ESE-2023 Mains Test Series

Detailed Solutions

Electrical Engineering Test No : 4

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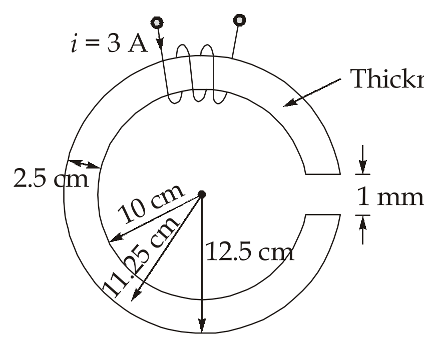
Section A : Electrical Machines

Q.1 (a) Solution:



$$\text{MMF} = Ni = 500 \times 3 = 1500 \text{ AT}$$

$$B_c = B_g = 1.2 \text{ T (no fringing)}$$

$$\text{Mean radius of ring} = \frac{12.5 + 10}{2} = 11.25 \text{ cm}$$

(i)

$$H_g = \frac{B}{\mu_o} = \frac{1.2}{4\pi \times 10^{-7}} = 9.549 \times 10^5 \text{ AT/m}$$

$$Ni = H_g l_g + H_c l_c$$

$$500 \times 3 = 9.549 \times 10^5 \times 1 \times 10^{-3} + H_c \times 2\pi \times 11.25 \times 10^{-2}$$

$$\Rightarrow \quad H_c = 771.16 \text{ AT/m}$$

(ii)
$$H_c = \frac{B_c}{\mu_o \mu_r}$$

$$\therefore \quad \mu_r = \frac{B_c}{\mu_o H_c} = \frac{1.2}{4\pi \times 10^{-7} \times 771.16} = 1238.3$$

(iii)
$$R_{\text{total}} = R_g + R_c$$

$$R_{\text{total}} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 2 \times 2.5 \times 10^{-4}} + \frac{2\pi \times 11.25 \times 10^{-2}}{4\pi \times 10^{-7} \times 1238.3 \times 2 \times 2.5 \times 10^{-4}}$$

$$= 1.592 \times 10^6 + 0.909 \times 10^6 = 2.5 \times 10^6 \text{ AT/Wb}$$

Q.1 (b) Solution:

$$P_c = P_e + P_n = K_e f^2 B_m^2 V + K_h f B_m^n V$$

$$= K_e' f^2 + K_h' f \qquad (\because B_m \text{ is constant})$$

$$\frac{P_c}{f} = K_e' f + K_h'$$

For $P_c = 2000$, $\quad f = 50$ Hz

or $$\frac{2000}{50} = 50K_e' + K_h'$$

$$50K_e' + K_h' = 40 \qquad ...(1)$$

For $P_c = 3200$, $\quad f = 75$ Hz

$$\frac{3200}{75} = 75K_e' + K_h'$$

$$75K_e' + K_h' = \frac{128}{3} \qquad ...(2)$$

On solving eqn. (1) and (2)

$$K_e' = \frac{8}{75} \text{ and } K_h' = \frac{104}{3}$$

At $f = 50$ Hz,

$$P_e = K_e' f^2 = 266.7 \text{ W} : P_h = 1733.33 \text{ W}$$

At $f = 75$ Hz

$$P_e = 600 \text{ W and } P_h = 2600 \text{ Watts}$$

Q.1 (c) Solution:

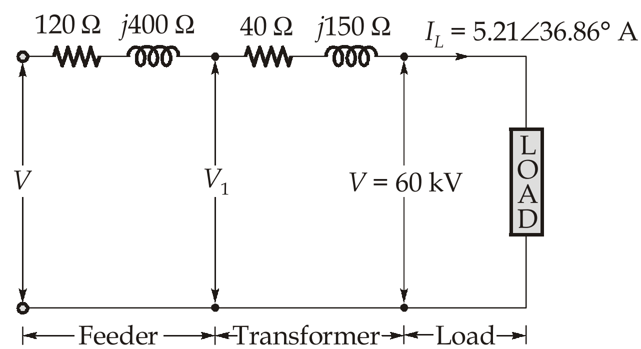
Impedance of 66 kV feeder = $(120 + j400)\ \Omega$

Equivalent impedance of transformer

referred to LV side = $(0.4 + j1.5)\ \Omega$

referred to HV side = $(0.4 + j1.5) \times 10^2\ \Omega$

= $(40 + j150)\ \Omega$



Load of 250 kW at 0.8 pf leading at 6 kV

$$I_L = \frac{250}{6 \times 0.8} = 52.08 \text{ A}$$

$$I'_L(\text{HV side}) = \frac{52.08}{10} = 5.21$$

(i) Primary voltage of transformer,

$$\vec{V}_1 = \vec{V} + \vec{Z}_T \vec{I}_L$$

$$\vec{V}_1 = 60\angle 0° + (40 + j150)(5.21\angle 36.86°) \times 10^{-3}$$

$$\vec{V}_1 = 59.70\angle 0.72° \text{ kV}$$

Sending end voltage of feeder

$$\vec{V} = \vec{V}_1 + \vec{Z}_f \vec{I}_L$$

$$= 59.70\angle 0.72° + (120 + j400)(5.21\angle 36.86°)$$

$$\vec{V} = 59.01\angle 2.71° \text{ kV}$$

(iii) Complex power input at the sending end of feeder,

$$\vec{S}_{in} = VI_L^*$$

$$= (59.01\angle 2.71) \times (5.21\angle -36.86°)$$

$$= (254.44 - j172.57) \text{ kVA}$$

Q.1 (d) Solution:

Reluctance of air gap

$$R = \frac{l}{\mu_o \mu_r A} = \frac{2(D-x)}{\mu_o A}$$

$$L(x) = \frac{N^2}{R} = \frac{\mu_o A N^2}{2(D-x)}$$

$$\lambda = N\phi = NAB_m \sin \omega t$$

(i)
$$e = \frac{d\lambda}{dt} = NAB_m \omega \cos \omega t$$

(ii)
$$W_f(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L} = \frac{1}{2} \times \frac{N^2 A^2 B_m^2 \sin^2 \omega t}{(\mu_o N^2 A)/2(D-x)}$$

$$W_f(\lambda, x) = \frac{AB_m^2}{\mu_o}(D-x)\cdot \sin^2 \omega t$$

$$F_f = -\frac{\partial W_f}{\partial x} = \frac{AB_m^2 \sin^2 \omega t}{\mu_o}$$

$$F_f = \frac{1}{2\mu_o} AB_m^2 (1 - \cos 2\omega t)$$

(iii)
$$F_f = M\frac{d^2 x_1}{dt^2} + B\frac{dx_1}{dt} + K(x_0 + x_1)$$

$$x_o = \frac{AB_m^2}{2K}$$

x can be obtained from

$$-\frac{1}{2\mu_o} AB_m^2 \cos 2\omega t = M\frac{d^2 x_1}{dt^2} + B\frac{dx_1}{dt} + Kx_1$$

$$H(j\omega) = \frac{1}{M(j2\omega)^2 + B(j2\omega) + K}$$

$$= \frac{1}{\sqrt{(K - 4M\omega)^2 + 4B^2\omega^2}} \angle -\tan^{-1}\left(\frac{K - 4M\omega}{2B\omega}\right)$$

$$\therefore \qquad x_1(t) = \frac{-\mu_o^{-1} A B_m^2}{2\sqrt{(K-4M\omega)^2 + 4B^2\omega^2}} \cos(2\omega t - \psi)$$

$$\left(\because \psi = \tan^{-1}\left(\frac{K-4M\omega}{2B\omega}\right)\right)$$

Net moment is $\qquad x(t) = x_0 + x_1(t)$

Q.1 (e) Solution:

$$E_a = \frac{3.75 \times 10^{-3} I_a \times n \times 180}{60} \times 1$$

$$= 11.25 \times 10^{-3} n I_a \qquad ...(1)$$

$$T = \frac{1}{2\pi} \times 3.75 \times 10^{-3} I_a \times 180 \times I_a$$

$$= 107.4 \times 10^{-3} I_a^2 \qquad ...(2)$$

$$\frac{250 - E_a}{1} = I_a \qquad ...(3)$$

Under steady condition,

$$T = T_L$$

$$107.4 \times 10^{-3} I_a^2 = 10^{-4} n^2$$

$$I_a = \frac{n}{32.77} \qquad ...(4)$$

Substituting (3) and (4) in eqn. (1)

$$250 - \frac{n}{32.77} = 11.25 \times 10^{-3} \times \frac{n^2}{32.77}$$

$$n^2 + \frac{10^3}{11.25} n - \frac{250 \times 32.77 \times 1000}{11.25} = 0$$

$$n^2 + 88.9n - 72.8 \times 10^4 = 0$$

$$n = \frac{-89 \pm \sqrt{0.79 \times 10^4 + 291.2 \times 10^4}}{2}$$

$$n = 810 \text{ rpm}$$

$$I_a = \frac{810}{32.77} = 24.7 \text{ A}$$

Q.2 (a) Solution:

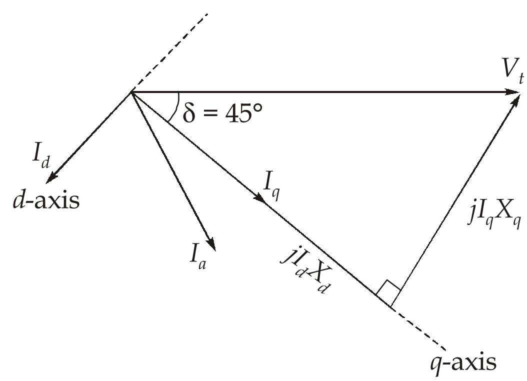
(i) For no field excitation ($E_f = 0$) the machine behaves as reluctance motor.

$$P_{\text{in}} = P_r = \frac{V_t^2(X_d - X_q)}{2X_d X_q}\sin 2\delta$$

For maximum $2\delta = 90°$ or $\delta = 45°$

$$P_{\text{in}} = \frac{1\times(1.2-0.6)}{2\times 1.2\times 0.6} = 0.4167 \text{ pu}$$

Phasor diagram :



Now from the phasor diagram,

$$|I_d X_d| = |I_q X_q| = V_t \cos 45° = \frac{1}{\sqrt{2}}$$

$$I_d = \frac{1}{\sqrt{2}}\times\frac{1}{X_d} = \frac{1}{1.2\sqrt{2}} = 0.59 \text{ pu}$$

$$I_q = \frac{1}{0.6\times\sqrt{2}} = 1.18 \text{ pu}$$

$$|I_a| = \sqrt{I_d^2 + I_q^2} = \sqrt{0.59^2 + 1.18^2} = 1.32 \text{ pu}$$

$$\psi = \tan^{-1}\left(\frac{I_d}{I_q}\right) = \tan^{-1}\left(\frac{0.59}{1.18}\right) = 26.6°$$

$$\phi = \psi + \delta = 45 + 26.6 = 71.6°$$

$$\text{Power factor} = \cos 71.6° = 0.32 \text{ lagging}$$

(ii)
$$P = \frac{E_f\cdot V_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta$$

On putting the values,

$$P = 0.833E_f \sin\delta + 0.4167 \sin 2\delta \quad ...(1)$$

For maximum power

$$\frac{dP}{d\delta} = 0$$

$$0.833E_f \cos\delta + 0.833 \cos 2\delta = 0$$

$$E_f = \frac{\cos 2\delta}{\cos\delta} \quad ...(2)$$

From eqn. (1) and (2)

$$P_{max} = \frac{-0.833\cos 2\delta}{\cos\delta}\sin\delta + 0.4167\sin 2\delta$$

$$= -0.833(2\cos^2\delta - 1)\tan\delta + 0.4167 \sin 2\delta$$

$$= -0.833 \sin 2\delta + 0.833 \tan\delta + 0.4167 \sin 2\delta$$

$$P_{max} = P_{rated} = 1 \text{ pu}$$

$$1 = -0.833 \sin 2\delta + 0.833 \tan\delta + 0.4167 \sin 2\delta$$

or $$\tan\delta = 1.2 + 0.5 \sin 2\delta \quad ...(3)$$

Now this equation, by trial and error

$$\delta = 58.69°$$

Now, from eqn. (2)

$$E_f = \frac{\cos(2\times 58.69°)}{\cos 58.69°} = 0.8848 \text{ pu}$$

Q.2 (b) Solution:

Machines parameters :

$$R_1 = 0.015\ \Omega,\ R'_2 = 0.035\ \Omega$$

$$X_1 = 120\pi \times 0.385 \times 10^{-3} = 0.1451\ \Omega$$

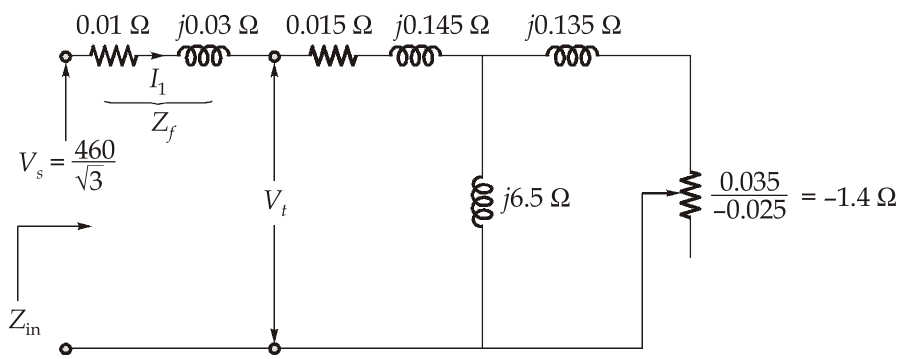
$$X'_2 = 120\pi \times 0.358 \times 10^{-3} = 0.135\ \Omega$$

$$X_m = 120\pi \times 17.24 \times 10^{-3} = 6.5\ \Omega$$

(i) Speed, $$N_s = \frac{120\times f}{P} = \frac{120\times 60}{8} = 900 \text{ rpm}$$

$$N_r = (1 + 0.025) \times 900 = 922.5 \text{ rpm}$$

(ii) Equivalent circuit of given system



Input impedance, $Z_{in} = (0.01 + j0.03) + (0.015 + j0.145) + (j6.5 \,||\, (-1.4 + j0.135)$

$Z_{in} = 1.39\angle 155.35° \ \Omega$

Taking V_s as reference, then

$$V_s = \frac{460}{\sqrt{3}} = 265.5811\angle 0° \text{ V}$$

$$I_1 = \frac{V_s}{Z} = \frac{265.58\angle 0°}{1.38\angle 155.35°} = 191.0\angle -155.35° \text{ A}$$

$$V_t = V_s - Z_{in}I_1$$

$$= 265.58\angle 0° - (191.06\angle -155.35°)(0.01 + j0.03)$$

$$V_t = 265\angle 1.299° \text{ V (L-N)}$$

(iii) Power from source,

$$P_s = 3 \times V_s \times I_1 \cos\theta_1$$

$$= 3 \times 265.5811 \times 191.066 \times (-0.9089)$$

$$P_s = -138.5913 \text{ kW}$$

Negative sign means the power is delivered to the source and machine is working as induction generator.

Input power factor $= \cos\theta_1 = \cos(-155.35°) = 0.9089$ leading

(iv) Air-gap power, $P_{ag} = P_s + 3I_1^2(0.01 + 0.015)$

$$P_{ag} = 138.591 \times 10^3 + 3 \times (191.066)^2 \times 0.025$$

$$P_{ag} = 141.3383 \text{ kW}$$

Mechanical power developed,

$$P_{md} = (1 - s)P_{ag} = (1 + 0.025) \times 141.3383 \times 10^3$$

$$P_{md} = 144.862 \text{ kW}$$

Rotational losses, $P_{rot} = 3$ kW

$$P_{shaft} = P_{mech} + P_{rot} = 144.862 + 3 = 147.862 \text{ kW}$$

$$P_{in} = P_{shaft} = 147.862 \text{ kW}$$

$$\text{Efficiency, } \eta = \frac{P_s}{P_{in}} = \frac{138.5913}{147.862} = 0.9372 \text{ or } 93.72\%$$

Q.2 (c) Solution:

(i)

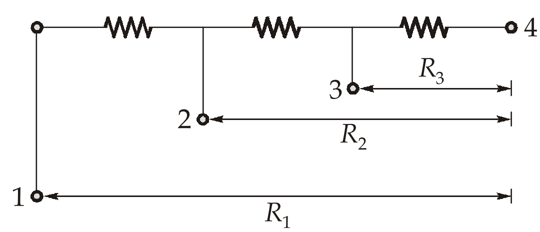
$$I_{a(\text{rated})} = \frac{P}{V} = \frac{10000}{100} = 100 \text{ A}$$

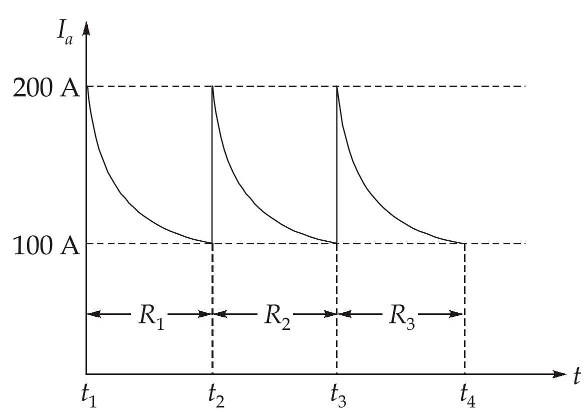
$$I_{a(\text{start})} = \frac{V_t}{R_a} = \frac{100}{0.1} = 1000 \text{ A} = 10\, I_{a(\text{rated})} = 10 \text{ pu}$$

(ii)

$$200 = \frac{100}{R_a + R_{se}} \Rightarrow R_{se} = 0.4\ \Omega$$

(iii) An arrangement of resistances in the starter box is shown in figure.





where R_1, R_2, R_3 represents total resistances of the box for position 1, 2, 3 respectively. The handle will be moved to a new position when I_a decreases to 100 A.

Now, from part (ii), $R_1 = 0.4\ \Omega$ = Total resistance in starter box

R_2 at any speed,

$$\underset{\text{fixed}}{\underset{\downarrow}{V_t}} = \underset{\text{increases with speed}}{\underset{\downarrow}{E_a}} + \underset{\text{decrease with speed}}{\underset{\downarrow}{I_a}}(R_a + R)$$

At $t = t_2^-$ (before the handle is moved to position 2)

$$I_a = 100 \text{ A}$$

and
$$E_{a2} = V_t - I_a(R_a + R_1)$$
$$= 100 - 100(0.1 + 0.4)$$
$$= 50 \text{ V}$$

At $t = t_2^+$,

$$I_a = 200 \text{ A} = \frac{V_t - E_{a2}}{R_a + R_2}$$

$$200 = \frac{100 - 50}{0.1 + R_2}$$

$$R_2 = 0.15\ \Omega$$

R_3, At $t = t_3^-$,

$$I_a = 100 \text{ A}$$
$$E_{a3} = 100 - 100(0.1 + 0.15)$$
$$E_{a3} = 100 - 25 = 75 \text{ V}$$

At $t = t_3^+$,

$$I_a = 200 \text{ A} = \frac{100 - 75}{0.1 + R_3}$$

$$R_3 = 0.025\ \Omega$$

R_4, At $t = t_4^-$,

$$I_a = 100 \text{ A}$$
$$E_{a4} = 100 - 100(0.1 + 0.025)$$
$$= 87.50 \text{ U}$$

At $t = t_4^+$,

$$I_a = 200 = \frac{100 - 87.50}{0.1 + R_4}$$

$$R_4 = -0.0375\ \Omega \text{ (Not possible)}$$

The –ve value of R_4 indicates that it is not required that is $R_4 = 0$. At $T = t_4^+$, the armature current without any resistance in the box will not exceed 200 A.

At $t = t_4^+$,

$$I_a = \frac{100 - 87.5}{0.1} = 125 \text{ A (within limit)}$$

Therefore, three resistances in the starter box are required. Their values are

$$R_1' = R_1 - R_2 = 0.4 - 0.15 = 0.275$$
$$R_2' = R_2 - R_3 = 0.125\ \Omega$$
$$R_3' = R_3 - R_4 = 0.025 - 0 = 0.025\ \Omega$$

Q.3 (a) Solution:

(i) As we know,

$$e = N\frac{d\phi}{dt}$$
$$N.d\phi = edt$$
$$N.\Delta\phi = E\Delta t$$

Flux linkage change = Volt-time product

Flux linkage during the each half cycle is

$$\Delta\phi = \frac{1}{500}\left[45\times\frac{1}{360} + 90\times\frac{1}{360} + 45\times\frac{1}{360}\right]$$
$$\Delta\phi = 1\times10^{-3}\ \text{Wb}$$

$\because$ $V_{av} = 0, \quad \therefore \quad \phi_{av} = 0$

$\therefore$ During positive half cycle of the input voltage, the flux varies from -0.5×10^{-3} Wb to 0.5×10^{-3} Wb.

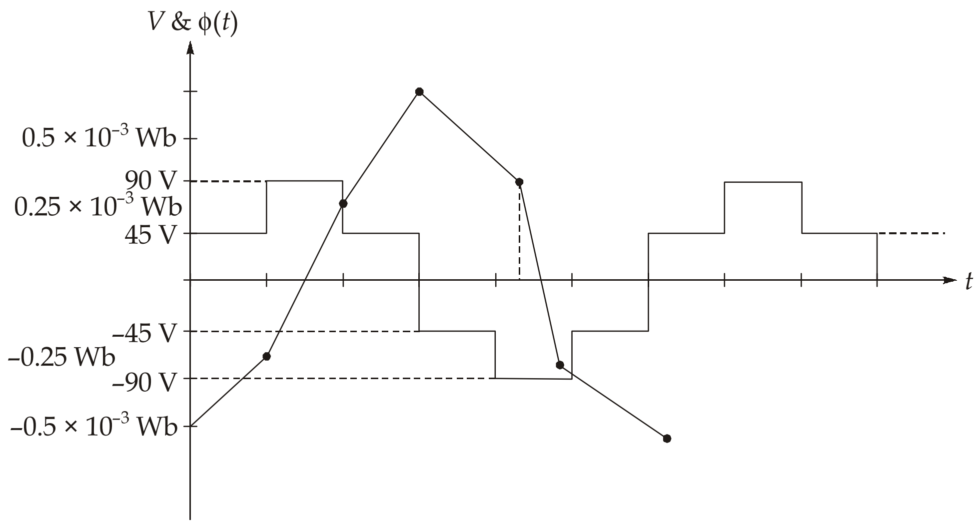
From $0 < t < \dfrac{1}{360}$

$$\phi(t) = -0.5\times10^{-3} + \frac{45}{500}t$$
$$\phi(t)\Big|_{t=\frac{1}{360}} = -0.5\times10^{-3} + \frac{45}{500}\times\frac{1}{360} = -0.25\times10^{-3}\ \text{Wb}$$

From $\dfrac{1}{180} < t < \dfrac{1}{120}$

$$\phi(t) = 0.25\times10^{-3} + \frac{45}{500}\left(t - \frac{1}{180}\right)$$
$$\phi(t)\Big|_{t=\frac{1}{120}} = 0.25\times10^{-3} + \frac{45}{500}\left(\frac{1}{120} - \frac{1}{180}\right)$$
$$= 0.5\times10^{-3}\ \text{Wb}$$

Now, since waveform is symmetrical about $t = \frac{1}{120}$, therefore, waveform in negative half cycle will symmetrical about $t = \frac{1}{120}$.



(ii) Slot/pole/phase,

$$m = \frac{90}{3 \times 10} = 3$$

$$\gamma = \frac{180°}{\text{slot/pole}} = \frac{180° \times 10}{90} = 20°$$

Distribution factor,

$$K_d = \frac{\sin\left(\frac{m\gamma}{2}\right)}{m \sin\left(\frac{\gamma}{2}\right)}$$

$$= \frac{\sin(30°)}{3\sin(10°)} = 0.9597$$

Winding is assumed to be full-pitched, then

$$E_p = 4.44 \times \phi_m \times f \times N_{Ph} \times K_d$$

$$\frac{11000}{\sqrt{3}} = 4.44 \times 0.9597 \times 50 \times 0.16 \times N_{ph}$$

$$N_{ph} = 186.30 \simeq 187 \text{ turns}$$

Q.3 (b) Solution:

The rating of windings are as follows :

$$V_{H(\text{rated})} = 2200 \text{ V}$$

$$V_{L(\text{rated})} = 220 \text{ V}$$

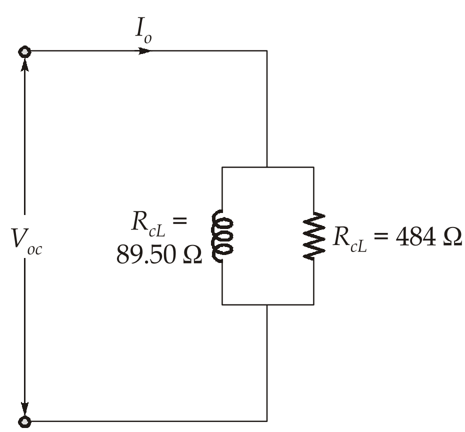
$$I_{H(\text{rated})} = \frac{10000}{2200} = 4.545 \text{ A} \simeq 4.55 \text{ A}$$

$$I_{L(\text{rated})} = \frac{10000}{220} = 45.50 \text{ A}$$

(i) The equivalent circuit at open circuit, on LV side.

$$P_{oc} = \frac{V_{LV}^2}{R_{CL}} \Rightarrow R_{CL} = \frac{220^2}{100} = 484 \ \Omega$$

$$X_{mL} = \frac{R_{CL}}{\tan\phi_o} = \frac{484}{\tan\left[\cos^{-1}\left(\frac{100}{2.5 \times 220}\right)\right]} = 89.50 \ \Omega$$



At short circuit test, the equivalent circuit is given as (on HV side)

$$P_{sc} = I_{\text{HV}}^2 \times R_{\text{eq(HV)}}$$

$$R_{\text{eq(HV)}} = \frac{215}{(4.55)^2} = 10.38 \ \Omega$$

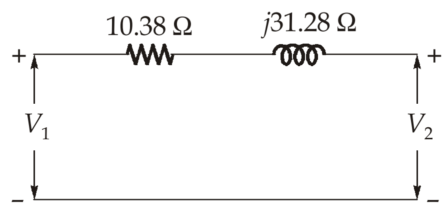
$$Z_{\text{eq(HV)}} = \frac{V_{sc}}{I_{sc}} = \frac{150}{4.55} = 32.96\angle 71.63° \ \Omega$$

Therefore,

$$Z_{\text{eq}} = 32.96\angle 71.63 = 10.38 + j31.28 \ \Omega$$

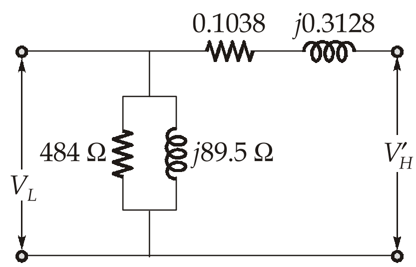
$$R_{\text{eq(HV)}} = 10.38 \ \Omega$$

$$X_{\text{eq(HV)}} = 31.28 \ \Omega$$

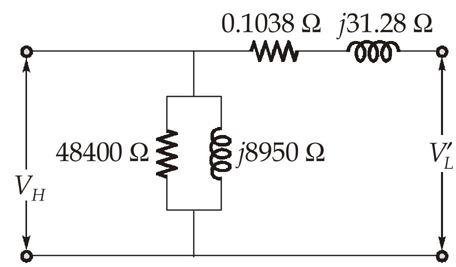


Therefore, equivalent :

On LV Side :



On HV Side :



(ii) Power factor at no-load,

$$\cos\phi_{nL} = \frac{P_{oc}}{V_{oc}I_{oc}} = \frac{100}{220 \times 2.5} = 0.182 \text{ lagging}$$

Power factor at short-circuit condition,

$$\cos\phi_{sc} = \frac{P_{sc}}{V_{sc}I_{sc}} = \frac{215}{150 \times 4.55} = 0.315 \text{ lagging}$$

(iii) Load current at 75% load and 0.6 pf lagging,

$$I_L = 0.75 \times I_{fL}\angle{-\cos^{-1}(0.6)}$$

$$I_L = 3.41\angle{-53.13°} \text{ A}$$

From the equivalent circuit on HV side

$$\vec{V}_H = \vec{V}_L' + \vec{I}_L \cdot \vec{Z}_{eq}$$

$$= 2200 + (3.41\angle{-53.13})(10.38 + j31.28)$$

$$\vec{V}_H = 2306.84\angle 0.88° \text{ V}$$

Therefore, voltage regulation

$$\% \text{ V.R.} = \frac{|\vec{V}_H| - |V_L'|}{|V_L'|} \times 100$$

$$= \frac{2306.84 - 2200}{2200} \times 100$$

$$\% \text{ V.R.} = 4.85\%$$

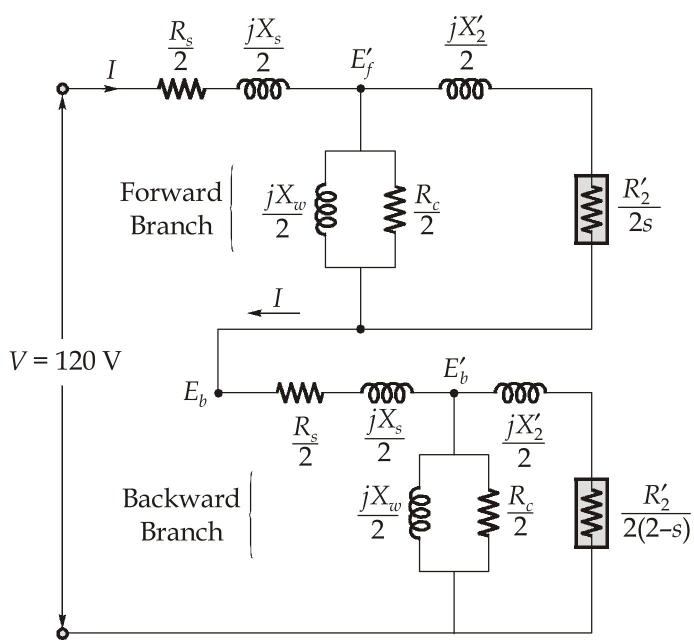
Q.3 (c) Solution:

Nearest possible synchronous speed is 1800 rpm with 4 poles.

$$\text{Slip of motor} = \frac{N_s - N_r}{N_s}$$

$$= \frac{1800 - 1725}{1800} = 0.0416$$

Equivalent Circuit :



Forward branch impedance,

$$Z_f = 1 + j1.5 + \cfrac{1}{\cfrac{1}{j30} + \cfrac{1}{300} + \cfrac{1}{48 + j1.5}}$$

$$Z_f = 25.79\angle 54.72°$$

Backboard branch impedance,

$$Z_b = 1 + j1.5 + \cfrac{1}{\cfrac{1}{j30} + \cfrac{1}{300} + \cfrac{1}{(1.02 + j1.5)}}$$

$$Z_b = 3.524\angle 56.82° \ \Omega$$

Stator current,

$$I = \frac{V}{Z_f + Z_b} = \frac{120}{25.79\angle 54.72° + 3.524\angle 56.82°}$$

$$I = 4.10\angle -54.98° \text{ A}$$

Forward branch voltage,

$$E_f = IZ_f = (4.10\angle -54.98°) \times (25.79\angle 54.72°)$$
$$= 105.739\angle -0.26° \text{ V}$$

Backward branch voltage,

$$E_b = IZ_b = (4.10\angle -54.98°) \times (3.524\angle 56.82°)$$
$$= 14.45\angle 1.84° \text{ V}$$

$$E'_f = E_f - \vec{I} \times (1 + j1.5)$$
$$= 105.739\angle -0.26 - (4.10\angle -54.98°)(1 + j1.5)$$
$$= 98.35\angle -0.38° \text{ V}$$

Forward branch rotor current

$$I_{Rf} = \frac{E'_f}{\frac{R'_2}{2(2-s)} + j\frac{X'_2}{2}} = \frac{98.35\angle -0.38°}{48 + j1.5}$$

$$I_{Rf} = 2.04\angle -2.17° \text{ A}$$

Now,

$$E'_b = E_b - \vec{I} \times (1 + j1.5)$$
$$= 14.45\angle 1.84 - (4.10\angle -54.98)(1 + j1.5)$$
$$= 7.06\angle 2.37° \text{ V}$$

Backward branch rotor current,

$$I_{Rb} = \frac{E'_b}{1.02 + j1.5} = \frac{7.06\angle 2.37°}{1.02 + j1.5}$$

$$I_{Rb} = 3.89\angle -53.41°$$

Forward power to rotor :

$$P_f = I_{Rf}^2 \times 48 = (2.04)^2 \times 48 = 200 \text{ W}$$

Backward power to rotor :

$$P_b = I_{Rb}^2 \times 1.02 = (3.89)^2 \times 1.02 = 15.43 \text{ W}$$

Net rotor power, $P_T = P_f - P_b = 200 - 15.43 = 184.57 \text{ W}$

Net mechanical power output,

$$P_M = P_T(1 - s) = 184.57(1 - 0.0416)$$
$$= 177 \text{ W}$$

Power input to motor,

$$P_{in} = VI \cos \phi_{in}$$
$$= 120 \times 4.10 \cos(54.98°)$$
$$= 282.34 \text{ Watts}$$

Power factor, $\cos \phi = \cos(54.98)$

$$= 0.573 \text{ lagging}$$

$$\text{Efficiency} = \frac{177}{282.34} = 0.627 \text{ or } 62.70\%$$

Q.4 (a) Solution:

(i) At the rated load, motor counter emf,

$$E_a = V_t - I_a r_a$$
$$K_m.\omega_r = 230 - 100 \times 0.5 = 180 \text{ V}$$

$\because$ ω_r = rated speed in rad/sec

Motor constant, $K_m = \dfrac{180}{\omega_r} = \dfrac{180 \times 60}{2\pi \times 250} = 6.875$ V-s/rad

Armature current at any speed ω is given by

$$I_a = \frac{V_t - E_a}{r_a} = \frac{230 - K_m\omega}{0.5}$$

∴ Motor torque, $T_e = K_m.I_a = \dfrac{K_m}{0.5}[230 - K_m\omega]$

Under steady state condition,

$$T_e = T_L$$
$$\frac{K_m}{0.5}[230 - K_m\omega] = 500 - 10\omega$$
$$\frac{6.875}{0.5}[230 - 6.875\omega] = 500 - 10\omega$$

On solving, $\omega = 31.495$ rad/sec

Speed, $N = \dfrac{31.495 \times 60}{2\pi} = 300.75$ rpm

The armature current taken by motor is given as

$$I_a = \frac{V_t - K_m\omega}{R_a}$$

$$= \frac{1}{0.5}[230 - 6.875 \times 31.495]$$

$$I_a = 26.91 \text{ A}$$

(ii) Constant field current, $I_f = \frac{200}{100} = 2 \text{ A}$

∴ Armature current, $I_{a1} = 22 - 2 = 20 \text{ A}$

The speed is to be reduced from 1000 rpm to 800 rpm, by armature resistance controlled method.

∵ $I_f = \text{Constant} \rightarrow \phi = \text{Constant}$

(a) Since load torque is independent, the electromagnetic torque will be constant at both the speeds.

$$T_e = \text{Constant}$$

As we know,

$$N \propto \frac{E_b}{\phi}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$E_{b2} = \frac{N_2}{N_1} \times E_{b1}$$

$$= \frac{800}{1000} \times [200 - 20 \times 0.1]$$

$$= 158.4 \text{ Volts}$$

$$E_{b2} = 158.40 \text{ Volt}$$

Now, $I_{a1} = I_{a2}$ (after connecting external resistance in armature)

(∵ T = Constant)

Therefore, $E_{b2} = V - I_{a2}(r_a + R_{ext})$

$$158.40 = 200 - 20(r_a + R_{ext})$$

$$R_{ext} = \frac{200 - 158.40}{20} - 0.1 = 1.98\ \Omega$$

$$R_{ext} = 1.98\ \Omega$$

(b) Now, $T_L \propto N$

and $T_e \propto I_a$ $(\because \phi = \text{Constant})$

Under steady state,

$$T_e = T_L$$

Therefore, $$\frac{N_2}{N_1} = \frac{I_{a2}}{I_{a1}}$$

$$I_{a2} = \frac{800}{1000} \times 20 = 16\ \text{A}$$

Also, $E_a \propto N$ $(\because \phi = \text{Constant})$

$$\frac{E_{a2}}{E_{a1}} = \frac{N_2}{N_1}$$

$$E_{a2} = (200 - 20 \times 0.1) \times \frac{800}{1000} = 158.40\ \text{V}$$

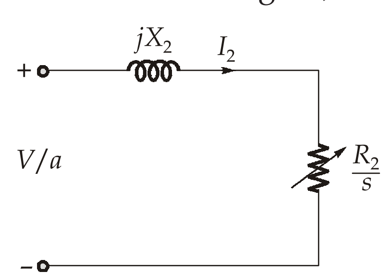
$$E_{a2} = V_t - I_{a2}(r_a + R_{ext})$$

$$R_{ext} = \frac{200 - 158.40}{16} - 0.1$$

$$R_{ext} = 2.50\ \Omega$$

Q.4 (b) Solution:

Motor circuit seen on rotor side is shown in figure, stator impedance been neglected



$$I_2 = \frac{V_1 / a}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}}$$

$$T = \frac{3}{\omega_s} \cdot I_2^2 \frac{R_2}{s}$$

$$= \frac{3}{\omega_s} \cdot \frac{\left(\frac{V}{a}\right)^2 \times \left(\frac{R_2}{s}\right)}{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}$$

$$T_{(start)} = \frac{3}{\omega_s} \times \frac{\left(\frac{V}{a}\right)^2 \times R_2}{R_2^2 + X_2^2} \qquad (\because s = 1)$$

Let the external resistance added to rotor circuit be R_{ext}, then

$$R_{2(total)} = R_{2t} = R_2 + R_{ext}$$

Then,
$$T_{(start)} = \frac{3}{\omega_s} \times \frac{\left(\frac{V}{a}\right)^2 \times R_{2t}}{R_{2t}^2 + X_2^2}$$

(i) $a = 2.5$,
$$X_2 = 0.4\ \Omega,\ R_2 = 0.08\ \Omega$$

$$T_{(start)} = T_{(load)} = 25\text{ Nm}$$

This is minimum starting torque. Actual starting torque must be sufficiently more than this.

$$V = \frac{400}{\sqrt{3}} = 231\text{ V},\ N_s = 750\text{ rpm},\ \omega_s = 78.54\text{ rad/sec}$$

On substituting, the respective values,

$$25 = \frac{3}{78.54} \times \frac{\left(\frac{231}{2.5}\right)^2 \times R_2}{R_{2t}^2 + 0.4^2}$$

$$R_{2t}^2 - 13.04R_{2t} + 0.16 = 0$$

$$R_{2t} = 13.0277\ \Omega \text{ or } 0.0123\ \Omega$$

Choose,
$$R_{2t} = 13.0277\ \Omega$$

(otherwise will increases copper losses and decreases full load torque)

Therefore,
$$R_{ext} = 13.0277 - 0.08 = 12.9477\ \Omega$$

(ii) Now,
$$25 = \frac{3}{(78.54)} \times \frac{\left(\frac{231}{2.5}\right)^2 \times \left(\frac{R_{2t}}{s}\right)}{\left(\frac{R_{2t}}{s}\right)^2 + (0.4)^2}$$

With R_{2t} = 0.137, we get s = 1061.125 or 0.99965

Therefore, motor speed, $N_r = N_s(1-s)$

$$N_r = 750(1 - 0.99965) = 0.2625 \text{ rpm}$$

Now, with external resistance cut out

$$25 = \left(\frac{3}{78.54}\right) \times \frac{\left(\frac{231}{2.5}\right)^2 \times \left(\frac{R_2}{s}\right)}{\left(\frac{R_2}{s}\right)^2 + (0.4)^2}$$

The solution would yield as before,

$$\frac{R_2}{s} = 13.0354, \quad \frac{R_2}{s} = 0.01227$$

With R_2 = 0.08, we get

$$s = 0.0061, 6.52$$

Again under running condition, slip must be as small as possible.

So, $s = 0.0061$

So, motor speed, $N_r = N_s(1-s)$

$$= 750(1 - 0.0061)$$

$$N_r = 745.397 \text{ rpm}$$

Q.4 (c) Solution:

(i) At a field current of 2.20 A, the line to line voltage on the air-gap line is

$$V_{a,ag} = \frac{202}{\sqrt{3}} = 116.70 \text{ V}$$

and for the same field current, the armature current on short circuit is

$$I_{a,sc} = 118 \text{ A}$$

Unsaturated synchronous reactance,

$$X_{\text{s.u.}} = \frac{116.70}{118} = 0.987\ \Omega/\text{phase}$$

Note that armature current is

$$I_{a,\text{rated}} = \frac{45000}{\sqrt{3} \times 220} = 118 \text{ A}$$

Therefore, $I_{a,\text{sc}}$ = 1.00 pu. The corresponding air-gap line voltage is

$$V_{a,ag} = \frac{202}{220} = 0.92 \text{ p.u.}$$

Now, $$X_{\text{s.u.}} = \frac{0.92}{1.00} = 0.92 \text{ p.u.}$$

The saturated synchronous reactance can be found from open and short-circuit characteristics,

$$X_s = \frac{V_{a,\text{rated}}}{I'_a} = \frac{\left(\frac{220}{\sqrt{3}}\right)}{152} = 0.836 \ \Omega/\text{phase}$$

In per unit, $$I'_a = \frac{152}{118} = 1.29$$

$$X_s = \frac{1.00}{1.29} = 0.775 \text{ p.u.}$$

Finally, from open and short-circuit characteristics, the short circuit ratio is given by

$$\text{SCR} = \frac{2.84}{2.20} = 1.29$$

As we know, the inverse of short-circuit ratio is equal to the per-unit saturated synchronous reactance,

$$X_s = \frac{1}{\text{SCR}} = \frac{1}{1.29} = 0.775 \text{ p.u.}$$

(ii) Let Base MVA = 325 MVA

Base voltage = 26 kV

Therefore, $V_t = 1.0$ pu

$$P = \frac{250}{325} = 0.7692 \text{ pu}$$

Therefore, $$I_{a(\text{pu})} = \frac{0.7692}{1 \times 0.89} = 0.8642\angle{-\cos^{-1} 10.89}$$

$$= 0.8642\angle{-27.12°} \text{ pu}$$

Now, $$E' = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi + I_a X_q)^2}$$

$$= \sqrt{(0.89)^2 + (0.456 + 0.8642 \times 1.18)^2}$$

$$E' = 1.723 \text{ pu}$$

Now, $\tan\psi = \dfrac{V_t \sin\phi + I_a X_q}{V_t \cos\phi + I_a R_a} = \dfrac{0.456 + 0.8642 \times 1.18}{0.89}$

$$\psi = \tan^{-1}(1.658) = 58.90°$$

Now, load angle

$$\delta = \psi - \phi = 58.90 - 27.12$$

$$\delta = 31.786°$$

Now, excitation emf,

$$E_f = E' + I_d(X_d - X_q)$$

$$= 1.723 + 0.74 \times 0.77$$

$$= 2.29 \text{ pu}$$

$$E_f = 2.29 \times 26 \text{ kV} = 59.62 \text{ kV (L–L)}$$

Section B : Power Systems-1 + Systems and Signal Processing-2 + Microprocessors-2

Q.5 (a) Solution:

For conductor 1, (self GMD)

$$D_1 = \sqrt[3]{0.7788 r_x \times D_{12} \times D_{13}}$$

$$= \sqrt[3]{0.7788 \times 0.03 \times 0.5 \times 2} = 0.2858 \text{ m}$$

For conductor 2,

$$D_2 = \sqrt[3]{0.7788 \times r_x \times D_{21} \times D_{23}}$$

$$= \sqrt[3]{0.7788 \times 0.5 \times 1.5 \times 0.03} = 0.2597 \text{ m}$$

For conductor 3,

$$D_3 = \sqrt[3]{0.7788 \times r_x \times D_{31} \times D_{32}}$$

$$= \sqrt[3]{0.7788 \times 1.5 \times 2 \times 0.03} = 0.4123 \text{ m}$$

Therefore, overall self GMD of conductor 'x',

Self GMD, $D_{sx} = \sqrt[3]{D_1 \times D_2 \times D_3} = 0.3128 \text{ m}$

Similarly, for conductor y,

$$D_{sy} = D_{1'} = D_{2'} = \sqrt{0.7788 \times r_y \times D_{1'2'}}$$

$$= \sqrt{0.7788 \times 0.04 \times 0.3} = 0.0966$$

Mutual GMD, $D_{xy} = \sqrt[6]{(D_{11'} D_{12'})(D_{21'} D_{22'})(D_{31'} D_{32'})}$

$$D_{xy} = \sqrt[6]{4 \times 4.3 \times 3.5 \times 3.8 \times 2 \times 2.3}$$

$$D_{xy} = 3.189 \text{ m}$$

Therefore, inductance of conductor 'x',

$$L_x = 0.2 \ln\left(\frac{D_{xy}}{D_{sx}}\right) \text{ mH/km}$$

$$= 0.2 \ln\left(\frac{3.189}{0.3128}\right) = 0.464 \text{ mH/km}$$

and

$$L_y = 0.2 \ln\left(\frac{D_{xy}}{D_{sy}}\right) \text{ mH/km}$$

$$= 0.2 \ln\left(\frac{3.189}{0.0966}\right) = 0.699 \text{ mH/km}$$

Therefore, total inductance of line,

$$L_T = L_x + L_y = 1.1633 \text{ mH/km}$$

Q.5 (b) Solution:

(i) Characteristics impedance

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j0.30}{j3.75 \times 10^{-6}}} = 282.84\ \Omega$$

Phase constant, $\beta = \sqrt{X_L X_C}$

$$\beta = \sqrt{0.30 \times 3.75 \times 10^{-6}} = 1.06 \times 10^{-3} \text{ rad/km}$$

(ii) Propagation constant, $j\beta = j1.06 \times 10^{-3}$ per km

(iii) ABCD constant,

$$A = \cos \beta l = \cos\left(400 \times 1.06 \times 10^{-3} \times \frac{180°}{\pi}\right) = 0.911$$

$$A = D = 0.911$$

$$B = jZ_c \sin(\beta l) = j116.36\ \Omega$$

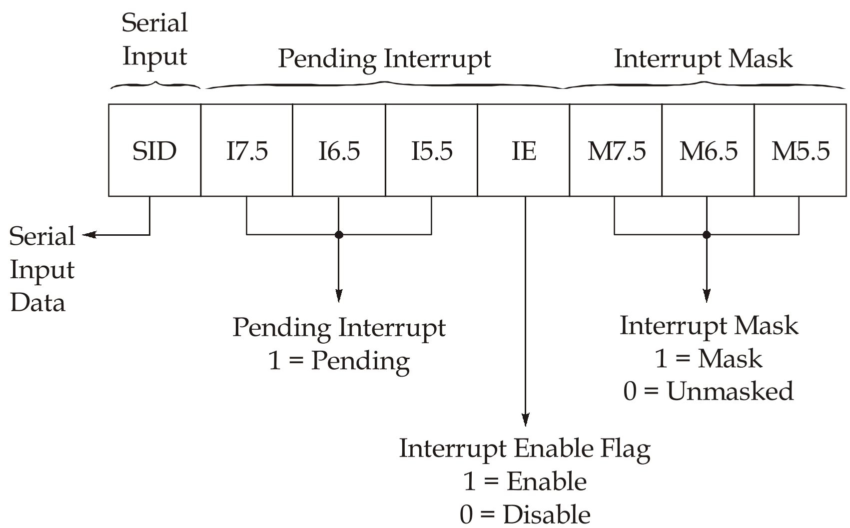
$$C = \frac{-j}{Z_c}\sin(\beta l) = -j1.45 \times 10^{-3} \text{ S}$$

(iv) Wavelength, $\lambda = \dfrac{C}{f} = \dfrac{3 \times 10^8}{50} = 6000 \text{ km}$

(v)
$$SIL = \frac{V_R^2}{Z_c} = \frac{(400)^2}{282.84} = 565.69 \text{ MW}$$

Q.5 (c) Solution:

RIM stands for 'Read Interrupt Mask' and its format is as follows :



When RIM instruction is executed in software, the status SID, pending interrupts and interrupt mask are loaded into the accumulator. Their status can be monitored.

It may so happen that when interrupt is being serviced, other interrupts may occur. The status of these pending interrupts can be monitored by the RIM instruction.

None of the flags are affected by RIM instruction.

Q.5 (d) Solution:

Given : $2y(n) + y(n-1) = x(n)$

Applying Z.T. on above expression

$$2y(z) + z^{-1}y(z) + y(-1) = X(z)$$
$$y(z)[2 + z^{-1}] = X(z) - y(-1) \qquad ...(1)$$

Now,
$$x(n) = \left(\frac{1}{4}\right)^n u(n)$$
$$\Big\downarrow\uparrow \text{ Z-T}$$
$$X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$$

Putting $X(z)$ in equation (1)

$$y(z) = \frac{1}{(2+z^{-1})\left(1-\frac{1}{4}z^{-1}\right)} - \frac{2}{(2+z^{-1})}$$

$$= \frac{1}{2}\left[\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}\right] - \frac{1}{1+\frac{z^{-1}}{2}}$$

$$y(z) = \frac{-\frac{1}{2}+\frac{1}{4}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = \frac{A_1}{1-\frac{1}{4}z^{-1}} + \frac{A_2}{1+\frac{1}{2}z^{-1}}$$

$$A_1 = \left(1-\frac{1}{4}z^{-1}\right)\cdot y(z)\Big|_{z=\frac{1}{4}} = \frac{-\frac{1}{2}+\frac{1}{4}z^{-1}}{1+\frac{1}{2}z^{-1}}\Bigg|_{z=\frac{1}{4}} = \frac{1}{6}$$

$$A_2 = \left(1+\frac{1}{2}z^{-1}\right)\cdot y(z)\Big|_{z=\frac{-1}{2}} \Rightarrow A_2 = -\frac{2}{3}$$

So,
$$y(z) = \frac{1}{6}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right) - \frac{2}{3}\left(\frac{1}{1+\frac{1}{2}z^{-1}}\right)$$

Taking the inverse unilateral z-transform of y, yields

$$y(n) = \frac{1}{6}\left(\frac{1}{4}\right)^n - \frac{2}{3}\left(-\frac{1}{2}\right)^n;\ \text{for } n \geq 0$$

Q.5 (e) Solution:

(i) Given :
$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s}{s^2+2s+1}$$

$$Y(s)[s^2+2s+1] = 2sX(s)$$

$$\Big\downarrow\uparrow \text{ I.L.T.}$$

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{2dx(t)}{dt}$$

with $T = 1$

$$\frac{d\eta(t)}{dt} \simeq \eta(n) - \eta(n-1)$$

$$\frac{d^2\eta}{dt^2} = \frac{d}{dt}\left[\frac{d\eta(t)}{dt}\right] \simeq [\eta(n) - \eta(n-1)] - [\eta(n-1) - \eta(n-2)]$$

$$= \eta(n) - 2\eta(n-1) + \eta(n-2)$$

So, the differential equation is approximated by,

$$(y(n) - 2y(n-1) + y(n-2)) + 2(y(n) - y(n-1)) + y(n) = 2[x(n) - x(n-1)]$$

$$y(n) - y(n-1) + 0.25y(n-2) = 0.5[x(n) - x(n-1)]$$

(ii) If $x(n) = u(n)$, then $x(n) - x(n-1) = \delta(n)$

$$y(n) - y(n-1) + 0.25y(n-2) = 0.5\delta(n)$$

Using z-transform

$$y(z)[1 - z^{-1} + 0.25z^{-2}] = 0.5$$

$$y(z) = \frac{0.5}{(1 - 0.5z^{-1})^2} = (n+1)(0.5)^{n+1}u(n)$$

Q.6 (a) Solution:

(i) Since line is lossless, therefore, receiving end real power can be given by

$$P_R = \frac{V_S \cdot V_R}{X}\sin\delta$$

$$30 = \frac{33 \times 33}{20}\sin\delta$$

$$\delta = \sin^{-1}\left[\frac{600}{(33)^2}\right]$$

Load angle, $\delta = 33.43°$

Receiving end reactive power is given as,

$$Q_R = \frac{-V_R^2}{X} + \frac{V_S V_R}{X}\cos\delta$$

$$Q_R = \frac{-33^2}{20} + \frac{33^2}{20}\cos(33.43°)$$

$$Q_R = -9 \text{ MVAR}$$

$\therefore$ $$Q_C = -Q_R = 9 \text{ MVAR}$$

(ii) Now if reactive power source is removed from the load end.

Suppose the active power transmitted is P at a power angle δ and magnitude of voltage at receiving end is V_R.

Now,
$$P = \frac{V_S \cdot V_R}{X}\sin\delta$$

$$P^2 = \left(\frac{V_S \cdot V_R}{X}\right)^2 \sin^2\delta \qquad \text{...(1)}$$

Since, no reactive power is transmitted,

$$\frac{-V_R^2}{x} + \frac{V_S \cdot V_R}{X}\cos\delta = 0$$

$$\frac{(V_S \cdot V_R)^2}{X^2}\cos^2\delta = \frac{V_R^4}{X^2} \qquad \text{...(2)}$$

Adding eqn. (1) and (2)

$$V_R^4 - V_S^2 V_R^2 + X^2P^2 = 0$$

Let
$$V_R^2 = v$$

$$v^2 - V_S^2.v + X^2P^2 = 0$$

$$v = \frac{V_s^2 \pm \sqrt{V_s^4 - 4X^2P^2}}{2}$$

$$v = \frac{(33)^2 \pm \sqrt{(33)^4 - 4 \times 400 \times P^2}}{2}$$

A solution of V_R can be obtained if V has a real solution. Hence, for maximum power,

$$(33)^4 - 4 \times 400 \times P^2 = 0$$

$$P^2 = \frac{(33)^4}{1600}$$

$$P = 27.225 \text{ MW}$$

and
$$v = \frac{(33)^2}{2} = 544.5$$

$$V_R = \sqrt{v} = 23.33 \text{ kV}$$

and power angle,

$$\delta = \sin^{-1}\left(\frac{XP}{v_S \cdot v_R}\right)$$

$$= \sin^{-1}\left(\frac{27.22 \times 20}{23.33 \times 33}\right)$$

$$\delta = 45°$$

Q.6 (b) Solution:

(i) Let $V_1(n) = u(n+5)$

so that $x(n) = \sum_{k=-\infty}^{n} [V_1(k) - u(k)]$

Let $V_2(n) = V_1(n) - u(n)$

$$x(n) = \sum_{k=-\infty}^{n} V_2(k)$$

$$\Big\downarrow\!\!\Big\uparrow \text{ Z-T}$$

$$X(z) = \frac{z}{z-1} V_2(z) \qquad \text{...(1)}$$

Now, $V_2(z) = V_1(z) - \frac{z}{z-1}$

and $V_1(z) = z^5\left[\frac{z}{z-1}\right]$

By substituting the values,

$$X(z) = \frac{z}{z-1} V_2(z)$$

$$= \frac{z}{z-1}\left[V_1(z) - \frac{z}{z-1}\right]$$

$$= \frac{z}{z-1}\left[z^5 \cdot \frac{z}{z-1} - \frac{z}{z-1}\right]$$

$$= \frac{z}{z-1}\left[\frac{z^6}{z-1} - \frac{z}{z-1}\right]$$

$$X(z) = \frac{z^2[z^5 - 1]}{(z-1)^2}$$

Since $x(n) = 0 \ \forall \ x \leq -6 \Rightarrow x(n)$ is right sided.

Therefore, the ROC of $X(z)$ must be outside the outermost pole.

Thus, $$X(z) = \frac{z^2[z^5 - 1]}{(z-1)^2}; \; |z| > 1$$

(ii) (a) We know that

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Now, let $n - k = v \Rightarrow k = n - v$

$$x(n) * h(n) = \sum_{v=-\infty}^{\infty} x(n-v) \cdot h(v)$$

$$x(n) * h(n) = \sum_{v=-\infty}^{\infty} h(v) \cdot x(n-v)$$

$$x(n) * h(n) = h(n) * x(n)$$

(b) Again,

$$[x(n) * h_1(n)] * h_2(n) = \sum_{l=-\infty}^{\infty} [x(l) * h_1(l)]h_2(n-l)$$

$$= \sum_{l=-\infty}^{\infty} (x(l) * h_1(l) \cdot h_2(n-l))$$

$$= \sum_{l=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x(k) \cdot h_1(l-k) \right) \cdot h_2(n-l)$$

Now, by changing the order of summation, we obtain

$$(x(n) * h_1(n)) * h_2(n) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} x(k) \cdot h_1(l-k) \cdot h_2(n-l)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot \sum_{l=-\infty}^{\infty} h_1(l-k)h_2(n-l)$$

Now, let $\lambda = l - k \Rightarrow l = \lambda + k$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot \sum_{\lambda=-\infty}^{\infty} h_1(\lambda) \cdot h_2(n-\lambda-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot \sum_{\lambda=-\infty}^{\infty} h_1(\lambda) \cdot h_2[(n-k)-\lambda]$$

$$= \sum_{k=-\infty}^{\infty} x(k)[h_1(n-k) * h_2(n-k)]$$

$$(x(n)*h_1(n))*h_2(n) = x(n) * (h_1(n) * h_2(n))$$

Q.6 (c) Solution:

(i) In order to find the parallel realisation of the given IIR digital filter, the partial fraction expansion of $\frac{H(z)}{z}$ is determined.

Given,
$$H(z) = \frac{3\left[2z^2+5z+4\right]}{(2z+1)(z+2)}$$

$$\therefore \quad \frac{H(z)}{z} = \frac{\frac{3}{2}\left[2z^2+5z+4\right]}{z\left(z+\frac{1}{2}\right)(z+2)}$$

Now using partial fraction expansion,

$$\frac{H(z)}{z} = \frac{A}{z}+\frac{B}{\left(z+\frac{1}{2}\right)}+\frac{C}{(z+2)}$$

$$A = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{\left(z+\frac{1}{2}\right)(z+2)}\right|_{z=0} = \frac{\frac{3}{2}[4]}{1} = 6$$

$$A = 6$$

$$B = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{(z+2)z}\right|_{z=-\frac{1}{2}} = \frac{\frac{3}{2}\left(2\times\frac{1}{4}-\frac{5}{2}+4\right)}{-0.5\times 1.5}$$

$$B = -4$$

$$C = \left.\frac{\frac{3}{2}(2z^2+5z+4)}{\left(z+\frac{1}{2}\right)z}\right|_{z=-2} = \frac{\frac{3}{2}[2\times4-5\times2+4]}{(-2)\times(-1.5)}$$

$$C = 1$$

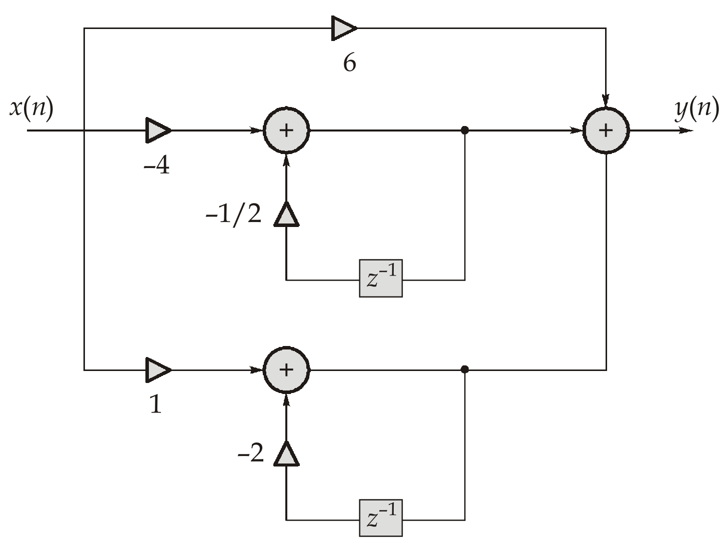
$$\therefore \quad \frac{H(z)}{z} = \frac{6}{z} + \frac{(-4)}{\left(z+\frac{1}{2}\right)} + \frac{1}{(z+2)}$$

$$\frac{H(z)}{z} = \frac{6}{z} - \frac{4}{\left(z+\frac{1}{2}\right)} + \frac{1}{(z+2)}$$

$$H(z) = 6 - \frac{4z}{\left(z+\frac{1}{2}\right)} + \frac{z}{(z+2)}$$

$$H(z) = 6 - \frac{4}{\left[1+\frac{1}{2}z^{-1}\right]} + \frac{1}{[1+2z^{-1}]}$$

∴ The parallel realization of $H(z)$ is



(ii) From the system diagram,

$$V(z) = X(z) + H_2(z)Y(z)$$

and

$$Y(z) = H_1(z)V(z)$$

$$\frac{Y(z)}{H_1(z)} = V(z)$$

$$\frac{Y(z)}{H_1(z)} = X(z) + H_2(z)Y(z)$$

$$Y(z) = H_1(z)X(z) + H_1(z)H_2(z)Y(z)$$

$$\frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 - H_1(z)H_2(z)}$$

Therefore, overall transfer function,

$$\frac{Y(z)}{X(z)} = \frac{(10\beta z)/(z-1)}{1-\frac{10\beta z}{z-1}\times 1}$$

$$= \frac{10\beta z}{z-1-10\beta z} = \frac{10\beta z}{z(1-10\beta)-1}$$

$$H(z) = \frac{10\beta}{1-10\beta}\times\left[\frac{z}{z-\frac{1}{1-10\beta}}\right]$$

In order to assess the BIBO stability of the system, we need to consider the poles of the system function $H(z)$. From the expression of $H(z)$, we can see that $H(z)$ has a single pole at $\frac{1}{1-10\beta}$. Since the system is causal, the system is BIBO stable if and only if, all of the poles are strictly inside a unit circle. Thus, we have

$$\left|\frac{1}{1-10\beta}\right| < 1$$

$$\Rightarrow \quad |1-10\beta| > 1$$

$$1-10\beta > 1 \text{ or } 1-10\beta < -1$$

$$10\beta < 0 \text{ or } 10\beta > 2$$

$$\beta < 0 \text{ or } \beta > \frac{1}{5}$$

Therefore, system is BIBO stable if and only if,

$$\beta < 0 \text{ OR } \beta > \frac{1}{5}$$

Q.7 (a) Solution:

(i) Fault at bus (1) with $Z_f = j0.2$ pu.

Since prefault voltages at buses are $1.0\angle 0°$ pu.

Therefore, $E_1 = E_2 = V_1 = V_2 = V_3 = 1.0\angle 0°$ pu

Now, fault current at bus (1),

$$I_f = \frac{V_1}{Z_{11}+Z_f} = \frac{1\angle 0°}{j0.0776 + j0.20} = -j3.60 \text{ pu}$$

Now, change in bus-voltages due to fault at bus (1)

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = j\begin{bmatrix} 0.0776 & 0.0448 & 0.0597 \\ 0.0448 & 0.1104 & 0.0806 \\ 0.0597 & 0.0806 & 0.2075 \end{bmatrix}\begin{bmatrix} -I_f \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = j\begin{bmatrix} 0.0776 & 0.0448 & 0.0597 \\ 0.0448 & 0.1104 & 0.0806 \\ 0.0597 & 0.0806 & 0.2075 \end{bmatrix}\begin{bmatrix} j3.60 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} -0.27936 \\ -0.16128 \\ -0.2150 \end{bmatrix}$$

Post fault voltages at buses,

$$\begin{bmatrix} V_1(F) \\ V_2(F) \\ V_3(F) \end{bmatrix} = \begin{bmatrix} V_1(P) \\ V_2(P) \\ V_3(P) \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1(F) \\ V_2(F) \\ V_3(F) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.27936 \\ -0.16128 \\ -0.2150 \end{bmatrix} = \begin{bmatrix} 0.72 \\ 0.838 \\ 0.785 \end{bmatrix}$$

Therefore,

$$I_{G1} = \frac{E_1 - V_1(F)}{j0.1} = \frac{1 - 0.72}{j0.1} = -j2.8 \text{ pu}$$

$$I_{G2} = \frac{E_2 - V_2(F)}{j0.2} = \frac{1 - 0.838}{j0.2} = -j0.81 \text{ pu}$$

Current between bus (2) and bus (3)

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.838 - 0.785}{j0.0806} = -j0.657 \text{ pu}$$

Current between bus (3) and bus (1)

$$I_{31} = \frac{V_3 - V_1}{Z_{31}} = \frac{0.785 - 0.72}{j0.0597} = -j1.088 \text{ pu}$$

Current between bus (2) and bus (1)

$$I_{21} = \frac{V_2 - V_1}{Z_{21}} = \frac{0.838 - 0.72}{j0.0448} = -j2.64 \text{ pu}$$

Q.7 (b) Solution:

(i) Given : $V_L = 11$ kV, $S_{rated} = 12$ MVA

Now, $I_L = \dfrac{S_{rated}}{\sqrt{3}V_L}$

$$I_L = \frac{12 \times 10^3}{11\sqrt{3}} = 629.83 \text{ A}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = 6350.85 \text{ Volt}$$

$$\% \text{ reactance} = \frac{IX}{V} \times 100$$

where, X = reactance per phase

$$10 = \frac{629.83X}{6350.82} \times 100$$

$\Rightarrow$ $X = 1.0083\ \Omega$

$\therefore$ Reactance of unprotected winding

= % of unprotected winding × X

$\Rightarrow$ $X_{un} = \dfrac{15}{100} \times 1.0083 = 0.1512\ \Omega$

Let the voltage induced in unprotected winding = v

Voltage induced in unprotected winding

$$= \frac{15}{100} \times 6350.85 = 952.6279 \text{ V}$$

i = fault current = 200 A

Z = Impedance offered to the fault

$$= \frac{v}{i} = \frac{952.6279}{200}$$

$= 4.7631$

$Z = r + j$ (reactance of unprotected winding)

$Z = (r + j0.1512)\ \Omega$

$$r = \sqrt{4.7631^2 - 0.1512^2}$$

$r = 4.7607\ \Omega$

Therefore, earthing resistance required is 4.7607 Ω.

(ii) The frequency of transient oscillations,

$$f_n = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{3}{2\pi \times 50} = 0.00954 \text{ H}$$

$$f_n = \frac{1}{2\pi\sqrt{0.00954 \times 0.015 \times 10^{-6}}} = 13.291 \text{ kHz}$$

The equation of restriking voltage is

$$V_c = V_m[1 - \cos \omega_n t]$$

$$V_c|_{max} = 2Vm \text{ corresponding to } \omega_n t = \pi$$

Therefore, maximum value of restriking

$$\text{Voltage} = 2V_m = 2 \times \sqrt{2} \times \frac{132}{\sqrt{3}} = 215.55 \text{ kV}$$

$\Rightarrow$ Maximum value of RRRV = $\omega_n V_m$

$$= 2\pi f_n \times \frac{132}{\sqrt{3}} \times \sqrt{2}$$

$$= 2\pi \times 13.291 \times 1000 \times \frac{132}{\sqrt{3}} \times \sqrt{2} \times 1000 \text{ V/s}$$

$$RRRV = 9.01045 \text{ kV/}\mu\text{S}$$

Q.7 (c) Solution:

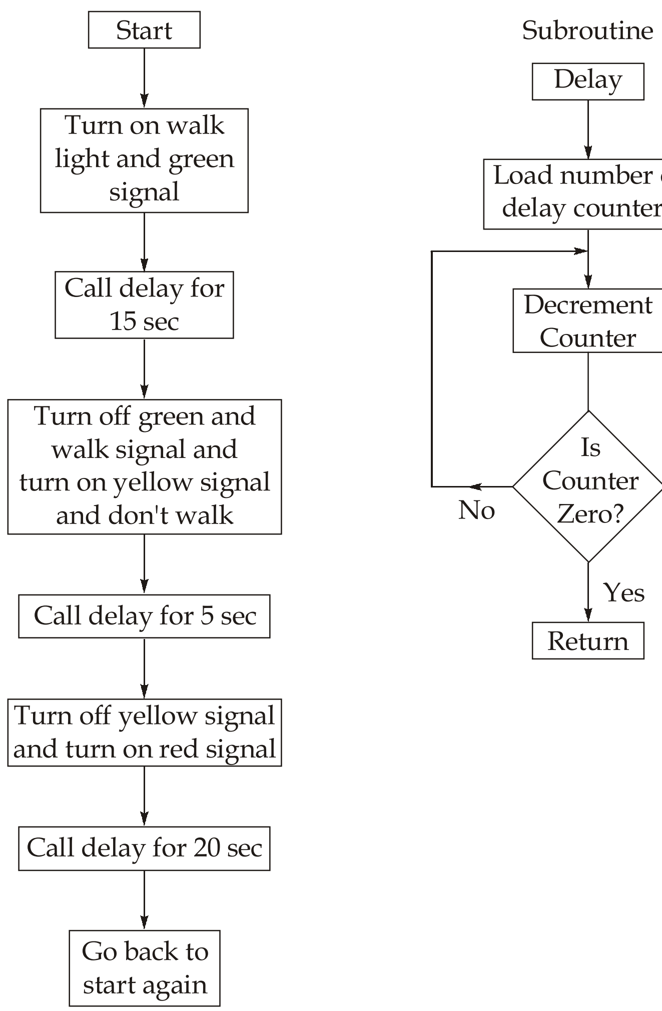
For providing delay sequence in given order for total sequence interval 40 sec, we can develop the truth table.

Time Sequence (in seconds)	Don't Walk	Walk							
0	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	
15 ↓									
15	0	1	0	0	0	0	0	1	⇒ 41 H
5 ↓									
20	1	0	0	0	0	1	0	0	⇒ 84 H
20 ↓									
40	1	0	0	1	0	0	0	0	⇒ 90 H

For 'Green' signal and 'Walk' signal, 41H is sent to output port.

15 sec delay is provided by using 1 sec subroutine and a counter with a count 15_{10}. Similar for $(84)_H$(Yellow) and $(90)_H$(Red) signal.

Flow chart for signal sequence program :



Program :

```
        LXI SP, XX99
Start:  MVI A, 41H
        OUT PORT #
        MVI B, 0FH
        CALL DELAY
        MVI A, 84H
```

```
        OUT PORT #
        MVI B, 05H
        CALL DELAY
        MVI A, 90H
        OUT PORT #
        MVI B, 14H
        CALL DELAY
        JMP START
```

Delay Subroutine Program :

1 sec delay subroutine provides delay according to parameter assigned in register B.

Input : No. of seconds specified in register B for delay.

```
DELAY:    PUSH D
          PUSH PSW
SECOND:   LXI D, COUNT
LOOP      DCX, D
          MOV A, D
          ORA E
          JNZ LOOP
          DEC B
          JNZ SECOND
          POP PSW
          POP D
          RET
```

Q.8 (a) Solution:

Assume $(\text{kVA})_{base} = 30 \text{ MVA}$

$(\text{kV})_{base} = 11 \text{ kV at generator terminal}$

For generator,

$$X_1 = X_2 = j0.1 \times \frac{30}{20} = j0.15 \text{ pu}$$

and

$$X_0 = j0.15 \times \frac{30}{20} = j0.225 \text{ pu}$$

For transformer (1)

$$X_1 = X_2 = X_0 = 0.12 \text{ pu}$$

For transformer (2)

$$X_0 = X_1 = X_2 = j0.05 \times \frac{30}{20} = j0.075$$

For transmission line,

$$Z_{\text{base}} = \frac{22^2}{30} = 16.133\ \Omega$$

Hence, pu impedance of transmission line is

$$Z_{TL(\text{pu})} = \frac{1 + j5}{16.133} = (0.062 + j0.31) \text{ pu}$$

Since load is unity power factor, so it can be represented by equivalent resistance R.

$$R_{\text{actual}} = \frac{V^2}{P} = \frac{(1.1)^2}{10} = 0.121$$

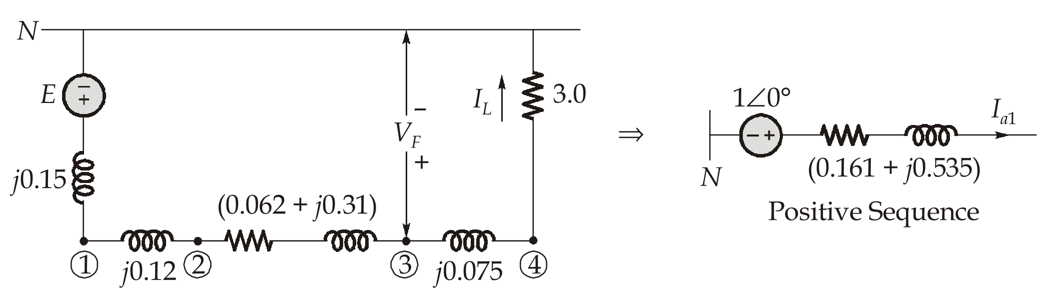
$$R_{\text{pu}} = \frac{R_{\text{actual}}}{(kV)_b^2} \times S_{\text{base}} = \frac{0.121}{(1.1)^2} \times 30 = 3 \text{ pu}$$

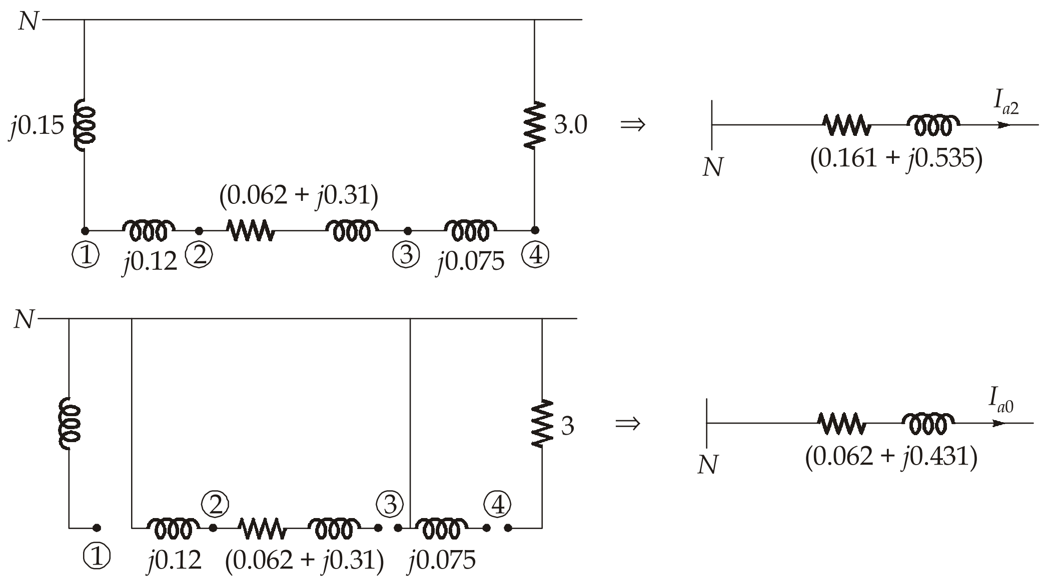
Per unit value of fault resistance,

$$R_{f(\text{pu})} = \frac{6.6 \times 30}{(22)^2} = 0.409$$

Load current, $$I_{L(\text{pu})} = \frac{P_{\text{pu}}}{V_{\text{pu}}} = \frac{\left(\frac{10}{30}\right)}{1.0} = 0.33\angle 0° \text{ pu}$$

Positive, negative and zero-sequence network are shown as





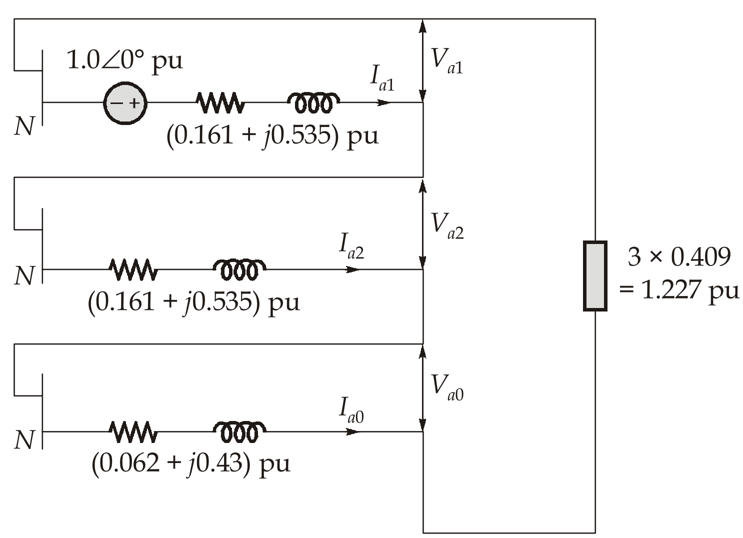
Assuming a prefault voltage of 1.0 pu at node (4), Thevenins equivalent voltage at node (3),

$$V_F = 1.0 + 0.33 \times j0.075 = 1.0\angle 1.46° \text{ pu}$$

Assuming that V_F is the reference phasor at the fault point, i.e., node (3).

$$V_F = 1.0\angle 0° \text{ pu}$$

The interconnected network for an SLG at node (3) is shown



From the interconnected network

$$I_{a0} = \frac{V_F}{Z_1 + Z_2 + Z_0 + 3R_f}$$

$$= \frac{1}{0.384 + j1.50 + 1.227}$$

$$I_{a0} = \frac{1}{1.611 + j1.501} = 0.459\angle{-43°} \text{ pu}$$

Fault current, $I_f = 3I_{a0} = 1.3624\angle{-43°}$ pu

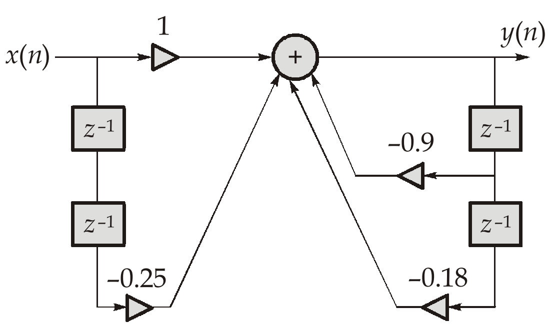
Actual fault current, $I_f = 1.3624 \times \frac{30 \times 10^3}{22\sqrt{3}} = 1072.66\angle{-43°}$ Amp

Q.8 (b) Solution:

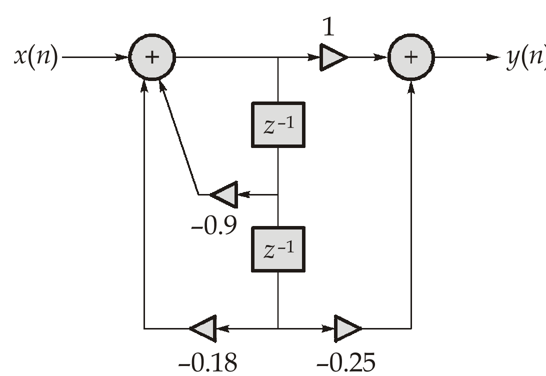
Given : $H(z) = \frac{1 - 0.25z^{-2}}{1 + 0.9z^{-1} + 0.18z^{-2}}$

(i) Direct Form Realization :

$$a_0 = 1,\ a_1 = 0.9,\ a_2 = 0.18,\ b_0 = 1,\ b_1 = 0,\ b_2 = -0.25$$



Direct Form-I



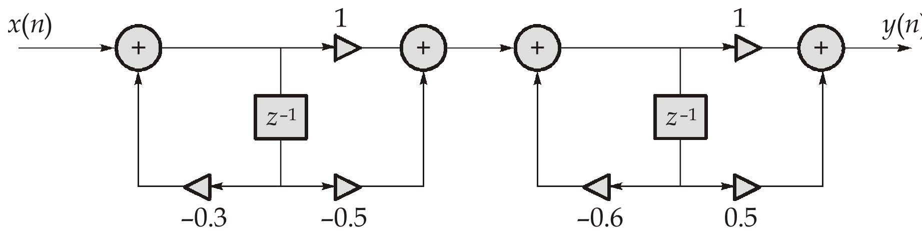
Direct Form-II

(ii) Cascade Realization :

$$H(z) = \frac{1 - 0.25z^{-2}}{1 + 0.9z^{-1} + 0.18z^{-2}}$$

$$= \frac{(1 - 0.5z^{-1})}{(1 + 0.3z^{-1})} \cdot \frac{(1 + 0.5z^{-1})}{(1 + 0.6z^{-1})}$$

Let $H_1(z) = \frac{1 - 0.5z^{-1}}{1 + 0.3z^{-1}}$ and $H_2(z) = \frac{1 + 0.5z^{-1}}{1 + 0.6z^{-1}}$



(iii) Parallel Realization :

$$H(z) = \frac{1-0.25z^{-2}}{(1+0.3z^{-1})(1+0.6z^{-1})} = \frac{z^2-0.25}{(z+0.3)(z+0.6)}$$

Using partial fraction

$$\frac{H(z)}{z} = \frac{A}{z} + \frac{B}{z+0.3} + \frac{C}{z+0.6}$$

where,

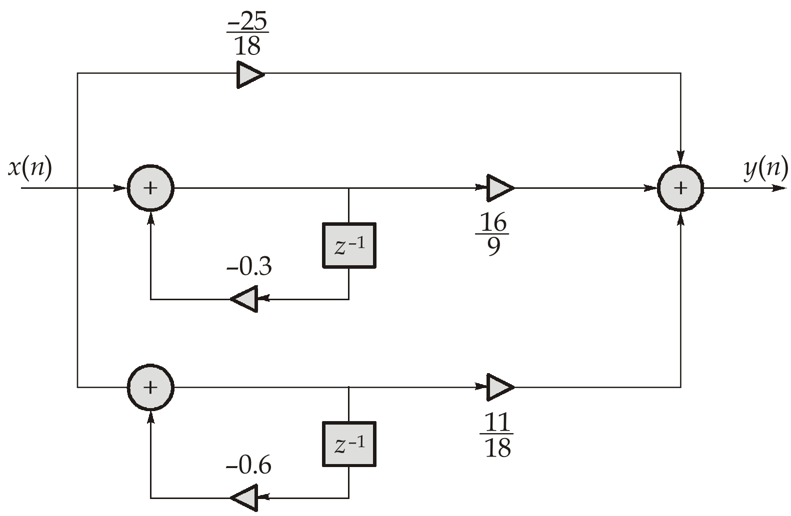
$$A = \left.\frac{z^2-0.25}{(z+0.3)(z+0.6)}\right|_{z=0} \Rightarrow A = \frac{-25}{18}$$

$$B = \left.\frac{z^2-0.25}{z(z+0.6)}\right|_{z=0.3} \Rightarrow B = \frac{16}{9}$$

$$C = \left.\frac{z^2-0.25}{z(z+0.3)}\right|_{z=-0.6} \Rightarrow C = \frac{11}{18}$$

$$H(z) = \frac{-25}{18} + \frac{16}{9}\left(\frac{z}{z+0.3}\right) + \frac{11}{18}\left(\frac{z}{z+0.6}\right)$$

$$H(z) = \frac{-25}{18} + \frac{16}{9}\left(\frac{1}{1+0.3z^{-1}}\right) + \frac{11}{18}\left(\frac{1}{1+0.6z^{-1}}\right)$$



Parallel Realization

Q.8 (c) Solution:

Given :

$$x_1(n) = \{1, 2, 3, 1\} \text{ and } x_2(n) = \{4, 3, 2, 2\}$$

The four point DFT of $x_1(n)$ is $X_1(k)$ and is given by

$$X_1(k) = [W_4\}x_{1N}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+3+1 \\ 1-2j-3+j \\ 1-2+3-1 \\ 1+2j-3-j \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$X_1(k) = \{7, -2-j, 1, -2+j\}$$

Similarly,

$$X_2(k) = [W_4\}X_{2N}$$

$$X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3+2+2 \\ 4-3j-2+2j \\ 4-3+2-2 \\ 4+3j-2-2j \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$X_2(k) = \{11,\ 2-j,\ 1,\ 2+j\}$$

Using circular convolution property,

$$y(n) = x_1(n)x_2(n)$$

i.e.,

$$Y(k) = X_1(k).X_2(k)$$

$$Y(k) = \{7, -2-j, 1, -2+j\}.\{11, 2-j, 1, 2+j]$$

$$Y(k) = \{77, -5, 1, -5\}$$

By applying Inverse Discrete Fourier Transform

$$y(n) = \frac{1}{N}\left[W_N^*\right]\cdot Y_N$$

$$y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 77-5+1-5 \\ 77-5j-1+5j \\ 77+5+1+5 \\ 77+5j-1-5j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

$$y(n) = \{17, 19, 22, 19\}$$

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