• Write all steps in detail

• Try to avoid calculation mistake

Revise Gain Margin concept

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ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Systems and Signal Processing + Microprocessors + Electrical Circuits-1 + Control Systems-1

Name : [redacted]

Roll No :

Test Centres	Student's Signature
Delhi ☑ Bhopal ☐ Jaipur ☐ Pune ☐ Kolkata ☐ Bhubaneswar ☐ Hyderabad ☐	[redacted]

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	33
Q.2	18
Q.3	
Q.4	
Section-B	
Q.5	46
Q.6	50
Q.7	42
Q.8	
Total Marks Obtained	189 [redacted]

Signature of Evaluator: Saurabh Kumar

Cross Checked by: ....

Sourabh kumar

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

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1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Systems and Signal Processing + Microprocessors

Q.1 (a) The input $x(n)$ and the impulse response $h(n)$ of a discrete-time LTI system are given by

$$x(n) = \alpha^n u(n),\ h(n) = \alpha^{-n} u(-n);\ 0 < \alpha < 1$$

Using z-transform, find the response $y(n)$.

[12 marks]

Solution

$y(n) = x(n) * h(n)$ —— ①

By taking Z-Transform

$Y(z) = X(z) \cdot H(z)$ —— ② (Convolution Become Multiplication in Z-Domain)

Given $x(n) = \alpha^n u(n)$

By taking Z-Transform

$X(z) = \frac{1}{1-\alpha z^{-1}}$ —— ③

and $h(n) = \alpha^{-n} u(-n)$

h(n) can be rewrite as.

$h(n) = \alpha^{-n} u(-n)$

$\Rightarrow (\alpha^{-1})^n u(-n)$

$= \left(\frac{1}{\alpha}\right)^n u(-n)$

$h(n) = (\alpha)^{-n} u(-n)$

Write in detail

as we know

$\alpha^n u(n) \overset{\text{Z-Transform}}{\rightleftharpoons} \frac{1}{1-\alpha z^{-1}}$

By substituting n = -n

$x(n) \longrightarrow X(z)$

$x(-n) \longrightarrow X(z^{-1})$

$h(n) = \alpha^{-n} u(-n) \overset{\text{ZT}}{\rightleftharpoons} \frac{1}{1-\alpha z}$ —— ④

By substituting value of eq ③ &④ in equation ②

$$Y(z) = \frac{1}{1-\alpha z^{-1}} \cdot \frac{1}{1-\alpha z} \quad \text{———} \; ⑤$$

$$Y(z) = \frac{z}{z-\alpha} \cdot \frac{1}{(1-\alpha z)}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-\alpha)(1-\alpha z)} \quad \text{———} \; ⑥$$

By taking partial fractions.

$$\frac{Y(z)}{z} = \frac{1}{1-\alpha^2} \cdot \frac{1}{z-\alpha} + \frac{\alpha}{1-\alpha^2} \cdot \frac{1}{1-\alpha z}$$

$$Y(z) = \left(\frac{1}{1-\alpha^2}\right)\left(\frac{z}{z-\alpha}\right) + \left(\frac{\alpha}{1-\alpha^2}\right) \frac{z}{1-\alpha z} \quad \text{———} \; ⑥$$

$$X(z) = \frac{z}{z-\alpha} \longrightarrow x(n) = \alpha^n u(n)$$

$$\frac{z}{1-\alpha z} \longrightarrow \alpha^{-(n+1)} u(-(n+1))$$

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By taking IZT of equation ⑥

$$y(n) = \frac{1}{1-\alpha^2} \cdot \alpha^n u(n) + \frac{\alpha}{1-\alpha^2} \cdot \alpha^{-n-1} u(-n-1)$$

$$y(n) = \frac{\alpha^n}{1-\alpha^2} u(n) + \frac{\alpha^{-n}}{1-\alpha^2} u(-n-1)$$

Answer

Good Approach

Q.1 (b) For an 8085 microprocessor, explain the followings :

(i) Logical operations

(ii) Branching operations

[8 + 4 = 12 marks]

Q.1 (c) Find the inverse laplace transform of the following :

(i) $X(s) = \dfrac{s^2+6s+7}{s^2+3s+2}$; Re{s} > −1

(ii) $X(s) = \dfrac{5s+13}{s(s^2+4s+13)}$; Re{s} > 0

[6 + 6 marks]

solution (i) $X(s) = \dfrac{s^2+6s+7}{s^2+3s+2} \quad \sigma > -1$

$X(s) = 1 + \dfrac{3s+5}{s^2+3s+2}$

$X(s) = 1 + \dfrac{2}{(s+1)} + \dfrac{1}{(s+2)}$ ——— ①

By taking ILT of equation ①

Poles are at −1, −2

and it is said $\sigma > -1$ so system will be right sided.

$x(t) = \delta(t) + 2e^{-t}u(t) + e^{-2t}u(t)$

$x(t) = \delta(t) + (2e^{-t} + e^{-2t})u(t)$

Ans

(ii)

$X(s) = \dfrac{5s+13}{s(s^2+4s+13)} \quad \sigma > 0$

$X(s) = \dfrac{5s+13}{s(s+2-3i)(s+2+3i)} \quad \sigma > 0$

$X(s) = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+4s+13} \quad \sigma > 0$

$X(s) = \dfrac{1}{s} + \dfrac{-s+1}{s^2+4s+13} \quad \sigma > 0$

$$X(s) = \frac{1}{s} - \frac{s-1}{s^2+4s+13} \; ; \; \sigma > 0$$

$$X(s) = \frac{1}{s} - \frac{s-1}{(s+2)^2+3^2} \; ; \; \sigma > 0$$

$$X(s) = \frac{1}{s} - \left(\frac{s+2-3}{(s+2)^2+3^2} \right) ; \sigma > 0$$

$$X(s) = \frac{1}{s} - \left[\frac{s+2}{(s+2)^2+3^2} - \frac{3}{(s+2)^2+3^2} \right] ; \sigma > 0$$

$$X(s) = \frac{1}{s} - \left[\frac{s+2}{(s+2)^2+3^2} - \frac{3}{(s+2)^2+3^2} \right] ; \sigma > 0$$

By taking ILT

$$x(t) = u(t) - \left(e^{-2t} \cos 3t - 3 e^{-2t} \sin 3t \right) u(t)$$

$$x(t) = \left(1 - e^{-2t} \cos 3t + 3e^{-2t} \sin 3t \right) u(t)$$

$$\left\{ \begin{array}{l} \because \; e^{-pt} \sin at \longrightarrow \dfrac{a}{(s+p)^2+a^2} \\ e^{-pt} \cos at \longrightarrow \dfrac{s+p}{(s+p)^2+a^2} \end{array} \right\}$$

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Good Approach

Q.1 (d) Explain the flag register of 8086 microprocessor.

[12 marks]

Q.1 (e) Consider the following discrete time system :

(i) $y(n) = |x(n)|$

(ii) $y(n) = \text{sgn}[x(n)]$

Check whether these systems are static or dynamic, linear or non-linear, time varying or time-invariant, causal or non-causal and stable or unstable.

[6 + 6 marks]

Solution (i) $y(n) = |x(n)|$

→ the given system Depends on only present Value of O/P on only present value of I/P So system is static.

→ System is non-linear since modulus operator applied to the O/P.

→ The system.

Delay in I/P

$y(n) = |x(n-n_0)|$ —①

Delay in O/P

$y(n-n_0) = |x(n-n_0)|$ —②

① = ②

So the system is time invarient.

⑥

→ System $y(2) = |x(2)|$

$y(-2) = |x(-2)|$

→ System Depends on only present value so causal.

→ System is stable since for all the value on n the O/P is finite. So given system is stable

(ii) $y(n) = sgn[x(n)]$

$y(1) = sgn[x(1)]$ → $sgn(+Ve) = 1$, $sgn(-Ve) = -1$

→ So system is ~~dynamic~~. system is static

→ system is non linear since sgn function applied to $x(n)$

→ By Delay in I/P

$y(n) = sgn[x(n-n_0)]$

By Delay on O/P

$y(n-n_0) = sgn[x(n-n_0)]$

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for same Delay in I/P, O/P also given same Delay so system is time invarient.

→ System always Depend ending on present Value of I/P so the given system is causal

→ Stable, for all +ve Value of $x(n)$ sgn fn given +1 as O/P.

for all -ve Value of $x(n)$ sgn fn give -1 as O/P

So system is stable.

Q.2 (a) Derive the DFT of the sample data sequence $x(n) = \{1, 1, 2, 2, 3, 3, 0, 0\}$.

[20 marks]

Q.2 (b) Write an assembly language program in 8085 to find 1's and 2's complement of 16-bit number. Assume that the number is stored at 2040 H and store the result at 2050 H and 2052 H respectively. Also give the algorithm of the program.

[20 marks]

Q.2 (c) **(i)** A causal and stable LTI system S has the frequency response :

$$H(\omega) = \frac{4 + j\omega}{6 - \omega^2 + 5j\omega}$$

(a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S.

(b) Determine the impulse response $h(t)$ of S.

(c) What is the output of S when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.

(ii) Compute the linear convolution of the following sequence to obtain $y(n)$

$$x(n) = \{1, \underset{\uparrow}{3}, 0, 4, -2\}$$

$$h(n) = \{2, 4, \underset{\uparrow}{-1}, -3\}$$

[15 + 5 marks]

Sol

$$H(\omega) = \frac{4 + j\omega}{6 + 5j\omega + (j\omega)^2} \text{ —— } ①$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{4 + j\omega}{6 + 5j\omega + (j\omega)^2}$$

$$(j\omega)^2 Y(\omega) + (5j\omega)\, Y(\omega) + 6Y(\omega) = 4X(\omega) + j\omega(X(\omega))$$

By taking IFT —

(a) $$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6\, y(t) = \frac{dx(t)}{dt} + 4x(t)$$

(b) $$H(\omega) = \frac{4 + j\omega}{6 - \omega^2 + 5j\omega}$$

$$H(\omega) = \frac{4 + j\omega}{(j\omega + 2)(j\omega + 3)}$$

$$H(\omega) = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 3}$$

$$H(\omega) = \frac{2}{j\omega + 2} - \frac{1}{j\omega + 3}$$

By taking inverse fourier Transform-

$$\frac{1}{a+j\omega} \xrightarrow{IFT} e^{-at}u(t)$$

(b) $h(t) = (2e^{-2t} - e^{-3t})u(t)$

(c) when i/p ⇒ $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$

By taking fourier Transform

$$X(\omega) = \frac{1}{4+j\omega} - \frac{1}{(4+j\omega)^2}$$

$$X(\omega) = \frac{4+j\omega-1}{(4+j\omega)^2}$$

$$X(\omega) = \frac{j\omega+3}{(4+j\omega)^2} \quad \text{——} ②$$

$$Y(\omega) = X(\omega)\cdot H(\omega) \quad \text{——} ③$$

$$\left\{ \because y(n) = x(n) * h(n)) \;\downarrow FT\; Y(\omega) = X(\omega).\, H(\omega)) \right\}$$

from eq ① ② ③

$$Y(\omega) = \frac{\cancel{4+j\omega}}{(2+j\omega)\cancel{(3+j\omega)}} \cdot \frac{\cancel{(j\omega+3)}}{(4+j\omega)^{\cancel{2}}}$$

$$Y(\omega) = \frac{1}{(2+j\omega)(4+j\omega)}$$

$$Y(\omega) = \frac{A}{2+j\omega} + \frac{B}{4+j\omega}$$

$$Y(\omega) = \frac{1}{2}\left[\frac{1}{2+j\omega} - \frac{1}{4+j\omega}\right]$$

By taking IFT

$$y(t) = \frac{1}{2}\left(e^{-2t} - e^{-4t}\right)u(t)$$ Ans

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Good Approach

(ii) $x(n) = \{1, \underset{\uparrow}{3}, 0, 4, -2\}$

$h(n) = \{2, 4, \underset{\uparrow}{-1}, -3\}$

By convolution

total length of convolution $= n_1 + n_2 - 1$

$= 4 + 5 - 1$

$= 8$

4

h(n) \ x(n)	1	3	0	4	-2
2	2	6	0	8	-4
4	4	12	0	16	-8
-1	-1	-3	0	-4	2
-3	-3	-9	0	-12	6

$y(n) = \{2, 10, 11, \underset{\uparrow}{2}, 3, -12, -10, 6\}$

Good Approach

Q.3 (a) **(i)** Explain the status pins ($\overline{S}_2, \overline{S}_1$ and $\overline{S}_0$) and queue status pins (Q_{S1} and Q_{S0}) of 8086 with their function.

(ii) Discuss the pointers and index group of registers of 8086.

[10 + 10 marks]

Q.3 (b) Design an ideal band reject filter with a desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1, & |\omega| < \dfrac{\pi}{3} \text{ and } |\omega| \geq \dfrac{2\pi}{3} \\ 0, & \text{otherwise} \end{cases}$$

Find the impulse response $h(n)$ and transfer function H(z) of the filter for length $M = 11$.

[20 marks]

Q.3 (c) Ten 8-bit numbers are stored starting from memory location 3000H. Write an 8085 assembly language program, by giving suitable flow chart to find the greatest of the ten numbers and store it at memory location 4000H.

[20 marks]

Q.4 (a) **(i)** The input to a linear shift-invariant system is $x(n) = 2\cos\left(\frac{n\pi}{4}\right) + 3\sin\left(\frac{3n\pi}{4} + \frac{\pi}{8}\right)$.

Find the output if the unit sample response of the system is $h(n) = \dfrac{2\sin(n-1)\frac{\pi}{2}}{(n-1)\pi}$.

(ii) Consider a system described by the difference equation

$$y(n) = y(n-1) - y(n-2) + 0.5x(n) + 0.5x(n-1).$$

Find the response of this system to the input $x(n) = (0.5)^n u(n)$, with initial conditions $y(-1) = 0.75$ and $y(-2) = 0.25$

[10 + 10 marks]

Q.4 (b) For $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$. Compute the DFT, $X(k)$ using DIF FFT algorithm.

[20 marks]

Q.4 (c) **(i)** Draw the lattice filter implementation of FIR filter $H(z) = 8 + 4z^{-1} + 2z^{-2} + z^{-3}$.

(ii) It is required to move a 16-byte long data string from offset 4000H to offset 5000H. Write an assembly language program to accomplish the above task for 8085 microprocessor.

[12 + 8 marks]

Section B : Electrical Circuits - 1 + Control Systems - 1

Q.5 (a) The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+20)}$$

Find the range of K for which the system is stable. Also show that the system response can oscillate at two different frequencies.

[12 marks]

Solution given OLTF ⇒ $G(s) = \dfrac{K(s+1)}{s(s-1)(s^2+4s+20)}$

$$q(s) = 1 + G(s) = 0$$

$$q(s) = s(s^3+4s^2+20s-s^2-4s-20) + K(s+1) = 0$$

$$q(s) = s^4 + 3s^3 + 16s^2 - 20s + Ks + K = 0$$

$$q(s) = s^4 + 3s^3 + 16s^2 + s(K-20) + K = 0$$

Stability of system can be find out using Routh array -

$$\begin{array}{c|ccc} s^4 & 1 & 16 & K \\ s^3 & 3 & K-20 & \\ s^2 & \dfrac{48-K+20}{3} & K & \\ s^1 & \dfrac{\left(\frac{68-K}{3}\right)(K-20)-3K}{\frac{68-K}{3}} & & \\ s^0 & K & & \end{array}$$

for stable system the first column of Routh array should have same sign of all entries.

so $\frac{48-k+20}{3} > 0$

$\frac{68-k}{3} > 0$

$\boxed{68>k}$ —— ①

and from necessary condition $\underline{K>0}$

$$\frac{\left(\frac{68-k}{3}\right)(k-20) - 3k}{\left(\frac{68-k}{3}\right)} > 0$$

$(68-k)(k-20) - 9k > 0$

$68k - 1360 - k^2 + 20k - 9k > 0$

$-k^2 + 79k - 1360 > 0$

$k^2 - 79k + 1360 < 0$

$(k-53.65)(k-25.34) < 0$

$K < 53.65$, $K < 25.34$ —— ②

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from equation ① & ②

$\boxed{K < 25.34}$

Good Approach

so condition for stability $\boxed{0<k<25.34}$

→ frequency of oscillation can find out by s^2 term polynomial

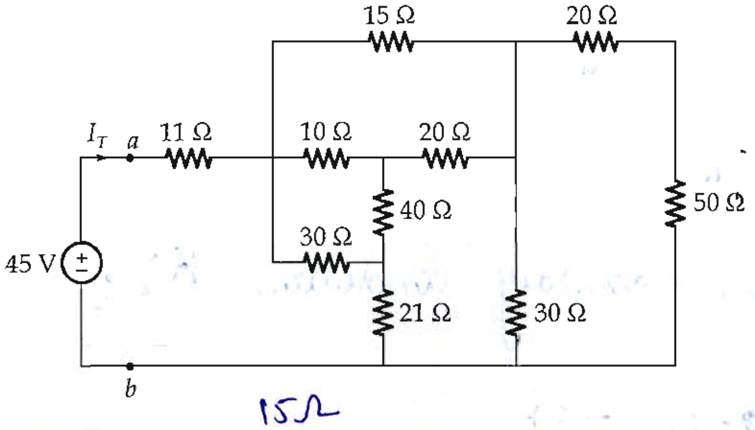
$\left(\frac{68-k}{3}\right)s^2 + k = 0$

$14.22 s^2 + 25.34 = 0$

$\boxed{s = \pm j1.3349}$

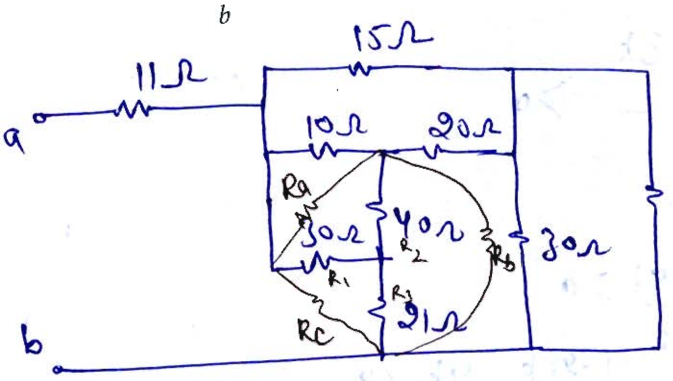
so freq. of oscillation $\underline{\omega_n = j1.3349}$ rad/sec

Q.5 (b) For the circuit shown in figure below, obtain the equivalent resistance at terminals $a - b$. Also find total current I_T as indicated below.



[12 marks]

Solution



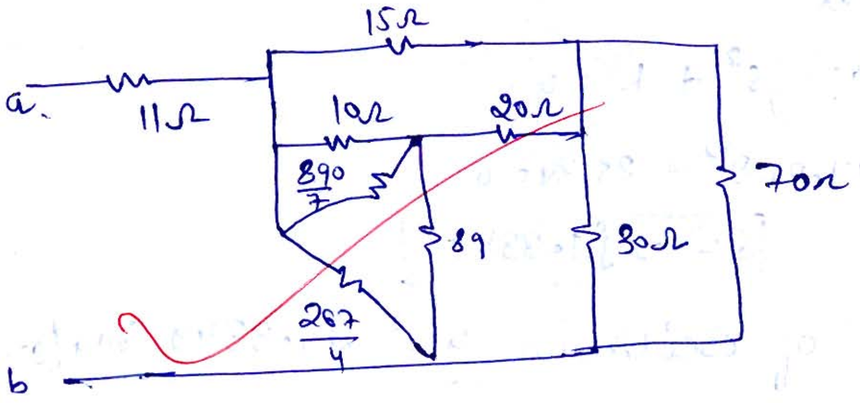
By making Star to Delta conversion.

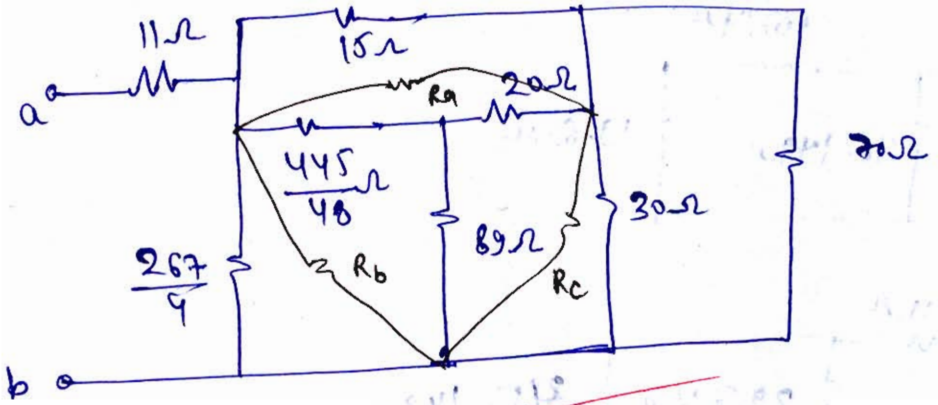
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_4}{R_3} \quad \text{and similarly -}$$

$$R_a = \frac{(30)(40)+(40)(21)+(21)(30)}{21} = \frac{2670}{21} = \frac{890}{7}\Omega$$

$$R_b = \frac{(30)(40)+(40)(21)+(21)(30)}{30} = \frac{2670}{30} = 89\Omega$$

$$R_c = \frac{(30)(40)+(40)(21)+(21)(30)}{40} = \frac{267}{4}\Omega$$



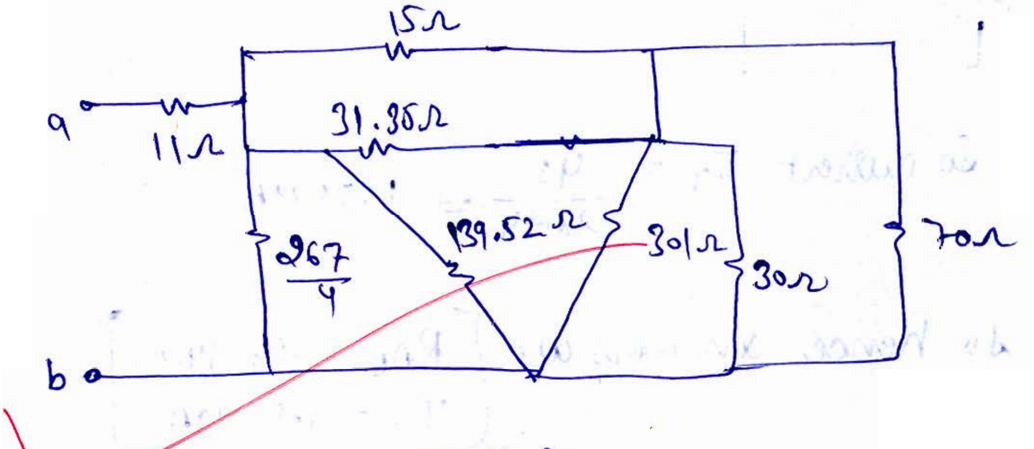


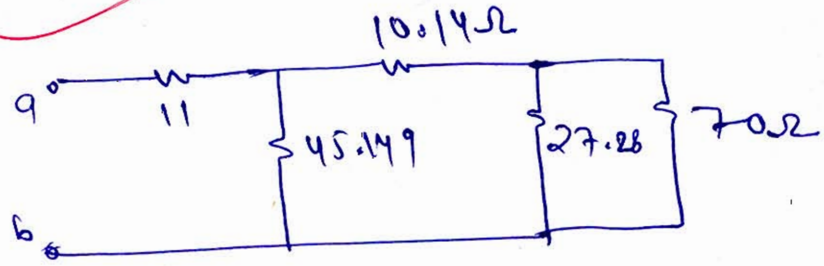
By again making star to Delta transformation

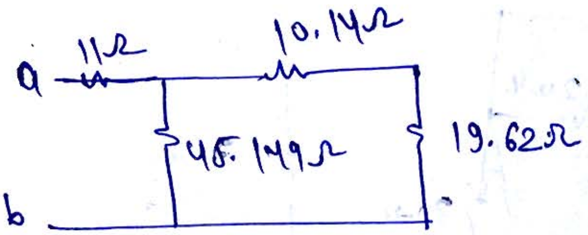
$$R_a = \frac{\left(\frac{445}{48}\right)(20) + (20)(89) + (89)\left(\frac{445}{48}\right)}{89} = 31.35\Omega$$

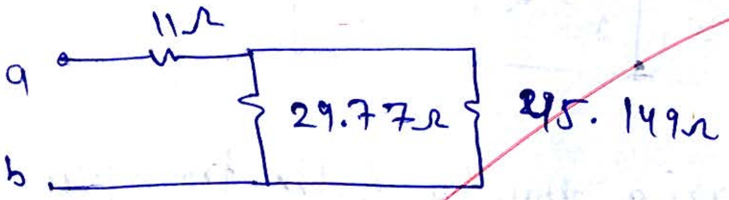
$$R_b = \frac{\left(\frac{445}{48}\right)(20) + (20)(89) + (89)\left(\frac{445}{48}\right)}{20} = 139.52\Omega$$

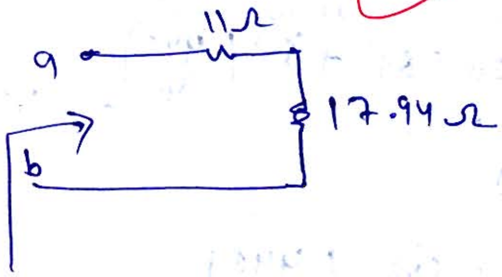
$$R_c = \frac{\left(\frac{445}{48}\right)(20) + (20)(89) + (89)\left(\frac{445}{48}\right)}{\left(\frac{445}{48}\right)} = 301$$





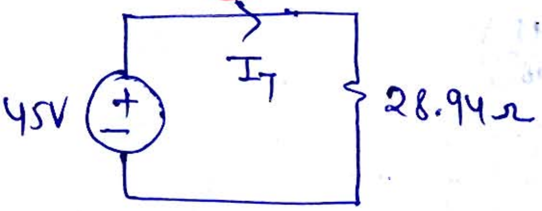






$R_{AB} = 28.94\Omega$

So the original signal can redrawn as



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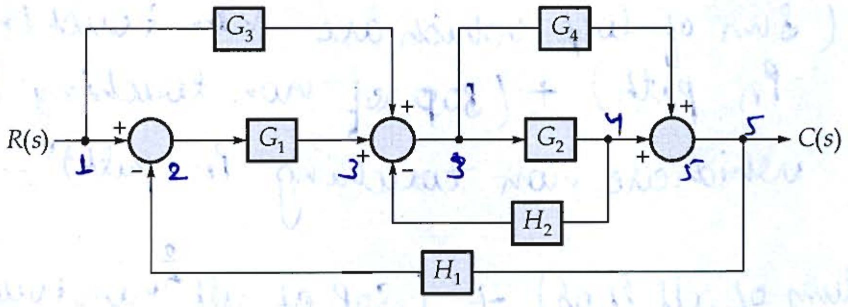
So current $I_T = \frac{45}{28.94} = 1.5549A$

Good Approach

So hence answers are $\left[\begin{array}{l} R_{AB} = 28.94\Omega \\ I_T = 1.5549A \end{array}\right]$

Q.5 (c) Determine the transfer function $\frac{C(s)}{R(s)}$ for a system represented by the block diagram shown below :

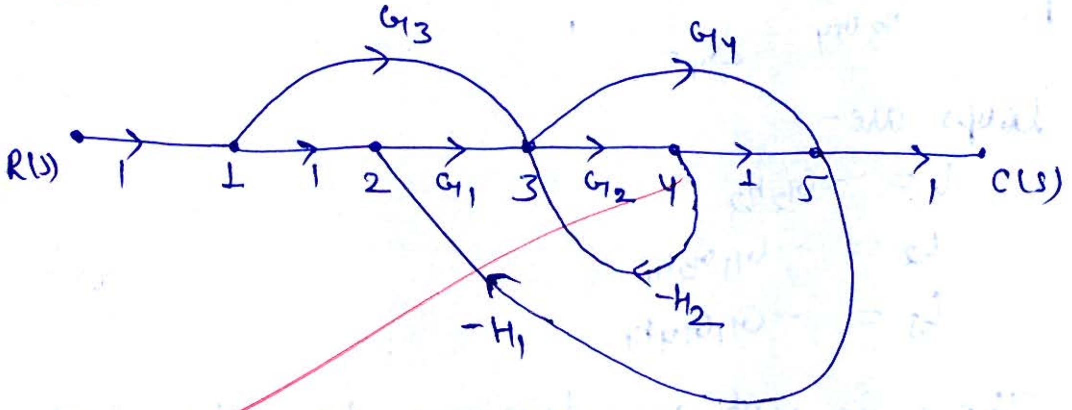


[12 marks]

Solution

By changing given block diagram into equivalent signal flow graph -

Here total 5 nodes are present in given system -



The ~~tr~~ansfer function can be obtain by using Manson's gain formula -

according to Mason's gain formula,

$$Tf = \frac{\sum_{n=1}^{k} P_n \Delta_n}{\Delta}$$

Here K→ total Number of forward path

$P_n \rightarrow$ gain of Pth forward path

Δ_n → Determinant corresponds to P_n forward Path.

$\Delta_n = 1 -$ (Sum of loop which are non touching to P_n path) + (SOP of non touching loop which are non touching P_n path) ---

$\Delta = 1 -$ (Sum of all loop) + (SOP of all 2 non touching loop) −(SOP of all 3 non touching loop) ---

For given system total 4 forward path present

$P_1 = G_1G_2 \quad \Delta_1 = 1$

$P_2 = G_3G_2 \quad \Delta_2 = 1$

$P_3 = G_1G_4 \quad \Delta_3 = 1$

$P_4 = G_3G_4 \quad \Delta_4 = 1$

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loops are-

$L_1 = -G_2H_2$

$L_2 = -G_1G_2H_1$

$L_3 = -G_1G_4H_1$

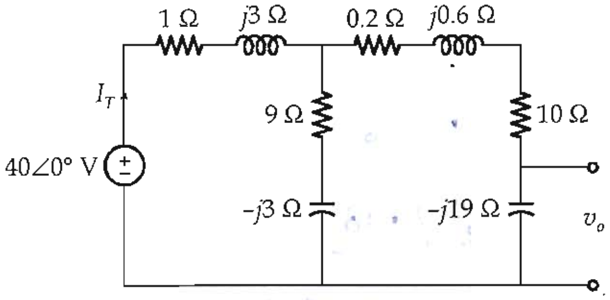
Good Approach

There is not any two non touching loop are present.

$$TF = \frac{G_1G_2 + G_3G_2 + G_1G_4 + G_3G_4}{1-(-G_2H_2 - G_1G_2H_1 - G_1G_4H_1)}$$

$$TF = \frac{G_1G_2 + G_3G_2 + G_1G_4 + G_3G_4}{1+G_2H_2 + G_1G_2H_1 + G_1G_4H_1}$$

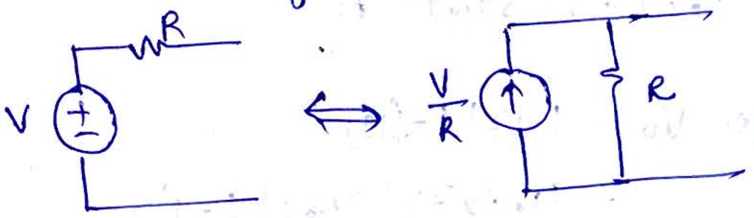
Q.5 (d) Use the concept of source transformation to find the phasor voltage v_o in the circuit shown below. Also, calculate the total current I_T of the circuit.



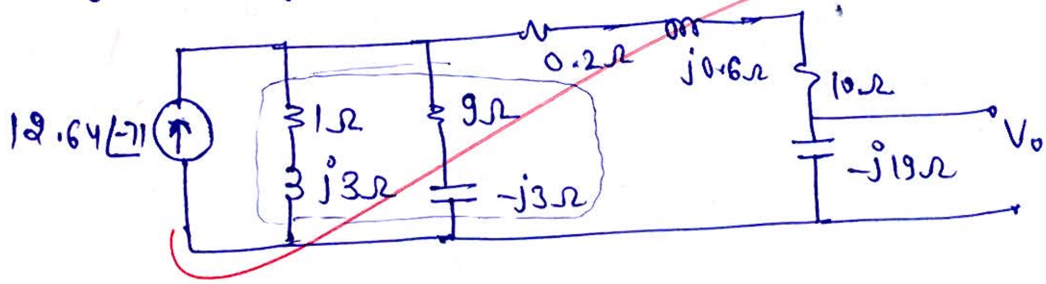
[12 marks]

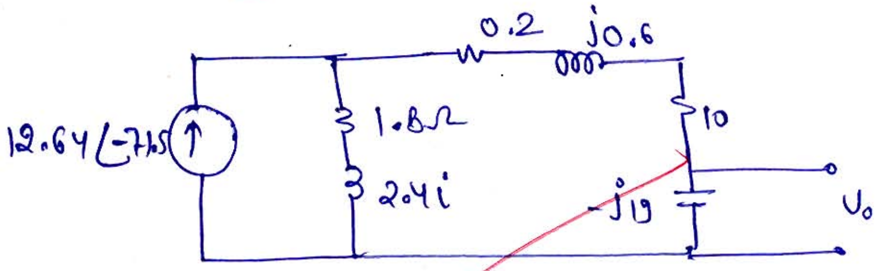
Solution

Source Transformation -

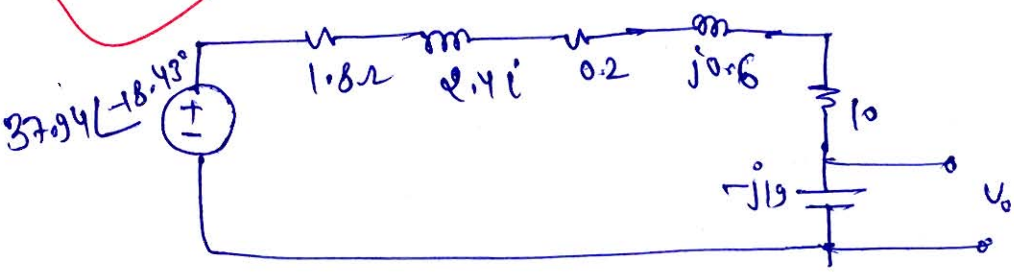


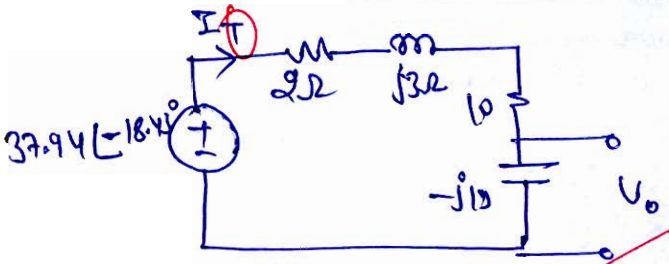
So by applying source Transformation-





again changing current source to equivalent voltage source-





$$I_T = \frac{37.94\angle -18.43°}{(2+j3)+(10-j19)}$$

$$\boxed{I_T = 1.8973\angle 34.69 \text{ A}}$$

voltage $V_o = (I_T)(-j19)$

$= (1.8973\angle 34.69)(-j19)$

$$\boxed{V_o = 36.04\angle -55.3° \text{ V}}$$

9

Good Approach

$I_T = ?$

Restore circuit and find I_T

Q.5 (e) A second-order servo-mechanism with unity feedback, has the open-loop transfer function $G(s) = \frac{K}{s(s+4)}$. Find the gain k so that the steady-state error shall not exceed 0.4 degree when the input shaft is rotated at 3 rpm. (Assume input $r(t) = \omega t$)

[12 marks]

Solution

$OLTF = GH = \frac{k}{S(S+4)}$

$i/P = \frac{3}{S^2}$

error = 0.4

the velocity error constant for a system

$K_V = \lim_{S \to 0} S(OLTF)$

$K_V = \lim_{S \to 0} S\left(\frac{k}{S(S+4)}\right)$

$K_V = \lim_{S \to 0} \frac{K}{(S+4)} = \frac{k}{4}$

~~3~~

4

and error ⇒ $e_{ss} = \frac{A}{K_V}$

$e_{ss} = 0.4 = \frac{3}{K_V}$

$0.4 = \frac{3}{\frac{K}{4}}$

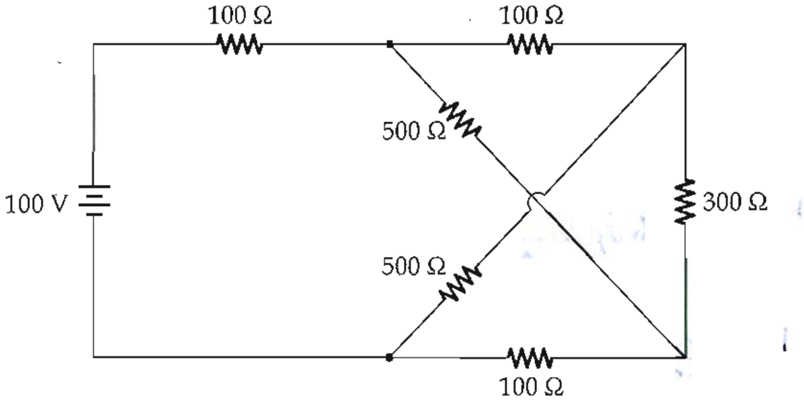
$0.4 = \frac{12}{K}$

$K = \frac{12}{0.4}$

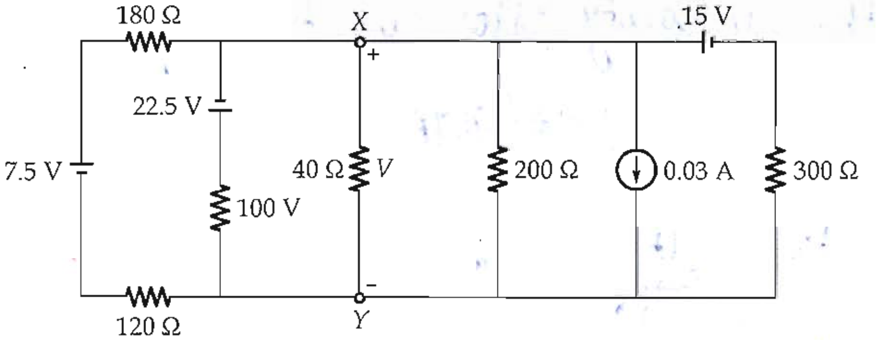
$\boxed{K = 30}$

so given OLTF will be $= \frac{30}{S(S+4)}$

Q.6 (a) (i) Determine the current supplied by the battery in the circuit shown below by using Mesh Analysis only.



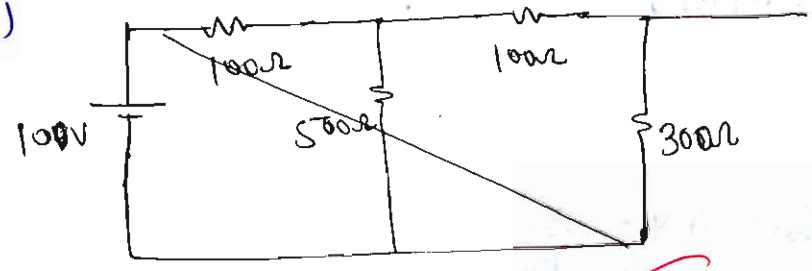
(ii) By constructing Millman equivalent voltage source with respect to terminals *x*-*y*, find the voltage across 40 Ω resistor.



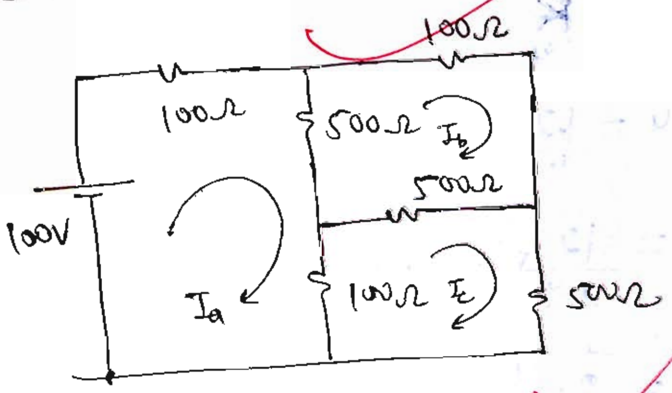
[10 + 10 marks]

Solution

(i)



Circuit can be Redrawn as –



By KVL in loop ①

$-100 + 700 i_a - 500 i_b - 100 i_c = 0$

$$700 i_a - 500 i_b - 100 i_c = 100 \quad —— ①$$

By KVL in loop ②

$$-500 i_a + 1100 i_b - 500 i_c = 0 \quad —— ②$$

By KVL in loop ③

$$-100 i_a - 500 i_b + 1100 i_c = 0 \quad —— ③$$

from equation ① ② & ③

$$\begin{bmatrix} 700 & -500 & -100 \\ -500 & 1100 & -500 \\ -100 & -500 & 1100 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

using Cramer's Rule -

$$i_a = \frac{\begin{vmatrix} 100 & -500 & -100 \\ 0 & 1100 & -500 \\ 0 & -500 & 1100 \end{vmatrix}}{\begin{vmatrix} 700 & -500 & -100 \\ -500 & 1100 & -500 \\ -100 & -500 & 1100 \end{vmatrix}} = 0.2857A$$

0.3A

8

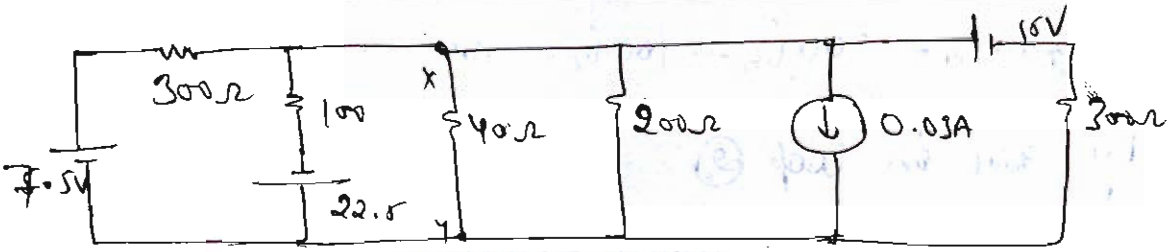
$$i_b = \frac{\begin{vmatrix} 700 & 100 & -100 \\ -500 & 0 & -500 \\ -100 & 0 & 1100 \end{vmatrix}}{336 \times 10^6} = 0.1785 A$$

0.2A

$$i_c = \frac{\begin{vmatrix} 700 & -500 & 100 \\ -500 & 1100 & 0 \\ -100 & -500 & 0 \end{vmatrix}}{336 \times 10^6} = 0.1071 A$$

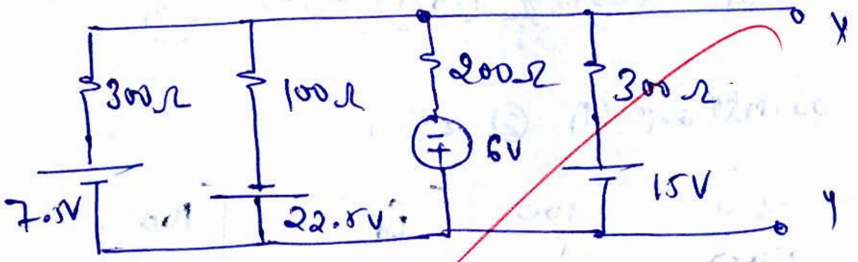
0.1A

(ii)



⇓

By converting into standard millmans form



$$V_{th} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

$$V_{th} = \frac{\frac{7.5}{300} + \frac{15}{300} - \frac{22.5}{100} - \frac{6}{200}}{\frac{1}{300} + \frac{1}{300} + \frac{1}{100} + \frac{1}{200}} = -8.3076V$$

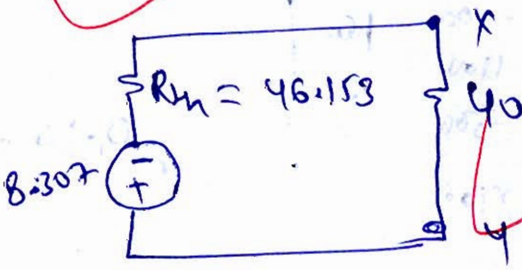
Good Approach

$$R_{th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{300} + \frac{1}{300} + \frac{1}{100} + \frac{1}{200}}$$

10

$$R_{th} = 46.153\Omega$$

So equivalent circuit



$$V_{xy} = \frac{40}{40+46.153} V_{th}$$

$$V_{xy} = -3.8571V$$

Q.6 (b) The open-loop transfer function of a system is $G(s) = \dfrac{K(s+4)}{(s+10)^2}$. What must be the value of K that the gain cross-over frequency is $\omega_{gc} = 30$ rad/s. Also find gain margin and phase margin for that value of K. Also comment on stability of system.

[20 marks]

Solution

$$G(s) = \frac{k(s+4)}{(s+10)^2}$$

at gain cross over frequency the mag of OLTF will be 1.

$$G(j\omega) = \frac{k(j\omega+4)}{(j\omega+10)^2}$$

$$|G(j\omega)| = \left.\frac{k\sqrt{\omega^2+16}}{(\omega^2+100)}\right|_{\omega_{gc}=30\text{rad/sec}} = 1$$

$$\frac{k\sqrt{30^2+16}}{(30^2+100)} = 1$$

$$k = \frac{1000}{\sqrt{916}} = 33.0409$$

for $k = 33.0409$

$$\text{OLTF} \quad G(j\omega) = \frac{33.04(j\omega+4)}{(j\omega+10)^2}$$

$$\text{gain margin} = 20\log\frac{1}{|G(j\omega)|_{\text{at }\omega_{pc}}}$$

$\omega_{pc} \rightarrow$ Phase Cross over frequency
freq. at which system phase become $-180°$

$$\phi = \tan^{-1}\left(\frac{\omega}{4}\right) - 2\tan^{-1}\left(\frac{\omega}{10}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega}{4}\right) - 2\tan^{-1}\left(\frac{\omega}{10}\right) = -180°$$

~~$\tan^{-1}\left(\frac{\omega}{4}\right) - \omega$~~

$$\tan^{-1}\left(\frac{\omega}{4}\right) - \left[\tan^{-1}\left(\frac{\frac{\omega}{10}+\frac{\omega}{10}}{1-\frac{\omega^2}{100}}\right)\right] = -180°$$

$$\tan^{-1}\left[\frac{\frac{\omega}{4} - \frac{20\omega}{100-\omega^2}}{1+\left(\frac{\omega}{4}\right)\left(\frac{20\omega}{100-\omega^2}\right)}\right] = -180°$$

$$\frac{\omega}{4} - \frac{20\omega}{100-\omega^2} = 0$$

$$\frac{\omega}{4} = \frac{20\omega}{100-\omega^2} \Rightarrow 100-\omega^2 = 80$$

$$\boxed{\omega_{pc} = 4.47}\ \text{rad/sec}$$

$$|G(j\omega)|_{\omega_{pc}} = \left.\frac{33.04\sqrt{\omega^2+4}}{(\omega^2+100)}\right|_{\omega=4.47} = 1.3488$$

$$\boxed{GM = 20\log\frac{1}{|G(j\omega)|} = -2.599\ dB}$$

$$PM = 180 + \phi|_{\omega_{gc}}$$

$$PM = 180 + \tan^{-1}\left(\frac{\omega}{4}\right) - 2\tan^{-1}\left(\frac{\omega}{10}\right)$$

$$\omega_{gc} = 30\ \text{rad/sec}$$

$$Pm = 180 + \tan^{-1}\left(\frac{30}{4}\right) - 2\tan^{-1}\left(\frac{30}{10}\right)$$

$$\boxed{Pm = 119.27°}$$

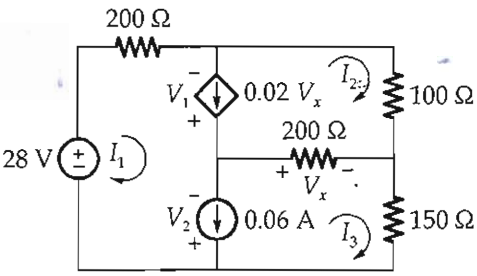
The system is stable for some condition since for the value K = 33.04 GM < 0 & PM > 0 so it show it is conditionally stable

ωpc and GM is undefined system is stable

Good Approach

13

Q.6 (c) Find the values for the loop currents I_1 , I_2 , I_3 and the power delivered by each independent source.



[20 marks]

Sol^n

loop ① & ~~loop~~ ② & loop ③

By K.V.L

$-28 + 200 i_1 + 100 i_2 + 150 i_3 = 0$

$200 i_1 + 100 i_2 + 150 i_3 = 28$ —— ①

By ohm's law -

$V_x = 200 (i_3 - i_2)$ —— ②

By Super Mesh -

$i_1 - i_2 = 0.02 V_x$ —— ③

& $i_1 - i_3 = 0.06$ —— ④

from eq ② & ③

$i_1 - i_2 = 0.02 (200 (i_3 - i_2))$

$i_1 - i_2 = 4 i_3 - 4 i_2$

$i_1 + 3 i_2 - 4 i_3 = 0$ —— ⑤

from equation ① ④ & ⑤

$$\begin{bmatrix} 200 & 100 & 150 \\ 1 & 0 & -1 \\ 1 & 3 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 28 \\ 0.06 \\ 0 \end{bmatrix}$$

$$i_1 = \frac{\begin{vmatrix} 28 & 100 & 150 \\ 0.06 & 0 & -1 \\ 0 & 3 & -4 \end{vmatrix}}{\begin{vmatrix} 200 & 100 & 150 \\ 1 & 0 & -1 \\ 1 & 3 & -4 \end{vmatrix}} = 0.1\ A$$

$$i_2 = \frac{\begin{vmatrix} 200 & 28 & 100 \\ 1 & 0.06 & -1 \\ 1 & 0 & -4 \end{vmatrix}}{\begin{vmatrix} 200 & 100 & 150 \\ 1 & 0 & -1 \\ 1 & 3 & -4 \end{vmatrix}} = 0.02\ A$$

$$i_3 = \frac{\begin{vmatrix} 200 & 100 & 28 \\ 1 & 0 & 0.06 \\ 1 & 3 & 0 \end{vmatrix}}{\begin{vmatrix} 200 & 100 & 150 \\ 1 & 0 & -1 \\ 1 & 3 & -4 \end{vmatrix}} = 0.04\ A$$

19

Good Approach

By independent source voltage-

$P_1 = 28 * I_1 = 28 \times 0.1 = 2.8W$ (Delivered)

By independent current source

$P_2 = V_2 * 0.06\ A$ —Ⓐ

By KVL in loop ③

$V_2 + V_x + 150 i_3 = 0$

$V_2 + 200(i_3 - i_2) + 150 i_3 = 0$

$V_2 = -10V$

from Ⓐ ⇒ so $P_2 = -10 \times 0.06 = -0.6W$

(0.6W absorbed.)

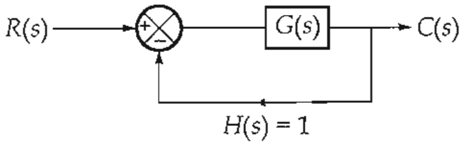
Q.7 (a) (i) The response of a feedback system to a unit step input is

$$C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}.$$

(a) Obtain the expression for the closed loop transfer function.

(b) Determine the undamped natural frequency and damping ratio of the system.

(ii) Consider the unity feedback system whose open loop transfer function $G(s)H(s) = \frac{4}{s(s+5)}$. When this system is excited by a unit step input then calculate the output response and comment on the peak overshoot of the system.



[10 + 10 marks]

Solution (i) $C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$

By taking laplace -

$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$C(s) = \frac{(s^2+70s+600) + 0.2(s^2+10s) - 1.2(s^2+60s)}{s(s+60)(s+10)}$$

$$C(s) = \frac{600}{s(s+60)(s+10)}$$

i/p = step (given)

$$R(s) = \frac{1}{s}$$

$$Tf = \frac{C(s)}{R(s)} = \frac{600}{(s+60)(s+10)}$$

$$Tf = \frac{600}{s^2+70s+600}$$

By comparing standard second order -

$$Tf = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\omega_n^2 = 600 \Rightarrow \omega_n = 24.49$ rad/sec

$2\xi\omega_n = 70$

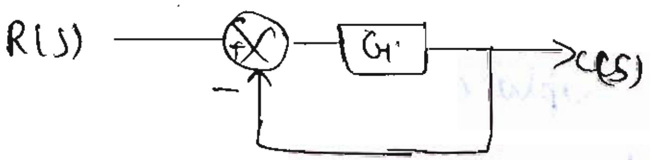
$\xi = \frac{70}{2\omega_n} = 1.4288$

9

Good Approach

(ii) $OLTF = GH = \frac{4}{S(S+5)}$

I/P = unit step



$Tf = \frac{GH}{1+GH} = \frac{\frac{4}{S(S+5)}}{1+\frac{4}{S(S+5)}}$

$Tf = \frac{4}{s^2+5s+4}$ —— ①

$I/P = \frac{1}{s}$

$C(s) = O/P = \frac{4}{S(S+1)(S+4)} = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+4}$

By taking ILT

$C(s) = \frac{1}{s} - \frac{4/3}{S+1} + \frac{1/3}{S+4}$

$c(t) = \left(1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}\right)u(t)$

$$TF = \frac{4}{s^2 + 5s + 4}$$

$\omega_n = 2$

$2\xi\omega_n = 5$

$$\xi = \frac{5}{2 \times 2} = 1.25$$

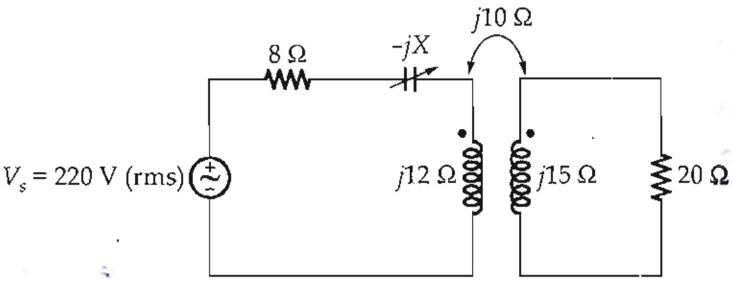
Peak overshoot $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

→ so for This system $\xi = 1.25$ which is greater than 1 so can't find out peak overshoot.

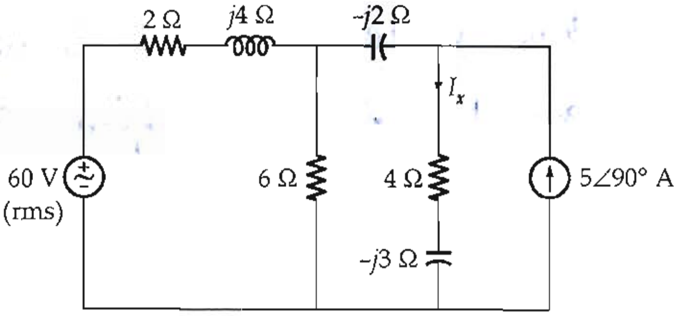
⑨

Good Approach

Q.7 (b) (i) For the circuit shown in figure, calculate the value of X that will give maximum power transfer to the 20 Ω load. Also calculate the maximum power delivered to load.

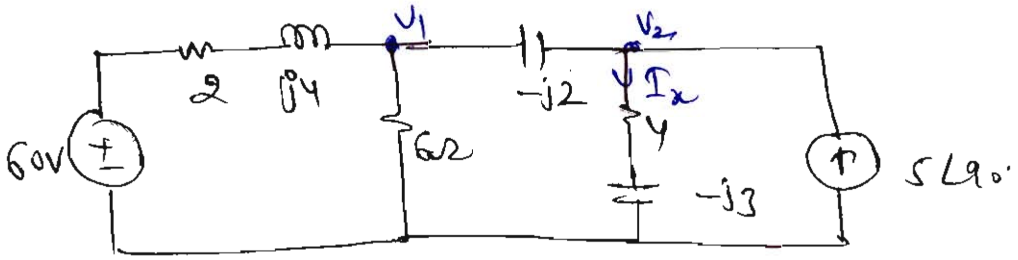


(ii) Calculate current I_X for the circuit shown below :



[12 + 8 marks]

Solution (ii)



By KCL at V_1 & V_2

$$\frac{V_1 - 60}{2+j4} + \frac{V_1}{6} + \frac{V_1 - V_2}{-j2} = 0 \quad \text{—①}$$

$$\frac{V_2 - V_1}{-j2} + \frac{V_2}{4-j3} = 5\angle 90 \quad \text{—②}$$

3

from equation ①

$$V_1\left(\frac{1}{2+j4} + \frac{1}{6} + \frac{1}{-j2}\right) + \frac{V_2}{j2} = \frac{60}{2+j4}$$

$$V_1\left(\frac{4}{15}+\frac{3}{10}i\right)+\frac{V_2}{j2}=6-12i \quad \text{——} ③$$

from eq^n ②

$$V_2\left(\frac{-1}{j2}+\frac{1}{4-j3}\right)+\frac{V_1}{j2}=5\angle 90^\circ$$

$$V_2\left(\frac{4}{25}+\frac{31}{50}i\right)+\frac{V_1}{j2}=5\angle 90 \quad \text{——} ④$$

By multiply eq ③ to $\left(\frac{1}{j2}\right)$ & eq^n ④ to $\left(\frac{4}{15}+\frac{3}{10}i\right)$ and then subtracting

$$V_1\left(\frac{1}{j2}\right)\left(\frac{4}{15}+\frac{3}{10}i\right)+\frac{V_2}{(j2)^2}=\frac{6-12i}{j2}$$

$$-\ V_1\left(\frac{1}{j2}\right)\left(\frac{4}{15}+\frac{3}{10}i\right)\ -\ V_2\left(\frac{4}{15}+\frac{3}{50}i\right)\left(\frac{4}{15}+\frac{3}{10}i\right)\ -\ 5\angle 90\left(\frac{4}{15}+\frac{3}{10}i\right)$$

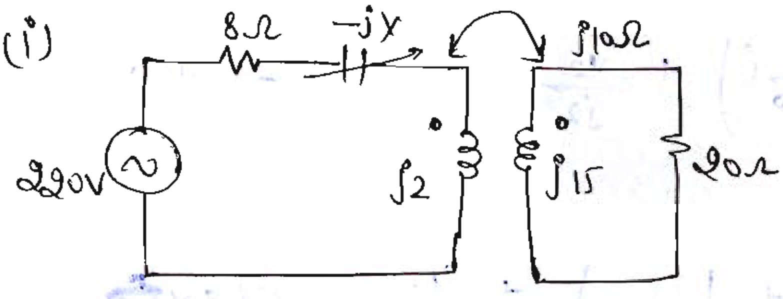
$$V_2\left[\frac{1}{(j2)^2}+\frac{43}{300}-\frac{16}{75}i\right)=(-3i+6)-\left(\frac{-3}{2}+\frac{4}{3}i\right)$$

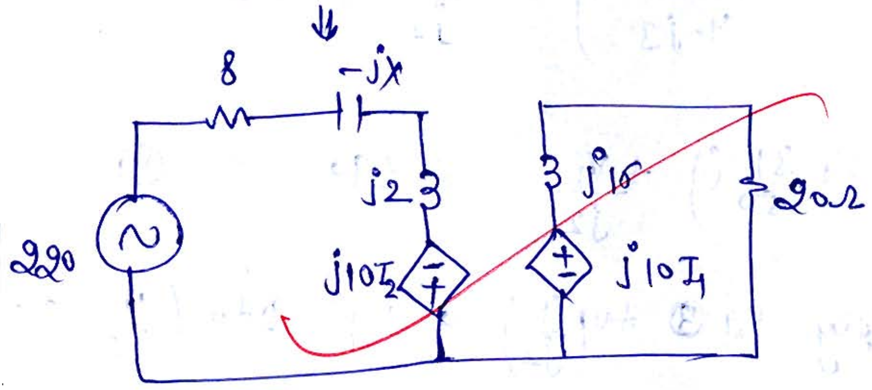
$$V_2\left(\frac{-8}{75}-\frac{16}{75}i\right)=\left(\frac{15}{2}-\frac{13}{3}i\right)$$

$$V_2=36.31\angle 86.54^\circ\ V$$

and $I_x=\dfrac{V_2}{4-j3}=7.26\angle 123.41^\circ$ A

(i)



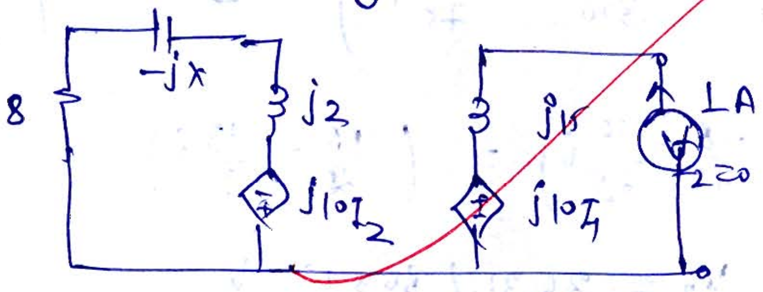


5

for loop ①

$$-220 + (8 + j2 - jx) I_1 - j10 I_2 = 0 \quad —①$$

for max^m transfer of power in $R_L = 20\Omega$ the o/p should open and calculate Z_{th} By Deactivating all the sources



$$\boxed{I_2 = 1A}$$

$$-j10 I_2 + (8 + j2 - jx) I_1 = 0$$

$$I_1 = \frac{j10}{8 + j(2-x)}$$

and $V = j10 I_1 + j15(1)$

$V = j10 I_1 + j15$

$V = j15 + j10\left(\frac{j10}{8+j(2-x)}\right)$

$V = j15 - \frac{100}{8+j(2-x)}$

$Z_{th} = \frac{V}{I} = \frac{V}{1} = V$

$Z_{th} = V = j15 - \frac{100}{8+j(2-x)}$ ⟵ ①

and V_{th} ⟶

From circuit ①

loop ①

$220 = [8+(j2-jx)]I_1 - j10I_2 \Rightarrow I_1 = \frac{220}{8+j(2-x)}$

& $I_2 = 0$ (since o/p ⇒ open)

$V_{th} = j10 I_1$

$V_{th} = j10\left(\frac{220}{8+j(2-x)}\right)$ ⟵ ②

so from equation ① for max^m power Transfer current ~~should~~ max^m and for max^m current

~~$2-x=0$~~

~~$\boxed{X=2}$~~

so $V_{th} = \frac{j2200}{8} =$ & $Z_{th} = j15 - \frac{100}{8}$

$\boxed{V_{th} = j275}$ $= j15 - 12.5$

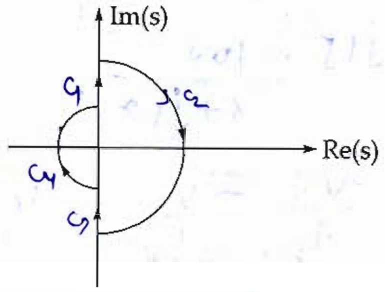
~~$P_{max} = \frac{V_{th}^2}{4X_{th}}$~~ $=$ ~~$= 19.52\angle 129.8$~~

~~$\boxed{P = 5377 \text{ watt}}$~~

Q.7 (c) The open loop transfer function of a unity negative feedback system is given as

$$G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$

The Nyquist contour in s-plane encloses the entire right half plane and a small neighbour around origin in left half plane, as shown in figure. Draw the Nyquist plot of the system and examine its closed loop stability.



[20 marks]

Solution (c)

$$G(j\omega) = \frac{1+4j\omega}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

The contour along C_1 :-

$S = j\omega$ & $\omega; 0 \longrightarrow \infty$.

→ 16

$$|G(j\omega)| = \frac{\sqrt{1+4\omega^2}}{-\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \quad \text{—} ①$$

$$\angle G(j\omega) = \tan^{-1}(4\omega) - 180° - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \quad ②$$

from equation ① & ②

$\omega : 0 \longrightarrow \infty$
$M : \infty \longrightarrow 0$
$\phi : -180 \longrightarrow -270°$ — (A)

for contour along C_2

$C_2 : \longrightarrow \quad j\omega \longrightarrow Re^{j\theta}$
$R \longrightarrow \infty$
$\theta \longrightarrow +90$ to $-90°$

$$G(Re^{j\theta}) = \frac{1+4Re^{j\theta}}{(R^2e^{2j\theta})(1+Re^{j\theta})(1+2Re^{j\theta})}$$

$R \longrightarrow \infty$

$$\underset{R\longrightarrow\infty}{G(Re^{j\theta})} = \frac{4Re^{j\theta}}{(R^2e^{2j\theta})(2R^2e^{2\theta})} \Rightarrow \cancel{\phi = -2\theta}$$

$\phi = -3\theta$

~~C_2: 0 ∠ -180 to 180~~ ——— Ⓑ

C_2: $0 \underline{\angle -270 \text{ to } 270'}$

along contour C_3

$s \rightarrow -j\omega$

$$|G(-j\omega)| = \frac{1-4j\omega}{(-j\omega)^2(1-j\omega)(1-2j\omega)}$$

~~$\phi = 180 - \tan$~~

$\phi = -\tan^{-1}(4\omega) + 180' + \tan^{-1}(\omega) + \tan^{-1}(2\omega)$

$S: \omega \rightarrow \infty \longrightarrow 0$

$M: 0 \longrightarrow \infty$

$\phi: 270 \longrightarrow 180'$ ——— ⓒ

along contour C_4.

C_4: $re^{j\theta}$

$r \rightarrow 0$

$\theta \rightarrow -90 \text{ to } 90'$

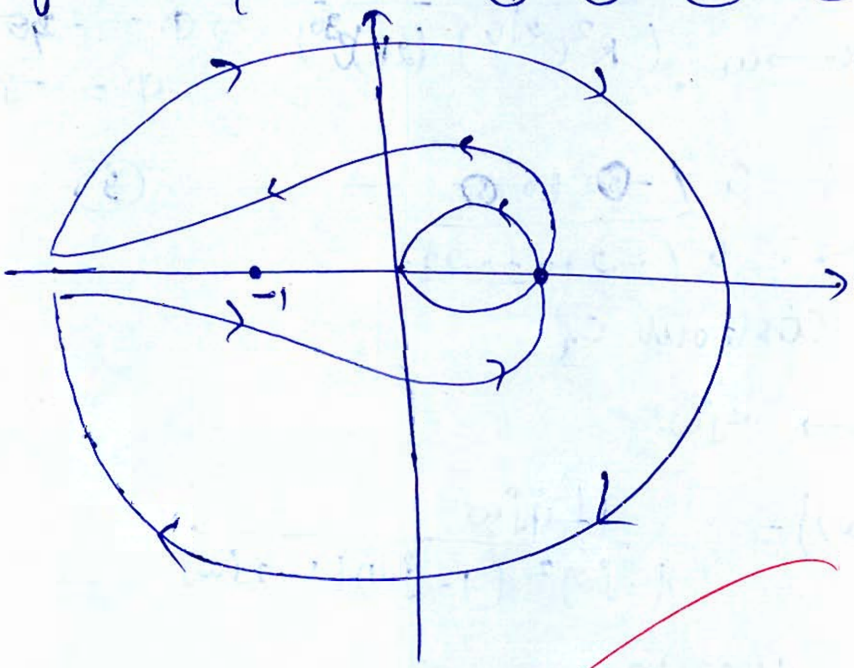
$$\underset{r\longrightarrow 0}{\text{lt}}\ G(re^{j\theta}) = \frac{1+4re^{j\theta}}{(r^2e^{2j\theta})(1+re^{j\theta})(1+2re^{j\theta})}$$

$$G(re^{j\theta}) = \frac{1}{(r^2e^{j2\theta})}$$

$\phi = -2\theta$

G_4 : $\lim_{r \to 0} = \infty \angle 180 \text{ to } -180^\circ$ ——— (D)

from equation (A) (B) (C) (D)



16

encirclement along $(-1, 0)$ →

$N = -1 + 1 = 0$

once in clockwise and once in ACW

so $N = 0$

as from question $P = 0$

since no pole present in right half of s-plane

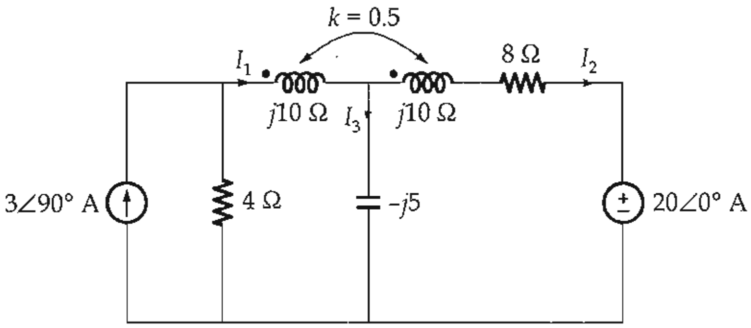
so $P = 0$

So for given question

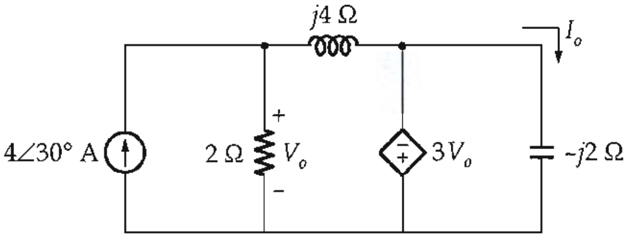
$\boxed{N = P}$

So the system is stable.

Q.8 (a) **(i)** Determine the current I_1, I_2 and I_3 in the circuit shown. Take $\omega = 1000$ rad/sec.



(ii) Calculate voltage V_o for the circuit shown below.



[15 + 5 marks]

Q.8 (b) Let $G(s) = \dfrac{K(s-1)}{(s+2)(s+3)}$ with unity negative feedback.

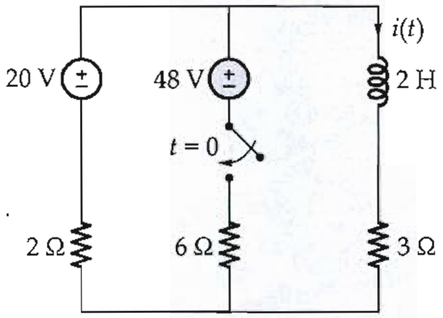
(i) Find the range of K for closed loop stability.

(ii) Plot the root locus for $K < 0$.

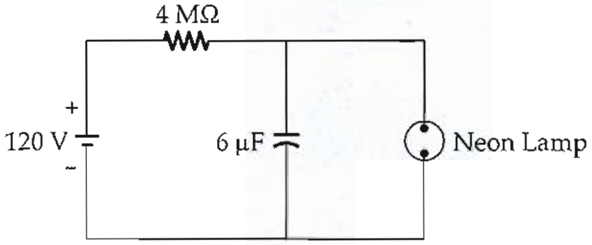
(iii) Assuming a step input, what value of K will result in the smallest attainable settling time?

[20 marks]

Q.8 (c) **(i)** Obtain the current $i(t)$ for both $t < 0$ and $t > 0$.



(ii) A simple relaxation oscillator circuit is shown in figure. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drop to 30 V. Its resistance is 120 Ω, when 'ON' and infinitely high when 'OFF'.



For how long is the lamp on each time the capacitor discharges? What is the time interval between two flashes?

[10 + 10 marks]

○○○○

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

$(s+2)^2$

s^2+

$$\frac{5s+13}{s(s^2+4s+13)}$$

$$\frac{5s+13}{s(s^2+\cancel{4}s+4+9)}$$

$$\frac{5s+13}{s[(s+2)^2+3^2]}$$

$\frac{A}{s}+$

$$\frac{A}{s}+\frac{Bs+C}{s^2+4s+13}$$

$As^2+4As+13A+Bs^2+Cs=5s+13$

$1+$

$A+B=0$

$4A+C=5$

$13A=13$

$\boxed{A=1}$

$4+C=5$

$$\frac{1}{(s+1)(s+2)}$$

$As+2A+Bs+B$

$A+B=3$

$2A+B=5$

$\boxed{A=2}$

$$\frac{\cancel{(s+6)}}{(s+1)(s+3)}$$

$$s^2+3s+2\overline{\smash{)}s^2+6s+7}$$

1

s^2+3s+2

$-\quad -$

$3s+5$

$=3s+5$

$$\frac{A}{s+1}+\frac{B}{s+3}$$

~~$As+$~~ $A+B=3$

$3A+B=5$

$2A=2$

$\boxed{A=1}$

$\boxed{B=2}$

$$\frac{1}{s+1}+\frac{2}{s+3}$$

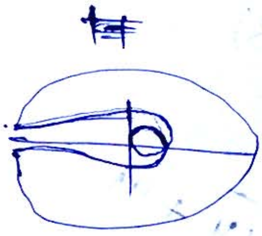
$s+3+2s+2$

Space for Rough Work

$1-\frac{4}{3}+\frac{1}{3}$ $\frac{3-4+1}{3}$

$\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+4}$

$A(s^2+5s+4)+B(s^2+4s)+C(s^2+s)$



$A+B+C=0$

$5A+4B+C=0$

$\boxed{A=1}$

$\frac{1}{s}+\frac{0.2}{s+60}-\frac{1.2}{s+10}$

$s^2+70s+600+0.2(s^2+10s)$

$-1.2(s^2+60s)$

$B+C=-1$

$4B+C=-5$

$-\ \ -\ \ +$

$-3B=4$

$\boxed{B=-4/3}$

$(s+10).(s+10)$

$\frac{4}{3}-1$

$\boxed{C=1/3}$

$\frac{z}{1-\alpha z}\longrightarrow$

$\alpha^{-n}u(-n)\to\frac{1}{1-\alpha z}$

$\frac{1}{1-\alpha^2}\frac{z}{z-\alpha}+\frac{\alpha}{1-\alpha^2}\boxed{\frac{z}{1-\alpha z}}$

$\alpha^n u(n)\ +$

$\cancel{(j)}-\frac{(j)}{(4+j\omega)^2}$

$\frac{A}{z-\alpha}+\frac{B}{1-\alpha z}$

$\cancel{\frac{z}{1-\alpha z}}$

$\boxed{\frac{z}{z^{-1}-\alpha}}$

$A-\alpha Az+Bz-\alpha B=1$

$A-\alpha B=1$

$\boxed{B=\alpha A}=0$

$A-\alpha(\alpha A)=1$

$A[1-\alpha^2]=1$

$A=\frac{1}{1-\alpha 2}$

$B=\frac{\alpha}{1-\alpha 2}$