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# ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

## Electronics & Telecommunication Engineering

### Test-1 : Network Theory + Control Systems [All Topics]

Name : ..........................................

Roll No :

| Test Centres | Student's Signature |
|---|---|
| Delhi ☑ Bhopal ☐ Jaipur ☐ Pune ☐ Kolkata ☐ Bhubaneswar ☐ Hyderabad ☐ | |

**Instructions for Candidates**

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

**FOR OFFICE USE**

| Question No. | Marks Obtained |
|---|---|
| Section-A | |
| Q.1 | 36 |
| Q.2 | 11 |
| Q.3 | |
| Q.4 | |
| Section-B | |
| Q.5 | 38 |
| Q.6 | 40 |
| Q.7 | |
| Q.8 | 47 |
| **Total Marks Obtained** | 172 |

Signature of Evaluator ........................ Cross Checked by ........................

- Impressive but still you can improve, keep it up.
- Avoid Calculation mistake.
- Selection of Question is Good.

# IMPORTANT INSTRUCTIONS

**CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.**

## DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

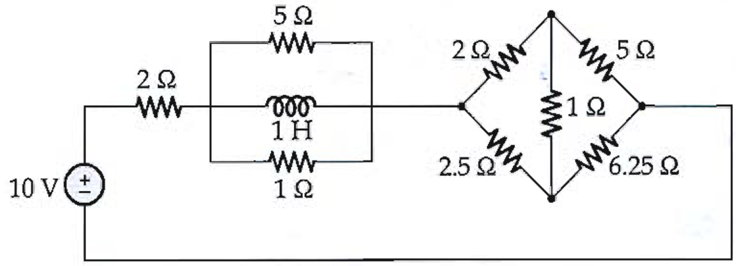
## DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

## Section A : Network Theory

Q.1 (a) Consider the circuit shown below:




Calculate the power supplied by source.

[10 marks]

here connect DC source so ~~resistance~~

$V_L = L\frac{di}{dt}$ $\left\{ \frac{di}{dt} = 0 \text{ due to DC source} \right\}$

Voltage across inductor is zero

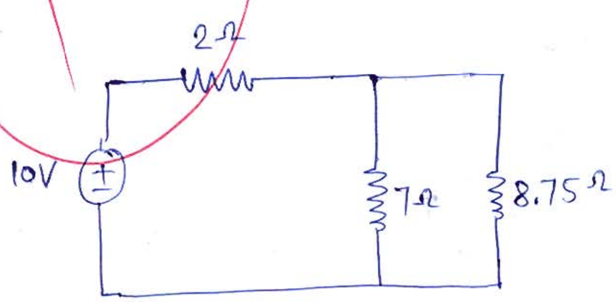
$X_L = j\omega L$ $\left\{ \text{in DC source have zero frequency} \right\}$

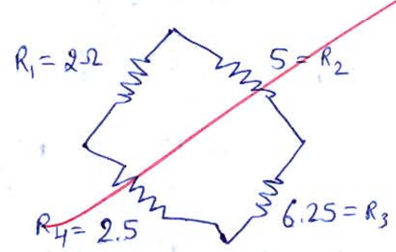
$X_L = 0$

short circuit

9

Bridge condition satisfy




$R_{eq} = 2 + \frac{7 \times 8.75}{7 + 8.75}$

$R_{eq} = 5.89\,\Omega$

$I = \frac{V}{R_{eq}} = \frac{10}{5.89} = 1.698\text{ Amp}$

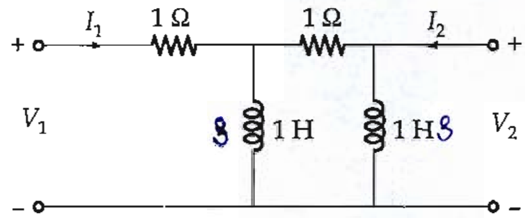
$\left\{ R_1R_3 = R_2R_4 \text{ so no current flow in } 1\Omega \text{ Resistance} \right\}$

Power supplied by the source is $P = V \times I$

$P = 10 \times 1.698$

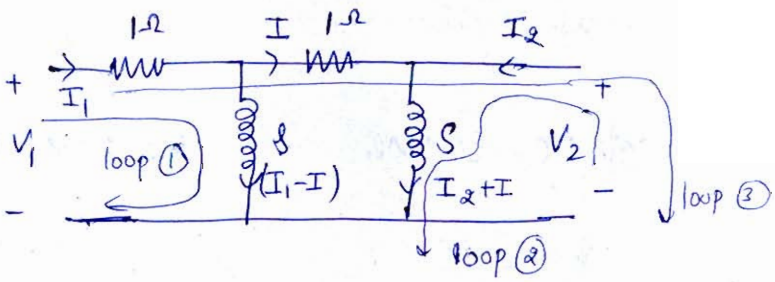
$\boxed{P = 16.98 \text{ watt}}$

**Q.1 (b)** Find the open circuit parameters for the two-port network shown below:




**[10 marks]**

taking lapalace transform




apply KVL

$V_1 - I_1 \times 1 - s(I_1 - I) = 0$

$V_1 - (1+s)I_1 + Is = 0$ — ① $\Rightarrow$ $I = \dfrac{(1+s)I_1 - V_1}{s}$

$V_2 = s(I_2 + I) \Rightarrow V_2 = I_2 s + Is$ — ②

now KVL apply in loop ③

$V_1 - I_1 - I - V_2 = 0$ — ③

put eq ② & eq ① into eq ③

$V_1 - I_1 - \left[\dfrac{(1+s)I_1}{s} - \dfrac{V_1}{s}\right] - \left[I_2 s + Is\right]$

$V_1 - I_1 - I - I_2 s - Is = 0$

$V_1 - I_1 - I_2 s - I(s+1) = 0$ put I value from eq ①

$V_1 - I_1 - I_2 s - \left[\dfrac{(1+s)I_1 - V_1}{s}\right](s+1) = 0$

$V_1 - I_1 - I_2 s - \dfrac{(s+1)^2}{s} I_1 + \left(\dfrac{s+1}{s}\right)V_1 = 0$

$\left(\dfrac{2s+1}{s}\right)V_1 = +\left(\dfrac{s + s^2 + 2s + 1}{s}\right) I_1 + I_2 s$

$V_1 = \left(\dfrac{s^2+3s+1}{2s+1}\right) I_1 + \left(\dfrac{s^2}{2s+1}\right) I_2$ — ④

So $\boxed{z_{11} = \dfrac{s^2+3s+1}{2s+1}}$

$\boxed{z_{12} = \dfrac{s^2}{2s+1}}$

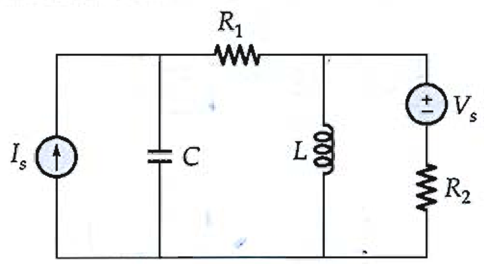
eq ④ put into eq ② $\quad V_2 = I_2 s + [(1+s)I_1 - V_1]$

$$V_2 = I_2 s + (1+s) I_1 - \left(\frac{s^2+3s+1}{2s}\right) I_1 - \left(\frac{s^2}{2s+1}\right) I_2$$

$$V_2 = \left(\frac{s^2-s-1}{2s}\right) I_1 + \left[\frac{s(s+1)}{(2s+1)}\right] I_2$$

$$\boxed{Z_{21} = \frac{s^2-s-1}{2s}} \qquad \boxed{Z_{22} = \frac{s(s+1)}{2s+1}}$$

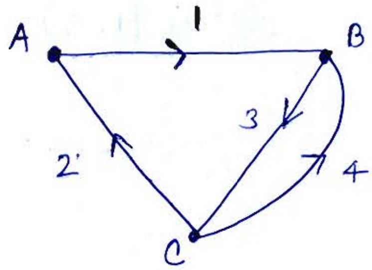
**Q.1 (c)** Draw the graph of the network shown in the figure below. How many trees are possible for this graph? Draw all the trees.




**[10 marks]**

Voltage source ⟶ shorts
current source ⟶ open

let give direction to graph by own




branches

| node | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 1 | -1 | 0 | 0 |
| B | -1 | 0 | 1 | -1 |
| C | 0 | 1 | -1 | 1 |

no. of tree possible from the graph is

$$\mathrm{Det}\left[A A^T\right]$$

where A is reduced matrix.

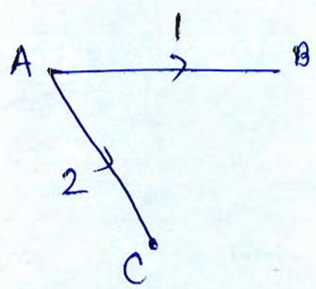
$$\text{Det}[A A^T] = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & -1 \end{bmatrix}_{2\times4} \begin{bmatrix} 1 & -1 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}_{4\times2}$$

$$\text{Det}[A A^T] = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

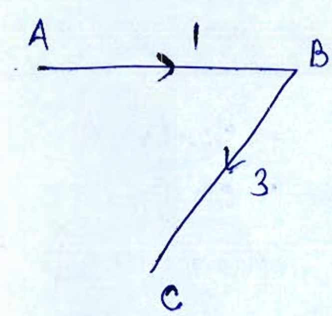
$$\det[A A^T] = 6 - (1) = 5$$

so total no. of tree is $= \boxed{5}$

Good

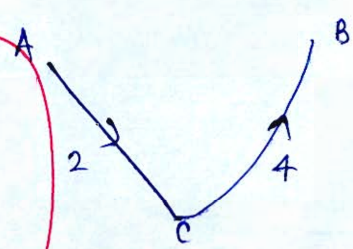



tree (1, 2)

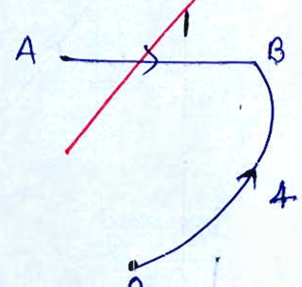



tree (1, 3)

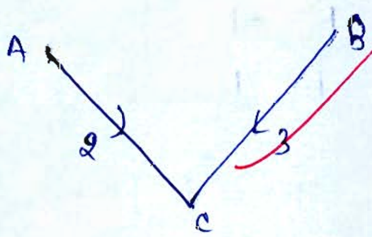
10




tree (2, 4)



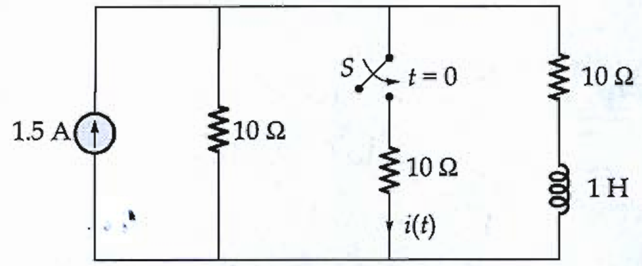

tree (1, 4)




tree (2, 3)

Do not write in this margin

**Q.1 (d)** Consider the network shown below:

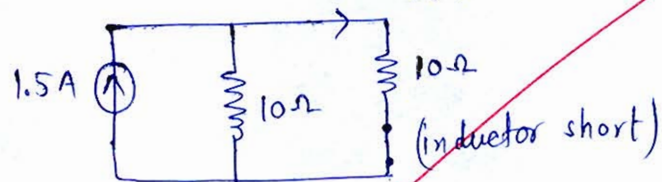



If switch $S$ is closed at $t = 0$, calculate $i(t)$ for $t > 0$ by using Laplace transform approach.

**[10 marks]**

before $t<0$ in steady state

inductor act as short circuit element

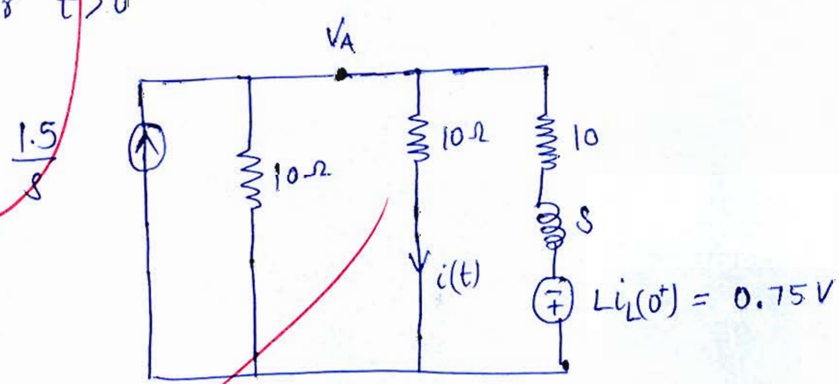



$$i_L(0^-) = 1.5 \times \frac{10}{10+10} = 0.75 \text{ Amp.}$$

inductor does not change current immedially so

$$i_L(0^-) = i_L(0^+) = 0.75 \text{ Amp}$$

after $t>0$




$$\frac{V_A}{10} + \frac{V_A}{10} + \frac{V_A + 0.75}{10+s} = \frac{1.5}{s}$$

$$(10+s) \times 2V_A + 10V_A + 7.5 = \frac{1.5 \times 10\,(s+10)}{s}$$

$$(2s+30)\,V_A = \frac{15s + 150 - 7.5s}{s} = \frac{7.5s + 150}{s}$$

$$V_A = \frac{7.5s + 150}{2s \times (15+s)} = \frac{3.75s + 75}{s\,(s+15)}$$

$i(s) = \frac{V_A}{10} = \frac{0.375s + 7.5}{s(s+15)} \doteq \frac{A}{s} + \frac{B}{(s+15)}$

$\Rightarrow \frac{(A+B)s + 15A}{s(s+15)}$ $A+B = 0.375$, $15A = 7.5$ by comparing

so we get $A = 0.5$, $B = -0.125$

$i(s) = \frac{0.5}{s} - \frac{0.125}{s+15}$ by taking inverse laplace transform

$$\boxed{i(t) = 0.5 - 0.125e^{-15t}}$$

**Q.1 (e)** Realise Cauer II form of the function

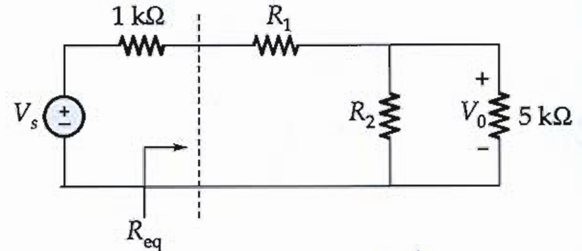
$$Z_{LC}(s) = \frac{s(s^4+3s^2+1)}{3s^4+4s^2+1}$$

**[10 marks]**

**Q.1 (f)** In a certain application, the circuit shown below must be designed to meet these two criteria:

(a) $\frac{V_0}{V_s} = 0.05$ (b) $R_{eq} = 39\ k\Omega$

If the load resistor 5 kΩ is fixed, find $R_1$ and $R_2$ to meet the criteria.




**[10 marks]**

$R_2 \| 5K$

$R_{eq} = 39k\Omega$ given =

$$R_{eq} = R_1 + \frac{R_2 \times 5K}{R_2 + 5K} = 39\ K \text{ --- } (1)$$

$V_0$ by voltage Divider rule is

$$V_0 = \frac{\frac{R_2 \times 5K}{R_2 + 5K}}{\frac{R_2 \times 5K}{R_2+5K} + R_1 + 1K} \times V_s \qquad \left\{ \frac{V_0}{V_s} = 0.05 \right\}$$

1

$$0.05\,x + R_1 \times 0.05 + 1K \times 0.05 = x$$

we consider $\left\{ \frac{R_2 \times 5K}{R_2 + 5K} = x \right\}$

$$R_1 \times 0.05 + 50 = 0.95\,x \text{ --- } (2)$$

$$R_1 + x = 39K \text{ --- } (3)$$

after solving eq ② & ③ we get

$x = 37K$ & $\boxed{R_1 = 2K\Omega}$

$$\frac{R_2 \times 5K}{R_2 + 5K} = 37K \Rightarrow 5R_2\cancel{K} = \cancel{K}[37R_2 + 5\times 37K]$$

$$32R_2 = -37\times 5K$$

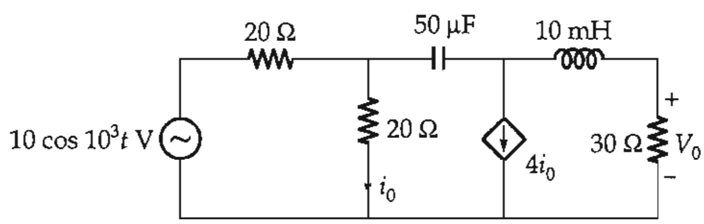
$$\boxed{R_2 = -5.78\ k\Omega}$$

**Q.2 (a)** Consider the circuit shown below and determine:

**(i)** $V_0$

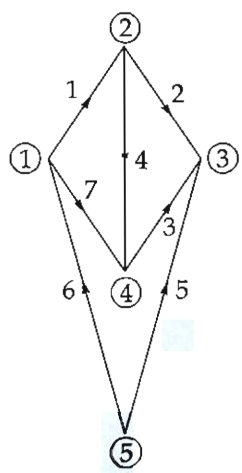
**(ii)** $i_0$

**(iii)** power factor between $V_0$ and $i_0$




**[20 marks]**

**Q.2 (b)** **(i)** Write the complete incidence matrix for the graph shown in the figure below. Find out how many trees are possible for the graph.




**(ii)** Draw the oriented graph corresponding to the reduced incidence matrix given by

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Also, find out how many tie sets are possible?

**[10 + 10 marks]**

node \ branch

$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ 2 & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \\ 5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

incoming branch = −ve = −1

outgoing branch = +ve = 1

2

A [reduced matrix] =

$$\det[AA^T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}_{4\times 7} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}_{7\times 4}$$

$$\text{Det} = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & +0 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}$$

$R_3 \rightarrow R_1 - 3R_3$

$$\cancel{\begin{matrix} 3 & -1 & 0 & -1 \\ -0 & 8 & -9 & \end{matrix}}$$

$$\cancel{[3]\,[(3)[3-1]+(-1)]}\,[\ .$$

$-1 + 3\times 3$ $\quad 0-3$

$$\begin{matrix} 3 & 0 & -3 & 0 \\ +3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \end{matrix}$$

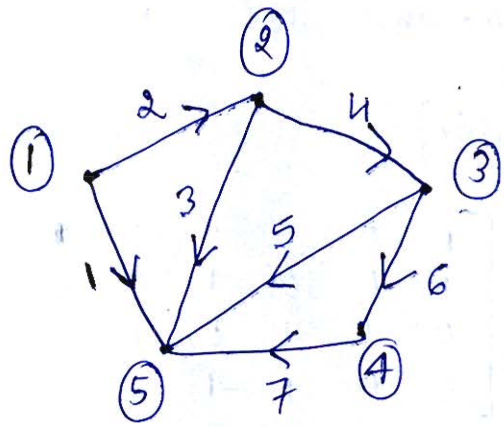
$$\begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 3 & 0 & -3 & 0 \\ -1 & 0 & -1 & 3 \end{bmatrix}$$

(ii)

$$\text{node}\quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$

branches




8

no. of node = 5

no. of branches = 7

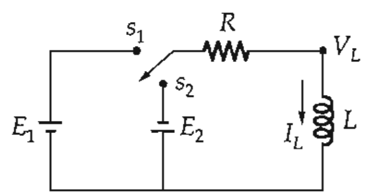
no. of tie set $= b-(n-1)$

$= 7-(5-1)$

$= 7-4$

$= \boxed{3}$

**Q.2 (c)** For the initially relaxed circuit shown below, the switch is closed on to position $s_1$ at time $t = 0$ and changed to position $s_2$ at time $t = 0.5$ ms.

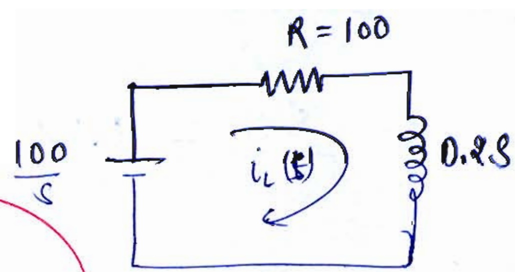



$E_1 = 100$ V; $E_2 = 50$ V
$R = 100\ \Omega$
$L = 0.2$ H

Obtain the equation for inductor current and voltage across the inductor in both the intervals and sketch the transients.

**[20 marks]**

at $t=0$




$$i_L(s) = \frac{\frac{100}{s}}{100 + 0.2s} = \frac{100}{s \times 0.2\left(s + \frac{100}{0.2}\right)}$$

$$i_L(s) = \frac{500}{s(s+500)} = \frac{A}{s} + \frac{B}{(s+500)}$$

$$\frac{As + 500A + Bs}{s(s+500)} \qquad (A+B) = 0$$

$$500A = 500$$

$$A = 1$$

$$B = -1$$

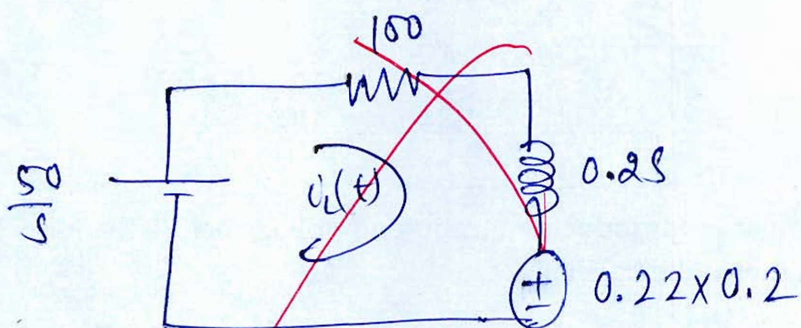
$$i_L(t) = \frac{1}{s} - \frac{1}{s+500}$$

$$i_L(t) = \left[1 - e^{-500t}\right] u(t)$$

at $t = 0.5$ msec

$$i_L(0.5\,msec) = 1 - e^{-500 \times 0.5 \times 10^{-3}}$$

$$i_L(0.5ms) = 0.221 \text{ Amp}$$




$$i_L(s) = \frac{\frac{50}{s} - 0.22 \times 0.2}{100 + 0.2s}$$

$$i_L(s) = \frac{50 - 0.442s}{(100 + 0.2s)s}$$

$$i_L(s) = \frac{50 - 0.442s}{0.2[s+500]s}$$

$$i_c(s) = \frac{250 - 2.11s}{s\,(s+500)} = \frac{A}{s} + \frac{B}{s+500}$$

$$(A+B)s + 500A$$

$500A = 250$ $\qquad$ $A+B = -2.11$

$A = 0.5$ $\qquad$ $B = -2.11 - 0.5$

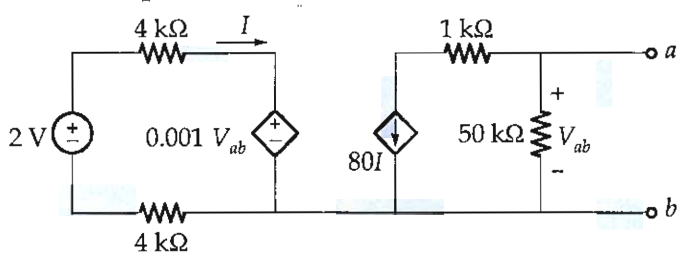
$B = -2.61$

$$i_c(s) = \frac{0.5}{s} - \frac{2.61}{s+500}$$

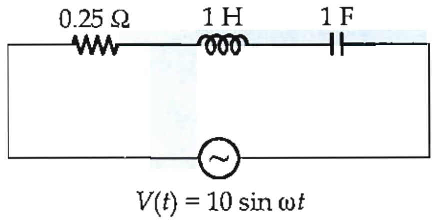
$$\boxed{\cancel{i_c(t) = 0.5 - 2.61e^{-500t}}}$$

$$V_c = L\frac{di_c}{dt}$$

**Q.3 (a)** **(i)** Obtain Norton's equivalent at terminals *a*-*b* of the circuit.




**(ii)** For given RLC series circuit,




Find:

1. Resonant frequency '$f_0$'.
2. Damping ratio '$\xi$'.
3. Maximum possible voltage across the inductor.

**[10 + 10 marks]**

**Q.3 (b)** **(i)** Using Foster form I, synthesize the function

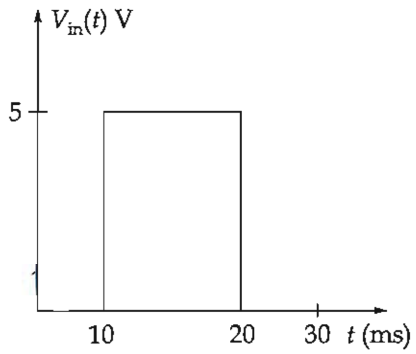
$$Z(s) = \frac{s(s^2+9)}{(s^2+5)(s^2+13)}$$

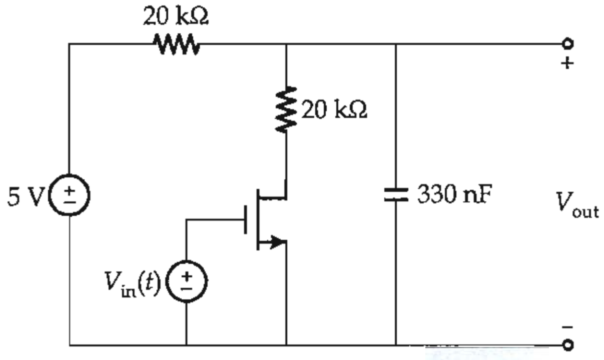
**(ii)** Using Foster form II, synthesize the function

$$Y(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)}$$

**[10 + 10 marks]**

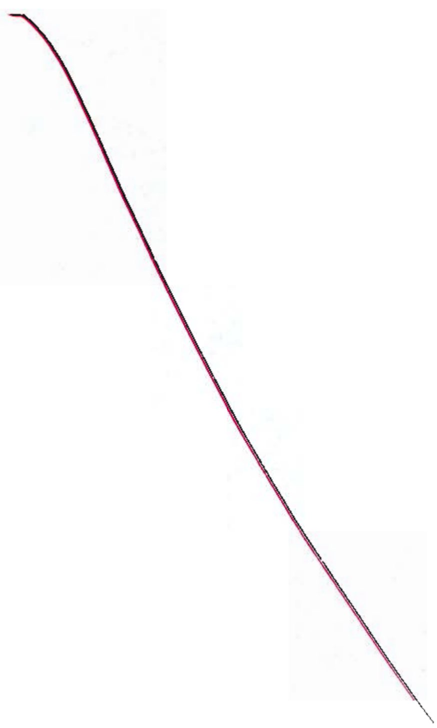
**Q.3 (c)** Consider the network shown below:



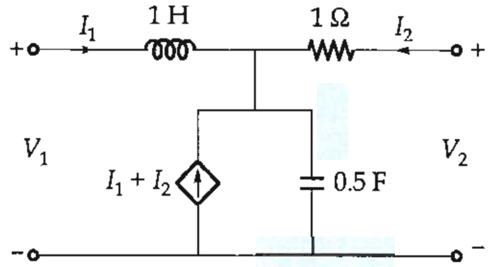




Draw the plot for $V_{out}(t)$ for $t > 0$.

**[20 marks]**



**Q.4 (a)** Determine the transmission parameters matrix for the two port network shown below.




**[20 marks]**

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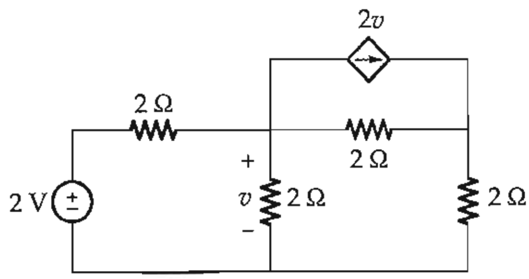
**Q.4 (b)** For the network shown in figure below, write down the f-cutset matrix and obtain the network equilibrium equation in matrix form using KCL and calculate $v$.




**[20 marks]**

**Q.4 (c)** (i) Derive expression for frequencies for maximum voltage across inductor in series RLC resonant circuit.

(ii) Calculate the maximum voltage across the inductor using result of **Q.4 (c) (i)** with constant voltage and variable frequency. Assume $R = 50\ \Omega$, $L = 0.05\text{H}$, $C = 20\ \mu\text{F}$ and $V = 100$ V.

**[10 + 10 marks]**

## Section B : Control Systems

**Q.5 (a)** Consider a negative feedback system having the characteristic equation

$$1+\frac{K}{(1+s)(1.5+s)(2+s)}=0$$

It is desired that all the roots of the characteristic equation have real parts less than –1. Extend the Nyquist stability criterion to find the largest value of $K$ satisfying the condition.

**[10 marks]**

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**Q.5 (b)** The loop transfer function of a negative feedback control system is given by

$$G(s)H(s) = \frac{2e^{-0.5s}(0.125s+1)}{s[0.5s+1]}$$

Determine the possible maximum phase margin and the frequency at which it occurs.

**[10 marks]**

$PM = 180 + \angle\phi\big|_{\omega_{gc}}$

gain crossover freq$^n$ at which gain of open loop TF is 1.

$$|G(s)H(s)| = \frac{2\sqrt{(0.125\omega)^2+1}}{\omega\sqrt{(\omega\times 0.5)^2+1}} = 1$$

square both side

$$4\left[(0.125\omega)^2+1\right] = \omega^2\left[(\omega\times 0.5)^2+1\right]$$

$$0.0625\omega^2 + 4 = 0.25\omega^4 + \omega^2$$

$$0.25\omega^4 + 0.9375\omega^2 - 4 = 0$$

let $\omega^2 = t$ then eq will be

$$0.25t^2 + 0.9375t - 4 = 0$$

$$t = 2.542$$

$$\omega = \pm\sqrt{2.542}$$

$$\boxed{\omega_{gc} = 1.59 \frac{rad}{sec}}$$

$$\angle G(s)H(s) = -0.5\omega + \tan^{-1}(0.125\omega) - \tan^{-1}(0.5\omega) - 90^\circ$$

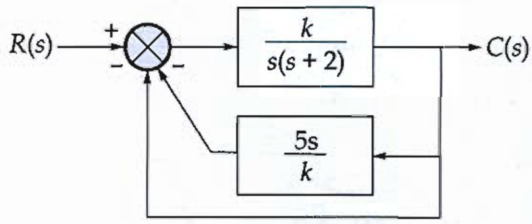
$$\angle G(s)H(s) = -0.5\times 1.59\times\frac{180}{\pi} + \tan^{-1}(0.125\times 1.59) - \tan^{-1}(0.5\times 0.159) - 90^\circ$$

$$\phi = -162.816$$

$$PM = 180 - 162.816$$

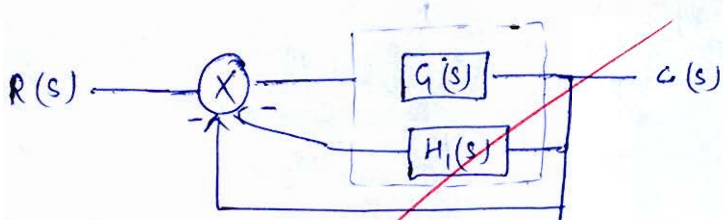
$$\boxed{PM = 17.184^\circ}$$

**Q.5 (c)** Consider the control system shown below:




Design the value of $k$ so that for an input of $100tu(t)$, there will be a 0.01 error in the steady state.

**[10 marks]**




$$\frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(s+2)}}{1+\frac{\cancel{K}}{s(s+2)}\times\frac{5s}{\cancel{K}}}$$

$$= \frac{K}{s^2+2s+5s} = \frac{K}{s(s+7)}$$

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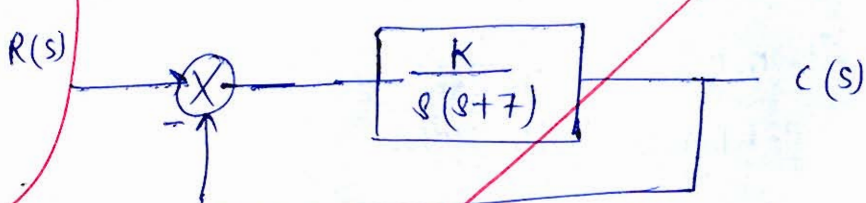



$$K_V = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} \cancel{s}\times\frac{K}{\cancel{s}(s+7)}$$

$$K_V = \frac{K}{7}$$

for Ramp function steady state error $= \frac{A}{K_V}$

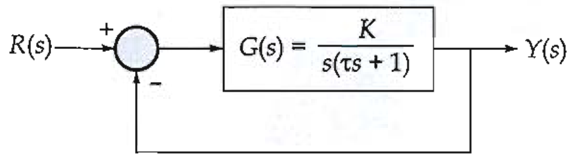
$$e_{ss} = 0.01 = \frac{100}{\frac{K}{7}}$$

$$\boxed{K = 70000}$$

**Q.5 (d)** For the feedback control system shown in figure, select the parameters $K$ and $\tau$ so that the following time-domain specifications will be satisfied:

**(i)** Peak overshoot of the response to a step input is 5% and

**(ii)** The settling time to within 2% of the final value is 4 seconds.




**[10 marks]**

close loop TF $= \dfrac{K}{s^2\tau + s + K}$

$$= \frac{\frac{K}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

compare to standard transfer function will be

$$\omega_n = \sqrt{\frac{K}{\tau}} \quad \& \quad \xi = \frac{1}{2\omega_n\tau}$$

given Peak overshoot $= 5\%$

$$0.05 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

taking ln both sides

$$\ln(0.05) = -\frac{3.14\,\xi}{\sqrt{1-\xi^2}}$$

$$-0.954 = \frac{\xi}{\sqrt{1-\xi^2}}$$

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now square both side

$$0.9102 - 0.9102\xi^2 = \xi^2$$

$$\xi^2 = 0.476 \qquad \boxed{\xi = 0.689}$$

$$t_s = \frac{4}{\xi\omega_n} = 4 \text{ sec (given)}$$

$$\xi\omega_n = 1$$

$$\omega_n = \frac{1}{\xi} = \frac{1}{0.689} = 1.44 \ \frac{\text{rad}}{\text{sec}}$$

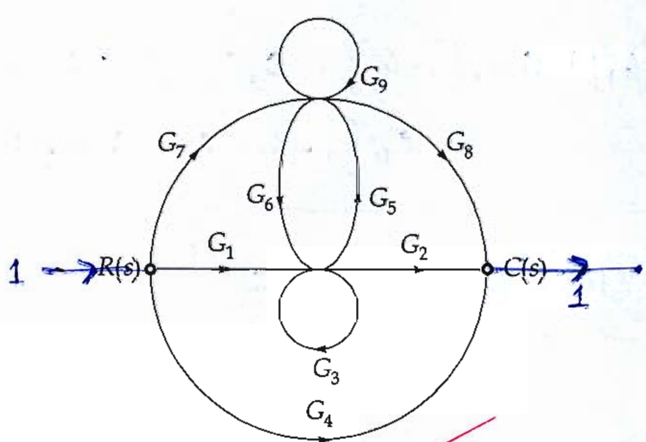
$$\tau = \frac{1}{2\xi\omega_n} = \frac{1}{2\times 0.689\times 1.44} =$$

$$\boxed{\tau = 0.5039}$$

$$K = \omega^2 \times \tau = (1.44)^2 \times 0.5039$$

$$\boxed{K = 1.044}$$

**Q.5 (e)** Find $\frac{C(s)}{R(s)}$ using Mason's gain formula for the following system with the signal flow graph shown below:




**[10 marks]**

forward path $P_1 = G_1 G_2$

$P_2 = G_7 G_8$

$P_3 = G_7 G_6 G_2$

$P_4 = G_1 G_5 G_8$

$P_5 = G_4$

$\Delta_1 = 1 - G_9$

$\Delta_2 = 1 - G_3$

$\Delta_3 = 1$

$\Delta_4 = 1$

$\Delta_5 = 1 - G_3 - G_9 - G_5 G_6 + G_3 G_9$

single loop $= L_1 = G_3$

$L_2 = G_9$

$L_3 = G_5 G_6$

two non touching loop $L_4 = G_3 G_9$

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Massan gain formula = $\frac{P_K \Delta_K}{\Delta}$

where $\Delta_k$ = $1 - \left(\begin{array}{l}\text{non touching single loop} \\ \text{which not touch} \\ \text{forward path}\end{array}\right) + \left(\begin{array}{l}\text{multipication of} \\ \text{two non touching} \\ \text{loop which not} \\ \text{touch together and} \\ \text{forward path}\end{array}\right)$

$\Delta = 1 - \left[\text{single loop}\right] + \left[\begin{array}{l}\text{multipication of} \\ \text{non touching loop}\end{array}\right] - \left[\begin{array}{l}\text{three multiple} \\ \text{loop which not} \\ \text{touch together}\end{array}\right]$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2[1-G_9] + G_7G_8[1-G_3] + G_2G_6G_7 + G_1G_5G_8 + G_4[1-G_3-G_9-G_5G_6+G_3G_6]}{[1-G_3-G_9-G_5G_6+G_3G_6]}}$$

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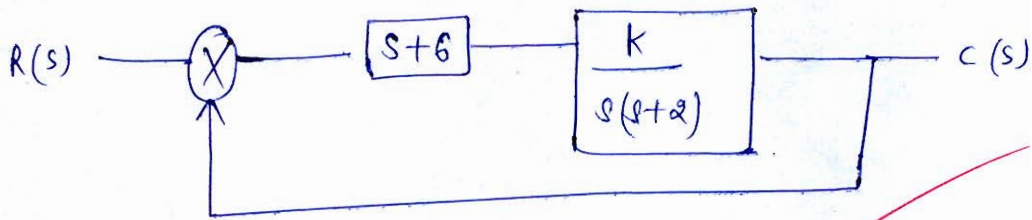
**Q.5 (f)** A unity negative feedback system has open-loop transfer function

$$G(s) = \frac{K}{s(s+2)}$$

The system is modified to include a forward path zero at $s = -6$. What is the value of $K$, if steady state error for input $r(t) = 2t\, u(t)$ to the modified system is 0.1?

**[10 marks]**




now open loop transfer function $= \dfrac{K(s+6)}{s(s+2)}$

steady state error for ramp input $= \dfrac{A}{K_V}$

$$K_V = \lim_{s \to 0} s\, G(s) H(s)$$

$$K_V = \lim_{s \to 0} \cancel{s} \times \frac{K(s+6)}{\cancel{s}(s+2)} = \frac{6K}{2} = 3K$$

so given input is $r(t) = 2t\, u(t)$

so $A = 2$

$e_{ss} = 0.1$

$$e_{ss} = 0.1 = \frac{A}{3K} = \frac{2}{3K}$$

$$K = \left(\frac{0.1 \times 3}{2}\right)^{-1}$$

$$\boxed{K = 6.67}$$

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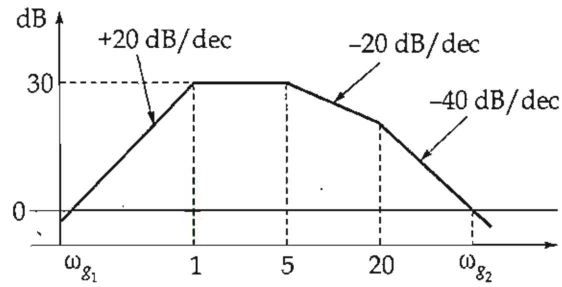
**Q.6 (a)** Consider a minimum phase system whose asymptotic amplitude frequency response is shown in figure below.

**(i)** Determine the transfer function $G(s)$ of the system.

**(ii)** Determine the two gain crossover frequencies $\omega_{g1}$ and $\omega_{g2}$.

**(iii)** Determine the phase margin at $\omega_{g2}$.




**[10 + 5 + 5 marks]**

**Q.6 (b)** Prove that a combination of two poles $s = -a_1$ and $s = -a_2$ and one zero $s = -b$ to the left of both of the poles on the real axis, results in a root locus whose complex root branches form a circle centered at the zero with radius given by $\sqrt{(b-a_1)(b-a_2)}$. Sketch the root locus plot with the gain ($K$) varying from 0 to $\infty$.

**[20 marks]**

**Q.6 (c)** A control system is represented by the state equation given below,

$$\dot{x}(t) = Ax(t)$$

The response of the system is $x(t) = \begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ when $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $x(t) = \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$ when $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Calculate the system matrix $A$ and the state transition matrix for the system.

**[20 marks]**

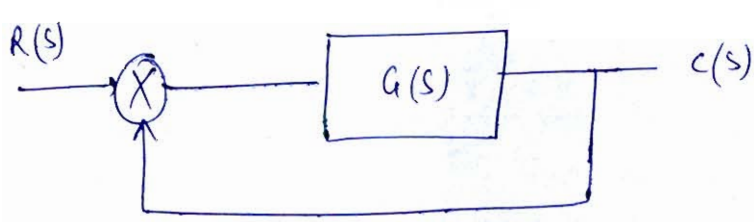
**Q.7 (a)** (i) The forward transfer function of a unity negative feedback type 1, second order system has a pole at –2 and zero at –2.5. If frequency of oscillation is $\sqrt{10}$ rad/s then determine:

1. Damping ratio
2. Damping factor
3. Steady state value, when input $r(t) = (5 + t)u(t)$ is applied.

(ii) For unit-step input to the second order system, define the following characteristics:

1. Rise time
2. Peak time
3. Peak overshoot
4. Settling time

**[15 + 5 marks]**




$\omega_n = \sqrt{10}$ rad/sec

$$G(s) = \frac{K(s+2.5)}{s(s+2)}$$

$$q(s) = 1 + G(s)H(s)$$

$$= (s+2)s + Ks + 2.5K$$

(i) $s^2 + 2s + Ks + 2.5K = 0$

$s^2 + (2+K)s + 2.5K = 0$

$\omega_n^2 = 2.5K$

given $\omega_n = \sqrt{10}$ so $\omega_n^2 = 10$

$K = \frac{10}{2.5} = 4$

$\boxed{K = 4}$

$2\xi\omega_n = (2+K) = 6$

$\xi = \frac{6}{2\omega_n} = \frac{6}{2\times\sqrt{10}}$

(ii) $\boxed{\xi = 0.9486}$

Damping factor $= \xi\omega_n = \alpha$

$\alpha = 0.9486 \times \sqrt{10} = 3$

(iii) steady state value $r(t) = 5u(t) + 5t\,u(t)$

$f_s = \lim_{s\to 0} s \; c(s)$

$\lim_{s\to 0} \cdot \cancel{s} \cdot \left[\frac{5}{\cancel{s}} + \frac{1}{s^2}\right] \times \frac{4\times(s+2.5)}{\cancel{s}\,(s+2)}$

steady state value $= \frac{5\times 4\times 2.5}{2} = 25$

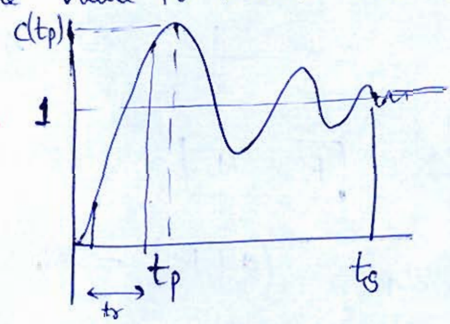
Rise time : time taken by the responce to reach 10% to 90% of its final value.

peak time : time taken by the responce to its max value.

peak overshoot : it is the ratio of difference of (value at $t_p$ - value at $t_s$) to value of $t_s$ $\quad \dfrac{c(t_p) - c(\infty)}{c(\infty)}$ or $\dfrac{c(t_p) - c(t_s)}{c(t_s)}$

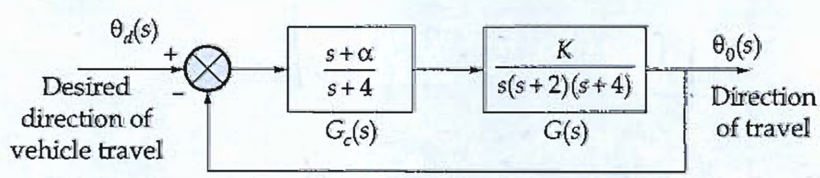
settling time : it is the value to react 98% of its final value.

$t_r \rightarrow$ 10% to 90%




$M_p\% = \dfrac{c(t_p) - 1}{1} \times 100$

**Q.7 (b)** Consider the block diagram which form a scheme for controlling (automatically) the direction of travel of a road vehicle. The controller (a compensating network) is represented by $G_c(s)$ and engine-vehicle dynamics by $G(s)$.




(i) Determine the necessary conditions linking $\alpha$, the controller parameter and gain $K$ of the engine vehicle part for the overall system to be stable.

(ii) Also suggest suitable values of $\alpha$ and $K$, while assuring that steady-state error due to unit ramp direction input $\left[\theta_d(s) = \dfrac{1}{s^2}\right]$ is no more than 20%.

**[20 marks]**

$$q(s) = 1 + G(s)H(s) = 0$$

$$1 + \left(\frac{s+\alpha}{s+4}\right)\frac{K}{s(s+2)(s+4)} = 0$$

$$s(s+2)(s^2+16+8s) + Ks + K\alpha = 0$$

$$(s^2+2s)(s^2+16+8s) + Ks + K\alpha = 0$$

$$s^4 + 16s^2 + 8s^3 + 2s^3 + 32s + 16s^2 + Ks + K\alpha = 0$$

$$s^4 + 10s^3 + 32s^2 + 32s + Ks + K\alpha = 0$$

$$s^4 + 10s^3 + 32s^2 + (32+K) + K\alpha = 0$$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 32 | $K\alpha$ |
| $s^3$ | 10 | $(32+K)$ | |
| $s^2$ | $\frac{320-32-K}{10}$ | $K\alpha$ | |
| $s^1$ | $\cancel{\frac{\left(\frac{288-K}{10}\right)(32+K) - 10K\alpha}{\frac{288-K}{10}}}$ | | |
| $s^0$ | $K\alpha$ | | |

to be stable $\quad 288-K > 0$

$\boxed{K < 288}$ — (1)

$K\alpha > 0$ — (2)

$(288-K)(32+K) - 100K\alpha > 0$

$9216 + 288K - 32K - K^2 - 100K\alpha > 0$

$-K^2 + 256K + 9216 - 100K\alpha > 0$

$K^2 - K(100\alpha - 256) + 9216 < 0$ — (3)

$(100\alpha - 256)^2 - 4 \times 9216 = 0$

$100 \times 100\alpha^2 + (256)^2 - 256 \times 100\alpha - 4 \times 9216 = 0$

$\boxed{\alpha > 5.76}$

(ii)

$K_V = \lim_{s \to 0} s \left(\frac{s+\alpha}{s+4}\right) \frac{K}{s(s+2)(s+4)}$

$K_V = \frac{\alpha K}{4 \times 2 \times 4} = \frac{K\alpha}{32}$

given that $e_{ss} \le \frac{20}{100}$

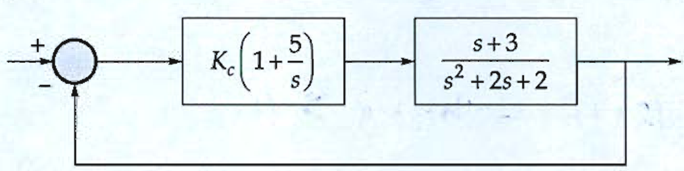
$$\frac{1}{K_V} \le 0.02$$

$$K_V \ge \frac{100}{2}$$

$$\boxed{K\alpha \ge 1600}$$

so let $\boxed{K = 160}$ then $\boxed{\alpha = 10}$

**Q.7 (c)** **(i)** Consider the closed loop system shown below. A PI controller controls a second order plant. Determine the range of $K_c$ for which the closed-loop poles satisfy Re($s$) < –2.




**(ii)** A unity-negative feedback control system has an open-loop transfer function,

$$G(s) = \frac{K}{s(s+1)(s+2)}; K \ge 0$$

Sketch the root locus plot of the system, explicitly identifying the centroid, the asymptotes, the breakaway points and $\pm j\omega$ axis crossover points.

**[10 + 10 marks]**

$$q(s) = 1 + G(s)H(s)$$

$$= 1 + K_c\frac{(s+5)}{s} \times \frac{s+3}{(s^2+2s+2)}$$

$$= 1 + \frac{K_c(s^2+8s+15)}{s(s^2+2s+2)}$$

$$= s^3 + 2s^2 + 2s + K_c s^2 + K_c 8s + 15K_c$$

$$= s^3 + (K_c+2)s^2 + (8K_c+2)s + 15K_c$$

put $s \longrightarrow (s+2)$

$$(s+2)^3 + (K_c+2)(s+2)^2 + (8K_c+2)(s+2) + 15K_c$$

$$(s^2+4s+4)(s+2) + (K_c+2)(s^2+4s+4) + 8K_c s + 16K_c + 2s + 4 + 15K_c$$

$$(s^3+6s^2+12s+8) + K_c s^2 + 2s^2 + 4K_c s + 8s + 4K_c + 8 + 8K_c s$$
$$+ 16K_c + 2s + 4 + 15K_c$$

$$s^3 + [6 + K_c + 2]s^2 + s[12 + 4K_c + 8 + 8K_c + 2]$$
$$+ 8 + 4K_c + 8 + 16K_c + 4 + 15K_c = 0$$

$$s^3 + (8+K_c)s^2 + [22 + 12K_c]s + [20 + 35K_c] = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 22+12K_c \\ s^2 & (8+K_c) & (20+35K_c) \\ s^1 & (8+K_c)(22+12K_c) - (20+35K_c) & \\ s^0 & 20+35K_c & \end{array}$$

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$8 + K_c > 0$ $\quad K_c > -8$ — ①

$20 + 35K_c > 0$ $\quad K_c > \frac{-20}{35}$ — ②

$$176 + 118K_c + 12K_c^2 - 20 - 35K_c > 0$$

$$12K_c^2 + 83K_c + 156 > 0$$

$K_c >$

(ii) $\dfrac{K}{s(s+1)(s+2)}$

no. of pole = 3 $\quad P_1 = 0 \quad P_2 = -1 \quad P_2 = -2$

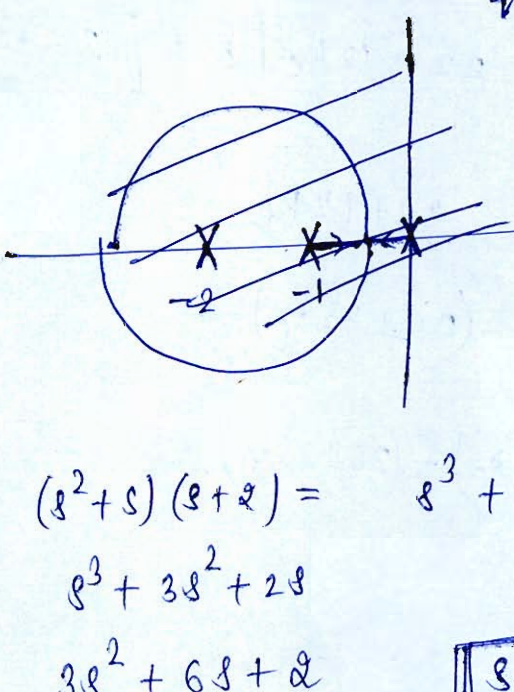
asymptot angle = $\dfrac{(2K+1)180}{P-Z} = \dfrac{(2K+1)180}{3}$

$\phi_1 = 60^\circ, \; 180^\circ, \; 300^\circ$.

centroid $\sigma_A = \dfrac{-1-2-0}{3} = -1$

Root locus start from pole at $K=0$ to end in zero or infinite at $K=\infty$

$q(s) = s^3 + 3s^2 + 2s + K$




| | | |
|---|---|---|
| $s^3$ | 1 | 2 |
| $s^2$ | 3 | K |
| $s^1$ | 6-K | |
| $s^0$ | K | |

at $K = 6$

$3s^2 + 6$

$\boxed{s = \pm j\sqrt{2}}$

$K = (s^2+s)(s+2) = s^3 + 2s^2 + s^2 + 2s$

$K = s^3 + 3s^2 + 2s$

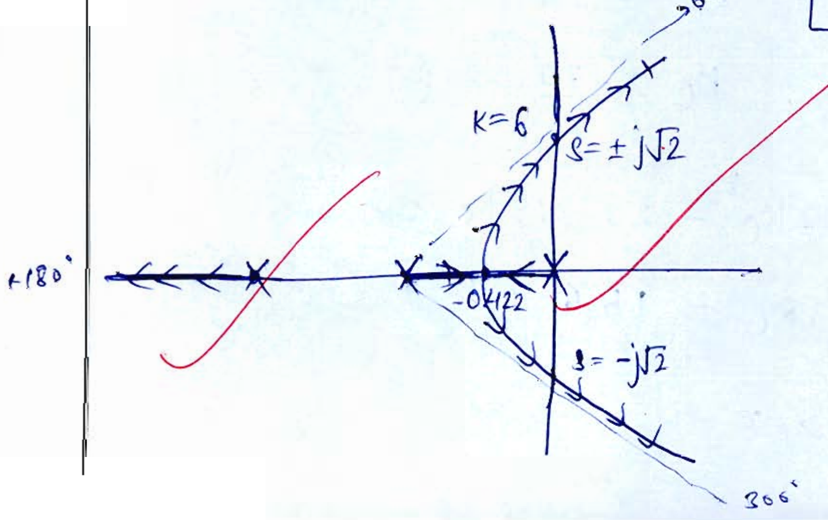
$\dfrac{dK}{ds} = 3s^2 + 6s + 2$

Breakaway point

$\boxed{s_1 = -0.422}$

$\boxed{s_2 = -1.577}$

~~Break in point~~



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.8 (a) A unity-negative feedback system is characterized by the open-loop transfer function

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

**(i)** Determine the steady-state errors to $2u(t)$, $5t\,u(t)$ and $5t^2u(t)$ input [$u(t)$ is step input].

**(ii)** Determine rise time, peak time, peak overshoot and settling time (±2%) of the unit-step response of the system.

**[8 + 12 marks]**

(i) steady state error due to unit step function

$$e_{ss} = \frac{A}{1+K_p}$$

where $K_p = \lim_{s \to 0} G(s)H(s)$

$$K_p = \lim_{s\to 0} \frac{1}{s(0.5s+1)(0.2s+1)} = \infty$$

$$e_{ss} = \frac{2}{1+\infty} = \boxed{0}$$

steady state error due to $5t\,u(t)$ ramp input

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$$e_{ss} = \frac{A}{K_V}$$

$$K_V = \lim_{s\to 0} s\,G(s)H(s) = s\,\frac{1}{s(s0.5+1)(0.2s+1)}$$

$$K_V = 1$$

$$e_{ss} = \frac{5}{1} = \boxed{5}$$

Steady state error due to $5t^2u(t)$ (parabola input)

$$e_{ss} = \frac{1}{K_a}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} s^2 \times \frac{1}{s(0.5s+1)(0.2s+1)}$$

$$K_a = 0$$

$$e_{ss} = \frac{1}{0} \Rightarrow e_{ss} = \boxed{\infty}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s(0.5s+1)(0.2s+1)} = \frac{R(s)}{0.5\times0.2[s\cdot(s+2)(s+5)]}$$

$$C(s) = \frac{10}{s^2(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{D}{s+5}$$

$$C(s) = \frac{A(s^2+2s)(s+5) + B(s^2+7s+10) + C(s^3+5s^2) + D(s^3+2s^2)}{s^2(s+2)(s+5)}$$

$$C(s) = \frac{(A+C+D)s^3 + [7A+B+5C+2D]s^2 + [10A+7B]s + 10B}{s^2(s+2)(s+5)}$$

$$\left.\begin{aligned} A+C+D &= 0 \\ 7A+B+5C+2D &= 0 \\ 10A+7B &= 0 \\ B &= 1 \end{aligned}\right\} \quad \begin{aligned} A &= -0.7 \\ B &= 1 \\ C &= 0.83 \\ D &= -0.134 \end{aligned}$$

$$C(s) = \frac{-0.7}{s} + \frac{1}{s^2} + \frac{0.83}{s+2} - \frac{0.134}{s+5}$$

$$\boxed{c(t) = \left[-0.7 + t + 0.83e^{-2t} - 0.134e^{-5t}\right]u(t)}$$

$$\frac{dc(t)}{dt} = 1 - 1.66e^{-2t} + 0.67e^{-5t} = 0 \quad \text{to find } t_p$$

rise time : when responce taken time to reach 10% to 90% of it final value.

setting time : responce take time to reach 98% of it final value.

peak time : it take time to max value

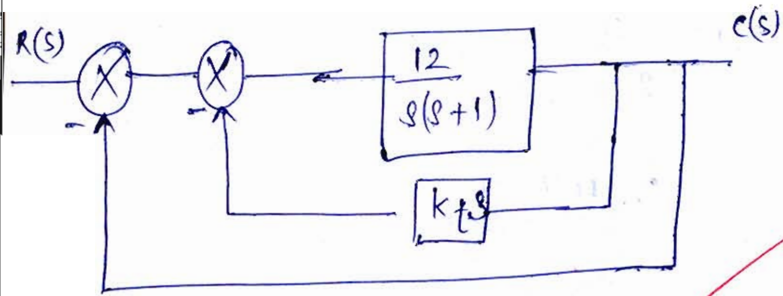
peak overshoot $= \dfrac{c(t_p) - c(\infty)}{c(\infty)}$

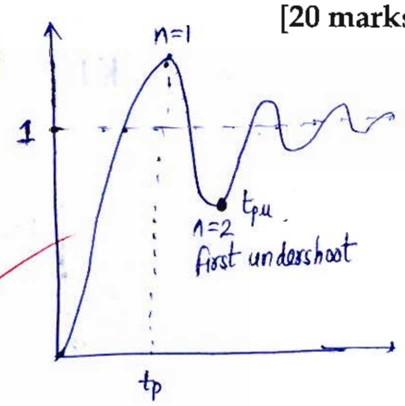
**.8 (b)** The open loop transfer function of a unity negative feedback control system is given by

$$G(s) = \frac{12}{s(s+1)}$$

The system is to have 6.5% first peak undershoot when compensated by tachometer feedback. Determine the tachometer feedback constant $K_t$.

**[20 marks]**




$$\text{open loop TF} = \frac{12}{s^2 + s + k_t \times 12 s}$$

$$t_{pu} = \frac{2\pi}{\omega_d} \qquad \frac{c(t_{pu}) - c(\infty)}{c(\infty)} = \frac{6.5}{100}$$

$$\text{open loop TF} = \frac{12}{s\left(s + (12k_t + 1)\right)}$$

Good

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$$\text{close loop TF} = \frac{12}{s^2 + (12k_t+1)s + 12}$$

$$m_p\% = e^{-\frac{n\pi\xi}{\sqrt{1-\xi^2}}}$$

$$m_p = e^{-\frac{2 \times 3.14 \times \xi}{\sqrt{1-\xi^2}}} = 0.065$$

$$\ln(0.065) = \frac{-2 \times 3.14 \times \xi}{\sqrt{1-\xi^2}}$$

$$0.189 = \frac{\xi^2}{1-\xi^2}$$

$$\boxed{\xi = 0.399}$$

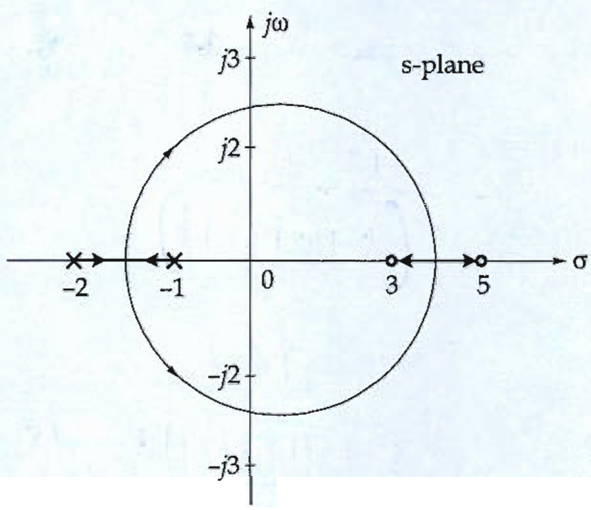
$$2\xi\omega_n = 12K_t + 1$$

$$2 \times 0.399 \times \sqrt{12} = 12K_t + 1$$

$$K_t = \frac{1.764}{12}$$

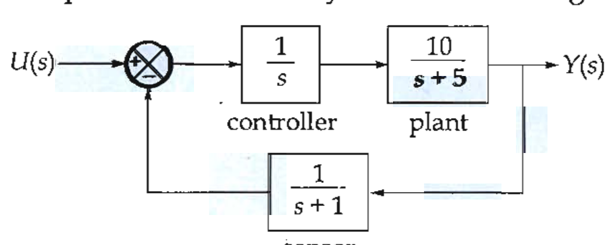
$$\boxed{K_t = 0.147}$$

**Q.8 (c)** **(i)** The root locus plot for the certain control system is shown below:




Find the break-away and break-in points for the above root locus plot.

**(ii)** Obtain a state-space model of the system shown in figure below:




**[10 + 10 marks]**

(i) from Root locus we can see

pole $P_1 = -2$ $P_2 = -1$

zero $Z_1 = 3$ $Z_2 = 5$

so Transfer function will be $= \dfrac{K(s-3)(s-5)}{(s+2)(s+1)}$

~~for~~ finding Breakaway or Breakin point we need to find $\dfrac{dK}{ds} = 0$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s-3)(s-5)}{(s+2)(s+1)} = 0$$

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$$K = -\frac{(s+1)(s+2)}{(s-3)(s-5)} = \frac{-(s^2+3s+2)}{(s^2-8s+15)}$$

$$\frac{dK}{ds} = -\left[\frac{(s^2-8s+15).(2s+3) - (s^2+3s+2).(2s-8)}{(s^2-8s+15)^2}\right] = 0$$

$$\frac{dK}{ds} = 2s^3+3s^2-16s^2-24s+30s+45 - [2s^3+6s^2+4s - 8s^2-24s-16] = 0$$

$$\frac{dK}{ds} = \cancel{2s^3}+\cancel{3s^2}-16s^2-24s+30s+45-\cancel{2s^3}-6s^2-4s+8s^2+24s+16=0$$

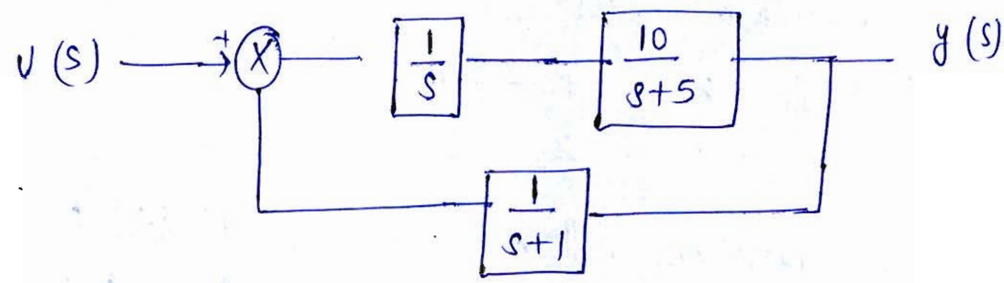
$$-11s^2 + 26s + 61 = 0$$

$$11s^2 - 26s - 61 = 0$$

$\boxed{s_1 = 3.816}$ → Break-in

$\boxed{s_2 = -1.4529}$ ← breakaway

(ii)




$$\frac{y(s)}{U(s)} = \frac{\dfrac{10}{s(s+5)}}{1 + \dfrac{10}{s(s+5)} \times \dfrac{1}{(s+1)}}$$

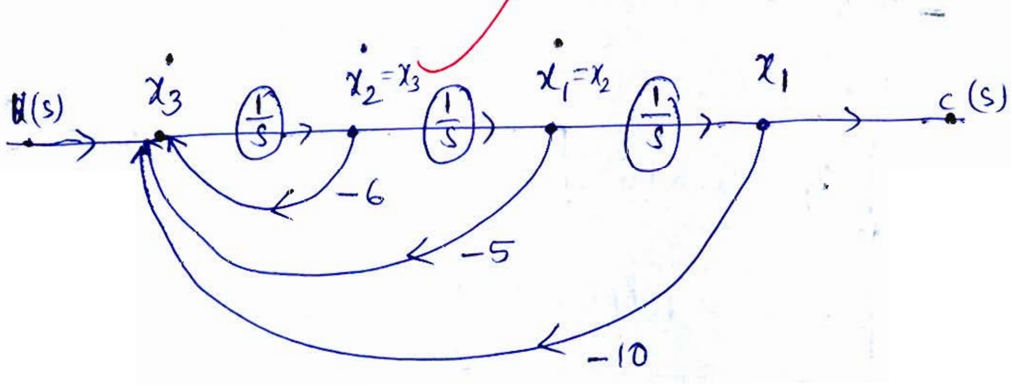
$$\frac{y(s)}{U(s)} = \frac{10}{s(s+5)(s+1) + 10} = \frac{10}{(s^2+5s)(s+1)+10}$$

$$\frac{y(s)}{U(s)} = \frac{10}{s^3 + s^2 + 5s^2 + 5s + 10}$$

$$\frac{y(s)}{U(s)} = \frac{10}{s^3 + 6s^2 + 5s + 10}$$

now taking $s^3$ common

$$\frac{y(s)}{U(s)} = \frac{\left[\dfrac{10}{s^3}\right]}{1 - \left[-\dfrac{6}{s} - \dfrac{5}{s^2} - \dfrac{10}{s^3}\right]}$$



$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -6x_3 - 5x_2 - 10x_1 + U(s)$

$C(s) = 1\, x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ -10 & -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

## Space for Rough Work

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