respectively, compute

(i) Sending end voltage and current

(ii) Voltage regulation

(iii) Transmission efficiency. Assume a power frequency of 50 Hz.

[12 marks]



$$R+jX = (0.08 + j\,100\pi \times 0.00125)\underset{\Omega/km}{} \times 60\,km$$

$$= 24.046\angle 78.48^\circ\ \Omega$$

$$|I_R| = \frac{P_{R3\phi}}{\sqrt{3}\,V_{RL}\cos\phi} = \frac{25\ MW}{\sqrt{3}\times 66\,kV \times 0.8} = 0.273\ kA$$

$$I_R = 0.273\angle -\cos^{-1}0.8\ kA = 0.273\angle -36.87^\circ\ kA$$

(i) Sending end Current, $I_S = I_R$

$$= 0.273\angle -36.87^\circ\ kA$$

(ii) Sending end phase voltage.

$$V_S = V_R + Z I_R$$

$$= \frac{66}{\sqrt{3}}\angle 0^\circ + (24.046\angle 78.48\ \Omega)(0.273\angle -36.87^\circ\ kA)$$

$$= 43.23\angle 5.78^\circ$$

Sending end line voltage,

$$V_{SL} = \sqrt{3}\times 43.23 = 74.88\ kV$$

(iii) For the above line,

$$A = 1$$

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voltage Regulation = $\dfrac{\left|\frac{V_{SL}}{A}\right| - |V_{RL}|}{|V_{RL}|} \times 100$

$= \dfrac{\frac{79.88}{1} - 66}{66} \times 100$

$= 13.45\%$

(iii) Transmission loss = $|I_s|^2 R_{Line}$

$= (0.273\ KA)^2 \times (24.046 \cos 78.8^\circ)$

$= 0.356\ MW$

η, efficiency $= \dfrac{P_{out}}{P_{out} + Loss} \times 100$

$= \dfrac{25\ MW}{25\ MW + 0.356} \times 100$

$= 98.59\%$

10

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Q.1 (b) A hydroelectric station is to be designed for catchment area of 150 km^2, rainfall for which is 120 cm/annum. The head availability is 30 m. 72% of total rainfall is available, rest is lost to evaporation. Penstock efficiency is 95%. Turbine efficiency is 85% and generator efficiency is 90% and load factor is 40%. Determine the capacity of the station.

[12 marks]

Overall efficiency

$= 0.72 \times 0.95 \times 0.85 \times 0.90 \times$

1

Incomplete Solution

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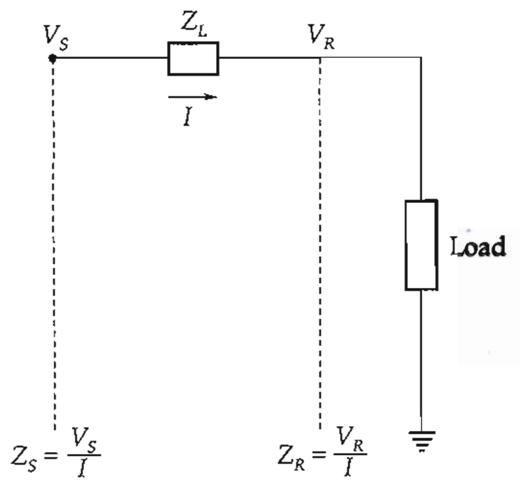
Q.1 (c) Consider the transmission line as shown in figure, with series impedance Z_L, negligible shunt admittance and a load impedance Z_R at the receiving end.



(i) Calculate Z_R for the given condition of $V_R = 1.0$ pu and $S_R = 2 + j0.8$ pu.
(ii) Construct the impedance diagram in R-X plane for $Z_L = (1 + j0.3)$ pu.
(iii) Find Z_S for this condition and angle between Z_S and Z_R.

[12 marks]

(i) $S_R = V_R I^*$

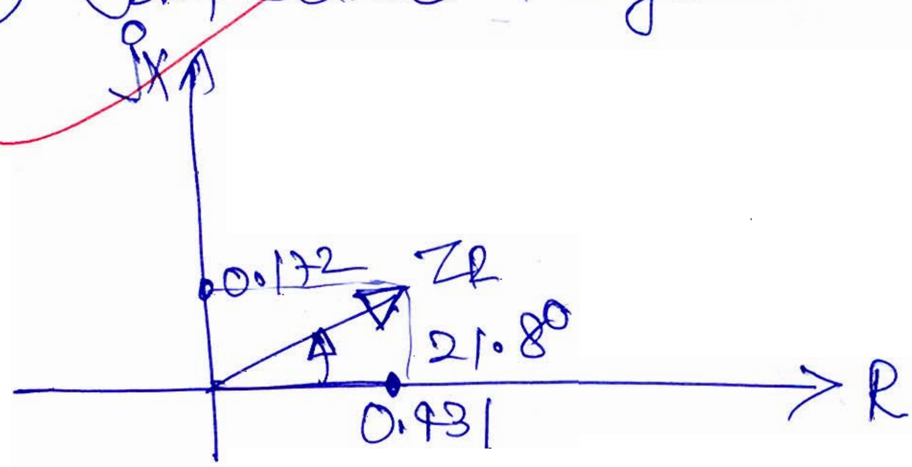
$\Rightarrow I = \frac{S_R^*}{V_R^*} = \frac{2 - j0.8}{1} = 2.154\angle -21.8^\circ$

$Z_R = \frac{V_R}{I} = \frac{1}{2.154\angle -21.8^\circ}$

$= 0.4642\angle 21.8^\circ$ pu

$= 0.431 + j0.172$

(ii) Impedance Diagram



(iii)

$$V_S = V_{Rf} + I Z_L$$

$$= 1 + (2.157\angle -20.8^\circ)(1 + j0.3)$$

$$= 3.297\angle -2.84^\circ \text{ PU}$$

(iii)

$$Z_S = \frac{V_S}{I_S}$$

$$= \frac{3.297\angle -2.84^\circ}{2.157\angle -20.8}$$

$$= 1.5079\angle 17.96^\circ$$

Angle B/w Z_S and Z_L

$= 17.96^\circ - 21.8^\circ$

$= -3.84^\circ$

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Good Approach

Q.1 (d) A 3-ϕ, 765 kV, 50 Hz, 300 km, completely transposed line has the following positive sequence impedance and admittance :

$$z = 0.0165 + j0.3306 = 0.3310\angle 87.14^\circ \ \Omega/\text{km}$$
$$y = 4.674 \times 10^{-6} \text{ S/km}$$

Assuming positive sequence operation, calculate exact $ABCD$ parameters of long line equation. Compare the exact B parameter with nominal π-circuit.

[12 marks]

Characteristic Impedance

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.3110\angle 87.14}{j4.674\times10^{-6}}} = 257.95\angle -1.43^\circ \ \Omega$$

Propagation Constant

$$\gamma = \sqrt{yz} = 1.2438\times10^{-3}\angle 88.57^\circ \ /\text{Km}$$

$$\gamma l = 1.2438\times10^{-3}\times 300\angle 88.57 = 0.37314\angle 88.57^\circ$$
$$= 9.31\times10^{-3} + j\,0.373$$

$$\cosh\gamma l = \cosh(\alpha l)\cos\beta l + j\sinh\alpha l\sin\beta l$$
$$= \cosh(9.31\times10^{-3})\cos(0.373) + j\sinh(9.31\times10^{-3})\sin(0.373)$$

$$\sinh\gamma l = \sinh(\alpha l)\cos\beta l + j\cosh\alpha l\sin\beta l$$

we get

$$\cosh\gamma l = 0.931 + j\,3.39\times10^{-3}$$
$$\sinh\gamma l = 8.66\times10^{-3} + j\,0.3644$$

$$A = \cosh\gamma l = 0.931\angle 0.208^\circ = D$$

$$B = Z_C \sinh\gamma l$$
$$= (257.95\angle -1.43)(8.66\times10^{-3} + j\,0.3644)$$
$$= 94\angle 87.2^\circ \ \Omega$$

97

$C = \frac{\sinh \gamma l}{Z_C} = 1.4 \times 3 \times 10^{-3} \angle 90.06^{\circ}$ ℧

1.37

Nominal π Circuit

$B = Z = z \angle$

$= (0.3316 \angle 87.14^{\circ}) \times 300$

$= 99.3 \angle 87.14^{\circ}\ \Omega$

As B from exact model

$\simeq 94 \angle 87.2^{\circ}\ \Omega$

from π model,

$B_{\pi} = 99.3 \angle 87.14^{\circ}\ \Omega$

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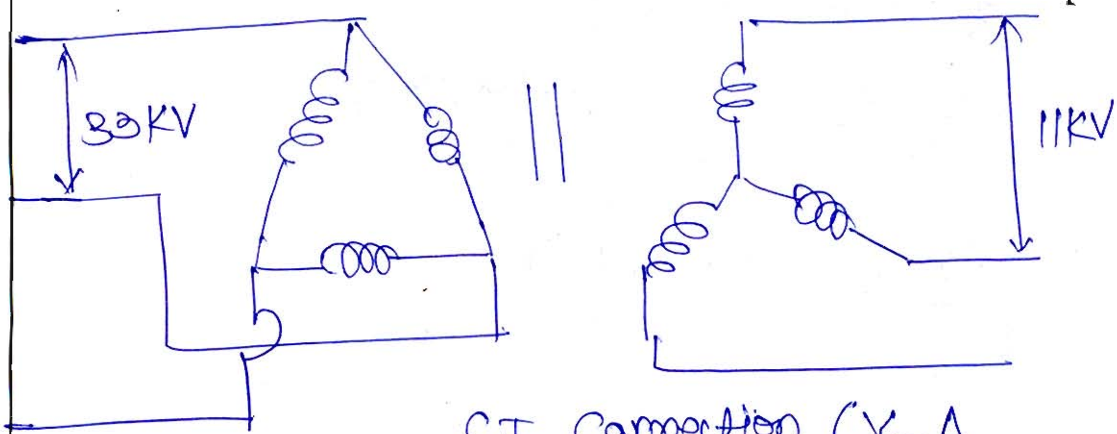
Q.1 (e) Consider a 3-phase, Δ-Y connected, 30 MVA, 33 : 11 kV transformer with differential relay protection. If the CT ratios are 500 : 5 on primary side and 2000 : 5 on secondary side, compute the relay current setting for faults drawing upto 200% of rated transformer current.

30 MVA

[12 marks]



CT Connection (Y–Δ

Primary Side

$$I_{ph,\Delta_{Trans}} = \frac{1}{\sqrt{3}}\left[\frac{30\,MVA}{\sqrt{3}\times 33\,KV}\right] = 303\,A$$

$$I_{ph,Y_{CT}} = 303\,A \times \frac{5}{500} = 3.03\,A$$

$$I_{Line,Y_{CT}} = 3.03\,A$$

For Secondary Side [on 100% loading]

$$I_{Line,Y_{Tr}} = I_{ph,Y_{Tr}} = \frac{30\,MVA}{\sqrt{3}\times 11\,KV} = 1574.6\,A$$

$$I_{ph,\Delta CT} = 1574.6 \times \frac{5}{2000} = 3.9364\,A$$

$$I_{Line,\Delta CT} = \sqrt{3}\times 3.9364 = 6.818\,A$$

we have

$\dot{i}_1 = 3.03\ A$

$\dot{i}_2 = 6.818\ A$

$\dot{i}_R = i_2 - \dot{i}_1 = 3.7888\ A$

$\frac{i_1 + \dot{i}_2}{2} = 4.924\ A$

As due to 200% overloading i_R as well as $\frac{i_1+\dot{i}_2}{2}$ proportionally changes.

So for not tripping upto 200%

$\dot{i}_R \leq K\left(\frac{i_1+\dot{i}_2}{2}\right)$

$\Rightarrow 3.7888 \leq K(4.924)$

$\boxed{K \geq 76.94\%}$

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Required Relay Setting, $K = 76.94\%$

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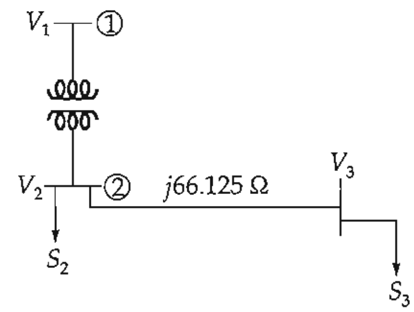
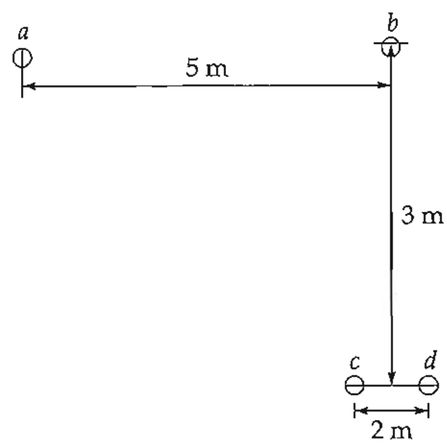
Q.2 (a) **(i)** The single line diagram of 3-phase power system is shown in figure. The transformer reactance is 20% on the base of 100 MVA, 23/115 kV and line impedance of $Z = j66.125\ \Omega$. The load at bus–2 is S_2 = 184.8 MW + j6.6 MVAR and at bus-3 is S_3 = 0 MW + j20 MVAR. It is required to hold the voltage at bus-3 at $115\angle 0°$ kV. Determine the voltages at bus-1 and bus-2.



(ii) A 50 Hz, 1-ϕ power line and telephone line are parallel to each other as shown in figure. The telephone line is symmetrically positioned directly below phase b. The power line carries a current of 226 A. Assume zero current flows in ungrounded telephone wires. Find the magnitude of voltage per km induced in the telephone line.



[10 + 10 marks]

Q.2 (b) **(i)** A 400 MVA synchronous machine has $H_1 = 4.6$ MJ/MVA and 1200 MVA machine has $H_2 = 3.0$ MJ/MVA. The two machines operate in parallel in a power plant. Find out H_{eq}, relative to a 100 MVA base.

(ii) The per unit bus impedance matrix for a power system is given by

$$Z_{\text{bus}} = j\begin{bmatrix} 0.0450 & 0.0075 & 0.030 \\ 0.0075 & 0.06375 & 0.030 \\ 0.030 & 0.030 & 0.21 \end{bmatrix}$$

A 3-ϕ fault occurs at bus-3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix, calculate the fault current, bus voltages and line currents during fault. Assume the pre-fault voltages at each bus is 1.0 pu.

[10 + 10 marks]

Q.2 (c) A single area consists of two generating units, rated at 400 MVA and 800 MVA with speed regulation of 4% and 5% on their respective ratings. The units are operating in parallel, sharing 700 MW. Unit-1 supplies 200 MW and unit-2 supplies 500 MW at 1.0 pu (60 Hz). The load increased by 130 MW.

(i) Assume there is no frequency-dependent load, i.e., $D = 0$. Find the steady-state frequency deviation and the new generation on each unit.

(ii) The load varies 0.804% for every 1% change in frequency, i.e., $D = 0.804$. Find the steady-state frequency deviation and the new generation on each unit.

[20 marks]

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Q.3 (a) A 3-ϕ overhead line has resistance and reactance per phase 5 Ω and 25 Ω respectively. The load of receiving end 15 MW, 33 kV, 0.8 pF lagging. Find the compensation equipment needed to deliver this load with sending end voltage of 33 kV.

Calculate the extra load of 0.8 lagging power factor delivered with the compensating equipment (of capacity as calculated above) installed, if the receiving end voltage is permitted to drop to 28 kV.

[20 marks]

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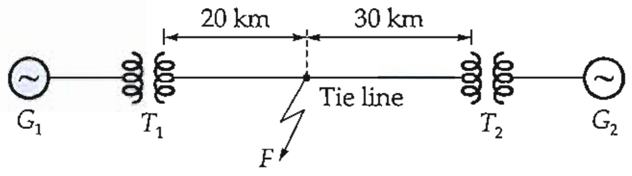
Do not write in this margin

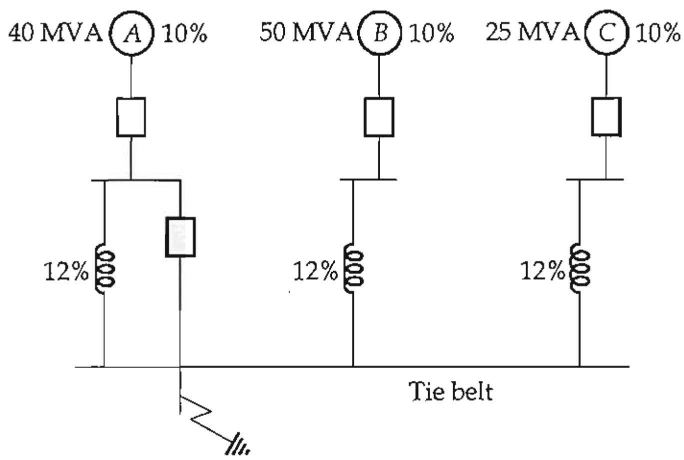
Q.3 (b) Generator G_1 and G_2 are identical and rated 11 kV, 20 MVA and have a transient reactance of 0.25 p.u at own MVA base. The transformers T_1 and T_2 are also identical and are rated 11/66 kV, 5 MVA and have a reactance of 0.06 p.u. to their own MVA base. The tie line is 50 km long, each conductor has a reactance of 0.848 Ω/km. The three phase fault is assumed at F, which is 20 km away from transformer T_1 as shown below. Find the short circuit current.



[20 marks]

Q.3 (c) **(i)** A single-core, lead sheathed cable joints has a conductor of 10 mm diameter and two layers of different insulating materials, each 10 mm thick. The relative permittivities are 3 (inner) and 2.5 (outer). Calculate the potential gradient at the surface of conductor when the potential difference between the conductor and the lead sheath is 60 kV.

(ii) Three 6.6 kV generators A, B and C, each of 10%, leakage reactance and MVA rating 40, 50 and 25 respectively are interconnected electrically as shown in figure, by a tie bar current limiting reactor, each of 12% reactance based upon the rating of machine to which it is connected. A 3-ϕ feed is supplied from the bus-bar of generator A at a line voltage of 6.6 kV. The feeder has resistance of 0.06 Ω/ph and an inductive reactance of 0.12 Ω/ph. Estimate the maximum MVA there can be fed into symmetrical short circuit at the far end of the feeder.



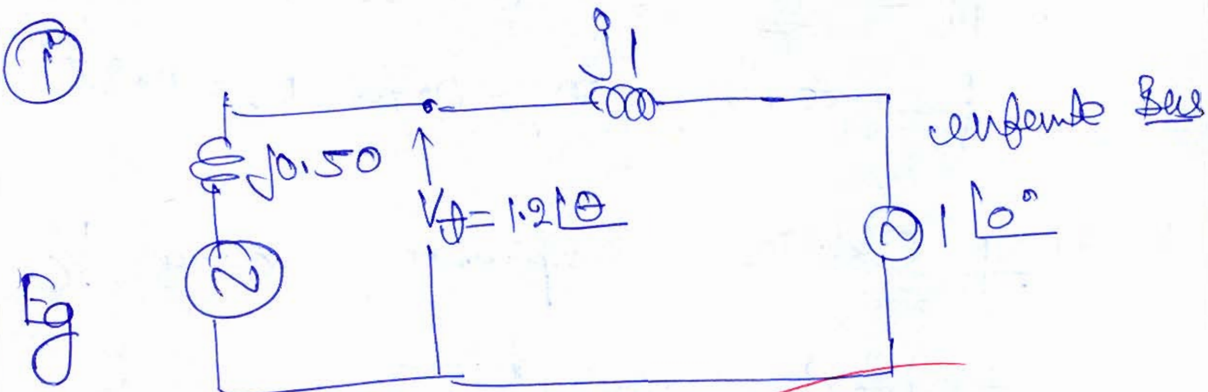
[8 + 12 marks]

Q.4 (a) **(i)** Find the steady state power limit of a system consisting of a generator with equivalent reactance 0.50 pu connected to an infinite bus through a series reactance of 1 pu. The terminal voltage of generator held at 1.20 pu and voltage of infinite bus is 1.0 pu.

(ii) Determine the corona characteristics of a 3-phase line 160 km long. Conductor diameter 1.036 cm, 2.44 m delta spacing, air temperature 26.67°C, altitude 2440 m, corresponding to an approximate barometric pressure of 73.15 cm, operating at 110 kV at 50 Hz. Surface irregulating factor is 0.85 and $m_V = 0.72$.

[10 + 10 marks]

(i)



For steady state power $E_g\angle\delta = E_g\angle 90°$

Thus we have

$$\frac{E_g\angle 90 - 1.2\angle\theta}{j0.50} = \frac{1.2\angle\theta - 1\angle 0°}{j1}$$

$$\Rightarrow 2(jE_g - 1.2\cos\theta - j1.2\sin\theta) = (1.2\cos\theta - 1) + j1.2\sin\theta$$

equal Real and Imaginary part

$-2.4\cos\theta = 1.2\cos\theta - 1$ we get $\theta = 73.87°$

$2E_g - 2\times 1.2\sin\theta = 1.2\sin\theta$

$\Rightarrow E_g = \frac{3.6\sin\theta}{2} = 1.729$ Pu

10

Good Approach

Thus steady state Power limit $= \frac{E_g \times 1}{0.50 + 1}$

$= \frac{1.729\times 1}{1.5} = 1.1527$ Pu

(ii) Given, $L = 160\ km$

$r = 0.518\ cm$

$d = 2.44m = 244\ cm$

$T = 26.67°C$, $P = 73.15\ cm$

$V_{ph} = \frac{110}{\sqrt{3}}\ kV = 63.508\ kV$

$f = 50\ Hz$, $m_v = 0.72$, $m_0 = 0.85$

Finding Critical disruptive voltage, V_C

$$V_c = m_0 g \delta r \ln\left(\frac{d}{r}\right)$$

where $\delta = \frac{3.92 \times 73.15}{273 + 26.67} = 0.9568$

$$V_c = 0.85 \times 21.2\ \frac{kV}{cm} \times 0.518\ cm \times 0.9568 \times \ln\left(\frac{244}{0.518}\right)$$

$= 54.97\ kV$ (phase)

Finding visual disruptive voltage

$$V_v = m_v g \delta r \left(1 + \frac{0.3}{\sqrt{r\delta}}\right) \ln\left(\frac{d}{r}\right)$$

$$= 0.72 \times 21.2 \times 0.518 \times 0.9568 \times \left(1 + \frac{0.3}{\sqrt{0.518 \times 0.9568}}\right) \times \ln\left(\frac{244}{0.518}\right)$$

$= 66.40\ kV.$

Corone loss

As $V_c > V_{ph}$ thus finding Corona loss in kW/km/ph

$$P_c = 242.2\times10^{-5}\times\left(\frac{f+25}{\delta}\right)\times\sqrt{\frac{r}{d}}\times\left(V_{ph}-V_c\right)^2$$

$$=242.2\times10^{-5}\times\left(\frac{50+25}{0.9588}\right)\times\sqrt{\frac{0.578}{244}}\left(63.508-59.97\right)^2$$

$$=813.49\times10^{-5}\ \text{kW}/\cancel{\text{km}}/\text{ph}$$

Total Corona loss for 160 km long line

$$=813.49\times10^{-5}\times160\times\underset{\hookrightarrow 3\phi}{3}$$

$$=\underline{3.905\ \text{kW}}$$

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Q.4 (b) A 50-Hz, 100 MVA, 4-pole, synchronous generator has inertia constant of 3.5 sec and supply 0.16 pu power on a system base of 500 MVA. The input to the generator is increased to 0.18 pu. Determine :

(i) Kinetic energy stored in the rotor.

(ii) Acceleration of the generator.

(iii) If acceleration continues for 7.5 cycles, calculate the change in rotor angle.

(iv) Speed in rpm at the end of the acceleration.

[20 marks]

(i) For $H = 3.5 \frac{MJ}{MVA}$

Kinetic Energy Stored in the Rotor

$= HS$ [S = MVA Base]

$= 3.5 \times 100 = 350$ MJoule

(ii) Using Swing Equation,

$$\frac{2H}{\omega s} \frac{d^2\delta}{dt^2} = (P_m - P_e)_{pu}$$

$$\Rightarrow \frac{2 \times 3.5}{100\pi} \frac{d^2\delta}{dt^2} = (P_m - P_e)$$

$$\frac{d^2\delta}{dt^2} = 44.88 (P_m - P_e)_{pu} \quad \frac{\text{elec rad}}{\text{sec}^2}$$

After input is increased to 0.18 pu

$P_m - P_e = 0.18 - 0.16 = 0.02$ pu

Thus acceleration in $\frac{\text{elec rad}}{\text{sec}^2} = 44.88 \times 0.02$

$= 0.8976 \frac{\text{elec rad}}{\text{sec}^2}$

(iii) 7.5 cycle $\Rightarrow \frac{7.5}{50}$ sec $= 0.15$ sec

As $\frac{d^2\delta}{dt^2} = 0.8976$

integrating we get

$\omega - \omega_s = \frac{d\delta}{dt} = 0.8976\,t$ —— (1)

Further integrating

$\delta = 0.4488\,t^2 + \delta(0)$ —— (2)

Change in rotor angle

$= \delta(0.15\text{ sec}) - \delta(0)$

$= 0.4488 \times (0.15)^2$

$= 0.01$ elec rad

$= 0.578^\circ$ elec deg

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(iv) from equation (1)

$\omega - \omega_s$ at $t = 0.15$ sec is

$\omega - \omega_s = 0.8976 \times 0.15$

$= 0.13464 \frac{\text{elec rad}}{\text{sec}}$

As $\omega_s = 100\pi \frac{\text{elec rad}}{\text{sec}}$

Thus $\omega = 100\pi + 0.13464 = 314.2939 \frac{\text{elec rad}}{\text{sec}}$

Speed of rotor in rpm

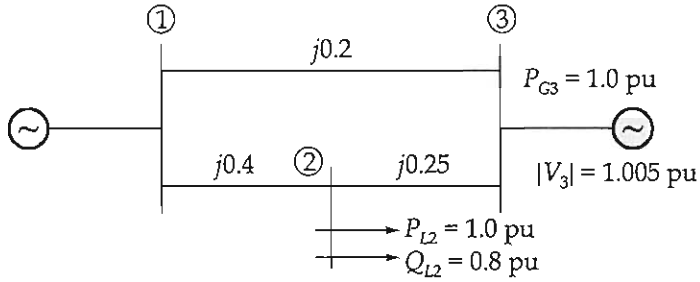
$$= \left(\omega \frac{2}{P}\right)\left(\frac{60}{2\pi}\right) \text{ rpm}$$

$$= \left(314.2939 \times \frac{2}{4}\right) \times \frac{60}{2\pi} \text{ rpm}$$

$$= \cancel{1500.6428 \text{ rpm}}$$

$N' = 1503.21$ rpm

Q.4 (c) For the power system network shown in figure, compute the bus voltages using the Gauss-Seidel iteration method. Line reactances and loads are shown in figure. Bus-1 is the slack bus ($V_1 = 1.04\angle 0°$) and bus-2 and bus-3 are the load and voltage-control buses respectively. Assume tolerance equal to 1×10^{-5}.



Compute V_1, V_2 and V_3 upto one iteration.

[20 marks]

As we have,

$$y_{12} = \frac{1}{j0.4} = -j2.5$$

$$y_{13} = \frac{1}{j0.2} = -j5$$

$$y_{23} = \frac{1}{j0.25} = -j4$$

Forming Y_{Bus} of above system

$$Y_{Bus} = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j6.5 & j4 \\ j5 & j4 & -j9 \end{bmatrix}$$

From the given information

$V_1 = 1.04\angle 0°$ slack Bus

For Load Bus 2

$$P_2 + jQ_2 = (0+j0) - (1+j0.8) = -1 - j0.8 \text{ pu}$$

For Voltage Controlled Bus ③

$|V_3| = 1.005$ $\quad P_3 = 1$ pu

Using Gauss Sidel iteration technique,

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{(0)*}} - Y_{21}V_1 - Y_{23}V_3^{(0)}\right]$$

Assuming initial guess as

$V_2^{(0)} = 1\angle 0^\circ$

$V_3^{(0)} = 1.005\angle 0^\circ \longrightarrow$ guess value

$$V_2^{(1)} = \frac{1}{-j6.5}\left[\frac{-1+j0.8}{1\angle 0^\circ} - (j2.5)(1\angle 0^\circ) - (j4)(1.005\angle 0^\circ)\right]$$

$= 0.8933\angle -9.91^\circ$ pu

Computing $Q_3^{(1)}$

$$P_3^0 - jQ_3 = V_3^{(0)*}\left(Y_{31}V_1 + Y_{32}V_2^{(1)} + Y_{33}V_3^{(0)}\right)$$

$$= (1.005\angle 0)\left(j5 \times 1\angle 0^\circ + j4 \times (0.8933\angle -9.91^\circ) - j9(1.005\angle 0^\circ)\right)$$

$= 0.6180 - j0.5277$

Thus $Q_3^{(1)} = 0.5277$ pu

we Compute $V_3^{(1)}$

$$V_3^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{31}V_1 - Y_{32}V_2^{(1)}\right]$$

$$= \frac{1}{(-j9)}\left[\frac{1 - j0.5277}{1.005\angle 0^\circ} - j5 \times 1\angle 0^\circ - j4 \times 0.8933\angle -9.91\right]$$

$$= 1.00588\angle 2.406^\circ \text{ pu}$$

modifieby $V_3^{(1)}$ to have magnitude 1.005 pu

$$V_3^{(1)}_{\text{modified}} = 1.005\angle 2.406^\circ \text{ pu}$$

Thus after one iteration

$$V_1 = 1\angle 0^\circ$$

$$V_2^{(1)} = 0.8933\angle -9.91^\circ \text{ pu}$$

$$V_3^{(1)} = 1.005\angle 2.406^\circ \text{ pu}$$

18

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Section B : Systems and Signal Processing-1 + Microprocessor-1 + Electrical Circuits-2 + Control Systems-2

Q.5 (a) Calculate the delay in the following loop, assuming the system clock frequency is 3 MHz.

```
         LXI B, 12FFH
DELAY :  DCX B
         XTHL
         XTHL
         NOP
         NOP
         MOV A, C
         ORA B
         JNZ DELAY
```

[12 marks]

As here,

LXI B, 12FFH → 10 T

DELAY: DCX B → 6T

XTHL → 16T

XTHL → 16 T

NOP → 4T

NOP → 4T

mov A, C → 4T

ORA B → 4T

JNZ DELAY → 10T | 7T

Content of B = 12FFH

(N) = $(4863)_{10}$

$t_{outside\ loop} = 10T$ states

$t_{another\ loop} = (n-1)[6+16+16+4+4+4+4+10] + [6+16+16+4+4+4+4+7]$

$= [64(n-1)+67]$ T states

$t_{delay} = (64(n-1)+61+10)T$

for $T = \frac{1}{3}\ \mu sec$

and $n = 4868$

$t_{delay} = (64 \times 4862 + 71) \times \frac{1}{3}\ \mu sec$

$= 103746\ \mu sec$

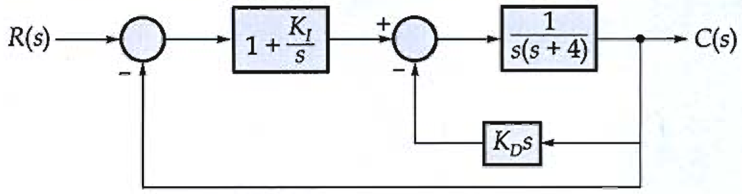
$=$

$0.1037\ sec$

11

Try to avoid over writing

Q.5 (b) Determine the ranges of controller gains (K_D, K_I) so that the system shown in figure below remains stable. Also determine the type of the system. Plot the region of stability.



[12 marks]

open loop transfer function,

$$G(s) = \left(1 + \frac{K_I}{s}\right)\left(\frac{\frac{1}{s(s+4)}}{1 + \frac{1}{s(s+4)} \cdot K_D s}\right)$$

$$= \left(\frac{s+K_I}{s}\right)\left(\frac{1}{s^2+(4+K_D)s}\right)$$

$$= \frac{(s+K_I)}{s^2(s+4+K_D)}$$

Type of Above System = 2

As OLTF has 2 poles at origin.

For stability,

$$q(s) = 1 + G(s) = 0$$

$$\Rightarrow s^3 + (4+K_D)s^2 + s + K_I$$

Forming Routh Array :-

Necessary Condition:

$$4 + K_D > 0 \Rightarrow K_D > -4$$

$$K_I > 0$$

s^3	1	1
s^2	$4+K_D$	K_I
s^1	$\dfrac{4+K_D-K_I}{4+K_D}$	0
s^0	K_I	0

11

Good Approach

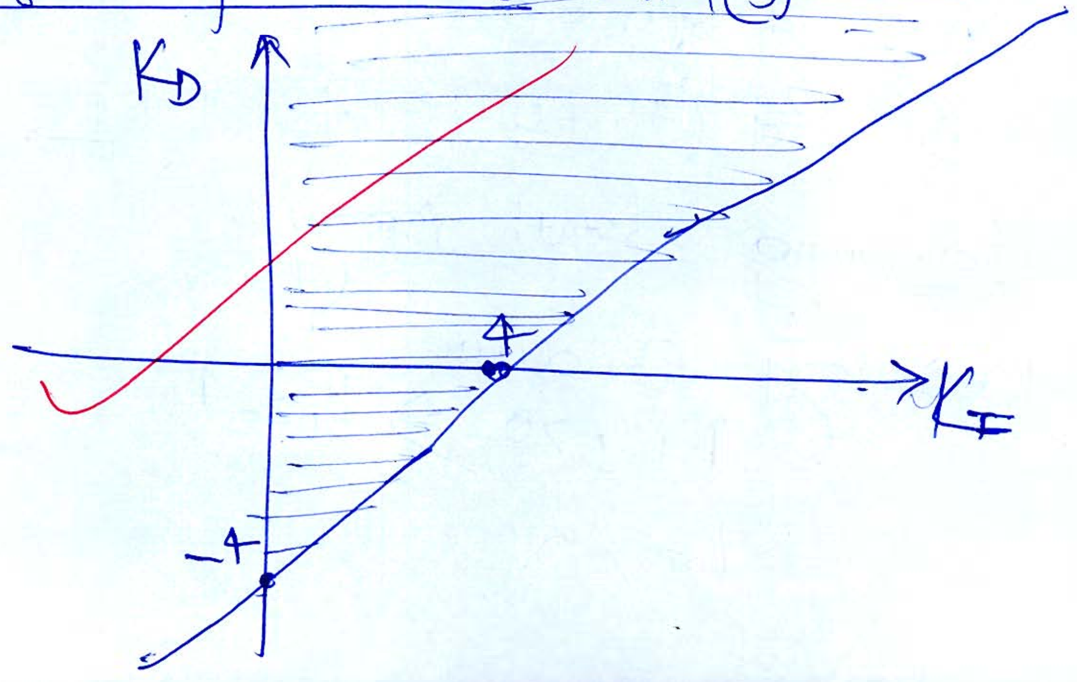
For stability there should be no sign change in first column.

i.e $4+K_D>0$ $K_D>-4$ —— (1)

$4+K_D-K_I>0$ —— (2)

$K_I>0$ —— (3)

Region of Stability : (satisfying (1), (2) and (3))



Q.5 (c) The reduced incidence matrix of an oriented graph is given as :

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

(i) Draw its graph.

(ii) Determine the number of trees are possible for this graph.

[12 marks]

(i)

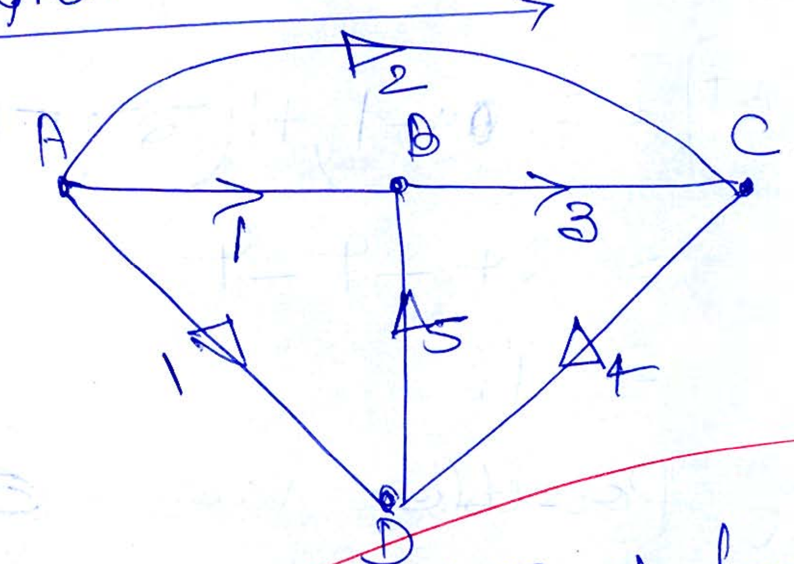
$$A_r = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Number of nodes = 4

no of Branches = no of columns = 6

We have assuming D as reference node.

~~Tree~~ Graph of above ~~(tree)~~ matrix



:- Oriented graph for A_r

(ii) No of Possible trees $= \det[A_r A_r^T]$

$$A_r A_r^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}_{3\times 3}$$

$$\det[A_r A_r^T] = 3(9-1) + 1(-3-1) - 1(1+3)$$

$$= 24 - 4 - 4$$

$$= 16$$

No of Possible trees = ~~8~~ 16

11

Good Approach

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Q.5 (d) A continuous-time linear system S with input $x(t)$ and output $y(t)$ yields the following input-output pairs.

$$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$$

$$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$$

(i) If $x_1(t) = \cos(2t)$, determine the corresponding output $y_1(t)$ for system S.

(ii) If $x_2(t) = \cos(2t - 1)$, determine the corresponding output $y_2(t)$ for system S.

[12 marks]

(i) we have

$$x_1(t) = \cos 2t$$

$$= \frac{e^{j2t} + e^{-j2t}}{2}$$

As for $e^{j2t} \rightarrow \boxed{sys} \rightarrow e^{j3t}$

and $e^{-j2t} \rightarrow \boxed{sys} \rightarrow e^{-j3t}$

$$y_1(t) = \frac{e^{j3t} + e^{-j3t}}{2}$$

$$= \cos 3t$$

(ii) we have,

$$x_2(t) = \cos(2t-1)$$

$$= \frac{e^{j(2t-1)} + e^{-j(2t-1)}}{2}$$

$$x_2(t) = \frac{e^{-j} \cdot e^{j2t} + e^{j} e^{-j2t}}{2}$$

Further ~~pro~~ with Same logic

$$y_2(t) = \frac{e^{-j} e^{j3t} + e^{j} e^{-j3t}}{2}$$

$$= \frac{e^{j(3t-1)} + e^{-j(3t-1)}}{2}$$

$$= \cos(3t-1)$$

Thus output is

Case (I) $y_1(t) = \cos 3t$

Case (II) $y_2(t) = \cos(3t-1)$

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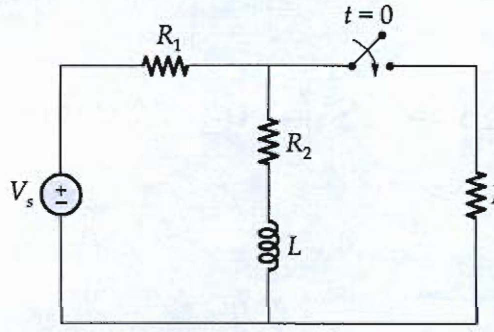
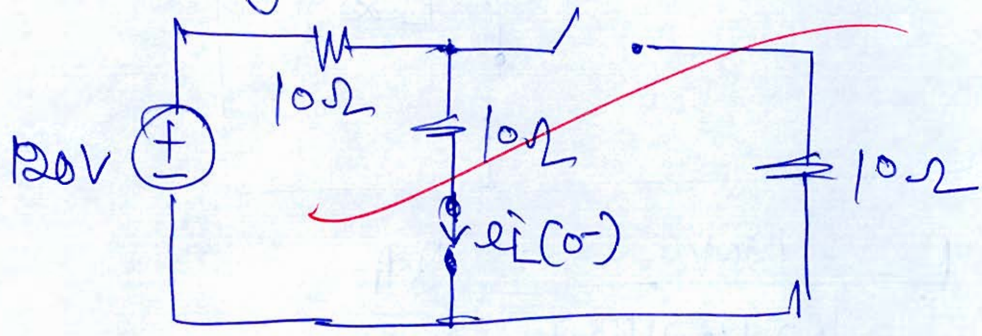
Good Approach

Q.5 (e) The switch in the circuit given below closes at $t = 0$, after being open for a long time. Find the inductor current $i_L(t)$, if $R_1 = R_2 = R_3 = 10\ \Omega$, $L = 0.01$ H and $V_s = 120$ V.



[12 marks]

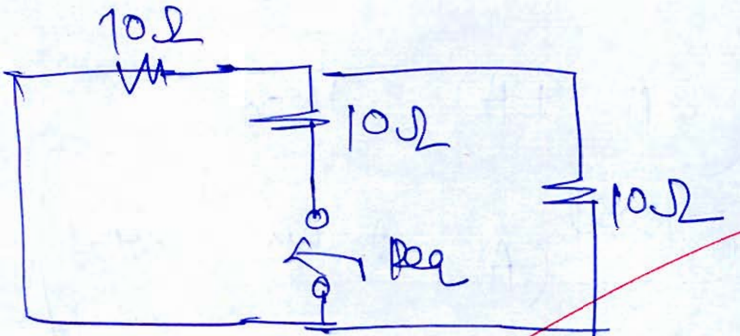
Before closing switch at $t=0^-$ in steady state ($L \to SC$)



$$i_L(0^-) = i_L(0^+) = \frac{120}{20} = 6A$$

for $t > 0$ (with sw closed)

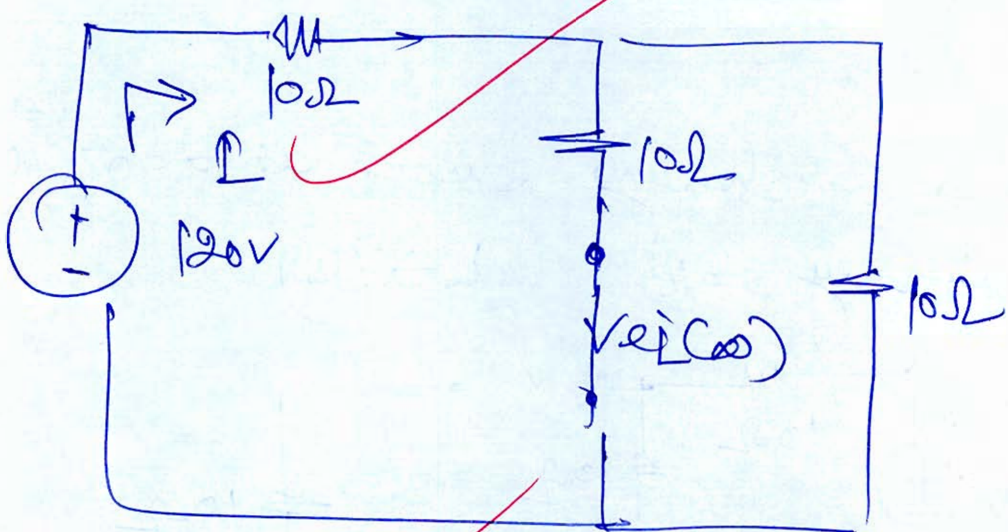
finding Req across 0.01H



$$R_{eq} = (10\Omega \parallel 10\Omega) + 10\Omega = 15\Omega$$

Time Constant $\tau_1 = \frac{L}{R_{eq}} = \frac{0.01}{15} = \frac{1}{1500}$ sec

in the steady state earth 'S' closed



$$I = \frac{120V}{10\Omega + 10\Omega \| 10\Omega} = 8A$$

$$i_L(\infty) = \frac{I}{2} = 4A$$

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$i_L(t)$ expression for $t>0$

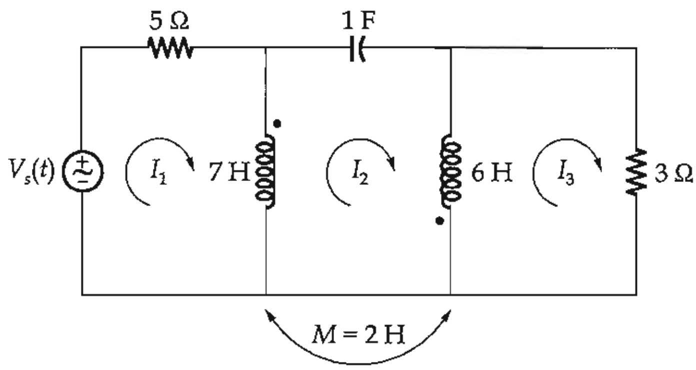
Good Approach

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-\frac{t}{\tau}}$$

$$= 4A + (6A - 4A)e^{\frac{-t}{1/1500}}$$

$$= 4 + 2e^{-1500t} \text{ (A)} \quad [t \text{ in second}]$$

Q.6 (a) For the magnetically coupled circuit shown in figure, find the loop current I_1, I_2 and I_3, if $V_s(t) = 2\cos(2t)$.



[20 marks]

Q.6 (b) Write a program to arrange first 10 numbers from memory address 2040H in ascending order. Write the comment of each instruction.

[20 marks]

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Q.6 (c) A system is represented by the state model,

$$\dot{X} = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \text{ and } y = [1 \quad -1]X$$

If the initial state vector is $X[0] = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, find the zero input response, zero state response and total output response for a unit step input.

[20 marks]

Q.7 (a) Consider the cascade of the following systems S_1 and S_2, as depicted in figure,

$$x(n) \longrightarrow \boxed{S_1} \xrightarrow{w(n)} \boxed{S_2} \longrightarrow y(n)$$

S_1 : Causal LTI

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

S_2 : Causal LTI

$$y(n) = \alpha y(n-1) + \beta w(n)$$

The difference equation relating $x(n)$ and $y(n)$ is

$$y(n) = \frac{-1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n)$$

(i) Determine α and β.

(ii) Find the impulse response of the cascaded connection S_1 and S_2.

[20 marks]

(i) For S1

$$x(n) \longrightarrow \boxed{S_1} \longrightarrow w(n)$$

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

Taking z-transform

$$W(z) = \frac{1}{2}z^{-1}W(z) + X(z)$$

we get $H_1(z) = \dfrac{W(z)}{X(z)} = \dfrac{1}{1-0.5z^{-1}}$

For S2

$$w(n) \longrightarrow \boxed{S_2} \longrightarrow y(n)$$

$$y(n) = \alpha y(n-1) + \beta w(n)$$

Taking Z transform

$$Y(z) = \alpha z^{-1}Y(z) + \beta W(z)$$

$$\frac{Y(z)}{W(z)} = H_2(z) = \frac{\beta}{1-\alpha z^{-1}}$$

For Cascaded System

$$y(n) = -\frac{1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n)$$

$$\Downarrow$$

$$Y(z) = -\frac{1}{8}z^{-2}Y(z) + \frac{3}{4}z^{-1}Y(z) + X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$\Rightarrow H_1(z)\,H_2(z) = \frac{1}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{2}{4}z^{-1}\right)}$$

$$\Rightarrow \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)} \cdot \frac{\beta}{\left(1 - \alpha z^{-1}\right)} = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)} \cdot \frac{1}{\left(1 - \frac{1}{4}z^{-1}\right)}$$

Comparing we get

$$\beta = 1$$

$$\alpha = \frac{1}{4} = 0.25$$

(ii) for Cascaded Connection

$$\frac{Y(z)}{X(z)} = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)} \cdot \frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}$$

$$= \frac{2}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$$

Taking inverse Z transform of $\frac{Y(z)}{X(z)}$, we get impulse response of Cascaded System,

$$h_{cas}(n) = 2(0.5)^n u(n) - (0.25)^n u(n))$$

$\hookrightarrow$ for Causal System.

18

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Q.7 (b) The open loop transfer function of a unity feedback system is $G_P(s) = \frac{K}{s(s+2)}$. Design a lead compensator to have a velocity-error constant of $20s^{-1}$ and a phase margin of at least 50°.

$$G_C(s) = \frac{1+Ts}{1+\alpha Ts}; \ \alpha < 1$$

[20 marks]

~~Assuming~~

we have OLTF of compensated system $= \left(\frac{1+TS}{1+\alpha TS}\right)\left(\frac{K}{S(S+2)}\right)$

for $K_v = 20$

ie $\lim_{s \to 0} s\,G(s) = 20$

$\Rightarrow \frac{K}{2} = 20 \Rightarrow K = 40$

Step 2

ω_{gc} of uncompensated system

$|G_P(\omega_{gc})| = 1$

$\Rightarrow \frac{40}{\omega_{gc}(\omega_{gc}^2+4)^{1/2}} = 1$

$\Rightarrow \omega_{gc}^4 + 4\omega_{gc}^2 - 1600 = 0$

$\omega_{gc} = 6.168$ rad/sec

$PM_{uncom} = 180° + \angle G_P(\omega_{gc})$

$= 180 - 90° - \tan^{-1}\left(\frac{6.168}{2}\right)$

$= \underline{18^\circ}$

(III) Step 3 [find PM]

For 50° PM,
Additional phase lead by compensator $= (50^\circ - 18^\circ) + 5^\circ$ → Tolerance

$= \underline{37^\circ}$

Thus $\alpha = \dfrac{1 - \sin 37^\circ}{1 + \sin 37^\circ} = 0.249$

(IV) $\underline{\omega_{gc}^{new}}$

At $\alpha = 0.249$ and $\omega = \dfrac{1}{T\sqrt{\alpha}}$

Amplification by $G_c(s) = \dfrac{1}{\sqrt{0.249}} = 2$

Thus at ω_{gc}^{new}, $|G_p(\omega_{gc}^{new})| = \frac{1}{2}$

$\Rightarrow \dfrac{40}{\omega_{gc}^{new}(\omega_{gcnew}^2 + 4)^{1/2}} = \dfrac{1}{2}$

$\Rightarrow \omega_{gc\,new}^4 + 4\omega_{gcnew}^2 - 6400 = 0$

$\omega_{gcnew} = 8.88 \frac{rad}{sec}$

(iv) find T,

$$\omega_m = \frac{1}{T\sqrt{\alpha}} = \omega_{gc}^{new} = 8.83$$

we get $$T = \frac{1}{8.83 \times \sqrt{0.249}}$$

$$T = 0.2269$$

Thus $T = 0.2269$

$\alpha T = 0.0565$

So $$G_c(s) = \frac{1 + 0.2269\, s}{1 + 0.0565\, s}$$

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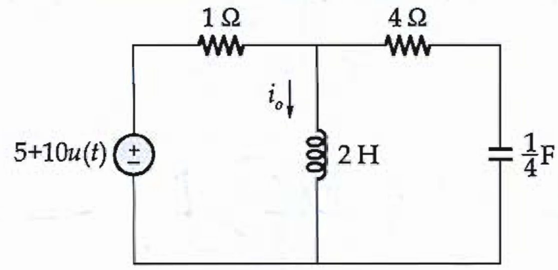
Good Approach

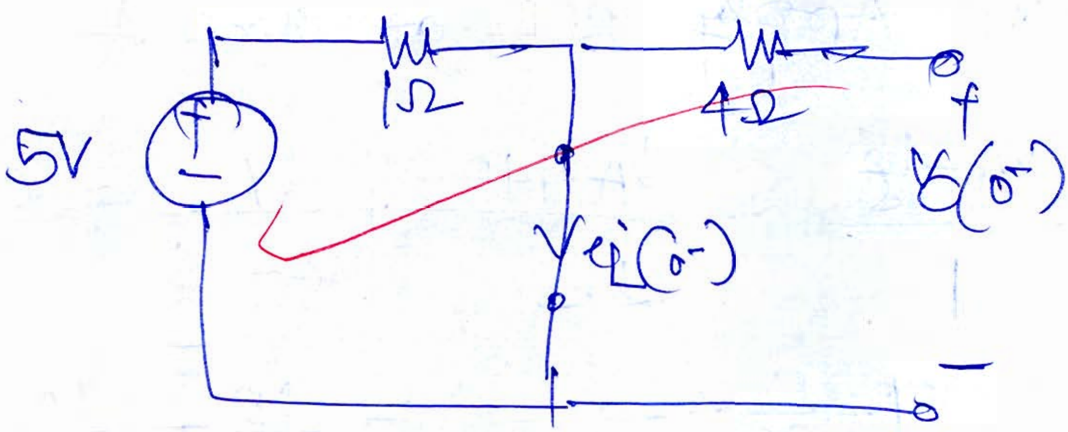
Q.7 (c) **(i)** Determine the current i_o in the circuit shown below :



(ii) Differentiate between memory mapped I/O and I/O mapped I/O.

[15 + 5 marks]

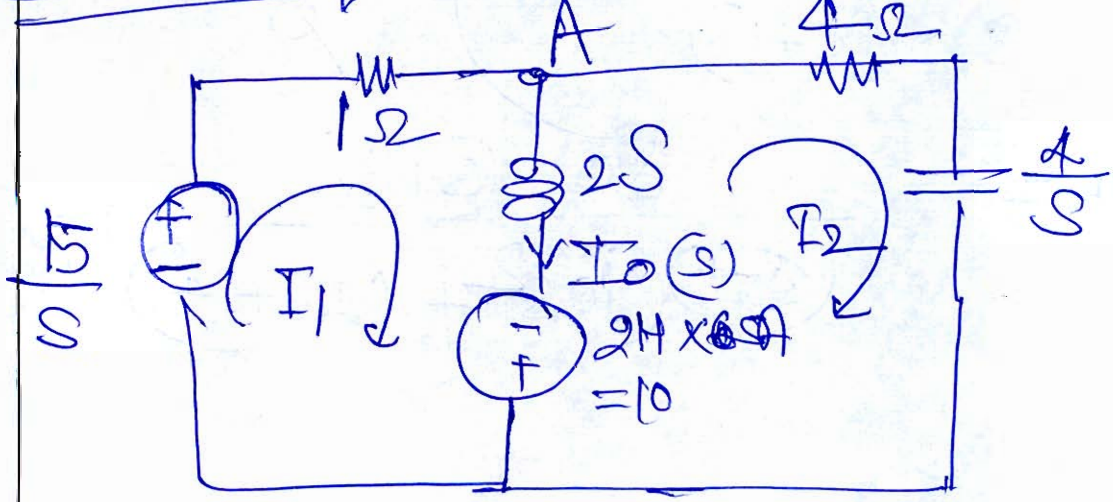
For $t<0$ [in steady state]



$i_L(0^-) = 5A = i_L(0^+)$

$v_C(0^-) = 0V = v_C(0^+)$

For $t>0$



Write your answer Ale to space.

KVL in mesh (1) and (2) gives,

$$\frac{15}{S}+10=(1+2S)I_1-2SI_2 \quad (1)$$

$$-10=(-2S)I_1+\left(4+\frac{4}{S}+2S\right)I_2 \quad (2)$$

इतना बड़ा बड़ा मत लिखो

Or Apply KCL at node A

$$\frac{V_A-\frac{15}{S}}{1}+\frac{V_A+10}{2S}+\frac{V_A}{4+\frac{4}{S}}=0$$

$$\Rightarrow V_A\left(1+\frac{1}{2S}+\frac{S}{4S+4}\right)=\frac{15}{S}-\frac{10}{2S}$$

$$\Rightarrow V_A\left(\frac{2S(4S+4)+4S+4+S\cdot 2S}{2S(4S+4)}\right)=\left(\frac{30-10}{2S}\right)$$

$$\Rightarrow V_A\left(\frac{8S^2+8S+4S+4+2S^2}{8S(S+1)}\right)=\frac{10}{\cancel{8}}$$

$$\Rightarrow V_A=\frac{80(S+1)}{(10S^2+12S+4)}$$

Never write in this margin.

You can use space smartly

Reduce your font size

$$I_o(s) = \frac{V_A + 10}{2s}$$

$$= \frac{\frac{80(s+1)}{10s^2+12s+4} + 10}{2s}$$

$$= \frac{80(s+1) + 10(10s^2+12s+4)}{2s(10s^2+12s+4)}$$

$$= \frac{100s^2 + 200s + 120}{2s(10s^2+12s+4)}$$

$$= \frac{5s^2 + 10s + 6}{s(s^2+1.2s+0.4)}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2+1.2s+0.4}$$

$$= \frac{As^2 + 1.2As + 0.4A + Bs^2 + Cs}{s(s^2+1.2s+0.4)}$$

Comparing $A+B=5$, $1.2A+C=10$, $0.4A=6$

$A=15$, $B=-10$, $C=-8$

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$$I_o(s) = \frac{15}{s} - \left(\frac{10s+6+2}{(s+0.6)^2+0.2^2}\right)$$

$$= \frac{15}{s} - 10\left(\frac{s+0.6}{(s+0.6)^2+0.2^2}\right) - \frac{2}{0.2}\left(\frac{0.2}{(s+0.6)^2+0.2^2}\right)$$

$$i_o(t) = 15u(t) - 10e^{-0.6t}\cos 0.2t - 10e^{-0.6t}\sin 0.2t$$

(A)

Memory mapped I/O

$\Rightarrow$ use 16 bit address

$\Rightarrow$ $\overline{MWR}$ $\overline{MRD}$ as control signals

$\Rightarrow$ STA, LDA instructions

$\Rightarrow$ Treated as memory

I/O mapped I/O

$\Rightarrow$ 8 bit address

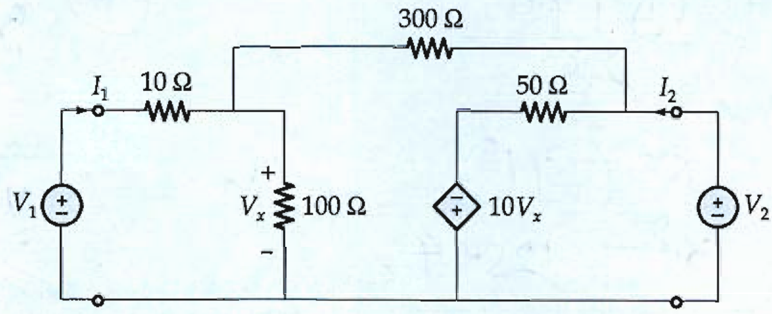
$\Rightarrow$ $\overline{IOR}$, $\overline{IOW}$ signals

$\Rightarrow$ IN, OUT instr

$\Rightarrow$ Treated as I/O Device

4

Q.8 (a) Obtain the h-parameter of the two-port network shown in figure below :



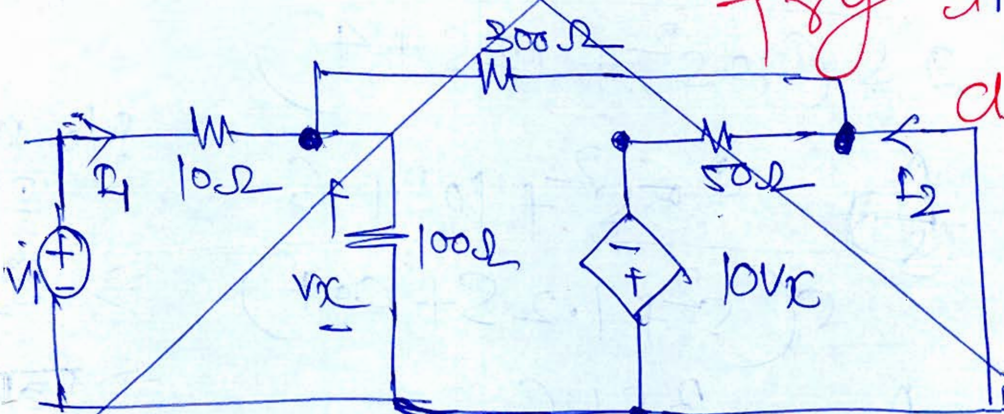
[20 marks]

h parameter relates V_i and I_i as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Case 1:- $V_2 = 0$ $\left[h_{11} = \frac{V_1}{I_1}, \; h_{21} = \frac{I_2}{I_1} \right]$

Try avoid



$V_1 = 10 I_1 + V_x$ —— (1)

Assuming the above the n/w as parrall- parall combination of 2 network with additonal 10Ω Ress

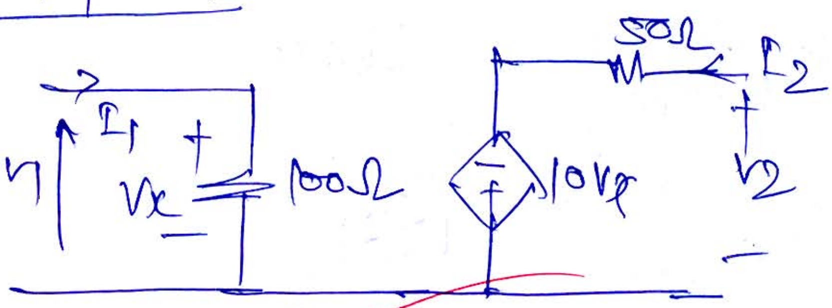
n/w

$I_1 = \frac{1}{300} V_1 - \frac{1}{300} V_2$

$I_2 = -\frac{1}{300} V_1 + \frac{1}{300} V_2$

$$y_1 = \begin{bmatrix} \frac{1}{300} & -\frac{1}{300} \\ -\frac{1}{300} & \frac{1}{300} \end{bmatrix}$$

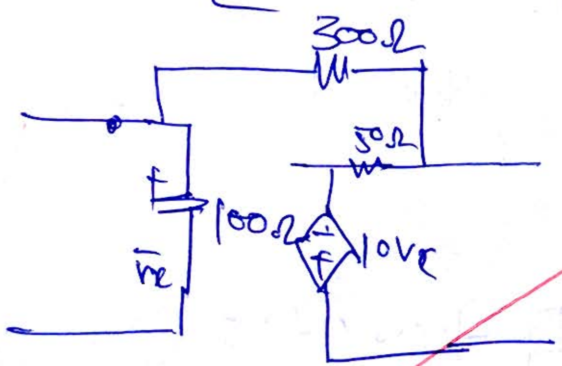
for N/w 2



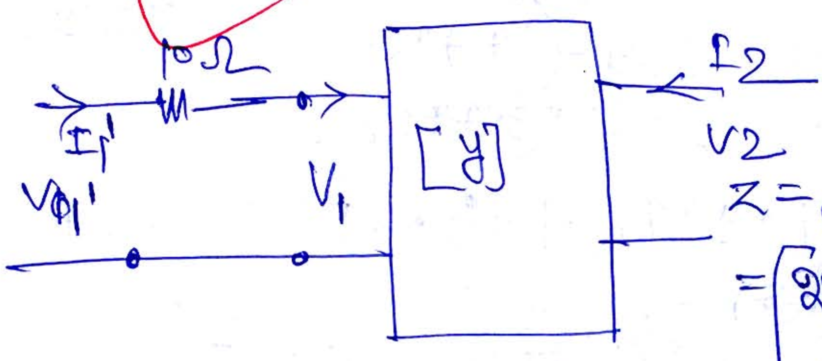
$V_1 = V_x = 100 I_1$

$V_2 = 50 I_2 - 10V_x = 50 I_2 - 10V_1 = 50 I_2 - 1000 I_1$

$$Z_2 = \begin{bmatrix} 100 & 0 \\ -1000 & 50 \end{bmatrix} \quad y_2 = [Z_2]^{-1} = \begin{bmatrix} 0.01 & 0 \\ 0.20 & 0.02 \end{bmatrix}$$



$$y = y_1 + y_2 = \begin{bmatrix} \frac{1}{75} & -\frac{1}{300} \\ \frac{59}{300} & \frac{7}{300} \end{bmatrix}$$



$$Z = y^{-1} = \begin{bmatrix} 27.137 & 3.4482 \\ -203.4 & 13.79 \end{bmatrix}$$

$$V_1' = 10 I_1' + V_1 = 10 I_1' + Z_{11} I_1' + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 = Z_{21} I_1' + Z_{22} I_2$$

Thus $\left[Z_{given\ N/w} \right] = \begin{bmatrix} Z_{11}+10 & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$$= \begin{bmatrix} 34.137 & 3.4482 \\ -203.4 & 13.79 \end{bmatrix} \Omega$$

So $V_1 = 34.137 I_1 + 3.4482 I_2$ —(1)

$V_2 = -203.4 I_1 + 13.79 I_2$ —(2)

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from (2)

$I_2 = 14.749 I_1 + 0.0725 V_2$ —(3)

from (1)

$V_1 = 34.137 I_1 + (14.749 I_1 + 0.0725 V_2) \times 3.4482$

$V_1 = 84.99 I_1 + 0.25 V_2$ —(4)

from (3) and (4)

$$h = \begin{bmatrix} 84.99\Omega & 0.25 \\ 14.749 & 0.0725 ℧ \end{bmatrix}$$

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Q.8 (b) (i) A system is represented by a state model as

$$\dot{x}_1 = -2x_1 - x_2 - 3x_3 + 2r$$
$$\dot{x}_2 = -2x_2 + x_3 + r$$
$$\dot{x}_3 = -7x_1 - 8x_2 - 9x_3 + 2r$$

The output, $y = 4x_1 + 6x_2 + 8x_3$

Check the controllability and observability of the system.

(ii) Explain the following instruction sets of 8086 microprocessor with example.
1. ROL; 2. ROR; 3. RCR; 4. RCL

[12 + 8 marks]

Forming Controllability matrix

$Q_C = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$ and

observability matrix $Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}^2$$

$$= \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix}$$

As $\dot{x} = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} r$

$y = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix} x$

$B = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix}$

$$AB = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -11 \\ 0 \\ -40 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 142 \\ -40 \\ 437 \end{bmatrix}$$

$$CA = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} =$$

$$= \begin{bmatrix} -38 & -12 & -118 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix}$$

$$= \begin{bmatrix} 524 & -140 & 1680 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 2 & -11 & 142 \\ 1 & 0 & -40 \\ 2 & -40 & 437 \end{bmatrix}$$

$$\det|Q_c| = -3193 \neq 0$$

So System is Controllable

$$\det Q_0 = \det \begin{bmatrix} 4 & 6 & 8 \\ -38 & -12 & -198 \\ 594 & -140 & 1630 \end{bmatrix} = -161928 \neq 0$$

∴ Observable.

10

Write Answer properly

.8 (c) Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose we are given the following facts :

1. $x(t)$ is real and non-negative.
2. $F^{-1}\{(1 + j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is independent of t.
3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi$

Determine a closed-form expression of $x(t)$.

[20 marks]

from (2)

$$(1+j\omega)\,X(\omega) = \frac{A}{2+j\omega}$$

Thus $$X(\omega) = \frac{A}{(1+j\omega)(2+j\omega)}$$

$$X(\omega) = A\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$$

Taking inverse fourier Transform

$$x(t) = A\left[e^{-t} - e^{-2t}\right]u(t)$$

As $x(t)$ is Real so A is Real

As $x(t)$ is positive so A must be positive.

From information (3) and using Parseval Energy Relation

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{2\pi}{2\pi} = 1$$

Do not write in this margin

$$\Rightarrow \int_0^\infty A^2\left[e^{-2t} + e^{-4t} - 2e^{-3t}\right]dt = 1$$

$$\Rightarrow A^2\left[\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right] = 1$$

$$A^2 = 12$$

Thus $\boxed{A = +\sqrt{12} = +3.464}$

15

Incomplete Solution

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work