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Detailed Solutions

ESE-2023 Mains Test Series

Electrical Engineering Test No : 5

Section A : Basic Electronics Engineering + Analog Electronics + Electrical Materials

Q.1 (a) Solution:

Given :

Forward voltage, $V_D = 0.5$ V

The current equation of diode,

$$I_D = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right]$$

I_o = Reverse saturation current

V_T = Thermal voltage

η = Ideality factor

$I_{o1} = I_{o2} \times 2^{10}$

At $T = 400°$ K, for diode D_1

Thermal voltage, $V_T = \dfrac{T}{11600}$

$$V_T = \frac{400}{11600} = 0.0345 \text{ V}$$

$$I_{D1} = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right]$$

$$= I_o\left[e^{\frac{0.5}{2\times 0.0345}} - 1\right]$$

$$I_{D1} = 1402.01 I_o$$

At T = 300°K; for diode D_2

Thermal voltage, $$V_T = \frac{T}{11600}$$

$$V_T = \frac{300}{11600} = 0.0258 \text{ V}$$

$$I_{D2} = I_o\left[e^{\frac{V_D}{\eta V_T}} - 1\right]$$

$$= I_o\left[e^{\frac{0.5}{2\times 0.0258}} - 1\right]$$

$$I_{D2} = 16152.99 I_o$$

Required ratio, $$\frac{I_{D1}}{I_{D2}} = \frac{1402.01 I_o}{16152.99 I_o}$$

$$\frac{I_{D1}}{I_{D2}} = 0.0868$$

Q.1 (b) Solution:

Given : $M = 3.2 \times 10^{-4}$ Bohr magnetron

T = 27° C, 1 Bohr magnetron = 9.27×10^{-24} A/m^2

The magnetization for a paramagnetic spin is given by Curie law, i.e.,

$$M = \frac{NP_B^2 \mu_o H}{kT}$$

Magnetization per spin in Bohr magnetron,

$$M' = \frac{M}{NP_B} = \frac{P_B \mu_o H}{kT}$$

$$P_B = \text{Bohr magnetron}$$

$$M' = \frac{9.27\times 10^{-24} \times 4\pi \times 10^{-7} \times H}{1.38 \times 10^{-23} \times 300}$$

$$H = \frac{3.2 \times 10^{-4} \times 1.38 \times 10^{-23} \times 300}{9.27 \times 10^{-24} \times 4\pi \times 10^{-7}}$$

$$H = 113726.25 \text{ A/m}$$

$$H = 1.14 \times 10^{5} \text{ A/m}$$

Q.1 (c) Solution:

Given : $V_{Th} = 0.9$ V, $\mu_n C_{ox} = 40\ \mu\text{A/V}^2$

Both the transistors are operating in saturation mode as drain is shorted to gate.

So, $I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{Th})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{Th})^2$$

$$V_{GS1} = 6 - V_o$$

$$V_{GS2} = V_o$$

$$60(6 - V_o - 0.9)^2 = 25(V_o - 0.9)^2$$

$$12(5.1 - V_o)^2 = 5(V_o - 0.9)^2$$

$$12(26.01 - 10.2V_o + V_o^2) = 5(V_o^2 - 1.8V_o + 0.81)$$

$$7V_o^2 - 113.4V_o + 308.07 = 0$$

On solving,

$$V_o = 12.75 \text{ V}, 3.45 \text{ V}$$

$$V_o = 3.45 \text{ V}$$

Q.1 (d) Solution:

Top-down Approach : In top-down approach, nano-scale objects are made by processing larger objects in size. Integrated circuit fabrication is an example for top down nanotechnology. For nano-material synthesis ball-milling is an important top-down approach where macro crystalline structures are broken down to nano-crystalline structures but original integrity of the material is retained. Sometimes this method is used to prepare nanostructured metal oxides by chemical reaction between two constituents during crushing. The crystallites are allowed to react with each other by the supply of kinetic energy during milling process to form the required nanostructured oxide.

Bottom-up Approach : Bottom-up approach in nanotechnology is making larger nanostructures from smaller building blocks such as atoms and molecules, therefore, very important for nano-fabrication. The bottom-up non-lithographic approach of nano-material synthesis is not completely proven in manufacturing yet, but has a great potential to become important alternative to lithographic process. Examples of bottom-up technique are self assembly of nanomaterials solgel technology, electrodeposition, physical and chemical vapour deposition (PVD and CVD), epitaxial growth, laser ablation etc.

Q.1 (e) Solution:

(i) Given :

$$R_H = -8.25 \times 10^{-5} \text{ m}^3/\text{C}$$

$$\sigma = 2.50 \ \Omega^{-1} \text{ cm}^{-1}$$

Here, R_H is negative. So, the type of semiconductor is n-type.

(ii) Density of charge carrier,

$$n = \left|\frac{-1}{R_H e}\right|$$

$$= \left|\frac{-1}{8.25 \times 10^{-5} \times 1.6 \times 10^{-19}} \text{m}^{-3}\right|$$

$$n = 7.575 \times 10^{22} \text{ m}^{-3}$$

(iii) We know,

$$\sigma = ne\mu_n$$

Mobility,

$$\mu_n = \frac{\sigma}{ne}$$

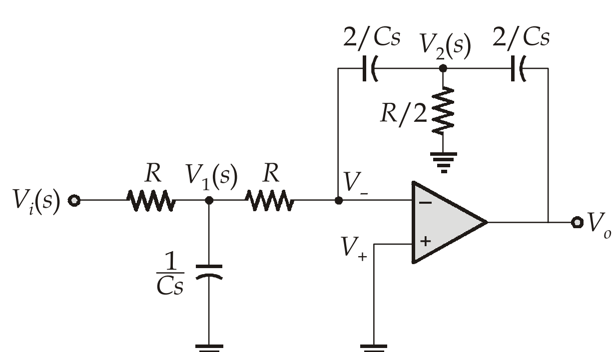
$$\mu_n = \frac{2.5 \times 100 \ \mho/\text{m}}{7.575 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$\mu_n = 0.020625 \text{ m}^2/\text{V-s}$$

$$\mu_n = 206.25 \text{ cm}^2/\text{V-s}$$

Q.2 (a) Solution:

The circuit is s-domain



Due to virtual short

$$V_+ = V_- = 0 \text{ V}$$

Apply nodal analysis at node $V_1(s)$

$$\frac{V_1(s) - V_i(s)}{R} + \frac{V_1(s)}{1/Cs} + \frac{V_1(s)}{R} = 0$$

$$V_1(s)\left[\frac{1}{R} + \frac{1}{R} + Cs\right] = \frac{V_i(s)}{R}$$

$$V_1(s) = \frac{V_i(s)}{(2 + RCs)} \quad \text{...(1)}$$

Apply nodal analysis at node V_-

$$\frac{0 - V_1(s)}{R} + \frac{0 - V_2(s)}{2/Cs} = 0$$

$$V_1(s) = -\frac{RCs}{2}V_2(s) \quad \text{...(2)}$$

From eqn. (1) and (2)

$$-\frac{RCs}{2}V_2(s) = \frac{V_i(s)}{(2 + RCs)}$$

$$V_2(s) = \frac{-2}{RCs(2 + RCs)}V_i(s) \quad \text{...(3)}$$

Apply nodal analysis at node $V_2(s)$

$$\frac{V_2(s) - 0}{2/Cs} + \frac{V_2(s)}{R/2} + \frac{V_2(s) - V_o(s)}{2/Cs} = 0$$

$$\left(\frac{Cs}{2} + \frac{Cs}{2} + \frac{2}{R}\right)V_2(s) = \frac{Cs}{2}V_o(s)$$

$$V_o(s) = \frac{2(2 + RCs)}{RCs}V_2(s) \quad \text{...(4)}$$

From eqn. (3) and (4)

$$V_o(s) = \frac{2(2 + RCs)}{RCs}\left[\frac{-2}{RCs(2 + RCs)}V_i(s)\right]$$

$$V_o(s) = \frac{-4}{(RCs)^2}V_i(s)$$

$$v_o(t) = \frac{-4}{(RC)^2}\int\left[\int v_i(t)dt\right]dt$$

Q.2 (b) Solution:

Density, $\rho = 6.51 \text{ g/cm}^3$

Atomic weight, $A_{zr} = 91.2 \text{ g/mol}$

HCP crystal structure, i.e., $n = 6$ atoms/unit cell.

(i) We know, Volume of zirconium,

$$V_c = \frac{nA_{zr}}{\rho N_A}$$

where, N_A = Avogadro number

$$V_c = \frac{6 \times 91.2}{6.51 \times 6.023 \times 10^{23}}$$

$$= 1.396 \times 10^{-22} \text{ cm}^3$$

$$V_c = 1.396 \times 10^{-28} \text{ m}^3$$

(ii) For HCP structure,

$$\frac{c}{a} = 1.593$$

$$c = 1.593a$$

The volume of Zr unit cell,

$$V_c = \frac{3\sqrt{3}}{2}a^2c = \frac{3\sqrt{3}}{2}(2R)^2c$$

$$a = 2R$$

$$V_c = \frac{3\sqrt{3}}{2}a^2(1.593a)$$

$$a = \sqrt[3]{\frac{2V_c}{3\sqrt{3} \times 1.593}}$$

$$= \sqrt[3]{\frac{2 \times 1.396 \times 10^{-28}}{3\sqrt{3} \times 1.593}}$$

$$a = 3.23 \times 10^{-10} \text{ m}$$

$$a = 0.323 \text{ nm}$$

$$c = 1.593a$$

$$= 1.593 \times 0.323 \times 10^{-9}$$

$$c = 0.5145 \text{ nm}$$

Q.2 (c) Solution:

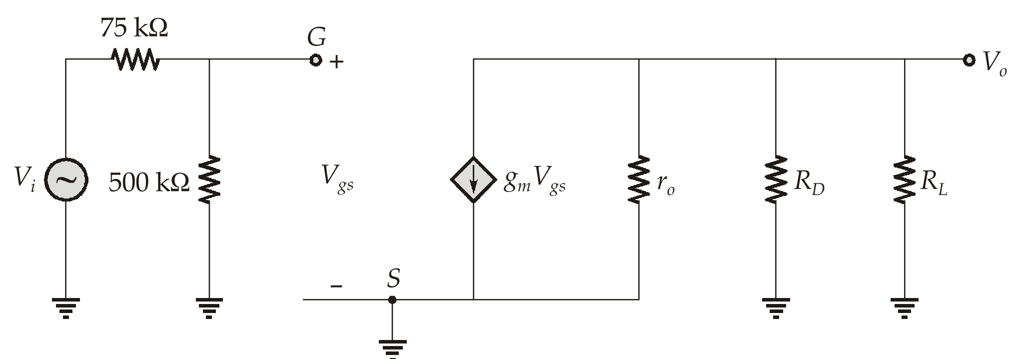
Given :
$$V_T = 0.8 \text{ V}, k = \frac{\mu_n C_{ox} W}{2L} = 1.2 \text{ mA/V}^2$$
$$\lambda = 0, I_{DQ} = 0.6 \text{ mA}$$
$$I_{DQ} = k(V_{GSQ} - V_T)^2$$
$$0.6 \times 10^{-3} = 1.2 \times 10^{-3}(V_{GSQ} - 0.8)^2$$
$$V_{GSQ} = \sqrt{\frac{0.6 \times 10^{-3}}{1.2 \times 10^{-6}}} + 0.8$$
$$V_{GSQ} = 1.507 \text{ V}$$

AC Analysis :

Trans-conductance,
$$g_m = 2k(V_{GSQ} - V_T)$$
$$g_m = 2 \times 1.2 \times 10^{-3}(1.507 - 0.8)$$
$$g_m = 1.697 \text{ m℧}$$

Output resistance,
$$r_o = \infty$$

Small signal equivalent circuit



Output voltage,
$$v_o = -g_m V_{gs}[r_o \parallel R_D \parallel R_L]$$
$$v_{gs} = \frac{500k}{(500+75)k} v_i$$

Voltage gain,
$$A_v = \frac{v_o}{v_i} = \frac{-g_m[r_o \parallel R_D \parallel R_L)500}{500+75}$$
$$A_v = \frac{-1.697 \times 10^{-3}[3.4 \times 10^3] \times 500}{575}$$
$$A_v = -5.017$$

Q.3 (a) Solution:

Given : $\mu_n = 7.5\ \text{cm}^2/\text{V-s};\ \rho = 9.43 \times 10^{-6}\ \Omega\text{-m}$

(i) We know, $\sigma = ne\mu_n$

$$n = \frac{1}{\rho\mu_n e}$$

$$= \frac{1}{9.43\times10^{-4}\times7.5\times1.6\times10^{-19}}$$

$$= 8.84 \times 10^{20}\ \text{cm}^{-3}$$

Atomic concentration, $n_{at} = \dfrac{dN_A}{M_{at}}$

$$n_{at} = \frac{7.3\times6.023\times10^{23}}{115}$$

$$= 3.82 \times 10^{22}\ \text{cm}^{-3}$$

Effective number of electrons denoted by per In atom

$$n_{\text{eff}} = \frac{n}{n_{at}} = \frac{8.84\times10^{26}}{3.82\times10^{22}}$$

$$\eta_{\text{eff}} = 23141.36$$

(ii) Given : $l_e = 8.2\ \text{nm}$

Drift mobility, $\mu_m = \dfrac{e\tau}{m_e}$

$$\tau = \frac{\mu_n m_e}{e} \qquad (m_e = \text{mass of electron})$$

$$\tau = \frac{7.5\times10^{-4}\times9.1\times10^{-31}}{1.6\times10^{-19}}$$

$$\tau = 4.266 \times 10^{-15}\ \text{s}$$

We know, $l_e = v_d\tau$

$$v_d = \frac{8.2\times10^{-9}}{4.266\times10^{-15}}\ \text{m/s}$$

$$v_d = 1.922 \times 10^{6}\ \text{m/s}$$

(iii) According to Wiedemann-Franz law,

Thermal conductivity, $K = \sigma T C_{\text{WFL}}$

where C_{WFL} = Lorentz number (or) Wiedemann-Frantz-Lorentz coefficient

$$C_{WFL} = \frac{\pi^2 K_B^2}{3e^2}$$

$$= \frac{\pi^2(1.38\times10^{-23})^2}{3\times(1.6\times10^{-19})^2}$$

$$= 2.45 \times 10^{-8}\ \text{W}\Omega/\text{K}^2$$

$$K = \sigma T C_{WFL} = \frac{TC_{WFL}}{\rho}$$

$$= \frac{300\times2.45\times10^{-8}}{9.43\times10^{-6}}$$

$$K = 0.78\ \text{W/K}$$

Q.3 (b) Solution:

Given : $V_{CEQ} = 11.5$ V, $I_C = 1.5$ mA, $\beta = 50$

$$I_E = I_C + I_B = I_C + \frac{I_C}{\beta}$$

$$I_E = \left(1.5 + \frac{1.5}{50}\right)\text{mA} = 1.53\ \text{mA}$$

Apply kVL in C-E loop

$$-V_{CC} + I_C R_C + V_{CEQ} + I_E R_E = 0$$

$$R_E = \frac{V_{CC} - V_{CEQ} - I_C R_C}{I_E}$$

$$R_E = \frac{20 - 11.5 - (1.5\times5)}{1.53}\ \text{k}\Omega$$

$$= 0.6536\ \text{k}\Omega$$

$$R_E = 653.6\ \Omega$$

Thevenin equivalent circuit

$$R_B = R_1 \parallel R_2$$

$$V_{Th} = V_{R2} = \frac{R_2 V_{CC}}{R_1 + R_2}$$

Also, $V_{Th} = V_{R2} = V_{BE} + I_E R_E$

$$V_{Th} = 0.7 + (1.53 \times 10^{-3} \times 653.6)$$

$$V_{R2} = V_{Th} = 1.7 \text{ V}$$

$$1.7 = \frac{R_2 \times 20}{R_1 + R_2}$$

$$\frac{R_1}{R_2} = \frac{20}{1.7} - 1$$

$$R_1 = 10.765 R_2$$

For self-bias circuit, stability factor,

$$S = \frac{1+\beta}{1 + \dfrac{\beta R_E}{R_{th} + R_E}}$$

$$2 = \frac{51}{1 + \dfrac{50 \times R_E}{R_{th} + R_E}}$$

$$R_{th} = 1.04 R_E = 680\ \Omega$$

$$V_{th} = I_B R_{th} + V_{BE} + I_E R_E$$

$$= 1.72 \text{ V}$$

$$\frac{R_{th}}{V_{th}} = \frac{\dfrac{R_1 R_2}{R_1 + R_2}}{\dfrac{V_{cc} R_2}{R_1 + R_2}} = \frac{R_1}{V_{cc}}$$

$$R_1 = \frac{V_{cc} R_{th}}{V_{th}} = 7.9 \text{ k}\Omega$$

$$R_2 = \frac{R_1 \times R_{th}}{R_1 - R_{th}} = 0.744 \text{ k}\Omega$$

Q.3 (c) Solution:

(i) Given :

Gain, $A = 800$

$f_U = 16 \text{ kHz}; \; f_L = 40 \text{ Hz}$

Bandwidth, $BW = f_U - f_L$

$= 16000 - 40$

$\text{BW} = 15.96 \text{ kHz}$

Feedback factor, $\beta = \frac{2}{100} = 0.02$

Due to negative feedback bandwidth increases, i.e., upper cut-off frequency increases and lower cut-off frequency decreases by a factor $(1 + A\beta)$

$$f_{Lf} = \frac{40}{1+800\times0.02} = \frac{40}{17}\text{ Hz} = 2.353\text{ Hz}$$

$$f_{Uf} = 16\times10^3(1+800\times0.02) = 16\times17\text{ kHz} = 272\text{ kHz}$$

$$BW = \left(272 - \frac{2.353}{1000}\right)\text{ kHz}$$

$$BW = 271.998\text{ kHz}$$

(ii) Superconductivity : The state of material at which resistivity reduces to zero is called superconductivity. The two independent condition for superconductivity are

1. Zero resistivity ($\rho = 0$)
2. Perfect diamagnetism, i.e., $\chi_m = -1$ or $\mu_r = 0$

Properties of Superconductor :

(i) The resistivity 'ρ' of superconducting materials are greater than other conductors at room temperature.

(ii) All the thermoelectric effects disappear in superconducting state.

(iii) When a sufficient strong magnetic field is applied to superconductor below critical temperature T_c, its superconducting property is destroyed.

(iv) The magnetic flux density in superconductor is zero.

(v) When current is passed through the superconducting material the heat loss (I^2R) is zero.

Q.4 (a) Solution:

(i) Given : $L_s = 0.33$ H, $C_s = 0.065$ pF, $C_P = 1$ pF, $R_s = 5.5$ kΩ

(a) Series resonant frequency

$$f_s = \frac{1}{2\pi\sqrt{L_sC_s}}$$

$$f_s = \frac{1}{2\pi\sqrt{0.33\times0.065\times10^{-12}}}$$

$$f_s = 1086.69\text{ kHz}$$

(b) Parallel resonant frequency

$$f_P = \frac{1}{2\pi\sqrt{L_s C_{eq}}}$$

$$C_{eq} = \frac{C_P C_s}{C_P + C_s} = \frac{0.065 \times 1}{0.065 + 1} \text{ pF}$$

$$C_{eq} = 0.061 \text{ pF}$$

$$f_P = \frac{1}{2\pi\sqrt{0.33 \times 0.061 \times 10^{-12}}}$$

$$f_P = 1121.76 \text{ kHz}$$

% increase in parallel resonant frequency

$$= \frac{1121.76 - 1086.69}{1086.69} \times 100 = 3.23\%$$

(c) Quality factor at series resonant frequency

$$Q = \frac{\omega_s L}{R} = \frac{2\pi \times 1086.69 \times 10^3 \times 0.33}{5.5 \times 10^3}$$

$$Q = 409.67$$

Quality factor at parallel resonant frequency

$$Q = \frac{\omega_P L}{R} = \frac{2\pi \times 1121.76 \times 10^3 \times 0.33}{5.5 \times 10^3}$$

$$Q = 422.89$$

(ii) Given : $\in_r = 4.1, \ \tan\delta = 0.001, \ f = 60 \text{ Hz}, \ E = 45 \text{ kV/cm}$

The heat generated,

$$W_E = \frac{E^2 f \varepsilon_r \tan\delta}{1.8 \times 10^{12}} \text{ W/cm}^3$$

$$= \frac{(45 \times 10^3)^2 \times 60 \times 4.1 \times 0.001}{1.8 \times 10^{12}}$$

$$W_E = 2.7675 \times 10^{-4} \text{ W/cm}^3$$

$$W_E = 276.75 \ \mu\text{W/cm}^3$$

Q.4 (b) Solution:

Given : $I_{D(ON)} = 3 \text{ mA at } V_{GS(ON)} = 10 \text{ V}$

$V_{GS(TH)} = 5 \text{ V}$

We know,

$$k = \frac{I_{D(ON)}}{[V_{GS(ON)} - V_{GS(TH)}]^2}$$

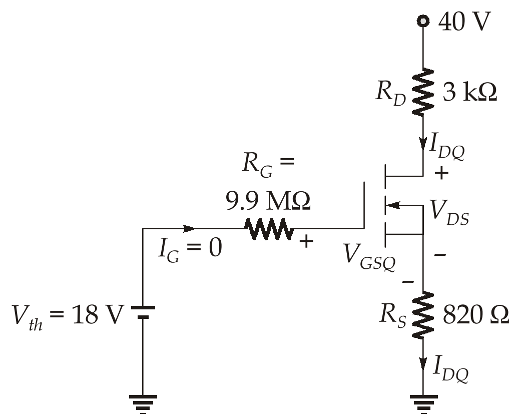
$$k = \frac{3}{(10-5)^2} = 0.12 \text{ mA/V}^2$$

Thevenin equivalent circuit,

$$V_{Th} = \frac{40 \times 18}{18+22} = 18 \text{ V}$$

$$R_{Th} = R_G = 18 \,||\, 22 = \frac{18 \times 22}{18+22} = 9.9 \text{ M}\Omega$$

Thevenin equivalent circuit,



Apply KVL in G-S loop,

$$-V_{Th} + I_G R_G + V_{GS} + I_{DQ} R_S = 0$$

$$I_D = \frac{18 - V_{GS}}{820} \text{ A} \quad \text{...(1)}$$

Also, the current in MOSFET

$$I_D = K[V_{GS} - V_{GS(TH)}]^2$$

$$I_D = 0.12 \times 10^{-3}[V_{GS} - 5]^2 \quad \text{...(2)}$$

From eqn. (1) and (2)

$$\frac{18 - V_{GS}}{820} = 0.12 \times 10{-}3[V_{GS}^2 - 10V_{GS} + 25]$$

$$9.84V_{GS}^2 - 98.4V_{GS} + 246 = 1800 - 100V_{GS}$$

$$9.84V_{GS}^2 + 1.6V_{GS} - 1554 = 0$$

$$V_{GS} = 12.486, -12.648$$

$$V_{GS} = 12.486 \text{ V}$$

$$I_{DQ} = \frac{18 - 12.486}{820} = 6.724 \times 10^{-3} \text{ A}$$

$$I_{DQ} = 6.724 \text{ mA}$$

Apply KVL in D-S loop,

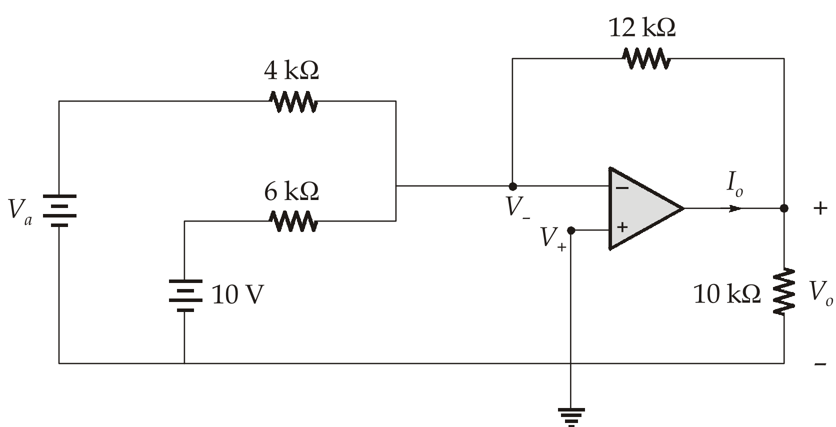
$$-40 + R_D I_{DQ} + V_{DS} + R_S I_{DQ} = 0$$

$$V_{DS} = 40 - (3 + 0.82) \times 10^3 \times 6.724 \times 10^{-3}$$

$$V_{DS} = 14.314 \text{ V}$$

Q.4 (c) Solution:

(i) For $V_a = 4$ V



Due to virtual ground,

$$V_+ = V_- = 0 \text{ V}$$

Apply KCL at node V_-

$$\frac{V_- - 4}{4k} + \frac{V_- - 10}{6k} + \frac{V_- - V_o}{12k} = 0$$

$$V_o = \frac{12}{4}(-4) - \frac{12}{6} \times 10$$

$$V_o = -32 \text{ V}$$

Current I_o : Apply KCL at node V_o

$$\frac{v_o}{10k} + \frac{v_o - V_-}{12k} = I_o$$

$$I_o = \left[\frac{-32}{10} + \frac{-32}{12}\right] \text{mA}$$

$$V_o = -32\text{ V}$$

$$I_o = -5.867\text{ mA}$$

(ii) For linear operation,

$$-12 < v_o < 12\text{ V}$$

Apply KCL at node V_-

$$\frac{V_- - V_a}{4k} + \frac{V_- - (+10)}{6k} + \frac{V_- - V_o}{12k} = 0$$

$$-3v_a - 20 = v_o$$

$$-12 < -3v_a - 20 < 12$$

$$-3v_a - 20 > -12$$

$$v_a < \frac{-8}{3} \Rightarrow v_a < -2.667$$

$$-3v_a - 20 < 12$$

$$v_a > -\left(\frac{20+12}{3}\right) \Rightarrow v_a > -10.667$$

Range : $-10.667 < v_a < -2.667$

Section B : Electrical Machine-1 + Power Systems-2

Q.5 (a) Solution:

For transformer T_1, π-model is

$$y_{13} = \frac{y_t}{a} = \frac{-j80}{0.8} = -j100$$

$$y_{10} = \left(\frac{1-a}{a^2}\right)y_t = \left(\frac{1-0.8}{0.64}\right) \times (-j80) = -j25$$

$$y_{30} = \left(\frac{a-1}{a}\right)y_t = \frac{0.8-1}{0.8} \times (-j80) = j20$$

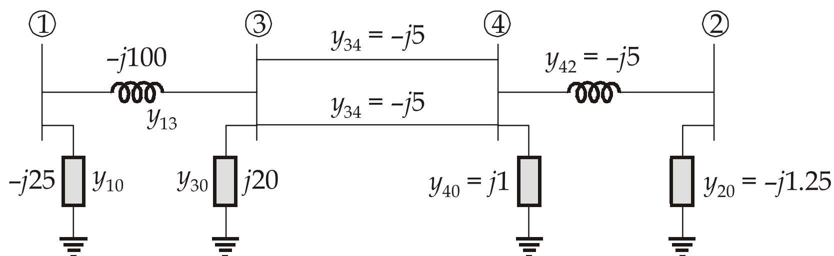
Similarly for transformer T_2, π-model is

$$y_{42} = \frac{y_t}{a} = \frac{-j100}{16 \times 1.25} = -j5$$

$$y_{40} = \left(\frac{1-a}{a^2}\right) \times y_t = \left(\frac{-0.25}{1.25}\right) \times \left(\frac{-j100}{16}\right) = j1$$

$$y_{20} = \left(\frac{a-1}{a}\right) \times y_t = \left(\frac{1.25-0.25}{1.25}\right) \times \left(\frac{-j100}{16}\right) = -j1.25$$

All line impedances are converted to admittances and the admittance diagram constructed as shown :



Now,

$$Y_{11} = y_{10} + y_{13} = -j125;\ Y_{12} = 0;\ Y_{13} = -y_{13} = j100;\ Y_{14} = 0$$
$$Y_{21} = -y_{12} = 0;\ Y_{22} = y_{20} + y_{24} = -j1.25 - j5 = -j6.25;$$
$$Y_{23} = 0;\ Y_{24} = -y_{24} = j5$$
$$Y_{31} = -y_{31} = j100;\ Y_{32} = 0;$$
$$Y_{33} = y_{30} + y_{31} + y_{34} = j20 - j10 - j100 = -j90;\ Y_{34} = -j9$$
$$Y_{41} = 0;\ Y_{42} = j5;\ Y_{43} = j10;\ Y_{44} = j1 - j5 - j10 = j14$$

Now, Bus-Admittance Matrix,

$$[Y_{Bus}] = \begin{bmatrix} -j125 & 0 & j100 & 0 \\ 0 & -j6.25 & 0 & j5 \\ j100 & 0 & -j90 & j10 \\ 0 & j5 & j10 & -j13 \end{bmatrix}$$

Q.5 (b) Solution:

Reluctance of path,

$$R = \frac{l}{\mu_o \mu_r A} = \frac{l}{\mu_o A} \qquad (\because l = 2x)$$

$$R = \frac{2x}{\mu_o A}$$

Therefore, inductance of the coil,

$$L(x) = \frac{N^2}{R} = \frac{N^2 \mu_o A}{2x}$$

Now, we know that energy stored in the coil is,

$$W_f(\lambda, x) = \frac{1}{2} \cdot \frac{\lambda^2}{L} = \frac{1}{2} L(x) I^2 \qquad (\because \lambda = LI)$$

$$W_f(\lambda, x) = \frac{1}{2}\cdot\frac{N^2\mu_o A I^2}{2x} = \frac{N^2\mu_o A I^2}{4x}$$

The force expression on electromechanical system can be given as

$$F_f = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

$$F_f = \frac{N^2\mu_o A I^2}{4x^2}$$

$$N^2 = \frac{4x^2 F_f}{\mu_o A I^2}$$

$$N^2 = \frac{4\times 1\times 10^{-6}\times 500}{4\pi\times 10^{-7}\times 600\times 10^{-6}\times 625}$$

$$N^2 = 4244.13$$

$$N = \sqrt{4244.65} = 65.14 \simeq 65 \text{ turns}$$

Q.5 (c) Solution:

(i) With direct-online starting,

Maximum line current, $I_L = 120$ A

Starting current, $I_{st} = 6I_{fL}$

Since maximum line current drawn from the supply is 120 A,

$$6I_{fL} = 120$$

$$I_{fL} = \frac{120}{6} = 20 \text{ A}$$

Maximum permissible rating of the motor is,

$$S = \sqrt{3}V_L I_{fL} = \sqrt{3}\times 400\times 20 = 13.856 \text{ kVA}$$

(ii) With auto-transformer starting,

$$I_{st} = x^2 I_{sc} = x^2 \times (6I_{fL})$$

$$I_{fL} = \frac{120}{(0.6)^2\times 6} = 55.55 \text{ A}$$

Maximum permissible rating of motor,

$$= \sqrt{3}V_L I_{fL}$$

$$= \sqrt{3}\times 400\times 55.55 = 38.49 \text{ kVA}$$

(iii) With star-delta starting,

$$I_{st} = \frac{1}{3} I_{sc} = \frac{1}{3} \times 6 I_{fL}$$

$$I_{fL} = \frac{I_{st}}{2} = \frac{120}{2} = 60 \text{ A}$$

Maximum permissible kVA rating of the motor is

$$= \sqrt{3} \times 400 \times 60 = 41.56 \text{ kVA}$$

Q.5 (d) Solution:

Given : 230 V, 20 hp, 60 Hz, 6 pole, 3-ϕ induction motor

$$T = \text{Constant at } f_{\text{rated}}, V_{\text{rated}} \text{ and } hp_{(\text{rated})}$$

$$T = \frac{3}{\omega_s} \cdot \frac{V_{ph}^2}{\left(\frac{r_2'}{s}\right)^2 + (x_2')^2} \quad \text{...(1)}$$

(∵ When stator impedance is neglected)

We know that when motor runs at full load, then full-load slip of motor is (2 – 5)%, therefore,

$$\left(\frac{r_2'}{s}\right) >> x_1' \Rightarrow \left(\frac{r_2'}{s}\right)^2 >> (x'_2)^2$$

Therefore, we can neglect $(x'^2)^2$ term in eqn. (1).

Now,
$$T = \frac{3}{\omega_s} \cdot \frac{V_{\text{ph}}^2 \left(\frac{r_2'}{s}\right)}{\left(\frac{r_2'}{s}\right)^2} \propto \frac{s V_{\text{ph}}^2}{r_2' f}$$

Now, due to disturbance, $V = 0.9 V_{\text{ph}}$

and $f = 0.94 f_{\text{rated}}$

Since, torque is constant.

Therefore, $T_1 = T_2$

$$\frac{s_1 V_1^2}{f_1} = \frac{s_2 V_2^2}{f_2} \quad (\because R_2 \text{ is constant})$$

$$s_2 = \left(\frac{V_1}{V_2}\right)^2 \times \left(\frac{f_2}{f_1}\right) \times s_1$$

$$= \left(\frac{1}{0.9}\right)^2 \times (0.94) \times \left(\frac{1200 - 1175}{1200}\right)$$

New slip, $s_2 = 0.0241$

New synchronous speed, $N_{s2} = \dfrac{120 \times f_2}{P}$

$$= \frac{120 \times 0.94 \times 60}{6} = 1128 \text{ rpm}$$

Therefore, new speed of motor,

$$N_{r2} = N_{s2}(1 - s_2)$$

$$N_{r2} = 1128(1 - 0.0241)$$

$$N_{r2} = 1100 \text{ rpm}$$

Q.5 (e) Solution:

Given : $y_{11} = y_{10} + y_{12} + y_{13} + y_{14} \Rightarrow -j6 = y_{10} - j2 - j2.5 - j0$

$\Rightarrow$ $y_{10} = -j1.5$

Now, $y_{22} = y_{20} + y_{21} + y_{23} + y_{24} \Rightarrow -j10 = -j2 - j2.5 - j4 + y_{20}$

$\Rightarrow$ $y_{20} = -j1.5$

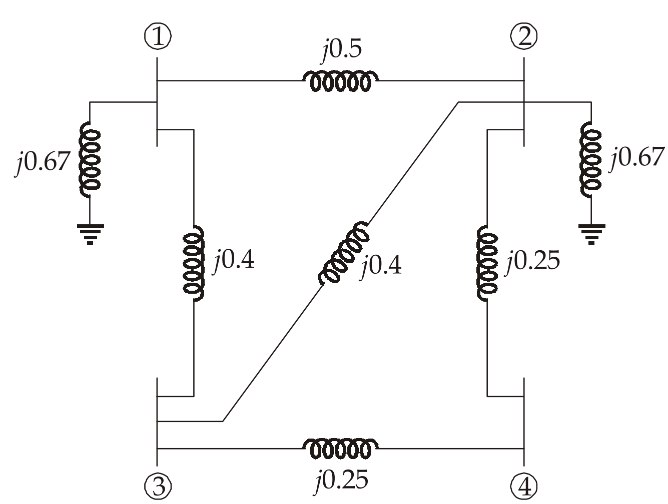
$\Rightarrow$ $y_{33} = y_{31} + y_{32} + y_{30} + y_{34} \Rightarrow -j9 = -j2.5 - j2.5 - j4 + y_{30}$

$\Rightarrow$ $y_{30} = 0$

$\Rightarrow$ $y_{44} = y_{41} + y_{42} + y_{43} + y_{40} \Rightarrow -j8 = -j0 - j4 - j4 + y_{40}$

$\Rightarrow$ $y_{40} = 0$

Now reactance diagram :



Q.6 (a) Solution:

By inspection, the bus admittance matrix in polar form is

$$[Y_{bus}] = \begin{bmatrix} -j60 & j40 & j20 \\ j40 & -j60 & j20 \\ j20 & j20 & -j40 \end{bmatrix}$$

The power flow equations with voltages and admittances expressed in polar form is,

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i| \, |V_j| \, |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of bus admittance matrix in above equations for P_2, P_3 and Q_3 will result in,

Elements of Jacobian Matrix :

$$P_2 = 40|V_1|\,|V_2| \cos\left(\frac{\pi}{2} - \delta_2 + \delta_1\right) + 20|V_2|\,|V_3|\cos\left(\frac{\pi}{2} - \delta_2 + \delta_3\right)$$

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_1|\,|V_2|\sin\left(\frac{\pi}{2} - \delta_2 + \delta_1\right) + 20|V_2|\,|V_3|\sin\left(\frac{\pi}{2} - \delta_2 + \delta_3\right)$$

$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2|\,|V_3|\sin\left(\frac{\pi}{2} - \delta_2 + \delta_3\right)$$

$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos\left(\frac{\pi}{2} - \delta_2 + \delta_3\right)$$

$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3|\,|V_2|\sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right)$$

$$\frac{\partial P_3}{\partial \delta_3} = 20|V_1|\,|V_3| \cdot \sin\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 20|V_2||V_3|\sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right)$$

$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 20|V_2|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_2|\,|V_3|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_1||V_3|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) + 20|V_2||V_3|\cos\left(\frac{\pi}{2} - \delta_3 + \delta_2\right)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin\left(\frac{\pi}{2} - \delta_3 + \delta_1\right) - 20|V_2|\sin\left(\frac{\pi}{2} - \delta_3 + \delta_2\right) + 80|V_3|$$

The load and generation expressed in per unit are

$$P_2^{sch} = \frac{400}{100} = 4.0 \text{ pu}$$

$$S_3^{sch} = -\left(\frac{500 + j400}{100}\right) = -5.0 - j4.0 \text{ pu}$$

Slack bus voltage is $V_1 = 1.0\angle 0°$ pu and bus (2) voltage magnitude is $|V_2| = 1.05$ pu. Starting with initial estimates of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$ and $\delta_3^{(0)} = 0.0$. the power residuals are

$$\Delta P_2^{(0)} = P_2^{Sch} - P_2^{(0)} = 4.0 - 0 = 4.0$$

$$\Delta P_3^{(0)} = P_3^{Sch} - P_3^{(0)} = -5 - 0 = -5.0$$

$$\Delta Q_3^{(0)} = Q_3^{Sch} - Q_3^{(0)} = -4.0 - 1.0 = -5.0$$

Evaluating the elements of Jacobian matrix with initial estimates, the set of linear equation in first iteration becomes

$$\begin{bmatrix} 4.0 \\ -5.0 \\ -5.0 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of above equation.

$$\Delta\delta_2^{(0)} = 0.0275 \Rightarrow \delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)} = 0.0275 \text{ rad} = 1.5782°$$

$$\Delta\delta_3^{(0)} = -0.1078 \Rightarrow \delta_3^{(1)} = 1 + (-0.1078) = -0.1078 \text{ rad} = -6.1790°$$

$$\Delta|V_3^{(0)}| = -0.1282 \Rightarrow |V_3^{(1)}| = 1 + (-0.1282) = 0.8718 \text{ pu}$$

Q.6 (b) Solution:

(i) The incremental fuel cost of the 2 units are given as :

For unit-1 : $$\frac{dF_1}{dP_1} = 0.010P_1 + 2 \text{ Rs/MWhr}$$

For unit-2 : $$\frac{dF_2}{dP_2} = 0.01P_2 + 1.5 \text{ Rs/MWhr}$$

From the coordination equation, we know that

$$\lambda = L_1 \times (\text{I.C.}_1) = L_2 \times (\text{I.C.}_2)$$

Now power loss expression,

$$P_L = B_{11}P_1^2 + 2B_{12}P_1P_2 + B_{22}P_2^2$$

$$P_L = 0.0015P_1^2 + 0.0025P_2^2 - 0.001P_1P_2 \text{ MW}$$

$$\frac{\partial P_L}{\partial P_1} = 0.003P_1 - 0.001P_2$$

$$\frac{\partial P_L}{\partial P_2} = 0.005P_2 - 0.001P_1$$

Now,

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.003P_1 + 0.001P_1}$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \frac{1}{1 - 0.005P_2 + 0.001P_1}$$

$$L_1 \times (\text{I.C.}_1) = \lambda$$

$$\Rightarrow \frac{1}{1 - 0.003P_1 + 0.001P_2} \times (0.01P_1 + 2) = 2.6$$

$$0.01P_1 + 2 = 2.6 - 0.0078P_1 + 0.0026P_2$$

$$0.0178P_1 - 0.0026P_2 = 0.6 \qquad \text{...(1)}$$

Again

$$L_2 \times (\text{I.C.}_2) = X$$

$$\Rightarrow \frac{1}{1 - 0.005P_2 + 0.001P_1} \times (0.01P_2 + 0.75) = 2.6$$

$$0.01P_2 + 0.75 = 2.6 - 0.013P_2 + 0.0026P_1$$

$$-0.0026P_1 + 0.023P_2 = 1.85 \qquad \text{...(2)}$$

Solving eqn. (1) and (2)

$$P_1 = 46.22 \text{ MW}$$

$$P_2 = 85.67 \text{ MW}$$

Power loss,

$$P_L = 0.0015 \times (46.22)^2 + 0.0025 \times (85.67)^2 - 2 \times 0.0005 \times 46.22 \times 85.67$$

$$P_L = 17.807 \text{ MW}$$

Total power generation,

$$P_g = P_D + P_L$$

$$P_1 + P_2 = P_D + P_L$$

Power demand, $P_D = P_1 + P_2 - P_L$

$$= 85.67 + 46.22 - 17.807$$

$$= 114.083 \text{ MW}$$

(ii) Average demand $= \dfrac{\text{Number of units generated or consumed}}{\text{Total number of hours}}$

$$P_{ag} = \frac{600 \times 10^6}{365 \times 24} = 68.50 \text{ MW}$$

Load factor = 0.60

We know that, Load factor $= \dfrac{\text{Average Demand } (P_{ag})}{\text{Maximum Demand } (P_{max})}$

Therefore, $P_{max} = \dfrac{68.50}{0.60}$

$$= 114.167 \text{ MW}$$

Plant utilization factor $= \dfrac{\text{Maximum demand}}{\text{Plant capacity}}$

$$= \frac{114.167}{200} = 0.57$$

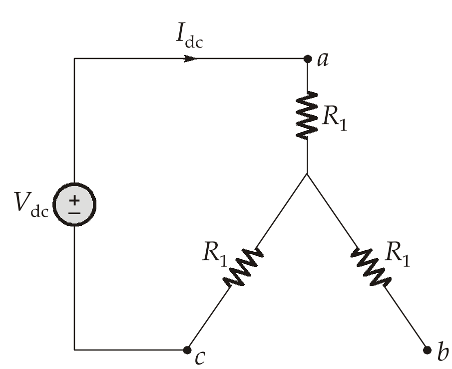
Plant capacity factor $= \dfrac{P_{ag}}{\text{Plant Capacity}} = \dfrac{68.50}{200} = 0.3425$

Reserve capacity = Plant capacity – Maximum demand

$$= 200 - 114.167 = 85.833 \text{ MW}$$

Q.6 (c) Solution:

(i) From the dc test



$$2R_1 = \frac{V_{dc}}{I_{dc}} = \frac{9.07}{28} = 0.3239$$

$$R_{ac} = 1.5 \times R_{dc}$$

Therefore, stator winding resistance,

$$R_1 = \frac{0.4858}{2} = 0.243\ \Omega$$

From no-load test,

$$I_{L,(avg)} = \frac{I_a + I_b + I_c}{3} = \frac{8.12 + 8.20 + 8.18}{3} = 8.17\ \text{A}$$

$$V_{ph(N\text{-}L)} = \frac{208}{\sqrt{3}} = 120\ \text{V}$$

Therefore, $$|Z_{NL}| = \frac{V_P}{I_L} = \frac{120}{8.17} = 14.687 \simeq 14.70\ \Omega$$

Now, stator copper losses

$$P_{sc} = 3I_1^2R_1 = 3 \times (8.17)^2 \times 0.243 = 48.66\ \text{W}$$

Therefore, no-load rotational losses are,

$$P_{rot} = P_{in} - P_{sc}$$

$$P_{rot} = 420 - 48.66 = 371.34\ \text{W}$$

Let the magnetizing branch reactance is X_m and stator reactance is X_1,

$\Rightarrow$ $$X_1 + X_m = \sqrt{Z_{N-L}^2 - R_1^2} = 14.684\ \Omega$$

From blocked-rotor test,

$$I_{L(avg)} = \frac{I_a + I_b + I_c}{3} = \frac{28.1 + 28.0 + 27.6}{3} = 27.9\ \text{A}$$

The blocked rotor impedance,

$$Z_{BR} = \frac{V_{BR}}{I_L\sqrt{3}} = \frac{25}{\sqrt{3} \times 27.9} = 0.5173\ \Omega$$

Impedance angle, $$\theta = \cos^{-1}\left(\frac{P_{in}}{\sqrt{3}V_{BR}I_L}\right)$$

$$\theta = \cos^{-1}\left(\frac{920}{\sqrt{3} \times 25 \times 27.9}\right) = 40.40°$$

$$Z_{BR} = R + jX; \quad R = R_1 + R_2; \quad X = X_1 + X_2$$

Then, $$R = Z_{BR}\cos\theta = 0.5173\cos(40.40°) = 0.40\ \Omega$$
$$X = Z_{BR}\sin\theta = 0.5173\sin(40.40°) = 0.3352\ \Omega$$
$$R_2 = R - R_1 = 0.40 - 0.243 = 0.157\ \Omega$$

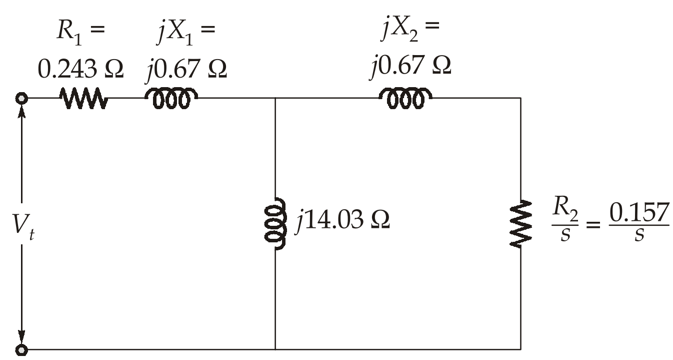
But, blocked rotor test is conducted at 15 Hz only, therefore,

$$X_{60\text{ Hz}} = \frac{60}{15}\times 0.3352 = 1.34\ \Omega$$

Therefore, $$X_1 = X_2 = \frac{X}{2} = 0.67\ \Omega$$

and $$X_m = X_{NL} - X_1$$
$$= 14.70 - 0.67 = 14.03\ \Omega$$

Hence, equivalent circuit of induction motor is



Rotational losses = 371.34 W

(ii) From the equivalent circuit,

$$V_{\text{th}} = \frac{jX_m}{R_1 + j(X_1 + X_m)}\times V_t = \frac{j14.03\times 120}{0.243 + j14.70} = 114.51\text{ Volt}$$
$$Z_{\text{th}} = R_{\text{th}} + jX_{\text{th}} = jX_m \parallel (R_1 + jX_1) = (0.243 + j0.67)\parallel(j14.03)$$
$$R_{\text{th}} = 0.221\ \Omega$$
$$X_{\text{th}} = 0.643\ \Omega$$

Therefore, slip at maximum torque is given as,

$$s_{\max,T} = \frac{R_2}{\sqrt{R_{\text{th}}^2 + (X_{\text{th}} + X_2)^2}} = \frac{0.157}{\sqrt{0.221^2 + 1.313^2}}$$
$$s_{\max,T} = 0.118$$

Therefore, maximum torque of motor is,

$$T_{\max} = \frac{3}{\omega_s} \times \frac{V_{th}^2 \times \left(\dfrac{R_2'}{s_{\max,T}}\right)}{\left(R_{th} + \dfrac{R_2'}{s_{\max,T}}\right)^2 + (X_2 + X_{th})^2}$$

$$= \frac{3}{60\pi} \times \frac{(114.5)^2 \times \left(\dfrac{0.157}{0.118}\right)}{\left(0.221 + \dfrac{0.157}{0.118}\right)^2 + (0.67 + 0.643)^2}$$

$$T_{\max} = 67.20 \text{ N-m}$$

Q.7 (a) Solution:

(i) At the initial operating point,

Let $\vec{V}_1 = V_1\angle\delta_1$

$\vec{V}_2 = V_2\angle\delta_2$

$P_e = 1.0,\ V_1 = V_2 = 1.0,\ X_t + X = 0.5$

We get $\delta_1 - \delta_2 = \sin^{-1}(0.5) = 30°$

Let the current $\vec{I}$ in line and reference voltage is

$$\vec{V}_2 = 1\angle 0° \text{ pu}$$

$$\vec{I} = \frac{\vec{V}_1 - \vec{V}_2}{j(X_T + X)} = \frac{1\angle 30° - 1}{j0.5} = 1.0352\angle 15° \text{ pu}$$

$$\vec{E}_b = \vec{V}_2 - j\vec{I}X_2$$

$$= 1 - (1.0352\angle 15°)(j0.1) = 1.032\angle -5.56° \text{ pu}$$

but $\vec{E}_b = 1.032\angle 0°$ (given as reference phasor)

therefore, $\delta_2 = 5.56$ and $\delta_1 = 35.56°$

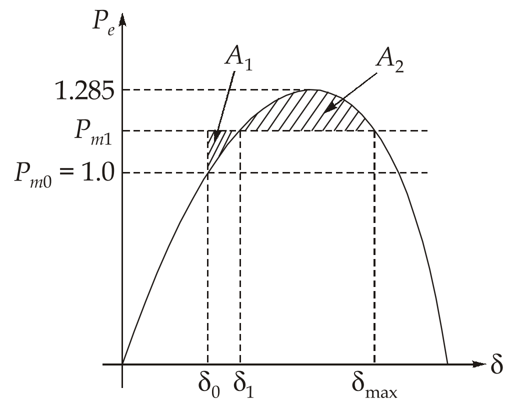
(shifting to E_b as reference phasor)

Hence, $\vec{E}_g = \vec{V}_1 + jX_g\vec{I} = 1.0\angle 35.56° + (j0.3)(1.0352\angle 15°)(1\angle 5.56°)$

$= 1.121\angle 51.1°$ pu

Therefore, $$P_{\max} = \frac{1.121 \times 1.03}{0.9} = 1.285$$

Power angle curve,



$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_{max}\sin\delta)\cdot d\delta$$

$$= -P_{max}[\cos\delta_0 - \cos\delta_1] + P_{m1}[\delta_1 - \delta_0] \quad ...(1)$$

$$A_2 = \int_{\delta_1}^{\pi-\delta_1} (P_{max}\sin\delta - P_{m1})\cdot d\delta$$

$$= 2P_{max}.\cos\delta_1 - P_{m1}(\pi - 2\delta_1) \quad ...(2)$$

Equating (1) and (2), we get

$$P_{m1}(\pi - \delta_1 - \delta_0) = P_{max}[\cos\delta_1 + \cos\delta_0]$$

$\because$ $$P_{m1} = P_{max}\sin\delta_1$$

$$\sin\delta_1(\pi - \delta_1 - \delta_0) = (\cos\delta_1 + \cos\delta_0)$$

$\because$ $$\delta_0 = \sin^{-1}\left(\frac{1}{1.285}\right) = 51.10° = 0.8918 \text{ rad}$$

$$\sin\delta_1(\pi - 0.8918 - \delta_1) = \cos\delta_1 + 0.6279$$

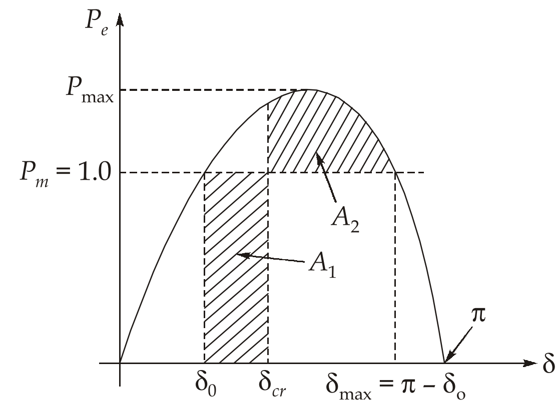
On solving above non-linear equation by hit and trial method,

$$\delta_1 \simeq 70.64°$$

Therefore, $$P_{m1} = 1.285\sin(70.64°) = 1.2123 \text{ pu}$$

Therefore, maximum step increase in mechanical power = 0.2123 pu.

(ii) Now 3-ϕ fault at generator terminals, therefore $P_e = 0$ during the fault.



$$A_1 = \int_{\delta_o}^{\delta_{cr}} (P_m - 0)d\delta = P_m(\delta_{cr} - \delta_o) \quad ...(1)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m)d\delta$$

$$A_2 = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr}) \quad ...(2)$$

Equating eqn. (1) and (2)

$$P_m(\delta_{cr} - \delta_o) = P_{max}\cos\delta_{cr} - P_{max}\cos\delta_{max} - P_m\delta_{max} + P_m\delta_{cr}$$

$$-P_m\delta_o = P_{max}\cos\delta_{cr} - P_{max}\cos\delta_{max} - P_m\delta_{max}$$

where, $\delta_o = 51.1°$ or 0.8918 rad

$$P_{max} = 1.285 \text{ pu}, P_m = 1.0, \delta_{max} = \pi - 0.8918 = 2.249 \text{ rad}$$

$$-0.8918 = 1.285\cos\delta_{cr} + 0.8062 - 2.249$$

On solving above equation,

$$\delta_{cr} = 64.60° \text{ or } 1.127 \text{ rad}$$

Now, critical clearing time

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_o)}{\pi P_m f_s}} = \sqrt{\frac{2 \times 4 \times (1.127 - 0.8918)}{\pi \times 1 \times 50}}$$

$$t_{cr} = 0.1094$$

Q.7 (b) Solution:

(i) For the two winding transformer, rated circuit for 11500 V winding,

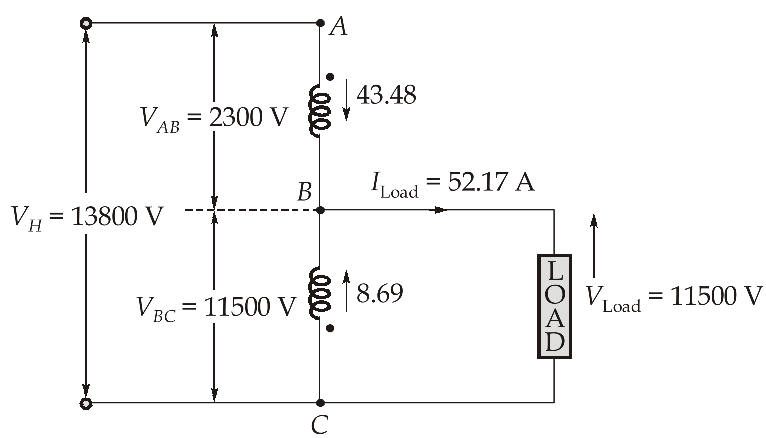
$$I_H = \frac{100 \times 1000}{11500} = 8.69 \text{ A}$$

Rated current for 2300 V winding,

$$I_L = \frac{100 \times 1000}{2300} = 43.48 \text{ A}$$

There are two possible connections of transformers.

First Configuration :



Here, $V_{AB} = 2300 \text{ V}, V_{BC} = 11500 \text{ V}$

$$V_H = V_{AB} + V_{BC} = 2300 + 11500 = 13800 \text{ V}$$

$$V_{Load} = V_{BC} = 11500 \text{ V}$$

Therefore, the voltage ratio for auto-transformer,

$$a_L = \frac{V_H}{V_L} = \frac{13800}{11500}$$

By KCL at point B,

$$I_L = I_{AB} + I_{CB} = 43.48 + 8.69 = 52.17 \text{ A}$$

kVA rating of auto-transformer,

$$V_H . I_{AB} = 13800 \times 43.48$$

$$(\text{kVA})_{auto} = 600 \text{ kVA}$$

Saving in conductor material $= \frac{1}{a_L} = \frac{11500}{13800} = 0.833$ pu or 83.33%

Power transferred through induction,

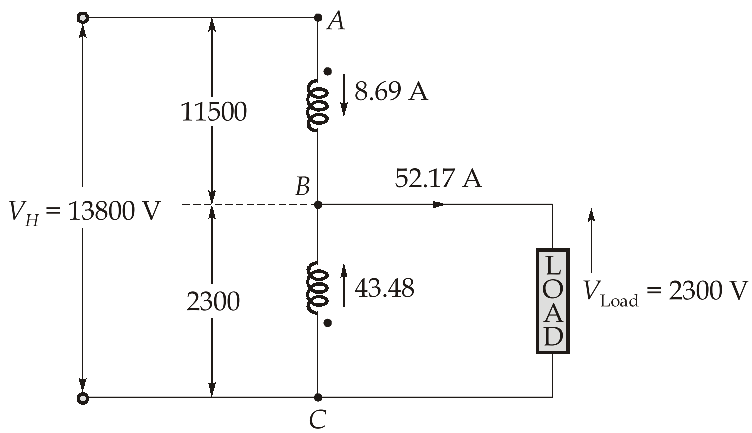
$$S_{ind} = V_{BC} . I_{BC} = 11500 \times 8.69 = 100 \text{ kVA}$$

Power transferred through conduction,

$$S_{cond} = (\text{kVA})_{auto} - S_{ind}$$

$$= 600 - 100 = 500 \text{ kVA}$$

Second Configuration :



$$V_H = V_{AB} + V_{BC} = 11500 + 2300 = 13800 \text{ V}$$
$$V_L = V_{BC} = 2300 \text{ V}$$
$$a_L = \frac{V_H}{V_L} = \frac{13800}{2300} = 6$$

By KCL at point B,

$$I_L = I_{AB} + I_{CB} = 8.69 + 43.48 = 52.17 \text{ A}$$

kVA rating of auto-transformer,

$$(\text{kVA})_{\text{auto}} = V_L . I_L = 2300 \times 52.17 = 120 \text{ kVA}$$

Percentage saving in conductor material

$$= \frac{1}{a_L} \times 100 = \frac{1}{6} \times 100 = 16.66\%$$

Power transferred through conduction,

$$S_{\text{cond}} = V_L . I_{CB} = 2300 \times 43.48 = 100 \text{ kVA}$$
$$S_{\text{ind}} = (\text{kVA})_{\text{auto}} - S_{\text{cond}} = 120 - 100 = 20 \text{ kVA}$$

(ii) Applications of auto-transformer :

1. Interconnection of power system of different voltage levels, i.e., 132 kV and 230 kV.
2. Boosting of supply voltage by a small amount in distribution systems to compensate voltage drop.
3. Auto-transformers with number of tappings are used for starting of induction motors and synchronous motors.
4. Auto-transformer is used as variable a.c. in laboratory applications that require continuously variable voltage over broad ranges.

Q.7 (c) Solution:

(i) Given :

Starting torque, $T_{st} = 1.6 \times T_{fL}$ ...(1)

Also, maximum torque, $T_{max} = 2 \times T_{fL}$ ...(2)

From eqn. (1) and (2)

$$\frac{T_{st}}{T_{max}} = \frac{1.6}{2} = 0.8$$

We know that,
$$\frac{T_{st}}{T_{max}} = \frac{2S_{M,T}}{S_{M,T}^2 + 1} = 0.8$$

$$S_{M,T}^2 - 2.5S_{M,T} + 1 = 0$$

$\Rightarrow$
$$S_{M,T} = 0.5$$

Now, for a full load slip of 5%,

$$s_{fL} = 0.05$$

Let the new slip at maximum torque is $s'_{M,T}$

$$\frac{T_{fL}}{T_{max}} = \frac{2 \cdot s_{fL} \cdot s'_{M,T}}{s_{fL}^2 + s'^2_{M,T}}$$

On putting the respective values,

$$s'^2_{M,T} - 0.2s'_{M,T} + 0.0025 = 0$$

On solving,
$$s'_{M,T} = 0.1866$$

We know that rotor resistance is,

$$r_2 \propto s_{M,T}$$

$$\frac{r'_2}{r_2} = \frac{s'_{M,T}}{s_{M,T}}$$

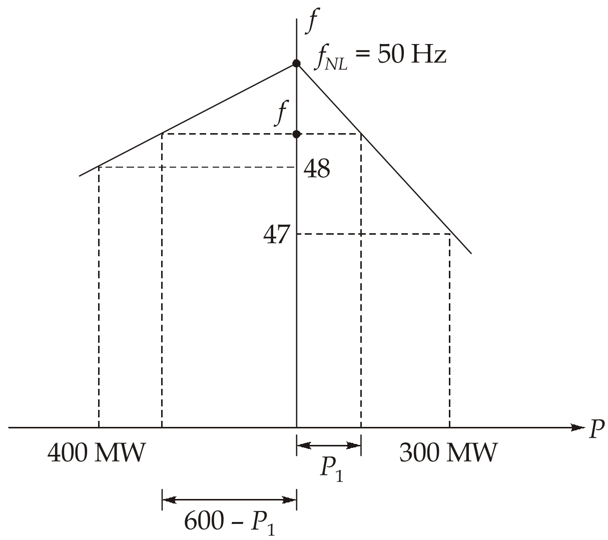
$$r'_2 = \frac{0.1866}{0.5} r_2 = 0.3732 r_2$$

Therefore, reduction in rotor resistance is

$$= (1 - 0.3732) \times 100\% = 62.68\%$$

(ii) Let both the generating units having same no-load frequency, i.e., $f_{NL} = 50$ Hz.

Now,



Using the similarity of triangles property

$$\frac{50-47}{300} = \frac{50-f}{P_1}$$

$$P_1 = 5000 - 100f \qquad ...(1)$$

Also,
$$\frac{50-48}{400} = \frac{50-f}{600-P_1}$$

$$\frac{600-P_1}{200} = 50 - f \qquad ...(2)$$

Using equation (1) and (2)

$$\frac{P_1}{100} = \frac{600-P_1}{200}$$

$$2P_1 = 600 - P_1$$

$$3P_1 = 600$$

$$P_1 = 200 \text{ MW}$$

Also, $P_1 + P_2 = 600$

$\Rightarrow$ $P_2 = 600 - P_1 = 400$ MW

Therefore, load shared by unit-1 = 200 MW

Load shared by unit-2 = 400 MW

Frequency of operation

$$\frac{P_1}{100} = 50 - f$$

$$\frac{200}{100} = 50 - f$$

$$f = 48 \text{ Hz}$$

Frequency of operation is 48 Hz.

Q.8 (a) Solution:

(i) The pu impedances on common base of 600 kVA are

$$\vec{Z}_1 = 0.012 + j0.06 = 0.061\angle 79° \text{ pu}$$

$$\vec{Z}_2 = \frac{600}{300}(0.014 + j0.045) = 0.094\angle 73° \text{ pu}$$

$$\vec{Z}_1 + \vec{Z}_2 = 0.04 + j0.15 = 0.155\angle 75° \text{ pu}$$

The load is

$$\vec{S}_L^* = 800(0.8 - j0.6) = 800\angle -36.86° \text{ kVA}$$

Load shared by transformer-1

$$\vec{S}_1 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \times \vec{S}_L$$

$$= \frac{0.094\angle 73°}{0.155\angle 75°} \times 800\angle -36.86°$$

$$\vec{S}_1^* = 485.16\angle -38.86° \text{ kVA}$$

$$\vec{S}_1 = 485.16\angle 38.86° \text{ kVA}$$

Similarly, load shared by transformer-2,

$$\vec{S}_2^* = \frac{\vec{Z}_1}{\vec{Z}_1 + \vec{Z}_2} \times \vec{S}_L^* = \frac{0.061\angle 79°}{0.155\angle 75°} \times 800\angle -36.86°$$

$$\vec{S}_2^* = 314.83\angle -32.86° \text{ kVA}$$

$$\vec{S}_2 = 314.83\angle 32.86 \text{ kVA}$$

It may be noted that the transformers are not loaded in proportion on their rating. At a load of 800 kVA, the 300 kVA transformer operates with 5% overload because of its pu impedance on common base being less than twice than that of the 600 kVA transformer.

(ii) Ohmic impedance referred to secondary

$$\vec{Z}_{1(\text{actual})} = (0.012 + j0.06) \times \frac{440}{\dfrac{600 \times 1000}{440}}$$

$$= (0.039 + j0.0194) = 0.0198\angle 79°\ \Omega$$

$$\vec{Z}_{2(\text{actual})} = (0.028 + j0.09) \times \frac{440}{\dfrac{600 \times 1000}{440}}$$

$$= (0.009 + j0.029) = 0.0304\angle 73°\ \Omega$$

$$\vec{Z}_1 + \vec{Z}_2 = 0.0129 + j0.0484 = 0.05\angle 75°\ \Omega$$

Load impedance,

$$\vec{Z}_L = \frac{(440)^2}{800 \times 10^3 \angle -37°} = 0.242\angle 36.86°\ \Omega$$

$$\vec{Z}_L = (0.1936 + j0.1452)\ \Omega$$

Now, current shared by transformer-1,

$$\vec{I}_1 = \frac{\vec{E}_1\vec{Z}_2 - (\vec{E}_1 - \vec{E}_2)\vec{Z}_L}{\vec{Z}_1\vec{Z}_2 + \vec{Z}_L(\vec{Z}_1 + \vec{Z}_2)}$$

$$= \frac{(455 \times 0.0304\angle 73°) - (455 - 445) \times 0.242\angle 36.86°}{(0.0198 \times 0.0304\angle 152°) + (0.242\angle 36.86°) \times (0.05\angle 75°)}$$

$$\vec{I}_1 = 940\angle -33.77°$$

Similarly, current shared by transformer-2,

$$\vec{I}_2 = \frac{\vec{E}_2 \cdot \vec{Z}_1 - (\vec{E}_1 - \vec{E}_2) \cdot \vec{Z}_L}{\vec{Z}_1 \cdot \vec{Z}_2 + \vec{Z}_L(\vec{Z}_1 + \vec{Z}_2)}$$

$$= \frac{445 \times 0.0198\angle 79° - 10 \times 0.242\angle 36.86°}{(0.0198 \times 0.0304\angle 152°) + (0.242\angle 36.86°) \times (0.05\angle 75°)}$$

$$= 883\angle -44°\ \text{A}$$

Corresponding kVAs are :

$$\vec{S}_1 = VI_1^*$$

$$= 440 \times 940 \times 10^{-3}\angle 34° = 413.6\angle 34°\ \text{kVA}$$

$$\vec{S}_2 = V\vec{I}_2^* = 440 \times 883 \times 10^{-3}\angle 44° = 388\angle 44°\ \text{kVA}$$

This is about 3% less than 800 × 0.8 = 640 kW required by the load because of assumption of the value of output voltage in order to calculate the load impedance.

The secondary circulating current on no-load is

$$\frac{(\vec{E}_1 - \vec{E}_2)}{\left|\vec{Z}_1 + \vec{Z}_2\right|} = \frac{-10}{0.05} = -200 \text{ A}$$

which is corresponding to about 88 kVA and considerable waste as copper loss.

Q.8 (b) Solution:

(i) There is a fundamental difference between the transmission of power in a d.c. and in an a.c. system. In an a.c. system power is given by $P = \frac{E_1 E_2}{X} \sin \delta$, where E_1 and E_2 are line voltages at the two ends, δ the electrical angle between E_1 and E_2 and X is the line reactance whereas in d.c. the power is given by

$$P = \frac{Ed_1 - Ed_2}{R} Ed_2$$

where Ed_1 and Ed_2 are the d.c. voltages at the two ends and R is the line resistance. From this it is clear that the d.c. power is proportional to the difference of the line voltages and thus will vary much more with the voltages than in the case of the a.c. transmission, where the power is proportional to the product of the line end voltages.

Line Circuit : The line construction is simpler as compared to a.c. transmission. A single conductor line with ground as return can be compared with a 3-phase single circuit line. Hence, the line is relatively cheaper and has the same reliability as that of a 3-phase single circuit line because 3-phase lines cannot operate, except for a short time when there is a single line to ground fault or a L-L fault as this creates unbalancing in the voltages and hence interfere with the communication lines and other sensitive apparatus on the system. It is claimed that a bipolar d.c. line has the same reliability index as a two-circuit 3-phase line having six line conductors.

Power per Conductor : For transmitting power both on a.c. and d.c. circuits let us assume that the two lines have the same number of conductors and insulators. Assuming that the current is limited by temperature rise, the direct current equals the r.m.s. alternating current. Since the crest voltage in both cases is same for the insulators the direct voltage is $\sqrt{2}$ times the r.m.s. alternating voltage.

The power per conductor in case of d.c. is

$$P_d = V_d I_d$$

and the power per conductor in a.c. is

$$P_a = V_a I_a \cos\phi$$

where I_a and I_d are the currents per conductor and V_a and V_d the line to ground voltages and cos ϕ the power factor.

Now since $V_d = \sqrt{2}V_a$ and $I_a = I_d$

$$\frac{P_d}{P_a} = \frac{V_d I_d}{\frac{V_d}{\sqrt{2}} \cdot I_d \cos\phi} = \frac{\sqrt{2}}{\cos\phi}$$

Since cos $\phi \le 1.0$, the power per conductor in case of d.c. is more as compared to a.c.

Power per Circuit : Let us compare the power transmission capabilities of a 3-phase single circuit line and a bipolar line. The power capabilities of the respective circuits are

$$P_d = 2p_d \text{ and } P_a = 3p_a$$

where p_d and p_a are the power transmitted per conductor of d.c. and a.c. lines. The ratio

$$\frac{P_d}{P_a} = \frac{2p_d}{3p_a} = \frac{2V_d I_d}{3V_a I_a \cos\phi} = \frac{2V_d I_d}{\frac{3}{\sqrt{2}} V_d I_d \cos\phi}$$

$$= \frac{2\sqrt{2}}{3\cos\phi} = \frac{2.828}{3\cos\phi}$$

Normally cos $\phi < 1$ and is of the order of 0.9. Therefore, the power transmission capability of the bipolar line is same as that of the 3-phase single circuit line. The d.c. line is cheaper and simpler as it requires two conductors instead of three and hence 2/3 as many insulators, and the towers are cheaper and narrower and hence a narrow right of way could be used.

No Charging Current : In case of a.c. the charging current flows in the cable conductor, a severe decrease in the value of load current transmittable occurs if thermal rating is not to be exceeded; in the higher voltage range lengths of the order of 32 km create a need for drastic derating. A further current loading reduction is caused by the appreciable magnitude of dielectric losses at high voltages. Since in case of d.c. the charging current is totally absent the length of transmission is not limited and the cable need not be derated.

No Skin Effect : The a.c. resistance of a conductor is somewhat higher than its d.c. resistance because in case of a.c. the current is not uniformly distributed over the section of the conductor. The current density is higher on the outer section of the conductor as compared to the inner section. This is known as skin effect. As a result of this the conductor section is not utilized fully. This effect is absent in case of d.c.

No Compensation Required : Long distance a.c. power transmission is feasible only with the use of series and shunt compensation, applied at intervals along the line. For such lines shunt compensation (shunt reactors) is required to absorb the line charging kVAs during light load conditions and series compensation (use of series capacitors) for stability reasons. Since d.c. line operate at unity power factor and charging currents are absent, thus no compensation is required.

Less Corona Loss and Radio Interference : The corona loss is directly proportional to (f + 25), where f is the frequency of supply. f being zero in case of d.c., the corona losses are less as compared to a.c. Corona loss and radio interference are directly related and hence radio interference in case of d.c. is less as compared to a.c. Also corona and radio interference slightly decreased by foul weather conditions (snow, rain or fog) in case of d.c. whereas they increase appreciably in case of a.c. supply.

Higher Operating Voltages Possible : The modern high voltage transmission lines are designed based on the expected switching surges rather than the lightning voltages because the former are more severe as compared to the latter. The level of switching surges due to d.c. is lower as compared to a.c. and hence, the same size of conductors and string insulators can be used for higher voltages in case of d.c. as compared to a.c. In cables, where the limiting factor is usually the normal working voltage the insulation will withstand a direct voltage higher than that of alternating voltage, which is already 1.4 times the r.m.s. value of the alternating voltage.

No Stability Problem : For a two machine system the power transmitted from one machine to another through a lossless system is given by

$$P = \frac{E_1 E_2}{X} \sin \delta$$

where X is the inductive reactance between the machines. The longer the length of the line, the higher is the value of X and hence lower will be the capability of the system to transmit power from one end to the other. With this the steady state stability limit of the system is reduced. The transient state stability limit is normally lower than the steady state; therefore, with longer lines used for transmission, the transient stability also becomes very low. A d.c. transmission line does not have any stability problem in itself because d.c. operation is an asynchronous operation of the machines

In fact, two separate a.c. systems interconnected only by a d.c. link do not operate in synchronism even if their nominal frequencies are equal and they can operate at different nominal frequencies e.g., one operating at 60 Hz and the other at 50 Hz.

Low Short Circuit Currents : The interconnection of a.c. system through an a.c. system increases the fault level to the extent that sometimes the existing switchgear has to be replaced. However, the interconnection of a.c. system with d.c. links does not increase the level so much and is limited automatically by the grid control to twice its rated current. As a result of this, faulty d.c. links do not draw large currents from the a.c. system.

(ii)

$$I_d = \frac{V_{or} - V_{oi}}{R}$$

where, V_{or} and V_{oi} are rectifier and inverter d.c. output voltage and R is loop resistance.

$$V_{or} = V_o \cos\alpha - \frac{3I_d X}{\pi}$$

$$= \frac{3\sqrt{2}\times 120}{\pi}\cos(15°) - \frac{45I_d}{\pi}$$

$$V_{oi} = V_o \cos\beta + \frac{3I_d X}{\pi} \qquad \text{(Here, } \beta = \delta + \Upsilon\text{)}$$

$$= \frac{3\sqrt{2}\times 120}{\pi}\cos(25°) + \frac{45I_d}{\pi}$$

$$I_d R = 10I_d = V_{or} - V_{oi}$$

$$= \frac{3\sqrt{2}\times 120}{\pi}(\cos 15° - \cos 25°)\times 1000 - \frac{90I_d}{\pi}$$

$$I_d\left(10 + \frac{90}{\pi}\right) = 9664$$

$$\Rightarrow \quad I_d = 249.96 \simeq 250 \text{ A}$$

Q.8 (c) Solution:

(i) Mechanical power,

$$P_m = (331100) \times 746 \times 10^{-6} = 247 \text{ MW}$$

$$P_m = \frac{247}{250} = 0.988 \text{ pu}$$

$$P_e = \frac{200}{250} = 0.8 \text{ pu}$$

$$9 \text{ cycles} = \frac{9}{60} = 0.15 \text{ sec}$$

Now, kinetic energy stored in rotor,

$$\text{K.E.} = \text{G.H.} = 250 \times 5.4 = 1350 \text{ MJ}$$

Using the swing equation

$$\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = (P_m - P_e) \text{ pu}$$

$$\frac{5.4}{180 \times 60} \cdot \frac{d^2\delta}{dt^2} = 0.988 - 0.8$$

Acceleration constant (δ),

$$\frac{d^2\delta}{dt^2} = \frac{0.188 \times 180 \times 60}{5.4} = 376 \text{ ele-deg/sec}^2$$

Now, we know that

$$\delta = \delta_o + \frac{1}{2}\alpha t^2$$

$$\Delta\delta = \delta - \delta_o = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 376 \times (0.15)^2$$

$$= 4.23 \text{ ele-degree}$$

$$\text{Change in power angle} = 4.23 \text{ ele-degree}$$

New frequency of operation,

$$f_{\text{new}} = f_o + \frac{\alpha}{2\pi} t$$

$$f_{\text{new}} = 60 + \frac{376}{360} \times 0.15$$

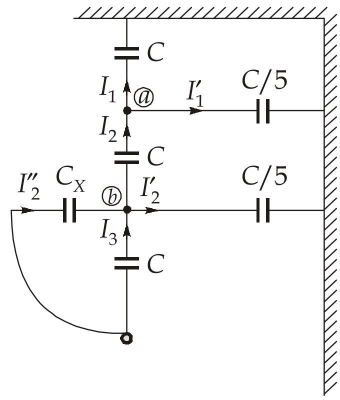
$$= 60.15 \text{ Hz}$$

New rotor speed,

$$N = \frac{120 \times f_{\text{new}}}{P} = \frac{120 \times 60.15}{2}$$

$$N = 3609 \text{ rpm}$$

(ii) Static capacitance 'C_X' of guard ring compensate the shunt capacitance (L/5) charging current, i.e.,



$$I'_2 = I''_2 \quad ...(1)$$

∴ $$I_3 = I_2 \neq I_1 \Rightarrow V_3 = V_2 \neq V_1 \quad ...(2)$$

Apply KCL at node (a),

$$I_2 = I_1 + I'_1$$

$$V_2\omega C = V_1\omega C + V_1.\omega(C/5)$$

$$V_2 = V_1 + \frac{V_1}{5} = V_1 \times \frac{6}{5} = V_3$$

∴ $$V_2 = V_3 = \frac{6V_1}{5}$$

Given that $$V_1 + V_2 + V_3 = 20 \text{ kV}$$

$$V_1 + 2\times\left(\frac{6V_1}{5}\right) = 20 \text{ kV}$$

$$3.4V_1 = 20 \text{ kV}$$

$$V_1 = 5.88 \text{ kV}$$

Therefore, $$V_2 = V_3 = \frac{6}{5}\times V_1 = 7.058 \text{ kV}$$

Now, from eqn. (1)

$$I'_2 = I''_2$$

$$(V_1 + V_2)\frac{\omega C}{5} = V_3\omega C_X$$

$$\frac{(V_1 + V_2)}{5V_3}C = C_X$$

⇒ $$C_X = \frac{(7.05 + 5.88)}{5\times 7.05} = 0.366C$$

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