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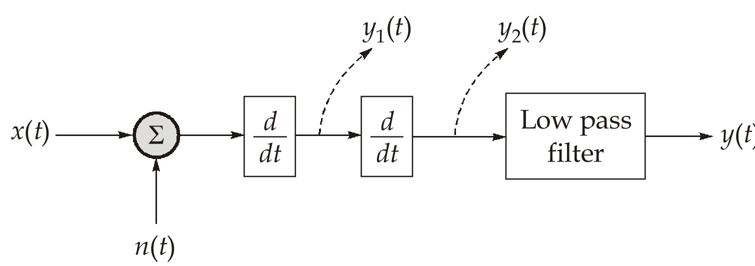
Detailed Solutions

# ESE-2023 Mains Test Series

# E & T Engineering Test No : 3

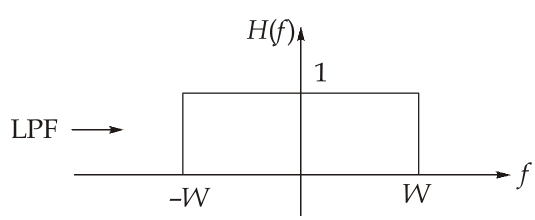
## Section A : Analog and Digital Communication Systems

**Q.1 (a) Solution:**




Given,
$$x(t) = A\cos 2\pi f_c t; \quad f_c < W$$

$$\text{PSD of } n(t) \rightarrow S_n(f) = \frac{N_0}{2}$$




Ignoring the additive noise component

$$y_1(t) = \frac{d}{dt}x(t) = -A(2\pi f_c)\sin 2\pi f_c t$$

$$y_2(t) = \frac{d}{dt}[y_1(t)] = -A(2\pi f_c)^2 \cos 2\pi f_c t$$

$$(\text{LPF})_{o/p} \rightarrow y(t) = -A(2\pi f_c)^2 \cos 2\pi f_c t \qquad [\because W > f_c \;\therefore \text{ LPF allows } y_2(t)]$$

Now, $$\text{Signal power} = S = \frac{A^2(2\pi f_c)^4}{2} \qquad ...(1)$$

Transfer function of cascaded differentiators,

$$H_d(f) = H_1(f) \times H_2(f)$$
$$= (j2\pi f)(j2\pi f)$$
$$= -(2\pi f)^2$$

PSD of noise at $(\text{LPF})_{\text{input}}$ $\Rightarrow$ $S_{ni}(f) = S_n(f) \cdot |H_d(f)|^2 = \frac{N_0}{2}[(2\pi f)]^4$

PSD of noise at $(\text{LPF})_{\text{output}}$ $\Rightarrow$ $S_{no}(f) = S_{ni}(f) \cdot |H(f)|^2$

$$S_{no}(f) = \frac{N_0}{2}(2\pi f)^4 \cdot 1; \; -W \le f \le W$$

Noise power at (LPF)output $\Rightarrow$ $N = \int_{-W}^{W} S_{no}(f)df$

$$= \frac{N_0}{2} \times (2\pi)^4 \int_{-W}^{W} f^4 df = \frac{N_0}{2}(2\pi)^4 \left[\frac{f^5}{5}\right]_{-W}^{W}$$

$$N = \frac{N_0}{10}(2\pi)^4 \times 2W^5 \qquad ...(2)$$

Now from equation (1) and (2)

$$\text{SNR} = \frac{S}{N} = \frac{\dfrac{A^2(2\pi f_c)^4}{2}}{\dfrac{N_0}{5}(2\pi)^4 \times W^5} = \frac{5A^2(f_c)^4}{2N_0 W^5}$$

**Q.1 (b) Solution:**

For continuous input signal applied to uniform quantizer, step size

$$\Delta = \frac{2V_{\max}}{L}$$

where, $L = 2^n [n \rightarrow \text{Number of bits/sample}]$

Now, $Q_e$ = Quantization error: $-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$ variation.

$$\sigma_Q^2 = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} Q_e^2 \, dQ_e$$

$$= \frac{1}{\Delta}\times\frac{Q_e^3}{3}\Bigg|_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left\{\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right\} = \frac{1}{3\Delta}\times 2\frac{\Delta^3}{8}$$

$$\sigma_Q^2 = \frac{\Delta^2}{12} = \frac{1}{12}\times\left(\frac{2V_{max}}{2^n}\right)^2$$

$$\sigma_Q^2 = \frac{1}{3}V_{max}^2\cdot 2^{-2n}$$ Hence proved.

Now, $$SNR_0 = \frac{\text{Signal power}}{\text{Noise power}} = \frac{P}{\frac{1}{3}V_{max}^2 2^{-2n}}$$

∴ $$(SNR)_0 = \frac{3P}{V_{max}^2}\cdot 2^{2n}$$ Hence proved

**Q.1 (c) Solution:**

Given $$X(t) = X\cos 2\pi f_0 t + Y\sin 2\pi f_0 t,\ m_X = 0,\ m_Y = 0$$

**(i)** $$m_X(t) = E[X(t)] = E[X\cos 2\pi f_0 t + Y\sin 2\pi f_0 t]$$

$$m_X(t) = E[X]\cos 2\pi f_0 t + E[Y]\cdot\sin 2\pi f_0 t$$

$$m_X(t) = 0$$

**(ii)** $$R_X(t+\tau, t) = E[(X\cos 2\pi f_0(t+\tau) + Y\sin 2\pi f_0(t+\tau))\cdot(X\cos 2\pi f_0 t + Y\sin 2\pi f_0 t)]$$

$$= E[X^2\cos(2\pi f_0(t+\tau))\cdot\cos 2\pi f_0 t + Y^2\sin 2\pi f_0(t+\tau)\cdot\sin 2\pi f_0 t$$
$$+ XY\cos(2\pi f_0(t+\tau))\cdot\sin 2\pi f_0 t + XY\sin 2\pi f_0(t+\tau)\cdot\cos 2\pi f_0 t]$$

$$= E[X^2]\cos 2\pi f_0(t+\tau)\cdot\cos 2\pi f_0 t + E[Y^2]\sin 2\pi f_0(t+\tau)\cdot\sin 2\pi f_0 t$$
$$+ E[XY][\cos(2\pi f_0(t+\tau))\cdot\sin 2\pi f_0 t + \sin 2\pi f_0(t+\tau)\cdot\cos(2\pi f_0 t)]$$

Given: $$m_X = 0,\quad m_Y = 0$$

$$\sigma_X^2 = E[X^2] - m_X^2 = \sigma^2$$

$$E[X^2] = \sigma^2$$

$$\sigma_Y^2 = E[Y^2] - m_Y^2 = \sigma^2$$

$$E[Y^2] = \sigma^2$$

Also, $X$ and $Y$ are independent RV

∴ $$E[XY] = E[X]\,E[Y] = 0$$

∴ $$R_X(t+T, t) = \sigma^2[\cos 2\pi f_0(t+\tau)\times\cos 2\pi f_0 t + \sin 2\pi f_0(t+\tau)\cdot\sin 2\pi f_0 t]$$

$$= \sigma^2\cos[2\pi f_0(t+\tau) - 2\pi f_0 t]$$

$$R_X(t+\tau, t) = \sigma^2\cos(2\pi f_0\tau)$$

As $R_X(t + \tau, t)$ depends only on $\tau$, hence the process is stationary.

As $m(t)$ is periodic with period $T = 2\pi$, we have $R_X(t + mT + t, t + mT) = R_X(t + T, t)$. Hence, the process is cyclostationary.

**Q.1 (d) Solution:**

**(i)** Let message signal frequency is $f_m$

We know, bit rate $(R_b) = n.f_s$

where;
$$f_s \geq 2{\cdot}f_m$$
$$R_b \geq n(2f_m)$$
$$f_m \leq \frac{R_b}{2n} = \frac{60\times10^6}{2\times8} = 3.75\text{ MHz}$$

**(ii)** For uniformly distributed sample of message signal having uniform quantization level.

$$\text{SQNR} = 6.02n = 6.02 \times 8 = 48.16\text{ dB}$$

**Q.1 (e) Solution:**

**Capture effect:** The inherent ability of FM to diminish the effects of interfering signals is called the capture effect. Unlike AM receiver, FM receivers have the ability to differentiate between two signals received at the same frequency. Therefore, if two stations are received simultaneously at the same or nearly the same frequency, the receiver locks onto the stronger station while suppressing the weaker station. The capture ratio of an FM receiver is the minimum dB difference in signal strength between two received signals necessary for the capture effect to suppress the weaker signal. Capture ratio of 1 dB are typical for high quality FM receivers.

**Threshold effect:** It is observed experimentally that when the signal to noise ratio $(S/N)_r$ at the FM receiver input becomes even slightly less than unity, an impulse or spike of noise is generated. This noise impulse appears at the output of the FM discriminator in the form of a "click" sound. When the $(S/N)_r$ is slightly less than unity, the frequency of spike generation is small, and each spike produces individual clicking sound at the discriminator output.

But, when the $(S/N)_r$ is moderately less than unity, the spikes are generated rapidly and the clicks merge into a sputtering sound. This phenomena is known as threshold effect in FM. The minimum $(S/N)_r$ for which the sputtering effect cannot cause distortion in the FM receiver is called as threshold of the FM receiver.

- The threshold effect is more serious in FM compared to AM. The process of lowering the threshold level is known as threshold improvement or threshold reduction.

The popularly used methods for threshold improvement in FM are,

**(a)** Using pre-emphasis and de-emphasis circuits.

**(b)** frequency modulation with feedback (FMFB) i.e., using PLL for FM demodulation.

**Q.2 (a) Solution:**

Given, $P_m$ = 0.1 W; BW of channel = 100 kHz; $f_m$ = 4 kHz

**(i)** $\mu = 0.85$

We know,

For FM system, output SNR is given as

$$(\text{SNR})_{o,\,\text{FM}} = \frac{3}{2}\frac{A_c^2 K_f^2 P_m}{N_0 f_m^3}$$

But for FM,

$$\beta_f = \frac{K_f\,|m(t)|_{\max}}{f_m}$$

$$K_f = \frac{\beta_f f_m}{|m(t)|_{\max}}$$

$\therefore$

$$(\text{SNR})_{o,\,\text{FM}} = \frac{3}{2}\frac{A_c^2 \beta_f^2 P_m}{N_0 f_m \left[|m(t)|_{\max}\right]^2} \quad \text{...(1)}$$

Given, BW = 100 kHz

Using Carson's rule

$$\text{BW} = 2f_m(1 + \beta_f)$$

$$100 = 2 \times 4(1 + \beta_f)$$

$$\beta_f = 11.5$$

Now for AM system, output SNR is given as

$$(\text{SNR})_{o,\,\text{AM}} = \frac{A_c^2 K_a^2 P_m}{2N_0 f_m}$$

But,

$$\mu = K_a\,|\,m(t)\,|_{\max}$$

$\therefore$

$$(\text{SNR})_{o,\,\text{AM}} = \frac{A_c^2 \mu^2 P_m}{2N_0 f_m \left[|m(t)|_{\max}\right]^2} \quad \text{...(2)}$$

Now, dividing eqn. (1) and (2), we get

$$\frac{(\text{SNR})_{o,\,\text{FM}}}{(\text{SNR})_{o,\,\text{AM}}} = \frac{\dfrac{3A_c^2\beta_f^2 P_m}{2N_0 f_m\left[|m(t)|_{\max}\right]^2}}{\dfrac{A_c^2\mu^2 P_m}{2N_0 f_m\left[|m(t)|_{\max}\right]^2}} = \frac{3\beta_f^2}{\mu^2}$$

$$\frac{(\text{SNR})_{o,\,\text{FM}}}{(\text{SNR})_{o,\,\text{AM}}} = \frac{3\times(11.5)^2}{(0.85)^2} = 549.139$$

In dB, $\Rightarrow$ $\dfrac{(\text{SNR})_{o,\,\text{FM}}}{(\text{SNR})_{o,\,\text{AM}}} = 10\log_{10} 549.139$

$= 27.3968$ dB

**(ii)** Given, $(\text{SNR})_{o,\,\text{FM}} = (\text{SNR})_{o,\,\text{PM}}$

We know, $$(\text{SNR})_{o,\,\text{FM}} = \frac{3}{2}\cdot\frac{A_c^2\beta_f^2 P_m}{N_0 f_m\left[|m(t)|_{\max}\right]^2} \quad ...(3)$$

We know, $$(\text{SNR})_{o,\,\text{PM}} = \frac{K_p^2 A_c^2 P_m}{2N_0 f_m}$$

But, $\beta_p = K_p|m(t)|_{\max}$

$\therefore$ $$(\text{SNR})_{o,\,\text{PM}} = \frac{A_c^2\beta_p^2 P_m}{2N_0 f_m\left[|m(t)|_{\max}\right]^2} \quad ...(4)$$

From equation (3) and (4)

$$\frac{3}{2}\cdot\frac{A_c^2\beta_f^2 P_m}{N_0 f_m\left[|m(t)|_{\max}\right]^2} = \frac{A_c^2\beta_p^2 P_m}{2N_0 f_m\left[|m(t)|_{\max}\right]^2}$$

$$3\beta_f^2 = \beta_p^2$$

$$\beta_p = \sqrt{3}\,\beta_f \quad ...(5)$$

Now, $$\frac{\text{BW}_{\text{PM}}}{\text{BW}_{\text{FM}}} = \frac{2f_m(\beta_p+1)}{2f_m(\beta_f+1)} = \frac{(\beta_p+1)}{(\beta_f+1)}$$

Using equation (5),

$$\frac{\text{BW}_{\text{PM}}}{\text{BW}_{\text{FM}}} = \frac{(\sqrt{3}\beta_f+1)}{(\beta_f+1)}$$ Hence proved.

**Q.2 (b) Solution:**

**(i)** The bandwidth of the signal is

$$B = 5 \text{ kHz}$$

$$\text{Nyquist rate} = 2B = 2 \times 5000 = 10,000 \text{ Hz}$$

Hence,
$$\text{sampling rate} = 2 \times \text{Nyquist rate} = 2 \times 10,000$$

$$\therefore \quad \text{Sampling rate } (r) = 20,000 \text{ samples/second}$$

Since the samples are quantized into 256 equally likely levels there will be,

$$n = \log_2(256) = 8 \text{ bits/sample}$$

The information rate (R) = $n \times r = 8 \times 20,000$

$$R = 160 \text{ kbps}$$

**(ii)** To check for error-free transmission on the AWGN channel with

$$B = 10 \text{ kHz and } \frac{S}{N} = 40 \text{ dB}$$

In decibels,
$$\frac{S}{N} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$40 = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\frac{S}{N} = 10^4$$

The capacity of AWGN channel is given by,

$$C = B\log_2\left[1+\frac{S}{N}\right]$$

$$C = 10000 \log_2[1 + 10^4]$$

$$C = 132.88 \text{ kbps}$$

If $R \le C$, then the error free transmission is possible but in this case $C = 132.88$ kbps and $R = 160$ kbps.

$\therefore$ The information rate is more than the capacity of the channel. Hence error free transmission is not possible in this case.

**(iii)** $\frac{S}{N}$ ratio for error-free transmission, with $B = 10$ kHz.

$$R \le C = B\log_2\left[1+\frac{S}{N}\right]$$

$$160 \text{ K} \le 10\text{K}\log_2\left[1+\frac{S}{N}\right]$$

$$\left(\frac{S}{N}\right) \geq 65535$$

In decibels,
$$\left(\frac{S}{N}\right)_{dB} \geq 10 \log_{10}(65535)$$

$$\left(\frac{S}{N}\right)_{dB} \geq 48.16 \text{ dB}$$

**(iv)** Given $\left(\frac{S}{N}\right)$ ratio is 40 dB

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$40 = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\frac{S}{N} = 10^4$$

For error-free transmission over AWGN channel,

$$R \leq C = B\log_2\left[1+\frac{S}{N}\right]$$

$$160 \times 10^3 \leq B \log_2[1 + 10^4]$$

$$\frac{160\times 10^3}{\log_2\left[1+10^4\right]} \leq B$$

$\therefore$
$$B \geq 12.04 \text{ kHz}$$

**Q.2 (c) Solution:**

**(i)** For binary modulation schemes, using correlator receivers with exact phase synchronisation and optimum threshold detection, the average symbol error probability can be given as

$$P_e = Q\left[\sqrt{\frac{d_{min}^2}{2N_0}}\right]$$

where $d_{min}$ = Minimum distance between adjacent message points in the constellation diagram.

$$\frac{N_0}{2} = \text{Two sided PSD of white noise} = 1 \times 10^{-11} \text{ W/Hz}$$

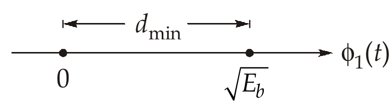
$$N_0 = 2 \times 10^{-11} \text{ W/Hz}$$

Let take reference energy $E_b = \dfrac{A^2}{2} T_b$

Where $A = 15 \times 10^{-3}$ V and $T_b = \dfrac{1}{R_b} = \dfrac{1}{0.5 \times 10^6} = 2 \times 10^{-6}$ sec

$$\therefore \quad E_b = \frac{(15 \times 10^{-3})^2}{2} \times \frac{1}{0.5 \times 10^6} = 2.25 \times 10^{-10}\, J$$

**For BASK Scheme**




$$d_{min} = \sqrt{E_b}$$

$$\therefore \quad P_e = Q \cdot \left[\sqrt{\frac{E_b}{2N_0}}\right] = Q\left[\sqrt{\frac{2.25 \times 10^{-10}}{2 \times 2 \times 10^{-11}}}\right]$$

$$P_e = Q(2.37)$$

**For BFSK scheme**

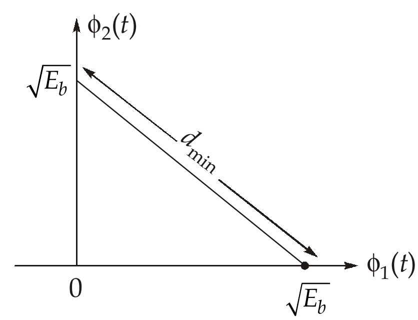
$$d_{min} = \sqrt{E_b + E_b}$$

$$d_{min} = \sqrt{2E_b}$$

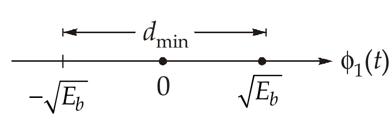
$$\therefore \quad P_e = Q\left[\sqrt{\frac{2E_b}{2N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2.25 \times 10^{-10}}{2 \times 10^{-11}}}\right]$$

$$P_e = Q(3.35)$$




**For BPSK scheme**




$$d_{min} = 2\sqrt{E_b}$$

$$\therefore \quad P_e = Q\left[\sqrt{\frac{4 \times E_b}{2 \times N_0}}\right] = Q\left[\sqrt{\frac{2 \times 2.25 \times 10^{-10}}{2 \times 10^{-11}}}\right]$$

$$P_e = Q[4.743]$$

**(ii) 1.** For '$s$' error to be detected:

$$d_{min} \geq s + 1$$
$$5 \geq s + 1$$
$$s \leq 4$$

So, upto 4 errors per codeword can be detected.

**2.** $d_{min} > (2t + 1)$... for '$t$' errors to be detected and corrected.

$$5 \geq 2t + 1$$
$$t \leq \frac{4}{2}$$
$$t \leq 2$$

$\therefore$ upto 2 errors per code word can be detected and corrected.

**Q.3 (a) Solution:**

The given Gaussian pulse is

$$x(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2\sigma^2} \quad \text{...(i)}$$

Now, the maximum signal to noise ratio of matched filter is given as,

$$\left(\frac{S}{N}\right)_{max} = \frac{2E}{N_0}$$

Here, $E$ is the energy of the signal $x(t)$.

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$E = \int_{-\infty}^{\infty} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2\sigma^2}\right]^2 dt$$

$$E = \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2} e^{-t^2/\sigma^2} dt$$

Let, $\frac{t^2}{\sigma^2} = y^2$

so that, $\frac{t}{\sigma} = y$

and $dt = \sigma\, dy$

$$E = \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2} e^{-y^2} (\sigma dy)$$

$$E = \frac{1}{2\pi\sigma} \int_{-\infty}^{\infty} e^{-y^2} dy \quad \text{...(ii)}$$

Using the standard result of integration

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \quad \text{...(iii)}$$

∴ From the result of equation (iv) and substituting in equation (iii) we get,

$$E = \frac{1}{2\pi\sigma}\sqrt{\pi}$$

$$E = \frac{1}{2\sigma\sqrt{\pi}}$$

Substitute value of $E$ in $\left(\frac{S}{N}\right)_{max}$ equation.

∴ $$\left(\frac{S}{N}\right)_{max} = \frac{2\times\left(\frac{1}{2\sigma\sqrt{\pi}}\right)}{N_0} = \frac{1}{N_0\sigma\sqrt{\pi}}$$

Substituting the value of σ = 1 and $\frac{N_0}{2} = 10^{-20}$ Watt/Hz in above equation,

$$\left(\frac{S}{N}\right)_{max} = \frac{1}{2\times 10^{-20}\times 1\times\sqrt{\pi}} = 2.8209\times 10^{19}$$

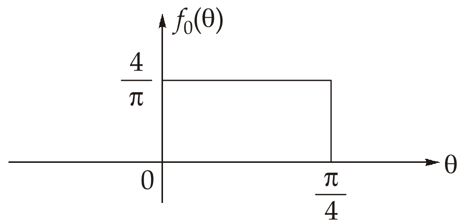
$$\left(\frac{S}{N}\right)_{max} = 2.82 \times 10^{19}$$

In decibels, $$\left(\frac{S}{N}\right)_{max} = 10 \log_{10} (2.82 \times 10^{19})$$

$$\left(\frac{S}{N}\right)_{max} = 194.50 \text{ dB}$$

**Q.3 (b) Solution:**

**(i)** Given θ is a RV uniformly distributed on [0,π/4], the probability density function can be sketched as below:




$$\therefore \quad E[X(t)] = \int_{-\infty}^{\infty} x(t)\cdot f_\theta(\theta)d\theta$$

$$= \int_0^{\pi/4} \left(\frac{4}{\pi}\right) A\cos(2\pi f_0 t + \theta)d\theta$$

$$= \frac{4A}{\pi}\left[\sin(2\pi f_0 t + \theta)\right]_0^{\pi/4}$$

$$= \frac{4A}{\pi}\left[\sin\left(2\pi f_0 t + \frac{\pi}{4}\right) - \sin(2\pi f_0 t)\right]$$

Thus $E[X(t)]$ is periodic with period $T = \frac{1}{f_0}$.

$$\therefore R_X(t+\tau, t) = E[A^2\cos(2\pi f_0(t+\tau)+\theta)\cdot\cos(2\pi f_0 t+\theta)]$$

$$= \frac{A^2}{2}E[\cos[2\pi f_0(2t+\tau)+2\theta] + \cos(2\pi f_0\tau)]$$

$$= \frac{A^2}{2}\cos(2\pi f_0\tau) + \frac{A^2}{2}E\left[\cos\left[2\pi f_0(2t+\tau)+2\theta\right]\right]$$

$$= \frac{A^2}{2}\cos(2\pi f_0\tau) + \frac{A^2}{2}\times\frac{4}{\pi}\int_0^{\pi/4}\cos(2\pi f_0(2t+\tau)+2\theta)d\theta$$

$$= \frac{A^2}{2}\cos(2\pi f_0\tau) + \frac{2A^2}{\pi}\left[\sin\left[2\pi f_0(2t+\tau)+2\theta\right]\right]_0^{\pi/4}\times\frac{1}{2}$$

$$= \frac{A^2}{2}\cos(2\pi f_0\tau) + \frac{A^2}{\pi}\left[\sin\left[2\pi f_0(2t+\tau)+\frac{\pi}{2}\right] - \sin\left[2\pi f_0(2t+\tau)\right]\right]$$

$$= \frac{A^2}{2}\cos(2\pi f_0\tau) + \frac{A^2}{\pi}\left[\cos\left[2\pi f_0(2t+\tau)\right] - \sin\left[2\pi f_0(2t+\tau)\right]\right]$$

which is periodic with period $T' = \frac{1}{2f_0}$. Thus the process is cyclostationary with period $T = \frac{1}{f_0}$.

Auto correlation function of cyclostationary process is (ACF) = $\frac{1}{T}\int_0^T R_X(t+\tau, t)dt$

Using the formula,

$$S_X(t) = FT[\text{ACF}]$$

$$S_X(t) = FT\left[\frac{1}{T}\int_0^T R_X(t+\tau, t)dt\right]$$

$$S_X(f) = F.T\left[\frac{A^2}{2}\cos[2\pi f_0\tau] + \frac{A^2}{\pi T}\int_0^T \left[\cos\left(2\pi f_0(2t+\tau)\right) - \sin\left(2\pi f_0(2t+\tau)\right)\right]dt\right]$$

$$S_X(f) = F.T\left[\frac{A^2}{2}\cos\left(2\pi f_0\tau\right) + 0\right]$$

$$S_X(f) = \frac{A^2}{4}\left[\delta(f - f_0) + \delta(f + f_0)\right]$$

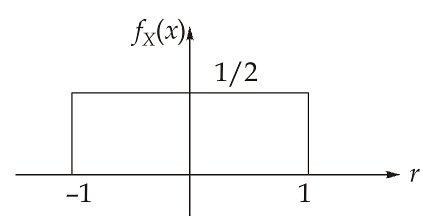
**(ii)**

$$E[X] = \int_{-1}^{1} x f_x(x)dx$$

$$= \frac{1}{2} \times \left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{4}[1-1]$$

$$E[X] = 0$$




Now, $R_X(t+\tau, t) = E[X(t+\tau)X(t)]$

$$= E[(X + Y)(X + Y)]$$

$$= E[X^2 + Y^2 + 2XY]$$

$$= E[X^2] + E[Y^2] + 2E[XY]$$

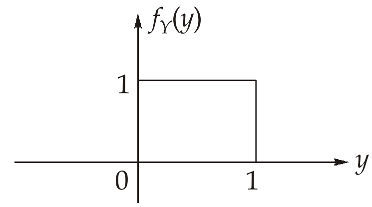
But $E[XY] = E[X]\cdot E[Y] = 0$ ($\because$ $X$ and $Y$ are independent)

$\therefore$ $R_X(t+\tau, t) = E[X^2] + E[Y^2]$ ($\because$ $E[X] = 0$)

Now, $$E[X^2] = \int_{-1}^{1} x^2 f_X(x)dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{6}[1+1]$$

$$E[X^2] = \frac{1}{3}$$

$$E[Y^2] = \int_{0}^{1} y^2 f_y(y)dy$$




$$= 1\left[\frac{y^2}{3}\right]_0^1$$

$$E[Y^2] = \frac{1}{3}$$

$\therefore$ $$R_X(t+\tau, t) = \frac{1}{3}+\frac{1}{3}$$

$$R_X(t+\tau, t) = \frac{2}{3}$$

The Fourier transform of $R_X(t + \tau, t)$ is the power spectral density of $X(t)$. Thus,

$$S_X(f) = FT[R_X(t+\tau, t)]$$

$$S_X(f) = \frac{2}{3}\delta(f)$$

**Q.3 (c) Solution:**

$$P[Y] = P[X]\cdot P\left[\frac{Y}{X}\right]$$

where; $P\left[\frac{Y}{X}\right]$ is channel matrix

$$P\left[\frac{Y}{X}\right] = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$P\begin{bmatrix} y_0 & y_1 & y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{24} & \frac{5}{12} & \frac{7}{24} \end{bmatrix}$$

We have;

$$\text{Mutual information } I(X; Y) = H(Y) - H\left(\frac{Y}{X}\right)$$

$$H(Y) = \sum_{i=0}^{2} P(y_i)\cdot \log_2 \frac{1}{P(y_i)}$$

$$= 2\times\frac{7}{24}\log_2\frac{24}{7} + \frac{5}{12}\log_2\frac{12}{5} = 1.563 \text{ bits/symbol}$$

Now, conditional probability $H\left(\frac{Y}{X}\right)$ is given as:

$$H\left(\frac{Y}{X}\right) = \sum_{i=0}^{2}\sum_{j=0}^{2} P(x_i, y_j)\cdot\log_2\frac{1}{P\left[\frac{y_j}{x_i}\right]}$$

The joint probability matrix is given as:

$$P(X, Y) = P[X]\big|_{\text{diagonal}} \cdot P\left[\frac{Y}{X}\right]$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{matrix} \\ x_0 \\ x_1 \\ x_2 \end{matrix}\begin{matrix} \quad y_0 \quad y_1 \quad y_2 \\ \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{8} & \frac{1}{8} \end{bmatrix} \end{matrix}$$

$$\therefore \quad H\left(\frac{Y}{X}\right) = -\left[\frac{4}{8}\log_2\left(\frac{1}{2}\right) + \frac{3}{6}\log_2\frac{1}{3}\right]$$

$$= 1.292 \text{ bits/symbol}$$

Now; Mutual information is given as;

$$I(X, Y) = H(Y) - H\left(\frac{Y}{X}\right) = 1.563 - 1.292$$

$$= 0.271 \text{ bits/symbol}$$

**Q.4 (a) Solution:**

Given AM signal,

$$u(t) = [20 + 2\cos 3000\pi t + 10\cos 6000\,\pi t]\cos 2\pi f_c t$$

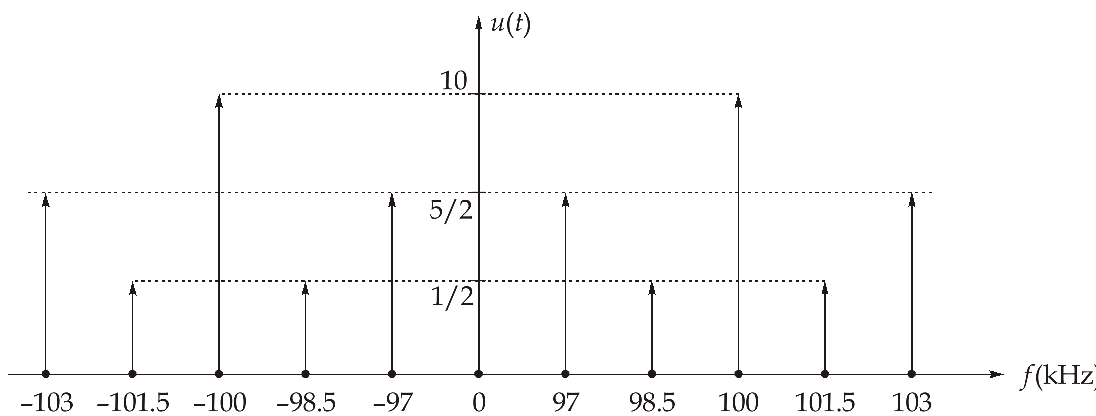
$$u(t) = 20\cos 2\pi f_c t + 2\cos 3000\pi t\cdot\cos 2\pi f_c t + 10\cos 6000\pi t\cdot\cos 2\pi f_c t$$

$$u(t) = 20\cos 2\pi f_c t + \cos[2\pi(f_c + 1500)t] + \cos[2\pi(f_c - 1500)t] + 5\cos[2\pi(f_c + 3000)t] + 5\cos[2\pi(f_c - 3000)t]$$

Spectrum of $u(t)$ is

$$U(f) = 10[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{2}[\delta[f - (f_c + 1500)] + \delta[f + (f_c + 1500)]]$$
$$+\frac{1}{2}\left[\delta\left[f - (f_c - 1500)\right] + \delta\left[f + (f_c + 1500)\right]\right]$$
$$+\frac{5}{2}\left[\delta\left[f - (f_c + 3000)\right] + \delta\left[f + (f_c + 3000)\right]\right]$$
$$+\frac{5}{2}\left[\delta\left[f - (f_c - 3000)\right] + \delta\left[f + (f_c + 3000)\right]\right]$$

**(i)** Given, $f_c = 100$ kHz




**(ii)** From the figure

⇒ Power content of carrier freq. $= \frac{(20)^2}{2} = 200$ W

⇒ Power content of the frequency $f_c + 1500$ is same as the power content of the frequency $f_c - 1500$, and is equal to,

$$P = \frac{(1)^2}{2} = 0.5 \text{ W}$$

⇒ Power content of the frequency $f_c + 3000$ is same as the power content of the frequency $f_c - 3000$, and it is equal to,

$$P = \frac{(5)^2}{2} = \frac{25}{2}\text{W} = 12.5 \text{ W}$$

**(iii)** $u(t) = 20\left[1+\frac{1}{20}(2\cos 3000\pi t + 10\cos 6000\pi t)\right]\cos 2\pi f_c t$

Comparing with standard AM signal, we get

$m(t)$ = message signal = 2 cos 3000$\pi t$ + 10 cos 6000 $\pi t$ = $m_1(t) + m_2(t)$

$$\therefore \quad \mu_1 = \frac{A_{m1}}{A_c}, \quad \mu_2 = \frac{A_{m2}}{A_c}$$

$$\mu_1 = \frac{1}{10} = 0.1, \quad \mu_2 = 0.5$$

$$\therefore \quad \text{Modulation index, } \mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$= \sqrt{(0.1)^2 + (0.5)^2} = \frac{\sqrt{26}}{10}$$

$$\mu = 0.5099$$

**(iv)** Total power $P_t = P_c\left(1+\frac{\mu^2}{2}\right)$

$$P_c = \frac{A_c^2}{2} = \frac{20\times 20}{2} = 200\text{ W}, \mu = 0.5099$$

$$\therefore \quad P_t = 200\left[1+\frac{(\mu)^2}{2}\right] = 226\text{ W}$$

$$\text{Side band power, } P_{SB} = \frac{P_c\mu^2}{2} = \frac{200\times(\mu)^2}{2} = 26\text{ W}$$

Ratio of side band power to total power = $\eta$

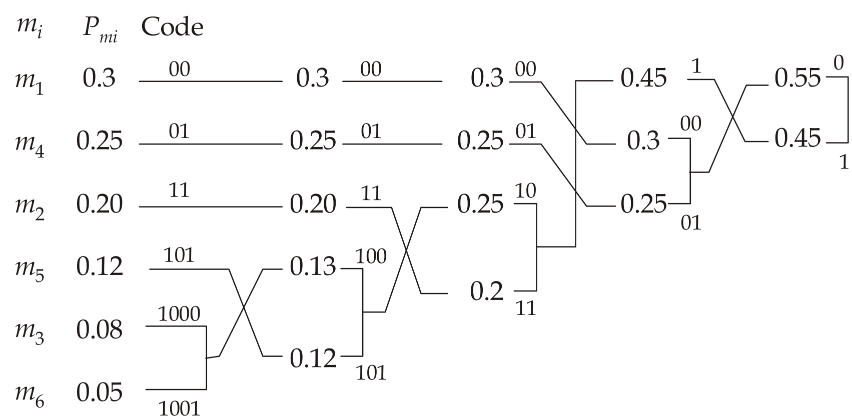
$$\eta = \frac{P_{SB}}{P_t} = \frac{26}{226} = 0.11504$$

$$\%\eta = 11.504\%$$

**Q.4 (b) Solution:**

**(i)** Given message with their probabilities:

$m_1 = 0.3$, $m_2 = 0.2$, $m_3 = 0.08$, $m_4 = 0.25$, $m_5 = 0.12$ and $m_6 = 0.05$




$$H(m) = -\sum_i P_{mi} \log_2(P_{mi})$$

$$= -[0.3 \log_2(0.3) + 0.25 \log_2(0.25) + 0.20 \log_2(0.2) + 0.12 \log_2(0.12) + 0.08 \log_2(0.08) + 0.05 \log_2(0.05)]$$

$$H(m) = 2.36 \text{ bits/symbol}$$

$$\text{Average length } (L_{avg}) = \sum_i n_i P_{mi} \quad \text{[where } n \text{ is code word length]}$$

$$= [(2 \times 0.3) + (2 \times 0.25) + (2 \times 0.2) + (3 \times 0.12) + (4 \times 0.08) + (4 \times 0.05)]$$

$$L_{avg} = 2.38 \text{ bits/symbol}$$

$$\text{Efficiency, } \eta = \frac{H(m)}{L_{avg}} = 0.99$$

$$\% \text{ Redundancy } (V) = (1 - \eta) \times 100 = (1 - 0.99) \times 100$$

$$\%V = 1\%$$

**(ii)** $f_c = 200$ kHz, $f_{m\,(max)} = 6$ kHz

**1.** The lower sideband extends from the lowest possible lower side frequency to the carrier frequency or

$$\text{LSB} = [f_c - f_{m\,(max)}] \text{ to } f_c$$

$$= (200 - 6) \text{ to } 200$$

$$\text{LSB} = 194 \text{ kHz to } 200 \text{ kHz}$$

The upper sideband extends from the carrier frequency to the highest possible upper side frequency or

$$\text{USB} = f_c \text{ to } (f_c + f_{m\,(\max)})$$
$$= 200 \text{ kHz to } (200 + 6)$$
$$\text{USB} = 200 \text{ kHz to } 206 \text{ kHz}$$

**2.** The bandwidth is equal to the difference between the maximum upper side frequency and the minimum lower side frequency or

$$\text{BW} = 2\, f_{m(\max)}$$
$$= 2 \times 6 = 12 \text{ kHz}$$

**3.** Considering the single frequency 2 kHz tone, we have $f_m$ = 2 kHz. The upper side frequency is the sum of the carrier and modulating frequency or

$$f_{usf} = f_c + f_m = 200 + 2 \text{ kHz} = 202 \text{ kHz}$$

The lower side frequency is the difference between the carrier and the modulating frequency or

$$f_{lsf} = f_c - f_m = 200 - 2 = 198 \text{ kHz}$$

**Q.4 (c) Solution:**

**(i)** Given $S(t) = \frac{1}{2} aA_m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_m A_c (1-a) \cos[2\pi(f_c - f_m)t]$

Expanding $S(t)$ we get,

$$S(t) = \frac{1}{2} aA_m A_c [\cos 2\pi f_c t \cdot \cos 2\pi f_m t - \sin 2\pi f_c t \cdot \sin 2\pi f_m t]$$
$$+ \frac{1}{2} A_m A_c (1-a)[\cos 2\pi f_c t \cdot \cos 2\pi f_m t + \sin 2\pi f_c t \cdot \sin 2\pi f_m t]$$
$$= \frac{1}{2} A_m A_c \cos(2\pi f_c t) \cdot \cos(2\pi f_m t) + \frac{1}{2} A_m A_c (1-2a) \sin(2\pi f_c t) \cdot \sin(2\pi f_m t)$$

Therefore, the quadrature component is given as

$$S_Q(t) = \frac{1}{2} A_m A_c (1-2a) \sin(2\pi f_m t)$$

**(ii)** After adding the carrier, the signal will be:

$$S(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} A_m A_c \cos(2\pi f_c t) \cos(2\pi f_m t) + \frac{1}{2} A_m A_c (1-2a) \sin(2\pi f_c t) \sin(2\pi f_m t)$$

$$S(t) = A_c \left[ \cos(2\pi f_c t) + \frac{1}{2} A_m \cos(2\pi f_m t) \cdot \cos(2\pi f_c t) + \frac{1}{2} A_m (1-2a) \sin(2\pi f_m t) \sin(2\pi f_c t) \right]$$

The envelope equals

$$a(t) = A_c\sqrt{\left[1+\frac{1}{2}A_m\cos(2\pi f_m t)\right]^2+\left[\frac{1}{2}A_m(1-2a)\sin(2\pi f_m t)\right]^2}$$

$$a(t) = A_c\left[1+\frac{1}{2}A_m\cos(2\pi f_m t)\right]\sqrt{1+\left[\frac{\frac{1}{2}A_m(1-2a)\sin(2\pi f_m t)}{\left[1+\frac{1}{2}A_m\cos(2\pi f_m t)\right]}\right]^2}$$

$$a(t) = A_c\left[1+\frac{1}{2}A_m\cos(2\pi f_m t)\right]\cdot d(t)$$

where $d(t)$ is the distortion, defined by

$$d(t) = \sqrt{1+\left[\frac{\frac{1}{2}A_m(1-2a)\sin(2\pi f_m t)}{1+\frac{1}{2}A_m\cos(2\pi f_m t)}\right]^2}$$

**(iii)** $d(t)$ = distortion reaches its worst possible condition i.e., greatest value when $a = 0$.

**Section B : Signals and Systems-1 + Microprocessors and Microcontroller-1 + Network Theory-2 + Control Systems-2**

**Q.5 (a) Solution:**

For Z.I.R [Zero input response], input is taken as zero and response is obtained only because of initial state.

i.e., $\ddot{y}(t)+2y(t)+3\dot{y}(t) = 0$

Taking Laplace transform,

$$s^2Y(s)-s\cdot y(0)-\dot{y}(0)+2Y(s)+3[sY(s)-y(0)] = 0$$

$$s^2Y(s) - 3s - 4 + 2Y(s) + 3[sY(s) - 3] = 0$$

$$s^2Y(s) - 3s - 4 + 2Y(s) + 3sY(s) - 9 = 0$$

$$Y(s)[s^2 + 2 + 3s] = 3s + 13$$

$$Y(s) = \frac{3s+13}{s^2+3s+2} = \frac{3s+13}{(s+1)(s+2)} = \frac{A}{s+1}+\frac{B}{s+2}$$

$$A = 10, B = -7$$

$$\therefore \quad Y(s) = \frac{10}{s+1}-\frac{7}{s+2}$$

Taking ILT

$$y(t) = (10e^{-t} - 7e^{-2t})\cdot u(t)$$

For zero state response, input is available and the initial conditions are set to zero.

We have;

$$\ddot{y}(t)+3\dot{y}(t)+2y(t) = x(t)$$

$$s^2Y(s) + 3sY(s) + 2Y(s) = \frac{3}{s+4}$$

$$Y(s)[s^2 + 3s + 2] = \frac{3}{s+4}$$

$$Y(s) = \frac{3}{(s+1)(s+2)(s+4)} = \frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+4}$$

$$A = 1, B = -3/2, C = 1/2$$

Now,

$$Y(s) = \frac{1}{s+1}-\frac{3/2}{s+2}+\frac{1/2}{s+4}$$

after taking ILT;

$$y(t) = \left(e^{-t}-\frac{3}{2}e^{-2t}+\frac{1}{2}e^{-4t}\right)\cdot u(t)$$

**Q.5 (b) Solution:**

**(i) LDS *Rd*, M :** Load pointer using DS register.

This instruction is used to load the double word stored in memory into specified register and DS register.

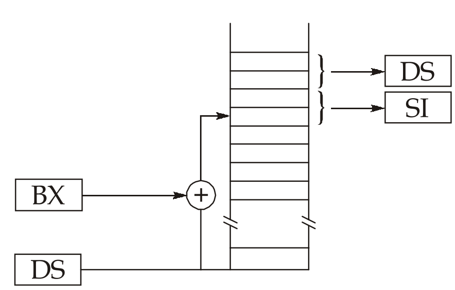
The general purpose register is loaded from the lower-order word of the memory operand and the segment register DS from the higher-order word.

The memory may be specified by any of the memory-related addressing mode.

For example,

LDS SI, [BX]

in which SI is set to [BX : BX + 1] and DS is set to [BX + 2 : BX + 3]



**(ii) AAM:** ASCII adjustment after multiplication

It converts the result of the multiplication of two valid unpacked BCD digits into a valid unpacked BCD number AX is the implicit operand in AAM.

AAM unpacks the result by dividing AX by 10, placing the quotient (Most significant Digit) in AH and the remainder (Least significant digit) in AL. In AAM, except carry and auxiliary carry flag bits, all the other flag bits are undefined.

**(iii) DAS:** Decimal adjust after substraction

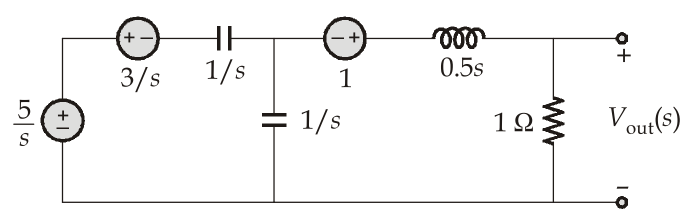
The DAS instruction adjusts the result of a subtraction to a packed BCD number (less than 100 decimal). It converts binary result of subtraction into packed BCD. If the sum is greater than 99H after adjustment, then the carry and auxiliary carry flags are set. Otherwise, carry and auxiliary carry flags are cleared. All flag bits are modified as per the result.

**(iv) CLI:** Clear the interrupt enable flag

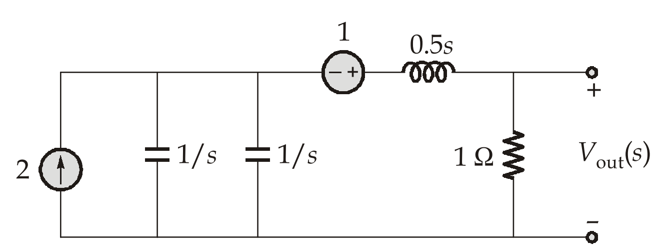
This instruction reset the interrupt flag to zero. No other flags are affected. If the interrupt flag is reset, the 8086 will not respond to an interrupt signal on its INTR input. This CLI instruction has no effect on the Non-maskable interrupt input, NMI.

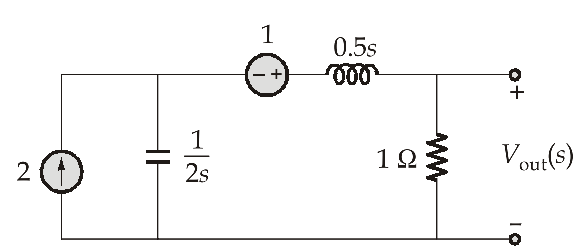
**Q.5 (c) Solution:**

Apply Laplace transform to the circuit.

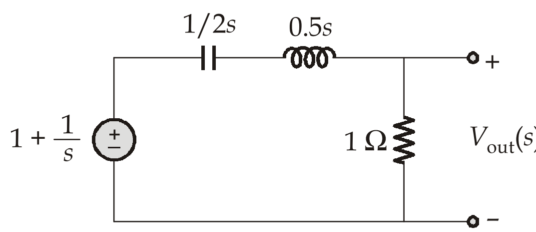



Perform a source transform and combine parallel capacitors.



Apply source transformation and combine series voltage source to produce a series circuit.




Use voltage division rule to find $V_{out}(s)$,

$$V_{out}(s) = \left(\frac{1}{\frac{1}{2s}+0.5s+1}\right)\left(1+\frac{1}{s}\right)$$

$$V_{out}(s) = \frac{2}{s+1}$$

Apply inverse Laplace transform

$$V_{out}(t) = L^{-1}\left[\frac{2}{s+1}\right] = 2e^{-t}u(t) \text{ Volts}$$

At $t = 1$ sec $V_{out}$ is

$$V_{out}(1) = 2e^{-1} = 0.736 \text{ Volts}$$

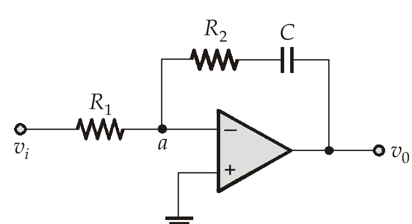
**Q.5 (d) Solution:**

The given transfer function is

$$G_c(s) = \frac{10s+4}{s} = 10+\frac{4}{s} \quad \text{...(i)}$$

$$G_c(s) = K_p + \frac{K_I}{s}$$

Thus, the given controller is a PI controller. It can be realised using operational amplifier as




Using KCL at '*a*' in *s*-domain, we get,

$$0 = \frac{0-V_i}{R_1} + \frac{0-V_0}{R_2+\frac{1}{Cs}}$$

or $$\frac{V_0(s)}{V_i(s)} = -\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$$

Here minus sign indicates inverting configuration

or, $$G_c(s) = \frac{R_2}{R_1}+\left(\frac{1}{R_1Cs}\right) \quad ...(ii)$$

On comparing equation (i) and (ii), we get,

$$\frac{R_2}{R_1} = 10$$

or $$R_2 = 10R_1 \quad ...(iii)$$

and $$\frac{1}{R_1C} = 4$$

or $$R_1C = \frac{1}{4} = 0.25 \quad ...(iv)$$

$\because$ $$C = 25\ \mu F \text{ (given)}$$

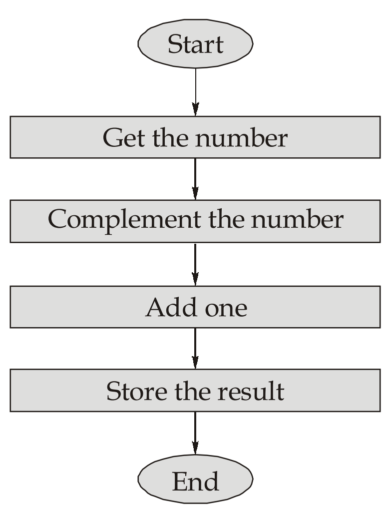
$\therefore$ $$R_1 = \frac{1}{4C} = \frac{0.25}{25\times10^{-6}} = 10\ k\Omega$$

and hence, from equation (iii)

$$R_2 = 100\ k\Omega$$

**Q.5 (e) Solution:**

**Flow chart**



**Program**

| Mnemonics | : | Comments |
|---|---|---|
| LDA 9000 H | : | Load the number to be complemented in the accumulator. |
| CMA | : | Complement the number |
| ADI 01 H | : | Add one to the complement |
| STA 9001 H | : | Store the result |
| HLT | : | End the program |

**Execution time ($T_E$)**

$$T_E = 13T + 4T + 7T + 13T + 5T$$

$$T_E = 42T$$

Where,
$$T = \frac{1}{f_{clk}} = \frac{1}{5\times 10^6} = 200\, n\text{sec}$$

$$T_E = 8.4\ \mu\text{ sec}$$

**Q.6 (a) Solution:**

**(i)** Basic machine cycles of 8085:

1. **Memory read machine cycle:**

   It refers to the process of reading one byte from memory. It requires 3T states.

2. **Memory write machine cycle:** If refers to the process of writing one byte into the memory. It requires 3T states.

3. **Opcode fetch machine cycle:**

   It is the first operation in any instruction. It involves reading the opcode from memory. It requires 4T states.

4. **I/O read/write machine cycle:**

   These involves reading from and writing to an input and output port respectively. Each requires 3T states.

5. **Interrupt Acknowledge:**

   This is the machine cycle to get the address of the interrupt service routine in order to service the interrupt device. It requires 10T states.

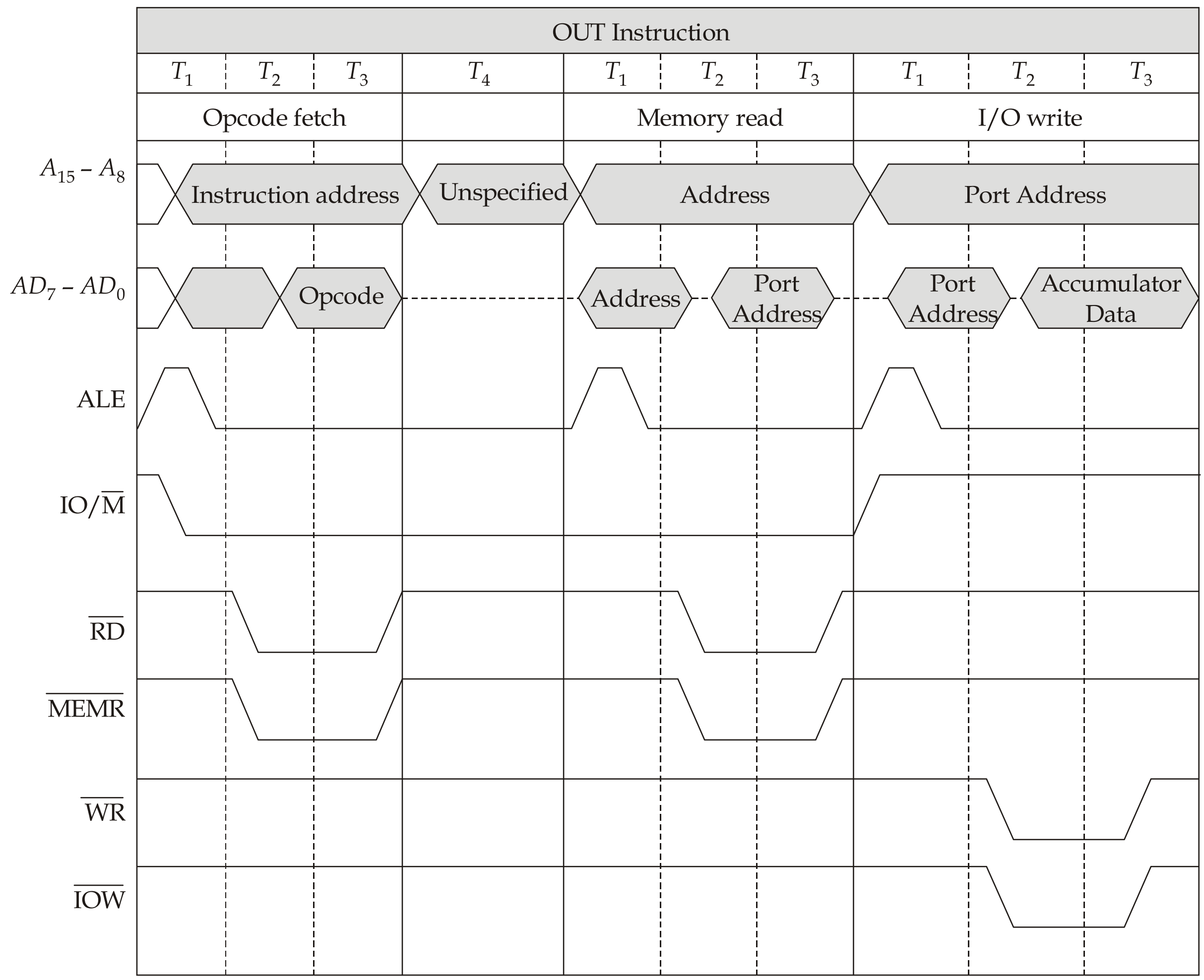
**Difference between Instruction cycle and Machine cycle:**

Instruction cycle: It is defined as the time required to complete the execution of an instruction. The 8085 instruction cycle consists of one of five machine cycles or one to five operations.

Machine cycle:

It is defined as the time required to complete one operation of accessing memory, I/O, or acknowledging an external request. It consists of three to six T-states i.e. three to six clock periods.

**(ii)**




**Q.6 (b) Solution:**

- For the given system, the control input and the plant output are not in same energy domain. Hence, the error signal will be,

$$E(s) = R(s) - H(s)C(s)$$

$$C(s) = \frac{K_1K_2}{(s+2)}E(s) + \frac{K_2}{(s+2)}D(s)$$

So,

$$E(s) = R(s) - \frac{K_1K_2(s+1)}{(s+2)}E(s) - \frac{K_2(s+1)}{(s+2)}D(s)$$

$$\left[1+\frac{K_1K_2(s+1)}{(s+2)}\right]E(s) = R(s)-\frac{K_2(s+1)}{(s+2)}D(s)$$

$$E(s) = \frac{(s+2)R(s)}{(s+2)+K_1K_2(s+1)}-\frac{K_2(s+1)D(s)}{(s+2)+K_1K_2(s+1)}$$

- Given that, both the input and the disturbance are unit step signals.

So, $R(s) = D(s)=\frac{1}{s}$

- The steady state error will be,

$$e_{ss} = \lim_{s\to 0} sE(s)=\lim_{s\to 0}\left[\frac{(s+2)}{(s+2)+K_1K_2(s+1)}-\frac{K_2(s+1)}{(s+2)+K_1K_2(s+1)}\right]$$

$$e_{ss} = \frac{2-K_2}{2+K_1K_2}$$

- The sensitivity of $e_{ss}$ for changes in $K_1$ will be,

$$S_{K_1}^{e_{ss}} = \frac{\partial e_{ss}}{\partial K_1}\times\frac{K_1}{e_{ss}}=\frac{-(2-K_2)K_2}{(2+K_1K_2)^2}\times\frac{K_1}{e_{ss}}=-\frac{K_1K_2}{2+K_1K_2}$$

- The sensitivity of $e_{ss}$ for changes in $K_2$ will be,

$$S_{K_2}^{e_{ss}} = \frac{\partial e_{ss}}{\partial K_2}\times\frac{K_2}{e_{ss}}=\left[\frac{-(2+K_1K_2)-K_1(2-K_2)}{(2+K_1K_2)^2}\right]\times\frac{K_2}{e_{ss}}$$

$$= -\frac{2(K_1+1)K_2}{(2-K_2)(2+K_1K_2)}$$

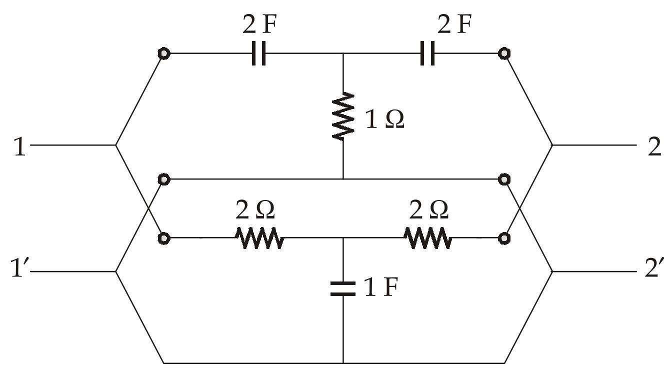
- When $K_1$ = 100 and $K_2$ = 0.1,

$$S_{K_1}^{e_{ss}} = -\frac{(100)(0.1)}{2+(100\times 0.1)}=-\frac{10}{12}=-0.833$$

$$S_{K_2}^{e_{ss}} = -\frac{2(100+1)(0.1)}{(2-0.1)(2+100\times 0.1)}=-\frac{0.2\times 101}{1.9\times 12}=-0.886$$

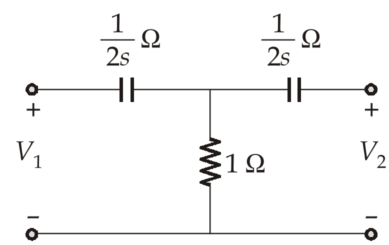
**Q.6 (c) Solution:**

**(i)** Given network is divided into two sub-networks and these networks are connected in parallel.




The total Y-parameters of the network is obtained by addition of two individual sub-network Y-parameter.

i.e., $$[Y]_T = [Y]_A + [Y]_B$$

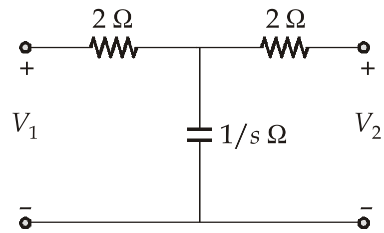



$Z$-parameters of the network is

$$[Z]_A = \begin{bmatrix} 1+\frac{1}{2s} & 1 \\ 1 & 1+\frac{1}{2s} \end{bmatrix}$$

$$[Z]_A = \begin{bmatrix} \frac{2s+1}{2s} & 1 \\ 1 & \frac{2s+1}{2s} \end{bmatrix}$$

$$[Y]_A = [Z]_A^{-1} = \begin{bmatrix} \frac{4s^2+2s}{4s+1} & \frac{-4s^2}{4s+1} \\ \frac{-4s^2}{4s+1} & \frac{4s^2+2s}{4s+1} \end{bmatrix}$$




Z-parameters of the network is

$$[Z]_B = \begin{bmatrix} 2+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & 2+\frac{1}{s} \end{bmatrix}$$

$$[Z]_B = \begin{bmatrix} \frac{2s+1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{2s+1}{s} \end{bmatrix}$$

$$[Y]_B = [Z]_B^{-1} \Rightarrow \begin{bmatrix} \frac{2s+1}{4s+4} & \frac{-1}{4s+4} \\ \frac{-1}{4s+4} & \frac{2s+1}{4s+4} \end{bmatrix}$$

$$[Y]_T = [Y]_A + [Y]_B$$

$$[Y] = \begin{bmatrix} \frac{16s^3+32s^2+14s+1}{(4s+1)(4s+4)} & \frac{-(16s^3+16s^2+4s+1)}{(4s+1)(4s+4)} \\ \frac{-(16s^3+16s^2+4s+1)}{(4s+1)(4s+4)} & \frac{16s^3+32s^2+14s+1}{(4s+1)(4s+4)} \end{bmatrix}$$

**(ii) 1.** At 10 MHz, the load impedance is

$$Z_L(j\omega) = \frac{1}{\left(\frac{1}{R_L}\right) + j\omega C}$$

$$Z_L(j2\pi \times 10^7) = \frac{1}{0.00005 + j2\pi \times 10^7 \times 40 \times 10^{-12}} = 397.8\angle -88.9^\circ\ \Omega$$

We have, $|V_{out}| = (g_m V_1)Z_L$

Therefore, since $|V_1| = |V_{in}| = 0.1$ V, the magnitude of the output voltage is

$$|V_{out}| = 0.1 \times 0.002 \times 397.8 = 0.0796 \text{ V}$$

Here the voltage gain is 0.0796/0.1 = 0.796 due to the low impedance of C at the high operating frequency.

2. By tuning out the effect of the capacitance, this poor gain can be improved. The inductance needed to tune out the capacitance is

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{4\pi^2 \times 10^{14} \times 40 \times 10^{-12}} = 6.33 \times 10^{-6} \text{H}$$

With a 6.33-μH inductor connected across the load, the parallel $LC$ behaves as an open circuit at 10 MHz and the load looks like a pure resistance of 20 kΩ to the amplifier. The new output voltage magnitude becomes

$$|V_{out}| = 0.1 \times 0.002 \times 20{,}000 = 4 \text{ V}$$

with a resultant voltage gain of 4/0.1 = 40.

**Q.7 (a) Solution:**

**(i)** Whenever an instruction is executed, it performs a specific operation on the operand. The way by which an operand is specified for an instruction is called addressing mode.

The 8086 consists of 8 addressing modes:

**1. Immediate Addressing mode:**

In this mode the operand is specified in the instruction itself.

Eg: MOV BX, 3598 H

**2. Direct Addressing Mode**

In this mode, operand's offset address is given in the instruction as an 8-bit or 16-bit displacement. In this addressing mode, the effective address of the data is the part of the instruction.

e.g. ADD BL, [0103H]

This instruction adds the content of offset address 0103 H to BL. The operand is placed at the given offset (01034) within the DS.

**3. Register Addressing Mode**

In this mode, the operand is placed in one of 8-bit or 16-bit general purpose registers.

e.g. MOV AX, CX;    ADD AL, BL etc.

**4. Register Indirect Addressing Mode**

The operand's offset address is placed in one of the register such as BX, Base pointer, source index or destination index register.

e.g. MOV BX, [BP]

5. **Register Relative Addressing Mode**

The operand's offset address is the sum of 8-bit or 16-bit displacement and the content of base register, index register or base pointer.

Offset (Effective address) = [BX + Displacement]

e.g. MOV CL, [BX + 05 H]

6. **Based Addressing Mode**

In this addressing mode, the offset address of the operand is given by the sum of contents of the BX/BP registers.

e.g. MOV DX, [BX]

7. **Indexed Addressing Mode**

The operand's offset address is the sum of 8-bit or 16-bit displacement and the content of index register SI or DI.

e.g. MOV BX, [SI + 1528 H]

8. **Based Index Addressing Mode**

The operand's offset address is the sum of content of base register and indexed register.

offset = [BX or BP] + [SI or DI]

e.g. ADD CX, [BX + SI]

9. **Relative Based Indexed Addressing Mode**

In this mode operand's offset address is given as

Offset = Base register + Indexed register + 8-bit/16-bit displacement

e.g. MOV AX, [BX + SI + 1234 H]

**(ii)** Once the effective/offset address is computed based on the addressing mode, the physical address is calculated as

Physical address (PA) = (Segment Address × 10H) + Offset (or) Effective address (EA)

1. **Register indirect addressing mode (assuming DI)**

$$\begin{aligned} EA &= [DI] = 4000\text{ H} \\ PA &= \text{Segment register : EA} \\ &= 7000\text{ H} : 4000\text{ H} \\ &= 70000 + 4000 = 74000\text{ H} \end{aligned}$$

2. **Based addressing mode (assuming BX)**

$$\begin{aligned} EA &= [BX] + \text{displacement} \\ &= 2000\text{ H} + 1000\text{ H} = 3000\text{ H} \end{aligned}$$

PA = Segment register : EA

= DS : EA

= 7000 H : 3000 H = 70,000 + 3000

= 73000 H

**3. Based index addressing mode**

EA = [BX] + [DI]

= 2000 H + 4000 H = 6000 H

PA = Segment register : EA

= 7000 H : 6000 H = 76000 H

**4. Based index with displacement addressing mode**

EA = [BX] + [DI] + Displacement

= 2000 H + 4000 H + 1000 H = 7000 H

PA = Segment register : EA

= 7000 H : 7000 H = 77000 H

**Q.7 (b) Solution:**

Selecting $x_1(t)$ and $x_2(t)$ as state variables,

$$\dot{x}_1(t) = \frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\dot{x}_2(t) = \frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$

$$\dot{x} = Ax + Bu$$

So, $$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y(t) = x_1(t)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$y = CX + DU$$

So, $$C = [1 \quad 0] \text{ and } D = 0$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

$$\text{Transfer function} = C[sI - A]^{-1} \text{B} + \text{D}$$

$$= [1 \quad 0] \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0$$

$$= \frac{[1 \quad 0] \begin{bmatrix} 2(s+2)+1 \\ (s+3) \end{bmatrix}}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)} = \frac{2s+5}{s^2+5s+6}$$

For state transition matrix,

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)} = \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix} = \begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\text{State transition matrix} = L^{-1}[(sI - A)^{-1}]$$

$$= L^{-1} \begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+2} - \frac{1}{s+3} \\ 0 & \frac{1}{s+2} \end{bmatrix} = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

**Q.7 (c) Solution:**

The switch is initially in position '$A$'.

The expression of $V_c(t)$ is

$$V_c(t) = \text{C.F} + \text{P.I.}$$

Particular integration gives steady state value. Using voltage division rule,

$$V_c(t) = 5 \sin t \times \frac{(-j1)}{2 - j1} \Rightarrow \sqrt{5} \sin(t - 63.43°) \text{ V}$$

Above equation indicates steady state voltage across the capacitor.

$$\text{C.F} = ke^{-t/\tau} \text{ where } \tau = \text{time constant}$$

i.e., $\tau = RC$; $R_{eq} = 2\ \Omega$; $C = 1$ F; $\tau = 2$ sec.

Expression for $V_c(t)$ for $0 \le t \le 1$ sec

$$V_c(t) = ke^{-t/2} + \sqrt{5}\sin(t - 63.43°)\text{ V}$$

Apply initial conditions to find the '$k$' value

$$V_c(0) = 0$$

$$0 = ke^{-0/2} + 2.236\sin(0 - 63.43°)$$

$$\therefore \quad k = 2$$

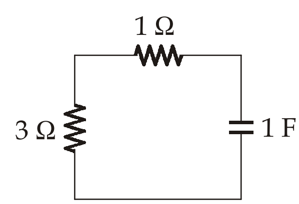
$$\therefore \quad V_c(t) = 2e^{-0.5t} + 2.236\sin(t - 63.43°)\text{ V} ;\ 0 \le t \le 1\text{ sec}$$

At $t = 1$ sec, the voltage across capacitor is

$$V_c(1) = 2e^{-0.5} + 2.236\sin\left(1 \times \frac{180°}{\pi} - 63.43°\right)$$

$$V_c(1) = 0.9739\text{ V}$$

Compute the response over $1 \le t < 2$. After switch moves from position '$A$' to position '$B$', the source is decoupled from the right half of the circuit.




It is a source free + first order circuit.

when $t \to \infty$, the voltage across the capacitor is '0'

The expression for $V_c(t)$ between the time interval $1 \le t \le 2$ is

$$V_c(t) = \text{final value} + (\text{initial value} - \text{final value})e^{-(t-1)/\tau}.$$

$$V_c(t) = 0 + (0.9739 - 0)e^{(t-1)/\tau}$$

$$\tau = R_{eq}\,C;\ R_{eq} = 3 + 1 = 4\ \Omega;\ C = 1\text{ F}$$

$$\tau = 4\text{ sec}$$

$$V_c(t) = 0.9739e^{-(t-1)/4}$$

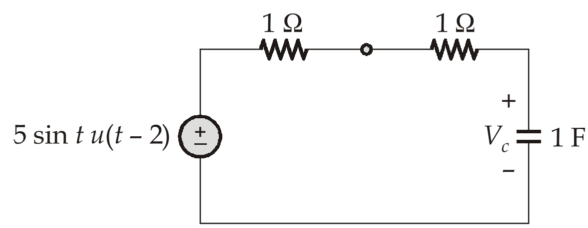
$$V_c(t) = 0.9739\ e^{-(t-1)0.25}\text{ V};\ 1 \le t < 2\text{ sec}$$

At $t = 2$ sec; the voltage across capacitor is

$$V_c(2) = 0.9739e^{-(2-1)(0.25)}\text{ V}$$

$$V_c(2) = 0.7585\text{ V}$$

Compute $V_c(t)$ for $t \geq 2$. The input for this time interval, 5 sin $t\, u\,(t - 2)$.




$$V_c(t) = \text{Complementary function + Particular integration}$$

(or)
$$V_c(t) = \text{transient + steady state}$$

The steady state voltage across the capacitor is

$$V_c(t) = 5\sin t \times \frac{(-j1)}{2-j1}$$

$$V_c(t) = 2.236 \sin(t - 63.43°)\text{ V}$$

(transient term) $V_c(t) = ke^{-(t-2)/\tau}$

$$\tau = R_{eq}\, C = 2 * 1 = 2 \text{ sec}$$

At $t = 2$ sec,
$$V_c(2) = ke^{-(2-2)/2} + 2.236\sin\left(2\times\frac{180°}{\pi} - 63.43°\right)$$

$$0.7585 = k + 2.236 \sin(51.16°)$$

$$k = -0.983$$

$$V_c(t) = -0.983e^{-(t-2)/2} + 2.236 \sin(t - 63.43°)\text{ V};\ t \geq 2 \text{ sec}$$

$$V_c(t) = \begin{cases} 2e^{-0.5t} + 2.236\sin(t-63.43°) & ;\ 0 \leq t < 1 \\ 0.974e^{-0.25(t-1)} & ; 1 \leq t < 2 \\ -0.983e^{-(t-2)0.5} + 2.236\sin(t-63.43°) & ;\ t \geq 2 \end{cases}$$

**Q.8 (a) Solution:**

Unilateral Laplace transform is defined as:

$$X(s) = \int_0^\infty x(t)\cdot e^{-st}\cdot dt$$

**(i)**
$$x(t) = [u(t-1) + u(-t-4)] * e^{-2t}u(t-1)$$

$$x(t) = u(t-1) * e^{-2t}u(t-1)$$

We know, $u(t-a) \xleftrightarrow{LT} \dfrac{e^{-as}}{s}$

$$e^{-at}u(t) \xleftrightarrow{LT} \frac{1}{s+a}$$

$$e^{-a(t-1)}u(t-1) \longleftrightarrow \frac{e^{-s}}{s+a}$$

Using the convolution property of Laplace transform,

$$x_1(t) * x_2(t) \longleftrightarrow X_1(s)\, X_2(s)$$

$\therefore$
$$X(s) = \frac{e^{-s}}{s} \cdot \frac{e^{-(s+2)}}{s+2} = \frac{e^{-2(s+1)}}{s(s+2)}$$

**(ii)**
$$x(t) = t \cdot \frac{d}{dt}\left[e^{-t} \cdot \cos t\, u(t) + e^{-(t+1)} u(-(t+1))\right]$$

$\because$ Unilateral Laplace transform exist for $0 < t < \infty$

$$x(t) = t \cdot \frac{d}{dt}\left[e^{-t} \cos t \cdot u(t)\right] = t \cdot y(t)$$

We know, $\cos at \cdot u(t) \xleftrightarrow{LT} \dfrac{s}{s^2 + a^2}$

$$e^{-bt} \cos at \cdot u(t) \xleftrightarrow{LT} \frac{s+b}{(s+b)^2 + a^2}$$

$$\frac{d}{dt} y'(t) \xleftrightarrow{LT} sY'(s)$$

$$t \cdot y(t) \xleftrightarrow{LT} -\frac{d}{ds} Y(s)$$

Now;
$$x(t) = t \frac{d}{dt} y'(t)$$

$\because$
$$y'(t) = e^{-t} \cos t \cdot u(t)$$

$$Y'(s) = \frac{s+1}{(s+1)^2 + 1}$$

$$y(t) = \frac{d}{dt} y'(t) \xleftrightarrow{LT} sY'(s)$$

$$t \cdot y(t) \xleftrightarrow{LT} -\frac{dY(s)}{ds}$$

$\therefore$
$$X(s) = -\frac{d}{ds}\left[\frac{s(s+1)}{(s+1)^2 + 1}\right] = \frac{-d}{ds}\left[\frac{s^2 + s}{s^2 + 2s + 2}\right]$$

$$X(s) = -\left[\frac{(s^2 + 2s + 2)(2s+1) - (s^2 + s)(2s+2)}{(s^2 + 2s + 2)^2}\right]$$

$$\therefore \qquad X(s) = \frac{-(s^2+4s+2)}{(s^2+2s+2)^2}$$

**Q.8 (b) Solution:**

Resistance of the circuit, $R = 300\ \Omega$

Inductive reactance of the circuit,

$$X_L = 2\pi fL = 314 \times 2.06 = 646.8\ \Omega$$

Capacitive reactance of the circuit,

$$X_c = \frac{1}{2\pi fC} = \frac{1}{314 \times 7.95 \times 10^{-6}} = 400\ \Omega$$

Impedance of the circuit,

$$Z = R + j(X_L - X_c)$$
$$= 300 + j(646.8 - 400)$$
$$= 300 + j246.8 = 388.5\ \angle 39.44°\ \Omega$$

Since, $$v(t) = 250\sqrt{2}\sin(314t + 30°)$$

$$V = \frac{250\sqrt{2}}{\sqrt{2}}\angle 30° = 250\angle 30°\ \text{V (rms)}$$

**(i)** The circuit current, $$I = \frac{V}{Z} = \frac{250\angle 30°}{388.5\angle 39.44°}$$
$$= 0.6435\ \angle -9.44° \text{A (rms)}$$

**(ii)** Voltage drop across the non-inductive resistor,

$$V_R = RI = 300 \times 0.6435\ \angle -9.44°\ \text{V}$$
$$= 193\ \angle -9.44°\ \text{V (rms)}$$

Also, $$v_R = \sqrt{2} \times 193 \sin(314t - 9.44°)\ \text{V}$$

Voltage drop across the inductor,

$$V_L = (jX_L)I = (646.8\ \angle 90°)(0.6435\ \angle -9.44°)$$
$$= 416.2\ \angle 80.56° \text{V (rms)}$$

Also, $$v_L = \sqrt{2} \times 416.2 \sin(314t + 88.56°)\ \text{V}$$

Voltage drop across the capacitor,

$$V_c = (-jX_c)I = 400\ \angle -90°)(0.6435\ \angle -9.44°)$$
$$= 257.4\ \angle -99.44° \text{V}$$

Also, $v_c = \sqrt{2} \times 257.4 \sin(314t - 99.44°)$ V

**(iii)** Power consumed in the circuit,

$$P = I^2R = (0.6435)^2 \times 300 = 124.2 \text{ W}$$

(or) $P = VI \cos \phi = 250 \times 0.6435 \cos[30° - (-9.44°)]$

$$= 124.2 \text{ W}$$

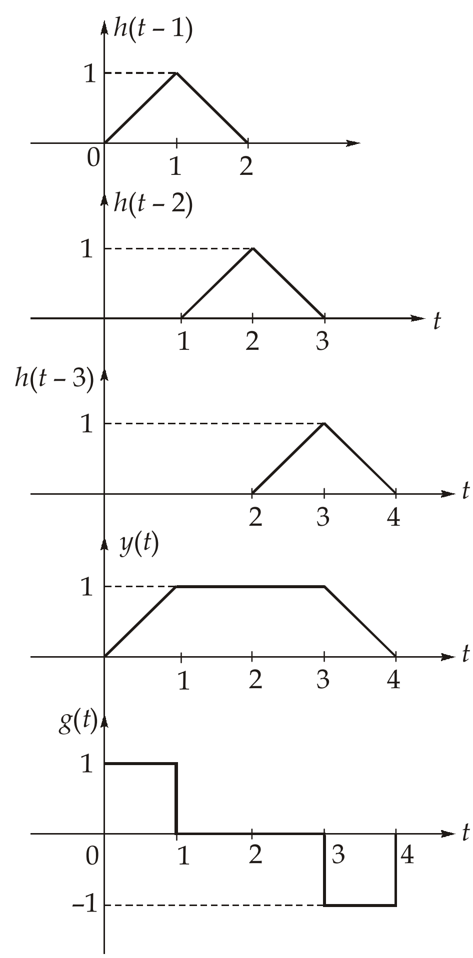
**Q.8 (c) Solution:**

**(i)** From given figure;

$$y(t) = x(t) * h(t)$$

$$= [\delta(t-1) + \delta(t-2) + \delta(t-3)] * h(t)$$

$$= h(t-1) + h(t-2) + h(t-3)$$



**(ii)** We know,

$$V(t) = \sum_{k=-\infty}^{\infty} C_k \cdot e^{jk\omega_0 t}; \omega_0 = \frac{2\pi}{2} = \pi$$

$$= 1 + C_1 \cdot e^{j\pi t} + C_{-1} \cdot e^{-j\pi t} + C_2 e^{j2\pi t} + C_{-2} \cdot e^{-j2\pi t}$$

$$= -4 + 2je^{j\pi t} - 2je^{-j\pi t} + 2e^{j2\pi t} + 2e^{-j2\pi t}$$

$$= 1 + 2j\left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right] \times 2j + 2\left[\frac{e^{j2\pi t} + e^{-j2\pi t}}{2}\right] \times 2$$

$$V(t) = 1 + -4 \sin\pi t + 4 \cos 2\pi t$$

at $t = 0$

$$V(0) = 1 + 4 = 5$$

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