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Detailed Solutions

ESE-2023 Mains Test Series

Mechanical Engineering Test No : 5

Section A : Production Engineering & Material Science [All Topics]

Section B : Theory of Machines-1 [Part Syllabus]

Fluid Mechanics & Turbo Machinery-2 [Part Syllabus]

Section : A

1. (a)

Linear density (LD) : It is defined as the number of atoms per unit length whose centres lie on the direction vector for a specific crystallographic direction.

$$LD = \frac{\text{Number of atoms centered on direction vector}}{\text{Length of direction vector}}$$

Planar density (PD) : It is defined as the number of atoms per unit area that are centered on a particular crystallographic plane.

$$PD = \frac{\text{Number of atoms centered on a plane}}{\text{Area of plane}}$$

Given : Atomic radius of zinc, $r = 0.135$ nm; Atomic weight of zinc, $A = 65.39$ g/mol

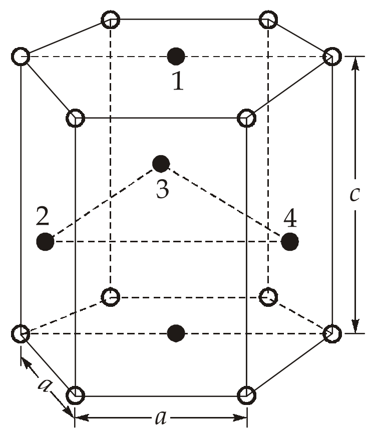
$\frac{c}{a} = 1.856$

Volume of the crystal structure,

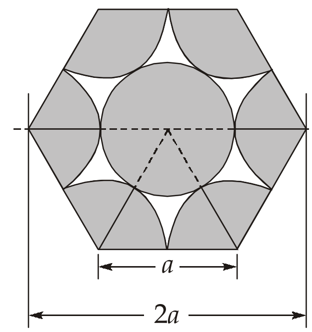
$$V = 6\times\frac{\sqrt{3}}{4}\times a^2\times c \qquad (a = \text{side of the hexagon})$$

$$= 6\times\frac{\sqrt{3}}{4}\times a^2\times 1.856a$$

$$= 4.822a^3$$



HCP crystal structure



Top view of HCP

For HCP crystal structure, $n = 6$

For HCP crystal structure, $a = 2r$

Avogadro's number, $N_A = 6.023 \times 10^{23}$ atoms/mol

$$\therefore \quad \text{Density} = \frac{n \times A}{N_A \times V} = \frac{6 \times 65.39}{6.023 \times 10^{23} \times 4.822 a^3}$$

$$= \frac{6 \times 65.39}{6.023 \times 10^{23} \times 4.822 \times (2 \times 0.135 \times 10^{-7})^3}$$

$$= 6.863 \text{ g/cc}$$

1. (b)

Given : Side of square hole, $a = 12$ mm; Thickness of steel plate, $t = 6$ mm; Resistance, $R = 60\ \Omega$; Capacitance, $C = 12\ \mu$F; Supply voltage, $V_0 = 220$ V;

Discharge voltage, $V_d = 150$ V; $Q \simeq 27.4\ \text{W}^{1.54}$

Amount of energy released per spark,

$$E = \frac{1}{2} \times C \times V_d^2$$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times (150)^2 = 0.135 \text{ J}$$

$$\text{Cycle time, } t_c = RC \ln\left(\frac{V_0}{V_0 - V_d}\right)$$

$$= 60 \times 12 \times 10^{-6} \ln\left(\frac{220}{220 - 150}\right)$$

$$= 8.245 \times 10^{-4} \text{ seconds}$$

So, Power input $= \frac{E}{t_c} = \frac{0.135}{8.245 \times 10^{-4}}$

$= 163.73$ Watts $= 0.163$ kW

Material removal rate, $Q = 27.4\ W^{1.54}$

$= 27.4 \times (0.163)^{1.54} = 1.677\ \text{mm}^3/\text{min}$

Total amount of material removed $= (12)^2 \times 6 = 864\ \text{mm}^3$

Therefore, time required to accomplish the operation,

$$t = \frac{864}{1.677} = 515.2 \text{ min}$$

1. (c)

Given : Diameter of hole, $d = 150$ mm; Thickness of plate, $t = 8$ mm;
Shear strength, $\tau = 560$ MPa; Penetration, $p = 40\% = 0.4$; $F_{max} = 250$ kN

Clearance per side, $c = 0.0032 t\sqrt{\tau}$

$= 0.0032 \times 8 \times \sqrt{560} = 0.6058$ mm

Punch diameter = Diameter of hole = 150 mm

Die diameter $= d + 2c$

$= 150 + 2 \times 0.6058 = 151.2116$ mm

Without any shear, the maximum punching load is given by

$$F = \tau \times \pi \times d \times t$$

$$= 560 \times \pi \times 150 \times 8 = 2111.15 \text{ kN}$$

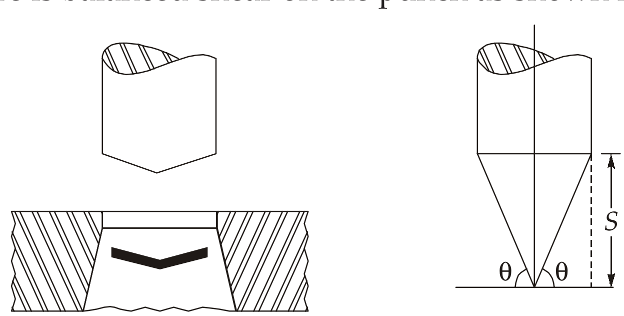
Let, S be the penetration of the punch in the form of shear to reduce the punch load to the available 250 kN and assuming $S > pt$

$$F_{S\max} \times pt = F_S \times S$$

$$2111.15 \times 0.4 \times 8 = 250 \times S$$

$$S = 27.023 \text{ mm}$$

It is given that there is balanced shear on the punch as shown in the figure below:

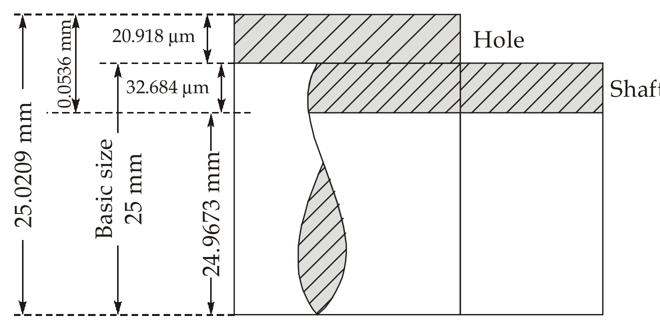


So, shear angle, $\theta = \tan^{-1}\left(\frac{S}{d/2}\right) = \tan^{-1}\left(\frac{27.023}{150/2}\right)$

$$\theta = 19.798°$$

1. (d)

Given : Fit is 25 H_7/h_8 ; 25 mm lies between 18 mm and 30 mm; Tolerance for IT7 = 16i ; Tolerance for IT8 = 25i



For hole H_7 and shaft h_8, the fundamental deviation is zero.

As 25 mm lies in the range of 18 mm and 30 mm

So, $$D = \sqrt{18 \times 30} = 23.238 \text{ mm}$$

The value of standard tolerance unit,

$$i = 0.45 \times \sqrt[3]{D} + 0.001D$$
$$= 0.45 \times \sqrt[3]{23.238} + 0.001 \times 23.238$$
$$= 1.30737 \ \mu\text{m}$$

Tolerance value for IT7 = 16i
= 16 × 1.30737
= 20.918 μm
= 0.0209 mm

∴ Minimum size of hole = 25 mm **Ans.**

Maximum size of hole = 25.0209 mm **Ans.**

Tolerance value of IT8 = 25i
= 25 × 1.30737
= 32.684 μm = 0.03268 mm

∴ Maximum size of shaft = 25 mm **Ans.**

Minimum size of shaft = 25 – 0.03268
= 24.9673 mm **Ans.**

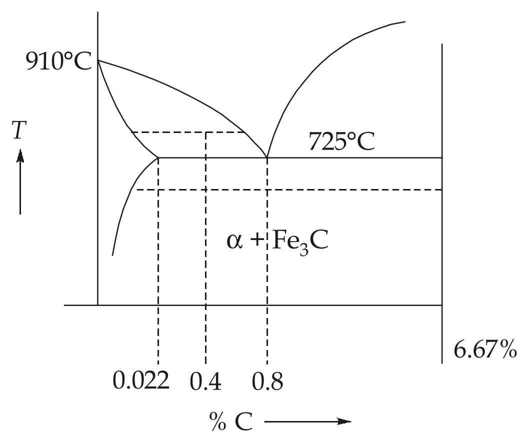
Allowance (or minimum clearance) = Minimum size of hole – Maximum size of shaft
= 25 – 25 = 0 **Ans.**

Maximum clearance = Maximum size of hole – Minimum size of shaft

= 25.0209 – 24.9673

= 0.0536 mm Ans.

1. (e)



(i)

This part is solved by employing a tie line (just below the eutectoid temperature) that extends all the way across α + Fe_3C phase.

Thus, at 0.4% of C

Mass fraction of total ferrite,

$$M_{\text{total-}\alpha} = \frac{6.67 - 0.4}{6.67 - 0.022} = 0.943$$

and mass fraction of total cementite,

$$M_{Fe_3C} = \frac{0.4 - 0.022}{6.67 - 0.022} = 0.0568$$

(ii)

The fractions of proeutectoid ferrite and pearlite are determined by using the lever rule and a tie line (just above the eutectoid temperature) that extends only to the eutectoid composition.

Thus, at 0.4% of C

Mass fraction of proeutectoid ferrite,

$$M_{\text{pro-}\alpha} = \frac{0.8 - 0.4}{0.8 - 0.022} = 0.514$$

Mass fraction of pearlite, $M_P = \dfrac{0.4 - 0.022}{0.8 - 0.022} = 0.486$

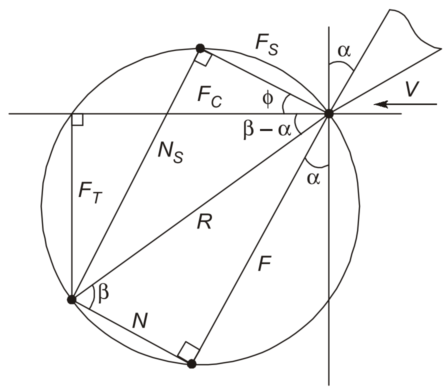
(iii)

All ferrite is either proeutectoid or eutectoid (in the pearlite), so the sum of these two ferrite fractions will be equal to the fraction of total ferrite.

So, $M_{\text{pro-}\alpha} + M_{\text{eut-}\alpha} = M_{\text{total-}\alpha}$

$\Rightarrow$ $M_{\text{eut-}\alpha} = 0.943 - 0.514 = 0.429$

2. (a)



Given : V = 240 m/min; α = 12°; Width of cut, b = 2 mm; Uncut thickness, t = 0.2 mm; Coefficient of friction, μ = 0.5; Shear stress, τ_s = 420 N/mm^2; Machining constant, $\cot^{-1}(k)$ = 75°; Angle of friction, $\beta = \tan^{-1}\mu = \tan^{-1}0.5 = 26.565°$

(i) Using Merchant's second analysis

$$2\phi + \beta - \alpha = \cot^{-1}(k)$$

$$\Rightarrow \quad 2\phi + 26.565 - 12 = 75°$$

$$\Rightarrow \quad \phi = 30.217° \qquad \text{Ans.}$$

$$\text{Shear stress, } \tau_s = \frac{F_s}{bt/\sin\phi}$$

$$\Rightarrow \quad F_s = \tau_s \times \frac{bt}{\sin\phi} = \frac{420 \times 2 \times 0.2}{\sin 30.217°} = 333.813 \text{ N}$$

$$\text{Resultant force, } R = \frac{F_s}{\cos(\phi + \beta - \alpha)}$$

$$= \frac{333.813}{\cos(30.217 + 26.565 - 12)} = 470.297 \text{ N}$$

(ii)

$$\text{Cutting force, } F_C = R\cos(\beta - \alpha)$$
$$= 470.297 \times \cos(26.565 - 12)$$
$$= 455.183 \text{ N} \qquad \textbf{Ans.}$$

$$\text{Thrust force, } F_T = R\sin(\beta - \alpha)$$
$$= 470.296 \times \sin(26.565 - 12)$$
$$= 118.269 \text{ N} \qquad \textbf{Ans.}$$

Now, using Lee and Shaffer theory:

$$\phi = 45° + \alpha - \beta$$

$\Rightarrow$ $$\text{Shear angle, } \phi = 45° + 12° - 26.565°$$
$$= 30.435° \qquad \textbf{Ans.}$$

$$\text{Shear force, } F_s = \tau_s \times \frac{bt}{\sin\phi} = 420 \times \frac{2 \times 0.2}{\sin 30.435°} = 331.648 \text{ N}$$

$$\text{Resultant force, } R = \frac{F_s}{\cos(\phi + \beta - \alpha)}$$
$$= \frac{331.648}{\cos(30.435 + 26.565 - 12)} = 469.021 \text{ N}$$

$$\text{Cutting force, } F_C = R\cos(\beta - \alpha)$$
$$= 469.021 \times \cos(26.565 - 12)$$
$$= 453.948 \text{ N} \qquad \textbf{Ans.}$$

$$\text{Thrust force, } F_T = R\sin(\beta - \alpha)$$
$$= 469.021 \times \sin(26.565 - 12)$$
$$= 117.948 \text{ N} \qquad \textbf{Ans.}$$

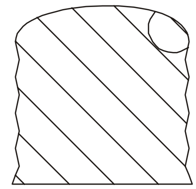
COMPARISON : It is found that results obtained by Merchant's second analysis and Lee and Shaffer method do not vary much.

2. (b)

(i)

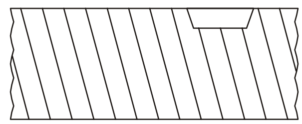
Some casting defects are as given below:

(I) Blow : It is a fairly large, well rounded cavity produced by the gases which displace the molten metal at the cope surface of a casting.



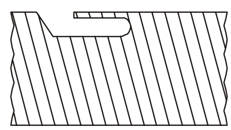
Blow

(II) **Scar** : A shallow blow, usually found on a flat casting surface is referred as scar.



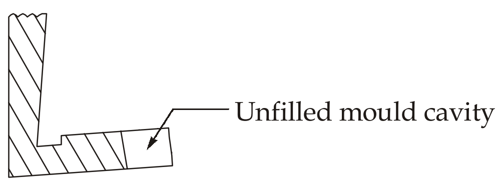
Scar

(III) **Blister** : This is a scar covered by the thin layer of a metal.



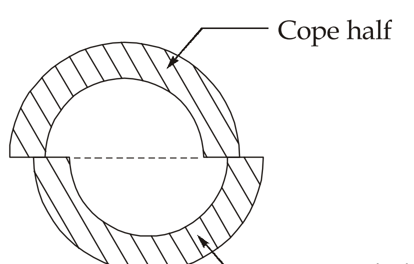
Blister

(IV) **Misrun** : Many a time, the liquid metal may, due to insufficient superheat, start freezing before reaching the farthest point of the mould cavity. This defect is called misrun.



Misrun

(V) **Mould shift** : A misalignment between two halves of a mould may give rise to a defect in casting known as mould shift.

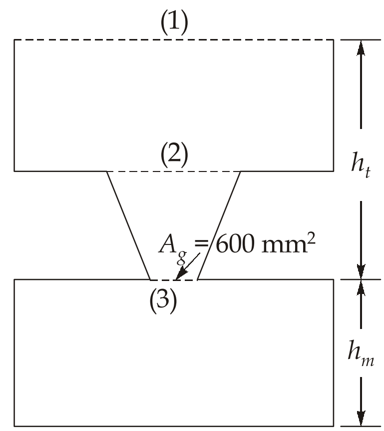


Mould shift

(ii)

Given : Casting area, A_m = 1200 mm × 1200 mm; Casting height, h_m = 260 mm; Manometric height, h_t = 260 mm; Gate area, A_g = 600 mm^2

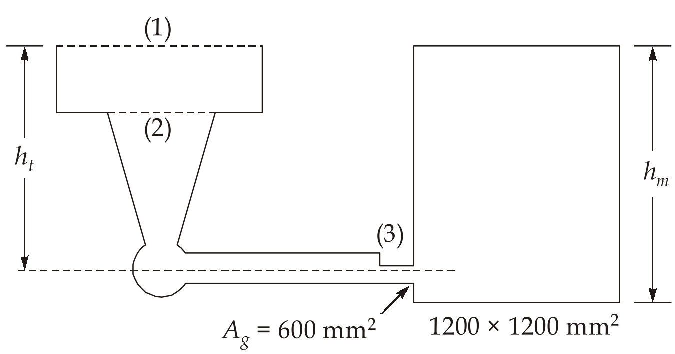
Top Gate:



$$t_f = \frac{V_m}{A_g \times \sqrt{2g \times h_t}}$$

$$= \frac{1200 \times 1200 \times 260 \times 10^{-3}}{600 \times \sqrt{2 \times 9.81 \times 0.26}} = 276.279 \text{ seconds}$$

Bottom Gate:



$$t_{bf} = \frac{2A_m}{A_g} \times \frac{1}{\sqrt{2g}} \times \left(\sqrt{h_t} - \sqrt{h_t - h_m}\right)$$

$$= \frac{2 \times 1200 \times 1200}{600} \times \frac{1}{\sqrt{2 \times 9.81}} \times \left(\sqrt{0.26} - \sqrt{0.26 - 0.26}\right)$$

$$= 552.558 \text{ seconds}$$

Note : It can be noted that $t_{bf} = 2t_f$. This will be the case when manometric height is equal to the height of casting.

2. (c)

Various basic forms of corrosion, which can occur in a system can be recognized as follows:

1. Uniform	2. Pitting
3. Crevice	4. Galvanic
5. Stress corrosion	6. Hydrogen Embrittlement
7. Intergranular attack	8. Dealloying

Pitting corrosion: Pitting is a local corrosion damage characterized by cavities. It is a particularly insidious form of corrosion, because even if one pit perforates the side of a tank, serviceability is lost until the tank is repaired. Chemical nature of the environment causes pitting which are as follows:

1. Halogen-containing solutions
2. Brackish water
3. Salt water
4. Chloride bleaches
5. Reducing inorganic acids are solutions that tend to produce pitting.

Stainless steels are particularly prone to pitting. Pitting of brass conductive tubes sometimes occurs due to dezincification. This consists in the solution of the brass followed by precipitation of copper by zinc in the brass. The net result is selective removal of zinc. A localized attack frequently occurs near the inlet ends of the condenser tubes, due to impingement of air bubbles, which carry away the corrosion products. Pumps and ship propellers are liable to an attack known as cavitation, an impact caused by the collapse of vapour bubbles.

Remedies :

1. Use of austenitic steels pit in salt water, so most designer tend to use copper alloys, bronzes, monels and other materials having lower pitting tendencies.
2. Use of carbon steels in salt water in which corrosion rate is much higher than with stainless steels, but attack is more uniform and no pitting takes places.

Stress corrosion: Stress corrosion cracking takes place under the combined action of action of applied tensile stress and corrosive environment. Some materials, which are inert in a particular corrosive medium become susceptible when a tensile stress is applied. Small cracks are developed, which propagate in a direction perpendicular to tensile stress, resulting in eventual failure of the material. Most alloys are susceptible to stress corrosion under specific environments. For example, stainless steels corrode in solution of chloride ions and brasses corrode when exposed to ammonia.

It is not necessary that the stress has to be externally applied. Even a residual stress can cause this type of corrosion. It is a form of environmentally assisted cracking (EAC). Hydrogen embrittlement, caustic embrittlement and liquid metal corrosion are all forms of EAC. This corrosion is time dependent, sometimes take months to occur.

Remedies :

1. Stress relieving heat treatment.
2. Use of pure metals, which are immune to corrosion.
3. To consult available corrosion data and to avoid the environment - material combination that tends to cause stress corrosion cracking.

Galvanic corrosion: If two dissimilar metals are connected electrically in an electrolyte, an electrochemical cell is formed. If the two metals are significantly dissimilar, then one metal will become anodic and corrode. Galvanic series of metals is established in sea water. Farther apart, two metals in the list (galvanic series of metals), the greater is the potential for corrosion, when they are coupled in an electrolyte. Magnesium to steel is a bad combination, while monel to stainless steel is a good combination (with negligible corrosion).

The other factor that controls galvanic corrosion is the relative size of the anode and the cathode. If the anode is smaller in comparison to the cathode, the attack on anode will be more, but if the size of the anode is bigger, then the situation is reversed. A steel bolt in an aluminium plate wherein aluminium becomes the anode, anode plate is large in comparison to steel bolt (cathode), and hence attack will be less.

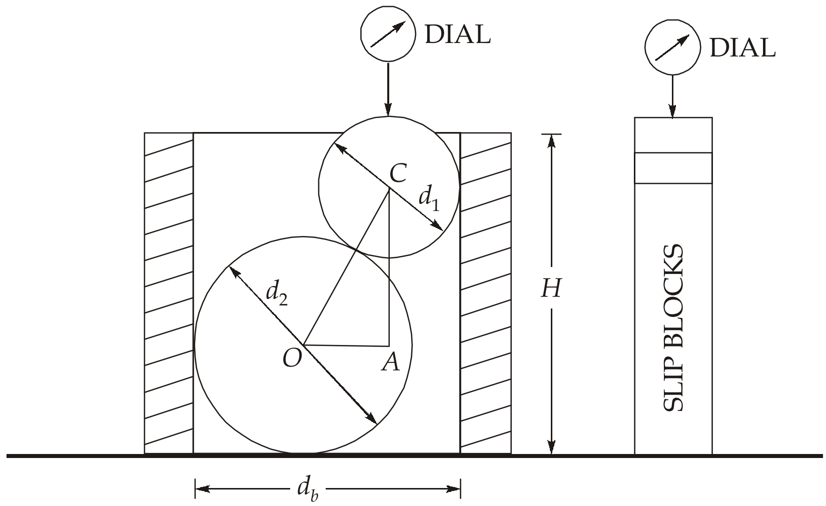
Remedy :

1. If two metals are immersed in electrolyte and they are not mixed, galvanic corrosion is avoided.

3. (a)

(i)

The balls of known diameters d_1 and d_2 are kept one over the other in bore as shown in the figure below. The sum of the values of d_1 and d_2 should be greater than diameter of bore. The height of highest point of upper ball from surface plate is measured with the help of slip gauges and dial indicator. The slip blocks are piled such that the dial indicator reading at slip blocks and highest point of ball is same. Knowing distance H, the diameter of bore can be calculated as follows:



$$\text{Diameter of bore, } d_b = \frac{d_1 + d_2}{2} + AO$$

$$AO^2 = OC^2 - CA^2$$

$$= \left(\frac{d_1 + d_2}{2}\right)^2 - \left(H - \frac{d_1 + d_2}{2}\right)^2$$

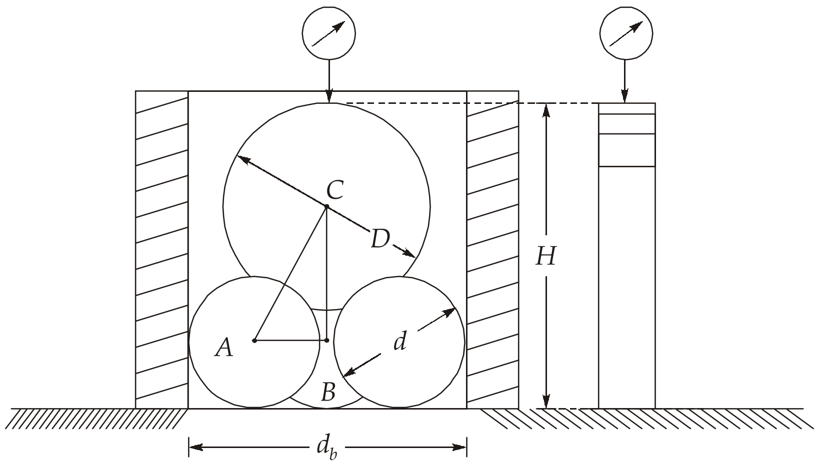
$$= -H^2 + (d_1 + d_2) \times H$$

So,
$$AO = \sqrt{(d_1 + d_2)H - H^2}$$

$\therefore$
$$d_b = \frac{d_1 + d_2}{2} + \sqrt{(d_1 + d_2)H - H^2}$$

(ii)

The diameter of bore can also be found by using three equal balls of diameter d less than half of the bore diameter and a fourth ball of any diameter D but less than 0.75 of the bore diameter. Balls are placed as shown in figure below. The height of highest point of upper ball is measured accurately with the help of slip gauges and dial mounted on a height gauge. It will be appreciated that bigger ball of diameter D resting over three balls of equal diameter d will be always be located in the centre of bore and therefore,



Diameter of bore, $d_b = d + 2AB$

$$AB^2 = AC^2 - BC^2$$

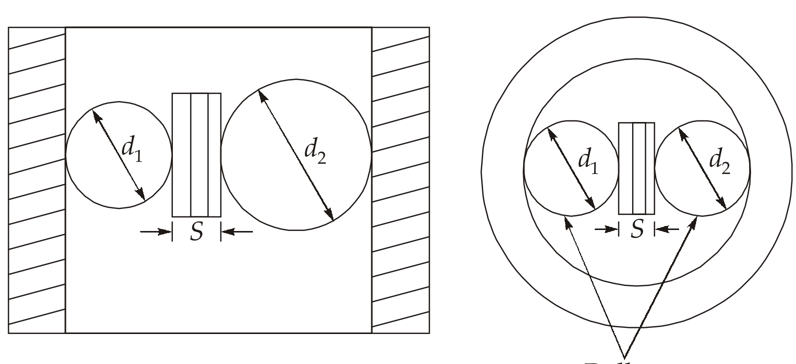
$$= \left(\frac{D+d}{2}\right)^2 - \left[H - \left(\frac{D+d}{2}\right)\right]^2$$

$\therefore$ $$AB = \sqrt{H(D+d) - H^2}$$

So, $$d_b = d + 2\sqrt{H(D+d) - H^2}$$

(iii)

The diameter of the bore can also be measured in the following way shown below.



Here two balls (or rollers) are taken such that $(d_1 + d_2)$ is less than the diameter of bore. Suitable slip blocks are inserted between the balls (or rollers).

Diameter of bore, $d_b = (d_1 + d_2)$ + slip inserted

This method is specially useful when the thickness of job is less.

3. (b)

Given : Initial diameter of rod, d_i = 8 mm; Final diameter of rod, d_f = 6 mm;
Speed, V = 0.5 m/s

(i)

$$\text{True strain, } \varepsilon_T = 2\ln\left(\frac{d_i}{d_f}\right)$$

$$= 2\ln\left(\frac{8}{6}\right) = 0.5753$$

$$\text{Average flow stress, } \bar{\sigma}_0 = \frac{k\varepsilon_T^n}{1+n} = \frac{900\times(0.5753)^{0.5}}{1+0.5} = 455.11 \text{ MPa}$$

$$\text{So, the drawing force, } F_d = \bar{\sigma}_0 A_f \times \varepsilon_T$$

$$= 455.11\times\frac{\pi}{4}\times(6)^2\times 0.5753$$

$$= 7402.92 \text{ N}$$

$$= 7.403 \text{ kN}$$

$$\text{Power} = F_d \times V = 7.403 \times 0.5 = 3.7 \text{ kW}$$

Since, frictional and redundant work constitutes 30% of the ideal work,
So, the actual power required, P_{actual} = 3.7 × 1.3 = 4.81 kW **Ans.**

(ii)

Based on the yield criteria, the die pressure along the die contact length can be obtained from

$$P = \sigma_f - \sigma \quad \text{...(i)}$$

where, σ is the tensile stress in the deformation zone at a particular diameter and σ_f is the flow stress of the material at that particular diameter. Note σ is thus equal to σ_d at the die exit and is zero at the die entry.

$$\therefore \quad \sigma_f = k\varepsilon_f^n = 900\times(0.5753)^{0.5} = 682.636 \text{ MPa}$$

From the above equation (i), σ is the drawing stress σ_d. Hence using the actual force, we have

$$\sigma_d = \frac{1.3\times 7.403\times 1000}{\frac{\pi}{4}\times(6)^2} = 340.37 \text{ MPa}$$

So, the pressure at the exit would be

$$P = 682.636 - 340.37 = 342.266 \text{ MPa}$$ **Ans.**

3. (c)

(i)

The coating on electrodes comprises of the following ingredients:

(a) **Cellulose :** It provides a reducing gas shield and increases arc voltage.

(b) **Potassium aluminium silicate :** It stabilizes the arc and gives strength to the coating.

(c) **Metal carbonates :** These produce a reducing atmosphere and adjust the basic nature of slag.

(d) **Mineral silicate :** These provide slag forming materials and strengthen the coating.

(e) **Ferromanganese and ferrosilicon :** These are used to deoxidise the weld metal.

(f) **Rutile :** It forms a highly fluid and quick freezing slag and adjusts the basic nature of slag.

(g) **Clays and gums :** These are used to produce a pasty material for extruding the coating during manufacturing of electrodes.

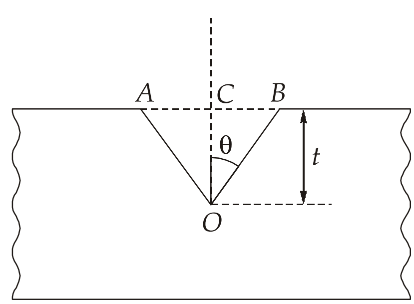
(h) **Iron powder :** It increases the amount of metal deposited and draws larger current and increases productivity. Deposition efficiency may by more than 100% with high yielding electrodes.

(i) **Calcium fluoride :** It produces shielding gas to protect the arc, provide fluidity, adjust the basicity of slag and solubility of metal oxides.

(j) **Titanium dioxide :** It helps in forming a highly fluid and quick freezing slag and provides ionization of the arc of the welding.

(k) **Manganese or iron oxide :** It helps in stabilizing the arc and adjusting the fluidity and properties of the slag.

(ii)



Current requirement for 15 mm thick plate = 200A

Current requirement for 22.5 mm thick plate = 420A

$$\text{Area of weld} = \frac{1}{2} \times AB \times OC$$

$$= \frac{1}{2} \times (2 \times t \times \tan\theta) \times t = t^2 \tan\theta$$

Let the welding speed be V

So, volume of the weld = $(t^2 \tan\theta) \times V$

As volume of the weld is directly proportional to the heat supplied and heat supplied is directly proportional to the square of the current.

So, $t^2 \times \tan\theta \times V \propto I^2$

$\Rightarrow$ $I \propto t$

or, $I = a + bt$

At t = 15 mm, I = 200 A

$\Rightarrow$ $200 = a + b \times 15$ ...(i)

At t = 22.5 mm; I = 420 A

$\Rightarrow$ $420 = a + b \times 22.5$ ...(ii)

From equation (i) and (ii)

$$7.5b = 220$$

$\Rightarrow$ $b = 29.33$ and $a = -240$

So, $I = -240 + 29.33t$

At t = 10 mm,

$$I = -240 + 29.33 \times 10 = 53.33 \text{ Amperes}$$

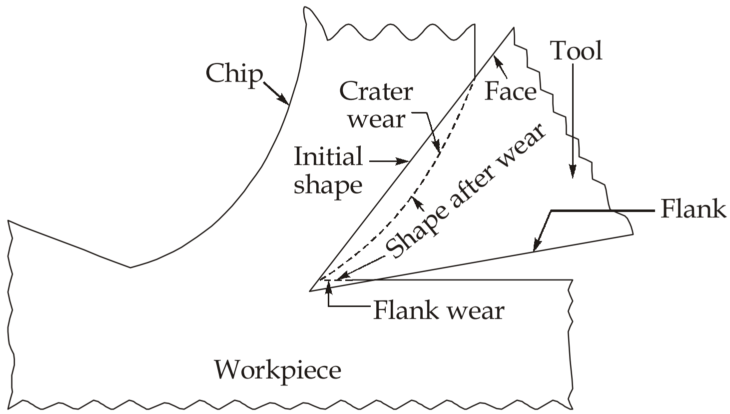
Hence, the welding current is 53.33 Amperes. Ans.

4. (a)

(i)

Flank wear : The gradual or progressive wear that develops on the flank surface of a cutting tool is called flank wear. Flank wear occurs as a result of friction between the progressively increasing contact area on the tool flank and the newly generated workpiece surface. Flank wear or wear land is on the clearance surface of the tool. The wear land can be characterised by the length of wear land. It notifies the tool geometry and changes the cutting parameters. With a high clearance angle, more flank wear is permissible before the critical wear land is reached; however excessive clearance weakens the cutting tool.

Crater wear : The gradual or progressive wear that develops on the rake surface of a cutting tool is called crater wear. Crater wear occurs as a result of the friction developed as the chip flows over the rake surface of the cutting tool. It is largely a temperature dependent phenomenon. The crater is on the rake face and is more or less circular. The crater does not always extend to the tool tip, but may end at a distance from the tool tip, but it increases the cutting forces, modifies the tool geometry and softens the tool tip.



(ii)

Given : For HSS tools :

$$VT^{1/8} = C_1$$

$$\Rightarrow \quad 30 \times (150)^{1/8} = C_1 \quad \text{...(i)}$$

For tungsten carbide tools:

$$VT^{1/5} = C_2$$

$$\Rightarrow \quad 30 \times (150)^{1/5} = C_1 \quad \text{...(ii)}$$

Now, at the cutting speed of 40 m/min;

For HSS tools:

$$30 \times (150)^{1/8} = 40 \times (T_1)^{1/8}$$

$$\Rightarrow \quad T_1 = 15.017 \text{ min}$$

For tungsten carbide tools:

$$30 \times (150)^{1/5} = 40 \times (T_2)^{1/5}$$

$$\Rightarrow \quad T_2 = 35.595 \text{ min}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{35.595}{15.017} = 2.37$$

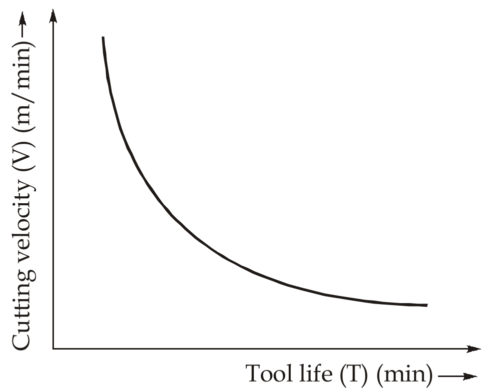
Hence, at 40 m/min, the life of tungsten carbide tools is 2.37 times more than the life of HSS tools.

(iii)

Factors affecting tool life are as follows:

- **Cutting speed :** It has greatest effect on tool life. Higher the cutting speed, smaller is the tool life. Based on work of F.W Taylor, the relationship between cutting speed and tool life can be given as $VT^n = c$.

  The figure shows the variation of tool life with cutting speed.



- **Feed and depth of cut :** An increase in feed or depth of cut will result in reduced tool life but not nearly as much as an increase in cutting speed. It can be noticed from modified Taylor's equation $VT^n f^a . d^b = c$.
- **Tool material :** Higher the hot hardness and toughness of a tool material, the longer the tool life.
- **Cutting fluids :** Cutting fluids with required properties and in adequate quantity helps in increasing tool life by keeping the cutting edge of tool cooler and lubricated.
- **Nature of cutting,** whether continuous cutting or intermittent cutting, tool is subjected to repeated impact loading. The continuous cutting is better for prolonged tool life.

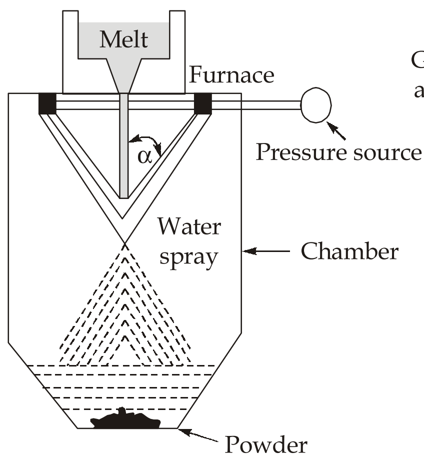
Some other factors that affect tool life are : type of the surface on the metal, profile of the cutting tool, microstructure of the material, surface finish required by the workpiece and cutting temperature.

4. (b)

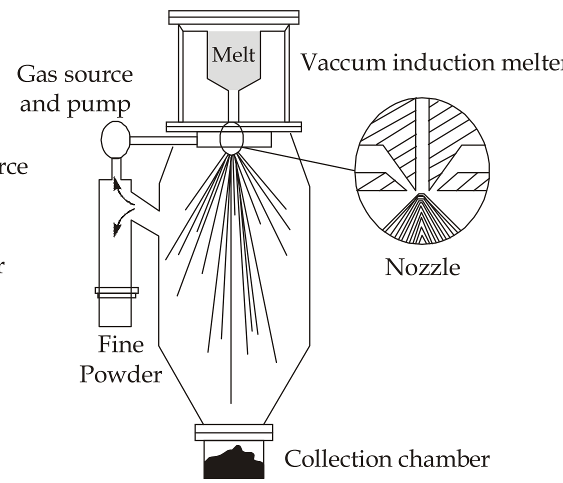
Various methods involved in the production of metal powders in powder metallurgy:

(i) Atomisation	(ii) Chemical reduction
(iii) Electrolytic process	(iv) Mechanical pulverisation
(v) Comminution	(vi) Machining
(vii) Milling	(viii) Condensation

(i) Atomisation : In this process, the molten metal is forced through an orifice into a stream of high velocity air, steam or inert gas. This causes extremely rapid cooling and disintegration into a very fine powder. The use of this process is usually limited to metals with low melting point.

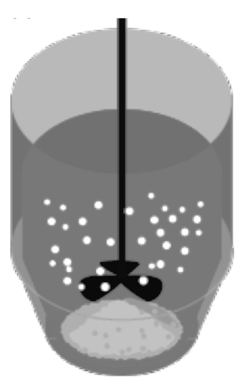


(a) Water atomization



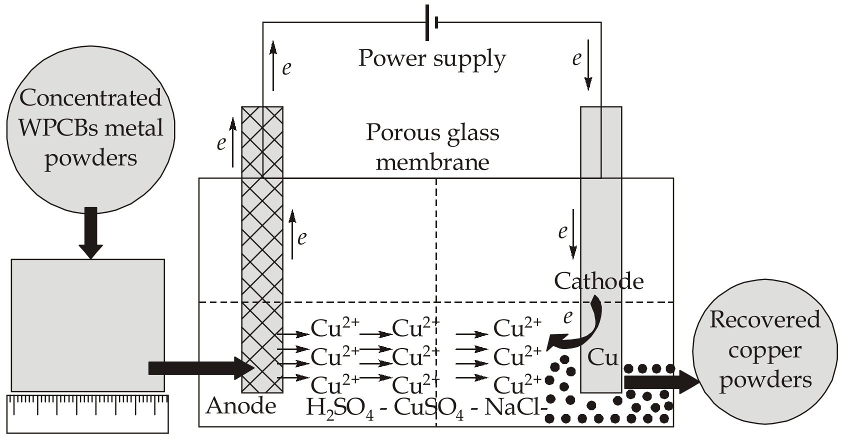
(b) Gas atomization

(ii) Chemical reduction : This process consists of grinding the metallic oxide to a finely divided state and then reducing it by hydrogen or carbon-monoxide. It is employed for metals such as iron, tungsten and copper (whose melting points are near or above 1100°C).



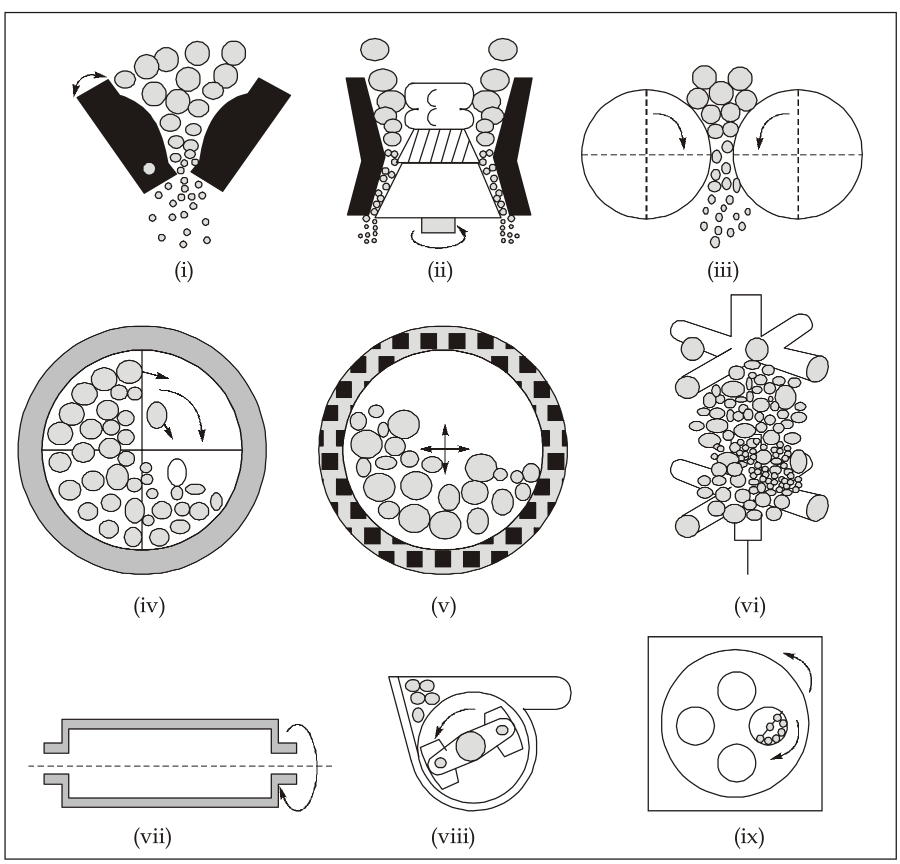
Chemical reduction

(iii) Electrolytic process : In this process of producing powder, the conditions of electrode position are controlled in such a way, that a soft spongy deposit is formed; which is then pulverised to form the powder. The particle size can be varied over a wide range by varying the electrolyte composition and various electrical parameters.



Electrolytic process

(iv) Mechanical pulverisation : This method is applicable to brittle materials like antimony. Brittle metals and alloys can be powdered down to a size of 0.001 mm. Many varieties of mechanical pulverisers are in use; some of these have counter-rotating plates or rapidly moving hammers. These methods carry out mechanical disintegration to the maximum fineness possible.



Mechanical Purlverisation Techniques

Applications of the powder metallurgy products:

(i) Porous and graphite containing metal bearings.

(ii) Electrical contacts consisting of a current and heat-conducting matrix in which wear resisting particles are embedded.

(iii) Tungsten wires.

(iv) Rotors of gear pump.

(v) Diamond impregnated tools.

(vi) Magnetic materials.

(vii) Refractory metal composites.

(viii) Metallic filters.

(ix) Motor brushes.

(x) Babbit bearings for automobiles.

4. (c)

Given : Rake angle, $\alpha = 12°$; Depth of cut, $d = 2$ mm; Feed rate, $f = 0.2$ mm/rev; Cutting speed, $V = 240$ m/min; Chip thickness ratio, $r = 0.36$; Vertical cutting force, $F_t = 1300$ N; Horizontal cutting force, $F_c = 700$ N; Thickness of primary shear zone, $t_s = 20$ µm

Shear plane angle, $$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = \frac{0.36\cos 12°}{1 - 0.36\sin 12°}$$

$$= 0.3806$$

$$\Rightarrow \qquad \phi = 20.837°$$

Shear force along the shear plane,

$$F_s = F_c\cos\phi - F_t\sin\phi$$
$$= 700\cos 20.837° - 1300\sin 20.837°$$
$$= 191.793 \text{ N}$$

Frictional force along the rake face,

$$F = F_c\sin\alpha + F_t\cos\alpha$$
$$= 700\sin 12° + 1300\cos 12°$$
$$= 1417.13 \text{ N}$$

Shear velocity, $$\frac{V_s}{V} = \frac{\cos\alpha}{\cos(\phi - \alpha)}$$

$$\Rightarrow \qquad V_s = \frac{240\cos 12°}{\cos(20.837 - 12)} = 237.575 \text{ m/min}$$

Chip velocity, $\frac{V_c}{V} = \frac{\sin\phi}{\cos(\phi-\alpha)}$

$\Rightarrow \quad V_c = \frac{240\sin 20.837}{\cos(20.837-12)} = 86.396 \text{ m/min}$

Total work done = $F_c \times V = (700 \times 240)$ N-m/min

(i) Percentage of shear work = $\frac{F_s \times V_s}{F_c \times V} = \frac{191.793 \times 237.575}{700 \times 240}$

$= 0.2712 = 27.12\%$ Ans.

(ii) Percentage of frictional work = $\frac{F \times V_c}{F_c \times V} = \frac{1417.13 \times 86.396}{700 \times 240}$

$= 0.7287 = 72.87\%$ Ans.

(iii) Shear stress, $\tau = \frac{F_s}{bt/\sin\phi} = \frac{F_s}{fd/\sin\phi}$

$\Rightarrow \quad \tau = \frac{191.793 \times \sin 20.837}{0.2 \times 2}$

$= 170.556$ MPa Ans.

Shear strain, $\gamma = \cot\phi + \tan(\phi - \alpha)$

$= \cot(20.837) + \tan(20.837 - 12)$

$= 2.7828$ Ans.

(iv) Rate of shear strain, $i = \frac{V_s}{t_s} = \frac{237.575}{20 \times 10^{-6} \times 60}$

$= 1.979 \times 10^5 s^{-1}$ Ans.

Section : B

5. (a)

The logarithmic decrement is given by,

$$\delta = \ln\frac{\theta_1}{\theta_2} = \ln\frac{6.75}{4.5^\circ} = 0.405$$ Ans.

Now, we have

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$0.405 = \frac{2\pi \times \zeta}{\sqrt{1-\zeta^2}}$$

or $$(0.405)^2 \times (1 - \xi^2) = 4\pi^2 \times \zeta^2$$

or $$0.405^2 = (4\pi^2 + 0.405^2)\zeta^2$$

∴ $$\zeta = \frac{0.405}{\sqrt{(4\pi^2 + 0.405^2)}}$$

∴ $$\zeta = 0.0643$$

Now, $$c_c = \sqrt{2k_t I}$$

$$I = 500 \text{ kg-cm}^2 = 0.05 \text{ kg.m}^2$$

$$k_t = \frac{GJ}{L} = \frac{4.5 \times 10^{10}}{0.5} \times \frac{\pi}{32} \times 0.2^4$$

$$k_t = 14.13 \times 10^6 \text{ Nm/rad}$$

∴ $$c = \zeta \cdot c_c = 0.0643 \times 2\sqrt{14.13 \times 10^6 \times 0.05}$$

∴ Damping torque at unit velocity = 108.093 × 1 = 108.093 Nm **Ans.**

Periodic time of vibration, $$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\xi^2} \times \omega_n}$$

$$\omega_n = \sqrt{\frac{k_t}{I}} = \sqrt{\frac{14.13 \times 10^6}{0.05}} = 16810.7 \text{ rad/s}$$

∴ $$T = \frac{2\pi}{16810.7 \times \sqrt{1 - 0.0643^2}}$$

$$T = 3.74 \times 10^{-4} \text{ sec}$$ **Ans.**

5. (b)

Given : Kinematic viscosity = 3.684 stokes = 3.684×10^{-4} m^2/s;

Density (ρ) = 950 kg/m^3; Diameter of pipe (D) = 120 mm = 0.12 m;

Length of pipe = 600 m; Discharge (Q) = 0.003 m^3/s

Dynamic viscosity (μ) = Kinematic viscosity × Density

or, $$\mu = 3.684 \times 10^{-4} \times 950 = 0.35 \text{ Pa-s}$$

and $$V = \frac{Q}{A} = \frac{0.003 \times 4}{\pi \times (0.12)^2} = 0.2652 \text{ m/s}$$

(i) Reynolds number, $Re = \frac{\rho VD}{\mu} = \frac{950 \times 0.2652 \times 0.12}{0.35}$

or $Re = 86.3794$ Ans.

As Re < 2000, the flow is laminar.

(ii) Head loss due to friction,

$$h_f = \frac{32\mu VL}{\rho g D^2}$$

$$h_f = \frac{32 \times 0.35 \times 0.2652 \times 600}{950 \times 9.81 \times (0.12)^2}$$

$$h_f = 13.2796 \text{ m}$$

If *A* is the pump level and *B* is outlet, then applying Bernoulli's equation between point *A* and *B*.

$$\frac{P_A}{\rho g} + \frac{V^2}{2g} + 0 = \frac{P_B}{\rho g} + 25 + \frac{V^2}{2g} + h_f$$

But $P_B = 0$ = atmospheric, as the outlet is free

Also, $h_f = 13.2796$ m

then, $P_A = (25 + 13.2796) \times 950 \times 9.81$

or $P_A = 356.746$ kPa Ans.

(iii) Power required for pumping the fluid

$$P = \rho g Q H$$

where, H = Overall head,

$$H = \text{Static head} + \text{Friction head}$$

$$H = 25 + 13.2796 = 38.2796 \text{ m}$$

$$\text{Power} = 950 \times 9.81 \times 0.003 \times 38.2796$$

$$P = 1070.2401 \text{ Watt}$$

As the overall efficiency of the pump set is 70%.

Power input required $= \frac{1070.2401}{0.7} = 1528.914$ Watt Ans.

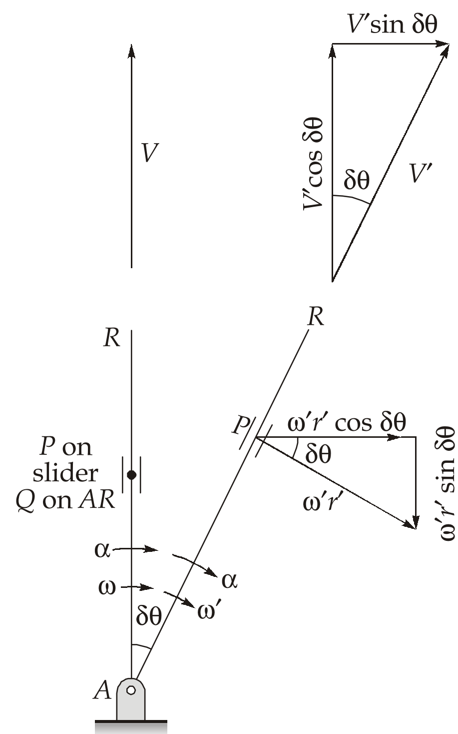
5. (c)

Let a link *AR* rotate about a fixed point *A*. On it is a point on a slider on the link.

At any given instant,

Let ω = angular velocity of the link; α = angular acceleration of the link; *f* = linear velocity of the slider on the link; *r* = radial distance of point P on the slider

In a short interval of time δ*t*, let *d*θ be the angular displacement of the link and δ*r*, the radial displacement of the slider in the outward direction.



After a short interval of time δ*t*, let

$$\omega' = \omega + \alpha\delta t = \text{angular velocity of the link}$$

$$V' = V + f\cdot\delta t = \text{linear velocity of the slider on the link}$$

$$r' = r + \delta r = \text{radial distance of the slider}$$

Acceleration of P parallel to AR

$$\text{Initial velocity of } P \text{ along } AR = v = v_{pq} = \omega r$$

$$\text{Final velocity of } P \text{ along } AR = v'\cos\delta\theta - \omega' r'\sin\delta\theta$$

$$\text{Change of velocity along } AR = (v'\cos\delta\theta - \omega' r'\sin\delta\theta) - v$$

$$\text{Acceleration of } P \text{ along } AR = \frac{(v + f\delta t)\cos\delta\theta - (\omega + \alpha\delta t)(r + \delta r)\sin\delta\theta - v}{\delta t}$$

In the limit, as $\delta t \to 0$

$\cos\delta\theta \to 1$ and $\sin\delta\theta \to \delta\theta$

Acceleration of P along AR = $f - \omega r \frac{d\theta}{dt}$

$= f - \omega r\omega = f - \omega^2 r$

= Acceleration of slider-centripetal acc.

This is the acceleration of P along AR in the radially outward direction. f will be negative if the slider has deceleration while moving in the outward direction or has acceleration while moving in the inward direction.

Acceleration of *P* perpendicular to AR

Initial velocity of P perpendicular to AR = ωr

Final velocity of P perpendicular to AR = $v' \sin\delta\theta + \omega' r' \cos\delta\theta$

Change of velocity perpendicular to AR = $(v' \sin\delta\theta + \omega' r' \cos\delta\theta) - \omega r$

Acceleration of P perpendicular to AR = $\frac{(v + f\delta t)\sin\delta\theta - (\omega + \alpha\delta t)(r + \delta r)\cos\delta\theta - \omega r}{\delta t}$

In the limit, as $\delta t \to 0$

$$\cos\delta\theta \to 1 \text{ and } \sin\delta\theta \to \delta\theta$$

Acceleration of P perpendicular to AR = $v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\alpha$

$= v\omega + \omega v + r\alpha = 2\omega v + r\alpha$

$= 2\omega v$ + tangential acc.

This is the acceleration of P perpendicular to AR. The component $2\omega v$ is known as the Coriolis acceleration component. It is positive if both ω and v are either positive or negative.

Thus, the coriolis component is positive if the

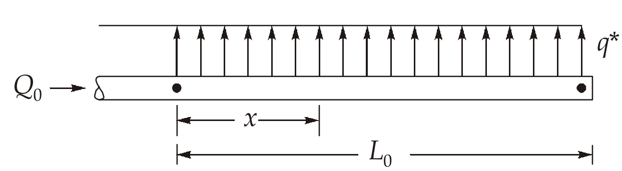
- link AR rotates clockwise and the slider moves radially outwards.
- link rotates counter-clockwise and the slider moves radially inwards.

Otherwise, the Coriolis component will be negative.

5. (d)

Given : Length of pipeline = 6000 m; Diameter of pipeline = 700 mm = 0.7 m

Friction factor, f = 0.03



First an expression for head loss in a pipe having a uniform withdrawal of q^* m^3/s per metre length is derived.

Consider a section at a distance x from the start of the uniform withdrawal at q^* per metre length.

$$\text{Discharge, } Q_x = Q_0 - q^*x$$

$$dh_f = \frac{fLV^2}{2gD} = \frac{f(Q_0 - q^*x)^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times D^5} dx$$

or
$$dh_f = \frac{8f}{\pi^2 gD^5}(Q_0 - q^* x)^2\, dx$$

Taking integration on both sides,

$$h_f = \int_0^{L_0} dh_f = \frac{-8f}{3\pi^2 gD^5}\frac{1}{q^*}\left[\left(Q_0 - q^*x\right)^3\right]_0^{L_0}$$

$$= \frac{8f}{3\pi^2 gD^5}\frac{1}{q^*}\left[Q_0^3 - \left(Q_0 - q^*L_0\right)^3\right]$$

In the question, it is given that,

$$Q_0 = \frac{1600}{350} \times 0.098 = 0.448 \text{ m}^3/\text{s}$$

$$q^* = \frac{0.098}{350} = 2.8 \times 10^{-4} \text{ m}^3/\text{s/m}$$

$$L_0 = 1600 \text{ m}$$

H_L = Total head loss = (Head lost in first (6000 – 1600) m with a discharge $Q_0 = 0.448$ m^3/s) + Head lost in 1600 m with a uniform withdrawal of q^*.

$$H_L = h_{f_1} + h_{f_2}$$

$$h_{f_1} = \frac{fLQ_0^2}{12.1 \times D^5} = \frac{0.03 \times 4400 \times (0.448)^2}{12.1 \times (0.7)^5}$$

$$h_{f_1} = 13.0273 \text{ m}$$

Now,
$$h_{f_2} = \frac{8f}{3\pi^2 gD^5}\frac{1}{q^*}\left[Q_0^3 - \left(Q_0 - q^*L_0\right)^3\right]$$

$$h_{f_2} = \frac{8 \times 0.03}{3\pi^2 \times 9.81 \times (0.7)^5} \times \frac{1}{2.8 \times 10^{-4}}\left[0.448^3 - (0.448 - 2.8 \times 10^{-4} \times 1600)^3\right]$$

$$h_{f_2} = 17.5579\left[0.448^3 - \left(0.448 - 2.8\times10^{-4}\times1600\right)^3\right]$$

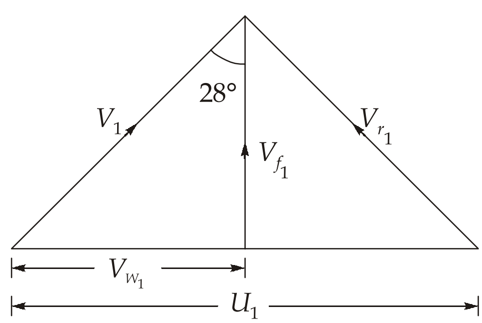
$$h_{f_2} = 17.5579 \times 0.448^3 = 1.5787 \text{ m}$$

$$\text{Total head loss} = h_{f_1} + h_{f_2} = 13.0273 + 1.5787 = 14.606 \text{ m}$$

Residual head at the dead end = 180 – 14.606 = 165.34 m

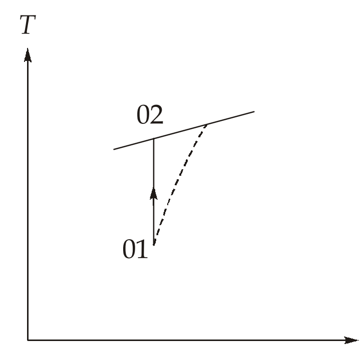
5. (e)

Given : Isentropic efficiency = 0.85; Speed (N) = 18000 rpm; Pressure ratio $\left(\frac{P_{02}}{P_{01}}\right) = 6$; Inducing air temperature (T_{01}) = 288 K; Prewhirl = 28°; Mean diameter of eye (D_1) = 0.28 m; Impeller tip diameter (D_2) = 0.65 m; Absolute velocity at inlet (V_1) = 180 m/s



Inlet velocity triangle

For process 02 - 01



$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

or, $$T_{02} = 288\times(6)^{0.286} = 480.777 \text{ K}$$

Isentropic temperature rise,

$$T_{02} - T_{01} = 480.777 - 288 = 192.777 \text{ K}$$

$$\text{Actual temperature rise, } \Delta T = \frac{\text{Isentropic temperature rise}}{\text{Isentropic efficiency}}$$

$$\Delta T = \frac{192.777}{0.85} = 226.796 \text{ K}$$

Power input per unit mass flow rate

$$= c_p \times \Delta T = 1.005\times226.796 = 227.93 \text{ kJ/kg}$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.28 \times 18000}{60} = 263.893 \text{ m/s}$$

From inlet velocity triangle

$$V_{w_1} = V_1 \sin 28° = 180 \sin(28°) = 84.504 \text{ m/s}$$

At exit,
$$U_2 = \frac{\pi \times D_2 \times N}{60} = \frac{\pi \times 0.65 \times 18000}{60}$$
$$= 612.6105 \text{ m/s}$$

Power input per unit mass flow rate = $V_{w_2} \cdot U_2 - V_{w_1} \cdot U_1 = 227.93$ kJ/kg

or,
$$V_{w_2} \times 612.6105 = 227.93 \times 10^3 + 84.504 \times 263.893$$

or,
$$V_{w_2} = 408.465 \text{ m/s}$$

We know,
$$\text{Slip factor } (\sigma) = \frac{V_{w_2}}{U_2} = \frac{408.465}{612.6105} = 0.667$$

6. (a)

Given : $\alpha = 25°$; $N_1 = 240$ rpm; $m = 75$ kg; $k = 150$ mm; $T_{mean} = 320$ Nm; $\theta = 45°$;

$N_{2max} - N_{2min} = 24$ rpm

Let suffix '1' and '2' denotes driving and driven shaft respectively.

Now, maximum and minimum speed of driven shaft

$$N_{2max} = \frac{N_1}{\cos\alpha} = \frac{240}{\cos 25°} = 264.81 \text{ rpm}$$ **Ans.**

$$N_{2min} = N_1 \cos\alpha = 240 \cos 25° = 217.5 \text{ rpm}$$ **Ans.**

Velocity ratio is unity when,

$$\tan\theta = \pm\sqrt{\cos\alpha} = \pm\sqrt{\cos 25°} = \pm 0.952$$

or
$$\theta = 43.59°, 223.6° \text{ or } 316.4°, 136.4°$$ **Ans.**

Now,
$$\omega_1 = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

Angular acceleration, at $\theta = 45°$

$$\alpha_2 = \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1-\cos^2\theta \sin^2\alpha)^2}$$

$$\alpha_2 = \frac{-(25.13)^2 \cos 25° \sin^2 25° \sin 90°}{(1-\cos^2 45° \sin^2 25°)^2}$$

$$\alpha_2 = -123.25 \text{ rad/s}^2$$

Torque required for retardation of the driven shaft,

$$\Delta T = I_2 \alpha_2$$

$$\Delta T = mk^2 \times \alpha_2$$

$$\Delta T = 72 \times 0.15^2 \times (-123.25)$$

$$= -199.66 \text{ Nm}$$

∴ Total torque required on the driven shaft

$$T_2 = T_{mean} + \Delta T$$

$$= 320 - 199.6$$

$$= 120.33 \text{ Nm}$$

Torque at driving shaft, $$T_1 = T_2 \cdot \frac{\omega_2}{\omega_1} = T_2 \times \frac{\cos\alpha}{1-\cos^2\theta \sin^2\alpha}$$

$$= 120.33 \times \frac{\cos 25°}{1-\cos^2 45° \sin^2 25°}$$

$$T_1 = 119.75 \text{ Nm}$$ **Ans.**

Now, for maximum variation in speed,

$$\frac{N_{2max} - N_{2min}}{N_{2mean}} = \frac{1-\cos^2\alpha}{\cos\alpha} \qquad \left[\because N_{2_{mean}} = N_1\right]$$

or $$\frac{24}{240} = \frac{1-\cos^2\alpha}{\cos\alpha}$$

∴ $$\cos^2\alpha + 0.1\cos\alpha = 1$$

On solving, $$\alpha = 17.96°$$ **Ans.**

6. (b)

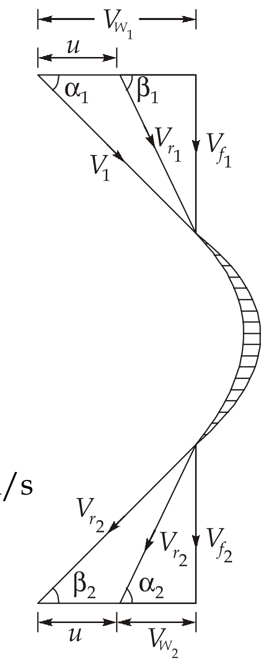
Given : Nozzle angle (α) = 22°, Entrance velocity (V_1) = 1200 m/s; Peripheral velocity (u) = 350 m/s

(i) As given in question

Blades are symmetrical ($\beta_1 = \beta_2$)

No friction effect $\left(V_{r_2} = V_{r_1}\right)$

So, from inlet velocity triangle



Velocity diagram

$$V_{w_1} = V_1 \cos\alpha_1$$
$$= 1200 \cos (22°)$$
$$= 1112.62 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha_1 = 1200 \sin(22°) = 449.527 \text{ m/s}$$

$$\tan\beta_1 = \frac{V_{f_1}}{V_{w_1} - u} = \frac{449.527}{1112.62 - 350}$$

or $$\beta_1 = 30.5173° = \beta_2$$

and $$V_{r_1} \sin\beta_1 = V_{f_1}$$

or, $$V_{r_1} = \frac{V_{f_1}}{\sin\beta_1} = \frac{449.527}{\sin(30.5173°)}$$

or, $$V_{r_1} = 885.246 \text{ m/s} = V_{r_2}$$

From exit velocity triangle,

$$V_{w_2} = V_{r_2} \cos(\beta_2) - u$$

$$V_{w_2} = 885.246 \cos(30.5173°) - 350$$

or, $$V_{w_2} = 412.6187 \text{ m/s}$$

$$V_{f_2} = V_{r_2} \sin(\beta_2) = 885.246 \sin(30.5173)$$

or, $$V_{f_2} = 449.5264 \text{ m/s}$$

$$\text{Tangential thrust } (F_t) = \dot{m} \times \left(V_{w_1} + V_{w_2}\right)$$
$$= 0.85(1112.62 + 412.6187)$$

or, $F_t = 1296.452$ N Ans.

$$\text{Diagram power } (P) = F_t \times u = 1296.452 \times 350$$

$$= 453.7585 \text{ kW}$$ Ans.

$$\text{Diagram efficiency} = \frac{453.7585 \times 10^3}{\frac{1}{2} \times 0.85 \times 1200^2}$$

$$\eta_D = 0.74143 = 74.143\%$$ Ans.

Axial thrust = 0 Ans.

(ii)

Given : $k = 0.88$; $V'_{r_2} = 0.88 V_{r_1}$

or, $$V'_{r_2} = 0.88 \times 885.246 = 779.0164 \text{ m/s}$$

$$V'_{w_2} = V'_{r_2} \cos(\beta_2) - u$$

$$V'_{w_2} = 779.0164 \cos(30.5173°) - 350$$

or $$V'_{w_2} = 321.1039 \text{ m/s}$$

$$V'_{f_2} = V'_{r_2} \sin(\beta_2)$$

$$= 779.0164 \sin(30.5173°)$$

or, $$V'_{f_2} = 395.5838 \text{ m/s}$$

$$\text{Axial thrust } (F_a) = \dot{m}\left(V_{f_1} - V'_{f_2}\right)$$

$$= 0.85(449.527 - 395.5838)$$

or, $F_a = 45.852$ N Ans.

$$\text{Diagram power} = \dot{m}\left(V_{w_1} + V'_{w_2}\right)u$$

$$P' = 0.85\,(1112.62 + 321.1039) \times 350$$

$$P' = 426.532 \text{ kW}$$ Ans.

Diagram efficiency, $\eta'_D = 0.6969 = 69.69\%$ Ans.

6. (c)

Given : $n = 10$; $d = 20$ mm; $\tau_{ut} = 250$ N/mm^2; $c_s = 0.2$; $D = 1.5$; $\eta = 0.9$; $\sigma = 6$ N/mm^2; $\rho = 7500$ kg/m^3; $b = 2t$

Maximum shear force required/punching

$$F_{max} = \tau_{ut} \times \pi dt$$
$$= 250 \times \pi \times 20 \times 30$$
$$= 471238.89 \text{ N}$$

Energy required per punching or stroke,

$$E_{hole} = \frac{F_{max}}{2} \times \text{Thickness of plate}$$

$$E_{hole} = \frac{471238.89}{2} \times 0.03$$

$$E_{hole} = 7068.58 \text{ Nm}$$

$$\text{Energy required per second} = E_{hole} \times \text{Number of strokes per second}$$

$$= 7068.58 \times \frac{10}{60} = 1178.097 \text{ Nm/s}$$

$$\text{Power of the motor, } P = \frac{\text{Energy required per second}}{\eta}$$

$$= \frac{1178.097}{0.9} = 1309 \text{ Nm/s}$$ **Ans.**

As the actual punching is done in $\frac{1}{10^{th}}$ of a cycle, the energy is stored in the flywheel during the $\frac{9}{10^{th}}$ of the cycle.

∴ Maximum fluctuation of energy,

$$\Delta e_{max} = \text{Energy stored in flywheel per stroke}$$
$$= 7068.58 \times 0.9$$
$$= 6361.72 \text{ Nm}$$

Since the hub and the spokes of the flywheel delivers 10% of the rotational inertia of the wheel, maximum fluctuation of energy provided by rim,

$$\Delta e_{rim} = 6361.72 \times 0.9 = 5725.5 \text{ Nm}$$

Mean angular speed of the flywheel,

$$\omega = \frac{2\pi(10 \times 15)}{60} = 15.7 \text{ rad/sec}$$

Now, $\Delta e_{\text{rim}} = Iw^2 c_s$

$$5725.5 = I \times 15.7^2 \times 0.2$$

$\therefore$ $I = 116.14 \text{ kg/m}^2$

or $m \times \left(\frac{D}{2}\right)^2 = 116.14$

$$m = \frac{116.14}{0.75^2} = 206.47 \text{ kg}$$

$$\text{Density} \times \text{Volume} = 206.47 \text{ kg}$$

$$7500 \times \pi \times 1.5 \times t \times 2t = 206.47$$

$\therefore$ $t = 0.054$ m

$$b = 2t = 0.108 \text{ m}$$

or $t = 54$ mm, b = 108 mm Ans.

7. (a)

The constants can be evaluated subject to the following compatibility (boundary) conditions:

*At the wall surface, $y = 0$ and $u = 0$

$\therefore$ $0 = a\sin(b \times 0) + c;$

and this gives $c = 0$

Thus, $u = a\sin(by)$ ...(i)

*At the outer edge of boundary layer ($y = \delta$)

$$u = U_0 \text{ (free stream velocity) and } \frac{\partial u}{\partial y} = 0$$

The application of these boundary conditions gives:

$$U_0 = a\sin(b\delta) \quad \text{...(ii)}$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=\delta} = ab\cos(b\delta) = 0 \quad \text{...(iii)}$$

From the equalities (ii) and (iii)

$$\frac{U_0}{\sin(b\delta)} \times b\cos(b\delta) = 0$$

$$U_0 b \cot(b\delta) = 0$$

A look at expression (i) would show that constant b cannot be equal to zero because that will make velocity zero at every cross-section throughout the boundary layer region. Obviously then,

$$\cot(b\,\delta) = 0;\ b\delta = \frac{\pi}{2};\ b = \frac{\pi}{2\delta}$$

Then from expression (ii),

$$U_0 = a\sin\left[\left(\frac{\pi}{2\delta}\right)\times\delta\right] = a\sin\frac{\pi}{2} = a$$

Substituting these values of constants a and b in expression (i), we obtain the velocity profile as:

$$u = U_0\sin\left(\frac{\pi y}{2\delta}\right)$$

Inserting the given velocity function into the momentum integral equation:

$$\tau_0 = \rho U_0^2\frac{\partial\theta}{\partial x}$$

$$= \rho U_0^2\frac{\partial}{\partial x}\left[\int_0^\delta\left(1-\frac{u}{U_0}\right)\frac{u}{U_0}dy\right]$$

$$= \rho U_0^2\frac{\partial}{\partial x}\left[\left\{\int_0^\delta 1-\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right\}\times\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy\right]$$

$$= \rho U_0^2\frac{\partial}{\partial x}\left[\int_0^\delta\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy-\int_0^\delta\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy\right]$$

Making the substitution,

$$\sin^2\theta = \frac{1-\cos2\theta}{2},\ \text{we get}$$

$$\tau_0 = \rho U_0^2\frac{\partial}{\partial x}\left[\int_0^\delta\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy-\frac{1}{2}\int_0^\delta\left\{1-\left(\cos\frac{\pi y}{\delta}\right)\right\}dy\right]$$

$$= \rho U_0^2 \frac{\partial}{\partial x}\left[\left\{\frac{-2\delta}{\pi}\cos\frac{\pi}{2}\frac{y}{\delta}\right\}_0^{\delta} - \frac{1}{2}\left\{y - \frac{\delta}{\pi}\sin\frac{\pi y}{\delta}\right\}_0^{\delta}\right]$$

$$= \rho U_0^2 \frac{\partial}{\partial x}\left[\frac{2\delta}{\pi} - \frac{\delta}{2}\right]$$

$$= 0.137\rho U_0^2 \frac{\partial\delta}{\partial x} \qquad ...(i)$$

At the solid surface, Newton's of law of viscosity gives:

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu\frac{\partial}{\partial y}\left\{U_0 \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right\}_{y=0}$$

$$= 1.57\frac{\mu U_0}{\delta} \qquad ...(ii)$$

Equating the expressions (i) and (ii) for wall shear stress,

$$0.137\rho U_0^2 \frac{\partial\delta}{\partial x} = 1.57\frac{\mu U_0}{\delta}$$

and this can be written in the differential form as

$$\delta\partial\delta = 11.46\frac{\mu}{\rho U_0}\partial x$$

Since δ is a function of x only, integration yields

$$\frac{\delta^2}{2} = 11.46\frac{\mu x}{\rho U_0} + C$$

The integration constant is obtained from the boundary condition : $\delta = 0$ at $x = 0$, and that gives $C = 0$. Therefore,

$$\frac{\delta^2}{2} = 11.46\frac{\mu x}{\rho U_0}$$

or

$$\delta^2 = 22.92\frac{\mu x}{\rho U_0}$$

This can be expressed in the non-dimensional form as

$$\frac{\delta}{x} = \sqrt{22.92}\sqrt{\frac{\mu}{x\rho U_0}}$$

$$= \frac{4.79}{\sqrt{Re_x}}$$

where $Re_x = \frac{x\rho U_0}{\mu}$ is the Reynolds number based on distance x from the leading edge.

(b) Shear stress and the local skin friction coefficient : An estimate of the wall shear stress can be made by substituting the value of boundary layer thickness in the expression for wall shear stress

$$\tau_0 = 1.57\frac{\mu U_0}{\delta} = \frac{1.57\mu U_0}{\frac{4.79x}{\sqrt{Re_x}}}$$

$$= \frac{\rho U_0^2}{2} \times \frac{0.655}{\sqrt{Re_x}}$$

In the non-dimensional form,

$$\frac{\tau_0}{\frac{1}{2}\rho U_0^2} = \frac{0.655}{\sqrt{Re_x}}$$

or
$$C_{fx} = \frac{0.655}{\sqrt{Re_x}}$$

where C_{fx} (ratio of local wall shear stress to the dynamic pressure of uniform flow stream) is the local skin friction (drag) coefficient.

(c) Drag force and the drag coefficient: The total drag on the one side of the plate with width b and length l is given by:

$$\text{Total drag force, } F_D = \int_0^l \tau_0 b\, dx$$

$$= \int_0^l \frac{\rho U_0^2}{2} \times \frac{0.655}{\sqrt{Re_x}} b dx$$

$$= \frac{\rho U_0^2}{2} \times \frac{0.655}{\sqrt{\frac{\rho U_0}{\mu}}} b\int_0^l \frac{dx}{\sqrt{x}}$$

$$= \frac{\rho U_0^2}{2} \times \frac{1.31}{\sqrt{\frac{\rho U_0 l}{\mu}}} \times bl = \frac{\rho U_0^2}{2} \times \frac{1.31}{\sqrt{Re_l}} \times bl$$

where $Re_l = \sqrt{\frac{\rho U_0 l}{\mu}}$ is the Reynolds number based on total length l of the plate. The total drag force F_D can be expressed in the drag coefficient C_D, dynamic head of undisturbed flow stream $\frac{\rho U_0^2}{2}$ and the area of plate.

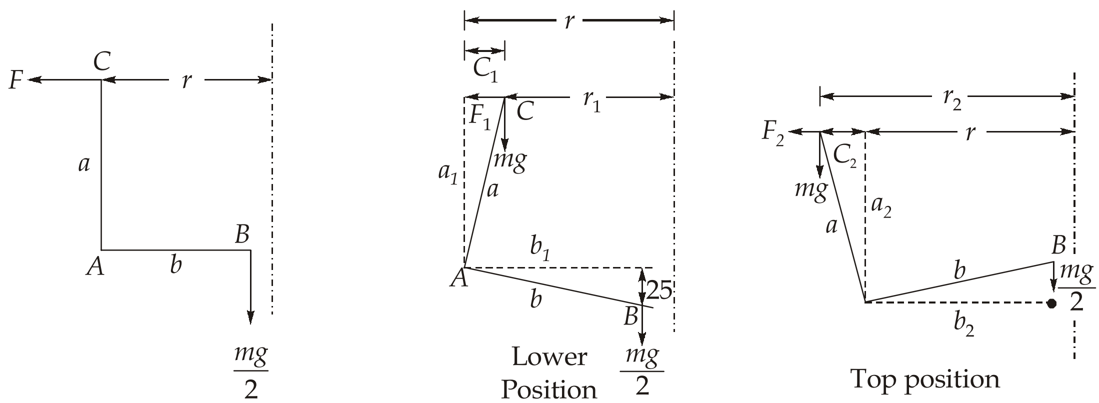
$$F_D = C_D \times \frac{\rho U_0^2}{2} \times \text{Area}$$

For the flow configuration under investigation, $A = bl$

$$\therefore \quad C_D = \frac{1.31}{\sqrt{Re_l}}$$

7. (b)

Refer figure as shown below,



Now, $$N_m = 15(N_2 - N_1)$$

or $$\frac{N_1 + N_2}{2} = 15(N_2 - N_1)$$

$$\frac{240 + N_2}{2} = 15(N_2 - 240)$$

$$240 + N_2 = 30(N_2 - 240)$$

$$240 + N_2 = 30N_2 - 7200$$

$$\therefore \quad N_2 = 256.5 \text{ rpm}$$

Angle turned by bell crank level between two extreme positions

$$\frac{\Delta x}{b} = \frac{C_1 + C_2}{a}$$

or $$\frac{50}{80} = \frac{C_1 + C_2}{100}$$

∴ $$C_1 + C_2 = 62.5 \text{ mm}$$

But $$C_1 = r - r_1 = 120 - 100 = 20 \text{ mm}$$

∴ $$C_2 = 42.5 \text{ mm}$$

∴ $$r_2 = r + C_2 = 120 + 42.5 = 162.5 \text{ mm}$$

$$\omega_1 = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

In the extreme positions,

$$mr_1\omega_1^2 a_1 = \frac{F_{s_1} \cdot b_1}{2} + mgC_1 \qquad [\because \; M = 0, f = 0]$$

$$a_1 = \sqrt{100^2 - 20^2} = 97.98 \text{ mm}$$

$$b_1 = \sqrt{80^2 - 25^2} = 76 \text{ mm} = b_2$$

$$a_2 = \sqrt{100^2 - 42.5^2} = 90.52 \text{ mm}$$

∴ At lower position,

$$5 \times 0.1 \times 25.13^2 \times 0.09798 = \frac{F_{s_1} \times 0.076}{2} + 5 \times 9.81 \times 0.02$$

$$30.93 = F_{s_1} \times 0.038 + 0.981$$

∴ $$F_{s_1} = 788.13 \text{ N}$$

At upper position, $$mr_2\omega_2^2 a_2 = \frac{F_{s_2} \cdot b_1}{2} - mgC_2$$

$$5 \times 0.1625 \times 26.86^2 \times 0.09052 = \frac{F_{s_2} \times 0.076}{2} - 9.81 \times 5 \times 0.0425$$

$$53.06 = F_{s_2} \times 0.038 - 2.084$$

∴ $$F_{s_2} = 1451.95 \text{ N}$$

∴ Spring stiffness, $$s = \frac{\Delta F_s}{\Delta x} = \frac{1451.95 - 788.13}{50}$$

$$s = 13.26 \text{ N/mm}$$

$\therefore$ Initial compression $= \dfrac{F_{s_1}}{s} = \dfrac{788.13}{13.26}$

$$x_1 = 59.43 \text{ mm}$$

Now, at mid position of sleeve

$$F_s = F_{s_1} + 25 \times s$$

$$F_s = 788.13 + 25 \times 13.261$$

$$F_s = 1119.65 \text{ N}$$

At the mid position, considering friction,

$$mr\omega_1^2 a = \frac{1}{2}(F_s + f)b$$

$$5 \times 0.12 \times \omega_1^2 \times 0.1 = \frac{(1119.65 + 130) \times 0.08}{2}$$

$\therefore$ $$\omega_1^2 = 833.103$$

$\Rightarrow$ $$\omega_1 = 28.86 \text{ rad/s}$$

Also, $$mr\omega_2^2 a = \frac{1}{2}(F_s - f)b$$

$$5 \times 0.12 \times \omega_2^2 \times 0.1 = \frac{(1119.65 - 130) \times 0.08}{2}$$

$$\omega_2^2 = 659.767$$

$$\omega_2 = 25.686 \text{ rad/s}$$

$\therefore$ $$N_2 = \frac{60 \times 25.686}{2 \times \pi} = 245.282 \text{ rpm}$$

and $$N_1 = \frac{60 \times 28.86}{2\pi} = 275.592 \text{ rpm}$$

$\therefore$ Alteration of speed $= N_1 - N_2$

$$= 275.592 - 245.282$$

$$= 30.31 \text{ rpm}$$

7. (c)

(i)

The fact of obtaining very high pressure ratios of about 8 to 10 by ram compression has made it possible to design a jet engine without a mechanical compressor. A deceleration of the air from Mach number 3 at diffusion inlet to Mach number 0.3 in combustion chamber would cause pressure ratio of more than 30. Due to shock and other losses inevitable at such velocities, all of this pressure rise is not available; still whatever we get is more than sufficient for raising the air pressure to the required combustion pressure. This principle of ram pressure rise is used in the ramjet engines. The ram pressure rise can be achieved in diffusers.

It may be noted that the simplest type of air breathing engine is the ramjet engine and a simplified sketch of the engine is illustrated in figure.

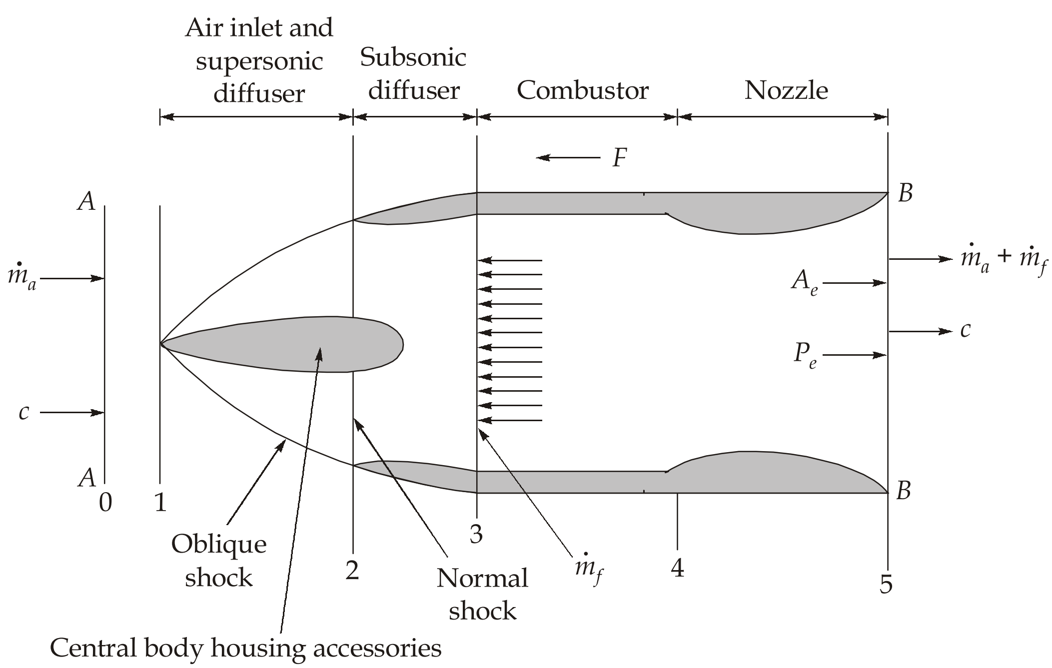
The engine consists of

(i) supersonic diffusion (1 - 2),

(ii) subsonic diffuser section (2 - 3),

(iii) combustion chamber (3 - 4), and

(iv) discharge nozzle section (4-5).



Ramjet Engine

Both supersonic and subsonic diffusers convert the kinetic energy of the entering air into pressure rise. This energy transformation is called ram effect and the pressure rise is called ram pressure. The principle of operation is as follows:

Air from the atmosphere enters the engine at a very high speed and its velocity gets reduced first in the supersonic diffuser, thereby its static pressure increases. The air then enters the subsonic diffuser wherein it is compressed further.

Afterwards, the air flows into the combustion chamber, the fuel is injected by suitable injectors and mixed with the unburnt air. The air is heated to a temperature of the order of 1500 – 2000 K by the continuous combustion of fuel. The fresh supply of air to the diffuser builds up pressure at the diffuser end so that these gases cannot expand towards the diffuser. Instead, the gases are made to expand in the combustion chamber towards the tail pipe. Further, they are allowed to expand in the exhaust nozzle section. The products will leave the engine with a speed exceeding that of the entering air. Because of the rate of increase in the momentum of the work fluid, a thrust, *F,* is developed in the direction of flight.

Advantages of Ramjet:

- Ramjet is very simple and does not have any moving part. It is very cheap to produce and requires almost no maintenance.
- Due to the fact that a turbine is not used to drive the mechanical compressor, the maximum temperature which can be allowed in ramjet is very high, about 2000°C as compared to about 900°C in turbojets. This allows a greater thrust to be obtained by burning fuel at air-fuel ratio of about 13 : 1, which gives higher temperatures.
- The specific fuel consumption is better than other gas turbine power plants at high speed and high altitudes.
- Theoretically, there seems to be no upper limit to the flight speed of the ramjet.

Disadvantages of Ramjet:

- Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ramjet without an external launching device.
- The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.
- Due to high air speed, the combustion chamber requires flame holder to stabilize the combustion.

Basic Characteristics and Applications:

The basic characteristics of the ramjet engine can be summarized as follows:

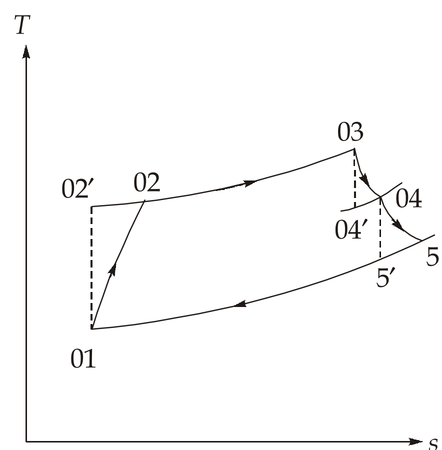
- It is a simple engine and should be adaptable for mass production at relatively low cost.
- It is independent of fuel technology and a wide range of liquids, and even solid fuels can be used.

- Its fuel consumption is comparatively very large for its application in aircraft propulsion or in missiles at low and moderate speeds.
- Its fuel consumption decreases with flight speed and approaches reasonable values when the flight Mach number is between 2 and 5 and therefore, it is suitable for propelling supersonic missiles.

Due to its high thrust at high operational speed, it is widely used in high-speed military aircrafts and missiles. Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

(ii)

Given : Air inlet velocity (C_i) = 275 m/s; Air inlet temperature = 290 K; Air inlet pressure = 1 bar; Mass flow rate $(\dot{m}_a)$ = 35 kg/s; Temperature after compression = 190°C = 463 K; Temperature at entry of turbine (T_{03}) = 920°C = 1193 K; Temperature at exit of turbine (T_{04}) = 715°C = 988 K; Compressor pressure ratio (r_p) = 4; Nozzle efficiency (η_n) = 0.92



For process 02 - 01

$$T_{02} - T_{01} = \frac{T_{01}}{\eta_c}\left[\left(r_p\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

or $$463 - 290 = \frac{290}{\eta_c}\left[(4)^{0.286} - 1\right]$$

or, $$\eta_c = 0.8156 = 81.56\%$$ Ans.

$$\text{Compressor work } (\dot{w}_c) = \dot{m}_a (c_p)_{air} (T_{02} - T_{01})$$

$$\dot{w}_c = 35 \times 1.005\,(463 - 290)$$

$$= 6085.275 \text{ kW}$$ Ans.

For process 03-04

$$T_{03} - T_{04} = \eta_T T_{03}\left(1 - \frac{1}{(r_t)^{\frac{1.33-1}{1.33}}}\right)$$

where r_t = pressure ratio across turbine; $\eta_c = \eta_T = 0.8156$ (given)

So, $$1193 - 988 = 1193 \times 0.8156\left(1 - \frac{1}{(r_t)^{0.248}}\right)$$

or, $$r_t = 2.596$$

then $$P_{04} = \frac{P_{03}}{r_t} = \frac{4}{2.596} = 1.5407$$

For process 04 - 5′

$$\frac{T_5'}{T_{04}} = \left(\frac{P_5}{P_{04}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}$$

or, $$T_5' = 988 \times \left(\frac{1}{1.5407}\right)^{\frac{0.33}{1.33}} = 887.524 \text{ K}$$

We know, $$\eta_n = \frac{T_{04} - T_5}{T_{04} - T_5'}$$

or, $$0.92 = \frac{988 - T_5}{988 - 887.524}$$

or, $$T_5 = 895.562 \text{ K}$$

Hence,

Exit velocity of nozzle, $$c_e = \sqrt{2 \times (c_p)_{gas} \times (T_{04} - T_5)}$$

or, $$c_e = \sqrt{2 \times 1147 \times (988 - 895.562)}$$

$$= 460.4918 \text{ m/s}$$ Ans.

Thrust developed $$(F) = \dot{m}_a(c_e - c_i)$$

$$F = 35(460.4918 - 275)$$

$$F = 6492.216 \text{ N}$$ Ans.

8. (a)

Given : m = 5 kg; L = 1 m; d = 0.02 m; e = 0.0025 m; c = 50 N-sec/m; N = 320 rpm; $E = 1.5 \times 10^{11}$ N/m²

For shaft,
$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64} \times 0.02^4$$
$$I = 7.85 \times 10^{-9} \text{ m}^4$$

∴ Static deflection,
$$\Delta = \frac{WL^3}{48EI} = \frac{5 \times 9.81 \times 1^3}{48 \times 1.5 \times 10^{11} \times 7.85 \times 10^{-9}}$$
$$\Delta = 8.67 \times 10^{-4} \text{ m}$$

∴ Critical speed,
$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{8.67 \times 10^{-4}}}$$
$$\omega_n = 106.32 \text{ rad/s}$$

or
$$k = m\omega_n^2 = 5 \times 106.32^2$$
$$k = 56519.7 \text{ N/m}$$

Angular speed of shaft,
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 320}{60} = 33.5 \text{ rad/s}$$

Frequency ratio,
$$r = \frac{\omega}{\omega_n} = \frac{33.5}{106.32} = 0.315$$

Damping factor,
$$\zeta = \frac{c}{2\sqrt{km}} = \frac{50}{2 \times \sqrt{56519.7 \times 5}}$$
$$\zeta = 0.047$$

Steady state amplitude is given by,

$$A = \frac{r^2 e}{\sqrt{\left(1 - r^2\right)^2 + (2\zeta r)^2}}$$

$$A = \frac{0.315^2 \times 0.0025}{\sqrt{\left(1 - 0.315^2\right)^2 + (2 \times 0.047 \times 0.315)^2}}$$

$$A = 2.75 \times 10^{-4} \text{ m}$$

Dynamic load on the bearings

$$F_D = A\sqrt{k^2 + c^2\omega^2}$$

$$F_D = 2.75\times10^{-4}\sqrt{(56519.7)^2 + (50\times33.5)^2}$$

$$= 15.56 \text{ N}$$

Dead load on shaft, $w = mg = 5 \times 9.81$

$$w = 49.05 \text{ N}$$

Total maximum load on the shaft,

$$F = 15.56 + 49.05$$

$$= 64.61 \text{ N}$$

Maximum bending moment on shaft,

$$M = \frac{F.L}{4} = \frac{64.61\times1}{4} = 16.15 \text{ Nm}$$

∴ Maximum stress under dynamic conditions,

$$\sigma_{max} = \frac{32M}{\pi d^3} = \frac{32\times16.15}{\pi\times(0.02)^3}$$

$$\sigma_{max} = 20.56 \text{ MPa}$$ Ans.

The maximum stress under dead load condition

$$(\sigma_{max})_{dead\ load} = \frac{8wL}{\pi d^3}$$

$$= \frac{8\times49.05\times1}{\pi\times0.02^3} = 15.61 \text{ MPa}$$

Damping torque, T = Damping force × Amplitude

$$= C\omega A \times A = C\omega A^2$$

$$= 50 \times 33.5 \times (2.75 \times 10^{-4})^2$$

$$= 1.267 \times 10^{-4} \text{ Nm}$$

∴ Power required to drive the shaft,

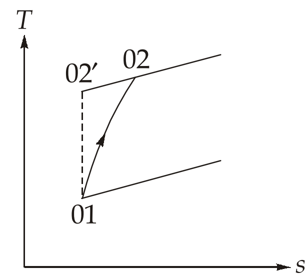
$$P = T\omega$$

$$= 1.267 \times 10^{-4} \times 33.5$$

$$= 4.24 \times 10^{-3} \text{ W}$$ Ans.

8. (b)

Given : Stagnation temperature rise (ΔT_s) = 20 K; Rotational speed (N) = 5000 rpm; Work done factor (F) = 0.92; Hub-tip ratio = 0.7; Isentropic efficiency (η_s) = 0.87; Ambient pressure (P_{01}) = 1 bar; Ambient temperature (T_{01}) = 290 K; Inlet velocity (V_1) = 160 m/s; Mach number (M_1) = 0.9



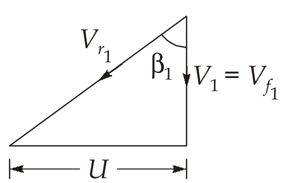
We know, $$T_1 = T_{01} - \frac{V_1^2}{2c_p}$$

$$T_1 = 290 - \frac{(160)^2}{2 \times 1005} = 277.263 \text{ K}$$

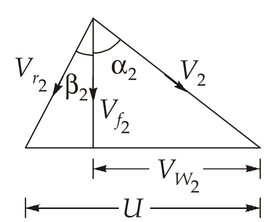
$$V_{r_1} = M_1\sqrt{\gamma R T_1} = 0.9\sqrt{1.4 \times 287 \times 277.263}$$

$$= 300.395 \text{ m/s}$$

Assuming axial inlet



Inlet velocity triangle



Exit velocity triangle

From inlet velocity triangle:

$$\cos\beta_1 = \frac{V_1}{V_{r_1}} = \frac{160}{300.395}$$

or, $$\beta_1 = 57.816°$$

$$U = V_{r_1} \sin\beta_1 = 300.395 \times \sin(57.816)$$

or, $$U = 254.238 \text{ m/s}$$

We know,

$$\text{Work input} = c_p \Delta T_s = FUV_f(\tan\beta_1 - \tan\beta_2)$$

or, $$\tan\beta_1 - \tan\beta_2 = \frac{c_p \Delta T_s}{FUV_f}$$

$$= \tan(57.816) - \frac{1005 \times 20}{0.92 \times 254.238 \times 160}$$

or, $\tan\beta_2 = 1.051866$

$\beta_2 = 46.4479°$ Ans.

For process 1 – 01

$$P_1 = \frac{P_{01}}{\left(\frac{T_{01}}{T_1}\right)^{3.5}} = \frac{1}{\left(\frac{290}{277.263}\right)^{3.5}} = 0.85453 \text{ bar}$$

and $$\rho_1 = \frac{P_1}{RT_1} = \frac{0.85463 \times 10^5}{287 \times 277.263} = 1.07387 \text{ kg/m}^3$$

$$\text{Radius at tip } (R_{tip}) = \frac{60 \times U}{2\pi N} = \frac{60 \times 254.238}{2\pi \times 5000}$$

or $R_{tip} = 0.48556$ m

So, Radius at root (R_{root}) = 0.7 × 0.48556 = 0.3398 m

$$\text{Mean radius } (R_m) = \frac{1}{2}(0.48556 + 0.3398) = 0.412725 \text{ m}$$

Blade height, $h = R_{tip} - R_{root}$

$= 0.48556 - 0.3398 = 0.14576$ m

Mass flow entering the stage $(\dot{m}) = \rho \times 2\pi \times R_m \times h \times V_{f1}$

or $\dot{m} = 1.07387 \times 2\pi \times 0.412725 \times 0.14576 \times 160$

$\dot{m} = 64.94574$ kg/sec

For process 01 – 02′

$$\frac{T_{02'}}{T_{01}} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{...(i)}$$

and $$\eta_s = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} \quad \text{...(ii)}$$

From (i) and (ii)

$$\text{Stagnation pressure ratio, } r_p = \left(1 + \frac{\eta_s \Delta T_s}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

or, $$r_p = \left(1 + \frac{0.87 \times 20}{290}\right)^{3.5} = 1.22622$$

$$\text{Input power} = \dot{m}c_p\Delta T_s = 64.94574 \times 1.005 \times 20 = 1305.409 \text{ kW}$$

Rotor air angle at root section,

$$U_{\text{root}} = \frac{2\pi \times R_{root} \times N}{60} = \frac{2\pi \times 0.3398 \times 5000}{60}$$

or, $$U_{\text{root}} = 177.9188 \text{ m/s}$$

then, $$\tan\beta_1' = \frac{177.9188}{160} = 1.11199$$

or, $$\beta_1' = 48.0353°$$

$$\tan\beta_2' = \tan\beta_1' - \frac{c_p\Delta T_s}{F \times U_{root} \times V_1}$$

$$= 1.11199 - \frac{1005 \times 20}{0.92 \times 177.9188 \times 160}$$

$$\beta_2' = 19.009°$$ Ans.

8. (c)

The kinetic energy of the barrel must be equal to the potential energy of the spring so,

$$\frac{1}{2} m\dot{x}^2 = \frac{1}{2} kx^2$$

∴ $$\dot{x}^2 = \frac{kx^2}{m} = \frac{235000 \times 1.5^2}{500}$$

∴ $$\dot{x} = 32.52 \text{ m/s}$$ ...Ans (i)

Critical damping coefficient, $c_c = 2\sqrt{km}$

$$= 2 \times \sqrt{235000 \times 500}$$

$$= 21679.5 \text{ N-s/m}$$ ...Ans (ii)

$$\text{Natural frequency, } \omega_n = \sqrt{\frac{235000}{500}} = 21.68 \text{ rad/s}$$

$$\text{Time period, } T = \frac{2\pi}{\omega_n} = \frac{2\pi}{21.68} = 0.29 \text{ sec}$$

$$\text{Time for recoil, } t_1 = \frac{T}{4} = \frac{0.29}{4} = 0.072 \text{ sec}$$

For critical damping, the displacement is given as,

$$x = (A_1 + A_2 t)e^{-\omega_n t}$$

The initial conditions are given as,

At $t = 0$, $x = 1.5$, $\dot{x} = 0$

$\therefore$ $A_1 = 1.5$

Now, $\dot{x} = A_2 - A_1\omega_n$ at $t = 0$,

or $0 = A_2 - 1.5 \times 21.68$

$\therefore$ $A_2 = 32.52$

So, the equation of motion can be written as,

$$x = (1.5 + 32.52t)e^{-21.68t}$$

At $x = 0.05$, we have

$$0.05 = (1.5 + 32.52t)e^{-21.68t}$$

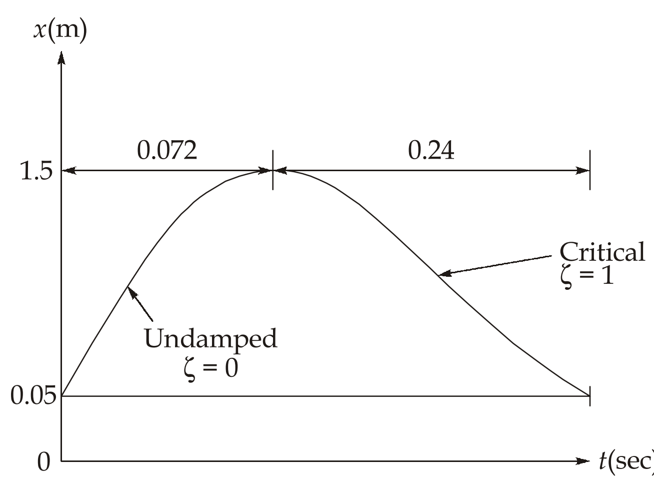
On solving, we get

$$t_2 = 0.24 \text{ sec}$$

$\therefore$ Total time $= t_1 + t_2$

$= 0.072 + 0.24$

$= 0.31$ sec



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