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Detailed Solutions

ESE-2023 Mains Test Series

Mechanical Engineering Test No : 4

Section A : Theory of Machines [All Topics]

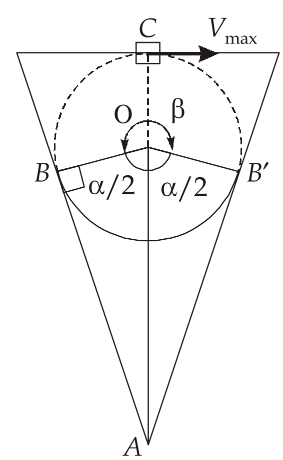
Section B : Fluid Mechanics & Turbo Machinery-1 [Part Syllabus]

Heat Transfer-2 + Refrigeration and Air-conditioning-2 [Part Syllabus]

Section : A

1. (a)

(i)



$$N = 240 \text{ rpm (cw)}$$

$$\therefore \quad w_{crank} = \frac{2\pi \times 240}{60} = 8\pi \text{ rad/s}$$

$$\text{QRR} = \frac{\alpha}{\beta} = \frac{1}{2}$$

or $$360 - \alpha = 2\alpha$$

$$\Rightarrow \quad \alpha = 120°$$

$$\cos\frac{\alpha}{2} = \frac{OB}{OA} = \frac{OB}{100}$$

$$\therefore \quad \frac{1}{2} = \frac{OB}{100}$$

$$OB = 50 \text{ cm (crank length)}$$

Velocity of slider, $$V_B = w_{crank} \times OB$$

$$= 8\pi \times 0.5 = 4\,\pi \text{ m/s}$$

Maximum velocity of slotted bar occurs at mid position.

$$\because \quad AC = \frac{100+50}{100} = 1.5 \text{ m}$$

At mid position,

$$V_C = V_B$$

$$\omega_{AC} \times 1.5 = 4\pi$$

$$\omega_{AC} = 8.37 \text{ rad/s}$$

(ii)

Cycloidal Teeth	Involute Teeth
(a) Pressure angle varies from maximum at the beginning of engagement, reduces to zero at the pitch point and again increases to maximum at the end of engagement resulting in less smooth running of the gears.	Pressure angle is constant throughout the engagement of teeth. This results in smooth running of the gears.
(b) It involves double curve for the teeth, epicycloid and hypocycloid. This complicates the manufacture.	It involves single curve for the teeth resulting in simplicity of manufacturing and of tools.
(c) Owing to difficulty of manufacture, these are costlier.	These are simple to manufacture and thus are cheaper.
(d) Exact centre-distance is required to transmit a constant velocity ratio.	A little variation in the centre distance does not affect the velocity ratio.
(e) Phenomenon of interference does not occur at all.	Interference can occur if the condition of minimum number of teeth on a gear is not followed.
(f) The teeth have spreading flanks and thus are stronger.	The teeth have radial flanks and thus are weaker as compared to the cycloidal form for the same pitch.
(g) In this, a convex flank always has contact with a concave face resulting in less wear.	Two convex surfaces are in contact and thus there is more wear.

1. (b)

Velocity of crank, $\vec{V}_{ao} = 0.32 \times 20 = 6.4$ m/s

Now, produce AC to R. Line AC passes through the pivot Q. Let B be a point on AR beneath Q. Writing the vector equation.

$$\vec{V}_{aq} = \vec{V}_{ab} + \vec{V}_{bq}$$

or $$\vec{V}_{ao} = \vec{V}_{bq} + \vec{V}_{ab}$$

or $$oa = qb + ba$$

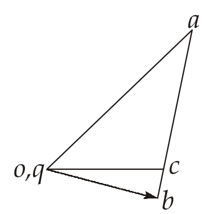
Steps :

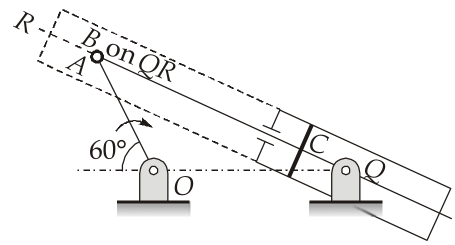
Take $\vec{V}_{ao}$ to a convenient scale.

$\vec{V}_{ba}$ is perpendicular to AB, draw a line perpendicular to AB through $\vec{V}_{bq}$ is along AB, draw a line perpendicular to AB through q.

The intersection locates the point b.

Complete the velocity triangle as shown in figure below.





Locate point C on ab such that $\frac{ac}{ab} = \frac{AC}{AB}$

Now,

(i)Angular velocity of cylinder = Angular velocity of AR or AB

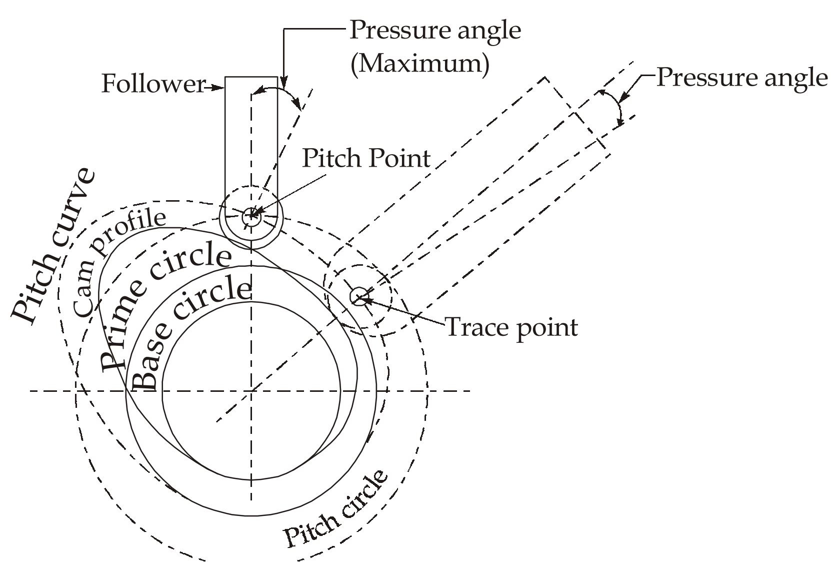
$$= \frac{\vec{V}_{ab}}{AB} = \frac{4.77}{0.85} = 5.61 \text{ rad/s (CW)}$$

(ii) Sliding velocity of plunger = Velocity of B relative to Q

$$= qb = 4.1 \text{ m/s}$$

(iii) Absolute velocity of plunger = OC or qc = 4.22 m/s

1. (c)



Base Circle : It is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.

Trace Point : It is a reference point on the follower to trace the cam profile such as the knife-edge of a knife-edged follower and centre of the roller of a roller follower.

Pitch Curve : It is the curve drawn by the trace point assuming that the cam is fixed, and the trace point of the follower rotates around the cam.

Pressure Angle : The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion. A high value of the maximum pressure angle is not desired as it might jam the follower in the bearings.

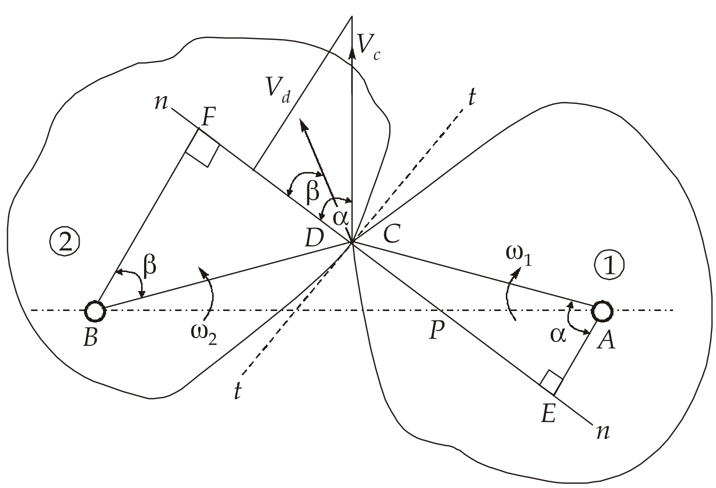
Pitch Point : It is the point on the pitch curve at which the pressure angle is maximum.

Pitch Circle : It is the circle passing through the pitch point and concentric with the base circle.

Prime Circle : The smallest circle drawn tangent to the pitch curve is known as the prime circle.

1. (d)

The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears. Figure shows two bodies 1 and 2 representing a portion of the two gears in mesh.



A point C on the tooth profile of the gear 1 is in contact with a point D on the tooth profile of the gear 2. The two curves in contact C or D must have a common normal at the point. Let it be $n - n$.

Let ω_1 = Instantaneous angular velocity of the gear 1 (Clockwise)

ω_2 = Instantaneous angular velocity of the gear 2 (Counterclockwise)

v_c = Linear velocity of C

v_d = Linear velocity of D

Then $v_c = \omega_1 \cdot AC$ in a direction perpendicular to AC or at an angle α to $n - n$.

$v_d = \omega_2 \cdot BD$ in a direction perpendicular to BD or at an angle β to $n - n$.

Now, if the curved surfaces of the teeth of two gears are to maintain contact, one surface may slide relative to the other along the common tangent $t - t$. The relative motion between the surfaces along the common normal n - n must be zero to avoid the separation, or the penetration of the two teeth into each other.

Component of v_c along $n - n = v_c \cos\alpha$

Component of v_d along $n - n = v_d \cos\beta$

Relative motion along $n - n = v_c \cos\alpha - v_d \cos\beta$

Draw perpendiculars AE and BF on $n - n$ from points A and B respectively. Then $\angle CAE = \alpha$ and $\angle DBF = \beta$. For proper contact,

$$v_c \cos\alpha - v_d \cos\beta = 0$$

or $$\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$$

or $$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

or $$\omega_1 AE - \omega_2 BF = 0$$

or $$\frac{\omega_1}{\omega_2} = \frac{BF}{AE}$$

$$= \frac{BP}{AP} \qquad [\because \Delta AEP \text{ and } BEP \text{ are similar}]$$

Thus, it is seen that the centre line AB is divided at P by the common normal in the inverse ratio of the angular velocities of the two gears. If it is desired that the angular velocities of two gears remain constant, the common normal at the point of contact of the two teeth should always pass through a fixed point P which divides the line of centres in the inverse ratio of angular velocities of two gears.

As seen earlier, P is also the point of contact of two pitch circles which divides the line of centres in the inverse ratio of the angular velocities of the two circles and is the pitch point.

Thus, for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Also, as the Δs *AEP* and *BFP* are similar,

$$\frac{BP}{AP} = \frac{FP}{EP}$$

or $$\frac{\omega_1}{\omega_2} = \frac{FP}{EP} \text{ or } \omega_1 EP = \omega_2 FP$$

1. (e)

Given : N_1 = 320 rpm; N_2 = 350 rpm; r_1 = 30 mm; r_2 = 70 mm;
m = 5 kg; a = 80 mm; b = 40 mm

$$\omega_1 = \frac{2\pi \times 320}{60} = 33.51 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi \times 350}{60} = 36.65 \text{ rad/s}$$

∴ $$F_1 = mr_1\omega_1^2 = 5 \times 0.03 \times 33.51^2$$

$$F_1 = 168.43 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 5 \times 0.07 \times 36.65^2$$

$$F_2 = 470.12 \text{ N}$$

∴ Spring constant, $$s = 2\left(\frac{a}{b}\right)^2\left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$

$$= 2\left(\frac{80}{40}\right)^2\left(\frac{470.12 - 168.43}{70 - 30}\right) = 60.34 \text{ N/mm}$$

We have, $$F_1 a = \frac{(Mg + F_{s_1} + f)b}{2}$$

∵ $M = 0;\ f = 0$

∴ $$168.43 \times 0.08 = \frac{F_{s_1}}{2} \times 0.04$$

∴ $$F_{s_1} = 673.72 \text{ N}$$

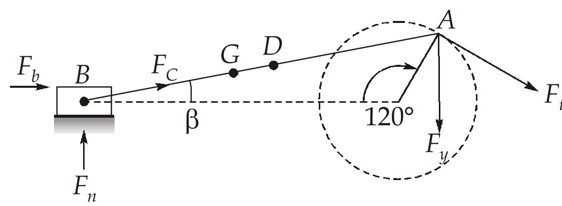
∴ Initial compression, $$x_1 = \frac{F_{s_1}}{s} = \frac{673.72}{60.34}$$

$$x_1 = 11.16 \text{ mm}$$

2. (a)

Given : $r = \frac{250}{2} = 125$ mm; $N = 320$ rpm, D = 120 mm; $n = 4.5$; $m_{CR} = 50$ kg; $m_{reci.} = 20$ kg, $AG = 160$ mm; $k = 130$ mm; $\theta = 120°$; $l = 562.5$ mm

Now,
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 320}{60} = 33.51 \text{ rad/s}$$



Divide the mass of connecting rod into two parts, refer figure,

$$m_a = \frac{50(562.5 - 160)}{562.5} = 35.77 \text{ kg}$$

$$m_b = 50 - 35.77 = 14.22 \text{ kg}$$

Total mass of reciprocating parts at 'B'.

$$m_{reci.} = 20 + 14.22 = 34.22 \text{ kg}$$

Acceleration of reciprocating parts

$$a = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Inertia force,
$$F_b = m_{reci} \cdot r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$F_b = 34.22 \times 0.125 \times 33.51^2\left(\cos 120° + \frac{\cos 240°}{4.5}\right)$$

$$F_b = -2935.34 \text{ N}$$

As θ is more than 90°, it is negative and towards right.

Inertia torque due to reciprocating parts,

$$T_b = F_b \cdot r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

$$T_b = -2935.34 \times 0.125\left(\sin 120° + \frac{\sin 240°}{2\sqrt{4.5^2 - (\sin 120°)^2}}\right)$$

$T_b = -2935.24 \times 0.768 \times 0.125$

$T_b = -281.78$ Nm (CW) or 281.78 Nm (ACW)

Corresponding couple due to assumed second mass of connecting rod at 'A'

$$\Delta T = m_{CR}\alpha_C b(l-L)$$

$$b = 562.5 - 160 = 402.5 \text{ mm}$$

$$L = b + \frac{K^2}{b} = 402.5 + \frac{130^2}{402.5}$$

$$L = 444.48 \text{ mm}$$

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}\right]$$

$$= -33.51^2 \sin 120^\circ\left[\frac{(4.5^2-1)}{(4.5^2-\sin^2 120^\circ)^{3/2}}\right]$$

$$= -217.4 \text{ rad/s}^2$$

$\therefore$ $$\Delta T = 50 \times (-217.4) \times (0.4025) \times (0.5625 - 0.444)$$

$$\Delta T = -518.46 \text{ Nm}$$

$\therefore$ Correction torque on the crankshaft,

$$T_c = \frac{\Delta T \cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$T_c = \frac{-518.46 \times \cos 120^\circ}{\sqrt{4.5^2 - \sin^2 120^\circ}}$$

$$T_c = 58.70 \text{ Nm}$$

Torque due to weight of mass at 'A'

$$T_a = m_a \cdot g \cdot r\cos\theta$$

$$T_a = 35.77 \times 9.81 \times 0.125 \times \cos 120^\circ$$

$T_a = -21.93$ Nm (CW) or 21.93 Nm (ACW)

The correct torque is to be deducted from the inertia torque on the crankshaft. Hence, the total inertia torque on the crankshaft,

$$T = T_b - T_c + T_a$$

$$= 281.78 - (-58.70) + 21.93$$

$$= 362.41 \text{ Nm (ACW)}$$

2. (b)

$$T_G = 32,\ T_F = 60,\ N_F = N_P = 0$$

Now,
$$T_F = 2\left[\frac{T_G}{2} + T_H\right]$$

$$60 = 2\left[\frac{32}{2} + T_H\right]$$

$$30 = 16 + T_H$$

$\Rightarrow$
$$T_H = 14$$

Now, prepare table

Action	C/E	D/G	H	F/P
C fixed, D + 1 rev	0	1	$-\frac{32}{14}$	$-\frac{32}{14}\times\frac{14}{60}$
C fixed, D + x rev	0	x	$-\frac{32x}{14}$	$-\frac{32}{60}x$
Add, y	y	$y+x$	$y-\frac{32x}{14}$	$y-\frac{32x}{60}$

$$N_F = y - \frac{32x}{60} = 0 \text{ or } y = \frac{32x}{60}$$

$\therefore$
$$\frac{T_C}{T_D} = \frac{N_D}{N_C} = \frac{y+x}{y} = \frac{\frac{32x}{60}+x}{\frac{32x}{60}}$$

$\therefore$
$$\frac{T_C}{T_D} = \frac{23}{8}$$

When B and C rotate at different speeds,

$$N_C = y = 120$$

$$N_A = 120\times\frac{T_C}{T_A}$$

$$N_B = \left(120\times\frac{T_C}{T_A}\right)\times 1.1$$

$$N_D = N_B\times\frac{T_B}{T_D} = \left(120\times\frac{T_C}{T_A}\times 1.1\right)\times\frac{T_B}{T_D}$$

$$= 120 \times 1.1 \times \frac{T_C}{T_D} = 120 \times 1.1 \times \frac{23}{8}$$

or $$y + x = 379.5$$

or $$x = 259.5$$

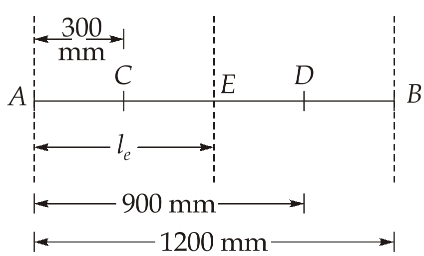
$\therefore$ $$N_P = y - \frac{32x}{60}$$

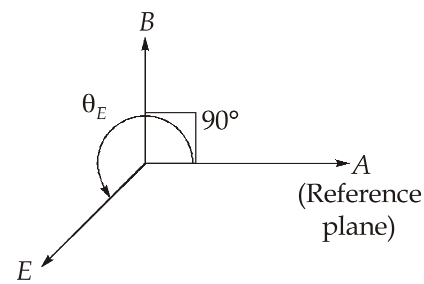
$$N_P = 120 - \frac{32 \times 259.5}{60} = -18.4 \text{ rpm}$$

$\therefore$ P rotates at 18.4 rpm in direction opposite to that of 'C'.

2. (c)

Figure below, shows the planes of masses and bearing as per given data:



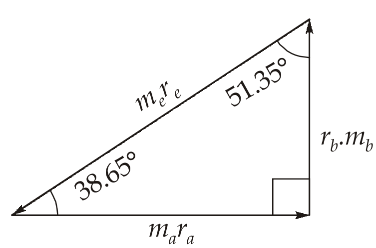


Graphical method:

Let plane A be the reference plane.

$$m_a r_a = 5 \times 2.5 = 12.5 \text{ kg.mm}$$
$$m_b r_b = 4 \times 2.5 = 10 \text{ kg.mm}$$
$$m_e r_e = m_e \times 25 = 25 m_e \text{ kg.mm}$$

Force polygon: Refer figure,

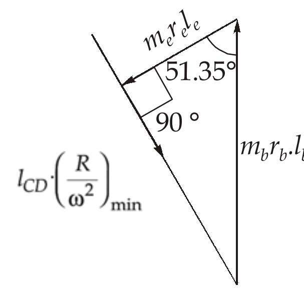


$$m_e r_e = \sqrt{(m_b r_b)^2 + (m_a r_a)^2}$$

$$m_e = \frac{\sqrt{12.5^2 + 10^2}}{25} = 0.64 \text{ kg}$$

$$\theta_e = 218.65°$$

Moment polygon : Refer figure,



$$m_e r_e l_e = m_b r_b l_b \cos 51.35$$

$$\therefore \quad l_e = \frac{10 \times 1200 \times \cos 51.35°}{25 \times 0.64}$$

$$l_e = 468.42 \text{ mm}$$

Also,
$$R = \frac{m_b r_b l_b w^2 \sin 51.35°}{l_{CD}}$$

$$\because \quad w = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.18 \text{ rad/s}$$

$$m_b r_b l_b = 10 \times 1200 = 12000 \text{ kg mm}^2$$

$$\therefore \quad R_{min} = \frac{12000 \times 26.18^2 \times \sin 51.35°}{600}$$

$$R_{min} = 10.7 \text{ N}$$

3. (a)

(i)

(a) Sensitiveness of a governor

A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

As a governor is used to limit the change of speed of the engine between minimum to full-load conditions, the sensitiveness of a governor is also defined as the ratio of the difference between the maximum and the minimum speeds (range of speed) to the mean equilibrium speed. Thus,

$$\text{Sensitiveness} = \frac{\text{Range of speed}}{\text{Mean speed}}$$

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

When N = Mean speed; N_1 = Minimum speed corresponding to full-load conditions; N_2 = Maximum speed corresponding to no-load conditions

(b) Hunting

Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously, because when the load on the engine falls, the sleeve rises rapidly to a maximum position. This shuts off the fuel supply to the extent to affect a sudden fall in the speed. As the speed falls to below the mean value, the sleeve again moves rapidly and falls to a minimum position to increase the fuel supply. The speed subsequently rises and becomes more than the average with the result that the sleeve again rises to reduce the fuel supply. This process continues and is known as hunting.

(c) Isochronism

A governor with a range of speed zero is known as an isochronous governor. This means that for all positions of a sleeve or the balls, the governor has the same equilibrium speed. Any change of speed results in moving the balls and the sleeve to their extreme positions. However, an isochronous governor is not practical due to friction at the sleeve.

For a porter governor, with all arms equal in length and intersecting on the axis (neglecting friction),

$$h_1 = \frac{g}{\omega_1^2}\left(1+\frac{M}{m}\right) \text{ and } h_2 = \frac{g}{\omega_2^2}\left(1+\frac{M}{m}\right)$$

For isochronism, $\omega_1 = \omega_2$ and thus $h_1 = h_2$. However, from the configuration of a porter governor, it can be judged that it is impossible to have two positions of the balls at the same speed. Thus, a pendulum type of governor cannot possibly be isochronous.
In the case of a Hartnell governor (neglecting friction).

At ω_1, $$mr_1\omega_1^2 a = \frac{1}{2}\left(Mg + F_{s_1}\right)b$$

At ω_2, $$mr_2\omega_2^2 a = \frac{1}{2}\left(Mg + F_{s_2}\right)b$$

For isochronism, $\omega_1 = \omega_2$

$\therefore$ $$\frac{Mg + F_{s_1}}{Mg + F_{s_2}} = \frac{r_1}{r_2}$$

Which is the required condition of isochronism.

(ii)

Path circle radius for pinion and gear,

$$R = \frac{mT}{2} = \frac{5\times 80}{2} = 200\,\text{mm}$$

$$r = \frac{m\times t}{2} = \frac{5\times 80}{2\times 2} = 100\,\text{mm}$$

Maximum possible length of path of approach

$$(\text{POA})_{max} = r\sin\phi$$

$$\text{Actual length, POA} = 0.6\ (\text{POA})_{max}$$

$$\text{POA} = 0.6 \times r\sin\phi$$

$\therefore$ $$\text{POA} = 20.52\text{ mm}$$

Maximum length of path recess,

$$(POR)_{max} = R\sin\phi$$

$$\text{Actual length, POR} = 0.6 \times R\sin\phi$$
$$= 0.6 \times 200 \times \sin 20°$$
$$= 41.04 \text{ mm}$$

We have,

$$POA = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$$

$$20.52 = \sqrt{R_a^2 - (200\cos 20°)^2} - 200\sin 20°$$

or $$R_a^2 - (200\cos 20°)^2 = (20.52 + 200\sin 20°)^2$$

or $$R_a^2 = 43228.37$$

or $$R_a = 207.91 \text{ mm}$$

∴ $$\text{Addendum of the wheel} = R_a - R = 207.91 - 200$$
$$= 7.91 \text{ mm}$$ Ans.

Also, $$POR = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$$

$$41.04 = \sqrt{r_a^2 - (100\cos 20°)^2} - 100\sin 20°$$

or $$r_a^2 = (100\cos 20°)^2 + (41.04 + 100\sin 20°)^2$$

$$r_a^2 = 14491.58$$

$$r_a = 120.38 \text{ mm}$$

$$\text{Addendum of the pinion} = r_a - r$$
$$= 120.38 - 100$$
$$= 20.38 \text{ mm}$$ Ans.

$$\text{Arc of contact, AOC} = \frac{POC}{\cos\phi}$$

$$AOC = \frac{0.6(r\sin\phi + R\sin\phi)}{\cos\phi}$$

or $$AOC = 0.6 \times (100 + 200)\tan 20°$$
$$= 65.5 \text{ mm}$$ Ans.

3. (b)

Equation of motion, for damped vibration,

$$m\ddot{x} + c\dot{x} + kx = 0$$

Now,
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{15700}{10}} = 39.62 \text{ rad/sec}$$

$$\text{Damping factor, } \xi = \frac{c}{2m\omega_n} = \frac{1570}{2 \times 10 \times 39.62}$$

$$\xi = 1.98 > 1$$

∴ System is overdamped, displacement,

$$x = A_1 \cdot \exp\left[-\xi + \sqrt{\xi^2 - 1}\right]\omega_n t + A_2 \exp\left[-\xi - \sqrt{\xi^2 - 1}\right]\omega_n t$$

Substituting the values of ε and ω_n, we get

$$x = A_1 \cdot \exp\left[-1.98 + \sqrt{1.98^2 - 1}\right]39.62t + A_2 \exp\left[-1.98 - \sqrt{1.98^2 - 1}\right]39.62t$$

∴ $$x = A_1 \exp(-10.74t) + A_2 \exp(-146.15)t$$

and $$\dot{x} = \frac{dx}{dt} = -10.74 A_1 \exp(-10.74t) - 146.15 A_2 \exp(-146.15)t$$

Initial conditions, are

$$x = 0.01 \text{ at } t = 0$$

∴ $$A_1 + A_2 = 0.01 \qquad \text{...(i)}$$

and, at $t = 0$, $\dot{x} = -V_0$

∴ $$-10.74A_1 - 146.15A_2 = -V_0 \qquad \text{...(ii)}$$

From equation (i) and (ii), solving and simplifying, we get

$$A_1 = -7.38 \times 10^{-3}\, V_0 + 0.0107$$
$$A_2 = 7.38 \times 10^{-3}\, V_0 - 7.93 \times 10^{-4}$$

∴ General expression for displacement,

$$x = (-7.38 \times 10^{-3} V_0 + 0.0107) \exp(-10.74t) + (7.38 \times 10^{-3} V_0 - 7.93 \times 10^{-4}) \exp(-146.15t)$$

Static equilibrium means $x = 0$ at $t = 0.01$ sec,

∴ $$0 = (-7.38 \times 10^{-3} V_0 + 0.0107)(0.898) + (7.38 \times 10^{-3} V_0 - 7.93 \times 10^{-4})(0.2318)$$

Solving, we get

$$4.91 \times 10^{-3} V_0 = 9.42 \times 10^{-3}$$

∴ $$V_0 = 1.92 \text{ m/s}$$

3. (c)

Given : $\delta = 1.5$ mm, $m = 500$ kg; $F = 2650$ N; $N = 2000$ rpm; $\xi = 0.25$

$$\text{Forcing frequency, } \omega = \frac{2\pi \times 2000}{60} = 209.44 \text{ rad/s}$$

$$\text{Stiffness, } k = \frac{Mg}{\delta} = \frac{500 \times 9.81}{1.5 \times 10^{-3}} = 32.7 \times 10^5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{32.7 \times 10^5}{500}} = 80.87 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = 2.59$$

$$\text{Transmissibility ratio, } \frac{F_T}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\frac{F_T}{2650} = \frac{\sqrt{1+(2\times 0.25\times 2.59)^2}}{\sqrt{(1-2.59^2)^2+(2\times 0.25\times 2.59)^2}}$$

$$F_T = 2650 \times 0.2795$$

$$= 740.76 \text{ N}$$ Ans. (i)

$$\text{For steady state vibrations, } A = \frac{F_0/k}{\sqrt{\left(1-r^2\right)^2+\left(2\xi r\right)^2}}$$

$$= \frac{2650}{32.7\times 10^5 \times \sqrt{\left(1-2.59^2\right)^2+\left(2\times 0.25\times 2.59\right)^2}}$$

$$= 1.38 \times 10^{-4} \text{ m or } 0.138 \text{ mm}$$ Ans. (ii)

$$\text{Phase lag, } \tan\phi = \frac{2\xi r}{1-r^2}$$

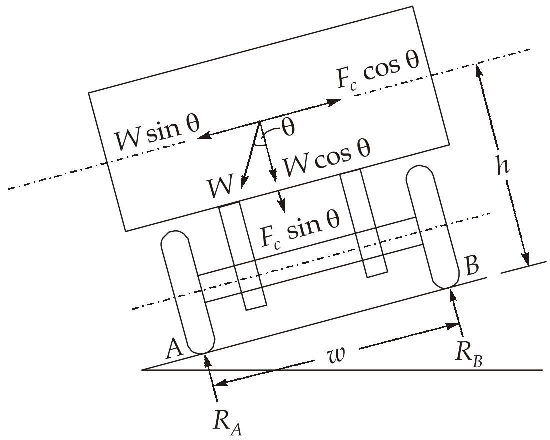
$$\tan\phi = \frac{2\times 0.25\times 2.59}{1-2.59^2}$$

$$= -12.78° \text{ or } 167.21°$$ Ans. (iii)

4. (a)

Given : $M = 1600$ kg; $w = 1.5$ m; $R = 20$ m; $V = \frac{36\times 5}{18} = 10$ m/s; $\theta = 15°$; $r = 0.325$ m;

$k = 0.25$ m; $m = 210$ kg; $h = 0.85$ m



Resolving the forces perpendicular to the track, refer figure

$$R_A + R_B = Mg\cos\theta + \frac{MV^2}{R}\sin\theta$$

$$R_A + R_B = 1600 \times 9.81 \times \cos 15 + \frac{1600 \times 10^2}{20} \times \sin 15°$$

$$R_A + R_B = 17231.72 \text{ N}$$

Taking moment about B,

$$R_A \times w = Mg\cos\theta\,\frac{w}{2} + F_c\sin\theta\,\frac{w}{2} + Mg\sin\theta \times h - F_c\cos\theta \times h$$

or $$R_A = \left(Mg\cos\theta + \frac{MV^2}{R}\sin\theta\right) \times \frac{1}{2} + \left[Mg\sin\theta - F_c\cos\theta\right] \times \frac{h}{w}$$

$\therefore$ $$R_A = \left(1600 \times 9.81 \times \cos 15° + \frac{1600 \times 10^2}{20} \times \sin 15°\right)$$

$$\times \frac{1}{2} + \left[1600 \times 9.81 \times \sin 15 - \frac{1600 \times 10^2}{20} \times \cos 15°\right] \times \frac{0.85}{1.5}$$

$\therefore$ $$R_A = 8615.86 - 3664.98 \times \frac{0.85}{1.5}$$

$$R_A = 6539.04 \text{ N}$$

$\therefore$ $$R_B = 10692.7 \text{ N}$$

Reaction due to gyroscopic couple

$$c_w = 2I_\omega \omega_\omega \cos\theta\, \omega_p$$

$$c_w = 2mk^2 \times \frac{V^2}{rR} \cos\theta$$

$$c_w = 2 \times 210 \times 0.25^2 \times \frac{10^2}{0.325 \times 20} \times \cos 15°$$

$$c_w = 390.085 \text{ Nm}$$

Reaction on each outer wheel,

$$R_{G_0} = \frac{c_w}{2w} = \frac{390.085}{2 \times 1.5} = 130 \text{ N (upwards)}$$

∴ On inner wheels,

∴ $R_{G_i} = 130$ N (Downwards)

Therefore,

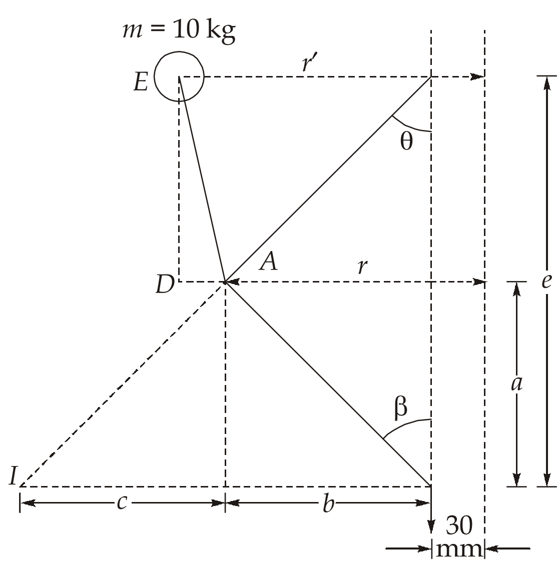
Pressure on outer rails = $R_B - R_{G_0}$
= 10692.7 + 130
= 10822.7 N (Downwards)

Pressure on inner rails = $R_A - R_{G_i}$
= 6539.04 – 130
= 6409.04 N (Upwards)

4. (b)

Given : m = 10 kg; M = 130 kg; L = 200 mm = 0.2 m; r' = 170 mm = 0.17 m

Length of extension arm = 80 mm = 0.08 m; $\theta = \beta = 30°$



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Refer figure,

$$b = c = 200 \sin 30° = 100 \text{ mm}$$

$$r = b + 30 = 130 \text{ mm}$$

$$a = 200 \cos 30 = 173.2 \text{ mm}$$

$$AD = r' - r = 170 - 130 = 40 \text{ mm}$$

$$\therefore \quad DE = \sqrt{80^2 - 40^2} = 69.28 \text{ mm}$$

$$\therefore \quad e = a + DE = 173.2 + 69.28 = 242.48 \text{ mm}$$

(i)

Taking moment about I

$$mr'\omega^2 e = mg(c + r - r') + \frac{Mg}{2}(b+c)$$

$$10 \times 0.17 \times \omega^2 \times 0.242 = 10 \times 9.81(0.1 + 0.13 - 0.17) + \frac{130 \times 9.81}{2}(0.1 + 0.1)$$

$$0.411\omega^2 = 5.88 + 127.53$$

$$\therefore \quad \omega = 18 \text{ rad/sec}$$

$$\therefore \quad N = \frac{60 \times 18}{2\pi} = 171.88 \text{ rpm}$$

(ii)

Considering the friction, let ω_1 and ω_2 be the minimum and maximum speeds respectively,

$$\therefore \quad 10 \times 0.17 \times \omega_1^2 \times 0.242 = 10 \times 9.81(0.1 + 0.13 - 0.17) + \frac{130 \times 9.81 - 50}{2}(0.1 + 0.1)$$

$$\therefore \quad 0.411\omega_1^2 = 5.88 + 122.53$$

$$\therefore \quad \omega_1 = 17.67 \text{ rad/s}$$

$$\Rightarrow \quad N_1 = 168.79 \text{ rpm}$$

For maximum speed,

$$10 \times 0.17 \times \omega_2^2 \times 0.242 = 10 \times 9.81(0.1 + 0.13 - 0.17) + \frac{130 \times 9.81 + 50}{2}(0.1 + 0.1)$$

$$\therefore \quad 0.411\omega_2^2 = 5.88 + 132.53$$

$$\therefore \quad \omega_2 = 18.35 \text{ rad/s}$$

$$\Rightarrow \quad N_2 = 175.23 \text{ rpm}$$

∴ Coefficient of insensitiveness,

$$C_s = \frac{N_2 - N_1}{N} = \frac{175.23 - 168.79}{171.8}$$

$$C_s = 0.0375 \text{ or } 3.75\%$$

(iii) Range of speed = 175.23 – 168.79

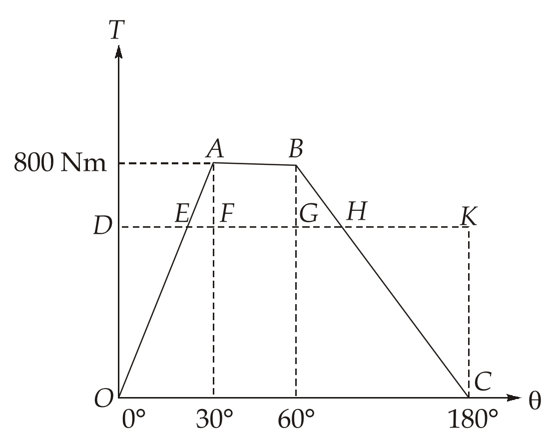
= 6.44 rpm

4. (c)

(i)

Flywheel	Governor
(a) Function of a flywheel is to control the cyclic variation of speed due to the variation in power produced by an engine in a cycle.	Function of a governor is to control the speed of an engine due to variation in external load on the engine by changing the energy supply in the form of fuel.
(b) Flywheel regulates the speed during each cycle of engine operation.	Governor regulates the speed of engine when load varies over a time period.
(c) Flywheel is operative in every cycle of the engine.	Governor operates only when load changes over the engine.
(d) Flywheel stores the energy itself and gives out to engine during each cycle.	Governor regulates the fuel supply to the engine as per the load.
(e) Flywheel has nothing to do for the quantity and quality of working medium.	Governor controls the speed by either quality or quantity variation of the working medium.
(f) Mathematically a flywheel controls the $\delta N/\delta t$.	A governor basically controls the δN.

(ii)



$N = 250$ rpm, $k = 200$ mm, $c_s = 1\%$

Refer figure,

Work done per cycle, W = Area of trapezium $OABC$

$$W = \frac{AB + OC}{2} \times 800$$

$$= \frac{\left(\frac{\pi}{6} + \pi\right) \times 800}{2} = 1466.07 \text{ Nm}$$

Mean torque, $T_{mean} = \frac{1466.07}{\pi} = 466.66$ Nm

Fluctuation of energy,

$$\Delta E = \text{max[Area (EABH)] or Area [(CKH)]}$$

Now, $$EF = \frac{800 - 466.66}{800} \times \frac{\pi}{6} = 0.218$$

$$GH = \frac{800 - 466.66}{800} \times \frac{2\pi}{3} = 0.873$$

Area $EABH$ = Area EAF + Area $ABGF$ + Area BHG

$$= (800 - 466.66)\left[\frac{0.218}{2} + \frac{\pi}{6} + \frac{0.873}{2}\right] = 356.37 \text{ Nm}$$

Also, area $CKH = CK \times KH = CK(GK - GH)$

$$= \frac{466.66}{2}\left[\frac{2\pi}{3} - 0.873\right]$$

$$= 284.98 \text{ Nm}$$

∴ $$\Delta E_{max} = \text{Area EABH} = 356.37 \text{ Nm}$$

$$mk^2\omega^2 C_S = 356.37$$

∴ $$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.18 \text{ rad/s}$$

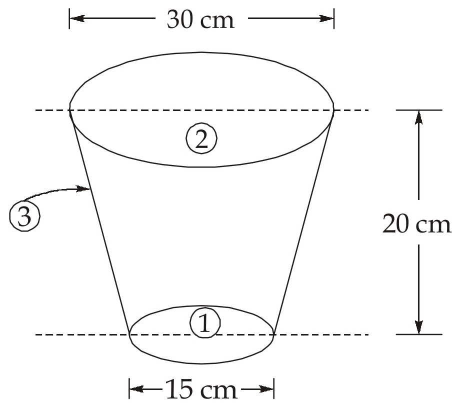
∴ $$m = \frac{356.37}{0.2^2 \times 26.18^2 \times 0.01}$$

$$m = 1299.875 \simeq 1300 \text{ kg}$$

Section : B

5. (a)

Refer the figure below for configuration and nomenclature.



Area of the curved surfaces,

$$A = \pi(r_1 + r_2)\sqrt{(r_2 - r_1)^2 + h^2}$$

$$= \pi(7.5 + 15)\sqrt{(15 - 7.5)^2 + 20^2}$$

$$= 1509.85 \text{ cm}^2$$

Now, $F_{21} = 0.15$ (Given)

By reciprocating theorem,

$$A_1F_{12} = A_2F_{21}$$

$\therefore$ $$F_{12} = \frac{A_2}{A_1} \times F_{21} = \frac{\pi \times 15^2}{\pi \times 7.5^2} \times 0.15$$

$$F_{12} = 0.6$$

Further, $F_{11} + F_{12} + F_{13} = 1$

$\therefore$ $F_{13} = 1 - F_{12}$ $\quad (\because \quad F_{11} = 0)$

$$F_{13} = 1 - 0.6 = 0.4$$

Again, $F_{21} + F_{22} + F_{23} = 1$

$\therefore$ $F_{23} = 1 - F_{21}$

$F_{23} = 1 - 0.15 = 0.85$ Ans. (i)

Now, by reciprocity theorem,

$$F_{32} = \frac{A_2}{A_3}F_{23}$$

$$F_{32} = \frac{\pi \times 15^2}{1509.85} \times 0.85 = 0.398$$

$$F_{31} = \frac{A_1}{A_3} \cdot F_{13} = \frac{\pi \times 7.5^2}{1509.85} \times 0.4 = 0.0468$$

$\therefore$ From the identity,

$$F_{31} + F_{32} + F_{33} = 1$$

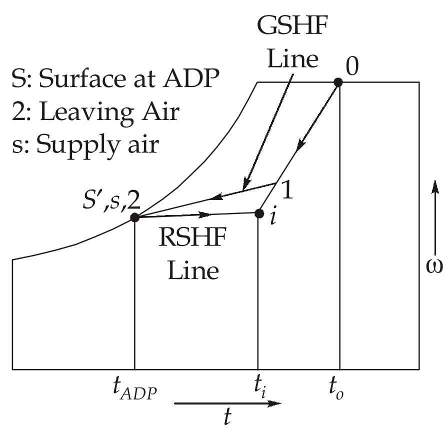
$$F_{33} = 1 - 0.0468 - 0.398$$

$$= 0.555$$

Ans. (ii)

5. (b)

(i)



Room sensible heat factor (RSHF) : It is the ratio of room sensible heat to the room total heat.

$$\text{RSHF} = \frac{RSH}{RSH + RLH} = \frac{RSH}{RTH}$$

In a cooling and dehumidification process, the temperature at which the RSHF line intersects the saturation curve is called the room apparatus dew point.

Grand sensible heat factor (GSHF) : It is the ratio of the total sensible heat to the grand total heat.

$$\text{GSHF} = \frac{TSH}{TSH + TLH} = \frac{TSH}{GTH}$$

Total sensible heat (TSH) = Room sensible heat (RSH) + Outside air sensible heat (OASH)

Total latent heat (TLH) = Room latent heat (RLH) + Outside air latent heat (OALH)

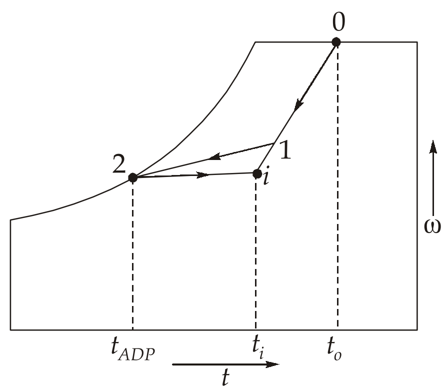
(ii)

From Psychrometric chart,

at 40°C DBT and 27°C WBT

$$h_0 = 85 \text{ kJ/kgd.a}$$

$$\omega_0 = 17.2 \text{ gw.v./kgd.a.}$$



At 25°C DBT and 50% RH

$$h_i = 50.8 \text{ kJ/kgd.a.}, \ \omega_i = 10 \text{ gw.v/kgd.a.}$$

$$h_2 = h_{ADP} = 37 \text{ kJ/kgd.a.}$$

Condition of air entering the coil,

$$\omega_1 = 0.25 \times \omega_o + 0.75 \times \omega_i$$

$$\omega_1 = 0.25 \times 17.2 + 0.75 \times 10 = 11.8 \text{ gw.v/kgd.a.}$$

$$h_1 = 0.25 \times h_o \times 0.75 \times h_i$$

$$= 0.25 \times 85 + 0.75 \times 50.8 = 59.35 \text{ kJ/kgd.a.}$$

$$t_1 = 0.25 \times t_o + 0.75 \times t_i$$

$$= 0.25 \times 40 + 0.75 \times 25 = 28.75°\text{C}$$

Specific volume of air entering the coil

$$v_1 = 0.869 \text{ m}^3\text{/kgd.a.}$$

Mass flow rate of air entering the cooling coil

$$\dot{m}_{a_1} = \frac{cmm}{60 \times v_1} = \frac{4800}{60 \times 0.869} = 92.059 \text{ kg/s}$$

$$\text{Total cooling load, } \dot{Q} = \dot{m}_{a_1}(h_1 - h_2) = 92.059(59.35 - 37)$$

$$\dot{Q} = 2057.518 \text{ kW}$$

$$\text{Fresh air load} = \dot{m}_{a_0}(h_o - h_i) = 0.25 \times 92.059(85 - 50.8)$$

$$\dot{Q}_0 = 787.104 \text{ kW}$$

$$\text{So, room heat gain (RTH)} = \dot{Q} - \dot{Q}_0 = 2057.518 - 787.104$$

$$\text{RTH} = 1270.414 \text{ kW}$$

5. (c)

Flow velocity in suction pipe (v_1) = 2 m/s

Suction pipe diameter (d_1) = 85 mm

Delivery pipe diameter (d_2) = 60 mm

Friction loss in piping system = 6 m

Pump efficiency (η) = 70%

Density (ρ) = 1750 kg/m^3

From continuity consideration, the flow velocity in the discharge pipe is

$$v_2 = \frac{A_1 v_1}{v_2} = \frac{\frac{\pi}{4} \times d_1^2 \times v_1}{\frac{\pi}{4} \times d_2^2} = \left(\frac{d_1}{d_2}\right)^2 v_1$$

or $$v_2 = \left(\frac{85}{60}\right)^2 \times 2 = 4.0138 \text{ m/s}$$

Also, velocity at the discharge end of 60 mm pipe is v_c = 4.0138 m/s.

(i)

Applying Bernoulli's equation between station b (surface of liquid in the tank) and station c (discharge end of delivery pipe).

$$\frac{P_b}{\rho g} + \frac{v_b^2}{2g} + z_a + \text{pump work} = \frac{v_c^2}{2g} + \frac{P_c}{\rho g} + z_c + \text{Friction loss} \quad \text{...(i)}$$

The velocity at station b is negligible because of large diameter of tank in comparison with that of pipe. Again with atmospheric pressure as the datum for pressure head $P_b = P_c$. Further, with datum plane for elevation through the station b;

$$z_b = 0 \text{ and } z_c = 18 \text{ m}$$

From equation (i)

$$\therefore \quad \text{Pump work head } (h_f) = \frac{(4.0138)^2}{19.6} + 18 + 6$$

$$h_f = 24.8219 \text{ m}$$

Discharge through the pump,

$$Q = A_1 v_1 = \frac{\pi}{4}(0.085)^2 \times 2$$

$$= 0.011349 \text{ m}^3/\text{s}$$

Power used by pump = $\rho g Q h_f$

$$P = (9.81 \times 1750) \times 0.011349 \times 24.8219$$

or $$P = 4836.149 \text{ Watt} = 4.836 \text{ kW}$$

Considering pump efficiency, the pump used should have power rating of

$$= \frac{4.836}{0.7} = 6.908 \text{ kW}$$

(ii)

The pressure developed by the pump can be found by applying Bernoulli's equation over the pump itself.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + \text{Pump work head} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

or $$P_2 - P_1 = \rho g\left(\frac{v_1^2 - v_2^2}{2g} + \text{Pump work head}\right)$$

$$P_2 - P_1 = \frac{1750 \times 9.81}{1000}\left(\frac{2^2 - 4.0138^2}{19.6} + 24.8219\right)$$

$$P_2 - P_1 = 415.5223 \text{ kPa}$$

5. (d)

As per given data, for vertical position,

$$\bar{h}_v = 0.943\left[\frac{k^3 \rho^2 g h_{fg}}{\mu l (t_{sat} - t_s)}\right]^{0.25}$$

$$\bar{h}_v = 0.943\left[\frac{0.68^3 \times 965.3^2 \times 9.81 \times 2285 \times 10^3}{3.14 \times 10^{-4} \times 2.5 \times (100 - 80)}\right]^{0.25}$$

$$\bar{h}_v = 4264.7 \text{ W/m}^2\text{K}$$

∴ Heat flow rate, in vertical position

$$Q_v = \bar{h}_v A (t_{sat} - t_s)$$

$$Q_v = 4264.7 \times \pi \times 0.1 \times 2.5(100 - 80)$$
$$= 66989.75 \text{ W}$$

Steam condensation rate, $\dot{m}_v = \dfrac{Q}{h_{fg}} = \dfrac{66989.75}{2285 \times 10^3}$

$$\dot{m}_v = 0.0298 \text{ kg/s}$$

Now, for horizontal position of tube, heat transfer coefficient is given by,

$$\bar{h}_h = \frac{0.725}{0.943}\left(\frac{l}{D}\right)^{0.25} \times \bar{h}_v$$

or $$\bar{h}_h = \frac{0.725}{0.943}\left(\frac{2.5}{0.1}\right)^{0.25} \times 4264.7$$

$$\bar{h}_h = 7331.61 \text{ W/m}^2\text{K}$$

Heat transfer rate, in horizontal position

$$Q_h = 7331.61 \times (\pi \times 0.1 \times 2.5) \times (100 - 80)$$
$$Q_h = 115164.78 \text{ W}$$

Steam condensation rate, in horizontal position,

$$\dot{m}_h = \frac{115164.78}{2285 \times 10^3} = 0.0512 \text{ kg/s}$$

$\therefore$ Percentage change in condensation rate $= \dfrac{0.0512 - 0.0298}{0.0298} \times 100 = 71.81\%$

5. (e)

The horizontal component of the resultant hydrostatic force acting on the cylinder is the horizontal force on the projected area of the curved surface on a vertical plane.

F_h = Resultant pressure force on projected OA of the curved surface.

$F_h = \rho g A y_c$

The projected area of curved surface on the vertical plane OA is

$$A = \text{Distance OA} \times \text{Cylinder length}$$
$$A = 3 \times 4 = 12 \text{ m}^2$$

Depth of centroid of OA, $y_c = 2.5 + \dfrac{3}{2} = 4 \text{ m}$

$\therefore$ $$F_h = 1000 \times 9.81 \times 12 \times 4 = 470.88 \text{ kN}$$

The line of action of F_h is

$$y_{ph} = y_c + \frac{I_c \sin^2\alpha}{A y_c}$$

$\sin\alpha = \sin 90° = 1$

$$I_c = \frac{bd^3}{12} = \frac{4\times 3^3}{12} = 9 \text{ m}^4$$

$$\therefore \quad y_{ph} = 4 + \frac{9\times 1}{12\times 4} = 4.1875 \text{ m}$$

Thus the horizontal component of the resultant hydrostatic force is 470.88 kN and it acts (5.5 – 4.1875) = 1.3125 m vertically above A.

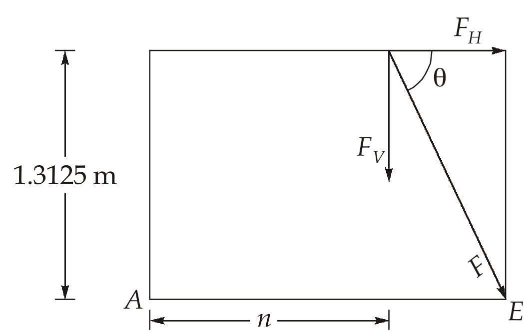
The vertical component of the resultant hydrostatic pressure is the weight of water above the face AB of the curved surface.

$F_v = \rho g$ [Volume of rectangular portion OBCD + Volume of region OAB]

$$F_v = 9.81\times 1000\left[2.5\times 3\times 4 + \frac{1}{4}\pi(3)^2\times 4\right]$$

$$F_v = 1000 \times (294.3 + 277.371)$$

or, $$F_v = 571.6712 \text{ kN}$$



The line of action of F_v may be obtained by taking moments of it's two components about the line OA.

$$571.6712 \times n = 294.3\times\frac{3}{2} + 277.371\times\frac{4\times 3}{3\pi}$$

where the term $\frac{4\times 3}{3\pi} = \frac{4r}{3\pi}$ represents the distance of centre of gravity from either straight line.

On solving above equation

$$n = 1.38997 \text{ m}$$

Thus the vertical component of the resultant hydrostatic pressure is 571.6712 kN and it acts 1.38997 m to the right of A.

The resultant hydrostatic pressure force,

$$F = \sqrt{F_h^2 + F_v^2} = \sqrt{(470.88)^2 + (571.6712)^2}$$

$$F = 740.6321 \text{ kN}$$

If θ is the angle of inclination of F to the horizontal

$$\tan\theta = \frac{F_v}{F_h} = \frac{571.6712}{470.88}$$

$$\theta = 50.522°$$

The distance of point E, where the resultant strikes the surface is

$$= 1.38997 + \frac{1.3125}{\tan\theta}$$

$$= 1.38997 + 1.08109 = 2.471 \text{ m from point A}$$

6. (a)

Given : $\in_1 = 0.7, \in_2 = 0.5, T_1 = 1200$ K, $T_2 = 800$ K

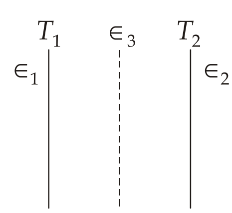
The rate of heat interchange between the two plates is,

$$Q_{1\text{-}2} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\frac{1}{\in_1} + \frac{1}{\in_2} - 1}$$

$$\therefore \quad Q_{1\text{-}2} = \frac{5.67 \times 10^{-8}\left(1200^4 - T_3{}^4\right)}{\frac{1}{0.7} + \frac{1}{0.5} - 1}$$

$$Q_{1\text{-}2} = 38849.5 \text{ W/m}^2$$

When a radiation shield with emissivity '$\in_3$' on both sides is placed between the two plates, then



$$Q_{\text{with shield}} = 0.3 Q_{\text{without shield}}$$

$$\therefore \quad \frac{5.67 \times 10^{-8}\left(1200^4 - 800^4\right)}{\left(\frac{1}{0.7} + \frac{1}{0.5} - 1\right) + \left(\frac{2}{\in_3} - 1\right)} = 0.3 \times 38849.5$$

$$\therefore \quad 1.428 + \frac{2}{\in_3} = 8$$

or $$\in_3 = 0.3$$

Now, under steady state conditions,

$$Q_{1\text{-}2} = Q_{1\text{-}3}$$

$\therefore$ $$0.3 \times 38849.5 = \frac{5.67 \times 10^{-8}\left(1200^4 - (T_3)^4\right)}{\left(\frac{1}{0.7} + \frac{1}{0.5} - 1\right)}$$

or $$11654.85 = 1.5 \times 10^{-8}\left(1200^4 - T_3^4\right)$$

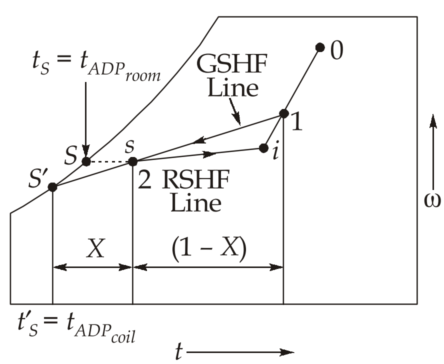
$\therefore$ $$T_3^4 = 1200^4 - \frac{11654.85}{1.5 \times 10^{-8}}$$

$\therefore$ $$T_3 = \left(1.296 \times 10^{12}\right)^{1/4}$$

$$T_3 = 1067.09 \text{ K}$$

6. (b)

(i)



From Psychrometric chart,

at 43°C DBT and 27.5°C WBT

$$h_0 = 87.5 \text{ kJ/kgd.a.}$$
$$\omega_0 = 17.0 \text{ gw.v/kgd.a.}$$
$$v_0 = 0.922 \text{ m}^3\text{/kgd.a.}$$

at 25°C DBT and 50% RH

$$h_i = 50.8 \text{ kJ/kgd.a.}$$
$$\omega_i = 10 \text{ gw.v/kgd.a.}$$

Condition of air entering the cooling coil,

$$h_1 = 0.75 \times h_i + 0.25 \times h_0$$
$$= 0.75 \times 50.8 + 0.25 \times 87.5 = 59.975 \text{ kJ/kgd.a.}$$
$$\omega_1 = 0.75 \times \omega_i + 0.25 \times \omega_0$$
$$= 0.75 \times 10 + 0.25 \times 17 = 11.75 \text{ gw.v/kgd.a.}$$
$$t_1 = 0.75 \times t_i + 0.25 \times t_0$$
$$= 0.75 \times 25 + 0.25 \times 43$$
$$= 29.5°\text{C}$$

(a)

We know

Room sensible heat (RSH) = $0.0204\,(t_1 - t_2) \times cmm$

Room latent heat (RLH) = $50 \times \text{cmm}(\omega_i - \omega_2)$

Now,
$$\frac{RSH}{RLH} = \frac{0.0204(25 - t_2)}{50(0.01 - \omega_2)} = \frac{40}{10} \quad \text{...(i)}$$

Relations for bypass factor

$$\frac{t_2 - t_{ADP}}{t_1 - t_{ADP}} = 0.15$$

Substitution $t_{ADP} = 12.8°\text{C}$

$$\frac{t_2 - 12.8}{29.5 - 12.8} = 0.15$$

$\Rightarrow$ $t_2 = 15.305°\text{C}$

We can write an other relation for bypass factor in terms of specific humidity

$$\frac{\omega_2 - \omega_{ADP}}{\omega_1 - \omega_{ADP}} = \frac{\omega_2 - \omega_{ADP}}{0.01175 - \omega_{ADP}} = 0.15 \quad \text{(ii)}$$

From Psychrometric chart $\omega_{ADP} = 9.5$ gw.v/kgd.a.

From equation (ii)

$$\frac{\omega_2 - 0.0095}{0.01175 - 0.0095} = 0.15$$

$\Rightarrow$ $\omega_2 = 0.0098375$ kgw.v/kgd.a.

$\therefore$ Condition of air leaving the coil is $t_2 = 15.3°\text{C}$ and $\omega_2 = 9.84$ gw.v/kgd.a.

(b)

Dehumidified air quantity,

$$(\text{cmm})_d = \frac{RSH}{0.0204(t_1 - t_2)}$$

$$(\text{cmm})_d = \frac{40}{0.0204(25 - 15.305)} = 202.247$$

(c)

From Psychrometric chart

Specific volume of supply air,

$$v_2 = 0.83 \text{ m}^3/\text{kgd.a.}$$

Mass flow rate of supply air,

$$\dot{m}_{a_s} = \frac{(cmm)_d}{60 \times v_2} = \frac{202.247}{60 \times 0.83} = 4.0612 \text{ kgd.a./sec}$$

Mass flow rate of fresh air, $\dot{m}_{a_0} = 0.25\dot{m}_{a_s} = 0.25 \times 4.0612 = 1.0153$ kgd.a./sec

Volume flow rate of fresh air,

$$Q_{V_0} = \dot{m}_{a_0} \times v_0 \times 60$$

$$Q_{V_0} = 1.0153 \times 0.922 \times 60$$

$$Q_{V_0} = 56.166 \text{ cmm}$$

(d)

Outside air total heat, $\text{OATH} = \dot{m}_{a_0}(h_i - h_0)$

$$= 1.0153(87.5 - 50.8)$$

$$= 37.262 \text{ kW}$$

Total refrigeration load on the air conditioning plant

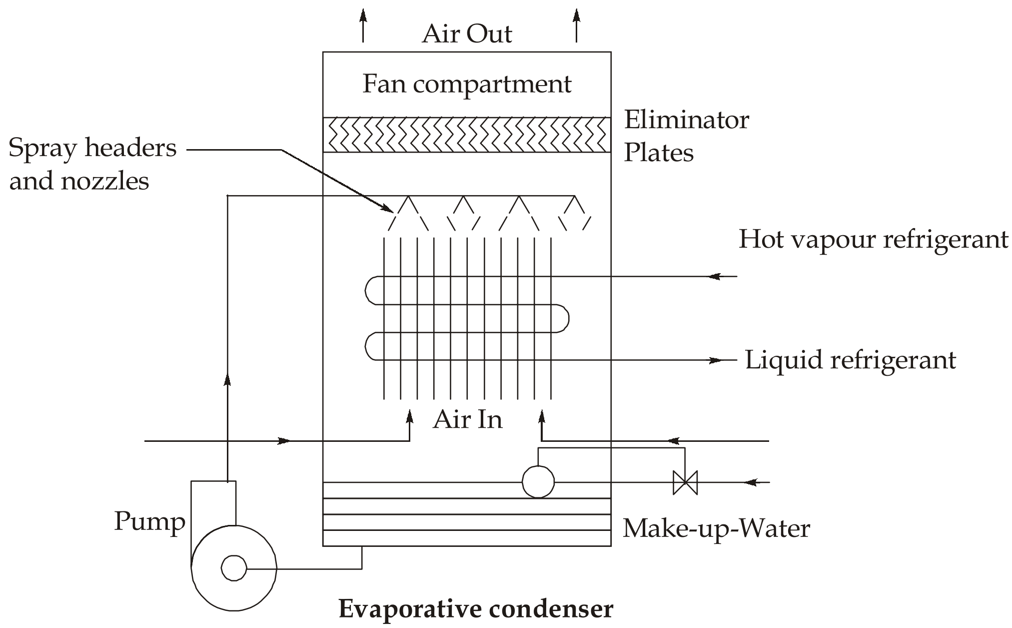
Grand total heat (GTH) = RTH + OATH

GTH = (RSH + RLH) + OATH

= 40 + 10 + 37.262

GTH = 87.262 kW

(ii)



Evaporative condenser

Figure shows the schematic diagram of an evaporative condenser. In an evaporative condenser, the refrigerant first rejects its heat to water and then water rejects its heat to air, mainly in the form of evaporated water. Air leaves with high humidity as in a cooling tower. Thus an evaporative condenser combines the functions of condenser and cooling tower.

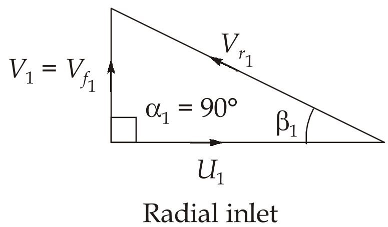
Evaporative condensers are commonly used on large ammonia plants as they are found to be cheaper. Such condensers require a larger amount of the refrigerant charge due to the longer length of the refrigerant piping. But in the case of ammonia systems, this is immaterial since the refrigerant is quite cheap.

6. (c)

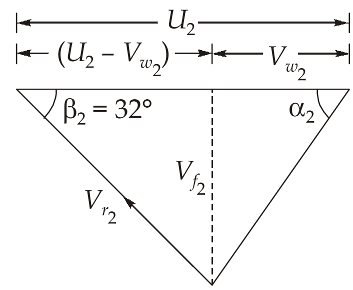
Given : Speed, $N = 800$ rpm; Outer diameter, $D_2 = 0.6$ m;

Discharge, $Q = \dfrac{10000}{1000 \times 60} = 0.167\ \text{m}^3/\text{s}$; $H_m = 35$ m; Inner diameter, $D_1 = 0.2$ m;

$\beta_2 = 32°$; Area of flow = $0.06\ \text{m}^2$



Inlet velocity triangle



Outlet velocity triangle

$$V_{f_1} = V_{f_2} = \frac{Q}{\text{Area}} = \frac{0.167}{0.06} = 2.7833 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 800}{60} = 25.132 \text{ m/s}$$

and
$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 800}{60} = 8.3775 \text{ m/s}$$

(i)

From the inlet velocity triangle,

$$\tan\beta_1 = \frac{V_{f_1}}{U_1} = \frac{2.7833}{8.3775}$$

or,
$$\beta_1 = 18.378°$$

(ii)

From outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f_2}}{\left(U_2 - V_{w_2}\right)} = \tan 32° = 0.6248$$

or,
$$0.6248\left(25.132 - V_{w_2}\right) = 2.7833$$

or,
$$15.7042 - 0.6248 V_{w_2} = 2.7833$$

or,
$$V_{w_2} = 20.68 \text{ m/s}$$

$$\text{Manometric efficiency } (\eta_m) = \frac{gH_m}{V_{w_2} \cdot U_2}$$

$$= \frac{9.81 \times 35}{20.68 \times 25.132} = 0.66063 = 66.06\%$$

(iii)

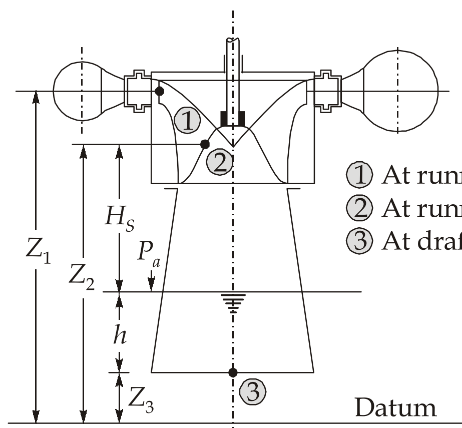
Minimum starting speed (N_m)

We know,

$$N_m = \frac{60\sqrt{2gH_m}}{\pi\sqrt{(D_2^2 - D_1^2)}} = \frac{60 \times \sqrt{2 \times 9.81 \times 35}}{\pi\sqrt{(0.6^2 - 0.2^2)}}$$

$$= 884.277 \text{ rpm}$$

7. (a)



Given : Diameter of penstock, D_P = 7 m; Speed, N = 140 rpm; Power, P = 64700 kW; Flow rate, Q = 120 m^3/s; Hydraulic efficiency, η_H = 0.94; Mean diameter of turbine at entry, D = 4.5 m; Mean blade width at entry, B = 1.2 m; Entry diameter of draft tube, D_t = 4.6 m; Velocity in tail race, V_3 = 2.5 m/s; Static pressure head at entry of runner $\left(\frac{P_1}{\rho g}\right) = 60.4$ m

$$z_1 - (z_3 + h) = 4 \text{ m}$$

$$\text{Loss in draft tube } (h_f) = 0.35\left(\frac{V_2^2}{2g}\right)$$

$$H_s = 2.5 \text{ m}$$

(i)

Net or effective head, $H = H_G - h_f$

$$H = \left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) - \left(z_3 + \frac{V_3^2}{2g} + h\right) \quad \text{...(i)}$$

$$V_1 = \frac{Q}{\frac{\pi}{4} \times (D_p)^2} = \frac{120 \times 4}{\pi \times (7)^2} = 3.11813 \text{ m/s}$$

and thus by substitution, we get

$$H = \left[60.4 + \frac{(3.11813)^2}{19.6} + 4 - \frac{(2.5)^2}{19.6}\right]$$

or $$H = 64.5771 \text{ m}$$

The overall efficiency is given by

$$\eta_0 = \frac{P}{\rho g Q H} = \frac{64700 \times 10^3}{9810 \times 120 \times 64.5771}$$

or $$\eta_0 = 0.851 \text{ or } 85.1\%$$

(ii)

Neglecting the vane thickness, the velocity of flow at inlet

$$V_f = \frac{Q}{\pi B D} = \frac{120}{\pi \times 1.2 \times 4.5} = 7.0735 \text{ m/s}$$

$$U = \frac{\pi D N}{60} = \frac{\pi \times 4.5 \times 140}{60}$$

or $$U = 32.9867 \text{ m/s}$$

We know,

$$\text{Hydraulic efficiency, } \eta_H = \frac{V_w U}{gH}$$

$$0.94 = \frac{V_w \times 32.9867}{9.81 \times 64.5771}$$

or $$V_w = 18.0524 \text{ m/s}$$

The direction of flow relative to the runner at inlet

$$\tan\theta = \frac{V_f}{U - V_w} = \frac{7.0735}{32.9867 - 18.0524}$$

or $$\theta = 25.344° \text{ or } 154.655°$$

(iii)

The pressure head at entry to the draft tube

Applying Bernoulli's equation between points 2 and 3.

$$\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 = \frac{P_3}{\rho g}+\frac{V_3^2}{2g}+z_3+h_f'$$

But $$\frac{P_3}{\rho g} = \frac{P_{atm}}{\rho g}+h$$

Since $(z_2 - z_3 - h) = H_s$, the height of runner exit above tail race level, thus

$$\frac{P_2}{\rho g} = \frac{P_{atm}}{\rho g}-\left[H_s+\frac{V_2^2-V_3^2}{2g}\right]+h_f' \quad \text{...(ii)}$$

where h_f' = Head loss in draft tube

$$h_f' = 0.35\left(\frac{V_2^2}{2g}\right)$$

$$V_2 = \frac{Q}{\frac{\pi}{4}\times(D_t)^2} = \frac{120\times 4}{\pi\times(4.6)^2} = 7.2206 \text{ m/s}$$

then, $$h_f' = 0.35\times\frac{(7.2206)^2}{19.6} = 0.93102 \text{ m}$$

Thus by substitution, we get

$$\frac{P_2}{\rho g}(gauge) = -\left[2.5+\frac{(7.2206)^2-(2.5)^2}{19.6}\right]+0.93102$$

or $$\frac{P_2}{\rho g} = -3.91015 \text{ m (gauge)}$$

7. (b)

Let suffix 'h' and 'c' represent hot and cold fluid respectively.

Now, from energy balance,

$$\dot{m}_h c_h\left(T_{h_1}-T_{h_2}\right) = \dot{m}_c c_c\left(T_{c_2}-T_{c_1}\right)$$

$$2 \times 1.05(250 - 150) = 1.4 \times 4.18 \times \left(T_{c_2}-125\right)$$

$\therefore$ $$T_{c_2} = 160.8°\text{C}$$

$\therefore$ $$P = \frac{160.8-125}{250-125} = 0.28$$

$$R = \frac{250-150}{160.8-125} = 2.8$$

Using these parameters with figure, we read correction factor $F = 0.8$.

Now, for counter flow,

$$\theta_1 = T_{h_1} - T_{c_2} = 250 - 160.8 = 89.2°C$$

$$\theta_2 = T_{h_2} - T_{c_1} = 150 - 125 = 25°C$$

$$\therefore \quad \text{LMTD, } \theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{89.2 - 25}{\ln\left(\frac{89.2}{25}\right)} = 50.47°C$$

Now, from energy balance,

$$Q = FUA\theta_m$$

$$\therefore \quad UA = \frac{Q}{F\theta_m} = \frac{2 \times 1.05 \times (250 - 150)}{0.8 \times 50.47}$$

$$= 5.2 \text{ kJ/sK}$$

Thermal capacity rates of hot fluid and cold fluid are,

$$C_h = \dot{m}_h c_h = 2 \times 1.05 = 2.1 \text{ kJ/sK} = C_{min}$$

$$C_c = \dot{m}_c c_c = 1.4 \times 4.18 = 5.85 \text{ kJ/sK} = C_{max}$$

$$\therefore \quad \text{NTU} = \frac{UA}{C_{min}} = \frac{5.2}{2.1} = 2.47$$

$$\text{Effectiveness, } \in = \frac{T_{h_2} - T_{h_1}}{T_{h_1} - T_{c_1}} = \frac{250 - 150}{250 - 125} = 0.8$$

Now, with effectiveness remains constant, the heat transfer with new set of operating conditions is

$$Q = \in C_{min}\left(T_{h_1} - T_{c_1}\right)$$

$$Q = 0.8 \times 2.1 \times (250 - 60) = 319.2 \text{ kJ/s}$$

From energy balance,

$$Q = \dot{m}_c c_c \left(T_{c_2} - T_{c_1}\right)$$

$$319.2 = 0.5 \times 4.18\left(T_{c_2} - 60\right)$$

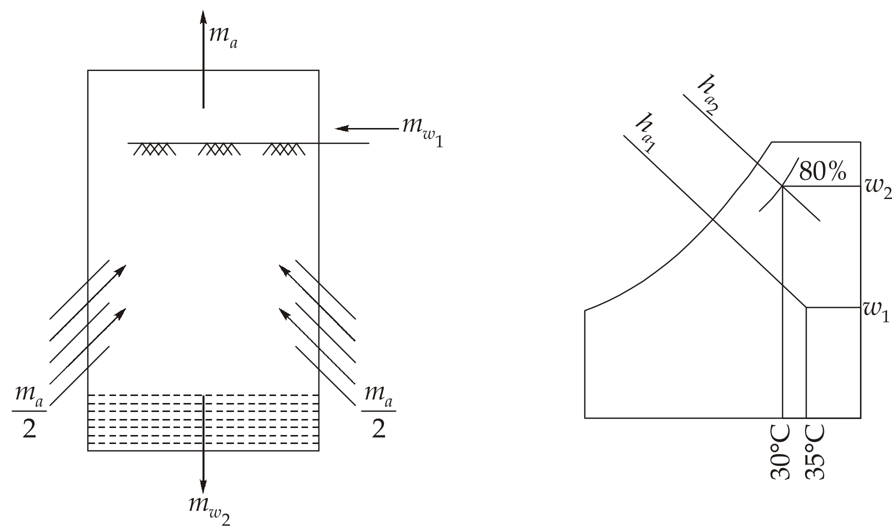
$$\therefore \quad T_{c_2} = 152.72 + 60$$

$$T_{c_2} = 212.72°C$$

7. (c)

(i)

Given : Water flow rate, m_{w_1} = 1200 kg/min; T_{w_1} = 35°C; T_{w_2} = 30°C



From Psychrometric chart:

$$h_{a_1} = 76.4 \text{ kJ/kg}; \; h_{a_2} = 85 \text{ kJ/kg}$$

$$w_1 = 16 \text{ grams/kg of dry air}; \; w_2 = 21.5 \text{ grams/kg of dry air}$$

$$v_{s_1} = 0.895 \text{ m}^3\text{/kg}$$

Energy balance:

Heat lost by the water = Heat gained by air

$$m_{w_1} h_{w_1} - m_{w_2} h_{w_2} = \frac{V}{v_{s_1}}\left(h_{a_2} - h_{a_1}\right)$$

where V is the volume of free air circulated per minute.

$$\therefore \quad m_{w_1} h_{w_1} - \left[m_{w_1} - \frac{V}{v_{s_1}}(w_2 - w_1)\right] h_{w_2} = \frac{V}{v_{s_1}}\left(h_{a_2} - h_{a_1}\right)$$

$$\therefore \quad m_{w_1}\left(h_{w_1} - h_{w_2}\right) + h_{w_2}\frac{V}{v_{s_1}}(w_2 - w_1) = \frac{V}{v_{s_1}}\left(h_{a_2} - h_{a_1}\right)$$

$$\Rightarrow \quad m_{w_1} \times c_{p_w}\left(T_{w_1} - T_{w_2}\right) + T_{w_2} \times c_{p_w} \times \frac{V}{v_{s_1}}(w_2 - w_1) = \frac{V}{v_{s_1}}\left(h_{a_2} - h_{a_1}\right)$$

Substituting the values in the above equation

$$1200\times 4.2(35-30)+30\times 4.2\times \frac{V}{0.895}(21.5-16)\times \frac{1}{1000} = \frac{V}{0.895}\times (85-76.4)$$

$$25200 + 0.7743V = 9.6089V$$

$$\Rightarrow \quad V = \frac{25200}{8.8346} = 2852.421\ \text{m}^3/\text{min}$$

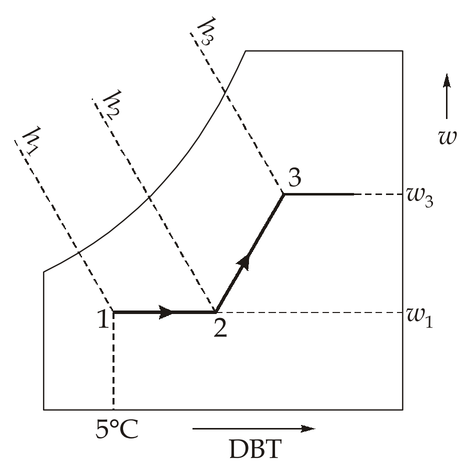
Make up water = Evaporated water carried with air

$$= \frac{V}{v_{s_1}}(w_2 - w_1) = \frac{2852.421}{0.895}(21.5-16)\times \frac{1}{1000}$$

Make up water = 17.528 kg/min

(ii)

Given : Air flow rate = 100 m^3/min; Heating capacity of coil = 42.7 kW; Flow rate of steam = 50 kg/hr



Mark the point 1 on Psychrometric chart as DBT and WBT are known

So, from Psychrometric chart

$$v_1 = 0.792\ \text{m}^3/\text{kg}$$

$$w_1 = 3.5\ \text{gm/kg of dry air}$$

$$h_1 = 13.82\ \text{kJ/kg}$$

The mass of air passed through the system is given as

$$m_a = \frac{100\times 60}{0.792} = 7575.75\ \text{kg/hr} = 2.104\ \text{kg/sec}$$

$$h_2 = h_1 + \frac{42.7}{2.104} = 13.82 + 20.2946$$

$$h_2 = 34.114 \text{ kJ/kg}$$

Draw a horizontal line through point 1 till it cuts line h_2 and then mark the point 2.

$$w_3 = w_2 + \frac{50}{7575.75} \times 1000$$

$$w_3 = w_1 + \frac{50}{7575.75} \times 1000 \quad [\text{as } w_2 = w_1]$$

$$w_3 = 3.5 + 6.6 = 10.1 \text{ gm/kg of dry air}$$

$$h_3 = h_2 + \frac{50}{7575.75} \times h_s$$

$$h_3 = 34.144 + \frac{50}{7575.75} \times 2691 = 51.9 \text{ kJ/kg}$$

As w_3 and h_3 are known, mark the point '3' and read the DBT and WBT from the Psychrometric chart.

$$\text{DBT} = 27°\text{C and WBT} = 19°\text{C}$$

8. (a)

Given : $H = 30$ m; $N = 170$ rpm; $P = 25$ MW; $D_1 = 500$ cm $= 5$ m; $D_h = 250$ cm $= 2.5$ m; $\eta_0 = 0.94$; $\eta_h = 0.96$

We know,

$$\text{Overall efficiency, } \eta_0 = \frac{\text{Power}}{\rho g Q H}$$

or,
$$P = \eta_0 \times \rho g \times Q \times H$$

or,
$$Q = \frac{P}{\eta_0 \times \rho g \times H} = \frac{25000 \times 10^3}{0.94 \times 9810 \times 30}$$

or,
$$Q = 90.3695 \text{ m}^3/\text{s}$$

Also, we can write,

$$Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f_1}$$

$$90.3695 = \frac{\pi}{4}\left(5^2 - 2.5^2\right) \times V_{f_1}$$

or,
$$V_{f_1} = 6.13664 \text{ m/s}$$

Consider a section at the tip of the blade.

$$V_{f_1} = V_{f_2} = 6.13664$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 5 \times 170}{60} = 44.5058 \text{ m/s}$$

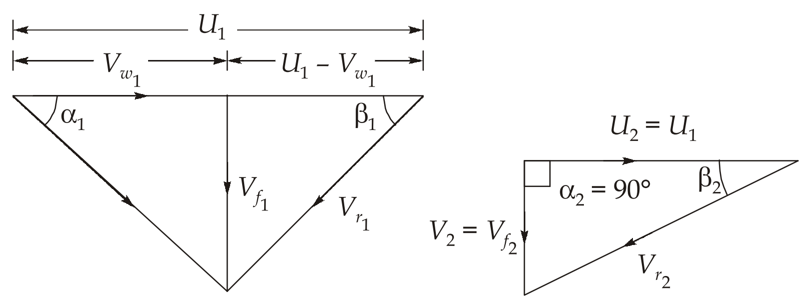
Hydraulic efficiency, $\eta_h = \dfrac{V_{w_1} U_1}{gH}$

$$0.96 = \frac{V_{w_1} \times 44.5058}{9.81 \times 30}$$

or, $V_{w_1} = 6.34811$ m/s

Whirl velocity at inlet at the tip of the blade $V_{w_1} = 6.34811$ m/s.

Since $V_{w_1} < U_1$, the inlet velocity triangle is an acute-angled triangle.



Inlet at tip Outlet at tip

From the inlet velocity triangle

$$\tan\alpha_1 = \frac{V_{f_1}}{V_{w_1}} = \frac{6.13664}{6.34811}$$

or, $\alpha_1 = 44.0296°$

Further, $\tan\beta_1 = \dfrac{V_{f_1}}{(U_1 - V_{w_1})} = \dfrac{6.13664}{44.5058 - 6.34811}$

or, $\beta_1 = 9.13625°$

Considering the outlet velocity triangle at the tip of the blade,

$$U_2 = U_1 = 44.5058 \text{ m/s}$$

$$V_{f_2} = V_{f_1} = 6.13664 \text{ m/s}$$

$$\tan\beta_2 = \frac{V_{f_2}}{U_2} = \frac{6.13664}{44.5058}$$

or, $\beta_2 = 7.8506°$

8. (b)

Given : Net head (H) = 200 m; Speed (N) = 520 rpm; Power (P) = 1500 kW; Bucket angle (β_2) = 170°; C_V = 0.96; Bucket friction coefficient (K) = 0.92; Friction and windage losses (H_{fw}) = 7% of velocity head; Pitch diameter of runner (D) = 1.4 m

$$V_1 = C_V\sqrt{2gH} = 0.96\sqrt{19.6\times 200} = 60.1055 \text{ m/s}$$

or, $$\frac{V_1^2}{2g} = \frac{(60.1055)^2}{19.6} = 184.32 \text{ m}$$

(a) Head loss in the nozzle, $H_N = 200 - 184.32 = 15.68$ m

(b) Friction and windage losses $= H_{fw} = \frac{7}{100}\times 184.32 = 12.9024$ m

$$U = \frac{\pi DN}{60} = \frac{\pi\times 1.4\times 520}{60} = 38.118 \text{ m/s}$$

Relative velocity at inlet $= V_{r_1} = (V_1 - U) = 60.1055 - 38.118$

or, $$V_{r_1} = 21.9875 \text{ m/s}$$

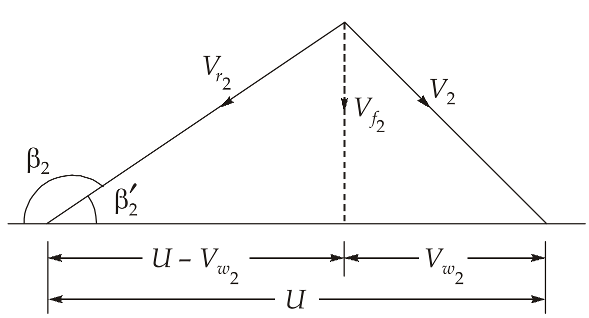
Relative velocity at exit $= V_{r_2} = kV_{r_1}$

or $$V_{r_2} = 0.92 \times 21.9875 = 20.2285 \text{ m/s}$$

(c) Energy loss in buckets (H_b) $= \frac{V_{r_1}^2}{2g} - \frac{V_{r_2}^2}{2g}$

or $$H_b = \frac{1}{19.6}\left((21.9875)^2 - (20.2285)^2\right)$$

$$H_b = 3.7886 \text{ m}$$



Outlet velocity triangle

From outlet velocity triangle:

$$V_{r_2}\cos\beta_2' = 20.2285 \times \cos 10° = 19.9211 \text{ m/s}$$

$$V_{f_2} = V_{r_2}\sin\beta_2' = 20.2285 \times \sin(10°) = 3.5126 \text{ m/s}$$

Since, $V_{r_2}\cos\beta_2' < U$, the outlet velocity triangle is an acute-angled triangle

Hence, $$(U - V_{w_2}) = V_{r_2}\cos\beta_2' = 19.9211 \text{ m/s}$$

or, $$V_{w_2} = U - 19.9211 = 38.118 - 19.9211$$

or, $$V_{w_2} = 18.1969 \text{ m/s}$$

From velocity triangle,

$$V_2^2 = \left(V_{w_2}\right)^2 + \left(V_{f_2}\right)^2$$

or, $$V_2 = \sqrt{(18.1969)^2 + (3.5126)^2}$$

$$V_2 = 18.5328 \text{ m/s}$$

(d) Velocity head of discharge at the exit = $\dfrac{V_2^2}{2g} = \dfrac{(18.5328)^2}{19.6} = 17.523$ m

Energy Balance:

Input head = 200 m	Losses	(m)
	At nozzle	15.68
	Friction and windage	12.9024
	At buckets	3.7886
	Discharge energy head	17.523
		49.894

Shaft energy = Input energy – Total losses
= 200 – 49.894 = 150.106 m

Efficiencies:

$$\text{Hydraulic efficiency } (\eta_H) = \frac{[200 - (15.68 + 3.7886 + 17.523)]}{200} = \frac{163.0084}{200}$$

or, $$\eta_H = 0.81504 = 81.50\%$$

$$\text{Mechanical efficiency } (\eta_m) = \frac{163.0084 - 12.9024}{163.0084} = 0.92084$$

or, $$\eta_m = 92.084\%$$

$$\text{Overall efficiency } (\eta_0) = \frac{(200 - 49.894)}{200} = 0.75053$$

or, $$\eta = 75.053\%$$

8. (c)

Given : $u = \frac{y^3}{3} + 4x - x^2 y\,;\quad v = xy^2 - 4y - \frac{x^3}{3}$

(a)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{y} = (4 - 2xy) + (2xy - 4) = 0$$

The given velocity components satisfy the continuity equation and thus represent a physically possible flow.

The differential $d\psi$ for the stream function is,

$$d\psi = \frac{\partial \psi}{\partial x}\cdot dx + \frac{\partial \psi}{\partial y}\cdot dy$$

$$= -v\cdot dx + u\cdot dy$$

$$= -\left(xy^2 - 4y - \frac{x^3}{3}\right)\cdot dx + \left(\frac{y^3}{3} + 4x - x^2 y\right) dy$$

$$d\psi = \frac{x^3}{3} dx + \frac{y^3}{3}\cdot dy + 4d(xy) - d\left(\frac{x^2 y^2}{2}\right)$$

Integrating on both sides

$$\psi = \frac{x^4}{12} + \frac{y^4}{12} + 4xy - \frac{x^2 y^2}{2} \quad \text{...(i)}$$

(b)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= \left(\frac{y^3}{3} + 4x - x^2 y\right)(4 - 2xy) + \left(xy^2 - 4y - \frac{x^3}{3}\right)(y^2 - x^2)$$

∴ Acceleration in the x-direction at $x = 2$ and $y = 3$

$$a_{x\,(2,\,3)} = \left(\frac{(3)^3}{3} + 4\times 2 - 2^2 \times 3\right)(4 - 2\times 2\times 3) + \left(2\times 3^2 - 4\times 3 - \frac{2^3}{3}\right)(3^2 - 2^2)$$

or $$a_{x\,(2,\,3)} = (5) \times (-8) + 3.333 \times 5$$

$$a_{x\,(2,\,3)} = -40 + 16.67 = -23.33 \text{ m/s}^2$$

Further,

$$a_y = u\frac{\partial y}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= \left(\frac{y^3}{3} + 4x - x^2 y\right)\left(y^2 - x^2\right) + \left(xy^2 - 4y - \frac{x^3}{3}\right)(2xy - 4)$$

$$a_{y\,(2,\,3)} = 5 \times (3^2 - 2^2) + 3.33 \times (2 \times 2 \times 3 - 4)$$

or

$$a_{y\,(2,\,3)} = 25 + 26.664 = 51.664 \text{ m/s}^2$$

Thus, the resultant acceleration,

$$a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{(-23.33)^2 + (51.664)^2} = 56.6886 \text{ m/s}^2$$

(c) From equation (i),

$$\psi_{(2,3)} = \frac{2^4 + 3^4}{12} + 4 \times 2 \times 3 - \frac{2^2 \times 3^2}{2} = 14.0833 \text{ m}^3/\text{s}$$

$$\psi_{(3,4)} = \frac{3^4 + 4^4}{12} + 4 \times 2 \times 4 - \frac{3^2 \times 4^2}{2} = 4.0833 \text{ m}^3/\text{s}$$

Hence the flow between the streamlines through (2, 3) and (3, 4).

$$= 14.0833 - 4.0833 = 10 \text{ m}^3/\text{s}$$

(d) Rotation (angular velocity)

$$w_x = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$w_x = \frac{1}{2}\left[\left(y^2 - x^2\right) - \left(y^2 - x^2\right)\right]$$

$$w_x = 0$$

So, the flow is irrotational and velocity potential does exist.

then

$$d\phi = \frac{\partial \phi}{\partial x}\,\partial x + \frac{\partial \phi}{\partial y}\,\partial y$$

$$= -udx - vdy$$

$$= -\left(\frac{y^3}{3} + 4x - x^2 y\right)dx - \left(xy^2 - 4y - \frac{x^3}{3}\right)dy$$

$$d\phi = -4x\,dx + 4y\,dy - \left(\frac{y^3}{3} \times dx + xy^2\cdot dy\right) + \left(\frac{x^3}{3} \times dy + x^2 y\,dx\right)$$

$$d\phi = -4xdx + 4ydy - \frac{1}{3}d\left(xy^3\right) + \frac{1}{3}d\left(x^3y\right)$$

Taking integration on both side

$$\phi = -2x^2 + 2y^2 - \frac{xy^3}{3} + \frac{x^3y}{3} \quad \text{...(ii)}$$

(e)

From equation (i) for stream function

$$\frac{\partial\psi}{\partial x} = \frac{x^3}{3} + 4y - xy^2$$

$$\frac{\partial^2\psi}{\partial x^2} = x^2 - y^2$$

$$\frac{\partial\psi}{\partial x} = \frac{y^3}{3} + 4x - yx^2$$

$$\frac{\partial^2\psi}{\partial y^2} = y^2 - x^2$$

$$\therefore \quad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = x^2 - y^2 + y^2 - x^2 = 0$$

Hence, stream function satisfy laplace equation.

From equation (ii) for velocity potential function,

$$\frac{\partial\phi}{\partial x} = 4x + \frac{y^3}{3} - x^2y$$

$$\frac{\partial^2\phi}{\partial x^2} = 4 - 2xy$$

$$\frac{\partial\phi}{\partial y} = -4y + xy^2 - \frac{x^3}{3}$$

$$\frac{\partial^2\phi}{\partial y^2} = -4 + 2xy$$

$$\therefore \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 2 - 2xy - 2 + 2xy = 0$$

Hence, the potential function satisfy Laplace equation.

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