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Detailed Solutions

ESE-2023 Mains Test Series

Civil Engineering Test No : 4

Section A : Design of Concrete and Masonry Structure

Q.1 (a) Solution:

As per IS 456-2000, the design for the limit state of collapse in flexure shall be based on the following assumptions:

- Plane sections normal to the longitudinal axis remain plane after bending.
- The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
- The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoidal, parabola or any other shape which results in prediction of strength in substantial agreement with results of test.

  For design purposes, the compressive strength of the concrete in structure shall be assumed to be 0.67 times the characteristics strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.
- The tensile strength of concrete is ignored.
- The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. For design purpose, the partial safety factor $\gamma_m = 1.15$ shall be applied.
- The maximum strain in the tension reinforcement in the section at failure shall not be less than $\dfrac{f_y}{1.15E_s} + 0.002$

where, f_y = yield strength of steel,

E_s = Modulus of elasticity of steel

Limiting percentage of tension steel:

$$C = T$$

$$0.36 f_{ck} B x_{u,\,\text{lim}} = 0.87 f_y A_{st}$$

$$\Rightarrow \quad \frac{A_{st}}{Bd} = \frac{0.36 f_{ck} x_{u,\text{lim}}}{0.87 f_y \times d}$$

For M30 and Fe500

$$\frac{A_{st}}{Bd} = \frac{0.36 \times 30 \times 0.46d}{0.87 \times 500 \times d}$$

$$\Rightarrow \quad p_{t\ \text{lim}} = \frac{A_{st}}{Bd} \times 100 = \left(\frac{0.36 \times 30 \times 0.46}{0.87 \times 500}\right) \times 100 = 1.142\%$$

Q.1 (b) Solution:

Given, b = 200 mm, d = 350 mm,

A_{sc} = 1245 mm^2, A_{st} = 1600 mm^2

f_{ck} = 20 N/mm^2, f_y = 250 N/mm^2

Stress in compression steel, $f_{sc} = E_s \varepsilon_{sc}$

$$\Rightarrow \quad f_{sc} = 2 \times 10^5 \times 0.0035\left(1 - \frac{d'}{x_u}\right) = 700\left(1 - \frac{50}{x_u}\right)$$

$$C = T$$

$$\Rightarrow 0.36 \times 20 \times 200 \times x_u + 700\left(1 - \frac{50}{x_u}\right) \times 1245 = 0.87 \times 250 \times 1600$$

$$\Rightarrow \quad 1440 x_u + 871500 - \frac{43575000}{x_u} = 348000$$

$$\Rightarrow \quad 1440\, x_u^2 + 523500\, x_u - 43575000 = 0$$

$$\Rightarrow \quad x_u = 69.83 \text{ mm}$$

$$x_{u\ \text{lim}} = 0.53d = 0.53 \times 350 = 185.5 \text{ mm}$$

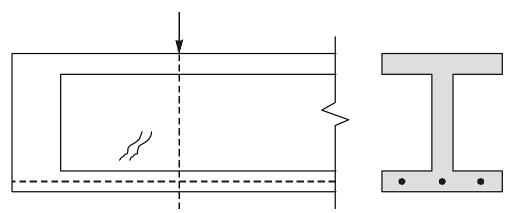
$$\therefore \quad x_u < x_{u\ \text{lim}}$$

$$\therefore \quad f_{sc} = 700\left(1 - \frac{50}{69.83}\right) = 198.78 \text{ N/min}^2$$

$$\therefore \quad M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$
$$= 0.36 \times 20 \times 200 \times 69.83(350 - 0.42 \times 69.83)$$
$$+ 198.78 \times 1245 \times (350 - 50)$$
$$= 106.49 \text{ kN-m}$$

Q.1 (c) Solution:

(i) For beams having relatively high depth and thin webs, subjected to high shear stresses and low bending stresses, the maximum principle tensile stress is located at the neutral axis level at an inclination of 45° to the longitudinal axis of the beam. The resulting cracks are termed as web shear crack or diagonal tension cracks.



Web-shear crack

In beams subjected to both flexural and shear stresses, a biaxial state of combined tension and compression exists. In such a case the flexure crack forms first and due to the increased shear stresses at the tip of the cracks, this flexural crack extends into a diagonal tension crack.

Appropriate shear reinforcement is required to prevent propagation of these cracks.

(ii) Limit state method as the term suggests design the members for limit state condition which is a hypothetical situation. So to incorporate loading at that state, load factor is taken in this method. This concept of load factor is absent in working stress method.

Working stress method emphasises on design of structures on loading upto elastic limit.

1. Limit state method design is more durable than working stress method because WSM is limited only upto elastic limit.
2. Depth of members designed under LSM is lesser than that of by WSM. Thus it significantly reduces weight of member and bring economy in design.
3. Amount of reinforcement required in LSM design is more than that deduced by WSM.
4. Materials in LSM are used way beyond the elastic limit. Thus material are used more meticulously and economically in LSM.

Q.1 (d) Solution:

Anchorage length of bar at simply supported end of beam can be determined by,

Development length, $$L_d \leq \frac{1.30M_u}{V_u} + l_0$$

Anchorage length, $$l_0 \geq L_d - \frac{1.3M_u}{V_u}$$

where, $$L_d = \frac{0.87 \times f_y \times \phi}{4 \times \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940 \text{ mm}$$

$$M_u = f_s A_{st} (d - 0.42x_u)$$

where $$x_u = \frac{f_{st} A_{st}}{0.36 f_{ck} b}$$

$f_{st} = 0.87 f_y$, if the section is balanced/under-reinforced.

$< 0.87 f_y$, if the section is over-reinforced.

Assume $x_u \leq x_{u,lim}$ (balanced/under-reinforced section)

$\therefore$ $$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

where $A_{st} = 3 \times 20\phi = 942 \text{ mm}^2$

$\therefore$ $$x_u = \frac{0.87 \times 415 \times 942}{0.36 \times 20 \times 300} = 157.458 \text{ mm}$$

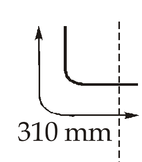
and $x_{u,lim} = 0.48d = 0.48 \times 500 = 240$ mm

$\because$ $x_u < x_{u,lim}$, therefore section is under-reinforced and computed value of x_u is correct.

$\therefore$ $$M_u = 0.87 \times 415 \times 942 \, (500 - 0.42 \times 157.458) \text{ Nmm}$$
$$= 147.56 \text{ kNm}$$

Hence, $$l_0 \geq 940 - \frac{1.3 \times 147.56 \times 10^6}{300 \times 10^3} \geq 300.57 \text{ mm}$$

$\therefore$ Therefore, anchorage length of 310 mm is provided.



Q.1 (e) Solution:

$$\text{Area of the beam section} = 300 \times 600 = 18 \times 10^4 \text{ mm}^2$$

$$\text{Section modulus of section, } Z = \frac{300 \times 600^2}{6} = 18 \times 10^6 \text{ mm}^3$$

$$\text{D.L. moment at mid span} = \frac{4.5 \times 12^2}{8} = 81 \text{ kN-m}$$

$$\text{L.L. moment at mid span} = \frac{7.5 \times 12^2}{8} = 135 \text{ kN-m}$$

(i) Analysis of the end section:

$$\text{Direct stress due to prestressing force} = +\frac{P}{A} = \frac{980 \times 10^3}{18 \times 10^4} = +5.44 \text{ N/mm}^2$$

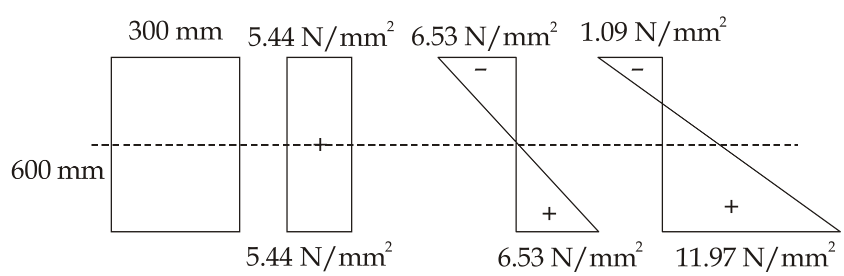
Extreme stress due to eccentricity of the prestressing force

$$= \mp\frac{Pe}{Z} = \mp\frac{980 \times 10^3 \times 120}{18 \times 10^6}$$

$$= \mp 6.53 \text{ N/mm}^2$$

$$\text{Resultant stress at the top edge} = 5.44 - 6.53 = -1.09 \text{ N/mm}^2$$

$$\text{Resultant stress at the bottom edge} = 5.44 + 6.53 = 11.97 \text{ N/mm}^2$$



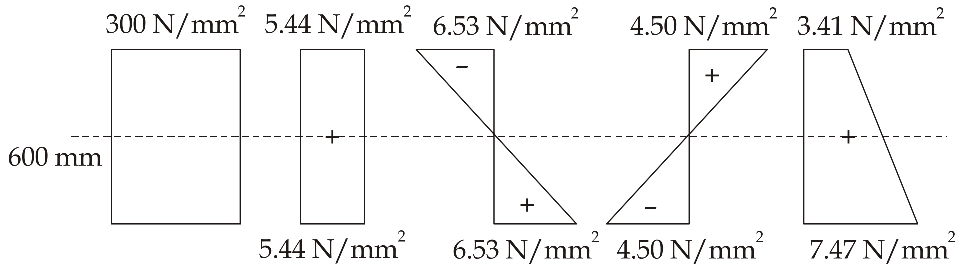
(ii) Analysis of midsection subjected to prestressing force and dead load:

$$\text{Direct stress} = +5.44 \text{ N/mm}^2$$

Extreme stress due to eccentricity of the prestressing force

$$= \mp 6.53 \text{ N/mm}^2$$

$$\text{Extreme stress due to dead load moment} = \pm\frac{M_d}{Z} = \pm\frac{81 \times 10^6}{18 \times 10^6} = \pm 4.5 \text{ N/mm}^2$$



$$\text{Resultant stress at the top edge} = 5.44 - 6.53 + 4.5 = 3.41 \text{ N/mm}^2$$

$$\text{Resultant stress at the bottom edge} = 5.44 + 6.53 - 4.5 = 7.47 \text{ N/mm}^2$$

(iii) Analysis of the mid-section subjected to prestressing force, dead load and live dead

$$\text{Direct stress} = +5.44 \text{ N/mm}^2$$

Extreme stress due to eccentricity of the prestressing force

$$= \mp 6.53 \text{ N/mm}^2$$

Extreme stress due to dead load moment

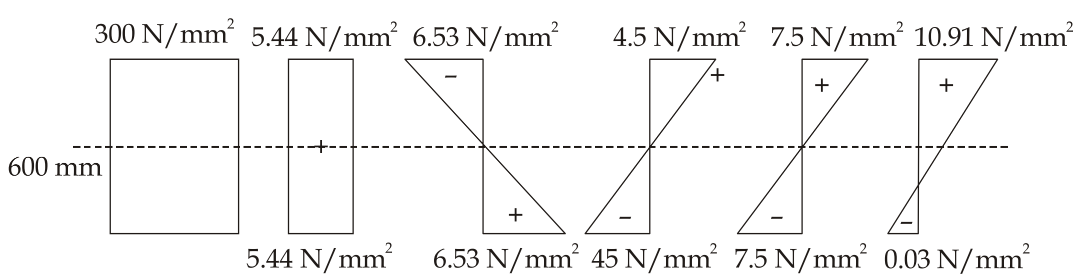
$$= \pm 4.50 \text{ N/mm}^2$$

Extreme stress due to live load moment

$$= \pm\frac{M_l}{Z} = \pm\frac{135\times 10^6}{18\times 10^6} = \pm 7.50 \text{ N/mm}^2$$

$$\text{Resultant stress at the top edge} = 5.44 - 6.53 + 4.5 + 7.5 = +10.91 \text{ N/mm}^2$$

$$\text{Resultant stress at the bottom edge} = 5.44 + 6.53 - 4.5 - 7.5 = -0.03 \text{ N/mm}^2$$



Q.2 (a) Solution:

1. Slab is supported on all four edges

$$L_{y_{cl}} = 7.5 \text{ m}, \qquad L_{x_{cl}} = 5.0 \text{ m}$$

$$\frac{L_{y_{cl}}}{L_{x_{cl}}} = \frac{7.5}{5} = 1.5 < 2$$

∴ Two way slab

2. **Effective depth,** $d = \dfrac{\text{Effective span}}{A \times MF_t} = \dfrac{5000}{26 \times 1.2} = 160.25$ mm

(Using modification factor of 1.2 for preliminary design)

Consider, $d = 170$ mm

$$\text{Width of support} = 400\,\text{mm} < \frac{1}{12}(5000)$$

$$L_{e_x} = L_{cl} + d = 5.0 + 0.17 = 5.17 \text{ m}$$

$$= L_{cl} + w = 5.0 + 0.4 = 5.4 \text{ m}$$

$\therefore$ $L_{e_x} = 5.17$ m

$$\text{Effective depth} = \frac{5170}{26 \times 1.2} = 166.71 \text{ mm}$$

$\therefore$ $d = 170$ mm is OK

Let, overall depth, $D = 170 + 30 = 200$ mm

Also, $L_{ey} = \text{min of} \begin{Bmatrix} 7.5 + 0.17 = 7.67 \text{ m} \\ 7.5 + 0.4 = 7.9 \text{ m} \end{Bmatrix} \text{ss} = 7.67$ m

Now, $r = \dfrac{L_{y_e}}{L_{x_e}} = \dfrac{7.67}{5.17} = 1.48$ say 1.50

3. **Loads,** $DL = 0.2 \times 1 \times 1 \times 25 = 5$ kN/m^2

$LL = 8$ kN/m^2

Finishing load = 1.5 kN/m^2

Total load, w = 14.5 kN/m^2

Factored load, $w_u = 1.5 \times 14.5 = 21.75$ kN/m^2

	α	**Moments (kN-m)**
$M_{ux} \ominus$	0.053	30.82
$M_{ux} \oplus$	0.041	23.84
$M_{uy} \ominus$	0.032	18.60
$M_{uy} \oplus$	0.024	13.95

4. **Moments,** $M_{ux}(-) = \alpha_x(-) \times w_u \times L_{e_x}^2 = 0.053 \times 21.75 \times 5.17^2$

$$= 30.81 \text{ kN-m}$$

5. Effective depth,

$$d = \sqrt{\frac{M_u}{Q \cdot B}} = \sqrt{\frac{30.82 \times 10^6}{0.138 \times 25 \times 1000}}$$

$$d = 94.50 \text{ mm} < 170 \text{ mm} \qquad \therefore \text{ OK}$$

6. Area of sleel, $d_x = 170$ mm, $d_y = 170 - 10 = 160$ mm

$$Ast_x(-) = \frac{0.5 \times f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d_x^2}}\right] B d_x$$

$$= \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 30.81 \times 10^6}{25 \times 1000 \times 170^2}}\right] \times 1000 \times 170$$

$$= 529.8 \text{ mm}^2$$

7. Spacing of reinforcement, $S_b = \dfrac{1000 \times \frac{\pi}{4} \times (10)^2}{529.8} = 148.24 \text{ mm} \approx 140 \text{ mm}$

Similarly,

	α	Moments (kN-m)	Ast $(mm)^2$	dia (mm)	Spacing required (mm)	Spacing provided (mm)
$M_{ux} \ominus$	0.053	30.82	529.8	10	148.24	140
$M_{ux} \oplus$	0.041	23.84	404.6	10	194.12	190
$M_{uy} \ominus$	0.032	18.60	333.7	8	150.63	150
$M_{uy} \oplus$	0.024	13.95	248	8	202.7	200

8. Minimum reinforcement

$$Ast_{min} = \frac{0.12}{100} \times 1000 \times 200 = 240 \text{ mm}^2$$

9. Check for deflection

$$Ast_x \text{ (+) required} = 404.6 \text{ mm}^2$$

$$Ast_x \text{ (+) provided} = \frac{100}{190} \times \frac{\pi}{4} \times (10)^2 = 413.4 \text{ mm}^2$$

$$\text{Percentage provided} = \frac{413.4}{1000 \times 170} \times 100 = 0.24\%$$

$$f_s = 0.58 \times f_y \times \frac{A_{st\,req}}{A_{st\,provided}} = 0.58 \times 415 \times \frac{404.6}{413.4} = 235.6$$

∴ Modification factor for tension reinforcement = 1.6

$$\text{Effective depth, } d = \frac{5170}{26 \times 1.6} = 124 \text{ mm} < 170 \text{ mm} \qquad \therefore \text{Safe}$$

10. Check for shear,

$$V_{u_1} = w_u \times L_x \times \frac{r}{2+r} = 21.75 \times 5 \times \frac{1.5}{2+1.5} = 46.61 \text{ kN}$$

$$\tau_{v_1} = \frac{V_{u_1}}{Bd_x} = \frac{46.61 \times 10^3}{1000 \times 170} = 0.27 \text{ N/mm}^2$$

$$\tau_{v_1} = 0.27 \text{ N/mm}^2 < (\tau_{c\,\min} = 0.29 \text{ N/mm}^2 \text{ for } P_t \le 0.15\%)$$

$\therefore$ Safe

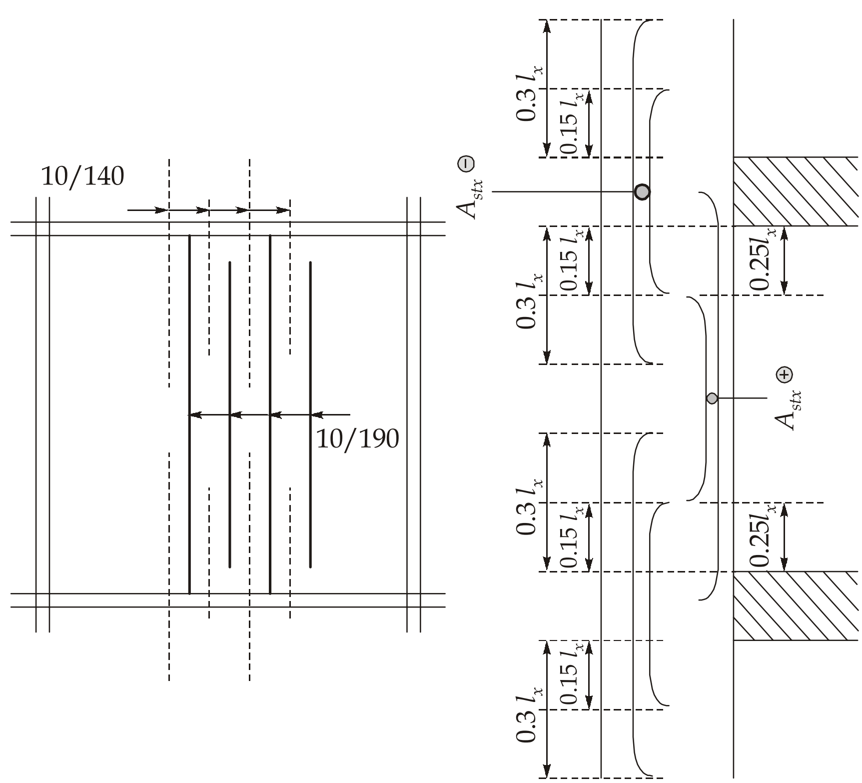
$$V_{u_2} = \frac{w_u \cdot L_x}{3} = \frac{21.75 \times 5}{3} = 36.25 \text{ kN}$$

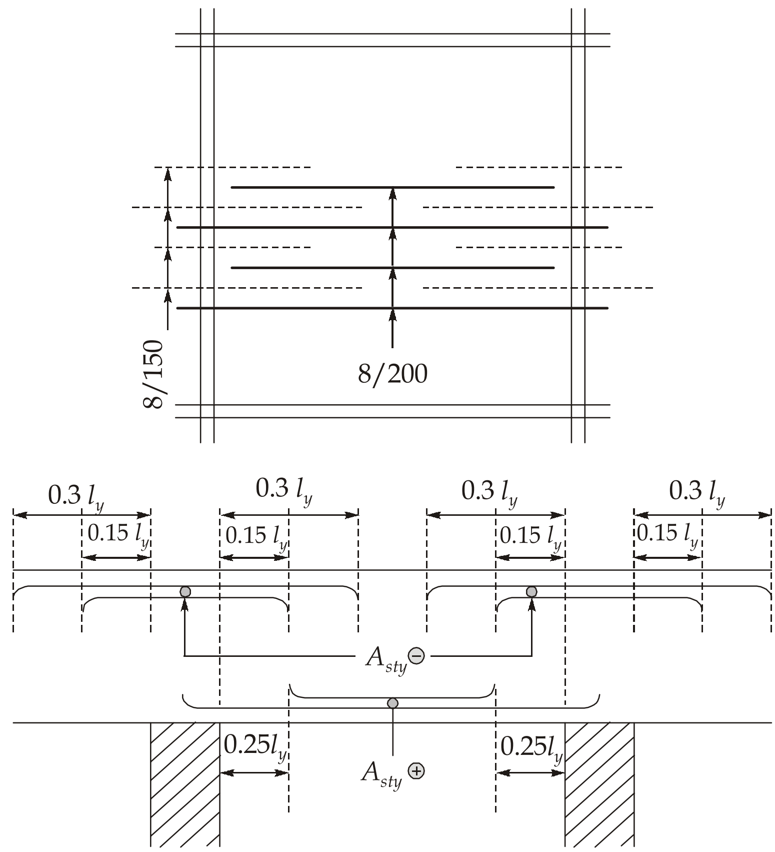
$$\tau_{v_2} = \frac{V_{u_2}}{B \cdot d_y} = \frac{36.25 \times 10^3}{1000 \times 160} = 0.23 \text{ N/mm}^2$$

$$\tau_{v_2} = 0.23 \text{ N/mm}^2 < (\tau_{c\,\min} = 0.29 \text{ N/mm}^2 \text{ for } P_t \le 0.15\%)$$

$\therefore$ Safe

Check for torsion reinforcement is not required since all the edges are continuous.





Q.2 (b) Solution:

Size of the tank

$$\text{Volume} = 6.5 \text{ lakh litres} = 650 \text{ m}^3$$

$$\text{Diameter of the tank} = 12 \text{ m}$$

Height,
$$H = \frac{650}{\frac{\pi}{4} \times 12^2} = 5.74 \approx 6 \text{ m}$$

Design for Hoop tension

$$t_{\text{approx}} = 30 \text{ H} + 50 = 30 \times 6 + 50 = 230 \text{ mm} \approx 300 \text{ mm}$$

$$\frac{H^2}{Dt} = \frac{6^2}{12 \times 0.3} = 10 \text{ m} \qquad \left\{\frac{H^2}{Dt} = 6 \text{ m to } 12 \text{ m}\right\}$$

$\therefore$
$$h = \text{maximum of } \begin{Bmatrix} \frac{H}{3} = \frac{6}{3} = 2\text{m} \\ 1\text{ m} \end{Bmatrix} = 2 \text{ m}$$

Maximum hoop tension

$$T_H = \frac{\gamma_w(H-h)D}{2} = \frac{10\times(6-2)\times 12}{2} = 240 \text{ kN}$$

Design steel reinforcement (Horizontal reinforcement)

$$A_{st_H} = \frac{T_H}{f_{st}} = \frac{240\times 10^3}{130} = 1846.2 \text{ mm}^2$$

$$Ast_H \text{ in each layer} = \frac{1846.2}{2} = 923.1 \text{ mm}^2$$

$$\text{Spacing of 12 mm dia bars} = \frac{1000\times\frac{\pi}{4}\times 12^2}{923.1} = 122.52 \text{ mm} \approx 120 \text{ mm}$$

∴ provide 12 mm dia bars @120 mm c/c on both faces

Check stress developed in concrete due to hoop tension

$$f_{ct} = \frac{T_H}{1000t+(m-1)A_{st}}$$

$$= \frac{240\times 10^3}{1000\times 300+(8.11-1)\times 1846.2}$$

$$= 0.76 \text{ N/mm}^2 < 1.6 \text{ N/mm}^2 \qquad \therefore \text{ OK}$$

Design for vertical cantilever moment

$$P_1 = \frac{1}{2}\times\gamma_\omega H\times h = \frac{1}{2}\times 10\times 6\times 2 = 60 \text{ kN}$$

Bending moment, $BM = P_1\times\frac{h}{3} = 60\times\frac{2}{3} = 40 \text{ kN-m}$

Stress in concrete (neglecting steel)

$$f_{cbt} = \frac{BM}{Z} \le 2.21 \text{ N/mm}^2$$

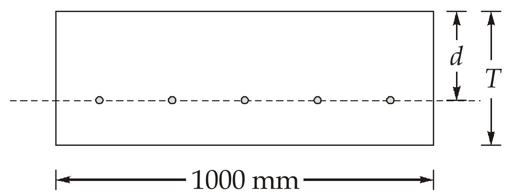
$$2.21 \ge \frac{40\times 10^6\times 6}{1000\times T^2}$$

$$T \ge 330 \text{ mm}$$

∴ Thickness required for bending = 330 mm

Area of steel (vertical)

$$A_{st} = \frac{BM}{\sigma_{st} \times j \times d}$$



$$j = \left(1 - \frac{K}{3}\right),$$

$$K = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = 0.418$$

$$j = 1 - \frac{0.418}{3} = 0.86$$

Let, effective cover = 60 mm

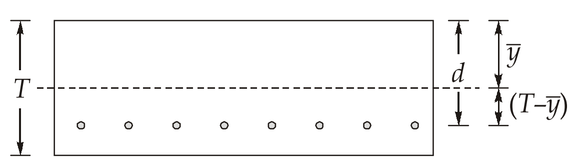
$$T = 330 - 60 = 270 \text{ mm}$$

$$A_{st_v} = \frac{40 \times 10^6}{130 \times 0.86 \times 270} = 1325.12 \text{ mm}^2$$

$$\text{Spacing of 16 mm dia bars} = \frac{1000 \times \frac{\pi}{4} \times 16^2}{1325.12} = 151.73 \text{ mm} \approx 150 \text{ mm}$$

$$A_{st_v} \text{ provided} = \frac{1000}{150} \times \frac{\pi}{4} \times 16^2 = 1340.413 \text{ mm}^2$$

Check stresses in concrete



$$\bar{y} = \frac{1000 \times 330 \times \frac{330}{2} + (8.11 - 1) \times 1340.413 \times 270}{1000 \times 330 + (8.11 - 1) \times 1340.413}$$

$$\bar{y} = 167.92 \text{ mm}$$

$$\therefore \quad (T - \bar{y}) = 330 - 167.92 = 162.08 \text{ mm}$$

$$I_{eq} \text{ about } NA = \left[\frac{1000 \times T^3}{12} + 1000 \times T\left(\bar{y} - \frac{T}{2}\right)^2\right] +$$

$$(m-1)\left[n \times \frac{\pi}{64} \times \phi^4 + A_{st}(d-\bar{y})^2\right]$$

$$I_{qq} = \left[\frac{1000 \times 330^3}{12} + 1000 \times 330(167.92 - 165)^2\right] +$$

$$(8.11 - 1)\left[\frac{1000}{150} \times \frac{\pi}{64} \times 16^4 + \frac{1000}{150} \times \frac{\pi}{4} \times 16^2(270 - 167.92)^2\right]$$

$$= 3.1 \times 10^9 \text{ mm}^4$$

$$f_{cbt\,(\text{developed})} = \frac{BM}{I_{eq}}(T - \bar{y}) = \frac{40 \times 10^6}{3.1 \times 10^9} \times (330 - 167.92)$$

$$= 2.09 \text{ N/mm}^2 < 2.21 \text{ N/mm}^2 \qquad \therefore \text{ OK}$$

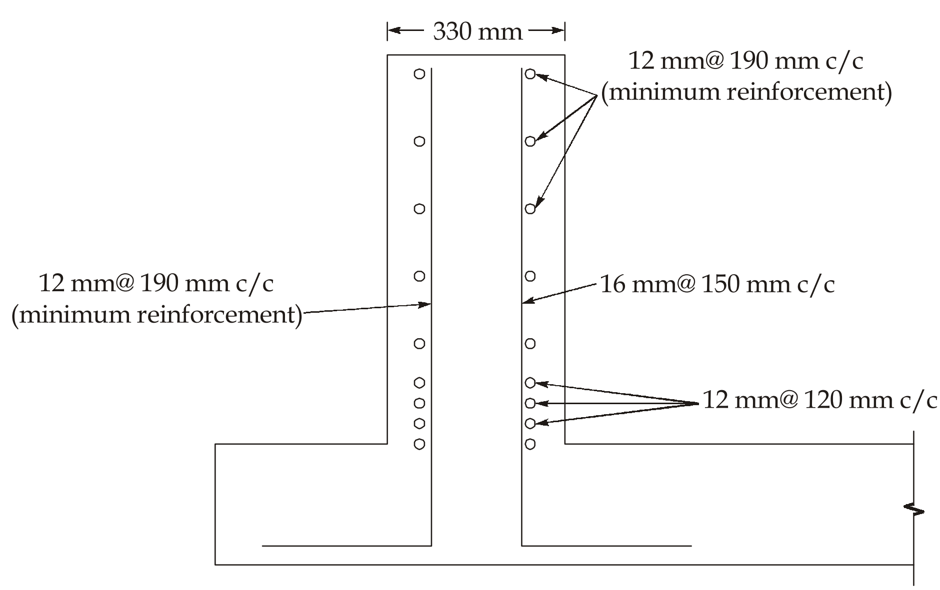
Minimum reinforcement

$$A_{\text{st min}} = \frac{0.35}{100} \times 1000 \times 330 = 1155 \text{ mm}^2$$

$$A_{\text{st min}} \text{ in each layer} = \frac{1155}{2} = 577.5 \text{ mm}^2$$

Spacing of 12 mm dia bars

$$= \frac{1000 \times \frac{\pi}{4} \times 12^2}{577.5} = 195.84 \text{ mm} \approx 190 \text{ mm}$$



Q.2 (c) Solution:

Given, $b = 350$ mm, $D = 750$ m,

$f_{ck} = 25$ MPa, $f_y = 415$ MPa,

$T_u = 140$ kN-m, $M_u = 200$ kN-m,

$V_u = 110$ kN, Effective cover = 50 mm,

$d = 750 - 50 = 700$ mm

Design of longitudinal reinforcement:

$$M_t = \frac{T_u\left(1+\frac{D}{b}\right)}{1.7} = \frac{140\times\left(1+\frac{750}{350}\right)}{1.7} = 258.82 \approx 259 \text{ kN-m}$$

Equivalent bending moments for design,

$$M_e = M_t \pm M_u$$

$$= 259 \pm 200$$

$$= \begin{cases} 459 \text{ kN-m} & \text{(Flexural tension at top)} \\ 59 \text{ kN-m} & \text{(Flexural tension at bottom)} \end{cases}$$

Design of top steel:

For Fe415, $M_{u\,\text{lim}} = 0.138 f_{ck}\, b\, d^2$

$= (0.138 \times 25 \times 350 \times 700^2) \times 10^{-6}$

$= 591.675$ kN-m

But $M_{e1} = 459 \text{ kN-m} < M_{u\,\text{lim}}$

$$\therefore \quad A_{st\,1} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6\, M_u}{f_{ck} B d^2}}\right]Bd$$

$$= \frac{0.5\times 25}{415}\left[1-\sqrt{1-\frac{4.6\times 459\times 10^6}{25\times 350\times 700^2}}\right]\times 350\times 700$$

$$= 2122 \text{ mm}^2$$

∴ Provide 2 - 28 ϕ + 2 - 25 ϕ at top

A_{st} provided $= (2 \times 615) + (2 \times 490) = 2210$ mm^2

Design of bottom steel:

$$A_{st\,2} = \frac{M_{e2}}{0.87 f_y (d - d_1)} = \frac{59\times 10^6}{0.87\times 415\times(700-50)} = 251.4 \text{ mm}^2$$

Minimum reinforcement, $\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$

$$\Rightarrow \quad A_{st\,min} = \frac{0.85 \times 350 \times 700}{415} = 501.8 \approx 502 \text{ mm}^2 > A_{st\,2}$$

∴ Provide minimum reinforcement

Provide 3 - 16 ϕ (A_{st} = 201 × 3 = 603 mm^2) at bottom

Side face reinforcement:

As D > 450 mm, side face reinforcement for torsion is required

$$(A_{st})_{req} = \frac{0.1}{100} \times bD = \frac{0.1}{100} \times 350 \times 750 = 262.5 \text{ mm}^2$$

Provide 4 - 10 ϕ (A_{st} = 78.5 × 4 = 314 mm^2), two bars on each side face

The vertical spacing between longitudinal bars will be less than 300 mm, as required by the code.

Design of transverse reinforcement:

Equivalent nominal shear stress,

$$\tau_{ve} = \frac{V_u + \frac{1.6T_u}{b}}{bd}$$

$$\Rightarrow \quad \tau_{ve} = \frac{(110 \times 10^3) + \frac{1.6 \times 140 \times 10^6}{350}}{350 \times 700} = 3.06 \text{ MPa}$$

$$< \tau_{c,\,max} = 0.625\sqrt{f_{ck}} = 0.625\sqrt{25}$$

$$= 3.1 \text{ MPa (for M25 concrete)}$$

Shear strength of concrete,

$$\text{For} \quad P_t = \frac{2210 \times 100}{350 \times 700} = 0.904\%$$

$$\tau_1 = 0.618 \text{ MPa (for M25 concrete)}$$

As torsional shear is relatively high, design of stirrups is done as follows.

$$0.87 f_y A_{sv} \times \frac{d_1}{s_v} = \frac{T_u}{b_1} + \frac{V_u}{2.5}$$

$$b_1 = B - 2 \times \text{(effective cover)} = 350 - 2 \times 50 = 250 \text{ mm}$$

$$d_1 = D - 2 \times \text{(effective cover)} = 750 - 2 \times 50 = 650 \text{ mm}$$

Assuming 10 ϕ 2-legged stirrups,

$$A_{sv} = 78.5 \times 2 = 157 \text{ mm}^2$$

$$0.87 \times 415 \times 157 \times \frac{650}{s_v} = \frac{140 \times 10^6}{250} + \frac{110 \times 10^3}{2.5}$$

$$\Rightarrow \quad s_v = 61.0 \text{ mm c/c (low)}$$

Alternatively, providing 12 ϕ , 2-legged stirrups,

$$A_{sv} = 113 \times 2 = 226 \text{ mm}^2$$

$$(s_v)_{\text{req}} = 61.0 \times \frac{226}{157} = 87.8 \text{ mm}$$

$$(s_v)_{\text{req}} = \frac{0.87 f_y A_{sv}}{(\tau_{ve} - \tau_c)b} = \frac{0.87 \times 415 \times 226}{(3.06 - 0.618) \times 350} = 95.46 \approx 95 \text{ mm c/c}$$

Maximum spacing:

$$x_1 = 250 + 28 + 12 = 290 \text{ mm}$$

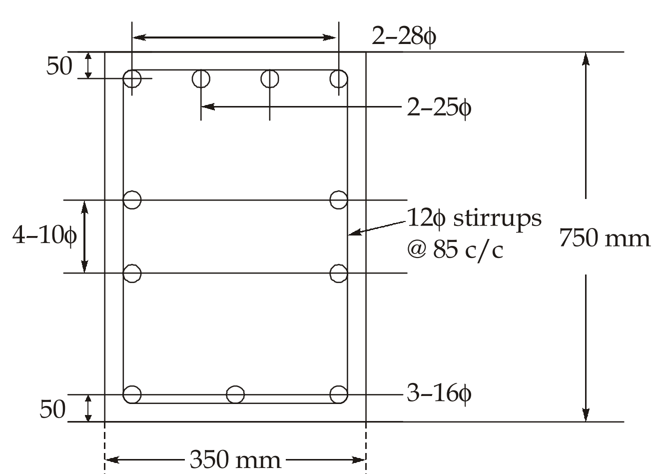
$$y_1 = 650 + \frac{28}{2} + \frac{16}{2} + 12 = 684 \text{ mm}$$

$$s_v \le \begin{cases} x_1 = 290 \text{ mm} \\ \frac{x_1 + y_1}{4} = \frac{290 + 684}{4} = 243.5 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

∴ Provide 12 ϕ, 2-legged stirrups at 85 mm c/c.

Check cover

With 50 mm effective cover, 12 ϕ stirrups and 28 ϕ longitudinal bars, clear cover to stirrups is $50 - 12 - \frac{28}{2} = 24 \text{ mm} > 20 \text{ mm}$ ∴ OK



Q.3 (a) Solution:

Size of footing:

Load from the column = 800 kN

Self weight of footing = 0.1 × 800 = 80 kN

Total load acting on soil = 880 kN

Given, SBC of soil = 200 kN/m^2

$$\text{Area of the foundation required} = \frac{880}{200} = 4.4 \text{ m}^2$$

Given, width, B = 2 m

$$\therefore \quad \text{Length of the footing, } L = \frac{4.4}{2} = 2.2 \text{ m}$$

∴ Provide 2.2 m × 2 m size of footing

Net design soil pressure:

$$\text{Net upward pressure,} \quad w_0 = \frac{P}{A} = \frac{800}{4.4}$$

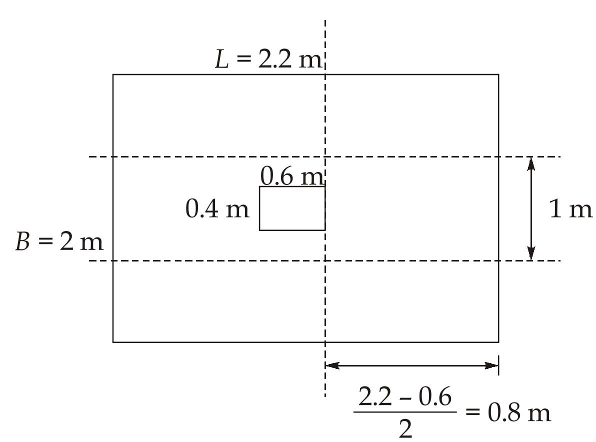
$$= 181.82 \text{ kN/m}^2$$

$$\text{Factored soil pressure,} \quad w_{u0} = 1.5 \times 181.82$$

$$= 272.73 \text{ kN/m}^2$$

Check for B.M. :

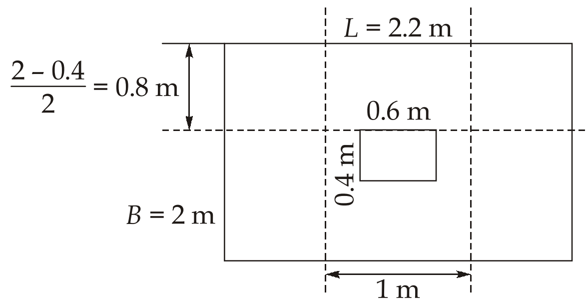
Critical section for BM



$$M_{u_x} = 272.73 \times 1 \times \frac{0.8 \times 0.8}{2}$$

$$= 87.3 \text{ kN-m}$$

Similarly, M_{u_y} :



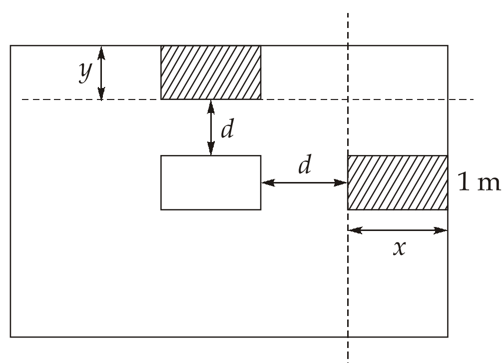
$$M_{u_y} = 272.73 \times 1 \times \frac{0.8 \times 0.8}{2} = 87.3 \text{ kN-m}$$

Effective depth required, $$d = \sqrt{\frac{M_u}{Q \cdot B}} = \sqrt{\frac{87.3 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$= 177.85 \text{ mm, say } 180 \text{ mm}$$

Check for one way shear:

Critical section is at 'd' distance from the face of column,



$$x = \frac{2.2 - 0.6}{2} - 0.18 = 0.62 \text{ m}$$

$$y = \frac{2 - 0.4}{2} - 0.18 = 0.62 \text{ m}$$

$\therefore$ $$V_u = 272.73 \times 1 \times 0.62 = 169.1 \text{ kN}$$

$$\tau_v = \frac{V_u}{Bd} = \frac{169.1 \times 10^3}{1000 \times 180} = 0.94 \text{ N/mm}^2 < K \cdot \tau_c$$

$$\tau_{c\,min} \text{ for M20} = 0.28 \text{ N/mm}^2$$

To ensure $\tau_v < \tau_{c\,min}$

$$V_u = 272.73 \times 1 \times (0.8 - d)$$

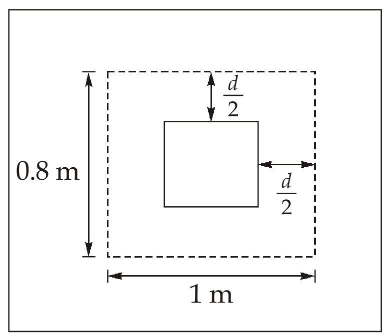
$$\tau_v = \frac{V_u}{Bd} = \frac{272.73 \times (0.8 - d) \times 10^3}{1000 \times d \times 10^3} = 0.28$$

$\Rightarrow$ $$272.73\,(0.8 - d) = 280d$$

$$d = 0.3947 \text{ m say } 0.4 \text{ m}$$

$\therefore$ $$d = 400 \text{ mm}$$

Check for punching shear:



$$P_{u\,net} = P_u - w_u \times (a + d)\,(b + d)$$
$$= (1.5 \times 800) - 272.73 \times 0.8 \times 1 = 981.816 \text{ kN}$$

Punching shear stress, $$\tau_{v_{p(dev)}} = \frac{981.816 \times 10^3}{2(1 + 0.8) \times 0.4 \times 10^6} = 0.682 \text{ N/mm}^2$$

Permissible punching shear stress = $K_\beta \times 0.25\sqrt{f_{ck}}$

where, $K_\beta = 0.5 + \frac{b}{a}, \geq 1$

$\therefore$ $$K_\beta = 0.5 + \frac{0.4}{0.6} = 1.16 > 1$$

$\therefore$ Take, $$K_\beta = 1$$

$\therefore$ $$\tau_{vp\,(perm)} = 1 \times 0.25 \times \sqrt{20} = 1.118 \text{ N/mm}^2$$

$\therefore$ $$\tau_{vp\,(dev)} < \tau_{vp\,(perm)} \qquad \therefore \text{ OK}$$

Design of reinforcement:

$$d = 400 \text{ mm}$$

Providing an effective cover of 80 mm,

$$D = 400 + 80 = 480 \text{ mm}$$

For M_{ux} :

Area of steel required for 1 m width

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6 M_{ux}}{f_{ck} B d_x^2}}\right] \times B \cdot d_x$$

$$\Rightarrow \quad A_{st} = \frac{0.5 \times 20}{415}\left[1-\sqrt{1-\frac{4.6 \times 87.3 \times 10^6}{20 \times 1000 \times 400^2}}\right] \times 1000 \times 400$$

$$\Rightarrow \quad A_{st} = 625 \text{ mm}^2$$

$$A_{st\,min} = \frac{0.12}{100} \times 1000 \times 480 = 576 \text{ mm}^2 < A_{st} \qquad \therefore \text{ OK}$$

For total width of 2 m,

$$\text{Total } A_{st} = 625 \times 2 = 1250 \text{ mm}^2$$

$$\text{Total no. of } 12\ \phi \text{ bars} = \frac{1250}{\frac{\pi}{4} \times 12^2} = 11.05 \approx 12$$

$$\text{Spacing} = \frac{2000}{12} = 166.67 \text{ mm} \approx 160 \text{ mm c/c (say)}$$

∴ Provide 12 mm ϕ bars @ 160 mm c/c.

For M_{uy} :

$$A_{sty} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6 M_{uy}}{f_{ck} B d_y^2}}\right] \times B \times d_y$$

$$= \frac{0.5 \times 20}{415}\left[1-\sqrt{1-\frac{4.6 \times 87.3 \times 10^6}{20 \times 1000 \times (400-12)^2}}\right] \times 1000 \times 388$$

$$= 645.8 \text{ mm}^2 > A_{st\,min} \qquad \therefore \text{ OK}$$

Total area of steel required for 2.2 m

$$= 645.8 \times 2.2 = 1420.76 \text{ mm}^2$$

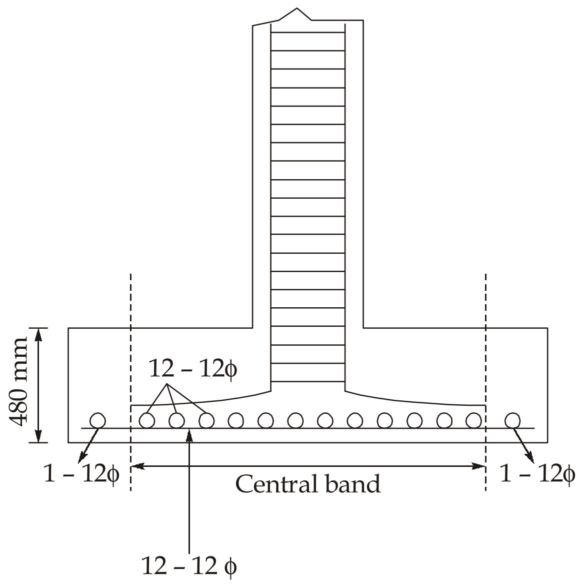
Reinforcement required in a central band of 2000 mm

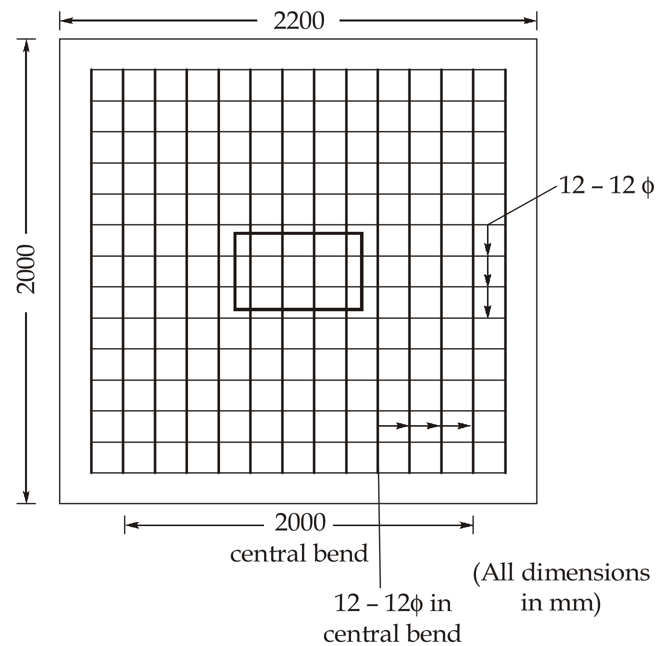
$$= \frac{2 \times 1420.76}{1 + \frac{2.2}{2}} = 1353.1 \text{ mm}^2$$

No. of 12 mm ϕ bars in the central band

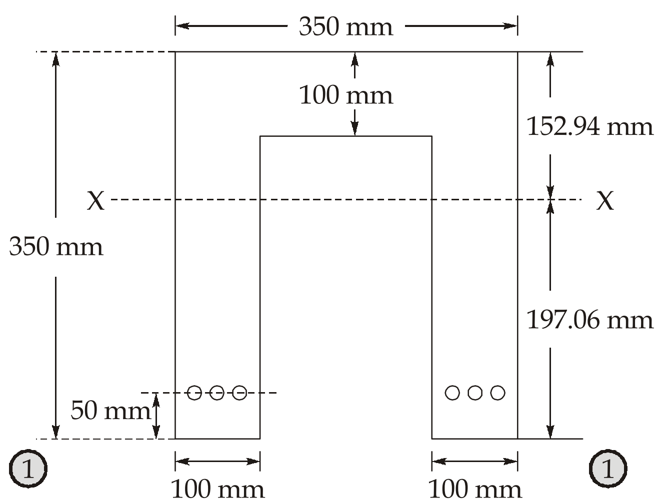
$$= \frac{1353.1}{\frac{\pi}{4} \times 12^2} = 11.96 \approx 12 \text{ (say)}$$

Provide 12- 12 mm ϕ bars in central band and an additional bar at each end.





Q.3 (b) Solution:



Total area of the section = 350 × 350 – 150 × 250 = 85000 mm^2

Distance of the centroidal axis from the bottom edge 1 - 1

$$= \frac{(350 \times 350 \times 175) - (150 \times 250 \times 125)}{85000} = 197.06 \text{ mm}$$

Moment of inertia of the section about the bottom edge 1 - 1

$$= \frac{350 \times 350^3}{3} - \frac{150 \times 250^3}{3} = 4.22083 \times 10^9 \text{ mm}^4$$

Let moment of inertial about the centroidal axis be $I_{C.G}$

$$\therefore \quad I_{1\text{-}1} = I_{C \cdot G} + A(\bar{y})^2$$

$$\Rightarrow \quad I_{C.G} = I_{1-1} - A(\bar{y})^2$$

$$\therefore \quad I_{C.G} = 4.22083 \times 10^9 - 85000 \times (197.06)^2$$

$$= 9.2 \times 10^8 \text{ mm}^4$$

$$\therefore \quad Z_t = \frac{9.2 \times 10^8}{152.94} = 6015430.88 \text{ mm}^3$$

$$Z_b = \frac{9.2 \times 10^8}{197.06} = 4668628.84 \text{ mm}^3$$

Initial prestress = 1250 N/mm^2

Final prestress = $0.85 \times 1250 = 1062.5 \text{ N/mm}^2$

Eccentricity, e = $197.06 - 50 = 147.06$ mm

Area of steel wires = $6 \times \frac{\pi}{4} \times 5^2 = 117.81 \text{ mm}^2$

Final prestressing force, P = $1062.5 \times 117.81 \text{ N} = 125173.13 \text{ N}$

For the condition that the maximum compressive stress at the top edge should reach 14 N/mm^2,

$$\frac{P}{A} - \frac{Pe}{Z_t} + \frac{M}{Z_b} = 14$$

$$\Rightarrow \frac{125173.13}{85000} - \frac{125173.13 \times 147.06}{6015430.88} + \frac{M}{6015430.88} = 14$$

$$M = 93.76 \text{ kN-m}$$

For the condition that the maximum tensile stress at the bottom edge should reach 0.75 N/mm^2,

$$\frac{P}{A} + \frac{Pe}{Z_b} - \frac{M}{Z_b} = -0.75$$

$$\Rightarrow \frac{125173.13}{85000} + \frac{125173.13 \times 147.06}{4668628.84} - \frac{M}{4668628.84} = -0.75$$

$$M = 28.78 \text{ kN-m}$$

$\therefore$ Maximum BM permissible = 28.78 kN-m

$$\therefore \quad \frac{wl^2}{8} = 28.78$$

$$\Rightarrow \quad w = \frac{28.78 \times 8}{8^2} = 3.6 \text{ kN/m}$$

$$\text{DL of the beam} = 25 \times \frac{85000}{10^6} = 2.125 \text{ kN/m}$$

$\therefore$ Additional safe superimposed load

$$= 3.6 - 2.125 = 1.475 \text{ kN/m}$$

Q.3 (c) Solution:

Unsupported length of the column,

$$L_0 = 6 \text{ m}, \quad L_{eff} = 0.65 \times 6 = 3.9 \text{ m}$$

Slenderness ratio:

$$SR_{X\text{-}X} = \frac{L_{eff\,x}}{D} = \frac{3900}{500} = 7.8 < 12$$

$$SR_{Y\text{-}Y} = \frac{L_{eff\,y}}{B} = \frac{3900}{340} = 11.47 < 12$$

$\therefore$ The given column is a short column.

Design values:

$$P_u = 1490 \text{ kN}$$

$$M_{ux} = P_u \cdot e_x = 1490 \times 0.08 = 119.2 \text{ kN-m}$$

$$M_{uy} = P_u \cdot e_y = 1490 \times 0.06 = 89.4 \text{ kN-m}$$

Minimum design moments:

$$(e_{min})_x = \frac{L_{x_0}}{500} + \frac{D}{30} = \frac{6000}{500} + \frac{500}{30} = 28.67 \text{ mm}$$

$$(M_{ux})_{min} = P_u \cdot (e_{min})_x$$

$$= 1490 \times \frac{28.67}{1000} = 42.72 \text{ kN-m}$$

But, $M_{ux} = 119.2$ kN-m $\therefore$ OK

$$(e_{\min})_y = \frac{L_{y_0}}{500} + \frac{B}{30} = \frac{6000}{500} + \frac{340}{30} = 23.33 \text{ mm}$$

$$(M_{uy})_{\min} = P_u \cdot (e_{\min})_y$$

$$= 1490 \times \frac{23.33}{1000} = 34.77 \text{ kN-m}$$

But, $M_{uy} = 89.4$ kN-m $\therefore$ OK

$$A_{sc} = 10 \times \frac{\pi}{4} \times 20^2 = 10 \times 314.16 = 3141.6 \text{ mm}^2$$

$$A_g = 340 \times 500 = 170000 \text{ mm}^2$$

$$A_c = A_g - A_{sc} = 166858.4 \text{ mm}^2$$

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$= (0.45 \times 20 \times 166858.4 + 0.75 \times 415 \times 3141.6)\text{N}$$

$$= 2479.5 \text{ kN}$$

$$\frac{P_u}{f_{ck}bD} = \frac{1490 \times 10^3}{20 \times 340 \times 500} = 0.438$$

$$\frac{d'}{D} = \frac{50}{500} = 0.1$$

Percentage of steel, $p = \frac{3141.6}{170000} \times 100 = 1.848\%$

$$\frac{p}{f_{ck}} = \frac{1.848}{20} = 0.0924 \approx 0.093$$

Referring to chart-44, SP:16,

For, $\frac{p}{f_{ck}} = 0.093$

and $\frac{P_u}{f_{ck}bD} = 0.438$

$$\frac{M_u}{f_{ck}bD^2} = 0.109$$

$\therefore$ $M_{u\,x1} = 0.109 \times 20 \times 340 \times 500^2$ N-mm $= 185.3$ kN-m

Now, $\frac{d'}{b} = \frac{50}{340} = 0.147$ say 0.15

Referring to chart-45, SP:16,

For $\frac{p}{f_{ck}} = 0.093$

and $\frac{P_u}{f_{ck}bD} = 0.438,$

$$\frac{M_u}{f_{ck}bD^2} = 0.116$$

$$M_{u\,y1} = 0.116 \times 20 \times 500 \times 340^2 \text{ N-mm} = 134.1 \text{ kN-m}$$

$$\frac{P_u}{P_{uz}} = \frac{1490}{2479.5} = 0.60$$

$$\alpha_n = 1 + \frac{(2-1)}{(0.8-0.2)} \times (0.6-0.2) = 1.667$$

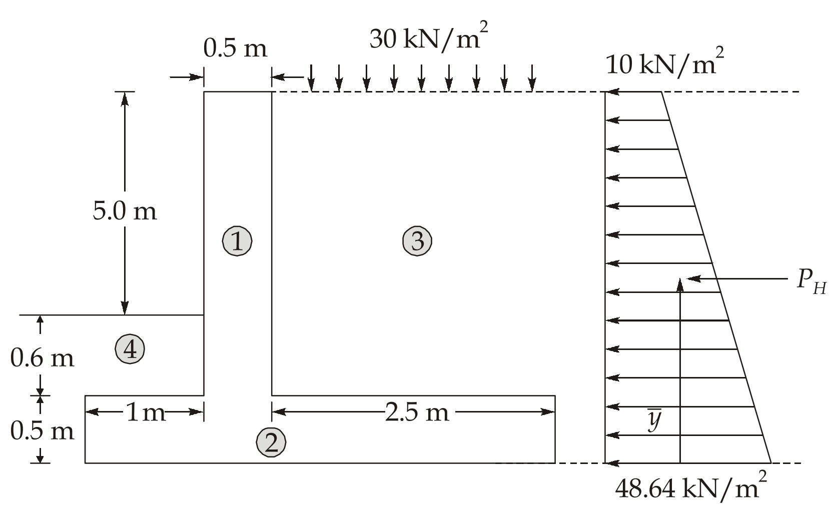
$$\left(\frac{M_{ux}}{M_{ux_1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy_1}}\right)^{\alpha_n} = \left(\frac{119.2}{185.3}\right)^{1.667} + \left(\frac{89.4}{134.1}\right)^{1.667}$$

$$= 0.48 + 0.51 = 0.99 \text{ (less than 1)}$$

Hence the column is safe under the given loading condition.

Q.4 (a) Solution:

(i)



$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

At top :

$$\sigma_H = K_a q = \frac{1}{3} \times 30 = 10 \text{ kN/m}^2$$

At bottom :

$$\sigma_H = K_a(q + \gamma H) = \frac{1}{3}(30 + 19 \times 6.1) = 48.63 \text{ kN/m}^2$$

$$P_H = \left(\frac{10 + 48.63}{2}\right) \times 6.1 \times 1 = 178.82 \text{ kN}$$

$$\bar{y} = \left(\frac{48.63 + 2(10)}{48.63 + 10}\right) \times \frac{6.1}{3} = 2.38 \text{ m}$$

Loads	Distance from toe	Moment about toe
$W_1 = 25 \times 0.5 \times 5.6 = 70$ kN	$1 + \frac{0.5}{2} = 1.25$ m	87.5 kN – m
$W_2 = 25 \times 4 \times 0.5 \times 1 = 50$ kN	$\frac{4}{2} = 2.0$ m	100 kN – m
Soil $W_3 = 2.5 \times 5.6 \times 19 = 266$ kN	$1 + 0.5 + \frac{2.5}{2} = 2.75$ m	731.5kN – m
$W_4 = 0.6 \times 1 \times 1 \times 19 = 11.4$ kN	$\frac{1}{2} = 0.5$ m	5.7 kN – m
$\Sigma W = 397.4$ kN		$\Sigma M_R = 924.7$ kN – m

Overturning moment, $M_0 = P_H \times \bar{y} = 178.82 \times 2.38 = 425.6$ kN-m

$$\text{FOS against overturning} = \frac{0.9 \times M_R}{M_o} = \frac{0.9 \times 924.7}{425.6} = 1.96 > 1.40 \quad \therefore \text{Safe}$$

(ii) Sliding force $P_H = 178.82$ kN

$$\text{Passive pressure} = \frac{1}{2} \times K_p \times \gamma \times H^2 = \frac{1}{2} \times 3 \times 19 \times 1.1^2 = 34.5 \text{ kN}$$

Balancing force $= 34.5 + (0.5 \times 397.4) = 233.2$ kN

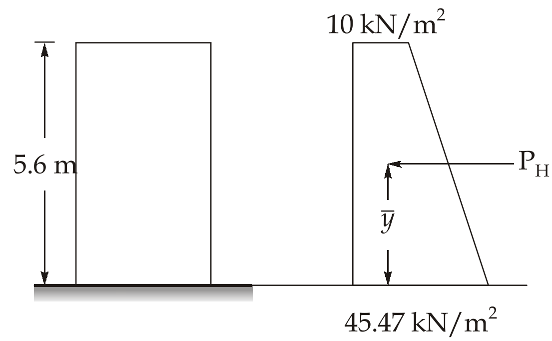
$$\text{FOS against sliding} = \frac{0.9 \times 233.2}{178.82} = 1.174 < 1.4$$

Hence a shear key needs to be provided to generate the balance force through passive resistance.

Design of vertical stem:

Height of cantilever above base

$$= 6.1 - 0.5 = 5.6 \text{ m}$$



$$\sigma_H \text{ at } 5.6 = \frac{1}{3}(30 + 19 \times 5.6) = 45.47 \text{ kN/m}^2$$

$$P_H = \frac{10 + 45.47}{2} \times 5.6 = 155.316 \text{ kN}$$

$$\bar{y} = \left[\frac{45.47 + (2 \times 10)}{45.47 + 10}\right] \times \frac{5.6}{3} = 2.2 \text{ m}$$

$$\text{B.M.} = P_H \times \bar{y} = 155.316 \times 2.2 = 341.7 \text{ kN-m}$$

Factored moment, $M_u = 1.5 \times 341.7 = 512.55$ kN-m

$$\therefore \quad d = \sqrt{\frac{M_u}{Q \cdot B}} = \sqrt{\frac{512.55 \times 10^6}{0.138 \times 30 \times 1000}} = 351.86 \text{ mm}$$

$\therefore$ Effective depth required = 351.86 mm

Effective depth provided $= 500 - 50 - \frac{20}{2} = 440 \text{ mm} > 351.86 \text{ mm (say)}$

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right) Bd$$

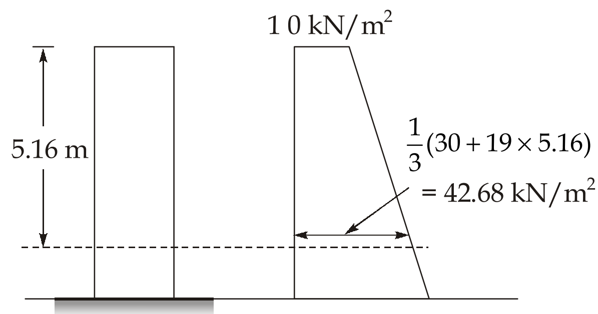
$$= \frac{0.5 \times 30}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 512.55 \times 10^6}{30 \times 1000 \times 440^2}}\right) \times 1000 \times 440$$

$$= 3645.912 \text{ mm}^2$$

Spacing of 20 mm ϕ bars $= \dfrac{1000 \times \frac{\pi}{4} \times 20^2}{3645.912} = 86.17$ mm say 80 mm c/c

Check for shear:

Critical section is at distance d = 0.44 m



$$V_u = 1.5\left(\frac{10+42.68}{2}\times 5.16\right) = 203.87 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{V_u}{Bd} = \frac{203.87\times 10^3}{1000\times 440} = 0.463 \text{ N/mm}^2$$

$$p_t = \frac{3645.912}{1000\times 440}\times 100 = 0.828\%$$

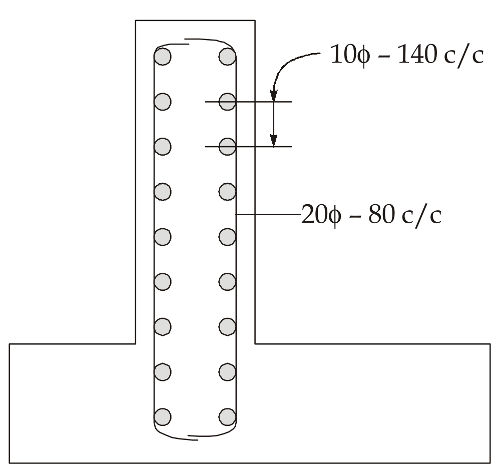
From table given,

$$\tau_c (\text{for } p_t = 0.75\%) = 0.59 \text{ N/mm}^2$$

$$\therefore \quad \tau_v < \tau_c \qquad \therefore \text{ Safe}$$

$$\text{Distribution steel} = \frac{0.12}{100}\times 1000\times 440 = 528 \text{ mm}^2$$

$$\text{Spacing of 10 mm } \phi \text{ bars} = \frac{1000\times\frac{\pi}{4}\times 10^2}{528} = 148.75 \text{ mm} \approx 140 \text{ mm c/c (say)}$$



Q.4 (b) Solution:

Step-1: Determination of natural period

Given that infill panels are provided.

$$\therefore \text{Fundamental natural period, } T = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times (3 \times 3.5)}{\sqrt{7}} = 0.357 \text{ sec}$$

Step-2: Other importance factors

For $T = 0.357$ sec, damping of 5% and for hard rock, $\frac{Sa}{g} = 2.5$, for Bhuj located in zone V, zone factor, $z = 0.36$, since the building is used as a school building, the importance factor $I = 1.5$

For a special moment resisting beam,

The response reduction factor, $R = 5.0$

Step-3: Determination of design horizontal seismic coefficient

The design horizontal seismic coefficient,

$$A_h = \left(\frac{Z}{2}\right)\left(\frac{I}{R}\right)\left(\frac{S_a}{g}\right)$$

$$= \left(\frac{0.36}{2}\right)\left(\frac{1.5}{5}\right)(2.5) = 0.135$$

Step-4: Determination of seismic weight

Weight of one storey = Total weight of beams + Slab + Columns + Walls + Live load

= 130 + 250 + 50 + 530 + 130

= 1090 kN

$\therefore$ Weight of I and II floor = 1090 kN

Weight of terrace floor = 655 kN

Total weight of building, W = 2 × 1090 + 655 = 2835 kN

Step-5 : Determination of base shear

$$\text{Design base shear, } V_B = A_h W$$

$$= 0.135 \times 2835$$

$$= 382.725 \text{ kN}$$

Step-6: Distribution of equivalent lateral load

$$Q_i = V_B\left(\frac{W_i h_i^2}{\sum_{i=1}^{n} W_i h_i^2}\right)$$

where, h_i is calculated from base

$$Q_1 = V_B\left(\frac{W_1 h_1^2}{W_1 h_1^2 + W_2 h_2^2 + W_3 h_3^2}\right)$$

$$= 382.725\left(\frac{1090 \times 3.5^2}{1090 \times 3.5^2 + 1090 \times 7^2 + 655 \times 10.5^2}\right)$$

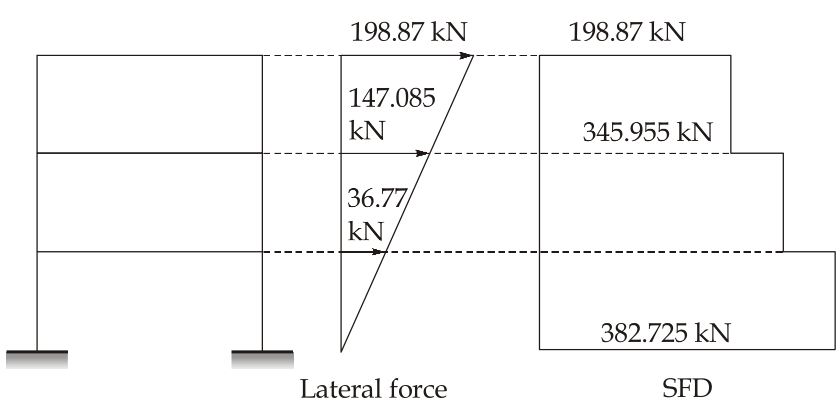
$$= 36.77 \text{ kN}$$

$$Q_2 = 382.725\left(\frac{1090 \times 7^2}{1090 \times 3.5^2 + 1090 \times 7^2 + 655 \times 10.5^2}\right)$$

$$= 147.085 \text{ kN}$$

$$Q_3 = 382.725\left(\frac{655 \times 10.5^2}{1090 \times 3.5^2 + 1090 \times 7^2 + 655 \times 10.5^2}\right)$$

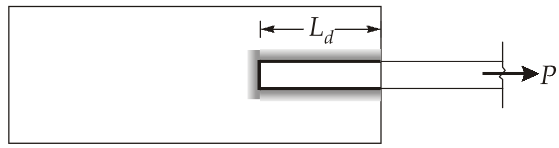
$$= 198.87 \text{ kN}$$



Check: $V_B = Q_1 + Q_2 + Q_3 = 382.725$ kN (Ok)

Q.4 (c) Solution:

(i)



$$L_d = \text{Development length}$$
$$\tau_{bd} = \text{Bond strength of concrete}$$

$\because$ τ_{bd} acts over the circumference of bar therefore

$$F_b = (\pi\phi)L_d \times \tau_{bd}$$

Maximum force carried by bar,

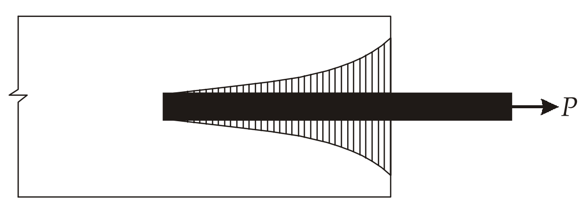
$$P = \frac{\pi}{4}\phi^2 \times (0.87 f_y)$$

For equilibrium of forces $P = F_b$

$$\Rightarrow \quad \frac{\pi}{4} \times \phi^2 \times 0.87 f_y = \pi\phi\, L_d \times \tau_{bd}$$

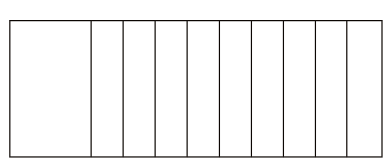
$$\Rightarrow \quad L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

Variation of bond stress along L_d

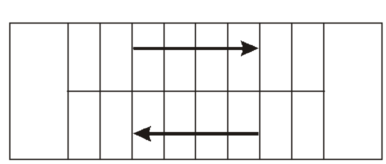


(ii) Types of staircase-based on geometrical configurations

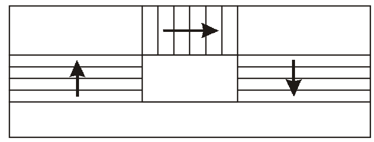
1. Single flight staircase



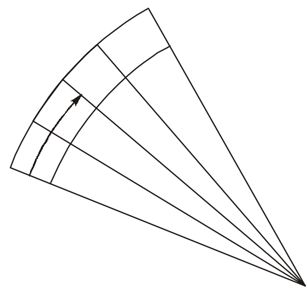
2. Two flight staircase



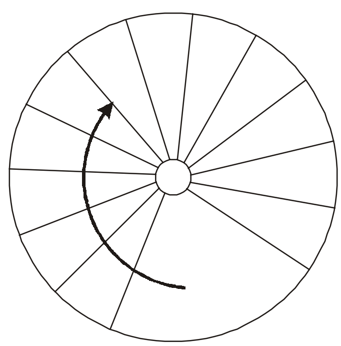
3. Open-well staircase

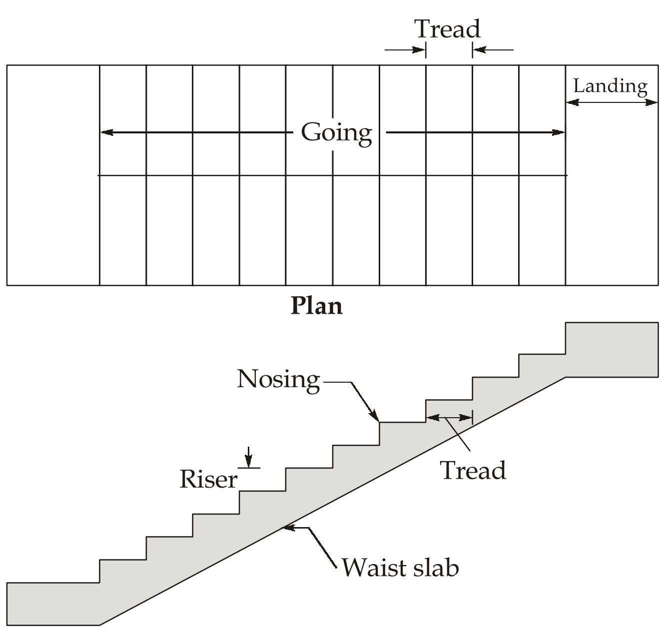


4. Helicoidal staircase



5. Spiral staircase





A typical stair case flight

Section B : Geo-technical & Foundation Engineering-1 + Highway Engineering-2 + Surveying and Geology-2

Q.5 (a) Solution:

(i) Given, Void ratio of sand = 0.61

Specific gravity of sand = 2.72

Now,
$$\gamma_{sat} = \left[\frac{G+e}{1+e}\right] \times \gamma_w$$
$$= \frac{2.72+0.61}{1+0.61} \times 9.81 = 20.29 \text{ kN/m}^3$$

At point A,

Total stress, $\sigma = 1.2 \times \gamma_w$
$$= 1.2 \times 9.81 = 11.772 \text{ kN/m}^2$$

Pore water pressure, $u = \left(1.2 + \frac{2.5}{3.5} \times 0\right) \times \gamma_w$
$$= 1.2 \times 9.81 = 11.772 \text{ kN/m}^2$$

$\therefore$ Effective stress, $\bar{\sigma} = \sigma - u$
$$= 11.772 - 11.772 = 0$$

At point B,

Total stress, $\sigma = 1.2 \cdot \gamma_w + 1.5 \times \gamma_{sat}$
$$= 1.2 \times 9.81 + 1.5 \times 20.29$$
$$= 42.21 \text{ kN/m}^2$$

Pore water pressure, $u = \left(1.2 + 1.5 + \frac{2.5}{3.5} \times 1.5\right) \times \gamma_w$
$$= 36.99 \text{ kN/m}^2 \approx 37 \text{ kN/m}^2$$

So, Effective stress, $\bar{\sigma} = \sigma - u$
$$= 42.21 - 37$$
$$= 5.21 \text{ kN/m}^2$$

At point C,

Total stress, $\sigma = 1.2 \cdot \gamma_w + 3.5 \times \gamma_{sat}$
$$= 1.2 \times 9.81 + 3.5 \times 20.29$$
$$= 82.787 \text{ kN/m}^2$$

Power water pressure, $u = \left(1.2 + 3.5 + \frac{2.5}{3.5} \times 3.5\right) \times \gamma_w = (1.2 + 3.5 + 2.5) \times 9.81$

$= 70.632 \text{ kN/m}^2$

So, Effective stress, $\bar{\sigma} = \sigma - u$

$= 82.787 - 70.632 = 12.155 \text{ kN/m}^2$

(ii) Hydraulic gradient $(i) = \frac{2.5}{3.5} = 0.714$

Seepage force per unit volume $= i \cdot \gamma_w$

$= 0.714 \times 9.81 = 7 \text{ kN/m}^3$

Q.5 (b) Solution:

Given normal flows on roads 1 and 2,

$q_1 = 440$ PCU/hr

$q_2 = 280$ PCU/hr

Saturation flows, $S_1 = 1300$ PCU/hr,

$S_2 = 1100$ PCU/hr

All red time, $R = 12$ sec

Number of phases, $n = 2$

$$y_1 = \frac{q_1}{S_1} = \frac{440}{1300} = 0.34$$

$$y_2 = \frac{q_2}{S_2} = \frac{280}{1100} = 0.26$$

$$y = y_1 + y_2 = 0.34 + 0.26 = 0.6$$

Total lost time, $L = 2n + R = 2 \times 2 + 12 = 16$ sec

Optimum cycle time, $C_0 = \frac{1.5L + 5}{1 - y} = \frac{1.5 \times 16 + 5}{1 - 0.6} = 72.5$ sec

$$G_1 = \frac{y_1}{y}(C_0 - L) = \frac{0.34}{0.6}(72.5 - 16) = 32 \text{ sec}$$

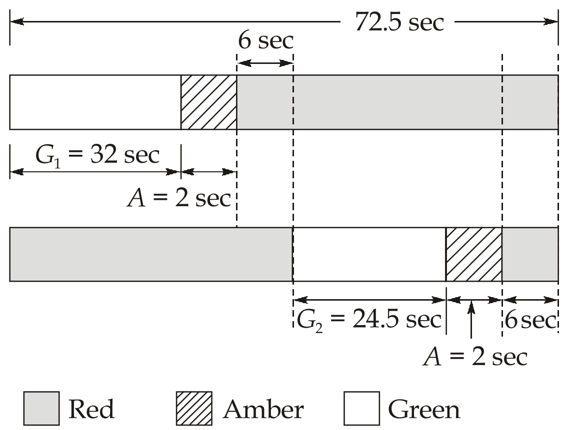
$$G_2 = \frac{y_2}{y}(C_0 - L) = \frac{0.26}{0.6}(72.5 - 16) = 24.5 \text{ sec}$$

Provide all-red time, R for pedestrian crossing = 12 sec

$$\text{all-red time per phase} = \frac{12}{2} = 6 \text{ sec}$$

Providing amber time of 2.0 sec each for clearance,

Total cycle time, $C_0 = 32 + 24.5 + 12 + 2 + 2 = 72.5$ sec



Q.5 (c) Solution:

(i) Given,

$$\text{Weight of soil + Paraffin} = 5.23 \times 10^{-3} \text{ kN}$$

$$\text{Weight of paraffin} = 1.71 \times 10^{-4} \text{ kN}$$

$$\text{Volume of soil + Paraffin} = 3.7 \times 10^{-4} \text{ m}^3$$

$$\text{Specific gravity of soil} = 2.72$$

$$\text{Specific gravity of paraffin} = 0.9$$

$$\text{Water content} = 11\%$$

Now,
$$\text{Weight of soil} = 5.23 \times 10^{-3} - 1.71 \times 10^{-4}$$
$$= 5.059 \times 10^{-3} \text{ kN}$$

and
$$\text{volume of paraffin} = \frac{\text{Weight of paraffin}}{G_{\text{paraffin}} \times \gamma_w} = \frac{1.71 \times 10^{-4}}{0.9 \times 9.81} = 1.937 \times 10^{-5} \text{ m}^3$$

$$\text{Volume of soil} = 3.7 \times 10^{-4} - 1.937 \times 10^{-5} = 3.506 \times 10^{-4} \text{ m}^3$$

$$\text{Bulk unit weight of soil, } \gamma_{\text{bulk}} = \frac{\text{Weight of soil}}{\text{Volume of soil}} = \frac{5.059 \times 10^{-3}}{3.506 \times 10^{-4}} = 14.43 \text{ kN/m}^3$$

Now,
$$\gamma_{\text{bulk}} = \frac{G + Se}{1 + e} \times \gamma_w$$

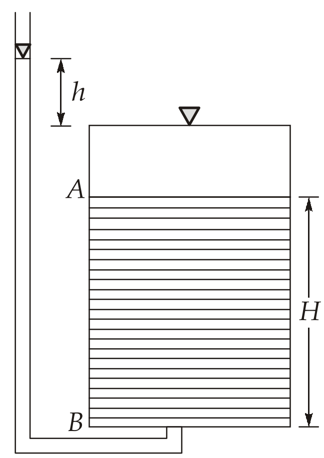
$$\Rightarrow \qquad \gamma_{bulk} = \left(\frac{G + Gw}{1+e}\right) \times \gamma_w \qquad [\because Se = Gw]$$

$$\Rightarrow \qquad 14.43 = \left(\frac{2.72 + 2.72 \times 0.11}{1+e}\right) \times 9.81$$

$$\Rightarrow \qquad e = 1.05$$

(ii) Effective stress is defined as the difference between total stress and pore water pressure, i.e.

$$\bar{\sigma}_B = H \cdot \gamma' - h \cdot \gamma_w \qquad \text{...(i)}$$



It is clear from equation (i) that by increasing total head difference h, it is possible to reach a condition, when effective stress in the soil becomes zero i.e.

$$H \cdot \gamma' - h \cdot \gamma_w = 0$$

$$\Rightarrow \qquad H \times \left(\frac{G-1}{1+e}\right)\gamma_w - \frac{h}{H} \times H \cdot \gamma_w = 0$$

$$\Rightarrow \qquad H \cdot \gamma_w \left(\frac{G-1}{1+e} - i\right) = 0 \qquad \left(\text{where } i = \frac{h}{H}\right)$$

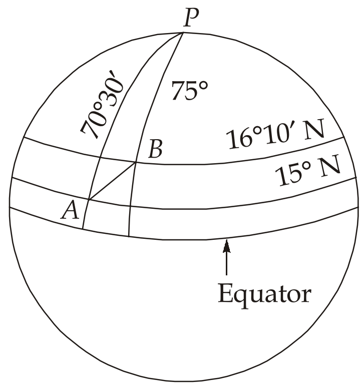
$$\Rightarrow \qquad i = i_{cr} = \frac{G-1}{1+e}$$

where i_{cr} is critical hydraulic gradient.

When upward flow takes place at the critical hydraulic gradient, a soil such as sand loses all its shearing strength and it cannot support any load. The soil is said to have become 'alive' or 'quick' and boiling will occur. This phenomenon is known as quick sand condition. It is not a type of sand but only a type of hydraulic condition.

Q.5 (d) Solution:

(i) In the spherical triangle PAB,



$$b = AP = 90° - \text{latitude of } A$$
$$= 90° - 15° = 75°$$
$$a = BP = 90° - \text{latitude of } B$$
$$= 90° - 16°10' = 73°50'$$
$$\angle P = \angle APB = \text{difference of longitudes of } A \text{ and } B$$
$$= 75° - 70°30' = 4°30'$$

Now, AB can be determined using cosine rule in spherical triangle APB

$$\cos AB = \cos a \cos b + \sin a \sin b \cos P$$
$$= \cos 73°50' \cos 75° + \sin 73°50' \sin 75° \cos 4°30'$$
$$= 0.99693$$
$$\Rightarrow \quad AB = 4°29' 26.67''$$

Now, from side AB, shortest distance can be determined as,

Shortest distance between A and B

$$= \text{Central angle} \times \text{Radius}$$
$$= 4°29' 26.67'' \times \frac{\pi}{180°} \times 6400 = 501.62 \text{ km}$$

(ii) Signal propagation errors:

GPS satellites transmit their timing information by radio signals. However radio signals do not behave as predictably desired.

It is assumed that radio signals travel at the speed of light, which is presumably a constant. However, the speed of light is constant only in vacuum. In the real world, light (or radio) speed slows down depending on what it is travelling through. As a GPS signal comes down through the charged particles in the ionosphere and then through the water vapor in the troposphere, it gets delayed a little. Since calculation of distance assumes a constant speed of light, this delay results into a miscalculation of the satellite's distance, which in turn translates into an error in position. This is know as signal propagation error. Good receivers add in a correction factor for a typical trip through the earth's atmosphere, which helps, but since the atmosphere varies from point to point and moment to moment, no correction factor can accurately compensate for the delay that actually occurs.

Q.5 (e) Solution:

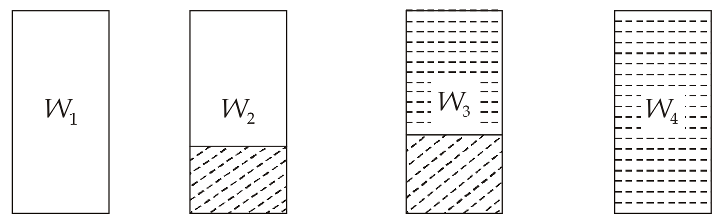
No rock possesses all the desirable properties so as to make it ideally suitable for various types of construction. For foundation purpose, rocks should be very strong and be able to withstand substantial loads, of superstructure of considerable magnitude, rocks should be easily workable and be available in plenty. Further they should be durable. For roofing purpose, rocks should be resistante to weathering; for flooring purposes rocks should be able to withstand wear and tear and for face work, rocks should have pleasant color, attractive appearance and the ability to take good polish. Igneous rocks in general, meet these requirements to a very satisfactory extent as discussed below.

- **Minerals:** The type of minerals present in the rocks influence color, durability and to some extent strength. The igneous rocks are composed mainly of silicate minerals. Among various minerals, silicate minerals are the most durable. Further, rocks rich in silica are pleasantly light colored, as in the case of granite. All minerals of common igneous rocks are relatively very hard and do not have cleavage to any harmful extent. These characters contribute to the strength and abrasion resistance of the rocks in a substantial way.
- **Structure:** Since igneous rocks are formed out of solidification of a melt, they are necessarily dense, compact and massive. In other words, these rocks do not have any internal openings or hollow nature. This contributes to the strength and heaviness of these rocks. The same factor is also responsible for non-porous and impermeable character of these rocks. They are also less susceptible to frost action owing to their impermeable nature. Further, igneous rocks do not have an inherent weakness unlike due to the occurrence of bedding planes or mineral alignment.
- **Joints:** From the workability point of view, massive igneous rocks are difficult to use because of their toughness and hardness. However, suitably placed joints enable these rocks to be worked out with less difficulty. This is so because joints are very common in all types of rocks. Mural joints in granites, columnar joints in basalt and sheet joints in all types of rocks are well known. These facilitate the dressing of quarried rocks into smaller blocks.

- **Texture:** Igneous rocks by virtue of their texture and minerals present in them have the ability to take a very good polish and thus have become increasing popular for face works too. Constructions done with these rocks, possess a majestic appearance.

Q.6 (a) Solution:

(i)



(1) Pycnometer (empty) (2) Pycnometer + moist soil (3) Pycnometer + moist soil +water (4) Pycnometer +water

As we know,

$$\text{Water content, } w = \frac{W_w}{W_d} \quad \text{...(i)}$$

where W_w is weight of water and W_d is weight of dry soil or soil solids

Now, weight of water in moist soil,

$$W_w = (W_2 - W_1) - W_d \quad \text{...(ii)}$$

As W_1 and W_2 are known in the pyncometer test, the weight of dry soil (W_d) is determined as follows:

$$G = \frac{\gamma_s}{\gamma_w} = \frac{W_d}{V_s \cdot \gamma_w}$$

$$\Rightarrow \quad V_s \cdot \gamma_w = \frac{W_d}{G} \quad \text{...(iii)}$$

Now, $$W_4 = W_3 - W_d + \gamma_w \cdot V_s \quad \text{...(iv)}$$

Putting equation (iii) in (iv), we get

$$\Rightarrow \quad W_4 = W_3 - W_d + \frac{W_d}{G}$$

$$\Rightarrow \quad W_d - \frac{W_d}{G} = W_3 - W_4$$

$$\Rightarrow \quad W_d\left(1 - \frac{1}{G}\right) = W_3 - W_4$$

$$\Rightarrow \quad W_d\left(\frac{G-1}{G}\right) = W_3 - W_4$$

$$\Rightarrow \qquad W_d = (W_3 - W_4) \times \left(\frac{G}{G-1}\right)$$

$\therefore$ Water content, $$w = \frac{W_w}{W_d} = \frac{W_2 - W_1 - W_d}{W_d} \qquad \text{...(v)}$$

Putting equation (ii) and (v) in equation (i), we get

$$\Rightarrow \qquad w = \frac{W_2 - W_1}{W_d} - 1$$

$$\Rightarrow \qquad w = \left(\left(\frac{W_2 - W_1}{W_3 - W_4}\right) \times \left(\frac{G-1}{G}\right)\right) - 1$$

(ii) Given,

Thickness of clay layer (H_1) = 5 m

Coefficient of volume compressibility (m_v)

$= 5.79 \times 10^{-4}$ m^2/kN

Change in stress, $\Delta\bar{\sigma} = 197.5 - 127.5$

$\Delta\bar{\sigma} = 70$ kN/m^2

Coefficient of permeability, $K = 1.6 \times 10^{-8}$ m/min

Now, total settlement, $\Delta H_1 = m_v \cdot H_1 \cdot \Delta\bar{\sigma}$

$$\Rightarrow \qquad \Delta H_1 = 5.79 \times 10^{-4} \times 5 \times 70$$

$$= 0.20265 \text{ m} = 20.27 \text{ cm}$$

Now, settlement at 50% consolidation is given by,

$$\frac{(\Delta h)_{50\%}}{(\Delta H_1)} = 0.5$$

$$\Rightarrow \qquad (\Delta h)_{50\%} = 0.5 \times 20.27 = 10.135 \text{ cm}$$

Now, for 90% consolidation, $T_V = 1.781 - 0.933\,[\log(100 - U\%)]$

$= 1.781 - 0.933\,[\log(100 - 90)]$

$= 0.848$

and, $$T_V = C_V \times \frac{t}{H_1^2}$$

But, $$C_V = \frac{K}{m_v \times \gamma_w} = \frac{1.6 \times 10^{-8}}{5.79 \times 10^{-4} \times 9.81}$$

$= 2.82 \times 10^{-6}$ m^2/min

∴ Under double drainage:

$$0.848 = 2.82 \times 10^{-6} \times \frac{t}{\left(\frac{5}{2}\right)^2}$$

⇒ $$t = 1879432.62 \text{ min} = 1305.16 \text{ days}$$

Q.6 (b) Solution:

(i) $$P = 5100 \text{ kg}, \qquad p = 7.0 \text{ kg/cm}^2$$

$$P = p \times \pi a^2$$

As we know,

Radius of loaded area, $$a = \left(\frac{P}{p\pi}\right)^{1/2} = \left(\frac{5100}{7 \times \pi}\right)^{1/2} = 15.23 \text{ cm}$$

$$E = 180 \text{ kg/cm}^2 \qquad [\because \text{given}]$$

Permissible deflection, $$\Delta = 0.25 \text{ cm} \qquad [\because \text{given}]$$

Pavement thickness, $$T = \left[\left(\frac{3P}{2\pi E\Delta}\right)^2 - a^2\right]^{1/2}$$

⇒ $$T = \left[\left(\frac{3 \times 5100}{2 \times \pi \times 180 \times 0.25}\right)^2 - (15.23)^2\right]^{1/2} = 51.93 \text{ cm}$$

(ii) Given, $$L = 4.5 \text{ m}, \qquad f = 1.3$$

$$\gamma = 2400 \text{ kg/m}^3$$

Friction developed by half length of slab = Resisting force by concrete

⇒ $$f \times \left(\gamma \times \frac{L}{2} \times B \times H\right) = S_f \times B \times H$$

∴ Tensile stress developed in the CC pavement due to contraction,

$$S_f = \frac{f \times \gamma \times L}{2} = \frac{1.3 \times 2400 \times 4.5}{2}$$

$$= 7020 \text{ kg/m}^2 = 0.702 \text{ kg/cm}^2$$

(iii) Given,

thickness of pavement, $h = 15$ cm,

Modulus of subgrade reaction, $K = 7.5 \text{ kg/cm}^3$

Poisson's ratio, $\mu = 0.15$

Modulus of elasticity of CC, $E = 2.1 \times 10^5 \text{ kg/cm}^2$

Center to centre spacing of dowel bars (δ) = 30 cm

Load to be transferred by joint is 50% of the wheel load (*P*)

Distance upto which dowel bars are effective = 1.8 × (radius of relative stiffness)

Now, radius of relative stiffness,

$$l = \left(\frac{Eh^3}{12k(1-\mu^2)}\right)^{1/4}$$

$$\Rightarrow \quad l = \left(\frac{2.1\times10^5\times(15)^3}{12\times7.5\times(1-0.15^2)}\right)^{1/4} = 53.3 \text{ cm}$$

Distance upto which dowel bars are effective

$$= 1.8\, l = 1.8 \times 53.3 = 95.94 \text{ cm}$$

If the load carried by a single dowel just below the wheel is *P*, then, the total load carried by the dowel system

$$= \left(1+\frac{95.94-30}{95.94}+\frac{95.94-60}{95.94}+\frac{95.94-90}{95.94}\right)P$$

$$= 2.124\, P$$

$$\therefore \quad 2.124P = 4100 \times 0.5$$

$$\Rightarrow \quad P = 965.16 \text{ kg}$$

Hence, load carried by the single dowel just below the wheel is about 965 kg.

Q.6 (c) (i) Solution:

Scale of the photograph, $S = \dfrac{f}{H-h}$

Here, f = Focal length = 200 mm = 0.2 m

H = Height of flight

h = average ground elevation

$$\therefore \quad S = \frac{1}{25000} = \frac{0.2}{H-330}$$

$$\Rightarrow \quad H - 330 = 0.2 \times 25000$$

$$\Rightarrow \quad H = 5330 \text{ m}$$

Thus, Height of flight, H = 5330 m

Length of photograph, l = 25 cm

Width of photograph, w = 25 cm

Percentage longitudinal overlap, $P_l = 60\%$

Percentage side overlap, $P_w = 40\%$

∴ Theoretical ground spacing of flight lines, $W = (1 - P_w)Sw$

$= (1 - 0.4) \times 25000 \times 25 \times 10^{-2} = 3750$ m

∴ Number of flight lines required, $N_2 = \dfrac{100 \times 1000}{3750} + 1 = 27.67 \approx 28$

⇒ Actual spacing of flight lines $= \dfrac{100 \times 10^3}{28} = 3571.43$ m

Thus, Spacing of flight lines = 3571.43 m

∴ Ground distance between exposure, $L = (1 - P_l)Sl$

$= (1 - 0.6) \times 25000 \times 25 \times 10^{-2}$

$= 2500$ m

⇒ Exposure interval $= \dfrac{2500}{270 \times \dfrac{1000}{3600}} = 33.33$ sec

Since, least count of invervalometer = 0.5 sec

∴ Exposure interval = 33 seconds

Adjusted ground distance between exposures,

$$L = \frac{270 \times 1000}{3600} \times 33 = 2475 \text{ m}$$

⇒ Ground distance between exposures = 2475 m

Number of photographs per flight line, $N_1 = \dfrac{150 \times 10^3}{2475} + 1 = 61.61 \approx 62$

Thus, total number of photographs required $= N_1 \times N_2$

$= 62 \times 28 = 1736$

Q.6 (c) (ii) Solution:

1. **Zenith and Nadir:** These are the poles of the celestial horizon. These points are obtained by producing the direction of gravity at any point on the earth's surface to pierce the celestial sphere in either direction. The corresponding point, on the celestial sphere, above the observer is called the zenith and the nadir is the point on this sphere, directly beneath the observer.

2. **Azimuth:** The azimuth of a heavenly body is the angle between the observer's meridian and the vertical circle passing through the body. It is generally measured on the horizon from 0° to 360° from the north point towards the east point.
3. **Prime vertical:** A vertical circle which is at right angles to the meridian is known as prime vertical. It intersects the horizon in the east and west points.
4. **Declination:** The declination of a celestial body is the angular distance measured on a star's meridian, north or south of the celestial equator. It can have any value from 0° to 90° and is marked plus or minus depending on whether the body is north or south of equator.

Q.7 (a) Solution:

(i) Let the shear strength parameters in terms of total stress are C and ϕ

For test - 1 :

As we know,
$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2C\tan\left(45° + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad 400 = 150\tan^2\left(45° + \frac{\phi}{2}\right) + 2C\tan\left(45° + \frac{\phi}{2}\right) \quad \text{...(i)}$$

For test-2 :

Similarly,
$$1000 = 450\tan^2\left(45° + \frac{\phi}{2}\right) + 2C\tan\left(45° + \frac{\phi}{2}\right) \quad \text{...(ii)}$$

Subtracting equation (i) from (ii), we get

$$600 = 300\tan^2\left(45° + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad \tan\left(45° + \frac{\phi}{2}\right) = \sqrt{2}$$

$$\Rightarrow \quad 45° + \frac{\phi}{2} = 54.736°$$

$$\Rightarrow \quad \phi = 19.47°$$

Putting the value of ϕ in equation (i), we get

$$400 = 150\tan^2\left(45° + \frac{19.47°}{2}\right) + 2 \times C\tan\left(45° + \frac{19.47°}{2}\right)$$

$$C = 35.36 \text{ kN/m}^2$$

(ii) Let the shear strength parameters in terms of effective stress are C' and ϕ'

For test-1 :

$$\sigma_1 = 400 \text{ kN/m}^2, \quad u = 30 \text{ kN/m}^2$$
$$\sigma_3 = 150 \text{ kN/m}^2$$

∴ Effective stresses will be,

$$\sigma_1' = \sigma_1 - u = 400 - 30 = 370 \text{ kN/m}^2$$
$$\sigma_3' = \sigma_3 - u = 150 - 30 = 120 \text{ kN/m}^2$$

Now, $$\sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi'}{2}\right) + 2C' \tan\left(45° + \frac{\phi}{2}\right)$$

⇒ $$370 = 120 \tan^2\left(45° + \frac{\phi'}{2}\right) + 2C' \tan\left(45° + \frac{\phi}{2}\right) \quad \text{...(iii)}$$

For test-2 :

$$\sigma_1 = 1000 \text{ kN/m}^2,$$
$$u = 125 \text{ kN/m}^2$$
$$\sigma_3 = 450 \text{ kN/m}^2$$

∴ Effective stresses will be,

$$\sigma_1' = \sigma_1 - u = 1000 - 125 = 875 \text{ kN/m}^2$$
$$\sigma_3' = \sigma_3 - u = 450 - 125 = 325 \text{ kN/m}^2$$

Now, $$875 = 325 \tan^2\left(45° + \frac{\phi'}{2}\right) + 2C' \tan\left(45° + \frac{\phi'}{2}\right) \quad \text{...(iv)}$$

Subtracting (iii) from (iv), we get

⇒ $$505 = 205 \tan^2\left(45° + \frac{\phi'}{2}\right)$$

⇒ $$2.46 = \tan^2\left[45° + \frac{\phi'}{2}\right]$$

⇒ $$\tan\left[45° + \frac{\phi'}{2}\right] = 1.568$$

⇒ $$\phi' = 24.94°$$

Put the value of ϕ' in equation (iii), we get

$$370 = 120 \tan^2\left(45° + \frac{24.94°}{2}\right) + 2C \tan\left(45° + \frac{24.94°}{2}\right)$$

$$C = 23.68 \text{ kN/m}^2$$

Q.7 (b) Solution:

Direction of trip	Journey time, min-sec	Total stopped delay, min-sec	No. of vehicles overtaking	No. of vehicles overtaken	No. of vehicles from opposite direction
N - S	6 - 48	1 - 50	3	7	270
	7 - 10	1 - 30	5	3	290
	6 - 10	1 - 30	3	6	270
	6 - 32	1 - 50	2	5	320
Total	26 - 40	6 - 40	13	21	1150
Mean :	**6 - 40**	**1 - 40**	**3.25**	**5.25**	**287.5**
S - N	7 - 20	1 - 40	4	3	190
	7 - 40	2 - 00	3	1	220
	8 - 00	2 - 30	2	2	190
	7 - 40	1 - 30	3	2	190
Total	30 - 40	7 - 40	12	8	790
Mean:	**7 - 40**	**1 - 55**	**3**	**2**	**197.5**

(*a*) North-South direction

n_y = Average no. of vehicles overtaking minus overtaken

= 3.25 – 5.25 = –2.0

n_a = Average no. of vehicles during trips in opposite direction (*S* - *N*)

= 197.5

t_w = Average journey time with the stream

= 6 min 40 sec = 6.67 min

t_a = Average journey time during trips against the steam

= 7 min 40 sec = 7.67 min

(i) Average volume,

$$q = \frac{n_a + n_y}{t_a + t_w} = \frac{197.5 - 2}{7.67 + 6.67} = 13.63 \text{ veh/min}$$

(ii) Average journey time,

$$\bar{t} = t_w - \frac{n_y}{q} = 6.67 - \frac{(-2)}{13.63} = 6.82 \text{ min}$$

$$\text{Average journey speed} = \frac{3.5 \times 60}{6.82} = 30.8 \text{ kmph}$$

(iii)

$$\text{Average stopped delay} = 1.67 \text{ min}$$

$$\begin{aligned}\text{Average running time} &= \text{Average journey time} - \text{average stopped delay}\\ &= 6.82 - 1.67\\ &= 5.15 \text{ min}\end{aligned}$$

$$\text{Average running speed} = \frac{3.5\times 60}{5.15} = 40.78 \text{ kmph}$$

(*b*) South-North direction

$$n_y = 3 - 2 = 1$$

$$n_a \text{ (for } N\text{-}S \text{ trips)} = 287.5$$

$$t_w = 7.67 \text{ min}$$

$$t_a = 6.67 \text{ min}$$

(i)

$$q = \frac{n_a + n_y}{t_a + t_w} = \frac{287.5+1}{6.67+7.67} = 20.12 \text{ veh/min}$$

(ii)

$$\bar{t} = t_w - \frac{n_y}{q} = 7.67 - \frac{1}{20.12} = 7.62 \text{ min}$$

$$\text{Average journey speed} = \frac{3.5\times 60}{7.62} = 27.56 \text{ kmph}$$

(iii)

$$\text{Average stopped delay} = 1.917 \text{ min}$$

$$\text{Average running time} = 7.62 - 1.917 = 5.693 \text{ min}$$

$$\text{Average running speed} = \frac{3.5\times 60}{5.694} = 36.88 \text{ kmph}$$

Q.7 (c) (i) Solution:

The word 'metamorphism' means change of form. It indicates the effects of temperature, pressure and chemically active solutions over the texture, minerals and composition of parent rocks. Igneous and sedimentary rocks which serve as parent rocks are formed under a certain physiochemical environment i.e. at the time of formation, they were in equilibrium with their surroundings in terms of temperature, pressure and chemically active fluids. Subsequent to their formation, if any of these factors change significantly, then the equilibrium gets upset and thus necessary metamorphism, i.e, textural, compositional and mineralogical changes take place to create a new equilibrium.

The process of metamorphism occurs in rock due to the effect of high temperature, pressure and chemically active fluids. They are known as metamorphic agents. They are described as follows:

- **Temperature:** The source of temperature which is responsible for metamorphism is either due to depth or due to contact with magma. The metamorphic changes mainly take place in the temperature range of 350° - 850° C. The temperature rise also increases the chemical activity in rocks and facilitaties reactions during metamorphism.
- **Pressure:** The pressure which causes metamorphism is of two different kinds, namely, uniform pressure and directed pressure.

  Uniform pressure increases with depth. It acts vertically downwards and affects the volume of both liquids and solids. Its effect is significant only at great depths. The temperature is also high at greater depths. Thus, high uniform pressure along with high temperature acts together to bring about metamorphism in rocks.

  The directed pressure is due to tectonic forces. Such pressure acts in any direction i.e. upwards, downwards or sidewards. It acts on solids and affects the shape of rocks or minerals. It is effective in the upper layers of the crust and increases with depth to some extent. With further increase in depth, it decreases and finally disappears.
- **Chemical active fluids:** Chemically active fluids plays a key role in causing metamorphism. Firstly, since metamorphism of any type cannot take place for solid minerals in a perfectly dry state, the presence of a liquid medium of some kind is indispensable. Secondary, the huge quantities of volatiles that are associated with magmatic bodies ultimately permeate through the surrounding country rocks by means of diffusion and lastly the magma may directly react with those rocks with which it comes in contact and cause metamorphism.

Q.7 (c) (ii) Solution:

Dykes: Dykes are discordant sheet-like, vertical or steeply inclined, intrusive igneous bodies. They are sheet-like, narrow in width and have nearly uniform thickness. They occur cutting across the bedding planes of the country rocks in which they are found.

During the forceful upward journey, magma intrudes through the discordant fractures, cracks, crevices, joints etc. Subsequent solidification of this gives rise to dykes. The contact of dykes with country rocks may be sharp, but sometimes when dykes are large, heat effect or chemical reaction or diffusion may occur along the sides.

Dykes may be horizontal or inclined or vertical. Steeply inclined or vertical dykes extending to great depths are common in nature. The dimensions of dykes vary widely. They may be very long and thick or may be very short and thin. But a majority of these are less than 5 meters thick.

Sills: Sills are similar to dykes in being sheet- like, intrusive igneous bodies but unlike dykes, these are concordant. Sills are formed due to the penetration of magma into bedding planes of country rocks and their spreading capacity depends on the viscosity of magma, its temperature and the weight of the overlying rocks. Basic magmas being more fluid and more hot, usually occur as sills.

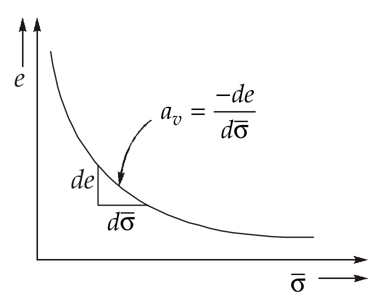
The sills, which spread over large area are generally thin and show uniform thickness. However, when traced to great lengths, they appear lenticular because they have tapering ends. Sills vary widely in their magnitude. Some are small while other are very large. They may occur as horizontal, inclined or vertical bodies but generally they are horizontal or gently inclined.

Dykes are important from civil engineering point of view for the following reasons:

- They are undesirable at the sites of foundations of dams because they introduce heterogeneity in the region and also their sides turn out to be weak planes.
- Dykes are like walls and acts as barriers for the flow of underground water and may result in good or bad potential of ground water in a region.
- Dykes may give rise to new springs or seal old springs.
- As dykes are hard and durable, they are commonly used in making statues, sculptures etc.
- Dykes may cause oil accumulation and thereby contribute to the occurrence of oil and gas deposits under favorable conditions.

Q.8 (a) (i) Solution:

1. **Coefficient of compressibility:** The coefficient of compressibility (a_v) is defined as decrease in void ratio per unit increase in effective stress. It is equal to the slope of the $e - \bar{\sigma}$ curve at the point under consideration.



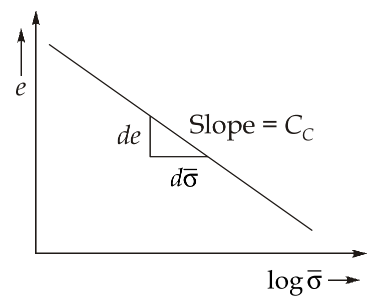
Thus,
$$a_v = -\left[\frac{e_2 - e_1}{\bar{\sigma}_2 - \bar{\sigma}_1}\right] = -\left[\frac{\Delta e}{\Delta \bar{\sigma}}\right]$$

As the effective stress increases, void ratio decreases, and therefore the ratio $\Delta e / \Delta\bar{\sigma}$ is negative. The minus sign is to make a_v positive. For convenience, the coefficient of compressibility a_v is reported as positive.

2. **Coefficient of volume change:** The coefficient of volume change (or coefficient of volume compressibility) is defined as the volumetric strain per unit increase in effective stress.

Thus, $$m_v = \frac{-\Delta V / V_0}{\Delta \bar{\sigma}}$$

3. **Compression Index:** The compression index (C_C) is equal to the slope of the linear portion of the void ratio-$\log\bar{\sigma}$ plot.



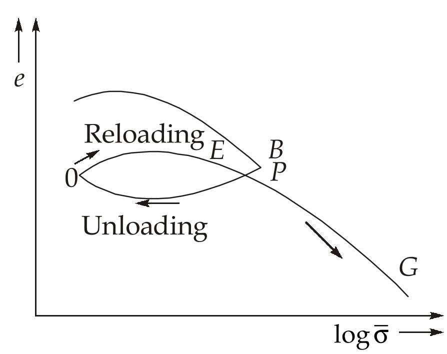
Thus, $$C_C = \frac{-\Delta e}{\log\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}$$

where, $\Delta\bar{\sigma}_0$ = Change in effective stress

$\bar{\sigma}$ = Final effective stress

Δe = Change in void ratio

4. **Expansion index:** The expansion index or swelling index (C_C) is the slope of the $e - \log\bar{\sigma}$ plot obtained during unloading.



As it is evident, the expansion index is much smaller than the compression index.

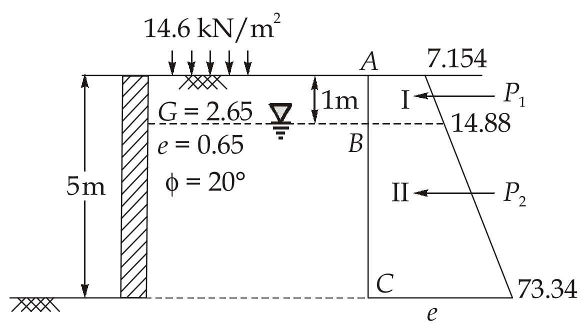
5. **Recompression index:** Recompression is the compression of a soil which had already been loaded and unloaded. The load during recompression is less than the load to which the soil has been subjected previously. The slope of the recompression curve

obtained during reloading, when plotted as $e - \log\bar{\sigma}$ is equal to the recompression index (C_r).

Thus,
$$C_r = \frac{-\Delta e}{\log\left(\frac{\bar{\sigma} + \Delta\bar{\sigma}}{\bar{\sigma}}\right)}$$

The recompression index is appreciably smaller than the compression index C_C.

Q.8 (a) (ii) Solution:



Given, surcharge (q) = 14.6 kN/m^2

Specific gravity (G) = 2.65

Void ratio (e) = 0.65

Angle of internal friction (ϕ) = 20°

Height of wall (H) = 5 m

Now,
$$\gamma_{dry} = \frac{G}{1+e} \times \gamma_w = \frac{2.65}{1+0.65} \times 9.81 = 15.76 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{G+e}{1+e} \times \gamma_w = \frac{2.65+0.65}{1+0.65} \times 9.81 = 19.62 \text{ kN/m}^2$$

Active earth pressure coefficient,

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 20°}{1+\sin 20°} = 0.49$$

Now, active earth pressure from A to B

$$p_{A\text{-}B} = K_a \cdot q + K_a \cdot \gamma_{dry} \cdot Z$$

At point A, $Z = 0$

$\therefore$ $p_A = 0.49 \times 14.6 + 0.49 \times 15.76 \times 0 = 7.154$ kN/m^2

At point B, $Z = 1$ m

$\therefore$ $p_B = 0.49 \times 14.6 + 0.49 \times 15.76 \times 1] = 14.88$ kN/m^2

Now, active earth pressure from B to C

$$p_{B-C} = K_a \cdot q + K_a \gamma_{dry} \cdot H_1 + K_a \cdot \gamma_{sub} \cdot Z + \gamma_w \cdot Z$$

at point B, $Z = 0$

$\therefore$ $p_B = 0.49 \times 14.6 + 0.49 \times 15.76 \times 1 + 0.49 \times (19.62 - 9.81) \times 0 + 9.81 \times 0$

$= 14.88 \text{ kN/m}^2$

at point C, $Z = 4 \text{ m}$

$\therefore$ $p_C = 0.49 \times 14.6 + 0.49 \times 15.76 \times 1 + 0.49 \times (19.62 - 9.81) \times 4 + 9.81 \times 4$

$= 73.34 \text{ kN/m}^2$

Now,

Total horizontal pressure $= P_1 + P_2$

Here, $$P_1 = \left[\frac{7.154 + 14.88}{2}\right] \times 1 = 11.017 \text{ kN/m}$$

and, $$P_2 = \left[\frac{14.88 + 73.34}{2}\right] \times 4 = 176.44 \text{ kN/m}$$

Total load per unit length of wall,

$$P_A = P_1 + P_2$$
$$= 11.02 + 176.44$$
$$= 187.46 \text{ kN/m}$$

Q.8 (b) Solution:

(i) The following angular methods are commonly used for setting out curves:

1. Rankine's method of deflection angle (one-theodolite method).
2. Two-theodolite method
3. Tacheometric method

1. Rankine's method of deflection angle: This method is useful for setting out a circular curve of long length and of large radius. It uses a theodolite for the measurement of angles and the distances are measured with the help of a tape. It yields good results except when the chords are long as compared to the radius, so that the variation between the length of an arc and its chord become considerable. It is quite accurate and is frequently used on highways and railways.

2. Two-theodolite method: This method is most convenient when the ground is undulating, rough and not suitable for linear measurements. In this method, two theodolites are used and linear measurements are completely eliminated. It is based on the principle that the angle between the tangent and the chord is equal to the angle subtended by the chord in the opposite segment of a circle.

3. **Tacheometric method:** In this method, similar to the two-theodolite method, chaining is dispensed with. Distance is measured using the principle of tachometery. A theodolite with a stadia diaphragm or a tachometer is used for conducting the survey. A point on the curve is established by feeding the deflection angle in the tacheometer and measuring the distance of a point on the curve by placing a staff on it.

(ii) The chain used is of 30 m length and degree of curve is 3°.

Radius of curve, $R = \frac{1718.87}{D} = \frac{1718.87}{3} = 572.96 \text{ m}$

Deflection angle, $\Delta = 180° - 150° = 30°$

Tangent length, $BT_1 = R\tan\left(\frac{\Delta}{2}\right) = 572.96 \tan 15° = 153.52 \text{ m}$

Chainage of intersection point, $B = 4274 \text{ m}$

Chainage of point of curve, $T_1 = 4274 - 153.52 = 4120.48 \text{ m}$

Length of curve $= R \times \Delta \times \frac{\pi}{180°} = 572.96 \times 30° \times \frac{\pi}{180°} = 300 \text{ m}$

Chainage of point of tangency, T_2 = Chainage of T_1 + Length of curve

$= 4120.48 + 300 = 4420.48 \text{ m}$

Length of the chords:

First subchord, $C_1 = 4140 - 4120.48 = 19.52 \text{ m}$

Last subchord, $C_{11} = 4420.48 - 4410 = 10.48 \text{ m}$

There are nine full unit chords of 30 m length.

Hence, there will be eleven chords altogether including two sub-chords viz. C_1 and C_{11}.

The offsets are,

$$O_1 = \frac{C_1^2}{2R} = \frac{19.52^2}{2 \times 572.96} = 0.33 \text{ m}$$

$$O_2 = \frac{C_2^2 + C_1C_2}{2R} = \frac{30^2 + 19.52 \times 30}{2 \times 572.96} = 1.3 \text{ m}$$

$$O_3 = O_4 = O_5 \ldots\ldots = O_{10} = \frac{C^2}{R} = \frac{30^2}{572.96} = 1.57 \text{ m}$$

$$O_{11} = \frac{C_{11}^2 + C_{10}C_{11}}{2R} = \frac{10.48^2 + 30 \times 10.48}{2 \times 572.96} = 0.37 \text{ m}$$

(iii) Drift: In the absence of wind, it is assured that the aircraft fly in a straight course along the predetermined flight lines. However, if there is wind, the aircraft will move bodily and fly in a course inclined at an angle to the proposed course. This effect is known as drift, the angle being called the drift angle. It reduces the effective forward and lateral overlap and creates possibility of gap in an aerial photograph.

Crab: When the camera is not square with the direction of flight at the time of exposure, an angle is formed between the flight line and the edges of photographs. This effect is known as crab, the angle being called as the crab angle. It also causes loss of longitudinal and side overlaps.

Q.8 (c) (i) Solution:

Given, $h = 26$ cm, $E = 3 \times 10^5$ kg/cm^2,

$\mu = 0.15$, $a = 15$ cm

$\alpha = 10 \times 10^{-6}$ per °C; $L_x = 450$ cm,

$L_y = 350$ cm

Temperature differential during the day (for warping stresses at interior and edge)

$= 0.6 \times 26 = 15.6°$ C

Temperature differential during the night (for warping stresses at corner)

$= 0.4 \times 26 = 10.4°$ C

$$\text{Radius of relative stiffness, } l = \left[\frac{Eh^3}{12k(1-\mu^2)}\right]^{1/4} = \left[\frac{3 \times 10^5 \times (26)^3}{12 \times 15 \times (1 - 0.15^2)}\right]^{1/4}$$

$= 73.99 \text{ cm} \approx 74$ cm

$$\therefore \quad \frac{L_x}{l} = \frac{450}{74} = 6.08$$

Referring to the given Bradbury's chart for warping stress coefficient corresponding to

$\frac{L_x}{l} = 6.08$, $C_x = 0.90$.

Similarly, for $\frac{L_y}{l} = \frac{350}{74} = 4.73$,

$C_y = 0.74$

Warping stress at interior region of the slab, during the day is,

$$S_{t(i)} = \frac{E\alpha t}{2}\left(\frac{C_x + \mu C_y}{1-\mu^2}\right)$$

$$= \frac{3\times10^5 \times 10\times10^{-6}\times15.6}{2}\left(\frac{0.9+0.15\times0.74}{1-0.15^2}\right)$$

$$= 24.2 \text{ kg/cm}^2$$

Warping stress at edge region, during the day

$$S_{t(e)} = \frac{C_x E\alpha t}{2} = \frac{0.90\times3\times10^5\times10^{-5}\times15.6}{2}$$

$$= 21.06 \text{ kg/cm}^2$$

[Note: Higher of the two values of warping stress coefficient is used.]

Warping stresses at corner region, during the night,

$$S_{t(c)} = \frac{E\alpha t}{3(1-\mu)}\sqrt{\frac{a}{l}}$$

$$= \frac{3\times10^5\times10\times10^{-6}\times10.4}{3\times(1-0.15)}\sqrt{\frac{15}{74}} = 5.51 \text{ kg/cm}^2$$

Q.8 (c) (ii) Solution:

When a bituminous wearing surface is to be placed on an existing untreated foundation layer like earth, gravel, etc, or on W.B.M road surface which is an absorbent surface, it becomes necessary to apply in the initial stage a coat of low viscosity cutback bituminous material. Such a coat is known as prime coat.

Purpose of prime coat:

- It binds the dust and loose particles to form a hard and tough road surface.
- It improves the adhesion between the base and the surface course.
- The capillary voids are plugged and the upward movement of water is restricted.
- When the construction work is carried out by road-mix procedure, it provides a table on which mixing can be done.
- When traffic has to use half of the roadway during construction, it serves as protection layer for the base before the pavement is placed.

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