- Do not left question incomplete
- Try to avoid calculation mistake

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ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-3 : Power Systems + Systems and Signal Processing-1 + Microprocessors-1 + Electrical Circuits-2 + Control Systems-2

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	35
Q.2	29
Q.3	
Q.4	44
Section-B	
Q.5	35
Q.6	
Q.7	
Q.8	54
Total Marks Obtained	197

Signature of Evaluator: Sourabh kumar

Cross Checked by: ....

Corp. office : 44 - A/1, Kalu Sarai, New Delhi-110016 | **Ph:** 9021300500 | **Web:** www.madeeasy.in

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1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
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4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
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6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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Section A : Power Systems

Q.1 (a) A 66 kV, 60 km long, transmission line delivers a load of 25 MW at 0.8 lagging power factor. If the line have series resistance and inductance of 0.08 Ω/km and 1.25 mH/km respectively, compute

(i) Sending end voltage and current

(ii) Voltage regulation

(iii) Transmission efficiency. Assume a power frequency of 50 Hz.

[12 marks]

Ans

given, $\underset{(L-L)}{V_R} = 66\,kV$, $P_R = 25\,MW$ at 0.8 lagging power factor

$R = 0.08 \times 60 = 4.8\,\Omega$

$X = \omega l \cdot (60) = 100\pi \times 1.25 \times 10^{-3} \times 60 = 23.52\,\Omega$

$\therefore \quad Z = R + jX = (4.8 + j\,23.5)\,\Omega$

from information of power,

$$(V_R)_{ph} = \frac{66}{\sqrt{3}} = 38.105\,kV$$

$$I_R = \frac{P_R}{\sqrt{3} \times (V_R)_{L-L} \times \cos\theta} = \frac{25 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0.8}$$

$$= 273.366\,A$$

• Write all quantities in phasor, take V_R as reference, $\therefore \; (\bar{V}_r)_{ph} = 38.105 \angle 0\,kV$

$$\bar{I}_r = 273.366 \angle -36.86\,A$$

$\{\because \theta = \cos^{-1}(0.8)\}$

i) ABCD matrix of line $= \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

$\therefore \quad (V_s)_{ph} = (V_R)_{ph} + Z I_r$

$= (38.105 \angle 0° + 6.556 \angle 41.59)$ kV

$= 43.228 \angle 5.77$ kV

$(V_s)_{ph} = 43.228$ kV

$(V_s)_{LL} = 74.87$ kV — Ans

sending end current $=$ ~~tha~~ $I_s = C(V_R)_{ph} + D I_r$

$\therefore \quad I_s = I_r$

$I_s = 273.366 \angle -36.87$ A — Ans

ii) Voltage Regulation $= \dfrac{|V_{R0}| - |V_R|}{V_{rated}} \times 100\% = \dfrac{\left|\frac{V_s}{A}\right| - |V_R|}{V_{rated}} \times 100\%$

$= \dfrac{|V_s| - |V_R|}{V_{rated}} = \dfrac{43.228 - 38.105}{38.105} \times 100\%$

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Good Approach

$= 13.44\%$ — Ans

iii) Power loss in transmission line, $P_L = 3 I_r^2 R$

$P_L = 3(273.366)^2 \, 4.8 = 1.076$ MW

$\therefore$ Efficiency $= \dfrac{P_R}{P_R + P_L} \times 100\% = \dfrac{25}{25 + 1.076} \times 100\% = 95.87\%$

Ans

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Q.1 (b) A hydroelectric station is to be designed for catchment area of 150 km^2, rainfall for which is 120 cm/annum. The head availability is 30 m. 72% of total rainfall is available, rest is lost to evaporation. Penstock efficiency is 95%. Turbine efficiency is 85% and generator efficiency is 90% and load factor is 40%. Determine the capacity of the station.

[12 marks]

Ans

Catchment area = A = 150 km^2 = 150×10^6 m^2

Height of head = H = 30m

Rainfall per year = 120 cm = 120×10^{-2} m

yield of rainfall = 0.72

penstock efficiency, $\eta_p = 0.95$

turbine efficiency, $\eta_T = 0.85$

gen^r efficiency, $\eta_g = 0.9$

Available volume of water per year

$$Q = 150 \times 10^6 \times 120 \times 10^{-2} \times 0.72 \text{ m}^3$$

$$= 129.6 \times 10^6 \text{ m}^3$$

overall efficiency of station $\eta = \eta_p \times \eta_T \times \eta_g$

$$\eta = 0.727$$

Energy generated, $E = w Q g h \times \eta$

$$E = 1000 \times 129.6 \times 10^6 \times 9.81 \times 30 \times 0.727$$

{ w: density of water = 1000 kg/m^3 }

$$= 38141280 \text{ MW sec} \times 0.727$$

$$E = 27728710.5 \text{ MW sec / yr}$$

$$E = \frac{27728710.56}{60 \times 60} \text{ MWhr/year}$$

$$E = 7702.42 \text{ MWhr/year}$$

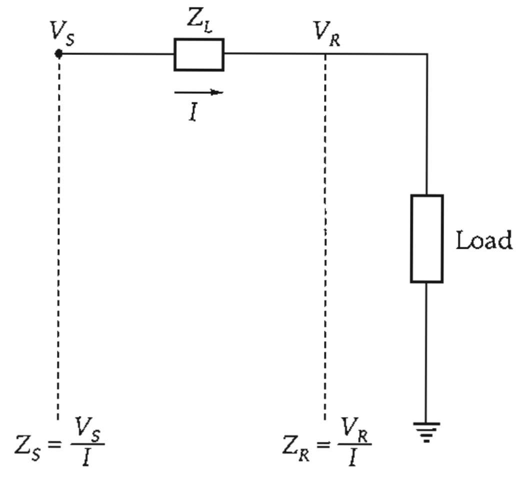
given, load factor $= 0.4 = \dfrac{\text{units generated}}{\text{installed capacity}}$

$$\therefore \text{capacity} = \frac{E}{0.4} = 19256.05 \text{ MWhr}$$

$$\therefore \text{capacity} = 19.26 \text{ GWhr}$$

Ans

Q.1 (c) Consider the transmission line as shown in figure, with series impedance Z_L, negligible shunt admittance and a load impedance Z_R at the receiving end.



(i) Calculate Z_R for the given condition of $V_R = 1.0$ pu and $S_R = 2 + j0.8$ pu.
(ii) Construct the impedance diagram in R-X plane for $Z_L = (1 + j0.3)$ pu.
(iii) Find Z_S for this condition and angle between Z_S and Z_R.

[12 marks]

Ans i) given $V_R = 1$ pu & $I_R = \frac{S_R}{V_R}$, let $Z_R = R + jX$

$\therefore I_R = 2 + j0.8 =$ {Take $\bar{V}_R = 1\angle 0°$}

$= 2.154 \angle 21.8°$ pu

from S_R, we have. $P_R = 2$ pu & $Q_R = 0.8$ pu

$\therefore P_R = I_R^2 \times R \Rightarrow R = \frac{2}{(2.154)^2} = 0.43$ pu

Similarly, $Q_R = X_R \cdot I_R^2 \Rightarrow X = \frac{0.8}{(2.154)^2} = 0.17$ pu

$\therefore Z_R = (0.43 + j0.17)$ pu = Ans

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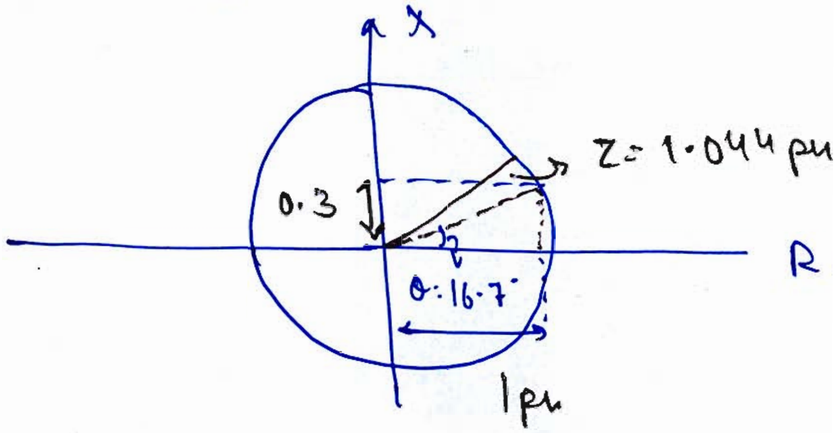
ii) given, $Z_L = 1+j0.3$



Incomplete Solution

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Q.1 (d) A 3-ϕ, 765 kV, 50 Hz, 300 km, completely transposed line has the following positive sequence impedance and admittance :

$$z = 0.0165 + j0.3306 = 0.3310\angle 87.14° \ \Omega/\text{km}$$
$$y = 4.674 \times 10^{-6} \ \text{S/km}$$

Assuming positive sequence operation, calculate exact *ABCD* parameters of long line equation. Compare the exact *B* parameter with nominal π-circuit.

[12 marks]

Ans given, $z = 0.3310 \angle 87.14° \ \Omega/\text{km}$

$\therefore \ Z = 0.3310 \times 300 \angle 87.14 = 99.3\,\Omega \angle 87.14\,\Omega$

$y = 4.674 \times 10^{-6} \text{S/km} \Rightarrow Y = 1.402 \times 10^{-3} \text{S}$

Exact ABCD parameters.

$A = \cosh(\gamma l) \qquad B = Z_c \sinh(\gamma l)$

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$C = \dfrac{1}{Z_c} \sinh(\gamma l) \qquad D = \cosh(\gamma l)$

$\therefore$ from given information,

$Z_c = \sqrt{\dfrac{z}{y}} = \sqrt{\dfrac{0.3310 \angle 87.14°}{j4.67 \times 10^{-6}}} = 266.23\,\Omega$

$\gamma = \sqrt{zy} = \sqrt{(0.3310 \angle 87.14)(4.674 \times 10^{-6} j)}$

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Q.1 (e) Consider a 3-phase, Δ-Y connected, 30 MVA, 33 : 11 kV transformer with differential relay protection. If the CT ratios are 500 : 5 on primary side and 2000 : 5 on secondary side, compute the relay current setting for faults drawing upto 200% of rated transformer current.

[12 marks]

Ans

High Voltage Transformer: 30 MVA, 33:11 kV (Δ-Y)

- let us assume I_L be line current on Δ-side.

	High voltage side of T/F	low voltage side of T/F
rated line current :	$I_L = \cancel{\frac{30\,MVA}{\sqrt{3}\,33\,kV}}$ $= I_L$	$\sqrt{3}\times 33\times I_L = \sqrt{3}\times 11\times I_L'$ $\therefore I_L' = 3I_L$
C.T. ratio :	500/5	2000/5
C.T. phase current :	$\frac{5I_L}{500}$	$\frac{5}{2000}\times 3I_L = \frac{15}{2000}I_L$
C.T. conn. :	Y	Δ
C.T. line current :	$\frac{5I_L}{500}$	$\frac{15\sqrt{3}}{2000}I_L$

$\therefore$ difference in both side CT relay

$$I_{op} = \left(\frac{5}{500} - \frac{15\sqrt{3}}{2000}\right) I_L$$

$$I_{op} = \left(\frac{1}{100} - \frac{3\sqrt{3}}{400}\right) I_L$$

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$I_{op} = 3\times10^{-3} I_L$

$I_{op} = 3\times10^{-3} I_L$

I_L: fault current = 200% of rated transformer current

$= 2\times \frac{\sqrt{3}\ 30 MVA}{\sqrt{3}\times 33\times10^{3}}$

$= 1049.728$ A

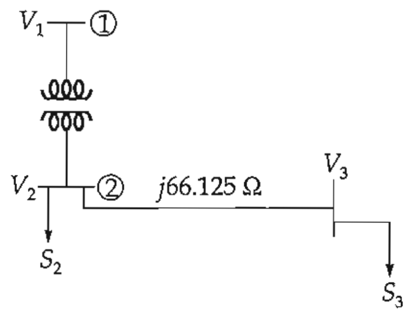
$\therefore I_{op} = 3\times10^{-3} \times 1049.728$ A

$\boxed{I_{op} = 3.15 \text{ A}}$ on C.T. side.

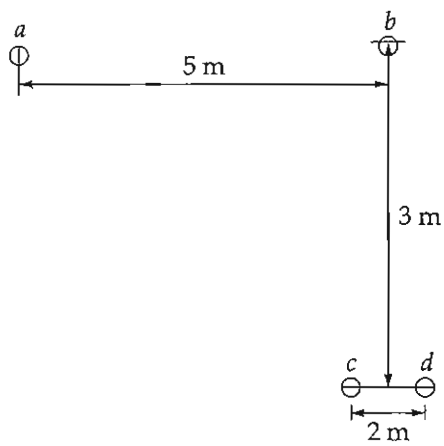
Ans

10

Q.2 (a) (i) The single line diagram of 3-phase power system is shown in figure. The transformer reactance is 20% on the base of 100 MVA, 23/115 kV and line impedance of $Z = j66.125\ \Omega$. The load at bus–2 is S_2 = 184.8 MW + j6.6 MVAR and at bus-3 is S_3 = 0 MW + j20 MVAR. It is required to hold the voltage at bus-3 at $115\angle 0°$ kV. Determine the voltages at bus-1 and bus-2.

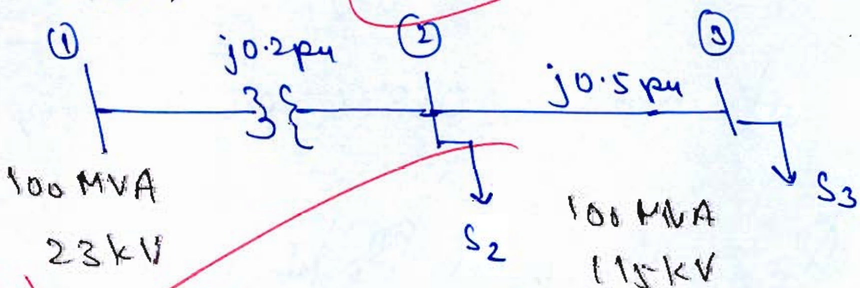


(ii) A 50 Hz, 1-ϕ power line and telephone line are parallel to each other as shown in figure. The telephone line is symmetrically positioned directly below phase b. The power line carries a current of 226 A. Assume zero current flows in ungrounded telephone wires. Find the magnitude of voltage per km induced in the telephone line.



[10 + 10 marks]

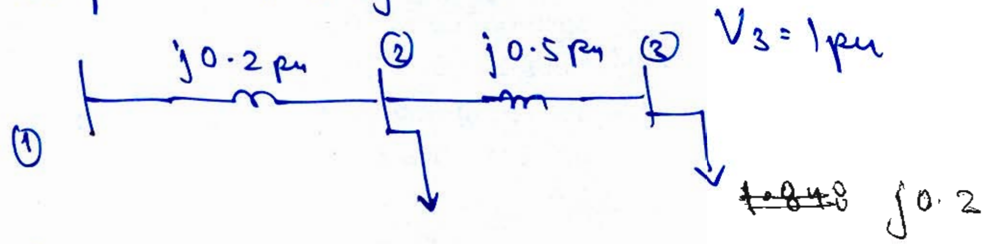
Ans i) select S_{base} = 100 MVA and V_{base} = 23 kV at bus ①.



$\therefore$ Transform, $X_T = j0.2$ pu

line, $X_l = j\dfrac{66.125}{132.25} = 0.5$ pu $\left\{ Z_{base} = \dfrac{115^2}{100} = 132.25\ \Omega \right\}$

per-phase per unit diagram



$$S_2 = \frac{184.8 + j6.6}{100} = 1.848 + j0.066 \text{ pu}$$

$S_3 = 0 + j0.2$ pu

let $\bar{V}_2 = V_2 \angle 0°$ & $\bar{V}_3 = 1\angle 0°$

due to $P_3 = 0$ pu

$$\because \quad Q_3 = 0.2 = \frac{|V_3|}{X}\left[|V_2|\cos\delta - |V_3|\right]$$

$$0.2 \times 0.5 = V_2 - 1$$

$$V_2 = 1 + 0.1 = \underline{\underline{1.1 \text{ pu}}}$$

from $\underline{S_2}$:- let $\bar{V}_1 = V_1 \angle \delta$

$$P_2 = 1.848 = \frac{V_1 \cdot (1.1)\sin\delta}{0.2}$$

$$V_1 \sin\delta = 0.336 \text{ pu} \quad -①$$

$$Q_2 = (Q_2)_{load} + (Q_3)_{load}$$

$$= 0.066 + 0.2$$

$$= 0.266 \text{ pu}$$

$$Q_2 = \frac{|V_2|}{X}\left(|V_1|\overset{\cos\delta}{} - |V_2|\right) = 0.266$$

$$= \frac{1.1}{0.2}\left(V_1\cos\delta - 1.1\right) = 0.266$$

$$V_1 \cos\delta = 1.148 \quad -②$$

from ① & ②

$$V_1^2 = 0.336^2 + 1.148^2$$

$$V_1 = 1.196\ pu$$

$\therefore$ $V_1 = 1.196\ pu$ (or) $1.196 \times 23\,kV$

$\cancel{V_1 = 27}$ $V_1 = 27.512\ kV$

& $V_2 = 1.1\ pu$ (or) $1.1 \times 115\ kV$

$$V_2 = 126.5\ kV$$

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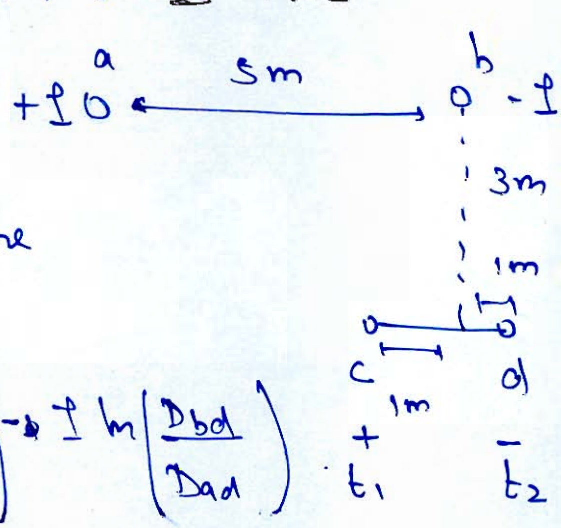
$\therefore$ $V_1 = 27.512\ kV$

$V_2 = 126.5\ kV$ — Ans

ii) given, $I = 226\ A$



induced flux in telephone lines

$$\lambda = \lambda_{t_1} - \lambda_2 = I \ln\left(\frac{D_{bc}}{D_{ac}}\right) - I \ln\left(\frac{D_{bd}}{D_{ad}}\right)$$

$\hookrightarrow$ ①

$\therefore \quad D_{ac} = \sqrt{5^2 + 3^2} = 5.83 \text{ m}$

$D_{ad} = \sqrt{6^2 + 3^2} = 6.708 \text{ m}$

$D_{bc} = D_{bd} = \sqrt{3^2 + 1^2} = \sqrt{10} = 3.162$

from eqn. (1)

$\lambda = I \ln\left(\frac{3.162}{5.83}\right) \mp I \ln\left(\frac{3.162}{6.708}\right)$

3

$\lambda = I \ln\left(\frac{6.708}{5.83}\right)$

$\lambda = 226 \ln\left(\frac{6.708}{5.83}\right) = 31.704 \text{ Wb/km}$

$\therefore \quad |V| = \omega \lambda$

$= 100\pi \times 31.704 \text{ V/km} \quad (\omega = 2\pi f)$

$= 9.96 \text{ kV/km}$

Ans

Q.2 (b) (i) A 400 MVA synchronous machine has $H_1 = 4.6$ MJ/MVA and 1200 MVA machine has $H_2 = 3.0$ MJ/MVA. The two machines operate in parallel in a power plant. Find out H_{eq}, relative to a 100 MVA base.

(ii) The per unit bus impedance matrix for a power system is given by

$$Z_{bus} = j\begin{bmatrix} 0.0450 & 0.0075 & 0.030 \\ 0.0075 & 0.06375 & 0.030 \\ 0.030 & 0.030 & 0.21 \end{bmatrix}$$

A 3-ϕ fault occurs at bus-3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix, calculate the fault current, bus voltages and line currents during fault. Assume the pre-fault voltages at each bus is 1.0 pu.

[10 + 10 marks]

i) Given, machine-① , $H_1 = 4.6$ sec $G_1 = 400$ MVA

machine-② $G_2 = 1200$ MVA, $H_2 = 3$ MJ/MVA

given, both machines operating in parallel

$\therefore \quad M_{eq} = M_1 + M_2$

$$\frac{H_{eq}G_{eq}}{\pi f} = \frac{G_1 H_1}{\pi f} + \frac{G_2 H_2}{\pi f}$$

to find H_{eq} relative to 100 MVA base

i.e, $G_{eq} = 100$ MVA

$$\therefore \quad H_{eq} = \frac{G_1 H_1 + G_2 H_2}{100}$$

$$= \frac{400 \times 4.6 + 1200 \times 3}{100}$$

$$= 18.4 + 36$$

$H_{eq} = 54.4$ MJ/MVA. = Ans

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ii) given, pre-fault voltage = 1 pu

and $Z_f = j0.19$ per unit

fault occurs at bus-3,

$\therefore$ I_f = fault current = $I_f = \frac{V_f}{Z_{33}+Z_f}$

$I_f = \frac{1}{0.21+0.19} = 2.5$ pu

- $(V_1)_{new} = (V_1)_{old} - I_f Z_{13}$

$= 1 - 2.5(0.03) = 1 - 0.75 = 0.25$ pu

6

- $(V_2)_{new} = (V_2)_{old} - I_f Z_{23}$

$= 1 - 2.5(0.03) = 0.25$ pu

- $(V_3)_{new} = (V_3)_{old} - I_f Z_{33}$

$= 1 - 2.5(0.21) = 0.475$ pu

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Q.2 (c) A single area consists of two generating units, rated at 400 MVA and 800 MVA with speed regulation of 4% and 5% on their respective ratings. The units are operating in parallel, sharing 700 MW. Unit-1 supplies 200 MW and unit-2 supplies 500 MW at 1.0 pu (60 Hz). The load increased by 130 MW.

(i) Assume there is no frequency-dependent load, i.e., $D = 0$. Find the steady-state frequency deviation and the new generation on each unit.

(ii) The load varies 0.804% for every 1% change in frequency, i.e., $D = 0.804$. Find the steady-state frequency deviation and the new generation on each unit.

[20 marks]

Ans given, G_1: 400 MVA, speed regulation = 4%.

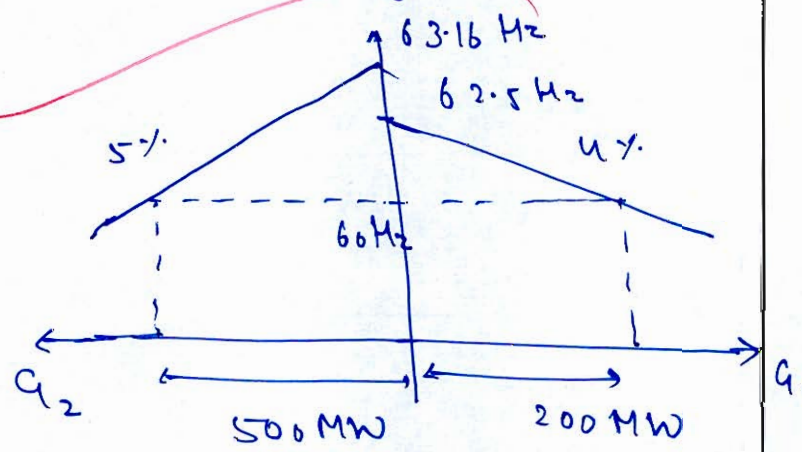
G_2: 800 MVA, speed regulation = 5%.

2

for G_1:

$$\frac{f_{NL} - 60}{f_{NL}} = 0.04$$

$f_{NL} = 62.5$ Hz



for G_2: $\frac{f_{NL} - 60}{f_{NL}} = 0.05 \Rightarrow 63.16$ Hz

i) ∵ load increased by 130 MW i.e.

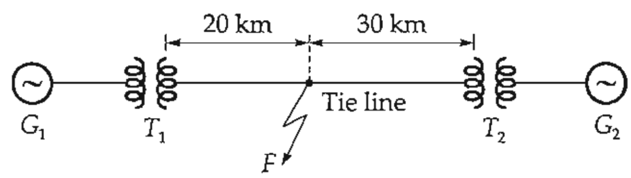
new load will be $P_L = 830$ MW

Q.3 (a) A 3-ϕ overhead line has resistance and reactance per phase 5 Ω and 25 Ω respectively. The load of receiving end 15 MW, 33 kV, 0.8 pF lagging. Find the compensation equipment needed to deliver this load with sending end voltage of 33 kV.

Calculate the extra load of 0.8 lagging power factor delivered with the compensating equipment (of capacity as calculated above) installed, if the receiving end voltage is permitted to drop to 28 kV.

[20 marks]

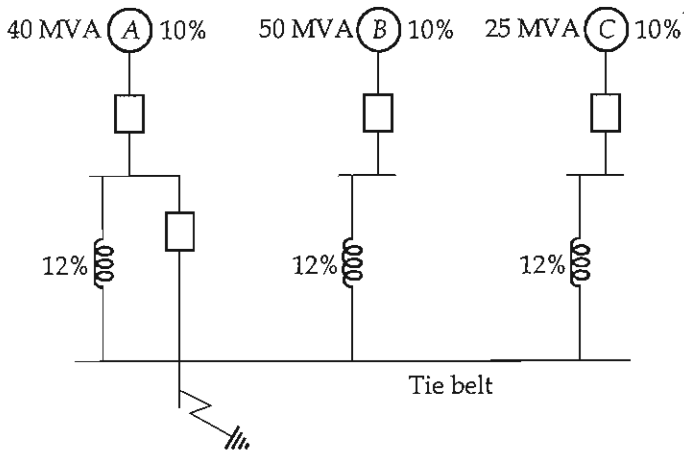
Q.3 (b) Generator G_1 and G_2 are identical and rated 11 kV, 20 MVA and have a transient reactance of 0.25 p.u at own MVA base. The transformers T_1 and T_2 are also identical and are rated 11/66 kV, 5 MVA and have a reactance of 0.06 p.u. to their own MVA base. The tie line is 50 km long, each conductor has a reactance of 0.848 Ω/km. The three phase fault is assumed at F, which is 20 km away from transformer T_1 as shown below. Find the short circuit current.



[20 marks]

Q.3 (c) (i) A single-core, lead sheathed cable joints has a conductor of 10 mm diameter and two layers of different insulating materials, each 10 mm thick. The relative permittivities are 3 (inner) and 2.5 (outer). Calculate the potential gradient at the surface of conductor when the potential difference between the conductor and the lead sheath is 60 kV.

(ii) Three 6.6 kV generators A, B and C, each of 10%, leakage reactance and MVA rating 40, 50 and 25 respectively are interconnected electrically as shown in figure, by a tie bar current limiting reactor, each of 12% reactance based upon the rating of machine to which it is connected. A 3-ϕ feed is supplied from the bus-bar of generator A at a line voltage of 6.6 kV. The feeder has resistance of 0.06 Ω/ph and an inductive reactance of 0.12 Ω/ph. Estimate the maximum MVA there can be fed into symmetrical short circuit at the far end of the feeder.



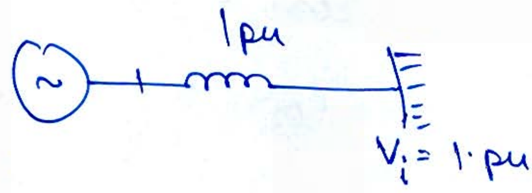
[8 + 12 marks]

Q.4 (a) (i) Find the steady state power limit of a system consisting of a generator with equivalent reactance 0.50 pu connected to an infinite bus through a series reactance of 1 pu. The terminal voltage of generator held at 1.20 pu and voltage of infinite bus is 1.0 pu.

(ii) Determine the corona characteristics of a 3-phase line 160 km long. Conductor diameter 1.036 cm, 2.44 m delta spacing, air temperature 26.67°C, altitude 2440 m, corresponding to an approximate barometric pressure of 73.15 cm, operating at 110 kV at 50 Hz. Surface irregulating factor is 0.85 and $m_V = 0.72$.

[10 + 10 marks]

Ans i) given,

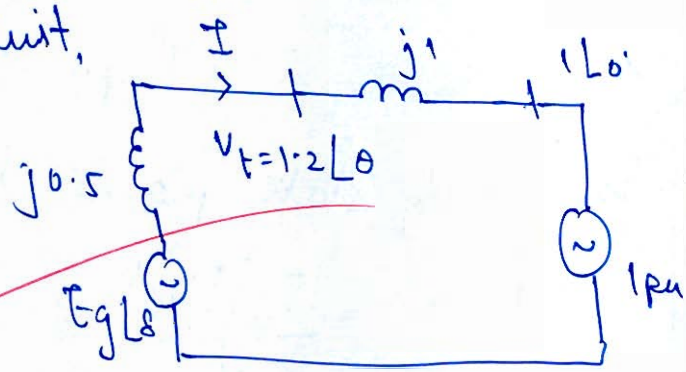


$X_g = 0.5$

$V_t = 1.2$, $V_i = 1\,pu$

equivalent dig^r. of circuit,

take infinite bus voltage as reference



$\therefore\ V_i = 1\angle 0^\circ\ pu$ $\qquad X_{eq} = 1.5\ pu$

for maximum power transfer (or) steady state limit, $\delta = 90°$.

$$\therefore\ I = \frac{1.2\angle\theta - 1}{j1} = \frac{E_g\angle\delta - 1}{j1.5}$$

$$\therefore\ 1.8\angle\theta - 1.5 = E_g\angle\delta - 1$$

$$E_g\angle\delta = 1.8\angle\theta - 0.5$$

$$\therefore\ E_g\cos\delta + jE_g\sin\delta = 1.8\cos\theta + j1.8\sin\theta - 0.5$$

~~equate~~ equate real & imaginary part

$E_g \cos\delta = 1.8\cos\theta - 0.5$ & $E_g \sin\delta = 1.8\sin\theta$

put $\delta = 90°$ for steady state power limit

$\therefore \quad 1.8\cos\theta = 0.5$

$\theta = \cos^{-1}\left(\frac{0.5}{1.8}\right) = 73.87°$

& $E_g = 1.8\sin\theta = 1.8\sin(73.87)$

$= 1.73\,pu$

9

So, steady state power limit

$P_{max} = \frac{|E_g||V_i|}{X_{eq}} = \frac{1.73 \times 1}{1.5} = 1.153\,pu$

$P_{max} = 1.153\,pu$ — Ans

ii) given, $d = 1.036\,cm \Rightarrow r = 0.518\,cm$

$D = 2.44\,m$

$t = 26.67°C$, $b = 73.15$

$m_0 = 0.85$ & $m_v = 0.72$

operating voltage $= 110\,kV$ (L-L)

at 50 Hz

dielectric strength of air, $g_0 = 21.1$ kV/cm

$$\therefore \; g_0 = 21.1 = \frac{V_{ph}}{r \ln\left(\frac{D}{r}\right)}$$

disruptive electric voltage,

$$V_{d_0} = m_0 g_0 a r \ln\left(\frac{D}{r}\right)$$

$$V_{d_0} = m_0 g_0 a r \ln(D/r)$$

$$\therefore \quad a = \frac{3.92b}{273+t} = \frac{3.92 \times 73.15}{273+26.67} \quad \left\{ \begin{array}{l} a : \text{atomospheric} \\ \text{cond}^n \text{ factor} \end{array} \right\}$$

$$a = 0.957$$

$$V_{d_0} = (0.85)(21.1)(0.957)(0.518) \times \ln\left(\frac{2.44}{0.518 \times 10^{-2}}\right)$$

$$= 54.72 \text{ kV/phase}$$

operating voltage $= V_0 = 110$ kV (L-L)

$$(V_0)_{ph} = \frac{110}{\sqrt{3}} = 63.51$$

7

$$\therefore \quad V_{d_0} < (V_0)_{ph}$$

$\therefore$ corona will not occur here.

Q.4 (b) A 50-Hz, 100 MVA, 4-pole, synchronous generator has inertia constant of 3.5 sec and supply 0.16 pu power on a system base of 500 MVA. The input to the generator is increased to 0.18 pu. Determine :

(i) Kinetic energy stored in the rotor.

(ii) Acceleration of the generator.

(iii) If acceleration continues for 7.5 cycles, calculate the change in rotor angle.

(iv) Speed in rpm at the end of the acceleration.

[20 marks]

Au given, $G = 100\,MVA$, $P = 4$, $H = 3.5$ sec

$P_e = 0.16 \times 500 = 80\,MW$ $\quad f = 50\,Hz$

$P_s = 0.18 \times 500 = 90\,MW$

i) ~~Kit~~ Kinetic energy stored $= GH$

$= 100 \times 3.5 = 350\,MWsec$ Ans

ii) from swing equation of generator,

$$M\frac{d^2\delta}{dt^2} = P_s - P_e, \quad \text{where } M = \frac{GH}{180 f} = MWs/\text{elec deg}$$

$$\frac{GH}{180f}\frac{d^2\delta}{dt^2} = \cancel{80}(90-80)\,MW$$

$$\frac{d^2\delta}{dt^2} = \frac{10 \times 180 f}{GH} = \frac{1800 \times 50}{100 \times 3.5}$$

$$\therefore \alpha = \frac{d^2\delta}{dt^2} = 257.14 \text{ elec.deg/sec}^2$$

Rotor Acceleration (or) $257.14 \times \frac{\pi}{180}$ elec.rad/sec^2

$$\therefore \alpha = \frac{d^2\delta}{dt^2} \quad 4.488 \text{ elec.rad/sec}^2$$ Ans

(iii) intial rotar angle, $\delta_0 = \sin^{-1}\left(\frac{P_c}{P_s}\right) = \sin^{-1}\left(\frac{0.16}{0.18}\right)$

$\delta_0 = 62.73^\circ$

from rotor angle equation,

$\delta = \delta_0 + \frac{1}{2}\alpha t^2$

given $t = 7.5T = \frac{7.5}{f\ \cancel{50}} = \frac{7.5}{50} = 0.15$ sec

$\therefore \delta = \delta_0 + \frac{1}{2}(257.44) \times (0.15)^2$

$\delta = 62.73 + 2.89^\circ$

$\delta_f = 65.62^\circ$

$\therefore$ ~~A~~ change in rotor angle

$= \Delta\delta = 2.89^\circ$

= Ans

iv) initial rpm, $N_s = \frac{\cancel{2\pi}60}{2\pi} \times \omega_s = \frac{60}{\cancel{2\pi}} \times \cancel{2\pi} f$

$\therefore N_s = 60 \times 50 \quad 3000$ rpm

from dynamics of synchronous machine,

$\omega = \omega_s + \alpha t^2 \qquad \alpha :$ rad/sec^2.

$\omega = (2\pi f) + (4.488)(0.15)$

$= 100\pi + 0.673 = 314.832$

Now, new speed in rpm, ~~N_r~~ =

$$N_s' = \frac{60}{2\pi} \times \omega$$

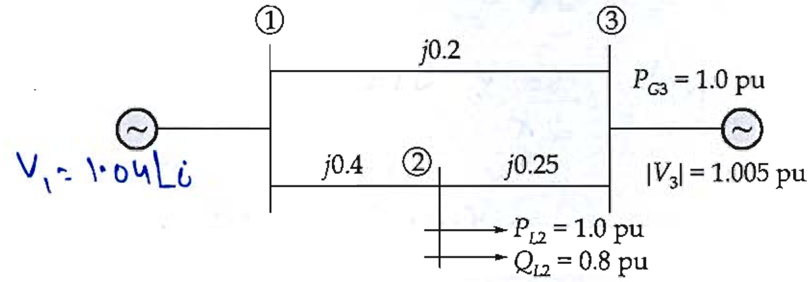
$$N' = \frac{60}{2\pi} \times 314.832$$

$$= 3006.42 \text{ rpm}.$$

Ans

15

Q.4 (c) For the power system network shown in figure, compute the bus voltages using the Gauss-Seidel iteration method. Line reactances and loads are shown in figure. Bus-1 is the slack bus ($V_1 = 1.04\angle 0°$) and bus-2 and bus-3 are the load and voltage-control buses respectively. Assume tolerance equal to 1×10^{-5}.

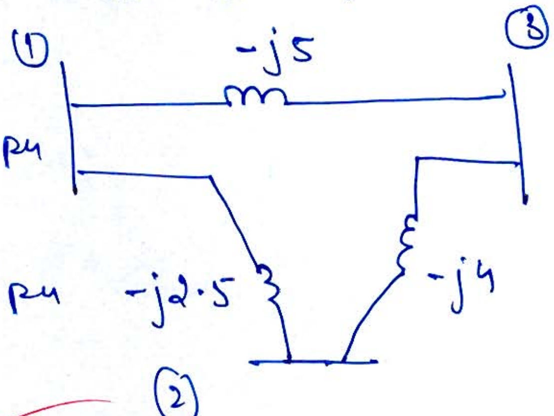


Compute V_1, V_2 and V_3 upto one iteration.

[20 marks]

Ans first construct Y-Bus matrix of system

• convert all impedances to admittance.



$Y_{11} = -j5 + (-j2.5) = -j7.5$ pu

$Y_{12} = Y_{21} = -(\cancel{-j2.5}) = j2.5$ pu

$Y_{13} = Y_{31} = -(-j5) = j5$ pu

$Y_{22} = -j2.5 - j4 = -j6.5$

$Y_{23} = Y_{32} = -(-j4) = j4$

$Y_{33} = -j5 - j4 = -j9$

$$[Y_{Bus}] = \begin{bmatrix} -j7.5 & j2.5 & j5 \\ j2.5 & -j6.5 & j4 \\ j5 & j4 & -j9 \end{bmatrix}$$

Table for information about buses ①, ② & ③

	$\lvert V \rvert$	δ	P	Q	
bus-1	1.04	0°	—	—	→ slack bus
bus-2	—	—	1pu	0.8pu	PQ bus
bus-3	1.005	—	1pu	—	→ PV bus

for voltage in gauss seidal iteration,

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{k\neq i}^{n} Y_{ik}V_k\right]$$

$$\& \quad Q_i = -\sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\sin(\theta_{ik}+\delta_k-\delta_i)$$

Now, to calculate V_2 & $V_3 \to \delta_3$

assume initial values as, $V_2^0 = 1\angle 0°$

& $V_3^0 = 1.005\angle 0°$ & $Q_3^0 = 1$pu

∴ <u>1st iteration,</u>

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0\right]$$

$$V_2^1 = \frac{1}{-j6.5}\left[\frac{1-j0.8}{1\angle 0°} - j2.5(1.04) - j4(1.005)\right]$$

$$\cancel{= \frac{j}{6.5}\left(1 - j0.8 - j2.6 - j4.02\right)}$$

$$\cancel{= 1.15\angle 7.67° \text{ pu}}$$

$$Q_3' = -V_3\left(V_1 Y_{13}\sin(\theta_{13}+\delta_1-\delta_3) + V_2' Y_{23}\sin(\theta_{23}+\delta_2-\delta_3)\right.$$
$$\left. + V_3 Y_{33}\sin(\theta_{33})\right]$$

$$= -1.005\left((1.04)(5)\sin(90+0-0) + 1.15\times 4\sin(90+7.67-0)\right.$$
$$\left. + 1.005\times 9\sin(-90)\right]$$

$$= -1.005\left[5.2 + 4.559 - 9.045\right]$$

$$Q_3' = -0.717 \ \cancel{pu}$$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3'}{(V_3^0)^*} - Y_{13}V_1 - Y_{23}V_2'\right]$$

$$= \frac{j}{9}\left[\frac{1+j0.717}{1.005} - j5(1.04) - j4(1.15)\right]$$

$$= \frac{j}{9}\left(1.224\angle 35.64 - j9.8\right)$$

$$V_3' = 1.016\angle 6.247$$

$\therefore$ $V_1 = 1.04\angle 0$

$V_2 = 1.15\angle \cancel{7.67}$ after 1st iteration.

$V_3 = 1.005\angle 6.247$ 2.41°

Ans

13

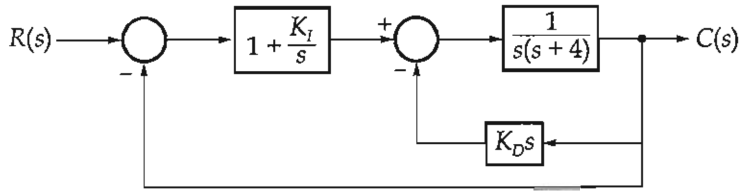
Section B : Systems and Signal Processing-1 + Microprocessor-1 + Electrical Circuits-2 + Control Systems-2

Q.5 (a) Calculate the delay in the following loop, assuming the system clock frequency is 3 MHz.

```
        LXI B, 12FFH
DELAY : DCX B
        XTHL
        XTHL
        NOP
        NOP
        MOV A, C
        ORA B
        JNZ DELAY
```

[12 marks]

Q.5 (b) Determine the ranges of controller gains (K_D, K_I) so that the system shown in figure below remains stable. Also determine the type of the system. Plot the region of stability.



[12 marks]

Ans

for given system,

$$G(s)\ \cancel{\frac{C(s)}{R(s)}} = \left(1+\frac{k_I}{s}\right)\left(\frac{1}{s^2+4s+k_D s}\right)$$

$$\cancel{\frac{C(s)}{R(s)} = \frac{\frac{1+k_I}{s^2(s+k_D+4)}}{1+(1+k_I)}}$$

~~∴ characteristic equation of system,~~

$$\cancel{q(s) = 1+GH = s}$$

$$G(s) = \frac{s+k_I}{s^2(s+k_D+4)}$$

characteristic equation of system,

$$q(s) = 1+GH = 0$$

$$= s^3+(k_D+4)s^2+s+k_I = 0$$

by Routh criteria,

system will be stable only if elements of column ~~a~~ have same sign in the Routh Table.

Routh table,

s^3	1	1
s^2	K_D+4	K_I
s^1	$\frac{K_I-K_D-4}{K_D+4}$	0
s^0	K_I	

for system to be stable,

$K_I > 0$, $K_D+4 > 0$

5

& $K_I > K_D+4$

assume $K_D = \cancel{4}\ -3$

$\therefore\ K_I > 1$

$\therefore$ $K_D = -3$ & $K_I = 2$

are one solution.

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Q.5 (c) The reduced incidence matrix of an oriented graph is given as :

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

(i) Draw its graph.

(ii) Determine the number of trees are possible for this graph.

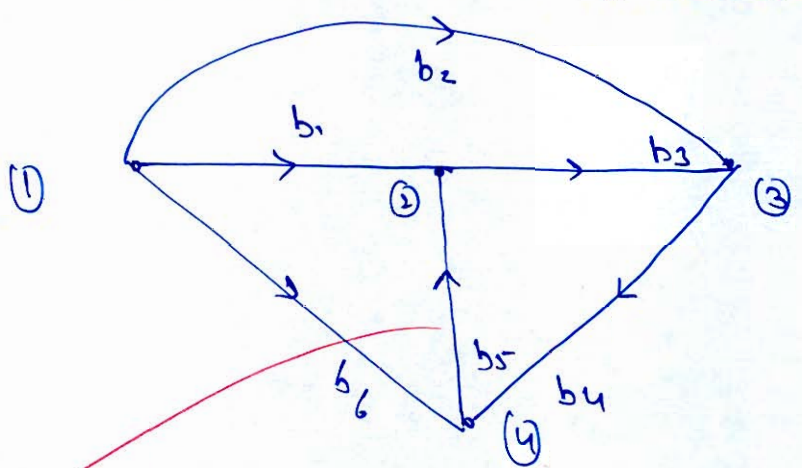
[12 marks]

Ans from given reduced incidence matrix. we form incidence matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}_{n \times b} \end{array}$$

$\therefore$ $n = 4$ nodes & $b = 6$ branches

i)



ii) given, ~~inci~~ incidence matrix is forming a complete graph i.e. from each node we can connect to any node directly.

$\therefore$ no. of trees possible $= n^{n-2}$

where n : no. of nodes

$\therefore$ no. of possible trees $= 4^{4-2}$

$\therefore$ Possible trees $= 4^2 = 16$

Given reduced incidence matrix will have 16 possible trees.

11

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Q.5 (d) A continuous-time linear system S with input $x(t)$ and output $y(t)$ yields the following input-output pairs.

$$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$$

$$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$$

(i) If $x_1(t) = \cos(2t)$, determine the corresponding output $y_1(t)$ for system S.

(ii) If $x_2(t) = \cos(2t - 1)$, determine the corresponding output $y_2(t)$ for system S.

[12 marks]

Ans i) given $e^{j2t} \xrightarrow{S} e^{j3t}$

S: system is a linear system.

$\therefore \quad x_1(t) = \dfrac{e^{j2t} + e^{-j2t}}{2} \xrightarrow{S} y_1(t)$

$\therefore$ from property of linearity i.e additive & homogenity.

we have $y_1(t) = \dfrac{e^{j3t} + e^{-j3t}}{2}$

$\therefore \quad y_1(t) = \cos 3t$ Ans

ii) $x_2(t) = \cos(2t-1) = \dfrac{e^{j(2t-1)} + e^{-j(2t-1)}}{2}$

$x_2(t) = \dfrac{e^{-1} \cdot e^{j2t} + e^{1} \cdot e^{-j2t}}{2}$

from property of ~~time~~ additivity & homogenity in a linear system

$x_2(t) \xrightarrow{S} y_2(t)$

$$y_2(t) = \frac{e^{-1} e^{j3t} + e^{1} \cdot e^{-j3t}}{2}$$

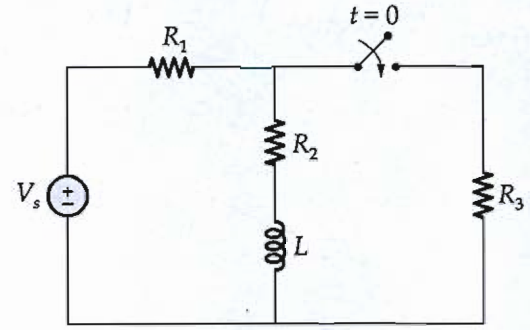
$$= \frac{e^{j(3t-1)} + e^{-j(3t-1)}}{2}$$

$$y_2(t) = \cos(3t-1)$$ Ans

11

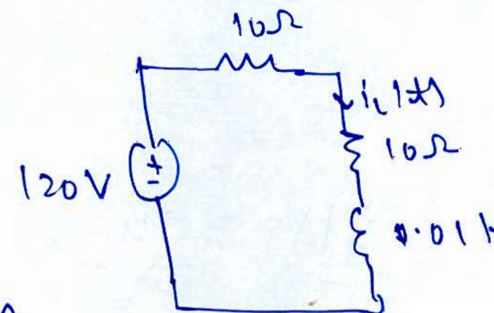
Good Approach

Q.5 (e) The switch in the circuit given below closes at $t = 0$, after being open for a long time. Find the inductor current $i_L(t)$, if $R_1 = R_2 = R_3 = 10\ \Omega$, $L = 0.01$ H and $V_s = 120$ V.



[12 marks]

Ans before switch closes,



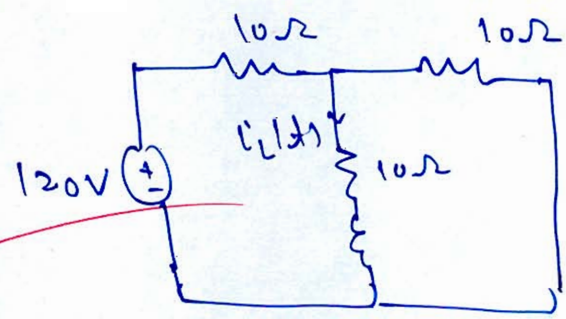
at steady state

$i_L(t) = \frac{120}{10+10} = \frac{120}{20} = 6\text{ A}$

$\therefore\ i_L(0^-) = 6\text{ A}$

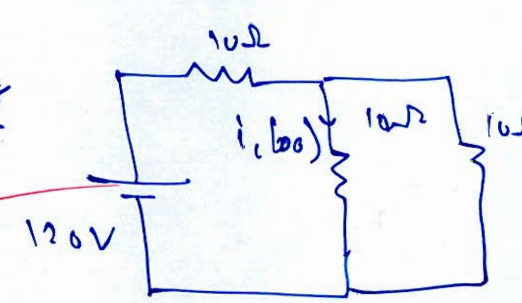
after switch closed,

$i_L(0^+) = i_L(0^-) = 6\text{ A}$



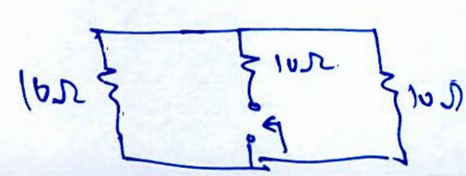
and now at steady state : inductor short circuited

$i_L(\infty) = \frac{V_L(\infty)}{10} = \frac{\cancel{120}^{8} \times 5/\cancel{15}}{10}$



$i_L(\infty) = 4\text{ A}$

time constant of circuit across inductor



$R_{eq} = 15\,\Omega$

$\therefore \quad \tau =$ time constant $= R_{eq} \cdot L$

$= 15 \times 0.01 = 0.15$ sec

now by eqn.

$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))\, e^{-t/\tau}$

$= 4 + (6-4)\, e^{-t/\tau}$

$6-4=2$

$i_L(t) = (4 - 2e^{-t/\tau})$ A

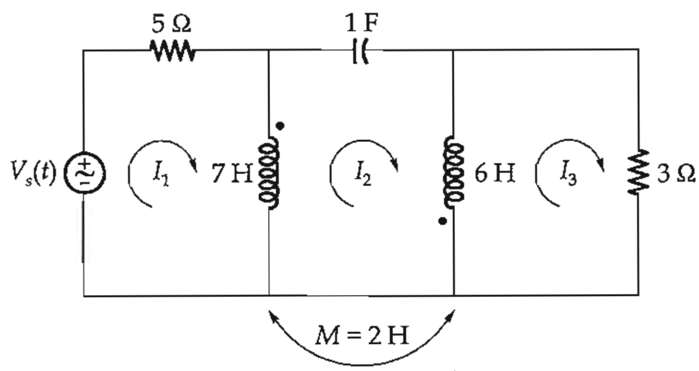
$= (4 - 2e^{-t/0.15})$ A

Ans

8

$4 + 2e^{-1500t}$

Q.6 (a) For the magnetically coupled circuit shown in figure, find the loop current I_1, I_2 and I_3, if $V_s(t) = 2\cos(2t)$.



[20 marks]

Q.6 (b) Write a program to arrange first 10 numbers from memory address 2040H in ascending order. Write the comment of each instruction.

[20 marks]

Q.6 (c) A system is represented by the state model,

$$\dot{X} = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \text{ and } y = [1 \quad -1]X$$

If the initial state vector is $X[0] = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, find the zero input response, zero state response and total output response for a unit step input.

[20 marks]

Q.7 (a) Consider the cascade of the following systems S_1 and S_2, as depicted in figure,

$$x(n) \longrightarrow \boxed{S_1} \xrightarrow{w(n)} \boxed{S_2} \longrightarrow y(n)$$

S_1 : Causal LTI

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

S_2 : Causal LTI

$$y(n) = \alpha y(n-1) + \beta w(n)$$

The difference equation relating $x(n)$ and $y(n)$ is

$$y(n) = \frac{-1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n)$$

(i) Determine α and β.

(ii) Find the impulse response of the cascaded connection S_1 and S_2.

[20 marks]

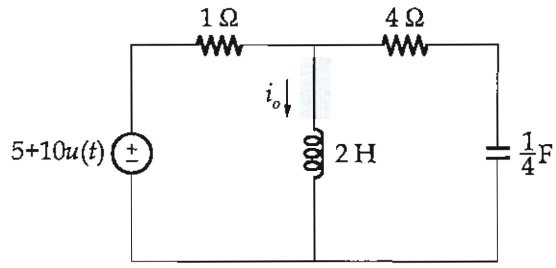
Do not write in this margin

Q.7 (b) The open loop transfer function of a unity feedback system is $G_P(s) = \dfrac{K}{s(s+2)}$. Design a lead compensator to have a velocity-error constant of $20s^{-1}$ and a phase margin of at least 50°.

$$G_C(s) = \frac{1+Ts}{1+\alpha Ts};\ \alpha < 1$$

[20 marks]

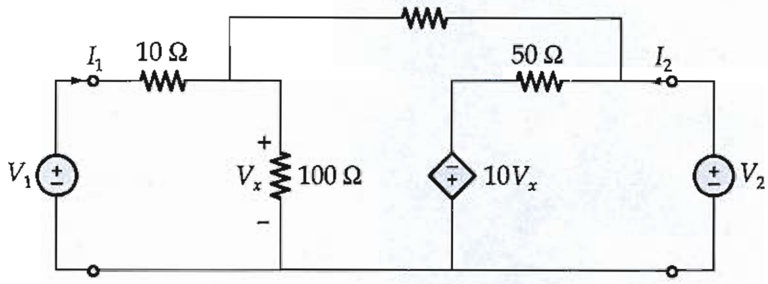
Q.7 (c) (i) Determine the current i_o in the circuit shown below :



(ii) Differentiate between memory mapped I/O and I/O mapped I/O.

[15 + 5 marks]

Q.8 (a) Obtain the h-parameter of the two-port network shown in figure below :



[20 marks]

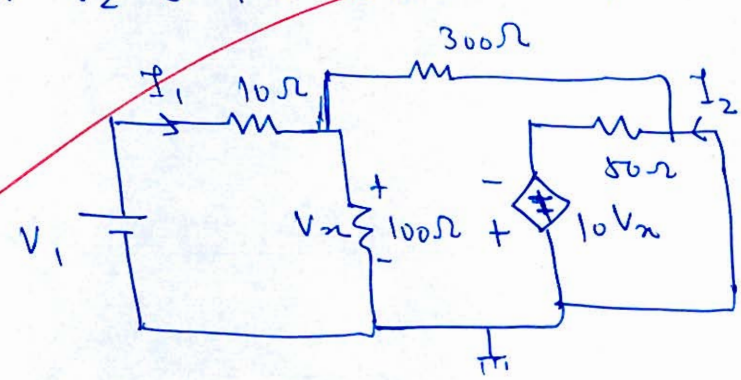
Ans

Equation of h-parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

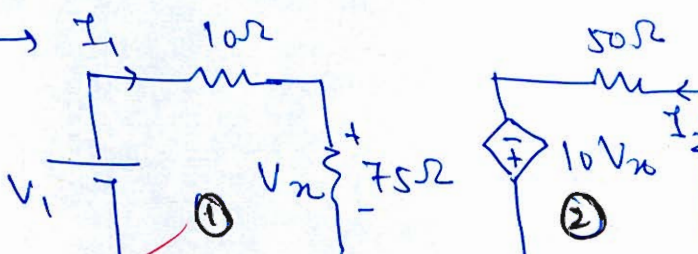
$$I_2 = h_{21} I_1 + h_{22} V_2$$

for h_{11} & h_{21}, put $V_2 = 0$ i.e short circuit port-②



∴ 300Ω will be parallel to 100Ω

∴ circuit looks like →



In loop ①,

$$I_1 = \frac{V_1}{85}$$

$$\therefore V_1 = 85 I_1 \quad \text{--- ①} \Rightarrow \underbrace{h_{11} = 85\Omega}$$

$$\& \; V_x = V_1 \times \frac{75}{85} \quad \text{(voltage divison)}$$

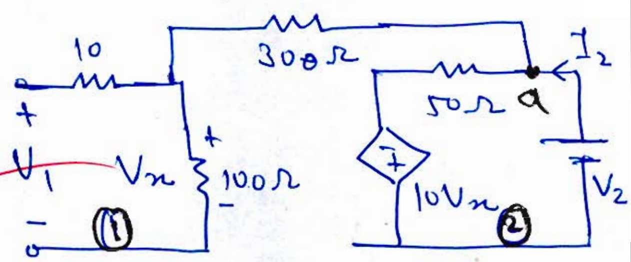
Loop-②

$$\therefore I_2 = \frac{10 V_x}{50} = \frac{V_1}{5} \times \frac{75}{85}$$

from eq^n ① $I_2 = \frac{15}{85} \times (85 I_1) = 15 I_1$

$\therefore$ $\underline{h_{21} = 15}$

- for h_{12} & h_{22}, open circuit port-① i.e, $I_1 = 0$



$\therefore$ from loop-①,

$V_x = V_1$ — ②

$\therefore$ by voltage division,

$$V_1 = V_2 \times \frac{100}{300+100}$$

$\therefore$ $V_1 = \frac{1}{4} V_2 = 0.25 V_2$ — ③

$\therefore$ $\underline{h_{12} = 0.25}$ $\cancel{V_2}$

apply KCL at node-ⓐ,

$$I_2 = \frac{V_2}{400} + \frac{V_2 + 10V_x}{50}$$

$$I_2 = \frac{V_2}{400} + \frac{V_2 + 10V_1}{50} \quad \text{(from eq. ②)}$$

$$I_2 = \frac{V_2}{400} + \frac{V_2 + 10(0.25V_2)}{50} \quad \text{(from eq}^n\text{ ③)}$$

$$I_2 = \frac{V_2}{400} + \frac{3.5V_2}{50} = 0.0725 V_2$$

$\therefore$ $\underline{h_{22} = 0.0725 \ ℧}$

Do not write in this margin

$\therefore$ h-parameters of given circuit are

$h_{11} = 35\,\Omega$, $h_{21} = 15$

$h_{12} = 0.25$, $h_{22} = 0.0725\,\mho$

Ans

18

Good Approach

Q.8 (b) (i) A system is represented by a state model as

$$\dot{x}_1 = -2x_1 - x_2 - 3x_3 + 2r$$
$$\dot{x}_2 = -2x_2 + x_3 + r$$
$$\dot{x}_3 = -7x_1 - 8x_2 - 9x_3 + 2r$$

The output, $y = 4x_1 + 6x_2 + 8x_3$

Check the controllability and observability of the system.

(ii) Explain the following instruction sets of 8086 microprocessor with example.
1. ROL; 2. ROR; 3. RCR; 4. RCL

[12 + 8 marks]

i) from equation form to state space model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} r$$

$$y = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] r$$

$$\therefore A = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$C = [4\,6\,8] \qquad D = [0]$$

Using kalman's controllability test,

controllability matrix

$$Q_c = [B \quad AB \quad A^2B]$$

$$\therefore B = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}_{3\times 1} \qquad AB = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -11 \\ 0 \\ -40 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 142 \\ -40 \\ 437 \end{bmatrix}$$

$$\therefore\ Q_c = \begin{bmatrix} 2 & -11 & 142 \\ 1 & 0 & -40 \\ 2 & -40 & 437 \end{bmatrix} \Rightarrow |Q_c| = -3193 \neq 0$$

$\because$ $|Q_c|$ is non-zero which ~~Q~~ means

Q_c is non-singular matrix.

$\therefore$ System is fully controllable.

• Kalman's test for observability

$$Q_0 = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix}$$

$$CA = \underset{1\times3}{\begin{bmatrix} 4 & 6 & 8 \end{bmatrix}} \underset{3\times3}{\begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}}$$

$$CA^2 = \begin{bmatrix} 674 & 848 & 814 \end{bmatrix}$$

$$= \begin{bmatrix} -64 & -80 & -78 \end{bmatrix}$$

$$\therefore\ Q_0 = \begin{bmatrix} 4 & 6 & 8 \\ -64 & -80 & -78 \\ 674 & 848 & 814 \end{bmatrix}$$

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$$\Rightarrow |Q_0| = -1576 \neq 0$$

$\because$ $|Q_0|$ is non-zero i.e. Q_0 is non-~~sig~~ singular matrix.

$\therefore$ System is observable too.

So, system is fully controllable and observable.

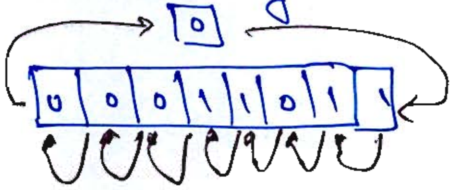
Ans

(ii) ~~ROL: Righ Accumulator left shift without~~ carry.

① ROL: ~~Ri~~ Left shift accumulator with carry

let say A = [00011011]

and cy = [0]



- Here, left shift each bit by one bit and D_0 is replaced by carry bit here.

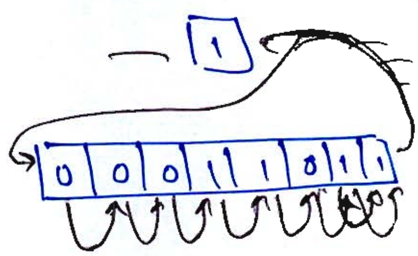
② ROR:- Shift Accumulator Right with carry

- As of the above, same function will occur with carry but here only instead of left we shift to Right side by one carry.

③ RCR:- Shift Accumulator Right without carry,

Let say A = [00011011]

& cy = [1]



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- Here, shifting accumulator bits right by one adjacent bit and in this series we won't use carry bit.

④ RCL:- Shift Accumulator Left w/o carry

- Similar to above one and here only shifting will be left instead of right side.

Do not write in this margin

Q.8 (c) Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose we are given the following facts :

1. $x(t)$ is real and non-negative.
2. $F^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is independent of t.
3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi$

Determine a closed-form expression of $x(t)$.

[20 marks]

Ans given, $x(t) \xrightarrow{F.T.} X(j\omega)$

(1) $x(t)$ is real $\xrightarrow{F.T.}$ $X(j\omega)$ will be Conjugate symmetry.

~~(2) (3) $\therefore$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 e^{j\omega t} d\omega$~~

$\therefore$ ~~at $t=0$~~

~~$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2$~~

(3) by Parseval's theorem, of energy,

$$P_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$\therefore \int_{-\infty}^{\infty} |x(t)|^2 dt = 1 \quad \text{— (1)}$$

(2) (2) from information (2),

$$A e^{-2t} u(t) \xrightarrow{F.T.} \frac{A}{2+j\omega}$$

$$\therefore (1+j\omega) X(j\omega) = \frac{A}{2+j\omega}$$

$$\therefore \quad X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)} = A\left(\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right)$$

apply inverse laplace transform

$$x(t) = \left(Ae^{-t} - Ae^{-2t}\right)u(t)$$

from eq^n. (1).

$$\int_{0}^{\infty} \left| Ae^{-t} - Ae^{-2t} \right|^2 dt = 1$$

$$A^2\left[\int_0^\infty \left(e^{-2t} + e^{-4t} - 2e^{-3t}\right)dt\right] = 1$$

$$A^2\left[\frac{e^{-2t}}{2} + \frac{e^{-4t}}{4} - \frac{2}{3}e^{-3t}\right]_\infty^0 = 1$$

$$A^2\left(\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right) = 1$$

$$A^2\left(\frac{6+3-8}{12}\right) = 1$$

$$A^2 = 12$$

$$A = +\sqrt{12}$$

$\therefore \quad x(t) = \sqrt{12}\left(e^{-t} - e^{-2t}\right) u(t)$

Ans

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Space for Rough Work