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Detailed Solutions

ESE-2023 Mains Test Series

Electrical Engineering Test No : 3

Section A : Power Systems

Q.1 (a) Solution:

Series impedance of the line is given by,

$$Z = (r + j\omega L) \times l$$

$$= (0.08 + j \times 2\pi \times 50 \times 1.25 \times 10^{-3}) \times 60$$

$$= 24.04\angle 78.48° \ \Omega$$

Taking the receiving end voltage as reference,

$$V_R = \frac{66}{\sqrt{3}} = 38.11 \text{ kV/phase}$$

The receiving end current,

$$I_R = \frac{25000}{\sqrt{3} \times 66 \times 0.8} \angle \cos^{-1}(-0.8) = 273.37\angle -36.87° \text{ A}$$

Since, the line length is 60 km, therefore, it is classified as short transmission line. Hence, *ABCD* parameters of line $A = D = 1$, $B = Z$, $C = 0$.

$$\begin{bmatrix} V_S \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & 24.04\angle 78.48 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 38.11\angle 0° \text{ kV} \\ 273.37\angle -36.87° \text{ A} \end{bmatrix} \quad ...(1)$$

(i) On solving eqn. (1)

Revised Solutions as on 27-04-2023

$$V_s = 38.11\angle 0° + (0.27337\angle -36.86°) \times (24.04\angle 78.48°)$$

$$V_s = 43.24\angle 5.80° \text{ kV/phase or } 74.90 \text{ kV (line)}$$

$$I_s = 273.37\angle -36.86° \text{ A}$$

(ii) % Voltage Regulation,

$$\text{V.R.} = \frac{|V_S| - |V_R|}{|V_R|} \times 100\%$$

$$\text{V.R.} = \frac{43.24 - 38.11}{38.11} \times 100 = 13.46\%$$

(iii) Sending end power,

$$P_S = \sqrt{3} \times 74.90 \times 273.37 \cos(42.66°)$$

$$P_S = 26.08 \text{ MW}$$

$$\% \eta = \frac{P_R}{P_S} \times 100 = \frac{25}{26.08} \times 100 = 95.86\%$$

Q.1 (b) Solution:

Quantity of water available for utilization per second

$$= \frac{\text{Catchment area (m}^2) \times \text{Average rainfall (m)} \times \text{Yield factor}}{365 \times 24 \times 60 \times 60}$$

$$= \frac{150 \times 10^6 \times 1.20 \times 0.72}{8760 \times 3600} = 4.11 \text{ m}^3$$

Available head, $H = 30$ m

Overall efficiency of power plant,

η = Penstock efficiency × turbine efficiency × generator efficiency

$= 0.95 \times 0.85 \times 0.90 = 0.72675$

Average power, $P = WQH \times 9.81 \times \eta \times 10^{-6}$ MW

$= 1000 \times 4.11 \times 30 \times 9.81 \times 0.72675 \times 10^{-6}$

$= 0.8790$ MW

$$\text{Capacity of power station} = \frac{P}{\text{Load factor}} = \frac{0.8790}{0.40} = 2.19 \text{ MW}$$

Q.1 (c) Solution:

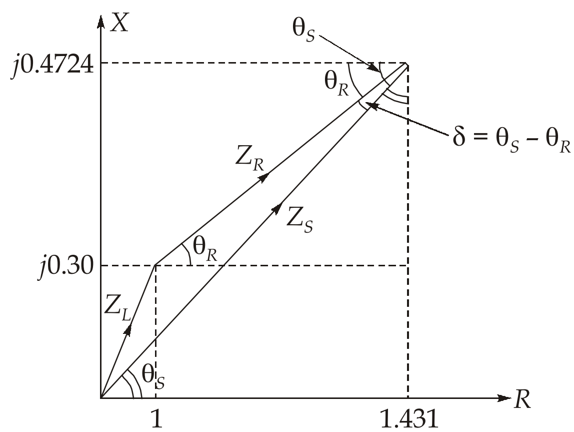
(i) As we know,

$$S^*_R = \frac{V_R^2}{Z_R}$$

$$Z_R = \frac{V_R^2}{S_R^*} = \frac{(1)^2}{(2 + j0.8)^*}$$

$$Z_R = (0.4310 + j0.1724) \text{ pu}$$

(ii)



(iii) Based on diagram, $\vec{Z}_S$ can be obtained analytically or graphically,

$$\vec{Z}_S = \vec{Z}_L + \vec{Z}_R = (1 + 0.431) + j(0.3 + 0.1724)$$

$$\vec{Z}_S = 1.50\angle 18.27° \text{ pu}$$

$$\delta = \theta_S - \theta_R = 18.27° - \tan^{-1}\left(\frac{0.1724}{0.431}\right)$$

$$\delta = -3.73°$$

Q.1 (d) Solution:

Characteristics impedance of lines,

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.3310\angle 87.14°}{4.674 \times 10^{-6}\angle 90°}} = 266.15\angle -1.43°\ \Omega$$

and

$$\gamma l = \sqrt{YZ} = l\sqrt{yz}$$

$$= 300 \times \sqrt{(0.3310\angle 87.14°) \times (4.674 \times 10^{-6}\angle 90°)}$$

$$= 0.3731\angle 88.57° = (0.00931 + j0.3730) \text{ pu}$$

$\Rightarrow$
$$e^{\gamma l} = e^{0.00931} \times e^{j0.3730} = 1.0094\angle 0.3730 \text{ radians}$$

[Note : $e^{\alpha + j\beta} = e^{\alpha}\angle\beta^{\text{rad}}$]

$$= 0.9400 + j0.3678$$

and
$$e^{-\gamma l} = e^{-0.00931} \times e^{-j0.3730} = 0.9907\angle -0.3730 \text{ radians}$$
$$= 0.9226 - j0.3610 \text{ radians}$$

Now,
$$\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{(0.94 + j0.3678) + (0.9226 - j0.3610)}{2}$$
$$\cosh \gamma l = 0.9313\angle 0.209°$$
$$\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{(0.94 + j0.3678) - (0.9226 - j0.3610)}{2}$$
$$\sinh \gamma l = 0.3645\angle 88.63°$$

Now, for long transmission line
$$A = D = \cosh \gamma l = 0.9313\angle 0.209° \text{ pu}$$
$$B = Z_c \sinh \gamma l = 97\angle 87.20°\ \Omega$$
$$C = \frac{\sinh \gamma l}{Z_c} = 1.37 \times 10^{-3}\angle 90.06° \text{ S}$$

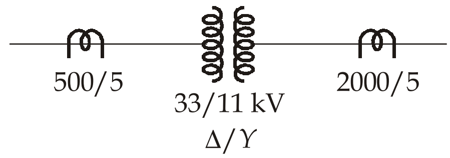
In nominal π-circuit
$$B_{\text{nomial-}\pi} = Z = (0.3310\angle 87.14°) \times (300)$$
$$= 99.3\angle 87.14°\ \Omega$$

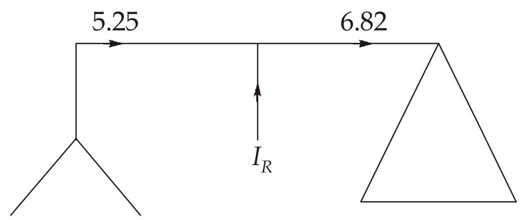
$B_{\text{nomial-}\pi}$ is 2.37% larger than the exact value.

Q.1 (e) Solution:

The primary line current is,
$$I_P = \frac{30 \times 10^6}{\sqrt{3} \times (33 \times 10^3)} = 524.86 \text{ A}$$

Secondary line current is, $\quad I_S = 3I_P = 1574.64 \text{ A}$





The CT secondary current on primary side (star),

$$I_1 = 524.88 \times \left(\frac{5}{500}\right) = 5.25 \text{ A}$$

and that on the secondary side (delta) is,

$$I_2 = 1574.64 \times \left(\frac{5}{2000}\right) \times \sqrt{3} = 6.82 \text{ A}$$

The relay current at 200% of the rated current is then,

$$I_R = 2[I_2 - I_1] = 2 \times (6.82 - 5.25)$$

$$I_R = 3.14 \text{ A}$$

Q.2 (a) Solution:

(i) Let base MVA = 100 MVA and base kV = 115 kV

Given :

$$S_2 = (184.8 + j6.6) \text{ MVA} = (1.848 + j0.066) \text{ pu}$$

$$S_3 = (0 + j20) \text{ MVA} = j0.2 \text{ pu}$$

$$V_3 = \frac{115\angle 0°}{115} = 1\angle 0° \text{ pu}$$

$$I_3 = \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1\angle 0°} = -j0.2 \text{ pu}$$

$\because$

$$Z_L(\text{pu}) = \frac{j66.125}{(115)^2} \times 100 = j0.5 \text{ pu}$$

$$V_2 = V_3 + Z_L(\text{pu})I_3$$

$$= 1\angle 0° + (j0.5)(-j0.2)$$

$$V_2 = 1.1\angle 0° \text{ pu}$$

Therefore, line-to-line voltage at bus-2 is,

$$V_2 = (115) \times (1.1) = 126.5 \text{ kV}$$

Similarly, $$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1\angle 0°}(1.68 - j0.06)\text{pu}$$

$$\vec{I}_{12} = \vec{I}_2 + \vec{I}_3 = (1.68 - j0.06) + (-j0.2)$$
$$= (1.68 - j0.26) \text{ pu}$$

Therefore, $$V_1 = V_2 + \vec{I}_{12} \times Z_T (\text{pu})$$
$$= 1.1\angle 0° + (j0.2)(1.68 - j0.26)$$
$$= 1.2\angle 16.26° \text{ pu}$$

Hence, the line-to-line voltage at bus-1 is

$$V_1 = (23 \times 1.2) = 27.6\angle 16.26° \text{ kV}$$

(ii) $$D_{ac} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$D_{ad} = \sqrt{6^2 + 3^2} = 6.7082 \text{ m}$$

Flux linkage in telephase line due to phase *a* :

$$\lambda_{cd}I_a = \left[0.2 \ln \frac{D_{ad}}{D_{ac}}\right] \times 226 = 13.28 \text{ mWb/km}$$

Similarly due to phase *b*,

$$\lambda_{cd}I_b = \left[0.2 \ln \frac{D_{bc}}{D_{bd}}\right] \times 226 = 0 \qquad [\because D_{bc} = D_{bd}]$$

Total flux linkage is

$$\Psi_{cd} = 13.28 \text{ mWb/km}$$

The voltage induced in the telephone line per km is

$$V = \omega\psi_{cd} = 2\pi \times 50 \times 13.28 \times 10^{-3}$$
$$= 4.173 \text{ V/km}$$

Q.2 (b) Solution:

(i) Kinetic energy stored = GH

Kinetic of equivalent machine = sum of kinetic energy individual machines,

$$\text{K.E.} = 4.6 \times 400 + 3 \times 1200 = 5440 \text{ MJ}$$

Equivalent inertia constant of multimachine system,

$$H_{eq} = \frac{G_1H_1 + G_2H_2}{G_{\text{base}}}$$

$$= \left[\frac{400}{100}\times 4.6 + \frac{1200}{100}\times 3\right] = 54.4 \text{ MJ/MVA}$$

or equivalent inertia relative to a 100 MVA/base,

$$H_{eq} = \frac{\text{KE}}{\text{System base}} = \frac{5440}{100} = 54.4 \text{ MJ/MVA}$$

(ii) For a fault at bus-3 with fault impedance $Z_f = j0.19$ pu, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1}{j0.21 + j0.19} = -j2.5 \text{ pu}$$

Bus voltage during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1 - (j0.03)(-j2.5) = 0.925 \text{ pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1 - (j0.03)(-j2.5) = 0.925 \text{ pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1 - (j0.21)(-j2.5) = 0.475 \text{ pu}$$

Now, the short circuit current in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{Z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \text{ pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{Z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \text{ pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{Z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \text{ pu}$$

Q.2 (c) Solution:

(i) Expressing the governor speed regulation of each unit to a common 1000 MVA base,

$$R_1 = \frac{1000}{400}\times 0.04 = 0.1 \text{ pu}$$

$$R_2 = \frac{1000}{800}\times 0.05 = 0.0625 \text{ pu}$$

The per unit load change is

$$\Delta P_L = \frac{130}{1000} = 0.13 \text{ pu}$$

When $D = 0$, the per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{-0.13}{10 + 16} = -0.005 \text{ pu}$$

The steady-state frequency deviation in Hz is,

$$\Delta f = (-0.005) \times 60 = -0.30 \text{ Hz}$$

New frequency of operation is

$$f = f_o + \Delta f = 59.70 \text{ Hz}$$

The change in generation for each unit is

$$\Delta P_1 = \frac{-\Delta\omega}{R_1} = -\frac{-0.005}{0.10} = 0.05 \text{ pu} = 50 \text{ MW}$$

$$\Delta P_1 = \frac{-\Delta\omega}{R_2} = -\frac{-0.005}{0.0625} = 0.08 = 80 \text{ MW}$$

Therefore,

$$P_1 = 200 + 50 = 250 \text{ MW}$$

$$P_2 = 500 + 80 = 580 \text{ MW}$$

$$f = 59.70 \text{ Hz}$$

(ii) For $D = 0.804$, per unit steady state frequency deviation.

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2} + D} = \frac{-0.13}{10 + 16 + 0.804} = -0.00485 \text{ pu}$$

$$\Delta f = (-0.00485) \times 60 = -0.291 \text{ Hz}$$

New frequency of operation is

$$f = f_o + \Delta f = 60 - 0.291 = 59.709 \text{ Hz}$$

The change in generation for each unit is

$$\Delta P_1 = \frac{-\Delta\omega}{R_1} = \frac{-0.00485}{0.10} = 0.0485$$

$$= 48.5 \text{ MW}$$

$$\Delta P_2 = \frac{-\Delta\omega}{R_2} = -\frac{-0.00485}{0.0625} = 0.0776 \text{ pu}$$

$$= 77.6 \text{ MW}$$

Therefore,

$$P_1 = 248.5 \text{ MW}$$

$$P_2 = 577.6 \text{ MW}$$

$$f = 59.709 \text{ Hz}$$

The total charge in generation is 126.1, which is 3.9 MW less than 130 MW load change. This is because of change in load due to frequency drop which is given by

$$\Delta\omega D = (-0.00485)(0.81) = -0.0039 \text{ pu} = -3.90 \text{ MW}$$

Q.3 (a) Solution:

Given : $Z = 5 + j25 = 25.50\angle 78.7° \ \Omega$

Receiving end power,

$$S_R = P_D + jQ_D = 15 + j15\tan(36.87°) = (15 + j11.25) \text{ MVA}$$

As we know,

$$P_R = \frac{V_S V_R}{|Z|}\cos(\theta - \delta) - \frac{V_R^2}{Z_S}\cos(\theta)$$

$$\Rightarrow \quad 15 = \frac{(33)^2}{25.50}\cos(78.7° - \delta) - \frac{(33)^2}{25.50}\cos(78.7°)$$

$$\cos(78.7 - \delta) = \frac{25.5 \times 15}{(33)^2} + \cos(78.7°)$$

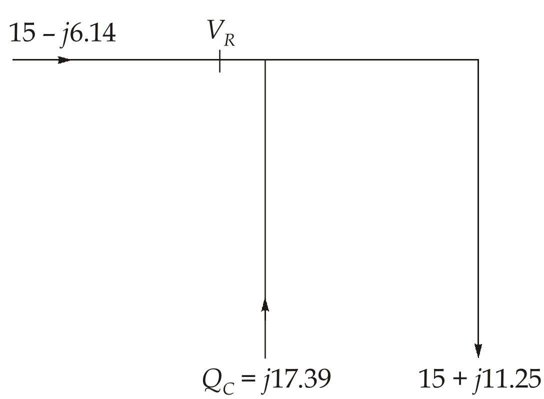
$$\Rightarrow \quad \delta = 21.90°$$

Now, receiving end reactive power,

$$Q_R = \frac{(33)^2}{25.5}\sin(78.7 - \delta) - \frac{(33)^2}{25.50}\sin(78.7°)$$

$$Q_R = \frac{(33)^2}{25.5}\sin(78.7 - 21.9°) - \frac{(33)^2}{25.50}\sin(78.70°)$$

$$= -6.14 \text{ MVAR}$$



Therefore, compensation element rating,

$$Q_C = Q_R - Q_D$$

$$Q_C = -6.14 - 11.25 = 17.39 \text{ MVAR leading}$$

(capacitive compensation)

Now, $|V_R| = 28 \text{ kV}$

$$P_D + jQ_D = P_D(1 + j\tan\phi)$$
$$= P_D(1 + j\tan 36.87°) = P_D(1 + j0.75)$$
$$P_R + jQ_R = P_D + j(0.75P_D - 17.39)$$
$$P_R = P_D = \frac{33 \times 28}{25.5}\cos(78.7° - \delta) - \frac{(28)^2}{25.50}\cos(78.7°)$$

And
$$Q_R = 0.75P_D - 17.39$$
$$= \frac{33 \times 28}{25.5}\sin(78.7 - \delta) - \frac{(28)^2}{25.50}\sin(78.7°)$$

or
$$\cos(78.7° - \delta) = \frac{25.50}{33 \times 28}P_D + \frac{28}{33}\cos 78.7°$$
$$= 0.0276P_D + 0.1663 \quad ...(1)$$

Also,
$$\sin(78.7 - \delta) = \frac{25.5 \times 0.75}{33 \times 28}P_D - \frac{25.50 \times 17.39}{33 \times 28} + \frac{28}{33}\sin(78.7°)$$
$$= 0.0207P_D + 0.352 \quad ...(2)$$

Eqn. $(1)^2$ + Eqn. $(2)^2$

$$1 = 1.19 \times 10^{-3}P_D^2 + 23.7 \times 10^{-3}P_D + 0.1516$$

On solving,

$$= 18.54 \text{ MW (negative sign is neglected)}$$
$$\text{Extra power transmitted} = 18.54 - 15 = 3.54 \text{ MW}$$

Note : It is assumed that as receiving-end voltage drops, the compensating element draws the same MVAR (leading).

Q.3 (b) Solution:

Let the base MVA of the system be 20 MVA, base kV of generator side 11 kV and for transmission circuit side 66 kV. Per unit reactance of different elements corresponding to this common base MVA are

For generator A and B,

$$X_{\text{p.u. g}} = j0.25 \text{ p.u.}$$

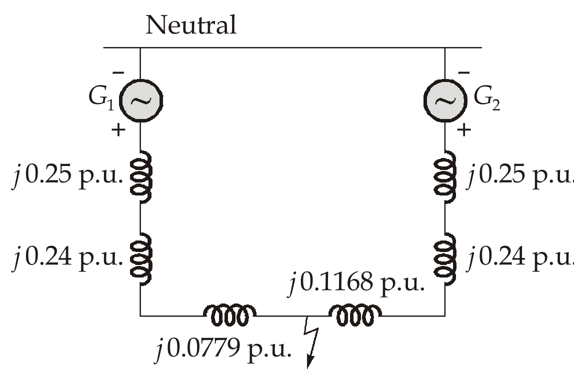
for transformers T_1 and T_2,

$$X_{\text{p.u.}} = j0.06 \times \frac{20}{5} = j0.24 \text{ p.u.}$$

$$\text{Per unit reactance of 20 km line} = \frac{20 \times 0.848 \times 20}{66^2} = 0.0779 \text{ p.u.}$$

$$\text{Per unit reactance of 30 km line} = \frac{30 \times 0.848 \times 30}{66^2} = 0.1168 \text{ p.u.}$$

Per unit reactance diagram is shown below,



$$X_{eq\ (p.u.)} = j(0.25 + 0.24 + 0.0779) \,||\, j(0.25 + 0.24 + 0.1168) \text{p.u.}$$

$$= j0.5679 \,||\, j0.6068 \text{ p.u.}$$

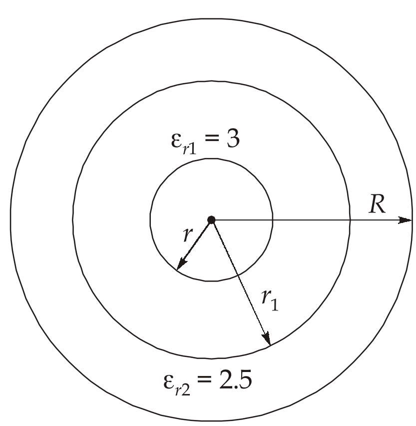
$$= \frac{j0.5679 \times j0.6068}{j0.5679 + j0.6068} = j0.2934 \text{ p.u.}$$

$$\text{Fault MVA} = \frac{\text{Base MVA}}{X_{eq(p.u.)}} = \frac{20}{0.2934} = 68.17 \text{ MVA}$$

$$\text{Fault current, } I_{SC} = \frac{68.17 \times 10^{-6}}{\sqrt{3} \times 66 \times 10^3} = 596.3 \text{ A}$$

Q.3 (c) Solution:

(i) Radius of conductor, $r = \frac{10}{2} = 5$ mm



$$t = 10 \text{ mm}$$

$$r_1 = r + t = 5 + 10 = 15 \text{ mm}$$

$$R_1 = r + 2t = 5 + 20 = 25 \text{ mm}$$

$$\varepsilon_{r1} = 3, \varepsilon_{r2} = 2.5$$

$$V = 60 \text{ kV}$$

$$g_{max1} = \frac{q}{2\pi\varepsilon_o\varepsilon_{r1}r} \text{ and } g_{max2} = \frac{q}{2\pi\varepsilon_o\varepsilon_{r2}r_1}$$

$$\frac{r_1 g_{max2}}{r g_{max1}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$V = V_1 + V_2$$

$$= g_{max1} r \ln\frac{r_1}{r} + g_{max2} \times r_1 \ln\frac{R}{r_1}$$

$$= r g_{max1}\left[\ln\frac{r_1}{r} + \frac{g_{max2} \times r_1}{g_{max} \times r}\ln\frac{R}{r_1}\right]$$

$$= r g_{max1}\left[\ln\frac{r_1}{r} + \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\ln\frac{R}{r_1}\right]$$

$$60 = 5 g_{max1}\left[\ln\frac{15}{5} + \frac{3}{2.5}\ln\frac{25}{15}\right]$$

$$= 5g_{max1}(1.098 + 0.613) = 8.558 g_{max1}$$

$$g_{max1} = \frac{60}{8.558} = 7.01 \text{ kV/mm}$$

(ii) Choose as base 50 MVA, 6.6 kV

Feeder impedance, $$Z_f = \frac{(0.06 + j0.12) \times 50}{(6.6)^2}$$

$$= (0.069 + j0.138) \text{ pu}$$

Generator A reactance $= \dfrac{0.1 \times 50}{40} = 0.125$ pu

Generator B reactance $= 0.1$ pu

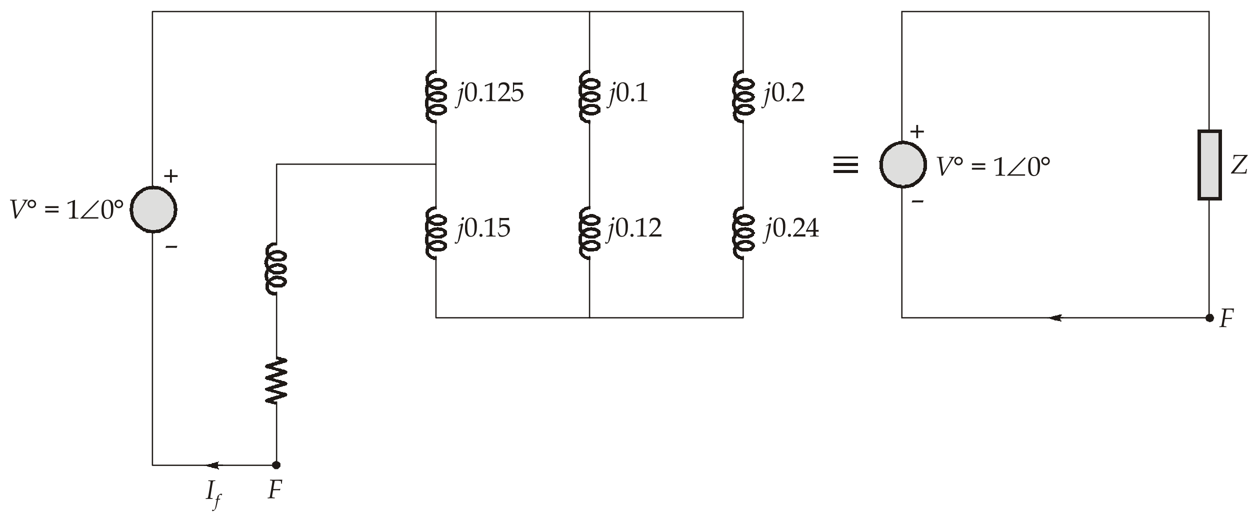
Generator C reactance $= \dfrac{0.1 \times 50}{25} = 0.2$ pu

Reactor A reactance $= \dfrac{0.12 \times 50}{40} = 0.15$ pu

$$\text{Reactor B reactance} = 0.12 \text{ pu}$$

$$\text{Reactor C reactance} = \frac{0.12 \times 50}{25} = 0.24 \text{ pu}$$

Now,



Assuming no-load prefault conditions, i.e., prefault currents are zero. Post-fault current can be calculated by the circuit model as shown.

Now,
$$Z = (0.069 + j0.138) + j0.125 \,||\, (j0.15 + (j0.22 \,||\, j0.44))$$
$$= 0.069 + j0.226 = 0.236\angle 73° \text{ pu}$$

$$\text{SC MVA} = V°I_f = V°\left(\frac{V°}{Z}\right) = \frac{1}{Z} \text{ pu} \qquad \{\because V° = 1\angle 0° \text{ pu}\}$$

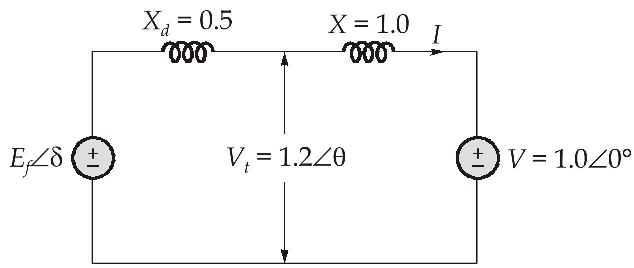
$$= \frac{1}{Z} \times (\text{MVA})_{\text{Base}}$$

$$= \frac{50}{0.236} = 212 \text{ MVA}$$

Q.4 (a) Solution:

(i) Let the voltage at infinite bus be taken as reference.

Then, $V = 1.0\angle 0°$ pu and $V_t = 1.2\angle 0°$ pu



Now,
$$I = \frac{V_t - V}{jX} = \frac{1.2\angle\theta - 1.0}{j1}$$
$$E_f = V_t + jX_d I$$
$$= 1.2\angle\theta + j0.5\left[\frac{1.2\angle\theta - 1.0}{j1}\right]$$
$$= 1.2\angle\theta + 0.6\angle\theta - 0.5$$
$$\vec{E}_f = 1.8\angle\theta - 0.5 \text{ pu}$$

As we know, steady state power limit is reached when E has an angle of $\delta = 90°$, i.e., its real part is zero.

Thus,
$$E_f = 1.8\cos\theta + j1.8\sin\theta - 0.5$$
$$\text{Real}[E_f] = 0$$
$$\Rightarrow \quad 1.8\cos\theta = 0.5$$
$$\theta = 73.87°$$
$$E_f = j1.8\sin\theta$$
$$|E_f| = 1.8\sin 73.87°$$
$$= 1.729 \text{ pu}$$
$$P_{\max} = \frac{E_f \cdot V}{X_T}$$
$$= \frac{1.729 \times 1.0}{1.0 + 0.5}$$
$$P_{\max} = 1.1527 \text{ pu}$$

(ii) Radius of conductor $= \dfrac{1.036}{2} = 0.518$ cm

The radio $\dfrac{d}{r} = \dfrac{2.44}{0.518} \times 100 = 471$

$$\sqrt{\frac{r}{d}} = \sqrt{\frac{1}{471}} = 0.046075$$

$$\delta = \frac{3.92b}{273+t} = \frac{392 \times 73.15 \times 10^{-2}}{273+26.67} = 0.957$$

Critical disruptive voltage

$$V_d = 21.1 \times 0.85\delta r \ln \frac{d}{r}$$

$$= 21.1 \times 0.85 \times 0.957 \times 0.518 \ln (471)$$

$$= 54.72 \text{ kV (line to neutral)}$$

The visual critical voltage,

$$V_v = 21.1 m_v \delta r\left(1+\frac{0.3}{\sqrt{r\delta}}\right)\ln\frac{d}{r}$$

$$V_v = 21.1 \times 0.72 \times 0.957 \times 0.518\left(1+\frac{0.3}{\sqrt{0.518 \times 0.957}}\right)\ln 471$$

$$= 66 \text{ kV}$$

$$\text{The power loss} = 241 \times 10^{-5} \times \frac{f+25}{\delta}\sqrt{\frac{r}{d}}(V-V_d)^2 \text{ kW/phase/km}$$

$$= 241 \times 10^{-5} \times \frac{75}{0.957} \times 0.046075(63.5-54.72)^2$$

$$= 0.671 \text{ kW/phase/km}$$

$$= 107.36 \text{ kW/phase}$$

$$= 322 \text{ kW for 3-}\phi \text{ phase}$$

The corona loss under foul weather condition will be when the disruptive voltage is taken as 0.8 × fair weather value, i.e.,

$$V_d = 0.8 \times 54.72 = 43.77 \text{ kV}$$

∴ Loss per phase/km will be

$$241 \times 10^{-5} \times \frac{75}{0.957} \times 0.046025(63.5-43.77)^2 = 3.3875 \text{ kW/km/phase}$$

or $= 542$ kW/phase

$$\text{Total loss} = 1626 \text{ kW for all 3-}\phi$$

Q.4 (b) Solution:

(i) Kinetic energy, K.E. = G.H

$= 3.5 \times 100 = 350$ MJ

(ii) System base(s) = 500 MVA

Then, the acceleration power,

$$P_a = P_i - P_u = (0.18 - 0.16) \times 500 = 10 \text{ MW}$$

$$M = \frac{2H}{\omega_s} \times S_{\text{rated}} = \frac{2 \times 3.5}{360 \times 50} \times 100 = \frac{3.5}{90}$$

Now,

$$M \cdot \frac{d^2\delta}{dt^2} = P_a = 10$$

$$\frac{d^2\delta}{dt^2} = \frac{10 \times 90}{3.5} = 257.143 \text{ ele-degree/sec}^2 \quad ...(1)$$

or 4.488 rad/sec^2

(iii) Acceleration period in seconds $= \frac{7.5}{50} = 0.15$ sec

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\frac{d^2\delta}{dt^2} = 4.488$$

$$\frac{d\delta}{dt} = 4.488t + K$$

$$K = \left.\frac{d\delta}{dt}\right|_{t=0} = 0$$

$$\delta = \frac{4.488t^2}{2} + \delta_o$$

$$\delta - \delta_o = \Delta\delta = \frac{4.488}{2} \times t^2$$

$$\Delta\delta = 0.05049 \text{ rad}$$

$$\Delta\delta = 2.89°$$

(iv) Rotor velocity is calculated as

$$\Delta\omega = \frac{d^2\delta}{dt^2} \times t$$

$$= 4.488 \times 0.15$$

$$= 0.6732 \text{ elec. rad/sec}$$

$$\Delta\omega = 0.6732 \times \frac{60}{2\pi} \text{ rpm} \times \frac{2}{P}$$

$$\Delta N = 6.429 \text{ rpm}$$

$$N' = N_s + \Delta N$$

$$N' = 1503.2145 \text{ rpm}$$

Q.4 (c) Solution:

The Y_{bus} matrix of system can be assembled as

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{11} = \text{Sum of all the admittances connected to bus-1}$$

$$= y_{12} + y_{13} = -j5 - j2.5 = -j7.5 \text{ pu}$$

$$Y_{12} = -y_{12} = j2.5 = Y_{21}$$

$$Y_{31} = Y_{13} = -y_{13} = j5$$

$$Y_{22} = y_{12} + y_{23} = -j4 - j2.5 = -j6.5$$

$$Y_{23} = -y_{23} = j4$$

$$Y_{33} = y_{31} + y_{32} = -j5 - j4 = -j9$$

Therefore,

$$Y_{bus} = j\begin{bmatrix} -7.5 & 2.5 & 5.0 \\ 2.5 & -6.5 & 4.0 \\ 5.0 & 4.0 & -9.0 \end{bmatrix}$$

Iteration 1 :

$$I_2^0 = \frac{S_2^*}{(V_2^0)^*} = \frac{P_2 - jQ_2}{(V_2^0)^*} = \frac{-1 + j0.8}{1.0 - j0} = -1 + j0.8$$

Now,

$$V_2^{(1)} = \frac{I_2^0 - (Y_{21}V_1 + Y_{23}V_3^0)}{Y_{22}}$$

$$\left[\because V_i^{K+1} = \frac{1}{Y_{11}}\left\{\frac{P_i - jQ_i}{(V_i^K)^*} - \sum_{m=1}^{i-1} Y_{im}V_m^{K-1} - \sum_{m=i+1}^{n} Y_{im}V_m^K\right\}\right]$$

$$= \frac{-1 + j0.8 + (j2.5\times 1.04 + j4\times 1.005)}{-j6.5}$$

$$V_2^{(1)} = (0.8953 - j0.1538)\text{ pu} = 0.9084\angle -9.75°\text{ pu}$$

Since, bus-3 is a voltage-controlled bus, real power P and magnitude of bus voltage are specified. Now, reactive power Q_3 at bus-3 can be computed as

$$Q_3^{(1)} = Im[V_3^{(0)} \times I_3^*]$$

$$= Im[1.005 \times (Y_{31}V_1^{(0)} + Y_{32}V_2^{(1)} + Y_{33}V_3^{(0)})^*]$$

$$= Im[(1.005) * \{(j5 \times 1.04) + j4 \times (0.8954 - j0.1538) + (-j9.0) \times (1.005)\}^*]$$

$$Q_3^{(1)} = 0.2647$$

$$I_3^{(0)} = \frac{S_3^*}{(V_3^0)^*} = \frac{1 - j0.2647}{1.005} = 0.9954 - j0.2633$$

$$V_3^{(1)} = \frac{[I_3^* - (Y_{31}V_1^{(1)} + Y_{32}V_2^{(1)})]}{Y_{33}}$$

Since, bus-1 is slack bus, so its voltage remains unchanged.

$$= \frac{(0.9950 - j0.2635) - [j5\times(1.04 + j0) + j4.0(0.8954 - j0.1538)]}{-j9.0}$$

$$V_3^{(1)} = 1.0050 + j0.0422$$

Since, the voltage magnitude at bus-3 is held constant, the real part of $V_3^{(1)}$ is modified as

$$e_3^{(1)} = \sqrt{(1.005)^2 - (0.0422)^2} = 1.0041$$

Hence, $V_3^{(1)} = 1.0041 + j0.0422 = 1.005\angle 2.41°\text{ pu}$

Section B : Systems and Signal Processing-1 + Microprocessor-1 + Electrical Circuits-2 + Control Systems-2

Q.5 (a) Solution:

		T-States
	LXI B, 12FFH	10 T
DELAY	DCX B	6 T
	XTHL	16 T
	XTHL	16 T

	NOR	4 T
	NOP	4 T
	MOV A, C	4 T
	ORA B	4 T
Count	12FFH = $1 \times 16^3 + 2 \times 16^2 + 15 \times 16^1 + 15 \times 16^0 = (4863)_{10}$	

The delay in the loop,

$$\text{Delay} = \text{T-states} \times \text{Clock period} \times \text{Count}$$

Clock period,
$$T = \frac{1}{f} = \frac{1}{3}\mu s$$

Total number of T-states in delay loop,

$$\text{T-states} = 6 + 16 \times 2 + 4 \times 4 + 10 \text{ (if JNZ is true)} = 64 \text{ T}$$

$$\text{T-states} = 6 + 16 \times 2 + 4 \times 4 + 7 \text{ (if JNZ is false)}$$

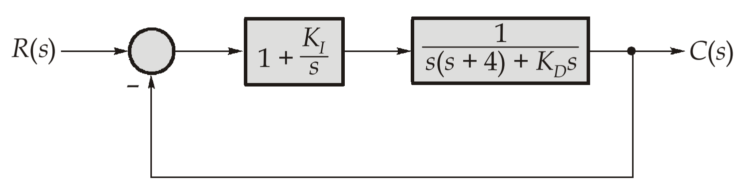
$$= 61 \text{ T}$$

$$\text{Delay} = \frac{1}{3} \times 10^{-6} \times (4862 \times 64 + 61)$$

$$= 103.743 \text{ ms}$$

Q.5 (b) Solution:

On solving inner feedback loop



The forward path transfer function,

$$G(s) = \frac{s + K_I}{s} \cdot \frac{1}{s(s + 4 + K_D)} = \frac{(s + K_I)}{s^2(s + 4 + K_D)}$$

The system type is type-2.

The characteristics equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{(s + K_I)}{s^2(s + 4 + K_D)} = 0$$

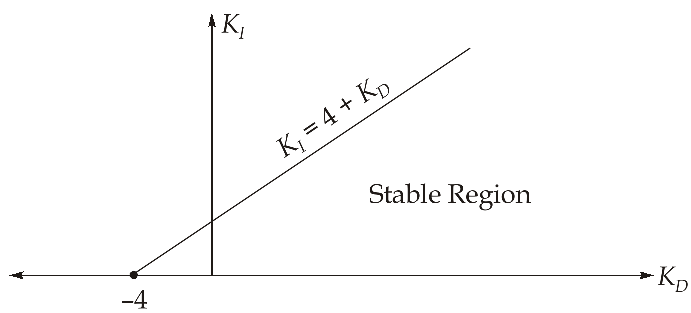
$$s^3 + (4 + K_D)s^2 + s + K_I = 0$$

Routh Array :

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 4+K_D & K_I \\ s^1 & \dfrac{4+K_D-K_I}{4+K_D} & \\ s^0 & K_I & \end{array}$$

The system remains stable if first column of Routh array have same sign.

i.e.,
$$4 + K_D > 0, \quad K_I > 0$$
$$4 + K_D - K_I > 0 \Rightarrow K_I < 4 + K_D$$
$$0 < K_I < 4 + K_D \quad \text{and} \quad K_D > -4$$



Q.5 (c) Solution:

(i) Given : Reduced incidence matrix

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

As algebraic sum of the column entries of an incidence matrix is 0. So, incidence matrix is

$$[A_1] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}$$

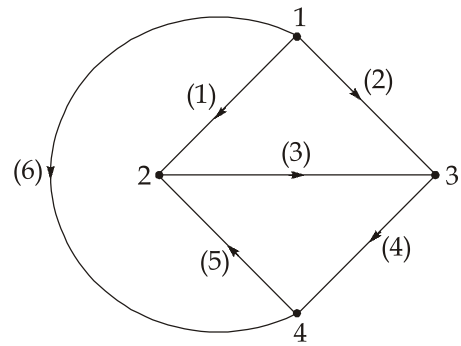
Number of nodes = 4

Further for A :

$$a_{ij} = 1; \text{ if branch } j \text{ is oriented away from node } i$$

a_{ij} = –1; if branch j is oriented towards from node i

a_{ij} = 0; if branch j is not incident on node i



(ii) Number of trees possible

$$T = \text{Determinant}[A]\,[A^T]$$

$$[A][A^T] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\text{Determinant }[A]\,[A^T] = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= 3(9 - 1) - 1(1 + 3) - 1(1 + 3)$$

$$= 24 - 4 - 4$$

$$= 16$$

Q.5 (d) Solution:

Given :

$$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$$

$$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$$

Since the system is linear,

(i)

$$x_1(t) = \cos(2t) = \frac{1}{2}(e^{j2t} + e^{-j2t})$$

$$x_1(t) = \frac{1}{2}(e^{j2t} + e^{-j2t}) \xrightarrow{S} y_1(t) = \frac{1}{2}(e^{j3t} + e^{-j3t})$$

$$y_1(t) = \frac{1}{2}(e^{j3t} + e^{-j3t}) = \cos(3t)$$

(ii)

$$x_2(t) = \cos(2t - 1) = \cos\left(2\left(t - \frac{1}{2}\right)\right)$$

$$= \frac{1}{2}[e^{-j}e^{j2t} + e^{j}e^{-j2t}]$$

$$x_2(t) = \frac{1}{2}[e^{-j}e^{j2t} + e^{j}e^{-j2t}] \xrightarrow{S} y_2(t) = \frac{1}{2}[e^{-j}e^{j3t} + e^{-j}e^{-j3t}]$$

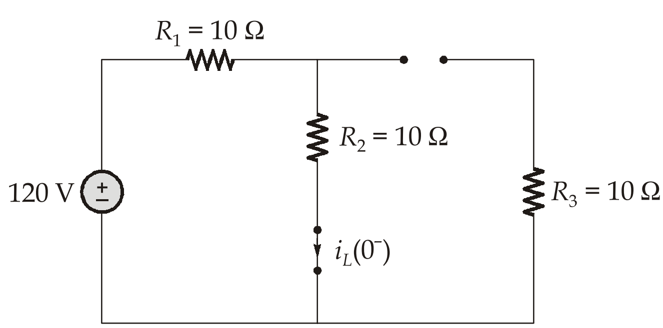
$$y_2(t) = \frac{1}{2}[e^{-j}e^{j3t} + e^{j}e^{-j3t}]$$

$$= \frac{1}{2}[e^{j(3t-1)} + e^{-j(3t-1)}]$$

$$y_2(t) = \cos(3t - 1)$$

Q.5 (e) Solution:

At $t = 0^-$; the switch was opened.



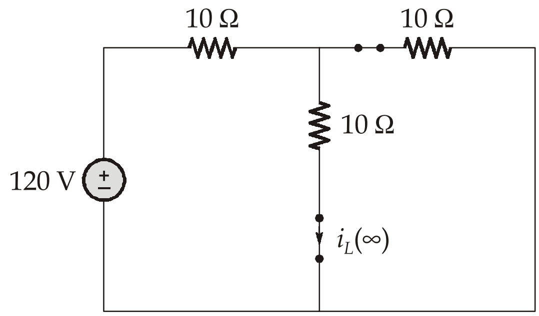
$$i_L(0^-) = \frac{120}{20} = 6 \text{ A}$$

The inductor does not allow sudden change in current.

$$i_L(0^+) = i_L(0^-) = 6 \text{ A}$$

At $t = 0^+$, the switch was closed.

At $t = \infty$



$$i_L(\infty) = \frac{120}{10 + (10 \parallel 10)} \times \frac{1}{2}$$

$$= \frac{120}{15} \times \frac{1}{2} = 4 \text{ A}$$

$$i_L(\infty) = 4 \text{ A}$$

Time constant, $$\tau = \frac{L}{R_{eq}}$$

$$R_{eq} = 10 + (10 \parallel 10) = 10 + 5 = 15\ \Omega$$

$$\tau = \frac{0.01}{15} = \frac{1}{1500}$$

$$i_L(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

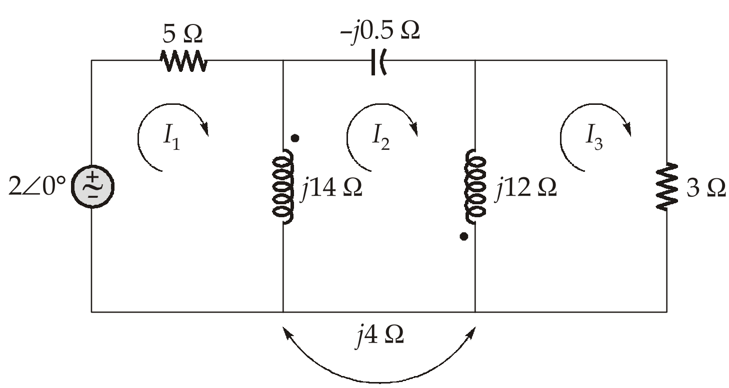
$$= 4 + (6 - 4)e^{-1500t}$$

$$i_L(t) = 4 + 2e^{-1500t} \text{ A}$$

Q.6 (a) Solution:

Redraw the circuit in phasor.

Here, $\omega = 2$ rad/sec



Apply KVL in mesh (1)

$$2\angle 0° = 5I_1 + j14(I_1 - I_2) + j4(I_3 - I_2)$$

$$(5 + j14)I_1 - j18I_2 + j4I_3 = 2\angle 0° \quad ...(1)$$

Apply KVL in mesh (2)

$$j14(I_2 - I_1) - j0.5I_2 + j12(I_2 - I_3) + j4(I_2 - I_1) + j4(I_2 - I_3) = 0$$

$$-j18I_1 + j33.5I_2 - j16I_3 = 0$$

$$18I_1 - 33.5I_2 + 16I_3 = 0 \quad ...(2)$$

Apply KVL in mesh (3)

$$3I_3 + j12(I_3 - I_2) + j4(I_1 - I_2) = 0$$

$$j4I_1 - j16I_2 + (3 + j12)I_3 = 0 \quad ...(3)$$

From eqn. (2)

$$I_2 = \frac{18I_1 + 16I_3}{33.5} \quad ...(4)$$

Put I_2 from eqn. (4) in eqn. (1) and (3)

Eqn. 1 :

$$(5 + j14)I_1 - j18\left[\frac{18I_1 + 16I_3}{33.5}\right] + j4I_3 = 2\angle 0°$$

$$6.613\angle 40.88° I_1 - 4.597\angle 90° I_3 = 2\angle 0° \quad ...(5)$$

Eqn. 3 :

$$j4I_1 - j16\left[\frac{18I_1 + 16I_3}{33.5}\right] + (3 + j12)I_3 = 0$$

$$-4.597\angle 90° I_1 + 5.29\angle 55.46° I_3 = 0 \quad ...(6)$$

From eqn. (6)

$$I_1 = 1.15\angle -34.54 I_3$$

Put I_1 in eqn. (5)

$$(6.613\angle 40.88)(1.15\angle -34.54 I_3) - 4.597\angle 90° I_3 = 2\angle 0°$$

$$I_3 = 0.237\angle 26.41° \text{ A}$$

$$I_1 = 1.15\angle -34.54 \times 0.237\angle 26.41$$

$$I_1 = 0.2723\angle -8.13 \text{ A}$$

From eqn. (4),

$$I_2 = \frac{18I_1 + 16I_3}{33.5}$$

$$= \frac{18(0.2723\angle -8.13°) + 16(0.237\angle 26.41°)}{33.5}$$

$$I_2 = 0.248\angle 6.87° \text{ A}$$

Q.6 (b) Solution:

```
START :    LXI H, 2040H   ;  Set up HL as a memory pointer for bytes
           MVI D, 00H     ;  Clear register D to setup a flag
           MVI C, 09H     ;  Initialize counter
CHECK :    MOV A, M       ;  Get data byte in Accumulator
           INX H          ;  Increment memory pointer
           CMP M          ;  Compare number with next number
           JC NXTBYT      ;  If (A) < second byte, do not exchange
           MOV B, M       ;  Get second byte for exchange
           MOV M, A       ;  Store first byte in second location
           DCX H          ;  Point to first location
           MOV M, B       ;  Get second byte in first location
           INX H          ;  Increment memory pointer
           MVI D, 01      ;  Load 1 in D register as reminder for exchange
NXTBYT :   DCR C          ;  Decrement counter
           JNZ CHECK      ;  If count ≠ 0; go to CHECK
           MOV A, D       ;  Move the data from D to Accumulator
           RRC            ;  Place the flag bit in carry
           JC START       ;  If carry = 1 jump to START
           HLT
```

Q.6 (c) Solution:

Given :
$$A = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad -1]$$

$$[sI - A]^{-1} = \begin{bmatrix} s & -2 \\ 3 & s+5 \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2 + 5s + 6}\begin{bmatrix} s+5 & 2 \\ -3 & s \end{bmatrix}$$

(i) Zero input response

$$Y_{ZIR}(s) = C[sI - A]^{-1}X(0)$$

$$= \frac{1}{s^2 + 5s + 6}[1 \quad -1]\begin{bmatrix} s+5 & 2 \\ -3 & s \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+6}[1 \quad -1]\begin{bmatrix} s+3 \\ -s-3 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+6}[s+3-(-s-3)]$$

$$= \frac{1}{s^2+5s+6}\cdot 2(s+3)$$

$$= \frac{2(s+3)}{(s+2)(s+3)} = \frac{2}{s+2}$$

Taking inverse laplace transform

$$y_{ZIR}(t) = 2e^{-2t}u(t)$$

(ii) Zero state response

$$Y_{ZSR}(s) = C[sI - A]^{-1}BR(s)$$

where, $R(s) = \frac{1}{s}$

$$Y_{ZSR}(s) = \frac{1}{s^2+5s+6}[1 \quad -1]\begin{bmatrix} s+5 & 2 \\ -3 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\frac{1}{s}\right)$$

$$= \frac{1}{s^2+5s+6}[1 \quad -1]\begin{bmatrix} 2 \\ s \end{bmatrix}\left(\frac{1}{s}\right)$$

$$= \frac{1}{s^2+5s+6}(-s+2)\left(\frac{1}{s}\right)$$

$$= \frac{(-s+2)}{s(s+2)(s+3)}$$

Using partial fraction

$$\frac{-s+2}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = \left.\frac{-s+2}{(s+2)(s+3)}\right|_{s=0} \Rightarrow A = \frac{1}{3}$$

$$B = \left.\frac{-s+2}{s(s+3)}\right|_{s=-2} \Rightarrow B = -2$$

$$C = \left.\frac{-s+2}{s(s+2)}\right|_{s=-3} \Rightarrow C = \frac{5}{3}$$

$$Y_{ZSR}(s) = \frac{1}{3s} - \frac{2}{s+2} + \frac{5/3}{s+3}$$

Taking inverse laplace transform,

$$y_{ZSR}(t) = \frac{1}{3}u(t) - 2e^{-2t}u(t) + \frac{5}{3}e^{-3t}u(t)$$

Total response,

$$y(t) = y_{ZIR}(t) + y_{ZSR}(t)$$

$$= 2e^{-2t}u(t) + \frac{1}{3}u(t) - 2e^{-2t}u(t) + \frac{5}{3}e^{-3t}u(t)$$

$$y(t) = \frac{1}{3}u(t) + \frac{5}{3}e^{-3t}u(t)$$

$$y(t) = \frac{1}{3}(1 + 5e^{-3t})u(t)$$

Q.7 (a) Solution:

(i) Given :

$$w(n) = \frac{1}{2}w(n-1) + x(n) \quad ...(1)$$

$$y(n) = \alpha y(n-1) + \beta w(n) \quad ...(2)$$

$$w(n) = \frac{1}{\beta}y(n) - \frac{\alpha}{\beta}y(n-1) \quad ...(3)$$

$$w(n-1) = \frac{1}{\beta}y(n-1) - \frac{\alpha}{\beta}y(n-2) \quad ...(4)$$

Multiplying eqn. (4) by $\frac{1}{2}$ and subtracting it from eqn. (3)

$$w(n) - \frac{1}{2}w(n-1) = \frac{1}{\beta}y(n) - \frac{\alpha}{\beta}y(n-1) - \frac{1}{2\beta}y(n-1) + \frac{\alpha}{2\beta}y(n-2)$$

$$w(n) - \frac{1}{2}w(n-1) = \frac{1}{\beta}y(n) - \frac{1}{\beta}\left(\alpha + \frac{1}{2}\right)y(n-1) + \frac{\alpha}{2\beta}y(n-2) \quad ...(5)$$

From eqn. (1)

$$x(n) = w(n) - \frac{1}{2}w(n-1) \quad ...(6)$$

From eqn. (5) and (6)

$$x(n) = \frac{1}{\beta}y(n) - \frac{1}{\beta}\left(\alpha + \frac{1}{2}\right)y(n-1) + \frac{\alpha}{2\beta}y(n-2)$$

$$y(n) = \left(\alpha + \frac{1}{2}\right) y(n-1) - \frac{\alpha}{2} y(n-2) + \beta x(n)$$

The difference equation,

$$y(n) = \frac{-1}{8} y(n-2) + \frac{3}{4} y(n-1) + x(n)$$

On comparing,

$$\alpha = \frac{1}{4}, \beta = 1$$

(ii) S_1 :

$$w(n) = \frac{1}{2} w(n-1) + x(n)$$

$$h_1(n) = \left(\frac{1}{2}\right)^n u(n)$$

S_2 :

$$y(n) = \frac{1}{4} y(n-1) + w(n)$$

$$h_2(n) = \left(\frac{1}{4}\right)^n u(n)$$

The overall impulse response,

$$h(n) = h_1(n) * h_2(n) = \sum_{h=-\infty}^{\infty} h_1(k) h_2(n-k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k u(k) \cdot \left(\frac{1}{4}\right)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \cdot \left(\frac{1}{4}\right)^{n-k}$$

$$h(n) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{2n-k}$$

$$= \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$$

Alternate :

$$H_1(z) = \frac{z}{z - \frac{1}{2}}$$

$$H_2(z) = \frac{z}{z - \frac{1}{4}}$$

$$H(z) = H_1(z).H_2(z)$$

$$\frac{H(z)}{z} = \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z - \frac{1}{4}} \text{ where } A = 2; B = -1$$

$$H(z) = \frac{2z}{z - \frac{1}{z}} - \frac{z}{z - \frac{1}{4}}$$

$$\Downarrow\Uparrow$$

$$h(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right]u(n)$$

Q.7 (b) Solution:

Given :

$$G_P(s) = \frac{K}{s(s+2)}$$

$$G_C(s) = \frac{1 + Ts}{1 + \alpha Ts}$$

$$G(s) = G_P(s).G_C(s) = \frac{K(1+Ts)}{s(s+2)(1+\alpha Ts)}$$

Velocity error constant,

$$K_v = \lim_{s\to 0} s \cdot G(s) = \lim_{s \to 0} s \cdot \frac{K(1+Ts)}{s(s+2)(1+\alpha Ts)}$$

$$K_v = \frac{K}{2} = 20$$

$$K = 40$$

Phase margin of uncompensated system

$$\text{P.M}|_{\omega_{gc}} = 180° + \angle G_P(j\omega)|_{\omega=\omega_{gc}}$$

$$\angle G_P(j\omega)|_{\omega=\omega_{gc}} = -90° - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right)$$

The gain cross-over frequency,

$$|G(j\omega)|\Big|_{\omega=\omega_{gc}} = 1$$

$$\frac{40}{\omega_{gc}\sqrt{\omega_{gc}^2 + 4}} = 1$$

$$1600 = \omega_{gc}^2(\omega_{gc}^2 + 4)$$

$$\omega_{gc}^4 + 4\omega_{gc}^2 - 1600 = 0$$

On calculating,
$$\omega_{gc}^2 = 38.05$$

$$\omega_{gc} = 6.17 \text{ rad/sec}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}} = -90° - \tan^{-1}\left(\frac{6.17}{2}\right)$$

$$= -162.036°$$

$$\text{P.M} = 180° - 162.036°$$

$$= 17.964°$$

Phase angle required by the compensator

$$\phi_m = 50 - 17.964 + \text{allowance}$$

$$= 50 - 17.964 + 5$$

$$\phi_m = 37.036°$$

Gain of the compensator,

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m}$$

$$\alpha = \frac{1 - \sin(37.036°)}{1 + \sin(37.036°)}$$

$$\alpha = 0.25$$

$$\text{Gain of the compensator} = \frac{1}{\sqrt{\alpha}}$$

$$\text{Gain of } |G_C(j\omega)| = 10\log\left(\frac{1}{\alpha}\right) = 6 \text{ dB}$$

So, gain of the compensated system must be increased by 6 dB.

So, now gain cross-over frequency, ω_m is

$$\frac{40}{\omega_m\sqrt{\omega_m^2+4}} = \sqrt{0.25} = 0.5$$

$$\omega_m^4 + 4\omega_m^2 - 6400 = 0$$

On solving, $\omega_m^2 = 78.025$

$$\omega_m = 8.83 \text{ rad/sec}$$

But $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$

$$1/T = 8.83 \times \sqrt{0.25} = 4.42;\ T = 0.226$$

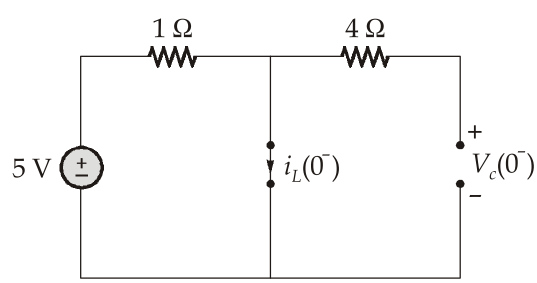
The equation of compensator

$$G_C(s) = \frac{1+Ts}{1+\alpha Ts} = \frac{(1+0.226s)}{(1+0.25\times 0.226s)}$$

$$G_C(s) = \frac{(1+0.226s)}{(1+0.057s)}$$

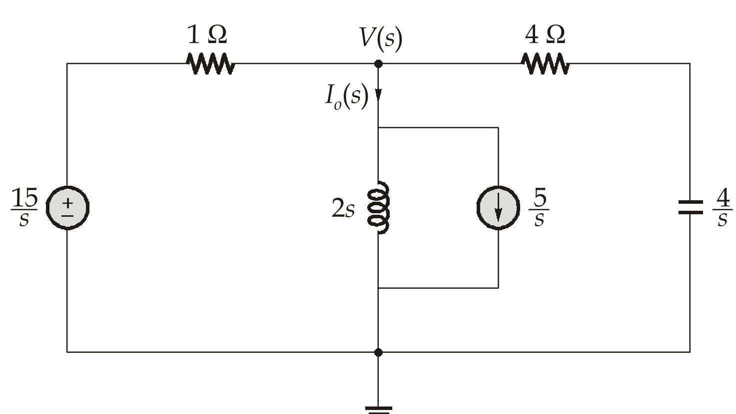
Q.7 (c) Solution:

(i) At $t < 0$; the circuit becomes



$$i_L(0^-) = \frac{5}{1} = 5 \text{ A},\ V_c(0^-) = 0 \text{ V}$$

At $t \geq 0$; the circuit becomes



Apply nodal at node (1)

$$\frac{V(s)-\frac{15}{s}}{1}+\frac{V(s)}{2s}+\frac{5}{s}+\frac{V(s)}{4+\frac{4}{s}} = 0$$

$$V(s)\left[1+\frac{1}{2s}+\frac{s}{4(s+1)}\right] = \frac{10}{s}$$

$$V(s)\left[\frac{4(s+1)s+2(s+1)+s^2}{4s(s+1)}\right] = \frac{10}{s}$$

$$V(s) = \frac{40(s+1)}{5s^2+6s+2}$$

$$I_o(s) = \frac{V(s)}{2s}+\frac{5}{s}$$

$$I_o(s) = \frac{40(s+1)}{2s(5s^2+6s+2)}+\frac{5}{s}$$

By using partial fraction

$$I'_o(s) = \frac{20(s+1)}{s(5s^2+6s+2)} = \frac{4(s+1)}{s(s^2+1.2s+0.4)}$$

$$I'_o(s) = \frac{4(s+1)}{s(s^2+1.2s+0.4)} = \frac{A}{s}+\frac{Bs+C}{s^2+1.2s+0.4}$$

$$4s+4 = (A+B)s^2+(1.2A+C)s+0.4A$$

On comparing,

$$A+B = 0 \Rightarrow A = -B$$

$$1.2A+C = 4$$

$$0.4A = 4 \Rightarrow A = 10$$

$$B = -10 \text{ and } C = -8$$

$$I_o(s) = \frac{10}{s}+\frac{-10s-8}{s^2+1.2s+0.4}+\frac{5}{s}$$

$$= \frac{15}{s}-\frac{(10s+8)}{(s+0.6)^2+(0.2)^2}$$

$$= \frac{15}{s}-\frac{10s+6}{(s+0.6)^2+(0.2)^2}-\frac{2}{(s+0.6)^2+(0.2)^2}$$

$$I_o(s) = \frac{15}{s} - \frac{10(s+0.6)}{(s+0.6)^2+(0.2)^2} - \frac{10(0.2)}{(s+0.6)^2+(0.2)^2}$$

Taking inverse laplace transform,

$$i_o(t) = [15 - 10e^{-0.6t}\cos(0.2t) - 10e^{-0.6t}\sin(0.2t)]u(t) \text{ A}$$

$$i_o(t) = [15 - 10e^{-0.6t}(\cos(0.2t) + \sin(0.2t))]u(t) \text{ A}$$

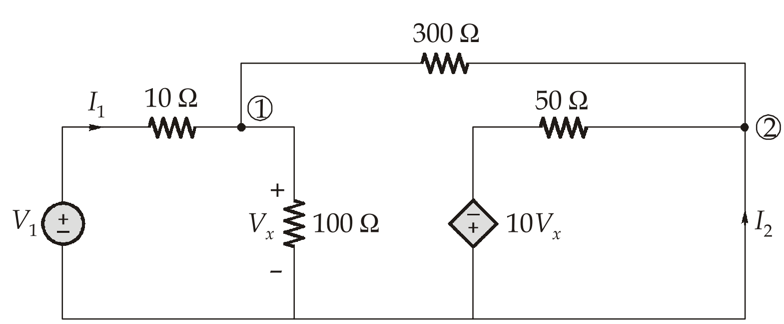
(ii)

Memory Mapped I/O	I/O Mapped I/O
1. In this scheme, the address is of 16-bit.	1. In this scheme, the address is of 8-bit.
2. $\overline{\text{MEMR}}$ and $\overline{\text{MEMW}}$ control signals are used to control read and write I/O operation.	2. $\overline{\text{IOR}}$ and $\overline{\text{IOW}}$ control signals are used to control read and write I/O operation.
3. Data transfer is between any register and I/O device.	3. Data transfer is between accumulator and I/O device.
4. Maximum number of I/O devices as 65536.	4. Maximum number of I/O devices are 256.
5. Decoding 16-bit address may require more hardware.	5. Decoding 8-bit address requires less hardware.

Q.8 (a) Solution:

To get h_{11} and h_{21}

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \text{ and } h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$



At node (1),

$$I_1 = \frac{V_x}{100} + \frac{V_x - 0}{300} \Rightarrow V_x = 75I_1 \qquad ...(1)$$

Also,

$$V_1 = 10I_1 + V_x = (10+75)I_1$$

$$\frac{V_1}{I_1} = h_{11} = 85\ \Omega$$

At node (2),

$$I_2 = \frac{0+10V_x}{50} + \frac{0-V_x}{300}$$

$$\Rightarrow \quad V_x = \frac{300}{59} I_2 \quad \text{...(2)}$$

From eqn. (1) and (2)

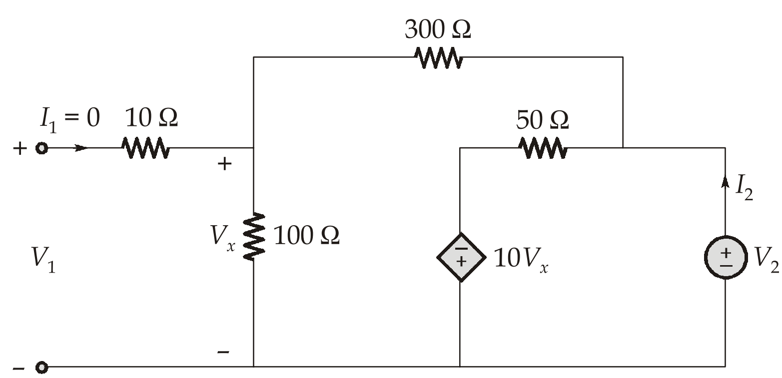
$$75I_1 = \frac{300}{59} I_2$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{75 \times 59}{300}$$

$$h_{21} = 14.75$$

To get h_{12} and h_{22}

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}, \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



At node (2),

$$I_2 = \frac{V_2}{300+100} + \frac{V_2 + 10V_x}{50}$$

$$400I_2 = 9V_2 + 80V_x \quad \text{...(3)}$$

But

$$V_x = \frac{100}{100+300} V_2$$

$$V_x = \frac{V_2}{4} \quad \text{...(4)}$$

From eqn. (3) and (4)

$$400I_2 = 9V_2 + 80 \times \frac{V_2}{4}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{29}{400} = 0.0725 \text{ s}$$

Also,
$$V_1 = V_x = \frac{V_2}{4}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{4} = 0.25$$

h-parameter,

$$[h] = \begin{bmatrix} 85\,\Omega & 0.25 \\ 14.75 & 0.0725s \end{bmatrix}$$

Q.8 (b) Solution:

(i) Given :
$$A = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix}$$

For controllability,
$$|Q_C| \neq 0$$
$$[Q_C] = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -11 \\ 0 \\ -40 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -2 & 1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}\begin{bmatrix} -2 & 1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} = \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 142 \\ -40 \\ 437 \end{bmatrix}$$

$$[Q_C] = \begin{bmatrix} 2 & -11 & 142 \\ 1 & 0 & -40 \\ 2 & -40 & 437 \end{bmatrix}$$

$$|Q_C| = -3193$$
$$|Q_C| \neq 0$$

Hence, the system is completely controllable.

For observability, $|Q_o| \neq 0$

$$[Q_o] = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

$$CA = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} = \begin{bmatrix} -64 & -80 & -78 \end{bmatrix}$$

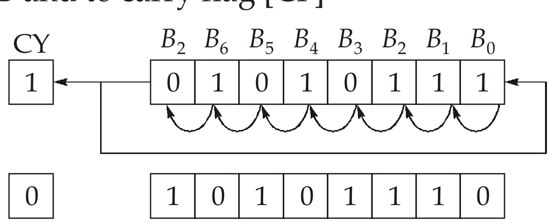
$$CA^2 = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix} = \begin{bmatrix} 674 & 848 & 814 \end{bmatrix}$$

$$[Q_o] = \begin{bmatrix} 4 & 6 & 8 \\ -64 & -80 & -78 \\ 674 & 848 & 814 \end{bmatrix}$$

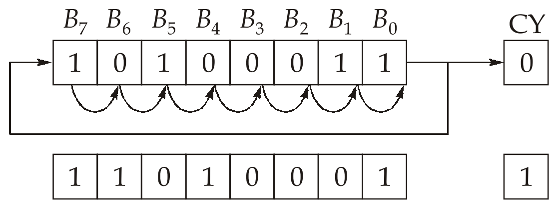
$$|Q_o| = -1576$$
$$|Q_o| \neq 0$$

Hence, the system is completely observable.

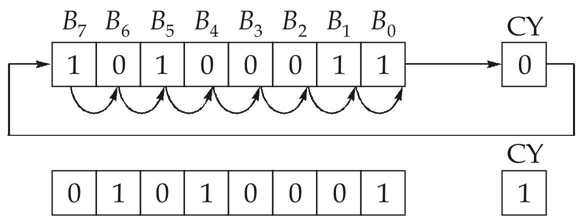
(ii) 1. **ROL :** This instruction rotate the data byte in register/memory towards the left, i.e., MSB to LSB and to carry flag [CF]



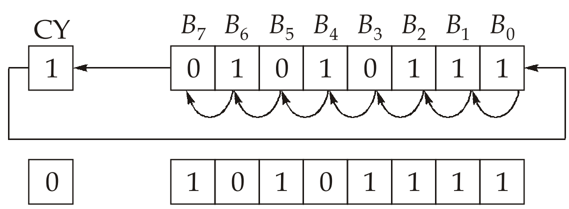
2. **ROR :** This instruction rotate the data byte in register/memory towards the right, i.e., LSB to MSB and to carry flag [CF].



3. **RCR :** This instruction is used to rotate the data byte/word towards the right through carry.



4. **RCL :** This instruction is used to rotate the data byte/word toward the left through carry.



Q.8 (c) Solution:

Given : From fact (2)

$$F^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$$

Taking fourier transform on both sides,

$$(1+j\omega)X(j\omega) = \frac{A}{2+j\omega}$$

$$X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)}$$

Taking inverse fourier transform,

$$X(j\omega) = \frac{A_1}{1+j\omega} + \frac{A_2}{1+j\omega}$$

where, $$A_1 = \left.\frac{A}{2+j\omega}\right|_{j\omega=-1} \Rightarrow A_1 = A$$

$$A_2 = \left.\frac{A}{1+j\omega}\right|_{j\omega=-2} \Rightarrow A_2 = -A$$

$$X(j\omega) = A\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$$

Taking inverse fourier transform,

$$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t) \qquad ...(1)$$

Using fact (3),

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 = 2\pi$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 = 2\pi \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

i.e.,

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = 1$$

$$\int_{-\infty}^{\infty} [Ae^{-t}u(t) - Ae^{-2t}u(t)]^2\, dt = 1$$

$$A^2 \int_{0}^{\infty} (e^{-2t} - 2e^{-3t} + e^{-4t})dt = 1$$

$$A^2\left[\frac{e^{-2t}}{(-2)} - \frac{2e^{-3t}}{(-3)} + \frac{e^{-4t}}{(-4)}\right]_0^{\infty} = 1$$

$$A^2\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] = 1$$

$$\frac{A^2}{12} = 1$$

$$A = \pm\sqrt{12}$$

Since, $x(t)$ is a non-negaitve.

So,

$$A = +\sqrt{12}$$

$$x(t) = \left[\sqrt{12}e^{-t} - \sqrt{12}e^{-2t}\right]u(t)$$

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