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Detailed Solutions

ESE-2023 Mains Test Series

Civil Engineering Test No : 5

Section A : Flow of fluids, Hydraulic machines and Hydro power

Q.1 (a) Solution:

Given, $\mu = 0.099$ Pa.s, $\omega = 100$ rad/s,

$t = 1.4$ mm

Due to the presence of viscous oil and relative motion between the frustum and the container, by virtue of the velocity gradient, shear stress will be generated at top, bottom and sides. Power is required to overcome this resistance generated as a result of shear stresses.

At top:

Consider an element at a distance 'r' from the axis of rotation,

$$dP = dT \cdot \omega$$

$$dP = \mu \frac{r\omega}{t} \cdot 2\pi r \cdot dr \cdot r \cdot \omega$$

$$P_{top} = \frac{2\pi\mu\omega^2}{t} \int_0^{r_{top}} r^3 dr \qquad \left[r_{top} = \frac{14}{2} = 7 \text{ cm}\right]$$

$$\Rightarrow \qquad P = \frac{2\pi\mu\omega^2}{t}\left(\frac{r^4}{4}\right)_0^{r_{top}} = \frac{2\pi\mu\omega^2}{t}\left(\frac{r_{top}^4}{4}\right)$$

At bottom:

Similarly, $$dP = \frac{\mu r\omega}{t} \cdot 2\pi r \cdot dr \cdot r \cdot \omega$$

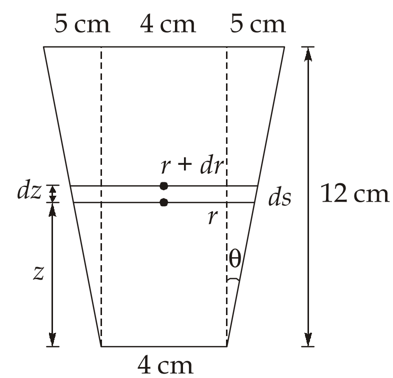
$$P_{bottom} = \frac{2\pi\mu\omega^2}{t}\int_0^{r_{bottom}} r^3 dr \qquad \left[r_{bottom} = \frac{4}{2} = 2\text{ cm}\right]$$

$$= \frac{2\pi\mu\omega^2}{t}\left(\frac{r^4}{4}\right)_0^{r_{bottom}} = \frac{2\pi\mu\omega^2}{t}\left(\frac{r_{bottom}^4}{4}\right)$$

At sides:

Consider an element of radius r at a height z from bottom

$$dP = \mu\frac{r\omega}{t}\cdot 2\pi r\, ds\cdot r\cdot\omega$$

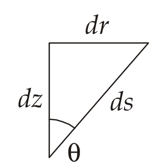


$$dP = \frac{2\pi\mu\omega^2}{t}\cdot r^3 ds$$

Also, from figure

$$\sin\theta = \frac{5}{13} = \frac{dr}{ds}$$

$\Rightarrow$
$$ds = \frac{dr}{\sin\theta} = \frac{13}{5}dr$$



$$P_{sides} = \frac{2\pi\mu\omega^2}{t}\int_{r_{bottom}}^{r_{top}} r^3\frac{dr}{\sin\theta}$$

$$= \frac{2\pi\mu\omega^2}{t\sin\theta}\left(\frac{r^4}{4}\right)_{r_{bottom}}^{r_{top}} = \frac{2\pi\mu\omega^2}{t\sin\theta}\left(\frac{r_{top}^4 - r_{bottom}^4}{4}\right)$$

Total power required, $P_{total} = P_{top} + P_{bottom} + P_{sides}$

$$= \frac{2\pi\mu\omega^2}{t}\left(\frac{r_{top}^4}{4}\right) + \frac{2\pi\mu\omega^2}{t}\left(\frac{r_{bottom}^4}{4}\right) + \frac{2\pi\mu\omega^2}{t\sin\theta}\left(\frac{r_{top}^4 - r_{bottom}^4}{4}\right)$$

$$= \frac{2\pi\mu\omega^2}{4t}\left(r_{top}^2 + r_{bottom}^4 + \frac{r_{top}^4 - r_{bottom}^4}{\sin\theta}\right)$$

$$= \frac{2\pi\mu\omega^2}{4t}\left(\frac{18r_{top}^4 - 8r_{bottom}^4}{5}\right) \qquad \left(\sin\theta = \frac{5}{13}\right)$$

$$= \frac{2\pi\times 0.099\times 100^2}{4\times 1.4\times 10^{-3}}\left(\frac{18(0.07)^4 - 8(0.02)^4}{5}\right) = 95.73\text{ W}$$

Q.1 (b) Solution:

Force due to oil, $F_{oil} = \rho_{oil} g h A$

$$= 0.9 \times 10^3 \times 9.81 \times \left(H - 1.5 + \frac{1.5}{2}\right) \times (1.5 \times 0.6)$$

$$= 7.9461\ (H - 0.75)\ \text{kN}$$

Centre of pressure of force F_{oil},

$$h_{c \cdot p_o} = h + \frac{I_{GXX}}{A\bar{h}} \qquad \text{(below the oil surface)}$$

$$= (H - 0.75) + \frac{0.6(1.5)^3}{12(0.6)(1.5)(H - 0.75)}$$

$$= (H - 0.75) + \frac{0.1875}{H - 0.75}$$

Force due to air, $F_{air} = P_{air} \times A$

$$= 35 \times 0.6 \times 1.5 = 31.5\ \text{kN}$$

Centre of pressure of force F_{air}

$$h_{c \cdot p_a} = H - 1.5 + \frac{1.5}{2} \qquad \text{(below the oil surface)}$$

$$= H - 0.75$$

Now, taking moment about the hinge,

$$F_{oil} \times \left(h_{c \cdot p_0} - (H - 1.5)\right) = F_{air} \times \left(H - 0.75 - (H - 1.5)\right)$$

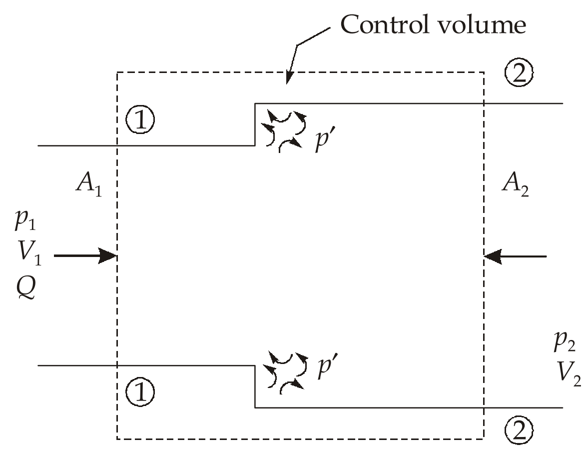
$$\Rightarrow \quad 7.9461(H - 0.75)\left(0.75 + \frac{0.1875}{H - 0.75}\right) = 31.5 \times 0.75$$

On solving, we get

$$H = 4.46\ \text{m}$$

Thus, the minimum height of oil at which gate rotates is 4.46 m.

Q.1 (c) Solution:



Applying momentum theorem in the control volume

$$p_1 A_1 + p'(A_2 - A_1) - p_2 A_2 = \rho Q (V_2 - V_1)$$

From experiment it is found that $p' = P_1$

$$\Rightarrow \quad (p_2 - p_1) A_2 = \rho Q (V_1 - V_2)$$

$$\Rightarrow \quad \Delta p = \frac{\rho Q}{A_2}(V_1 - V_2)$$

From continuity equation, $Q = A_2 V_2 = A_1 V_1$

$$\Rightarrow \quad \Delta p = \rho \times \frac{Q}{\frac{\pi}{4}\times d_2^2}\left(\frac{Q}{\frac{\pi}{4}\times d_1^2} - \frac{Q}{\frac{\pi}{4}\times d_2^2}\right)$$

$$\Delta p = \frac{\rho Q^2}{\left(\frac{\pi}{4}\right)^2}\left(\frac{1}{d_2^2 \times d_1^2} - \frac{1}{d_2^4}\right)$$

For maximum pressure rise,

$$\frac{d\Delta P}{dd_2} = 0$$

$$\Rightarrow \quad \left[\frac{-2d_2^{-3}}{d_1^2} + 4d_2^{-5}\right] = 0$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{1}{\sqrt{2}}$$

Corresponding pressure rise,

$$\Delta p = \frac{\rho Q}{\frac{\pi}{4}\times d_2^2}\left[\frac{Q}{\frac{\pi}{4}\times d_1^2} - \frac{Q}{\frac{\pi}{4}\times d_2^2}\right]$$

$$\Delta p = \frac{\rho Q}{\frac{\pi}{4}\times(2)(d_1^2)}\left(\frac{Q}{\frac{\pi}{4}\times(d_1^2)} - \frac{Q}{\frac{\pi}{4}\times(2)(d_1^2)}\right)$$

$$\Delta p = \rho\times\frac{V_1}{2}\left[V_1 - \frac{V_1}{2}\right]$$

$$\Delta p = \frac{\rho V_1^2}{4}$$

Q.1 (d) Solution:

(i) If the velocity of flow increases, as per Bernoulli's equation, the pressure will fall. If this pressure falls below vapour pressure, the liquid boils and large number of small bubbles of vapour are formed. As water goes from low pressure region to high pressure region, the bubbles collapse. This results in the formation of a cavity and the surrounding liquid rushes to fill it which results in rise of local pressure. It causes pitting on the metallic surface of runner blades or draft tube. This phenomenon is called as cavitation in reaction turbine.

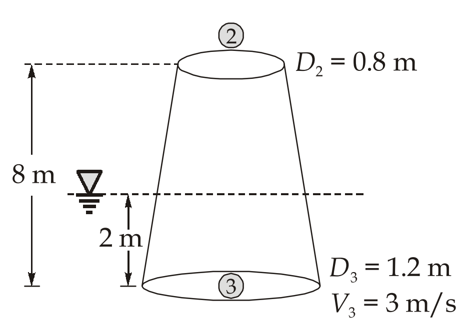
Thoma's cavitation factor is dimensionless number used to study the effect of cavitation.

For turbines, Thoma's cavitation factor is given by,

$$\sigma = \frac{(H_a - h_s) - H_v}{H}$$

where, H_a = Atmospheric head, H_v = Vapour pressure head,
h_s = Suction head, H = Working head

(ii)



Given,

$$\text{Atmosphere pressure} = 10.3 \text{ m of } H_2O \text{ column}$$

$$h_f = 0.25\frac{V_3^2}{2g}$$

Continuity equation, $A_2V_2 = A_3V_3$

$$\Rightarrow \quad V_2 = \frac{\frac{\pi}{4}\times 1.2^2 \times 3}{\frac{\pi}{4}\times 0.8^2} = 6.75 \text{ m/sec}$$

1. Applying energy equation between (2) and (3),

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 + h_f$$

$$\Rightarrow \quad \frac{p_2}{\rho g} + \frac{6.75^2}{2\times 9.81} + 8 = 10.3 + 2 + \frac{3^2}{2\times 9.81} + \frac{0.25\times 3^2}{2\times 9.81}$$

$$\Rightarrow \quad \frac{p_2}{\rho g} = 2.551 \text{ m}$$

$\therefore$ Pressure head at inlet = 2.551 m

2. Efficiency of draft tube,

$$\eta_{\text{draft}} = \frac{\frac{V_2^2}{2g} - \frac{V_3^2}{2g} - h_f}{\frac{V_2^2}{2g}} = \frac{\frac{6.75^2}{2g} - \frac{3^2}{2g} - \frac{0.25\times 3^2}{2g}}{\frac{6.75^2}{2g}}$$

$$= 0.7531 = 75.31\%$$

Q.1 (e) Solution:

(i) The following assumptions are made for the derivation of depth of hydraulic jump:

(a) The flow is steady and the pressure distribution is hydrostatic before and after the jump.

(b) Losses due to friction on the surface of the bed of the channel are small and hence neglected.

(c) The slope of the bed of the channel is small so that the component of the weight of the fluid in the direction of flow is negligible.

(ii) Given: Velocity of flow before hydraulic jump,

$$V_1 = 10 \text{ m/s}$$

Depth of flow before hydraulic jump,

$$y_1 = 1 \text{ m}$$

Discharge per unit width,

$$q_1 = V_1 y_1 = 10 \times 1 = 10 \text{ m}^2/\text{s}$$

$$\therefore \quad F_1^2 = \frac{V_1^2}{g y_1} = \frac{10^2}{9.81 \times 1} = 10.194$$

The depth of flow after the jump is given by

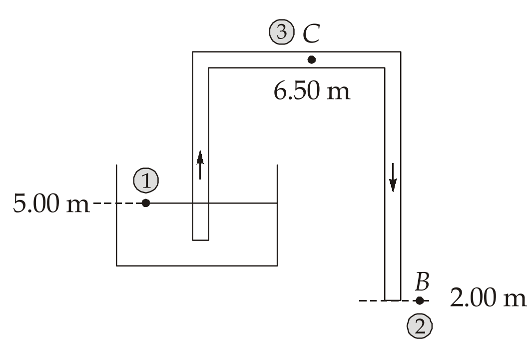
$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right]$$

$$\Rightarrow \quad y_2 = \frac{1}{2} \times \left[-1+\sqrt{1+8\times 10.194}\right] \times 1$$

$$\Rightarrow \quad y_2 = 4.043 \text{ m}$$

Loss in total head, $$h_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2} = \frac{(4.043-1)^3}{4 \times 4.043 \times 1} = 1.74 \text{ m}$$

Q.2 (a) (i) Solution:



Applying Bernoulli's equation between (1) and (2),

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_{L(1-2)}$$

$$\Rightarrow \quad 0 + 0 + 5 = 0 + \frac{V_2^2}{2g} + 2 + 0.4 + 1.2$$

$$\Rightarrow \quad \frac{V_2^2}{2g} = 1.4$$

$\Rightarrow \quad V_2 = \sqrt{1.4 \times 2 \times 9.81} = 5.24 \text{ m/s}$

1. Discharge,

$$Q = \frac{\pi}{4} \times (0.2)^2 \times 5.24$$
$$= 0.1646 \text{ m}^3/\text{s} = 164.6 \text{ L/S}$$

2. Applying Bernoulli's equation between (1) and (3),

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + H_{L(1-3)}$$

$$V_3 = V_2 = 5.24 \text{ m/s}$$

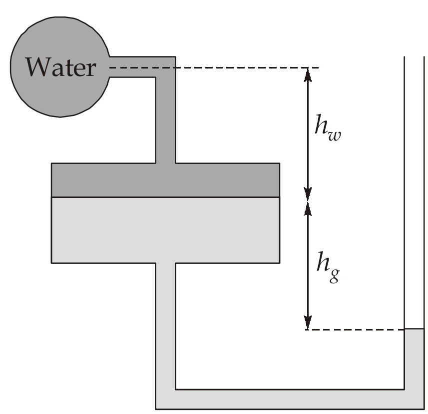
$$\therefore \quad 0 + 0 + 5 = \frac{P_3}{\gamma} + \frac{(5.24)^2}{2g} + 6.5 + 0.4$$

$$\Rightarrow \quad \frac{P_3}{\gamma} = -3.3 \text{ m}$$

$$P_3 = -3.3 \times 10^3 \times 0.85 \times 9.81 \times 10^{-3}$$
$$= -27.52 \text{ KPa (gauge)}$$

Q.2 (a) (ii) Solution:

Consider the figure as shown below,



Let P_W be the initial pressure in water pipe,

$$\rho_w = \text{density of water} = 1000 \text{ kg/m}^3$$
$$\rho_g = \text{density of glycerin} = 1300 \text{ kg/m}^3$$

At initial stage,

$$P_w + \rho_w g h_w + \rho_g g h g = P_{atm} \quad \text{...(i)}$$

Let ΔP be the pressure increment and x be the fall in glycerin level in the left tube.

Now from volume conservation,

$$\Rightarrow \quad \frac{\pi}{4}(3)^2 \times 70 = \frac{\pi}{4}(30)^2 \times x$$

$$\Rightarrow \quad x = 0.7 \text{ mm}$$

After pressure increment,

$$(P_w + \Delta P) + \rho_w g(h_w + x) + \rho_g g(h_g - x) = \rho_g g(0.07) + P_{atm} \quad \text{...(ii)}$$

From equation (ii) – (i),

$$\Rightarrow \quad \Delta P + \rho_w g(x) - \rho_g g(x) = \rho_g g(0.07)$$

$$\Rightarrow \quad \Delta P = \rho_g g(0.07) + \rho_g g(x) - \rho_w g(x)$$

$$= 1300 \times 9.81\ (0.07 + 0.7 \times 10^{-3}) - 1000 \times 9.81 \times 0.7 \times 10^{-3}$$

$$= 894.77 \text{ N}$$

So, Change in pressure in pipe = 894.77 N

Q.2 (b) Solution:

By Von Karman momentum integral equation,

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

Put $\frac{y}{\delta} = t$, $\quad y = 0, t = 0$

and $\quad y = \delta, t = 1$

$$\Rightarrow \quad dy = \delta\, dt$$

$$\theta = \int_0^1 (2t - 2t^3 + t^4)(1 - 2t + 2t^3 - t^4)\delta\, dt$$

$$= \int_0^1 (2t - 4t^2 + 4t^4 - 2t^5 - 2t^3 + 4t^4 - 4t^6 + 2t^7 + t^4 - 2t^5 + 2t^7 - t^8)\delta\, dt$$

$$= \int_0^1 (-t^8 + 4t^7 - 4t^6 - 4t^5 + 9t^4 - 2t^3 - 4t^2 + 2t)\delta\, dt = \frac{37\delta}{315}$$

$$\therefore \quad \frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left(\frac{37\,\delta}{315}\right) \quad \text{...(i)}$$

Also,

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$= U\mu\left[\frac{2}{\delta} - 2\times 3\left(\frac{y}{\delta}\right)^2 \cdot \frac{1}{\delta} + 4\left(\frac{y}{\delta}\right)^3 \cdot \frac{1}{\delta}\right]_{y=0} = \frac{2U\mu}{\delta}$$

Putting value of τ_0 in equation (i),

$$\frac{2\mu U}{\rho U^2 \delta} = \frac{d}{dx}\left(\frac{37\,\delta}{315}\right)$$

$$\frac{2\mu U}{\rho U^2} \times \frac{315}{37} dx = \delta\, d\delta$$

Integrating on both sides,

$$\frac{630}{37}\frac{\mu}{\rho U} x = \frac{\delta^2}{2} + C$$

At $x = 0$, $\delta = 0$

$\Rightarrow$ $C = 0$

$$\Rightarrow \quad \delta^2 = \frac{1260 \cdot x^2}{37 \dfrac{\rho U x}{\mu}}$$

$$\Rightarrow \quad \frac{\delta}{x} = \frac{5.836}{\sqrt{Re_x}} \qquad \left(\text{where, } Re_x = \frac{\rho U x}{\mu}\right) \quad \text{...(ii)}$$

Shear stress:

Now,

$$\tau_0 = \frac{2\mu U}{\delta}$$

Substituting value of δ from (ii),

$$\Rightarrow \quad \tau_0 = \frac{2\mu U \sqrt{Re_x}}{5.836 x} = \frac{0.3427\mu U}{x} \cdot \sqrt{\frac{\rho U x}{\mu}}$$

$$= \frac{0.3427 \mu^{1/2} U^{3/2} \rho^{1/2}}{x^{1/2}} \cdot \frac{\rho^{1/2} U^{1/2}}{\rho^{1/2} U^{1/2}} \cdot \frac{2}{2}$$

$\therefore \quad \tau_0 = 0.6854 \frac{PU^2}{2}\left(\frac{\mu}{\rho U x}\right)^{1/2}$

or $\quad \frac{\tau_0}{\frac{1}{2}\rho U^2} = \frac{0.6854}{\sqrt{Re_x}} = C_f$

Force on one side:

Consider a plate of unit width and length L

$$F_D = \int_0^L \tau_0 (1 \cdot dx)$$

$$= 0.6854 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U}\right)^{1/2} \int_0^L x^{-1/2} dx$$

$$= 0.6854 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U}\right)^{1/2} \left[2x^{1/2}\right]_0^L$$

$$= 0.6854 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U}\right)^{1/2} 2\sqrt{L}$$

$$F_D = 1.3708 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U L}\right)^{1/2} L$$

$$F_D = \frac{1}{2}\left(\frac{1.3708}{\sqrt{Re_L}}\right)\rho U^2 L$$

or $\quad \frac{F_D}{\frac{1}{2}\rho U^2 L} = C_{DF} = \frac{1.3708}{\sqrt{Re_L}}$

Q.2 (c) (i) Solution:

Let suffixes 1 and 2 represent the sections upstream and downstream of the transition respectively

$\therefore \quad V_1 = \frac{Q}{B_1 y_1} = \frac{15}{3 \times 2.5} = 2 \text{ m/sec}$

Froude number, $\quad F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2}{\sqrt{9.81 \times 2.5}} = 0.4038 < 1$

$\therefore$ The upstream flow is subcritical and the transition will cause a drop in the water surface.

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.5 + \frac{2^2}{2 \times 9.81} = 2.704 \text{ m}$$

Let, $B_{2\,min}$ = Minimum width at section 2 which does not cause choking.

Then, $E_{cm} = E_1 = 2.704$ m

$$y_{cm} = \frac{2}{3}E_{cm} = \frac{2}{3} \times 2.704 = 1.8026 \text{ m}$$

Since,
$$y_{cm} = \left[\frac{\left(\frac{Q}{B_{2min}}\right)^2}{g}\right]^{1/3}$$

$$\Rightarrow \quad (1.8026)^3 = \frac{15^2}{9.81 \times (B_{2min})^2}$$

$$\Rightarrow \quad B_{2\,min} = 1.978 \text{ m}$$

when $B_2 = 1.8$ m

Since, $B_2 < B_{2\,min}$, hence choking condition will prevail.

$\therefore$ The depth at the section-2, $y_2 = y_{c2}$

and the depth at upstream y_1 will increase to y_1'.

$\therefore$ At downstream section,

$$y_2 = y_{c2} = \left(\frac{q_2^2}{g}\right)^{1/3} = \left[\frac{\left(\frac{15}{1.8}\right)^2}{9.81}\right]^{1/3}$$

$$\therefore \quad y_2 = 1.92 \text{ m}$$

$$\therefore \quad E_{c2} = 1.5\, y_{c2} = 1.5 \times 1.92 = 2.88 \text{ m}$$

At upstream section 1,

$$E_1' = E_{c2} = 2.88 \text{ m}$$

$$\Rightarrow \quad y_1' + \frac{V_1'^2}{2g} = 2.88$$

$$\Rightarrow \quad y_1' + \frac{\left(\frac{15}{3}\right)^2}{2 \times 9.81 \times y_1'^2} = 2.88$$

$$\Rightarrow \qquad y_1' + \frac{1.274}{y_1'^2} = 2.88$$

$$\Rightarrow \quad y_1'^3 - 2.88y_1'^2 + 1.274 = 0$$

$$\therefore \qquad y_1' = 2.706 \text{ m}, 0.7786 \text{ m}, -0.6046 \text{ m}$$

Since, upstream flow is subcritical.

So, $y_1' > y_1$

Hence, water surface elevation at upstream section is 2.706 m and at downstream section is 1.92 m.

Q.2 (c) (ii) Solution:

A positive surge with a velocity ($-V_w$) i.e. travelling upstream, will be generated as a result of sudden closure of the sluice gate.

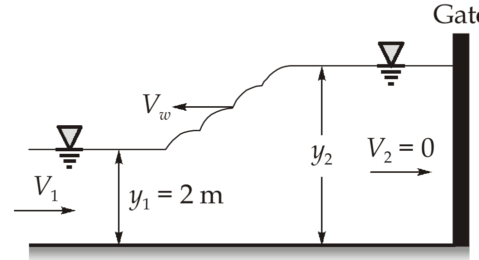


Fig. Positve surge moving upstream

$$\therefore \qquad V_1 = \frac{Q}{B_1 y_1} = \frac{15}{3.5 \times 2} = 2.1428 \approx 2.143 \text{ m/s}$$

$$\because \qquad V_2 = 0,$$

$$y_2 > 2 \text{ m}$$

By superimposing a velocity V_w on the system, a steady flow is simulated as shown in figure,

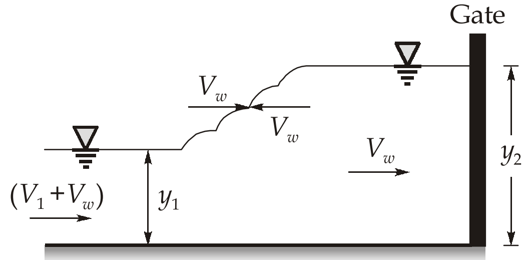


Fig. Stimulated steady flow

Using continuity equation,

$$y_1(V_1 + V_w) = y_2(V_2 + V_w)$$

$$\Rightarrow \quad 2(2.143 + V_w) = y_2 V_w$$

$$\Rightarrow \quad V_w = \frac{4.286}{y_2 - 2}$$

For a positive surge moving in upstream direction

$$\Rightarrow \quad \frac{(V_w + V_1)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1} + 1\right)$$

$$\Rightarrow \quad (V_w + V_1)^2 = \frac{g y_2}{2 y_1}(y_1 + y_2)$$

$$\Rightarrow \quad \left[\left(\frac{4.286}{y_2 - 2}\right) + 2.143\right]^2 = \frac{9.81 \times y_2}{2 \times 2}(2 + y_2)$$

$$\Rightarrow \quad y_2 = 3.064 \text{ m}$$

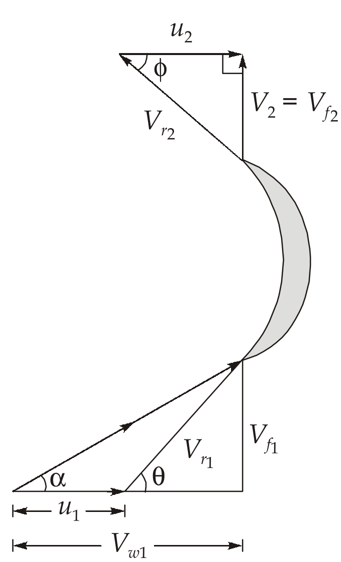
∴ Height of surge, $\Delta_y = y_2 - y_1 = 3.064 - 2 = 1.064$ m

Velocity of surge, $V_w = \dfrac{4.286}{3.064 - 2} = 4.028$ m/s

Q.3 (a) (i) Solution:

Given, $\eta_0 = 85\%$, $H = 8$ m,

$P = 160$ kW, $N = 160$ rpm



Peripheral velocity, $u_1 = 0.96\sqrt{2gH} = 0.96\sqrt{2\times 9.81\times 8}$

$= 12.027 \text{ m/s} \approx 12.03 \text{ m/s}$

Velocity of flow at inlet,

$$V_{f1} = 0.36\sqrt{2gH} = 0.36\sqrt{2\times 9.81\times 8} = 4.51 \text{ m/s}$$

Hydraulic loss = 24%

$\therefore$ Hydraulic efficiency, $\eta_h = (100 - 24) = 76\%$

For radial discharge,

$$\eta_h = \frac{V_{w_1}u_1}{gH}$$

$$\Rightarrow \quad 0.76 = \frac{V_{w1}\times 12.03}{9.81\times 8}$$

$$\Rightarrow \quad V_{w1} = 4.958 \approx 4.96 \text{ m/s}$$

1. From inlet velocity triangle,

$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{4.51}{4.96}$$

$$\Rightarrow \quad \alpha = 42.279° \approx 42.28°$$

$\therefore$ Angle of the guide blade at inlet = 42.28°

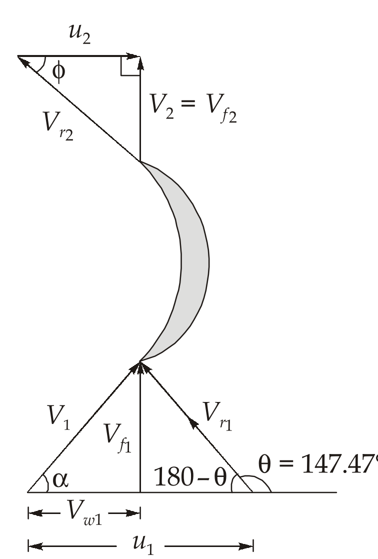
2. $$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}$$ (From inlet velocity triangle)

$$\Rightarrow \quad \tan\theta = \frac{4.51}{4.96 - 12.03}$$

$$\Rightarrow \quad \theta = -32.534°$$

$\therefore$ The wheel angle of inlet = 180 – 32.534°

= 147.466° ≈ 147.47°



3. $\because$ $$u_1 = \frac{\pi D_1 N}{60}$$

$$\Rightarrow \quad 12.03 = \frac{\pi D_1 \times 160}{60}$$

$$\Rightarrow \quad D_1 = 1.436 \text{ m}$$

$\therefore$ Diameter of wheel is 1.436 m

4. $\because$ $$\eta_0 = 85\% = \frac{P}{\rho Q g H}$$

$$\Rightarrow \quad 0.85 = \frac{160 \times 10^3}{1000 \times Q \times 9.81 \times 8}$$

$$\Rightarrow \quad Q = 2.398 \text{ m}^3/\text{sec}$$

Neglecting the vane thickness,

$$Q = \pi B_1 D_1 V_{f_1}$$

$$\Rightarrow \quad 2.398 = \pi \times B_1 \times 1.436 \times 4.51$$

$$\Rightarrow \quad B_1 = 0.1179 \text{ m} \approx 0.118 \text{ m}$$

$$\Rightarrow \quad B_1 = 118 \text{ mm}$$

$\therefore$ The width of wheel at inlet = 118 mm

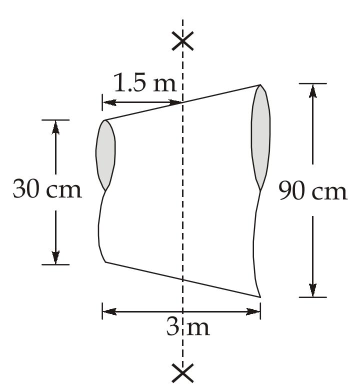
Q.3 (a) (ii) Solution:

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid (or water) may flow into the vessel or out the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out of the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single acting reciprocating pump:

- to obtain a continuous supply of liquid at uniform rate.
- to save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipe.
- to run the pump at a high speed without separation.

Q.3 (b) (i) Solution:



Diameter at section X-X

$$D_x = 0.3 + \left(\frac{0.9 - 0.3}{3}\right)x = 0.3 + 0.2x$$

At $x = 1.5$ m, $D_x = 0.6$ m

$$A_x = \frac{\pi}{4}D_x^2 = \frac{\pi}{4}(0.6)^2 = 0.2827 \text{ m}^2$$

- Local acceleration, $a_t = \dfrac{dV}{dt} = \dfrac{d}{dt}\left(\dfrac{Q}{A_x}\right) = \dfrac{1}{A_x}\dfrac{dQ}{dt}$

$$= \frac{1}{0.2827} \times 0.060 = 0.212 \text{ m/s}^2$$

- Convective acceleration, $a_s = V_x \dfrac{dV_x}{dx}$

$$V_x = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4}(0.2x + 0.3)^2} = \frac{0.3}{\frac{\pi}{4}(0.2x + 0.3)^2}$$

$$\frac{dV_x}{dx} = \frac{1.2}{\pi}\frac{(-2)(0.2)}{(0.2x + 0.3)^3} = \frac{-0.15279}{(0.2x + 0.3)^3}$$

$\Rightarrow$ $$a_s = \frac{0.3}{\frac{\pi}{4}(0.2x + 0.3)^2} \times \frac{-0.15279}{(0.2x + 0.3)^3}$$

At $x = 1.5$

$\Rightarrow$ $a_s = -0.751 \text{ m/s}^2$

- Total acceleration = Local acceleration + Convective acceleration

$= 0.212 - 0.751$

$= -0.539 \text{ m/s}^2$

Q.3 (b) (ii) Solution:

Horizontal scale, $L_r = \dfrac{1}{250}$

Vertical scale, $h_r = \dfrac{1}{50}$

For prototype

Discharge, $Q_P = 150 \text{ m}^3/\text{s}$

Width, $B_P = 100$ m

Depth, $y_p = 4 \text{ m}$

- **Discharge:**

$$\frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p}$$

$$\text{Froude's number} = \frac{V}{\sqrt{Lg}}$$

$$\Rightarrow \quad \frac{V_m}{\sqrt{y_m g}} = \frac{V_p}{\sqrt{y_p g}}$$

$$\Rightarrow \quad \frac{V_m}{V_p} = \sqrt{\frac{y_m}{y_p}} = \sqrt{h_r}$$

and $$\frac{A_m}{A_p} = \frac{B_m y_m}{B_p y_p} = L_r h_r$$

So, $$\frac{Q_m}{Q_p} = L_r h_r \sqrt{h_r}$$

$$\Rightarrow \quad Q_m = 150 \times \frac{1}{250} \times \left(\frac{1}{50}\right)^{1.5} = 1.697 \times 10^{-3} \text{ m}^3/\text{s}$$

- **Depth**

$$\Rightarrow \quad \frac{y_m}{y_p} = h_r$$

$$\Rightarrow \quad y_m = 4 \times \frac{1}{50} = 0.08 \text{ m}$$

Width

$$\Rightarrow \quad \frac{B_m}{B_p} = L_r$$

$$\Rightarrow \quad B_m = 100 \times \frac{1}{250} = 0.4 \text{ m}$$

- **Manning's n**

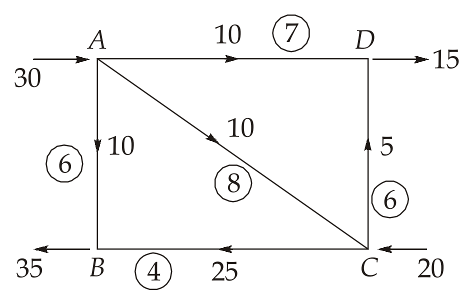
$$\frac{V_m}{V_p} = \frac{\frac{1}{n_m} R_m^{2/3} S_m^{1/2}}{\frac{1}{n_p} R_p^{2/3} S_p^{1/2}}$$

$$\Rightarrow \qquad \sqrt{h_r} = \frac{n_p}{n_m} \times h_r^{2/3} \times \frac{h_r^{1/2}}{L_r^{1/2}}$$

$$\Rightarrow \qquad n_m = \frac{n_p \times h_r^{2/3}}{L_r^{1/2}} = \frac{0.030 \times \left(\frac{1}{50}\right)^{2/3}}{\left(\frac{1}{250}\right)^{1/2}} = 0.0349$$

Q.3 (c) Solution:

Flow direction is assumed positive clockwise for all loops. A first trial set of discharges is selected to satisfy continuity at each node (flow into a node = flow out of the node) as shown below.



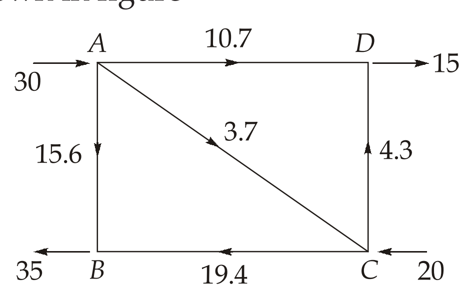
The Hardy-Cross method is now used to find the corrections, ΔQ.

I[st] trial:

Loop *ABC*		
Line	rQ^2	$\lvert 2rQ \rvert$
AB	$-6(10)^2$	$2 \times 6 \times 10$
CB	$4(25)^2$	$2 \times 4 \times 25$
AC	$8(10)^2$	$2 \times 8 \times 10$
	$\Sigma = 2700$	$\Sigma = 480$
Correction, $\Delta Q = \frac{-\Sigma rQ^2}{\Sigma \lvert 2rQ \rvert} = \frac{-2700}{480} = -5.6$		

Loop *ADC*		
Line	rQ^2	$\lvert 2rQ \rvert$
AD	$7(10)^2$	$2 \times 7 \times 10$
CD	$-6(5)^2$	$2 \times 6 \times 5$
AC	$-8(10)^2$	$2 \times 8 \times 10$
	$\Sigma = -250$	$\Sigma = 360$
Correction, $\Delta Q = \frac{250}{360} = 0.7$		

The corrected flows are shown in figure

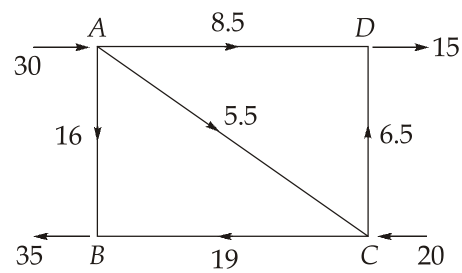


IInd trial:

Loop *ABC*		
Line	rQ^2	$\|2rQ\|$
AB	$-6(15.6)^2$	$2 \times 6 \times 15.6$
CB	$4(19.4)^2$	$2 \times 4 \times 19.4$
AC	$8(3.7)^2$	$2 \times 8 \times 3.7$
	$\Sigma = 154.8$	$\Sigma = 401.6$
Correction, $\Delta Q = \frac{-154.8}{401.6} = -0.4$		

Loop *ADC*		
Line	rQ^2	$\|2rQ\|$
AD	$7(10.7)^2$	$2 \times 7 \times 10.7$
CD	$-6(4.3)^2$	$2 \times 6 \times 4.3$
AC	$-8(3.7)^2$	$2 \times 8 \times 3.7$
	$\Sigma = 580.97$	$\Sigma = 260.6$
Correction, $\Delta Q = \frac{-580.97}{260.6} = -2.2$		

The corrected flows are shown in figure,

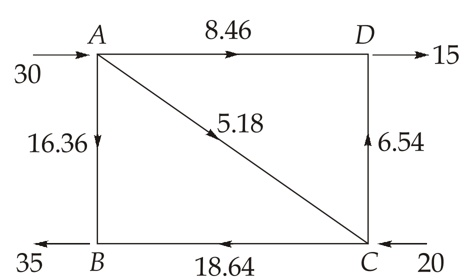


IIIrd trial:

Loop *ABC*		
Line	rQ^2	$\|2rQ\|$
AB	$-6(16)^2$	$2 \times 6 \times 16$
CB	$4(19)^2$	$2 \times 4 \times 19$
AC	$8(5.5)^2$	$2 \times 8 \times 5.5$
	$\Sigma = 150$	$\Sigma = 432$
Correction, $\Delta Q = \frac{-150}{432} = -0.36$		

Loop *ADC*		
Line	rQ^2	$\|2rQ\|$
AD	$7(8.5)^2$	$2 \times 7 \times 8.5$
CD	$-6(6.5)^2$	$2 \times 6 \times 6.5$
AC	$-8(5.5)^2$	$2 \times 8 \times 5.5$
	$\Sigma = 10.25$	$\Sigma = 285$
Correction, $\Delta Q = \frac{-10.25}{285} = -0.04$		

The corrected flows are shown in figure and iteration is stopped since ΔQ is very small.



Q.4 (a) (i) Solution:

Given, $B = 4$ m ; $n = 0.025$,

$Q = 6.0\ \text{m}^3/\text{sec}$

Let, the normal depth of flow be y

Area, $A = B \times y = 4y$

Perimeter, $P = B + 2y = 4 + 2y$

Hydraulic radius, $R = \frac{A}{P} = \frac{4y}{4+2y}$

Critical depth of flow, $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\left(\frac{6}{4}\right)^2}{9.81}\right)^{1/3} = 0.612$ m

1. From Manning's equation,

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

At $S = S_{01} = 0.0004$, normal depth be y_1

$$\Rightarrow \quad 6 = \frac{1}{0.025} \times (4y_1) \times \left(\frac{4y_1}{4+2y_1}\right)^{2/3} \times \sqrt{0.0004}$$

$$\Rightarrow \quad y_1 = 1.9057 \approx 1.906 \text{ m}$$

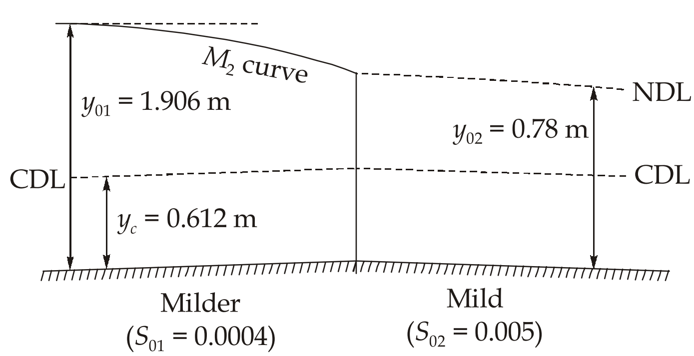
As $y_1 > y_c$, slope is mild slope

At $S = S_{02} = 0.005$, normal depth be y_2

$$\Rightarrow \quad 6 = \frac{1}{0.025} \times (4y_2) \times \left(\frac{4y_2}{4+2y_2}\right)^{2/3} \times \sqrt{0.005}$$

$$\Rightarrow \quad y_2 = 0.779 \approx 0.78 \text{ m}$$

As $y_2 > y_c$, slope is mild but $y_1 > y_2$, hence slope S_{01} is milder,



2. From Manning's equation,

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

At $S = S_{01} = 0.015$, normal depth by y_1.

$$\Rightarrow \quad 6 = \frac{1}{0.025}(4y_1)\left(\frac{4y_1}{4+2y_1}\right)^{2/3} \times \sqrt{0.015}$$

$$\Rightarrow \quad y_1 = 0.540 \text{ m}$$

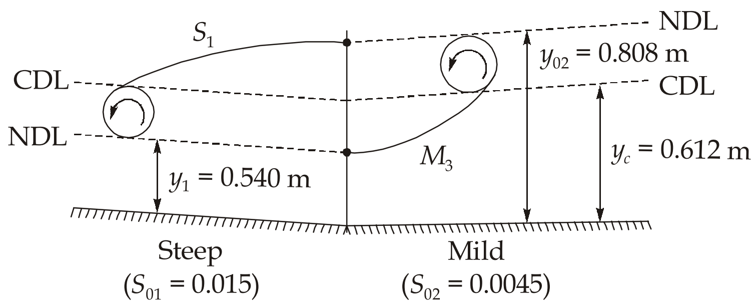
As $y_1 < y_c$, slope is steep.

At $S = S_{02} = 0.0045$, normal depth be y_2.

$$\Rightarrow \quad 6 = \frac{1}{0.025}(4y_2)\left(\frac{4y_2}{4+2y_2}\right)^{2/3} \times \sqrt{0.0045}$$

$$\Rightarrow \quad y_2 = 0.808 \text{ m}$$

As $y_2 > y_c$, slope is mild slope.



Q.4 (a) (ii) Solution:

Let suffixes 1 and 2 be the section upstream and downstream of the transition respectively.

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{Q}{B_1 y_1} = \frac{5}{1.5 \times 1.5} = 2.222 \text{ m/s}$$

Froude number, $$F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2.222}{\sqrt{9.81 \times 1.5}} = 0.579 < 1$$

$\therefore$ The upstream flow is subcritical and the transitions will cause a drop in the water surface.

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.5 + \frac{2.222^2}{2 \times 9.81} = 1.7516 \text{ m}$$

Since, the flow in triangular section is to be critical without changing upstream water surface.

$\therefore \quad E_{c2} = 1.25\, y_{c2}$

$$= 1.25\left(\frac{2Q^2}{gm^2}\right)^{1/5} = 1.25\left(\frac{2\times 5^2}{9.81\times 2^2}\right)^{1/5}$$

$\Rightarrow \quad E_{c2} = 1.312 \text{ m}$

and $\quad y_{c2} = \dfrac{1.312}{1.25} = 1.0496 \text{ m}$

$\therefore$ Location of vertex of triangular section relative to bed of rectangular channel is hump (Δz)

$$\therefore \quad \Delta z = E_1 - E_{c2}$$
$$= (1.7516 - 1.312)$$
$$= 0.4396 \text{ m} \approx 0.440 \text{ m}$$

Also, drop in water surface level $= y_1 - \Delta z - y_{c2}$
$$= 2.5 - 0.440 - 1.0496$$
$$= 1.0104 \text{ m}$$

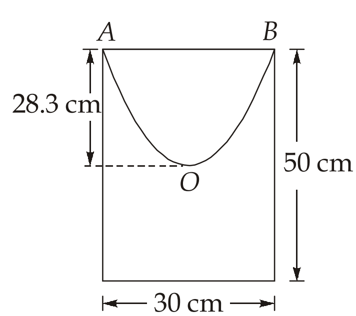
Q.4 (b) (i) Solution:

(*a*) When $\omega = 150$ rpm

$$\omega = \frac{150\times 2\pi}{60} = 15.71 \text{ rad/sec}$$

At the wall of cylinder, $r = 15 \text{ cm} = 0.15 \text{ m}$

$$Z_{max} = \frac{\omega^2 r^2}{2g} = \frac{(15.71)^2(0.15)^2}{2\times 9.81} = 0.283 \text{ m} = 28.3 \text{ cm}$$



Volume of water spilled = Volume of paraboloid *AOB*

and volume of paraboloid AOB $= \dfrac{1}{2}\times$ Volume of enclosing cylinder

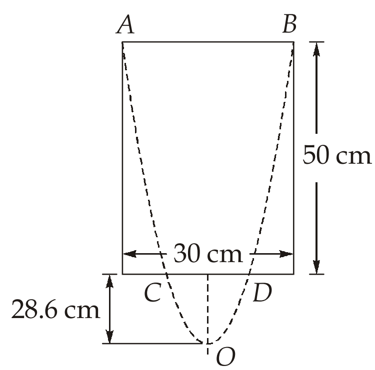
So, Volume of water spilled $= \frac{1}{2} \times \pi \times (0.15)^2 \times (0.283) = 0.0100\ m^3 = 10\ L$

(*b*) When $\omega = 250$ rpm

$$\omega = \frac{250 \times 2\pi}{60} = 26.18 \text{ rad/sec}$$

At the wall of cylinder, $r = 15\ cm = 0.15\ m$

$$Z_{max} = \frac{\omega^2 r^2}{2g} = \frac{(26.18)^2 (0.15)^2}{2 \times 9.81} = 0.786\ m = 78.6\ cm$$



Volume of water spilled = Volume of *ABCD*

and volume of *ABCD* = Volume of paraboloid AOB – Volume of paraboloid *COD*,

Let *x* be the radius of the exposed portion of the cylinder

For paraboloid *AOB*,

$$\frac{x^2}{28.6} = \frac{15^2}{78.6}$$

$\Rightarrow$ $x = 9.05$ cm

So, volume of water spilled $= \frac{1}{2} \times \pi \times (0.15)^2 \times (0.786) - \frac{1}{2} \times \pi \times (0.0905)^2 \times (0.286)$

$= 0.02410\ m^3 = 24.1\ L$

Q.4 (b) (ii) Solution:

For both smooth and rough turbulent flows

$$\frac{u - V}{u^*} = 5.75 \log \frac{y}{r_0} + 3.75$$

1. When $u = V$

$\Rightarrow$ $$5.75 \log \frac{y}{r_0} = -3.75$$

$$\Rightarrow \qquad \frac{y}{r_0} = 0.22275$$

$$\Rightarrow \qquad y = 0.22275 r_0$$

2. When $u = \frac{V}{2}$ and $u^* = \frac{V}{10}$

$$\Rightarrow \qquad \frac{\frac{V}{2} - V}{\frac{V}{10}} = 5.75 \log \frac{y}{r_0} + 3.75$$

$$\Rightarrow \qquad 5.75 \log \frac{y}{r_0} = -5 - 3.75$$

$$\Rightarrow \qquad \frac{y}{r_0} = 0.0301$$

$$\Rightarrow \qquad y = 0.0301 r_0$$

Q.4 (c) Solution:

Given,

$$B_2 = 15 \text{ mm} = 0.015 \text{ m}$$

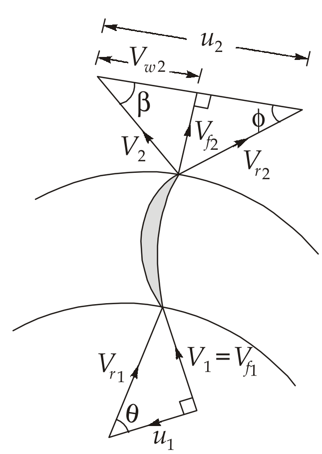
$$D_2 = 450 \text{ mm} = 0.450 \text{ m}$$

$$V_{f2} = 3.5 \text{ m/s}$$

(i) The discharge Q delivered by the pump is given by

$$Q = K\pi B_2 D_2 V_{f_2} = 1 \times \pi \times 0.015 \times 0.450 \times 3.5$$

$$= 0.07422 \text{ m}^3/\text{sec} = 4453.21 \text{ liters/minute}$$



(ii) Manometric efficiency is given by

$$\eta_{mano} = \frac{gH_m}{V_{w2} u_2}$$

$$\Rightarrow \qquad \frac{V_{w2}u_2}{g} = \frac{H_m}{\eta_{mano}}$$

Given, $H_m = 30$ m ;

$$\eta_{mano} = 80\% = 0.80$$

$$\therefore \qquad \frac{V_{w2}u_2}{g} = \frac{30}{0.80} = 37.5 \text{ m}$$

It means, total head developed by the pump

$$= 37.5 \text{ m}$$

The pressure rise through the impeller $\left(\frac{p_2 - p_1}{\rho g}\right)$ is 60% of total head developed by the pump.

$$\therefore \qquad \left(\frac{p_2 - p_1}{\rho g}\right) = 0.60 \times 37.5 = 22.5 \text{ m}$$

Applying Bernoulli's equation between the inlet and outlet tips of the impeller and neglecting head loss in the impeller, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} - \frac{V_{w2}u_2}{g}$$

$$\Rightarrow \qquad \left(\frac{p_2}{\rho g} - \frac{p_1}{\rho g}\right) = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{V_{w2}u_2}{g}$$

$$\because \qquad V_1 = V_{f1} = 3.5 \text{ m/s}$$

$$\therefore \qquad 22.5 = \frac{3.5^2}{2g} - \frac{V_2^2}{2g} + 37.5$$

$$\Rightarrow \qquad V_2 = 17.508 \approx 17.51 \text{ m/sec}$$

From outlet velocity triangle,

$$V_2^2 = V_{f2}^2 + V_{w2}^2$$

$$\Rightarrow \qquad V_{w2} = \sqrt{V_2^2 - V_{f2}^2} = \sqrt{17.51^2 - 3.5^2}$$

$$\therefore \qquad V_{w2} = 17.156 \approx 17.16 \text{ m/s}$$

Since, $$\frac{V_{w2}u_2}{g} = 37.5$$

$$\Rightarrow \quad u_2 = \frac{37.5 \times 9.81}{17.16} = 21.438 \text{ m/s}$$

$$\therefore \quad \frac{\pi D_2 N}{60} = 21.438$$

$$\Rightarrow \quad \frac{\pi \times 0.450 \times N}{60} = 21.438$$

$$\therefore \quad N = 909.86 \text{ rpm}$$

So, speed of the pump = 909.86 rpm

(iii) From outlet velocity triangle,

$$\tan \phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\Rightarrow \quad \tan \phi = \frac{3.5}{21.438 - 17.16}$$

$$\Rightarrow \quad \phi = 39.287° \approx 39.29°$$

$\therefore$ Blade angle at outlet = 39.29°

(iv) From inlet velocity triangle,

$$\tan \theta = \frac{V_{f1}}{u_1}$$

$$\Rightarrow \quad \tan 60° = \frac{3.5}{u_1}$$

$$\Rightarrow \quad u_1 = 2.0207 \text{ m/s}$$

$$\therefore \quad \frac{\pi D N}{60} = 2.0207$$

$$\frac{\pi \times D_1 \times 909.86}{60} = 2.0207$$

$$\Rightarrow \quad D_1 = 0.04242 \text{ m}$$

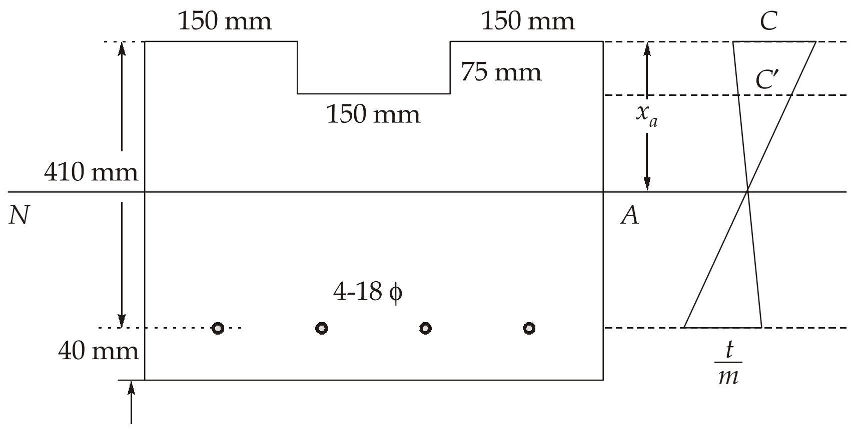
$$\therefore \quad D_1 = 42.42 \text{ mm}$$

So, the diameter of the impeller at inlet

= 42.42 mm

Section B : Design of concrete and Masonry Structures-1 + Geo-technical & Foundation Engineering-2

Q.5 (a) Solution:



$$A_{st} = 4\times\frac{\pi}{4}\times18^2 = 1017.88\ \text{mm}^2$$

Let x_a = depth of neutral axis. Taking moments about NA

$$\frac{300\times x_a^2}{2}+\frac{150\times(x_a-75)^2}{2} = 13.33 \times 1017.88 \times (410 - x_a)$$

$$150x_a^2+75(x_a-75)^2 = 13.33 \times 1017.88 \times (410 - x_a)$$

$$150x_a^2+75(x_a^2+5625-150x_a) = 5563019.564 - 13568.3404x_a$$

$$225x_a^2+2318.3404x_a-5141144.564 = 0$$

$$x_a = 146.1\ \text{mm}$$

From stress diagram, $\dfrac{C}{146.1} = \dfrac{C'}{(146.1-75)}$

∴ $C' = 0.487C$

Moment of resistance of the beam section.

$$= \frac{1}{2}\times C\times146.1\times300\times\left(410-\frac{146.1}{3}\right)+\frac{1}{2}\times C'\times(146.1-75)\times150\times\left(410-75-\frac{71.1}{3}\right)$$

$$= 7917889.5C + 1660007.25C'$$

$$= 7917889.5C + 160007.25(0.487C)$$

$$= 8726313.031C$$

Now, $8726313.031C = 50 \times 10^6$

$\Rightarrow$ $C = 5.73 \text{ N/mm}^2$

Corresponding stress in steel, $t = 13.33 \times \frac{5.73}{146.1} \times (410 - 146.1)$

$\Rightarrow$ $t = 137.97 \text{ N/mm}^2$

Q.5 (b) Solution:

Given

$$P_u = 1500 \text{ kN}$$
$$L_0 = 3.2 \text{ m} = 3200 \text{ mm}$$
$$L_{eff} = 3200 \text{ mm}$$

$$\text{Slenderness ratio} = \frac{3200}{400} = 8 < 12$$

$\therefore$ The column may be designed as a short column.

Minimum eccentricity:

$$e_{min} = \frac{L_0}{500} + \frac{D}{30} = \frac{3200}{500} + \frac{400}{30} = 19.73 \text{ mm} \quad (< 20 \text{ mm})$$

$\therefore$ $e_{min} = 20$ mm

$$0.05\,D = 0.05 \times 400 = 20 \text{ mm}$$

As e_{min} doesn't exceed 0.05 D, the codal formula for axially compressed short columns may be used.

$\therefore$

$$P_u = 1.05[0.4 f_{ck} Ac + 0.67 f_y A_{sc}]$$
$$P_u = 1.05[0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc}]$$
$$P_u = 1.05[0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}]$$
$$1500 \times 10^3 = 1.05\left[0.4 \times 25 \times \frac{\pi}{4} \times 400^2 + (0.67 \times 415 - 0.4 \times 25) \times A_{sc}\right]$$
$$A_{sc} = 641.43 \text{ mm}^2$$
$$\% \text{ of steel} = \frac{641.43}{\frac{\pi}{4} \times 400^2} \times 100 = 0.51\%$$

But, $A_{sc,\,min} = 0.8\%$ of Ag

$\therefore$ $$A_{sc} = \frac{0.8}{100} \times \frac{\pi}{4} \times 400^2 = 1005.31 \text{ mm}^2$$

Provide 6 no. of 16ϕ

$$\Rightarrow \quad A_{sc} \text{ provided} = 6\times\frac{\pi}{4}\times 16^2$$

$$= 1206.37 \text{ mm}^2 > 1005.31 \text{ mm}^2$$

$$A_{sc} \text{ maximum}, \quad 6\% \text{ of } A_g = 0.06\times\frac{\pi}{4}\times 400^2 = 7539.82 \text{ mm}^2$$

$\therefore$ OK

Design of helical reinforcement

$$\frac{V_h}{V_c} \geq 0.36\left(\frac{A_g}{A_c}-1\right)\frac{f_{ck}}{f_y}$$

$$A_g = \frac{\pi}{4}\times 400^2 = 125663.7061 \text{ mm}^2$$

Assuming a clear cover of 40 mm over spirals.

$$\therefore \quad \text{Core diameter, } D_c = 400 - (40 \times 2) = 320 \text{ mm}$$

$$A_c = \frac{\pi}{4}\times 320^2 = 80424.772 \text{ mm}^2$$

$$V_c = 1000 \times A_c = 80424772 \text{ mm}^3$$

Assuming bar dia. of 6 mm.

$$D_h = D_c - \phi_h = 320 - 6 = 314 \text{ mm}$$

$$V_h = (\text{no. of turns in one pitch length}) \times (\text{length in one turn}) \times (\text{c/s area of helical reinforcement})$$

$$= \frac{1000}{p}\times(\pi\times D_h)\times\frac{\pi}{4}\times\phi_h^2$$

$$= \frac{1000}{p}\times(\pi\times 314)\times\frac{\pi}{4}\times(6)^2 = \frac{27891502.04}{p}$$

$$\therefore \quad \frac{0.36\times 25}{415}\left[\frac{125663.7061}{80424.772}-1\right] \leq \frac{27891502.04}{p\times 80424772}$$

$$\Rightarrow \quad p \leq 28.43 \text{ mm}$$

Codal restrictions on pitch

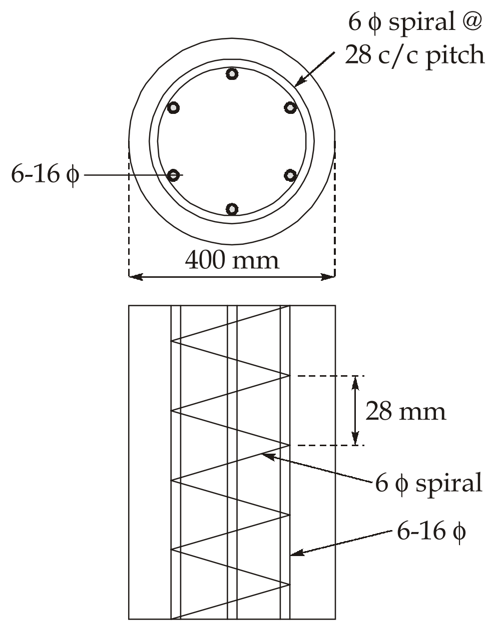
$$p < 75 \text{ mm}$$

$$< \frac{1}{6}\times D_c = \frac{1}{6}\times 320 = 53.33 \text{ mm}$$

$$p > 25 \text{ mm}$$

$$> 3\phi_h = 3 \times 6 = 18 \text{ mm}$$

∴ Provide 6ϕ spirals at 28 mm c/c pitch.



Reinforcement detailing

Q.5 (c) Solution:

(i) Arching effect in sand: Arching occurs when there is a difference of stiffness between the installed structure and the surrounding soil. If the structure is stiffer than the soil then load arches onto the structure. Otherwise, if the structure is less stiff than the soil then load arches away from the structure. For instances, if part of a rigid support of soil mass yields, the adjoining particles move with respect to the remainder of the soil mass. The movement is resisted by shearing stresses which reduces the pressure on the yielding portion of the support while increasing the pressure on the adjacent rigid zones. This phenomenon is called the arching effect.

(ii) Anchored bulkhead: The stability of an anchored sheet pile depends not only on the passive earth resistance but also on the anchor rod. The driving depth that is required in an anchored sheet pile is thus, considerably smaller than in a cantilever sheet pipe. The total length of the sheet pile is reduced and this leads to economy where the height of the sheet pile above the dredge line is not small.

A number of methods are used for the design of anchored bulkheads. However, the methods commonly used are the free earth support method and the fixed earth support method.

(iii) Coffer dams: A coffer dam is a temporary structure constructed usually in a river, lake etc. to keep the working area dry for construction of other structures. After the coffer dam is constructed, the area is rewatered by pumping. Through the coffer dam is meant to be developed, a certain amount of constant pumping is required as some water may leak through the coffer dam and the foundation. There are various types of coffer dams which can be used under different situations.

(iv) Geocells v/s Geoguids: Although both are similar geosynthetic products, geocells and geogrid differ based on their shape, lateral restriction and stiffness and load carrying capacity. The geocell is a deep, three dimensional mesh structure, while the geogrid is typically two dimensional. The 2D structure of geogrid makes it more flexible then geocells and ideal for use in instance where a flexible top support or separator is needed. A geogrid is a flatter, tighter structure that inhibits adjustability more so than a geocell. The 3D structure and the vertical support of each cell give geocells superior load-bearing capacity. Geocells are less likely to bend or buckle. Further more, almost any infill material can be used with geocells. The relatively flat nature of geogrids restricts the type of infill you can use with this product.

Q.5 (d) Solution:

In cantilever, critical section for shear should be taken at the support.

As the beam is of varying depth which increases with increase in bending moment, the shear force at the critical section,

$$V_u = 100 \times 3 = 300 \text{ kN}$$

∴ Nominal shear stress,

$$\tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}$$

M_u = Moment at the section under consideration i.e. at the support

$$= w_u \cdot \frac{L^2}{2} = 100 \times \frac{(3)^2}{2} = 450 \text{ kNm}$$

d = Effective depth of the beam at the section under consideration

= 500 mm

$\tan\beta$ = Tangent of slope at the bottom edge of the beam

$$= \frac{550 - 300}{3000}$$

On substituting values,

$$\therefore \quad \tau_v = \frac{300 \times 10^3 - \frac{450 \times 10^6}{500} \times \frac{550 - 300}{3000}}{300 \times 500}$$

$\Rightarrow \quad \tau_v = 1.5 \text{ N/mm}^2.$

$< (\tau_{c,\,max} = 0.625\sqrt{f_{ck}} = 0.625\sqrt{20} = 2.8 \text{ N/mm}^2)$ OK

Also, percentage of tensile reinforcement

$$P_t(\%) = \frac{A_{st}}{bd}\times 100 = \frac{6\times\frac{\pi}{4}(25)^2}{300\times 500}\times 100 = 1.96\%$$

From table given,
Design shear stress of concrete

$$\tau_c = 0.75 + \frac{0.79-0.75}{2.00-1.75}\times(1.96-1.75) = 0.784 \text{ N/mm}^2$$

As $\tau_v > \tau_c$, hence shear reinforcement is provided to carry the design shear force

$$V_{us} = V_u - V_c$$
$$= (1.5 \times 300 \times 500 - 0.784 \times 300 \times 500) \times 10^{-3} \text{ kN}$$
$$= 107.4 \text{ kN}$$

Spacing of shear stirrups:

2-legged 8 mmϕ stirrups are to be used

$$\therefore \quad V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

$$\Rightarrow \quad s_v = \frac{0.87\times 415\times 2\times\frac{\pi}{4}(8)^2\times 500}{107.4\times 10^3} = 168.98 \text{ mm}$$

From minimum shear reinforcement consideration,

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \quad s_v \leq \frac{0.87 f_y A_{sv}}{0.4 b}$$

$$\leq \frac{0.87\times 415\times 2\times\frac{\pi}{4}\times(8)^2}{0.4\times 300}$$

$$\leq 302.47 \text{ mm c/c}$$

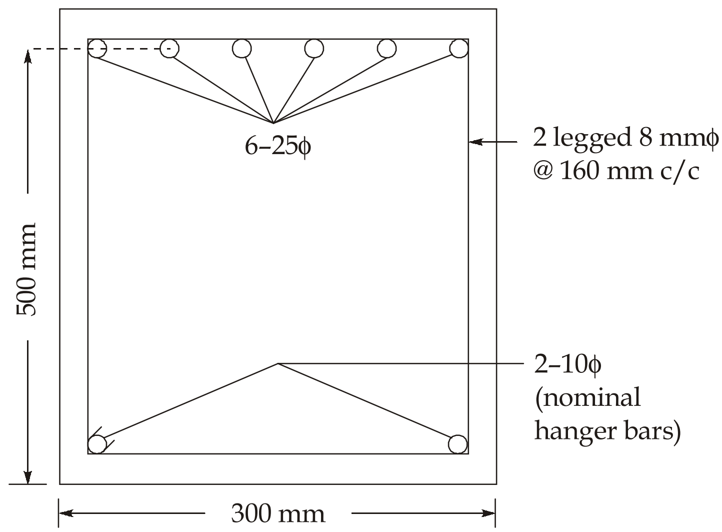
But, $s_v \not> 0.75d = 0.75 \times 500 = 375$ mm

$\not> 300$ mm

Use minimum of all above.

$\therefore$ Provide 2-legged 8 mmϕ @ 160 mm c/c.

Reinforcement detailing



Q.5 (e) Solution:

Given, $q = 125\ \text{kN/m}^2$

$\mu = 0.35$

For flexible footing, $(I_f) = 1.12$

⇒ For rigid footing, $I_f \approx (0.8 \times I_f \text{ of flexible footing})$

$$[I_f]_{\text{rigid}} = 0.8 \times 1.12 = 0.896$$

(i) 5 m size square flexible footing:

We determine the weighted value of E for the zone from ground surface to a depth of 5 times the width of the footing i.e. 25 m

$$\therefore \quad E_{\text{avg}} = \frac{E_1 h_1 + E_2 h_2}{(h_1 + h_2)}$$

$$= \frac{30 \times 10^3 \times 10 + 60 \times 10^3 \times 15}{25} = 48000\ \text{kN/m}^2$$

Now, immediate settlement,

$$S_i = qB(1-\mu^2)\frac{I}{E}$$

$$S_i = 125 \times 5 \times (1 - 0.35^2) \times \frac{1.12}{48 \times 10^3}\ \text{m} = 12.80\ \text{mm}$$

(ii) 4.5 m size square rigid footing:

In this, weighted value of E is calculated for 5 times the size of footing i.e. 22.5 m

$$\therefore \quad E_{avg} = \frac{E_1 \times h_1 + E_2 \times h_2}{h_1 + h_2}$$

$$= \frac{30 \times 10^3 \times 10 + 60 \times 10^3 \times 12.5}{22.5} = 46666.67 \text{ kN/m}^2$$

$$\therefore \quad S_i = \frac{125 \times 4.5 \times (1 - 0.35^2) \times 0.896}{46666.67} \text{ m} = 9.48 \text{ mm}$$

Q.6 (a) Solution:

Self weight of beam = 25 × 0.25 × 0.4 = 2.5 kN/m

Superimposed load = 50 kN/m

Total load = 50 + 2.5 = 52.5 kN/m

Factored load = 1.5 × 52.5 = 78.75 kN-m

$$\text{Factored BM} = \frac{wl^2}{8} = \frac{78.75 \times (4)^2}{8} = 157.5 \text{ kN-m}$$

Effective cover = 40 mm (Given)

$\therefore$ d = 400 – 40 = 360 mm

For Fe415, $x_{u\,lim} = 0.48d = 0.48 \times 360 = 172.8$ mm

$$M_{u\,lim} = 0.138 f_{ck} bd^2$$

$$= 0.138 \times 20 \times 250 \times 360^2 \times 10^{-6} \text{ kN-m}$$

$$= 89.424 \text{ kN-m} < (M_u = 157.5 \text{ kN-m})$$

$\therefore$ Doubly reinforced section is required.

Area of steel corresponding to the limiting bending moment.

$$A_{st1} = \frac{89.424 \times 10^6}{0.87 \times 415 \times (360 - 0.42 \times 172.8)}$$

$$A_{st1} = 861.72 \text{ mm}^2$$

$$A_{st2} = \frac{(157.5 - 89.424) \times 10^6}{0.87 \times 415 \times (360 - 40)} = 589.22 \text{ mm}^2$$

$$\text{Total } A_{st} = A_{st1} + A_{st2}$$

$$= 861.72 + 589.22 = 1450.94 \text{ mm}^2$$

$\therefore$ Provide 5-20 mm ϕ bars

Now,
$$\frac{0.0035}{x_{u\lim}} = \frac{\varepsilon_{sc}}{(x_{u\lim} - d')}$$

$$\varepsilon_{sc} = \frac{0.0035 \times (172.8 - 40)}{172.8} = 0.00268$$

$$\therefore\ f_{sc} = \left[0.9625 + \frac{(0.975 - 0.9625)}{(0.00276 - 0.00259)} \times (0.00268 - 0.00259)\right] \times 0.87\, f_y$$

$$= 0.97 \times 0.87\, f_y$$

$$= 0.97 \times 0.87 \times 415$$

$$= 350.2\ \text{N/mm}^2$$

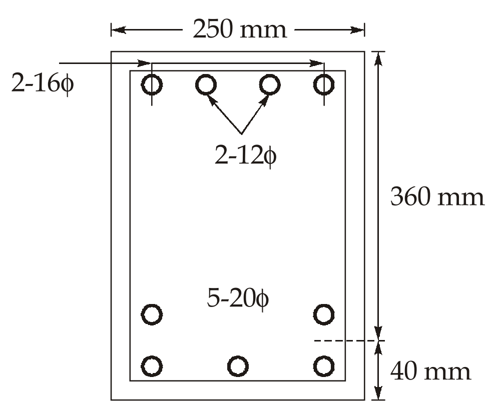
Area of compression steel $A_{sc} = \dfrac{(157.5 - 89.424) \times 10^6}{(350.2 - 0.446 \times 20) \times (360 - 40)}$

$$A_{sc} = 623.35\ \text{mm}^2$$

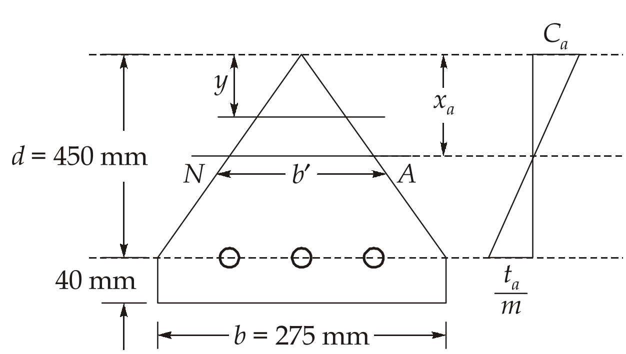
Provide 2-16 mm ϕ and 2-12 mm ϕ bars as compression reinforcement.

$$A_{sc}\ \text{provided} = 2\left[\frac{\pi}{4} \times 16^2 + \frac{\pi}{4} \times 12^2\right] = 628.32\ \text{mm}^2$$

> Asc required



Q.6 (b) Solution:



Area of steel, $A_{st} = 3 \times \frac{\pi}{4} \times 12^2 = 339.3 \text{ mm}^2$

Taking moment about NA,

$$\frac{1}{2} \times b' \times x_a \times \frac{x_a}{3} = m \cdot A_{st}(d - x_a)$$

$$b' = \text{Width of beam at the level of NA}$$

$$= \frac{275}{450} \times x_a = \frac{11}{18} x_a$$

$$\frac{1}{2} \times \frac{11}{18} \times \frac{x_a^3}{3} = 13.33 \times 339.3 \times (450 - x_a)$$

$$x_a^3 + 44406.35 x_a - 19982857.58 = 0$$

$$x_a = 217.69 \text{ mm}$$

Critical depth of NA, $x_c = \left(\frac{mc}{mc + t}\right) \times d = \left(\frac{13.33 \times 7}{13.33 \times 7 + 230}\right) \times 450$

$$= 129.87 \text{ mm}$$

$\because x_a > x_c \Rightarrow$ over reinforced section

Hence concrete attains its permissible stress earlier to steel

Corresponding stress reached in steel

$$= 13.33 \times \left(\frac{7}{217.69} \times 232.31\right) = 99.6 \text{ N/mm}^2$$

Width of beam at a depth 'y' from vertex

$$= \frac{275}{450} \times y = \frac{11}{18} y$$

Stress in concrete at a depth 'y' from vertex

$$= \frac{7}{217.69} \times (217.69 - y)$$

Consider an elemental strip of height 'dy' at a depth 'y' from the vertex

Force on the element strip $= \frac{11}{18} y \cdot dy \times \frac{7}{217.69} \times (217.69 - y)$

$$= \frac{77}{3918.42} \times y(217.69 - y) dy$$

Moment of this force about the line of action of the tension in steel

$$= \frac{77}{3918.42} \times y(217.69 - y)(450 - y)dy$$

$$= \frac{77}{3918.42}\left[97960.5y - 667.69y^2 + y^3\right]dy$$

$$\text{M.R of the section} = \frac{77}{3918.42}\int_0^{217.69}(97960.5y - 667.69y^2 + y^3)dy$$

$$\text{M.R.} = \frac{77}{3918.42}\left(\frac{97960.5y^2}{2} - \frac{667.69y^3}{3} + \frac{y^4}{4}\right)_0^{217.69}$$

$$= 11526452.61 \text{ N-m} = 11.526 \text{ kN-m}$$

Q.6 (c) (i) Solution:

Given, Slope angle, $\beta = 45°$

Cohesion, $c = 60$ kN/m^2

Angle of internal friction, $\phi_u = 0$

Unit weight, $\gamma_{sat} = 20$ kN/m^3

Area of failure wedge, $ABCD = 70$ m^2

(i) Now, length of the arc (ABC) $= r \cdot \theta$

$$L = 12 \times \left(90 \times \frac{\pi}{180} \text{radians}\right) = 18.84 \text{ m}$$

Area of wedge $ABCD = 70$ m^2

$\therefore$ Weight of wedge (W) = Area × Unit weight

= 70 × 20

= 1400 kN/m run

Weight of the wedge causes the disturbing force, where the stabilizing resistance is to be provided by the cohesion developed along the arc *ABC*.

$\therefore$ Total cohesive force per m run

$C = c \times L$

= 60 × 18.84 = 1130.4 kN-m/m-run

Now, taking moments about *O*, we get

Disturbing moment $= W \times 4.5$

= 1400 × 4.5 = 6300 kN-m/m run

Stabilizing moment $= C \times r$

$$= 1130.4 \text{ kN/m run} \times 12$$
$$= 13564.8 \text{ kN-m/m-run}$$

∴ Factor of safety (FOS) for trial failure surface

$$= \frac{\text{Stabilising moment}}{\text{Disturbing moment}} = \frac{13564.8}{6300} = 2.15$$

(ii) This failure surface is not the most critical surface, hence the factor of safety calculated above may not be minimum. The minimum factor of safety is to be calculated using stability number.

$$\therefore \quad S_n = \frac{C_{dev}}{\gamma \cdot H \cdot FOS}$$

Here, S_n for all values of β at $\phi = 0$ and $D = 0$ is taken 0.181

$$\therefore \quad 0.181 = \frac{60}{20 \times 8 \times F.O.S}$$

$$F.O.S = 2.07$$

Hence minimum factor of safety = 2.07

Q.6 (c) (ii) Solution:

The salient assumptions of the Rankine's earth pressure theory are:

- The backfill soil is isotropic, homogeneous, cohesionless and is semi-infinite.
- The soil is in a state of plastic equilibrium during active and passive earth condition.
- The rupture surface is a planar surface which is obtained by considering the plastic equilibrium of the soil.
- The backfill surface is horizontal.
- The back of the wall is vertical and smooth.

The important assumptions of Coulomb's theory of earth pressure are:

- The backfill soil is dry, cohesionless, homogeneous, isotropic and ideally plastic.
- The backfill surface is planar and can be inclined.
- The back of the wall can be inclined to the vertical.
- The failure surface is a plane surface which passes through the heel of the wall.
- The position and the line of action of the earth pressure are known.
- The sliding wedge is considered to be a rigid body and the earth pressure is obtained by considering the limiting equilibrium of the sliding wedge as a whole.

Q.7 (a) Solution:

1. **Thickness of tank required:**

Neglecting steel and assuming T = 180 mm

$$f_{ct} = \frac{P}{A_c} = \frac{60\times10^3}{1000\times180} = 0.33\ \text{N/mm}^2$$

$$f_{cbt} = \frac{BM}{t} = \frac{7.5\times10^6\times6}{1000\times180^2} = 1.38\ \text{N/mm}^2$$

$$\frac{f_{ct\,(\text{dev})}}{f_{ct\,(\text{perm})}} + \frac{f_{cbt\,(\text{dev})}}{f_{cbt(\text{perm})}} = \frac{0.33}{1.5} + \frac{1.38}{2} = 0.91 < 1.0$$

$\therefore$ OK

$\therefore$ Assume T = 180 mm

2. **Area of steel required:**

For pull of 60 kN. $$A_{st1} = \frac{P_u}{f_{st}} = \frac{60\times10^3}{130} = 461.54\ \text{mm}^2$$

For moment of 7.5 kN-m

$$A_{st2} = \frac{BM}{f_{st}\cdot j\cdot d}$$

$$d = 180 - 30 = 150\ \text{mm}$$

$$j = 1-\frac{k}{3}, \qquad \text{where, } k = \frac{9.33\times10}{(9.33\times10)+130} = 0.418$$

$$\Rightarrow \quad j = 1-\frac{0.418}{3} = 0.86$$

$$A_{st2} = \frac{7.5\times10^6}{130\times0.86\times150} = 447.23\ \text{mm}^2$$

$$A_{st} = 461.54 + 447.23 = 908.77\ \text{mm}^2$$

$$\text{Spacing of 16 mm } \phi \text{ bars} = \frac{1000\times\frac{\pi}{4}\times16^2}{908.77} = 221.24\ \text{mm}$$

say 220 mm c/c.

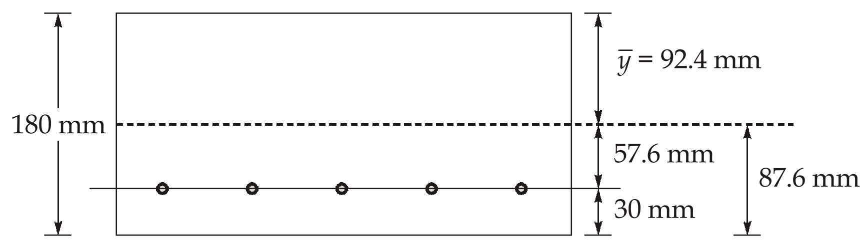
$$A_{st}\ \text{provided} = \frac{1000}{220}\times\frac{\pi}{4}\times16^2 = 913.92\ \text{mm}^2$$

3. **Stresses (uncracked section)**

$$\text{Due to direct pull, } f_{ct(\text{dev})} = \frac{P}{1000T+(m-1)A_{st}}$$

$$= \frac{60 \times 10^3}{(1000 \times 180) + (9.33 - 1)913.92} = 0.32 \text{ N/mm}^2$$

Due to bending moment:



$$\text{Depth of NA, } \bar{y} = \frac{\frac{1000 \times 180^2}{2} + (9.33 - 1) \times 913.92 \times 150}{(1000 \times 180) + (9.33 - 1) \times 913.92} = 92.4 \text{ mm}$$

$$I_{eq} = \frac{1000 \times 92.4^3}{3} + \frac{1000 \times 87.6^3}{3} + (9.33 - 1)\left[\frac{1000}{220} \times \frac{\pi \times 16^4}{64} + 913.92 \times 57.6^2\right]$$

$$I_{eq} = 5.124 \times 10^8 \text{ mm}^4$$

$$f_{cbt(\text{dev})} = \frac{B.M}{I_{eq}} \times (T - \bar{y})$$

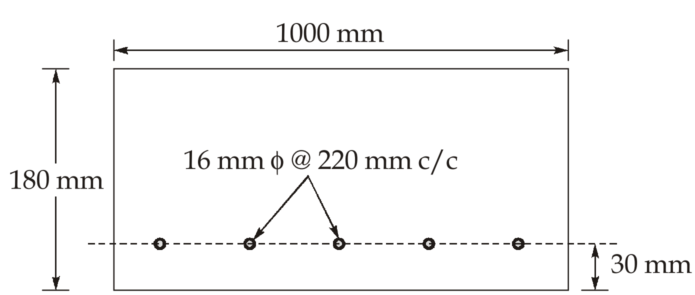
$$= \frac{7.5 \times 10^6}{5.124 \times 10^8} \times 87.6 = 1.282 \text{ N/mm}^2$$

Check:

$$\frac{f_{ct\,(\text{dev})}}{f_{ct\,(\text{perm})}} + \frac{f_{cbt(\text{dev})}}{f_{cbt\,(\text{perm})}} \le 1.0$$

$$\frac{0.32}{1.5} + \frac{1.282}{2} = 0.854 < 1.0$$

∴ **Safe**



Q.7 (b) Solution:

- **Layer-1: Filled-up soil:**

  Filled-up soil will give negative skin friction

  $$\therefore \quad Q_{nf_1} = \frac{1}{2} K \cdot \gamma \cdot L \tan\delta \cdot A_s$$

  As pile is bored pile,

  Therefore, $K = K_0 = 1 - \sin\phi$

  $\Rightarrow$ $K = 1 - \sin 25° = 0.577$

  As per IS2911,

  $$\text{take, } \delta = \frac{2}{3}\phi = \frac{2}{3} \times 25 = 16.67°$$

  $$\therefore \quad Q_{nf_1} = \frac{1}{2} \times 0.577 \times 16.5 \times 2 \tan 16.67° \times (\pi \times 0.6 \times 2)$$

  $$= 10.74 \text{ kN}$$

- **Layer-2: Very soft clay:**

  $$Q_{f_2} = \alpha \cdot C \cdot \pi d L$$

  $$\therefore \quad Q_{f2} = 0.7 \times 10 \times \pi \times 0.6 \times 3$$

  $$= 39.58 \text{ kN}$$

- **Layer-3: Stiff clay:**

  $$Q_{f_3} = \alpha \cdot C \cdot \pi d L$$

  $$= 0.4 \times 30 \times \pi \times 0.6 \times 8$$

  $$= 180.96 \text{ kN}$$

- **Layer-4: Dense sand:**

  $$Q_{f_4} = K \cdot \bar{\sigma}_{avg} \cdot \tan\delta \cdot As$$

  where $K = K_0 = 1 - \sin\phi$

  here, $\phi = 40°$

  $\therefore$ $K = 1 - \sin 40° = 0.36$

  $$\delta = \frac{2}{3}\phi = \frac{2}{3} \times 40° = 26.67°$$

  Now, $$\frac{L}{d} = \frac{6}{0.6} = 10 < 15 \Rightarrow \text{No arching effect}$$

$$\therefore \quad (\bar{\sigma}_v)_{\text{at 13m}} = 2\cdot\gamma_{\text{filled upsoil}} + 3\cdot\gamma'_{\text{soft clay}} + 8\cdot\gamma'_{\text{stiff clay}}$$

$$= 2 \times 16.5 + 3 \times (15 - 9.81) + 8 \times (18 - 9.81)$$

$$= 114.09 \text{ KPa}$$

Also,
$$(\bar{\sigma}_v)_{\text{at 19m}} = 114.09 + \gamma'_{\text{Densesand}} \times 6$$

$$= 114.09 + (20 - 9.81) \times 6 = 175.23 \text{ KPa}$$

Now,
$$Q_{f4} = 0.360 \times \left(\frac{114.09 + 175.23}{2}\right) \times \tan 26.67° \times \pi \times 0.6 \times 6$$

$$= 295.84 \text{ kN}$$

- **Ultimate point bearing resistance:**

For driven piles:
$$Q_b = A_b \times f_b + \left(\frac{1}{2}B\cdot\gamma N_\gamma\right)\cdot A_Q$$

$$f_b = \text{Ultimate bearing capacity at pile base}$$

$$= \bar{\sigma}_v N_q$$

where $(\bar{\sigma}_v)_{\text{at 19m}} = 175.23$ KPa and $N_q = 140$

$$\therefore \quad f_b = 175.23 \times 140 = 24532.2 \text{ KPa}$$

$$\Rightarrow \quad Q_b = 24532.2 \times \frac{\pi}{4} \times 0.6^2 + \frac{1}{2} \times 0.6 \times (20 - 9.81) \times 152 \times \frac{\pi}{4} \times 0.6^2$$

$$= 7067.7 \text{ kN}$$

Note: For bored pile, end bearing can be assumed as $\frac{1}{2}$ to $\frac{2}{3}$ of driven piles

$$\therefore \quad Q_b = \frac{1}{2} \times 7067.7 = 3533.85 \text{ kN}$$

Using static method:

$$Q_u = Q_b - Q_{f_1} - Q_{f_2} + Q_{f_3} + Q_{f_4}$$

$$= 3533.85 - 10.74 - 39.58 + 180.96 + 295.84]$$

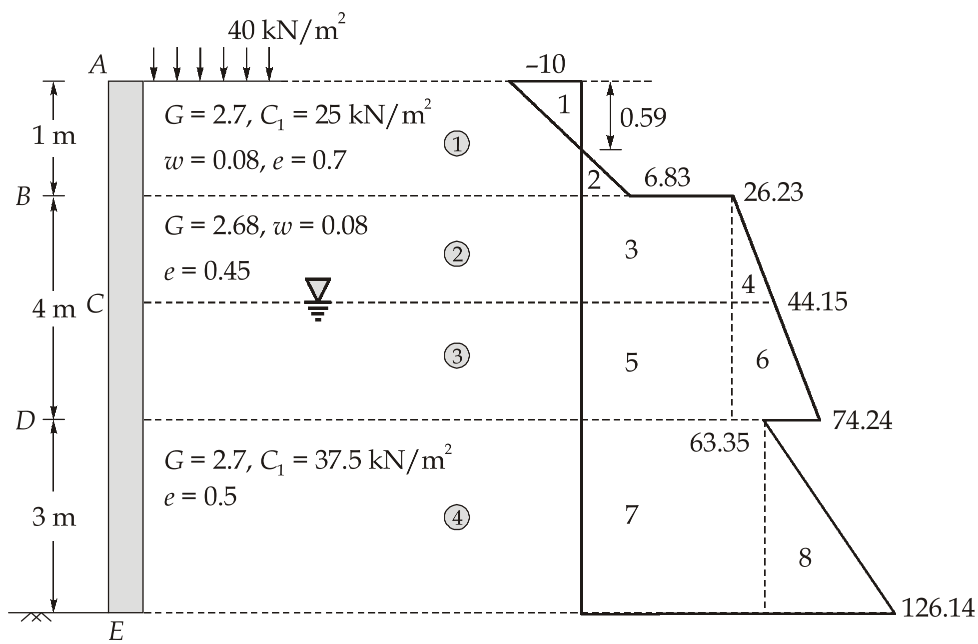
$$= 3960.33 \text{ kN}$$

Safe load carrying capacity:

$$Q_{\text{safe}} = \frac{Q_u}{\text{FOS}} = \frac{3960.33}{2.5} = 1584.132 \text{ kN}$$

Q.7 (c) Solution:

As per the statements given in the question, the figure is shown below.



$$C_{\text{Top clay}} = \frac{UCS_1}{2} = \frac{50}{2} = 25 \text{ kN/m}^2$$

$$C_{\text{bottom clay}} = \frac{UCS_2}{2} = \frac{75}{2} = 37.5 \text{ kN/m}^2$$

$$\gamma_1 = \frac{G+Se}{1+e} \times \gamma_w = \frac{G(1+w)}{1+e} \times \gamma_w$$

$$= \frac{2.7(1+0.08)}{1+0.7} \times 9.81 = 16.83 \text{ kN/m}^3$$

$$\gamma_2 = \frac{G(1+w)}{1+e} \times \gamma_w$$

$$= \frac{2.68(1+0.08)}{1+0.45} \times 9.81 = 19.58 \text{ kN/m}^3$$

$$(\gamma_{sat})_3 = \frac{G+e}{1+e} \times \gamma_w = \frac{2.68+0.45}{1+0.45} \times 9.81 = 21.18 \text{ kN/m}^2$$

$$(\gamma_{sat})_4 = \frac{G+e}{1+e} \times \gamma_w = \frac{2.7+0.5}{1+0.5} \times 9.81 = 20.93 \text{ kN/m}^2$$

For layer of sand, triaxial test is conducted,

$$\sigma_3 = 300 \text{ kN/m}^2$$

$$\sigma_d = 350 \text{ kN/m}^2$$

$$\therefore \quad \sigma_1 = \sigma_3 \tan^2\left[45 + \frac{\phi}{2}\right] + 2C\tan\left[45 + \frac{\phi}{2}\right]$$

$$\sigma_3 + \sigma_d = \sigma_3 \tan^2\left[45 + \frac{\phi}{2}\right] \quad [\because C = 0 \text{ for sand}]$$

$$650 = 300\tan^2\left[45 + \frac{\phi}{2}\right]$$

From here, $\phi = 21.62°$

Now, for sand layer, $K_{a2} = \dfrac{1 - \sin\phi}{1 + \sin\phi} = \dfrac{1 - \sin 21.62°}{1 + \sin 21.62°} = 0.46$

For clay layers, $K_{a1} = 1$

$$K_{a3} = 1$$

- Active earth pressures at different points:

$$P_A = qK_{a1} - 2C_1\sqrt{Ka_1}$$

$$= 40 \times 1 - 2 \times 25\sqrt{1} = -10 \text{ kN/m}^2$$

$$P_{B1} = (q + \bar{\sigma}_{v1})Ka_1 - 2C_1\sqrt{Ka_1}$$

$$= (q + \gamma_1 H_1)Ka_1 - 2C_1\sqrt{Ka_1}$$

$$= (40 + 16.83 \times 1) \times 1 - 2 \times 25\sqrt{1}$$

$$= +6.83 \text{ kN/m}^2$$

For calculating the depth of zero active earth pressure between A to B,

$$P_Z = 0$$

$$\therefore \quad P_Z = (q + \gamma_1 Z)Ka_1 - 2C\sqrt{Ka_1}$$

$$0 = (40 + 16.83 \cdot Z) \times 1 - 2 \times 25\sqrt{1}$$

From here, $Z = 0.59$ m

Now, $P_{B_2} = \left[q + \bar{\sigma}_{v1}\right] \times Ka_2$

$$= \left[q + \gamma_1 H_1\right] \times Ka_2$$

$$= [40 + 16.83 \times 1] \times 0.46 = 26.14 \text{ kN/m}^2$$

$$P_C = [q + \bar{\sigma}_{v1} + \bar{\sigma}_{v2}] \times Ka_2$$
$$= q \cdot Ka_2 + \gamma_1 H_1 Ka_2 + \gamma_2 H_2 Ka_2$$
$$= 40 \times 0.46 + 16.83 \times 1 \times 0.46 + 19.58 \times 2 \times 0.46$$
$$= 44.15 \text{ kN/m}^2$$

$$P_{D_1} = [q + \bar{\sigma}_{v1} + \bar{\sigma}_{v2}]Ka_2 + [\bar{\sigma}_{v3}]Ka_2 + \gamma_w \cdot H_3$$
$$= 44.15 + [21.18 - 9.81] \times 2 \times 0.46 + 9.81 \times 2$$
$$= 74.24 \text{ kN/m}^2$$

$$P_{D_2} = [q + \bar{\sigma}_{v1} + \bar{\sigma}_{v2} + \bar{\sigma}_{v3}]Ka_3 + [H_3 \cdot \gamma_w] - 2C_2\sqrt{Ka_3}$$
$$= [40 + 16.33 \times 1 + 19.58 \times 2 + (21.18 - 9.81) \times 2] \times 1$$
$$+ 9.81 \times 2 - 2 \times 37.5\sqrt{1}$$
$$= 62.85 \text{ kN/m}^2$$

$$P_E = [q + \bar{\sigma}_{v1} + \bar{\sigma}_{v2} + \bar{\sigma}_{v3} + \bar{\sigma}_{v4}]Ka_3 + [H_3 + H_4] \times \gamma_w - 2C\sqrt{Ka_3}$$
$$= [40 + 16.83 \times 1 + 19.58 \times 2 + (21.18 - 9.81) \times 2 + (20.93 - 9.81) \times 3]$$
$$\times 1 + [2 + 3] \times 9.81 - 2 \times 37.5\sqrt{1}$$
$$= 126.14 \text{ kN/m}^2$$

- Total active thrust on wall per meter length:

$$P_a = A_1 + A_2 + A_3 + A_4 + A_4 + A_5 + A_6 + A_7 + A_8$$

Here,
$$A_1 = \frac{1}{2}(-10) \times 0.59 = -2.95 \text{ kN/m}$$
$$A_2 = \frac{1}{2} \times 0.41 \times 6.83 = 1.4 \text{ kN/m}$$
$$A_3 = 26.14 \times 2 = 52.28 \text{ kN/m}$$
$$A_4 = \frac{1}{2}(44.15 - 26.14) \times 2 = 18.01 \text{ kN/m}$$
$$A_5 = 44.15 \times 2 = 88.3 \text{ kN/m}$$
$$A_6 = \frac{1}{2}(74.24 - 44.15) \times 2 = 30.09 \text{ kN/m}$$
$$A_7 = 62.85 \times 3 = 188.55 \text{ kN/m}$$

$$A_8 = \frac{1}{2}(126.14 - 63.35) \times 3 = 94.185 \text{ kN/m}$$

$$\therefore \quad P_a = 469.865 \text{ kN/m}$$

Now, location of active thrust from bottom

$$= \text{C.G. of positive earth pressure}$$

$$= \frac{\Sigma A_i Z_i}{\Sigma A_i}; \qquad \text{(where } Z_i = \text{location of respective area from bottom)}$$

$$\therefore \quad Z_1 = 3 + 4 + 0.41 + 0.59 \times \frac{2}{3} = 7.803 \text{ m}$$

$$Z_2 = 3 + 4 + \frac{0.41}{3} = 7.137 \text{ m}$$

$$Z_3 = 3 + 2 + 2 \times \frac{1}{2} = 6 \text{ m}$$

$$Z_4 = 3 + 2 + 2 \times \frac{1}{3} = 5.667 \text{ m}$$

$$Z_5 = 3 + \frac{2}{2} = 4 \text{ m}$$

$$Z_6 = 3 + \frac{2}{3} = 3.667 \text{ m}$$

$$Z_7 = 3 \times \frac{1}{2} = 1.5 \text{ m}$$

$$Z_8 = \frac{3}{3} = 1 \text{ m}$$

Putting all the value in above formula, we get C.G from bottom = 2.646 m

Q.8 (a) Solution:

1. **Effective span**

$$l_{\text{eff}} = 4.75 + \frac{0.23}{2} + \frac{0.23}{2} = 4.98 \text{ m}$$

2. **Effective depth**

$$d = \frac{\text{Effective span}}{A \times M_{ft}} = \frac{4980}{20 \times 1.2} = 207.5 \text{ mm say } 210 \text{ mm}$$

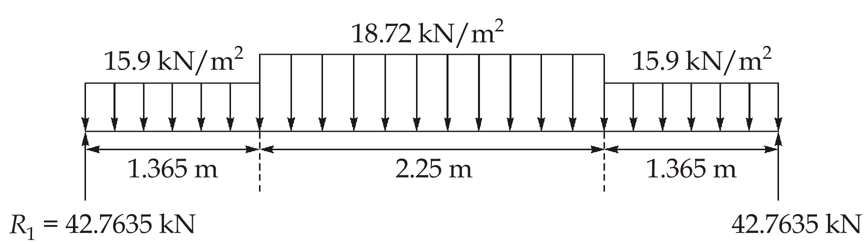
Assuming an effective cover of 30 mm, $D = 210 + 30 = 240$ mm. Let us assume thickness of slab for landing also as 240 mm.

3. Loads on landing:

$$LL = 4.0 \text{ kN/m}^2$$
$$\text{Finishes} = 0.6 \text{ kN/m}^2$$
$$\text{Self weight of slab} = 25 \times 0.24 = 6 \text{ kN/m}^2$$
$$\text{Total load} = 10.6 \text{ kN/m}^2$$
$$\text{Factored load} = 1.5 \times 10.6 = 15.9 \text{ kN/m}^2$$

4. Loads on waist slab (going)

$$LL = 4.0 \text{ kN/m}^2$$
$$\text{Finishes} = 0.6 \text{ kN/m}^2$$
$$\text{Self weight of waist slab} = 25 \times 0.24 = 6 \text{ kN/m}^2$$
$$\text{Self weight of steps} = \left(\frac{1}{2} \times 0.15 \times 0.25\right) \times \frac{1}{0.25} \times 25 = 1.875 \text{ kN/m}^2$$
$$\text{Total load} = 12.475 \text{ kN/m}^2$$
$$\text{Factored load} = 18.72 \text{ kN/m}^2$$



5. Maximum BM at mid span

$$= (42.7635 \times 2.49) - (15.9 \times 1.365 \times 1.8075) - \left(18.72 \times 1.125 \times \frac{1.125}{2}\right)$$
$$= 71 \text{ kN-m per m width}$$

$$\therefore \text{ Depth of slab required, } d = \sqrt{\frac{BM_u}{Q.B}} = \sqrt{\frac{71 \times 10^6}{0.138 \times 20 \times 1000}}$$
$$d = 160.4 \text{ mm}$$
$$d_{provided} = 210 \text{ mm} \quad \text{(OK)}$$

6. Design of flexural reinforcement

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 B.M_u}{f_{ck} B d^2}}\right] Bd$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 71 \times 10^6}{20 \times 1000 \times 210^2}}\right] \times 1000 \times 210$$

$$= 1044.8 \text{ mm}^2$$

$\therefore$ Spacing of 12 mm ϕ bars $= \dfrac{1000 \times \frac{\pi}{4} \times 12^2}{1044.8} = 108.24$ mm say 100 mm c/c

$\therefore$ Provide 12 mm ϕ bars @ 100 mm c/c

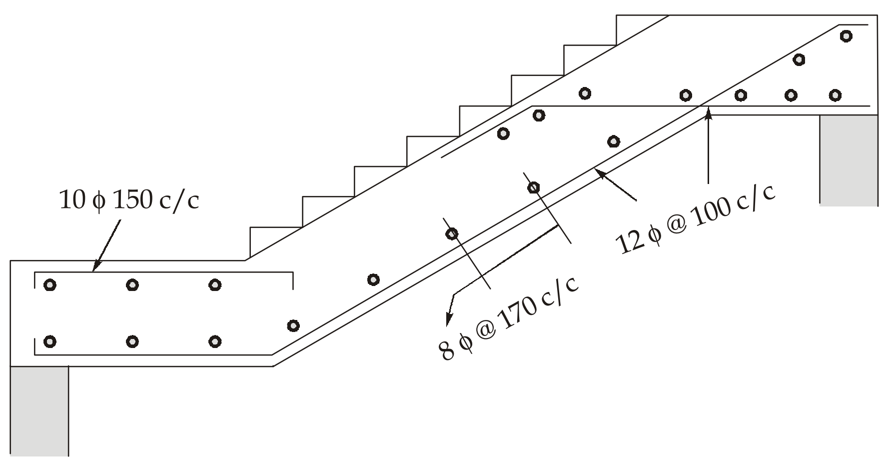
7. Minimum reinforcement

$$A_{st\,min} = \frac{0.12}{100} \times 1000 \times 240 = 288 \text{ mm}^2$$

$\therefore$ Spacing of 8 mm ϕ bars $= \dfrac{1000 \times \frac{\pi}{4} \times 8^2}{288} - 174.5$ mm say 170 mm c/c

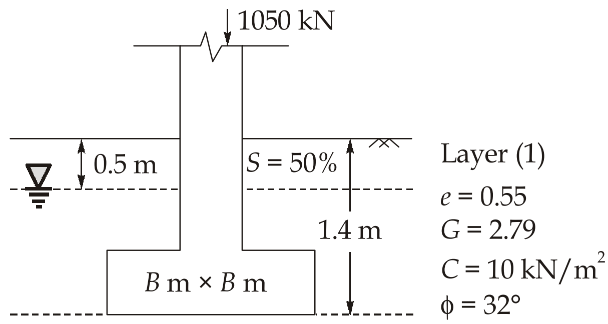
provide 8 mm ϕ bars @ 170 mm c/c.

8. Provide 10 mm ϕ bars at 150 c/c in the landing slab near the support at the top to resist any negative moment that may arise.



Reinforcement details

Q.8 (b) Solution:



Unit weight of soil for layer-1,

Above W.T: $$\gamma_{bulk_1} = \left(\frac{G+eS}{1+e}\right)\times\gamma_w$$

$$= \frac{2.79+0.55\times0.5}{1+0.55}\times 9.81 = 19.4\ \text{kN/m}^3$$

Below W.T. $$\gamma_{sat_1} = \left(\frac{G+e}{1+e}\right)\times 9.81 = \frac{2.79+0.55}{1+0.55}\times 9.81 = 21.1\ \text{kN/m}^3$$

Unit weight of soil for layer-2,

$$\gamma_{sat_1} = \frac{G+e}{1+e}\times 9.81 = \frac{2.67+0.64}{1+0.64}\times 9.81 = 19.8\ \text{kN/m}^3$$

Now, we know that ultimate bearing capacity of square footing as given by Terzaghi is,

$$q_u = 1.3cN_c + \gamma_{eff_1}\cdot D_f N_q + 0.4\cdot B\cdot\gamma_{eff_2}\cdot N_\gamma$$

Here, bearing capacity factors and cohesion is taken of bearing soil only

∴ For $\phi = 30°$, $N_c = 37.2$

$N_q = 22.5$

$N_\gamma = 19.7$

and $C = 12\ \text{kN/m}^2$

$$\gamma_{eff_1} = \frac{0.5\times\gamma_{bulk_1} + [1.4-0.5]\times\gamma_{sub_1}}{1.4}$$

$$= \frac{0.5 \times 19.4 + [1.4 - 0.5] \times (21.1 - 9.81)}{1.4} = 14.19 \text{ kN/m}^3$$

$$\gamma_{eff_2} = (\gamma_{sat})_2 - \gamma_w$$

$$= 19.8 - 9.81 = 9.99 \text{ kN/m}^3$$

$$\therefore \quad q_u = 1.3 \times 12 \times 37.2 + 14.19 \times 1.4 \times 22.5 + 0.4 \times B \times 9.99 \times 19.7$$

$$= 1027.305 + 78.72B$$

Net ultimate bearing capacity,

$$q_{nu} = q - \gamma_{eff\,1} \cdot D_f$$

$$= 1027.305 + 78.72B - 14.19 \times 1.4$$

$$= 1007.44 + 78.72B$$

Net safe bearing capacity, $q_{ns} = \dfrac{q_{nu}}{FOS} = \dfrac{1007.44 + 78.72B}{3}$

Safe bearing capacity, $q_s = q_{ns} + \gamma D_f$

$$= \frac{1007.44 + 78.72B}{3} + 14.19 \times 1.4 = 355.68 + 26.24B$$

Since q_s should be greater than $\dfrac{Q}{B^2}$

$$\therefore \quad q_{safe} \geq \frac{Q}{B^2}$$

$$\Rightarrow \quad 355.68 + 26.24B \geq \frac{1050}{B^2}$$

$$\Rightarrow \quad 355.68B^2 + 26.24B^3 - 1050 \geq 0$$

From here, $B = 1.624 \text{ m} \approx 1.63 \text{ m}$

Note: $\dfrac{D_f}{B} = \dfrac{1.4}{1.63} = 0.859 < 1$ (OK)

Q.8 (c) Solution:

(i) The commonly used samplers can be classified into three categories:

1. **Open-drive sampler:** The open-drive sampler is the simplest type of sampler for collection of samples. These are made up of seamless steel. The bottom of the tube is sharpened and beveled, which act as a cutting edge. The tube is connected through a head to the drill rod. The sampler head is provided with vents to permit water and air to escape during sampling and also a ball check valve to certain the

sample during the withdrawal of sampler. The sampling tube may be thick walled or thin walled. Thick walled samplers are used for obtaining disturbed but representative samples. They may be in the form of a solid tube or a split tube with or without a liner. The sample is collected by the thick walled sampler by the repeated blows of a falling weight.

Thin wall samplers are used for obtaining undisturbed samples. The area ratio is usually below 15 percent. The sampling tube for sampling of soil is pushed into the soil in a continuous rapid motion without impact or twisting.

2. **Piston sampler:** A piston sampler consists of two parts- (a) simpler cylinder (b) piston system. The piston rod fits easily inside the hollow drill rod. During the driving and upto the start of the sampling, the bottom of the piston is maintained flush with the cutting edge of the sampler. At the proposed sampling depth, the bottom of the piston is fixed in relation to the ground and the sampler cylinder forced in the soil independently, cutting a sample out of soil. As the sampler cylinder slides past the tight fitting portion during the sampling operation, a negative pressure develops above the sample which holds back the samples during withdrawal. After the cylinder is pushed to the required depth, both the cylinder and piston system are withdrawn with the sample inside the sample cylinder. Piston sampler is useful in sampling saturated sands and other soft and wet soils which cannot be sampled by open drive sampler.

3. **Rotary sampler:** A rotary sampler is a double-walled tube sampler with an inner removable liner. The outer tube or the rotating barrel is provided with a cutting bit. The bit cuts an annular ring when the barrel is rotated. The inner tube is stationary, slides over the cylindrical sample cut by the other rotating barrel. The sample is collected in the inner tube.

   Rotary sampler can be used for collection of undisturbed samples in stiff to hard clays, silts and sands with some cementation and also in rocks. The sampler is however undisturbed samples in stiff to hard clays, silts and sands with some cementation and also in rocks. The sampler is however unsuitable for gravelly soils and loose cohesionless soils.

(ii) Pressuremeter test: A pressuremeter is a device which is used to determine the stress-strain relations of insitu soil by pumping it into the certain depth of borehole. There are two basic types of pressuremeter:

- The manured pressuremeter which is lowered into a performed borehole.
- The self boring pressuremeter which forms its own borehole.

In both the cases, the pressure meter test consists of applying known stresses to the soil and measuring the resulting soil deformation. The pressuremeter test is conducted in a predrilled borehole normally at intervals of 1 m. A borchole that is sufficiently oversized is used. After drilling the borehole, the probe is lowered to the required depth using cables. The probe should be lowered without disturbing the surrounding soil. At the desired location the probe is fixed using clamping device.

With the probe in position, the control unit is used to audit water and gas to the test cell and guard cell respectively, to keep them at equal pressure.

- The pressure of water in the measuring cell in increased in increments until the soil fails. Usually failure is considered to have been reached when the total expanded volume of the test zone reached twice the volume of original cavity. Each increment of pressure is held for a fixed length of time, typically one minute and the related volume change readings are noted. A plot of pressure versus change in volume is made to obtain parameters necessary for foundation design. The curve obtained may contain some error. To overcome this, the pressuremeter should be calibrated for pressure loss, volume loss and hydrostatic pressure head before it is being used in design.

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