• Write all steps in detail
• write your answer in provided space only

MADE EASY

India's Best Institute for IES, GATE & PSUs

ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits + Control Systems [All Topics]

Name : ....

Roll No :

Test Centres **Student's Signature**

Delhi ☑ Bhopal ☐ Jaipur ☐ Pune ☐

Kolkata ☐ Bhubaneswar ☐ Hyderabad ☐

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE	
Question No.	Marks Obtained
Section-A	
Q.1	34
Q.2	
Q.3	40
Q.4	
Section-B	
Q.5	35
Q.6	35
Q.7	44
Q.8	
Total Marks Obtained	188

Signature of Evaluator Cross Checked by

Sourabh kumar ....

Avoid calculation mistake

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

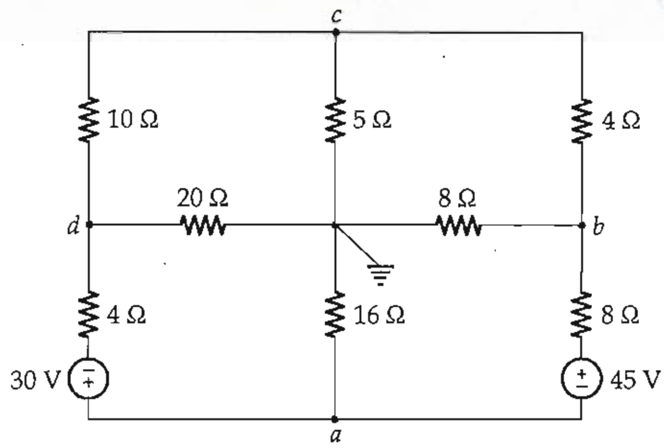
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough **notes** should be crossed through **afterwards**.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Electrical Circuits

Q.1 (a) Find the voltages at nodes a, b, c and d in the circuit shown.



[12 marks]

Sol^n

By KCL at Node-C.

$$\frac{V_c - V_d}{10} + \frac{V_c}{5} + \frac{V_c - V_b}{4} = 0.$$

$$2V_c - 2V_d + 4V_c + 5V_c - 5V_b = 0$$

$$11V_c - 2V_d - 5V_b = 0 \quad \text{——} \quad (1)$$

By KCL at Node-D

$$\frac{V_d - V_c}{10} + \frac{V_d}{20} + \frac{V_d + 30 - V_a}{4} = 0.$$

$$2V_d - 2V_c + V_d + 5V_d + 150 - 5V_a = 0.$$

$$-5V_a + 8V_d - 2V_c + 150 = 0 \quad \text{——} \quad (2)$$

By KCL at Node-A

$$\frac{V_a - 0}{16} + \frac{V_a - 30 - V_d}{4} + \frac{V_a + 45 - V_b}{8} = 0.$$

$$V_a + 4V_a - 120 - 4V_d + 2V_a + 90 - 2V_b = 0$$

$$7V_a - 2V_b - 4V_d = 30 \quad \text{——} \quad (3)$$

→ By KCL at Node - B -

$$\frac{V_b}{8} + \frac{V_b - V_c}{4} + \frac{V_b - (V_a + 45)}{8} = 0$$

$$V_b + 2V_b - 2V_c + V_b - V_a - 45 = 0.$$

$$5V_b - 2V_c - 45 = V_a \quad \text{——} \quad (4)$$

→ putting V_a from eq (4) to equation (2), (3), ~~(5)~~ -

$$-5(5V_b - 2V_c - 45) + 8V_d - 2V_c + 150 = 0.$$

$$-25V_b + 8V_c + 8V_d + 105 = 0 \quad \text{——} \quad (5)$$

and -.

$$7(5V_b - 2V_c - 45) - 2V_b - 4V_d = 30.$$

$$33V_b - 14V_c - 4V_d = 345\cancel{0} \quad \text{——} \quad (6)$$

from equation (1), (5), (6) -

$$\boxed{V_b = 22.987 \text{ volts}}$$

$$\boxed{V_c = 17.873 \text{ volts}}$$

$$\boxed{V_d = 40.836 \text{ volts}}$$

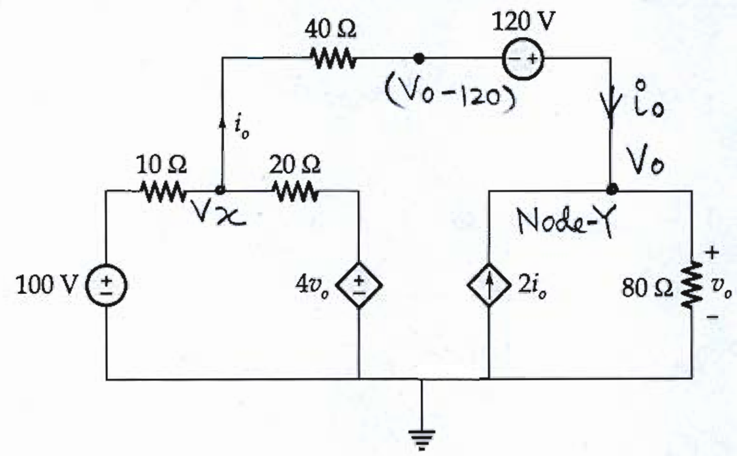
4

Calculation mistake

→ putting V_b & V_c values in equation (4)

$$\boxed{V_a = 34.189 \text{ volts}}$$

Q.1 (b) Using nodal analysis, find v_o and i_o in the circuit shown in figure.



[12 marks]

Sol^n-

By KCL at Node voltage - V_x -

$$\frac{V_x-100}{10}+\frac{V_x-4V_0}{20}+i_0=0.$$

$$2V_x-200+V_x-4V_0+20i_0=0.$$

$$3V_x-4V_0+20i_0=200 \quad \text{——} \ (1)$$

By KCL at Node voltage - V_0 (Node-Y)

$$\frac{V_0}{80}=i_0+2i_0.$$

$$V_0=240i_0 \quad \text{——} \ (2)$$

By ohm's law across 40Ω Resistor -

$$V_x-(V_0-120)=40\times i_0.$$

$$V_x-V_0+120=40i_0 \quad \text{——} \ (3).$$

putting $V_0=240i_0$ from equation (2) to equation (1) & (3) we get -

$$3V_x - 940 i_o = 200 \quad \text{—— (4)}$$

$$V_x - 120 = 280 i_o \quad \text{—— (5)}$$

→ put $V_x = 120 + 280 i_o$ in eq (4) –

$$3(120 + 280 i_o) - 940 i_o = 200$$

$$360 - 100 i_o = 200$$

$$\boxed{i_o = 1.6\ Amp}$$ Answer

(3)

→ put $i_o = 1.6$ in eq^n (2) –

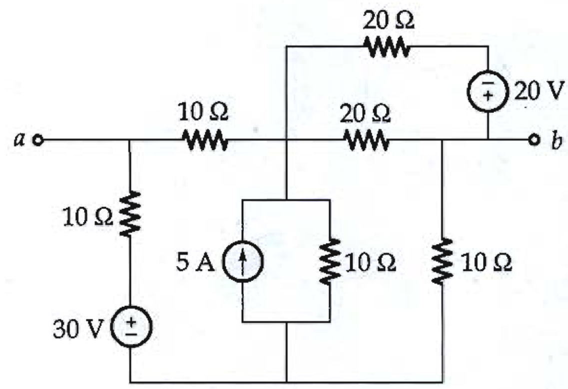
$$V_o = 240 i_o$$

$$V_o = 240 \times 1.6$$

$$\boxed{V_o = 384\ volts}$$ Answer

Calculation mistake

Q.1 (c) For the circuit shown, find the Thevenin's equivalent network between terminals *a* and *b*.

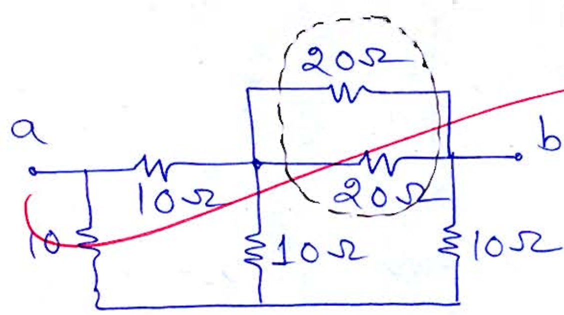


[12 marks]

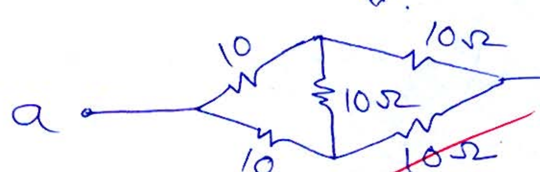
Sol^n- Step-1 Calculation of Rth -

(i) open circuit the ~~voltage~~ current sources.

(ii) short circuit the voltage sources.



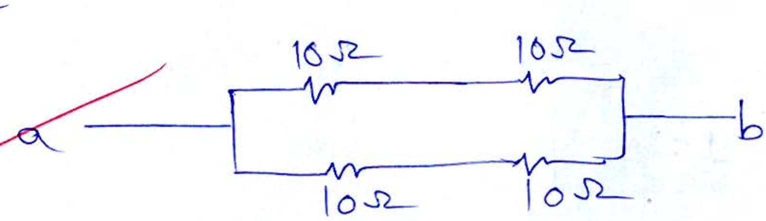
⇓



$\left(\because 20\Omega \| 20\Omega = 10\Omega\right)$

→ given circuit is a balanced wheatstone bridge.

thus-

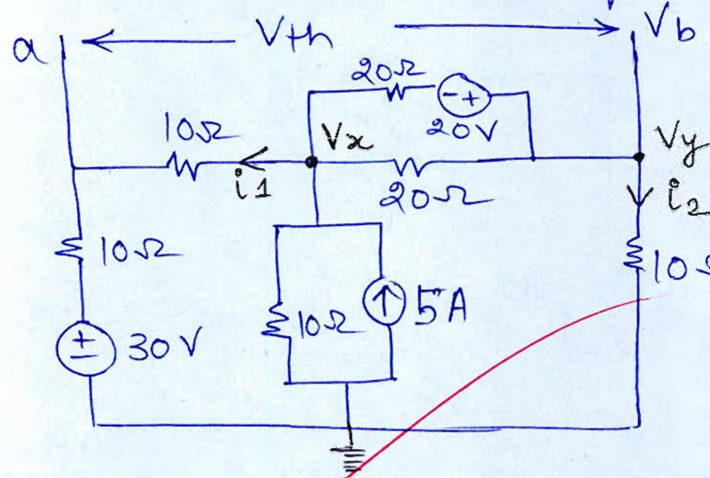


~~Rab = Rth = [(10Ω)‖(10Ω)] + [(10Ω)‖(10Ω)]~~

$$R_{th} = \frac{20\times20}{20+20} + \cancel{\frac{10\times10}{10+10}}$$

$$\boxed{R_{th} = 10\,\Omega}$$

Step-2 Calculation of Vth -



→ Applying KCL at Node - Vx

$$\frac{Vx-30}{20} + \frac{Vx-0}{10} + \frac{Vx-Vy}{20} + \frac{Vx-(Vy-20)}{20} = 0$$

$$Vx - 30 + 2Vx + Vx - Vy + Vx - Vy + 20 = 0$$

$$5Vx - 2Vy = 10 \quad \text{—— (1)}$$

→ KCL at Node - Vy -

$$\frac{Vy-0}{10} + \frac{Vy-20-Vx}{20} + \frac{Vy-Vx}{20} = 0$$

11

$$2Vy + Vy - 20 - Vx + Vy - Vx = 0$$

$$4Vy - 2Vx - 20 = 0 \quad \text{—— (2)}$$

from (1) & (2) - we get

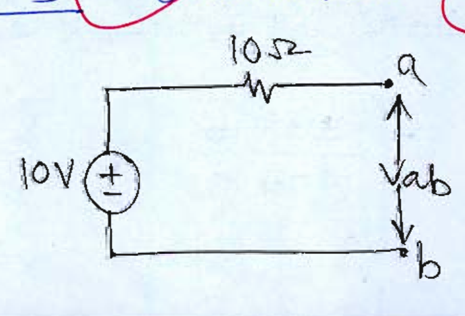
$$\boxed{Vx = 5V} \quad \boxed{Vy = 7.5V} \quad \boxed{i_1 = -1.25A}$$

$$Vth = Va - Vb = (Vx - 10i_1) - Vy$$

$$\boxed{Vth = 10V.}$$

Step-3 Thevenin's Ckt - equivalent Circuit

Good Approach

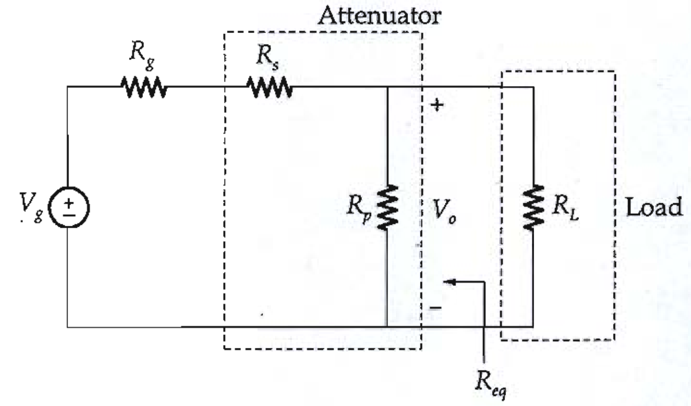


Answer

Q.1 (d) An attenuator is an interface circuit that reduces the voltage level without changing the output resistance. By specifying R_s and R_p of the interface circuit shown in figure, design an attenuator that will meet the following requirements :

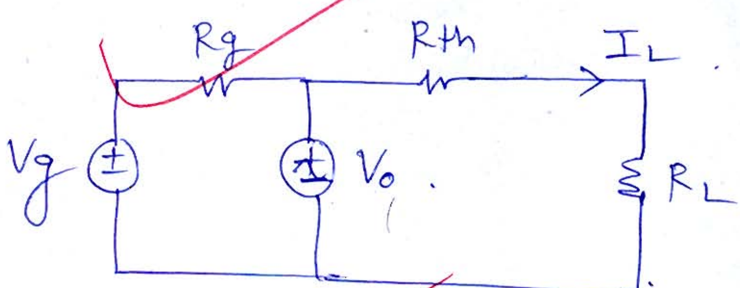
$$\frac{V_o}{V_g} = 0.125,\ R_{eq} = R_{th} = R_g = 100\ \Omega$$

Using the interface designed, also calculate the current through a load of $R_L = 50\ \Omega$ when $V_g = 12$ V.



[12 marks]

Sol^n- Replacing attenuator with thevenin's ckt -



where $V_o = 0.125\ V_g = 1.5$ volts.

$R_g = R_{th} = 100\,\Omega$.

$R_L = 50\,\Omega$.

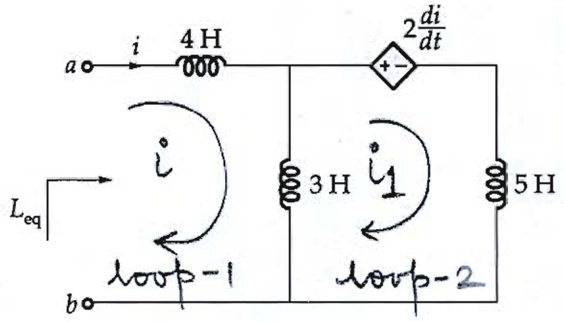
11

Good APProach

load current $I_L = \dfrac{1.5 - 0}{R_{th} + R_L} = \dfrac{1.5}{150}$.

$\boxed{I_L = 10\,mA}$ Answer

Q.1 (e) Determine the equivalent inductance, L_{eq} that can be used to represent the inductive network shown in figure.



[12 marks]

Sol^n Equivalent inductance of given circuit will be -

$$\boxed{L_{eq} = \frac{V_{ab}}{di/dt}}$$

→ By KVL in loop-1

$$V_{ab} - 4\frac{di}{dt} - 3\frac{d(i-i_1)}{dt} = 0$$

$$V_{ab} - 7\frac{di}{dt} = 3\frac{di_1}{dt} \quad \text{——— (1)}$$

By KVL in loop-2

$$-3\frac{d(i_1-i)}{dt} - 2\frac{di}{dt} - 5\frac{di_1}{dt} = 0$$

$$-8\frac{di_1}{dt} + \frac{di}{dt} = 0$$

$$\frac{di_1}{dt} = \frac{1}{8}\frac{di}{dt} \quad \text{——— (2)}$$

→ putting equation ② in equation ① -

$$V_{ab} - 7\frac{di}{dt} = \frac{3}{8}\frac{di}{dt}$$

$$V_{ab} = \frac{59}{8}\frac{di}{dt}.$$

$$\cancel{\frac{V_{ab}}{di/dt} = 59/8 = 7.375}$$

$$\frac{53}{8}$$

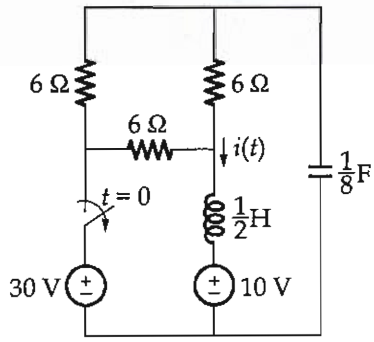
$$\cancel{\boxed{L_{eq} = 7.375\,H}}$$ Answer

$L_{eq} = 6.625H$

5

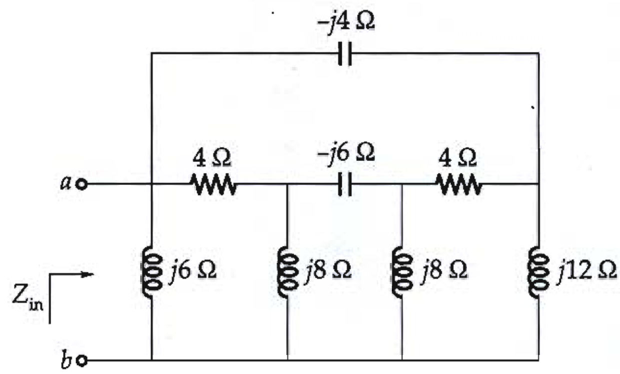
calculation mistake

Q.2 (a) For the network shown in figure, find the current $i(t)$ for $t > 0$.



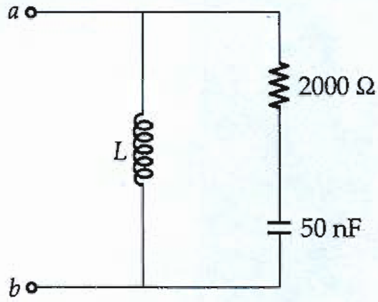
[20 marks]

Q.2 (b) Determine the equivalent impedance of the circuit shown



[20 marks]

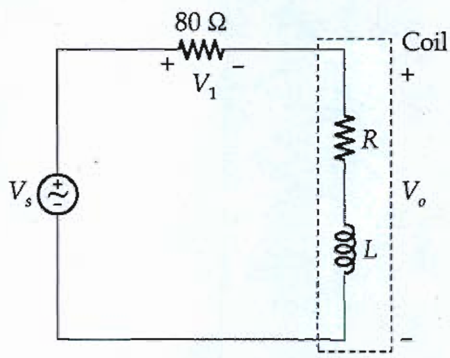
Q.2 (c) (i) An industrial load is modelled as a series combination of a capacitance and a resistance as shown in figure. Calculate the value of an inductance L across the series combination so that net impedance is resistive at a frequency of 50 kHz.



(ii) An industrial coil is modelled as a series combination of an inductance L and resistance R, as shown in figure. Since an AC voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz. When the circuit operates in steady state :

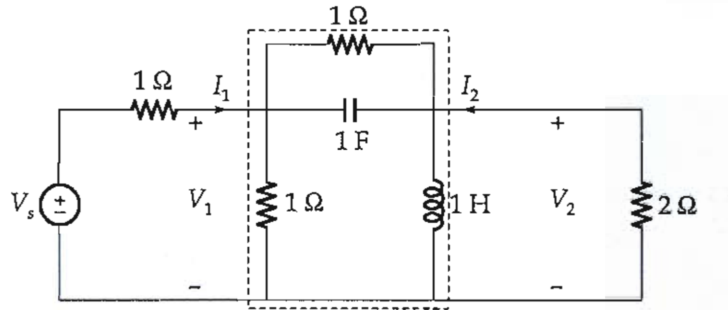
$$|V_s| = 145\text{ V}, |V_1| = 50\text{ V}, |V_o| = 110\text{ V}$$

Use these measurements to determine the values of L and R.



[10 + 10 marks]

Q.3 (a) Determine the y-parameters of two port network (1). Also determine $V_2(s)$ for $V_s = 2u(t)$ V.



Network (1)

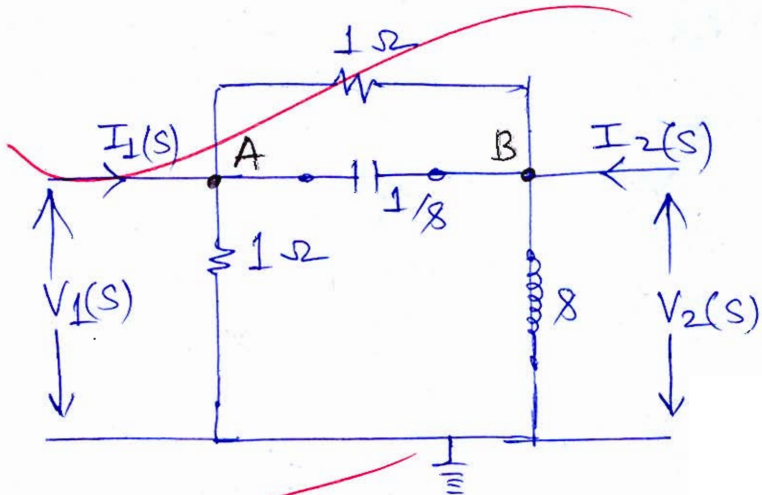
[10 + 10 marks]

Sol^n- Part-1 Y-parameter-

→ Transforming into s-domain -

→ ~~assuming that input to be DC the inductor gets short circuited and capacitor gets open circuited~~

Then the Network-1 becomes - Try to avoid



By KCL at Node-A

$\cancel{\frac{V_A - V_B}{1/8}}$ Try to avoid

$$\frac{V_1(s) - V_2(s)}{1/8} + \frac{V_1 - V_2(s)}{1} + \frac{V_1(s)}{1} = I_1(s).$$

$$I_1(s) = (s+2)V_1(s) + (-8-1)V_2(s) \quad \text{— (1)}$$

By KCL at Node - B

$$\frac{V_2(s)-V_1(s)}{1} + \frac{V_2(s)-V_1(s)}{1/s} + \frac{V_2(s)}{8} = I_2(s)$$

$$(-s-1)V_1(s) + (1+1/8+s)V_2(s) = I_2(s) \quad \text{--- } (2)$$

→ comparing equation (1) & (2) with standard Y-parameter equations -

$$I_1(s) = Y_{11}V_1(s) + Y_{12}V_2(s)$$

$$I_2(s) = Y_{21}V_1(s) + Y_{22}V_2(s)$$

weget -

$$[Y] = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} s+2 & -s-1 \\ -s-1 & 1+s+1/8 \end{pmatrix}$$

Part - 2 $\quad V_s = 2u(t)$

then - $\quad V_s(s) = 2/s$

By ohm's law -

$$V_s(s) - V_1(s) = I_1(s) \times 1 \quad \text{--- } (3)$$

and -

$$V_2(s) = -2I_2(s) \quad \text{--- } (4)$$

putting equation (3) to equation (1)

$$\frac{2}{s} - V_1(s) = sV_1(s) + 2V_1(s) + (-s-1)V_2(s) \quad \text{--- } (5)$$

putting $I_2(s)$ from eq^n ④ to eq^n ② -

$$(s+1)V_1(s) = \left(1+1/2+1/8+8\right)V_2(s)$$

$$V_1(s) = \frac{(2s+s+2+2s^2)}{2s(s+1)} \quad \text{—— ⑥}$$

putting $V_1(s)$ from equation ⑥ to equation ⑤ -

$$\frac{2}{s} = \left\{\frac{(s+3)(2s^2+3s+2)}{2s(s+1)} - s - 1\right\}V_2(s)$$

we get -

$$V_2(s) = \frac{2}{s} \times \frac{2(s)(s+1)}{\left((s+3)(2s^2+3s+2) - (s+1)^2 2s\right)}$$

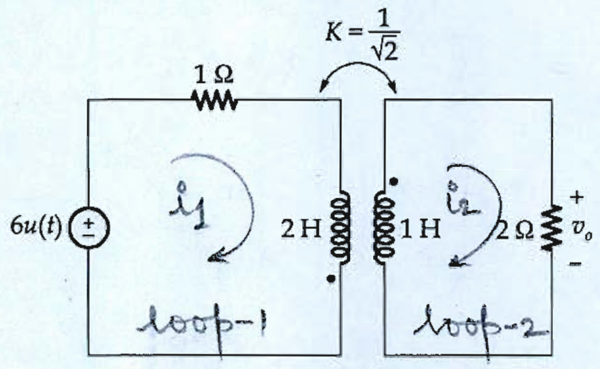
$$\boxed{V_2(s) = \frac{4(s+1)}{5s^2+9s+6}}$$ Answer

18

Good Approach

Do not write in this margin

Q.3 (b) For the circuit shown in figure, find $V_o(t)$ for $t > 0$.



[20 marks]

Soln. Given $K = 1/\sqrt{2}$ then mutual inductance M of given circuit will be -

$$M = K\sqrt{L_1L_2} = \frac{1}{\sqrt{2}}\sqrt{2\times 1} = 1 \text{ Henry}.$$

→ The output voltage - $V_o(t)$ will be -

$$\boxed{V_o(t) = 2\,i_2(t)} \quad \text{——— (1)}.$$

By KVL in loop-1

$$6 - i_1 - 2\frac{di_1}{dt} - 1\frac{di_2}{dt} = 0.$$

→ taking laplace transform -

$$6/s = I_1(s) + 2s\,I_1(s) + sI_2(s) \quad \text{——— (1)}$$

By KVL in loop-2

$$1.\frac{di_2}{dt} + 1\frac{di_1}{dt} + V_o = 0$$

taking laplace transform -

$sI_2(s) + I_1(s) + V_o(s) = 0$.

→ from eq^n 1 ⇒ $V_o(s) = 2I_2(s)$

we get -

$$I_1(s) = -(s+2)I_2(s) \quad \text{——— (3)}$$

→ put $I_1(s)$ from equation 3 to equation 2 -

$$6/s = -(1+2s)(s+2)I_2(s) + sI_2(s)$$

$$I_2(s) = \frac{6/s}{s-(1+2s)(s+2)} = \cancel{\frac{-6}{s[s^2+2s+1]\times 2}}$$

$$I_2(s) = \frac{-3}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

comparing the coefficient we get -

$\boxed{A = -3}$ $\boxed{B = 3}$ $\boxed{C = 3}$

$$I_2(s) = -\frac{3}{s} + \frac{3}{s+1} + \frac{3}{(s+1)^2}$$

→ taking inverse laplace transform -

$$\mathcal{L}^{-1} I_2(s) = i_2(t)$$

$$i_2(t) = (-3 + 3e^{-t} + 3te^{-t})\,u(t).$$

4

→ From eq^n 1 -

$\boxed{\cancel{V_o(t) = -6 + 6e^{-t} + 6te^{-t}}}$ for $t>0$

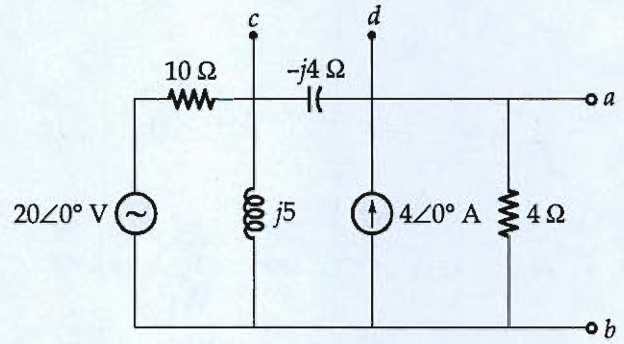
Answer

wrong value calculated

Q.3 (c) Find the Thevenin's equivalent of circuit shown in figure as seen from :

(i) terminals *a-b*

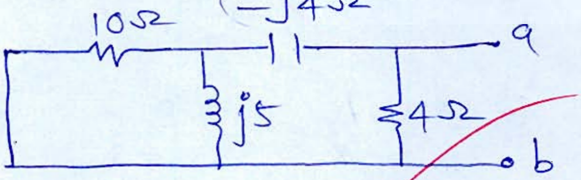
(ii) terminals *c-d*



[20 marks]

Sol^n (i) Across terminal ab -

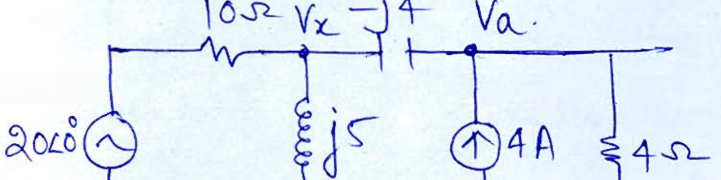
Calculation of ~~Rth~~ (Zth) -



$$Z_{ab} = \{[(10) \| (j5)] + (-j4)\} \| (4\Omega)$$

$$\boxed{Z_{ab} = 4/3\ \Omega}$$

Calculation of Vth -



KCL at Node-Vx.

$$\frac{V_x - 20}{10} + \frac{V_x}{j5} + \frac{V_x - V_a}{-j4} = 0 \quad \text{——— (1)}$$

KCL at Node-Va.

$$\frac{V_a - V_x}{-j4} + \frac{V_a}{4} = 4 \quad \text{——— (2)}$$

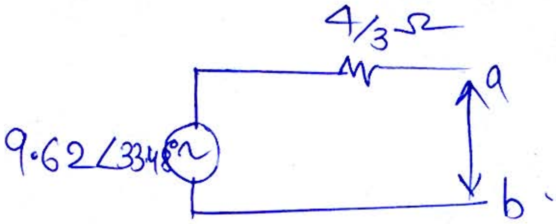
from equation ① & ② we get -.

$$\boxed{V_a = 9.6148 \angle 33.69^\circ}$$

Step-3

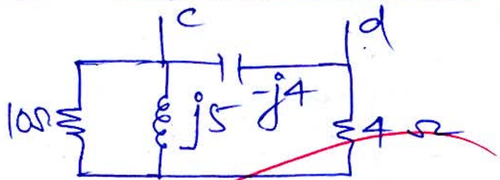
equivalent

Thevenin's circuit :-



(ii) across terminal c-d -

Step-1 Calculation of Zth -



$$Z_{cd} = [(10 \parallel j5) + 4] \parallel (-j4).$$

$$\boxed{Z_{cd} = 4.8 \angle -56.25^\circ}$$

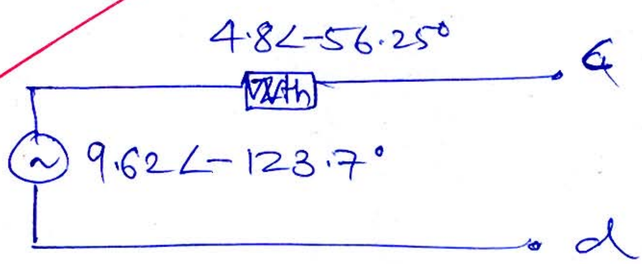
Step-2 . Calculation of Vth -

$$\boxed{V_{th} = 9.618 \angle -123.7^\circ}$$

Equivalent

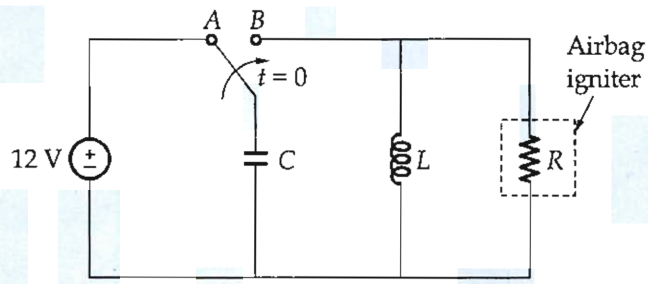
18

Step-3 Thevenin's ckt -



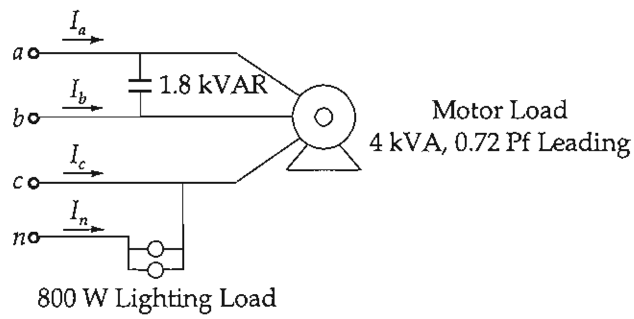
Good Approach

Q.4 (a) An automobile airbag igniter is modelled by the circuit shown in figure. Determine the time, it takes the voltage across the igniter to reach its first extreme (minimum or maximum) after switching from A to B. Let $R = 3\ \Omega$, $C = \frac{1}{30}F$ and $L = 60$ mH.



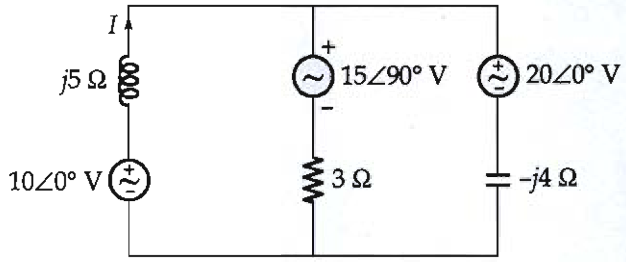
[20 marks]

Q.4 (b) In the figure shown, a 3-phase delta connected motor load which is connected to a line of 440 V, draws 4 kVA at a power factor of 0.72 leading. In addition, a single 1.8 kVAR capacitor is connected between line a and b, while 800 W lighting load is connected between line C and neutral. Assuming abc phase sequence and taking $V_{an} = V_p\angle 0°$, find the magnitude and phase angle of currents I_a, I_b, I_c and I_n.



[20 marks]

Q.4 (c) Find current I through $j5\ \Omega$ branch using superposition theorem for the network shown.

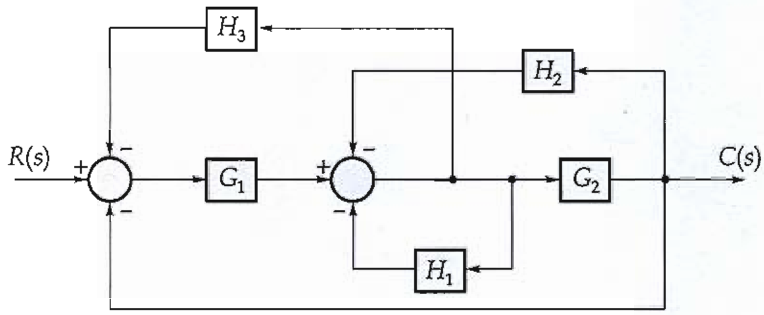


[20 marks]

Do not write in this margin

Section B : Control Systems

Q.5 (a) Using block-diagram reduction technique, find the transfer function $\frac{C(s)}{R(s)}$.



[12 marks]

Q.5 (b) The closed loop transfer function of a system is

$$\frac{C(s)}{R(s)} = \frac{100}{s^6 + 3s^5 + 8s^4 + 18s^3 + 20s^2 + 24s + 16}$$

Determine the number of poles on the RHP, LHP and on the $j\omega$-axis and comment on the stability of the system.

[12 marks]

Sol^n characteristic equation of given transfer function is -

$$q(s) = s^6 + 3s^5 + 8s^4 + 18s^3 + 20s^2 + 24s + 16 = 0$$

Routh stability criterian -

(i) Necessary condition -

→ coefficient of all terms in characteristic polynomial should have same sign.

(ii) Sufficient condition -

→ coefficient of first column in routh array should not have any sign change for stability.

Routh array -

s^6	1	8	20	16
s^5	3	18	24	
s^4	2	12	16	
s^3	0	0		
s^2				
s^1				
s^0				

→ as s^3 row is becoming zero thus there are 4 roots symmetric to origin, the coefficients in s^3 row will be given by $\frac{dA(s)}{ds} = 0$

where, $A(s) = 2s^4 + 12s^2 + 16 = 0$

$$\frac{dA(s)}{ds} = 8s^3 + 24s = 0$$

s^6	1	8	20	16
s^5	3	18	24	
s^4	2	12	16	
s^3	8	24		
s^2	6	16		
s^1	2.67			
s^0	16			

11

Good Approach

→ as there isn't any sign change so no roots are on RHS of s-plane.

→ Thus all the 4 symmetric roots of origin are on $j\omega$ axis.

→ After s^3 row = 0, no other row is becoming zero so roots on $j\omega$ axis are of non repeated roots.

Roots on RHP = 0
Roots on LHP = 2
Roots on $j\omega$ axis = 4

→ Given system is marginally stable.

Q.5 (c) The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K(s+\alpha)}{s(s^2+12s+32)}$$

Find the value of K and α so that the velocity error constant is 6.25 and the second-order response has a natural frequency of 5 rad/s. Assume that the system is stable.

[12 marks]

Solⁿ- Given velocity error constant $(K_v) = 6.25$

we know,

$$K_v = \lim_{s\to 0} s \cdot G(s)$$

$$K_v = \lim_{s\to 0} \frac{s \cdot K(s+\alpha)}{s(s^2+12s+32)}$$

$$K_v = \frac{K\alpha}{32} = 6.25$$

$$\boxed{K\alpha = 200} \quad \text{---} (1)$$

→ characteristic equation of given system will be-

$$q(s) = 1 + GH(s) = 0$$

$$q(s) = s^3 + 12s^2 + 32s + Ks + K\alpha = 0$$

$$q(s) = s^3 + 12s^2 + (32+K)s + 200 = 0$$

$$\text{---} (2) \qquad (\because K\alpha = 200, \text{ from eq-1})$$

→ given 3rd order system can be written as a 2nd order + 1st system as-

$$q(s) = (s+a)(s^2 + 2\xi\omega_n s + \omega_n^2)$$

$$q(s) = s^3 + (a + 2\xi\omega_n)s^2 + (\omega_n^2 + 2\xi\omega_n a)s + a\omega_n^2 \quad \text{---} (3)$$

→ comparing eqn ② & equation - ③ -

$a + 2\xi\omega_n = 12$

$\boxed{a + 10\xi = 12}$ $(\because \omega_n = 5)$

$\omega_n^2 + 2\xi\omega_n a = 32 + K$

$10\xi a + 25 = 32 + K$

$\boxed{K = 10\xi a - 7}$

$a\omega_n^2 = 200$

$a = 200/25 \Rightarrow \boxed{a = 8}$

→ on solving above identity we get -

$a = 8, \xi = 0.4, \quad K = 10 \times 0.4 \times 8 - 7$

$\boxed{K = 25}$ Answer

11

from equation - ① -

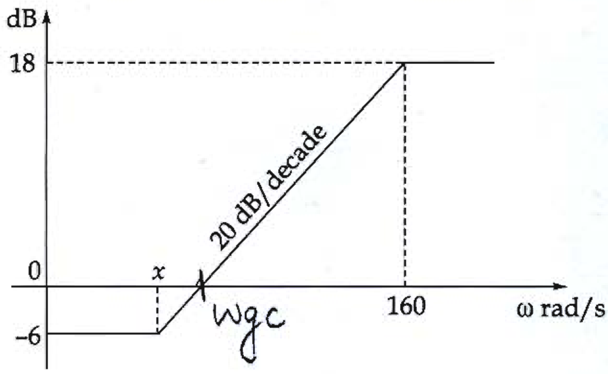
$K\alpha = 200$

$25 \times \alpha = 200$

$\boxed{\alpha = 8}$ Answer

Good Approach

Q.5 (d) An asymptotic bode magnitude plot is shown in figure.



Find the transfer function and gain cross-over frequency.

[12 marks]

Sol^n-

(i) Initial slope of given bode plot $= 0\,dB$.

→ hence there isn't any pole/zero at origin.

(ii) at frequency $(\omega_c = x)$ change in magnitude of slope of bode plot $= 20\,dB$

→ Thus there is a zero at freq $\omega = x$.

(iii) at frequency $(\omega_c = 160)$ change in magnitude of slope of bode plot $= -20\,dB$

→ Thus there is a zero at freq $\omega = 160$

Hence, Transfer function -.

$$T(s) = \frac{K}{(s/x + 1)(s/160 + 1)}$$

$$T(s) = \frac{160Kx}{(s+x)(s+160)} \quad \text{—— (1)}$$

(a) Calculation of x -

$$\text{slope} = 20\,dB/dec = \frac{18-(-6)}{\log_{10}160 - \log_{10}x}$$

$$\boxed{x = 10.095\ rad/sec}$$

(b) Calculation of K -

→ at lower frequency the transfer function can be approximated as -

$$T(s) \approx K$$

thus,

$$-6 = 20\log_{10}K \Rightarrow \boxed{K = 0.5}$$

→ Hence the transfer function -

$$\boxed{T(s) = \frac{807.6}{(s+10.095)(s+160)}}$$ Answer

11

(c) Gain ~~gross~~ cross over freq - (ω_{gc})

→ Gain ~~go~~ cross over frequency is the frequency at which magnitude of gain = 0dB.

Good Approach

From given bode plot -

$$\text{slope} = 20\,dB/dec = \frac{0-(-6)}{\log_{10}\omega_{gc} - \log_{10}x}$$

$$\boxed{\omega_{gc} = 20.142\ rad/sec}$$

Answer

Q.5 (e) A system is represented by the state model, $\dot{X} = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$ and $y = [1 \quad 2]X$. If the initial state vector is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find zero input response of the system.

[12 marks]

Sol^n- Transfer function of given system will be -

$$TF = C(SI-A)^{-1}B + D.$$

$$TF = (1 \quad 2)\begin{pmatrix} S & 0 \\ -3 & S+3 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0.$$

2

$$TF = \frac{(1 \quad 2)\begin{pmatrix} S+3 & 0 \\ 3 & S \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}}{S(S+3)}$$

$$TF = \frac{(1 \quad 2)\begin{pmatrix} S+3 \\ 3 \end{pmatrix}}{S(S+3)}$$

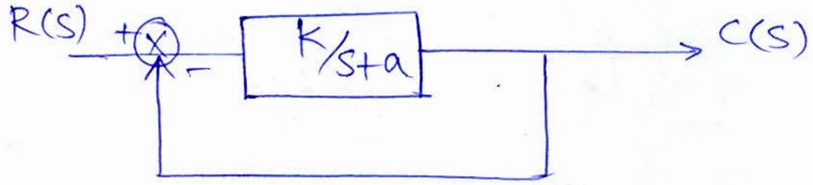
$$TF = \frac{S+9}{S(S+3)} = \frac{Y(S)}{X(S)}$$

$\cancel{SX(S) + 9X(S) = S^2Y(S) + 3SY(S)}$

Q.6 (a) The forward path gain of a first-order unity negative feedback system is $G(s) = \dfrac{K}{s+a}$. The unit step response reveals that the time constant is 1/6 sec. When the location of the pole is moved toward the origin by half its distance, the new time constant is found to be 1/4 sec. Find the value of a and K. For the time constant to be 1/8 sec, find the location of the closed-loop pole.

[20 marks]

Solⁿ–



Transfer function –

$$CLTF = \frac{C(s)}{R(S)} = \frac{K}{S+(a+K)} = \frac{K/a+K}{\frac{1}{a+K}s+1} \quad \text{—— (a)}$$

poles of given system are at $\boxed{s = -(a+K)}$

(i) Time constant = 1/6 sec

→ comparing with standard 1st order equation – $T(s) = \dfrac{K}{1+s\tau}$

we get – $\tau = \dfrac{1}{6} = \dfrac{1}{a+K}$ —— (1)

(ii) Time Constant = 1/4 sec

→ New location of poles = $\cancel{\frac{a+K}{2}}$. $\frac{a}{2}+K$

we know,

$$\text{time constant} = \frac{1}{\text{pole}}.$$

$$\frac{1}{4} = \frac{1}{\frac{a}{2}+K} \quad \text{—— (2)}$$

From equation 1 + 2 -

$$a+K=6$$
$$\frac{a}{2}+K=4$$

$\boxed{a=4}$ $\boxed{K=2}$ Answer -

→ (iii) Time constant = 1/8

→ Location of poles = $\frac{1}{\text{time constant}}$.

hence,

$\boxed{s=8}$ is a pole for time constant = 1/8

Answer.

18

Good Approach

Q.6 (b) A unity feedback system has open-loop transfer function

$$G(s) = \frac{3(2-s)}{(s+1)(s+5)}$$

Using Nyquist stability criterion, check whether the closed-loop system is stable or not. If the system is stable, find the gain margin and phase margin.

[20 marks]

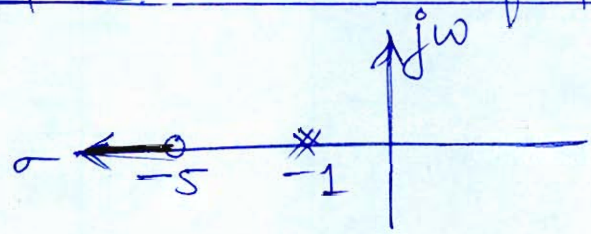
Q.6 (c) A feedback system has open-loop transfer function

$$G(s)H(s) = \frac{K(s+5)}{(s+1)^2}$$

Sketch the root locus.

[20 marks]

Sol^n - Step-1 Location of poles & zeros -.



Step-2 Asymptote -

(i) centroid $(\sigma_A) = \dfrac{-1-1-(-5)}{2-1} = 3$

(ii) No of asymptote $= P-Z = 2-1 = 1$.

(iii) angle of ~~depa~~ asymptote $(\phi_A) = \dfrac{(2K+1)\times 180^\circ}{P-Z}$

$\phi_A = 180^\circ$

Step-3 Saddle point -

$$-K = \frac{(s+1)^2}{(s+5)}$$

$$-\frac{dK}{ds} = \frac{2(s+1)(s+5) - (s+1)^2}{(s+5)^2} = 0.$$

$$\boxed{s = -1, -9}$$

Step-4 Intersection with ($j\omega$) axis -.

Routh array -

$$q(s) = (s+1)^2 + K(s+5)$$

$$= s^2 + (2+K)s + (5K+1)$$

s^2	1	5K+1
s^1	2+K	
s^0	5K+1	

→ putting coefficient of $s^1 = 0$ ⇒ $\boxed{K=-2}$

then–
$$A(s) = s^2 + (-9) = 0$$
$$(+j\omega)^2 - 9 = 0$$
$$\omega = \text{Non real value}$$

→ Thus there isn't any point of intersection with $j\omega$ axis –

Step-5 departure & arrival from poles and zeros –

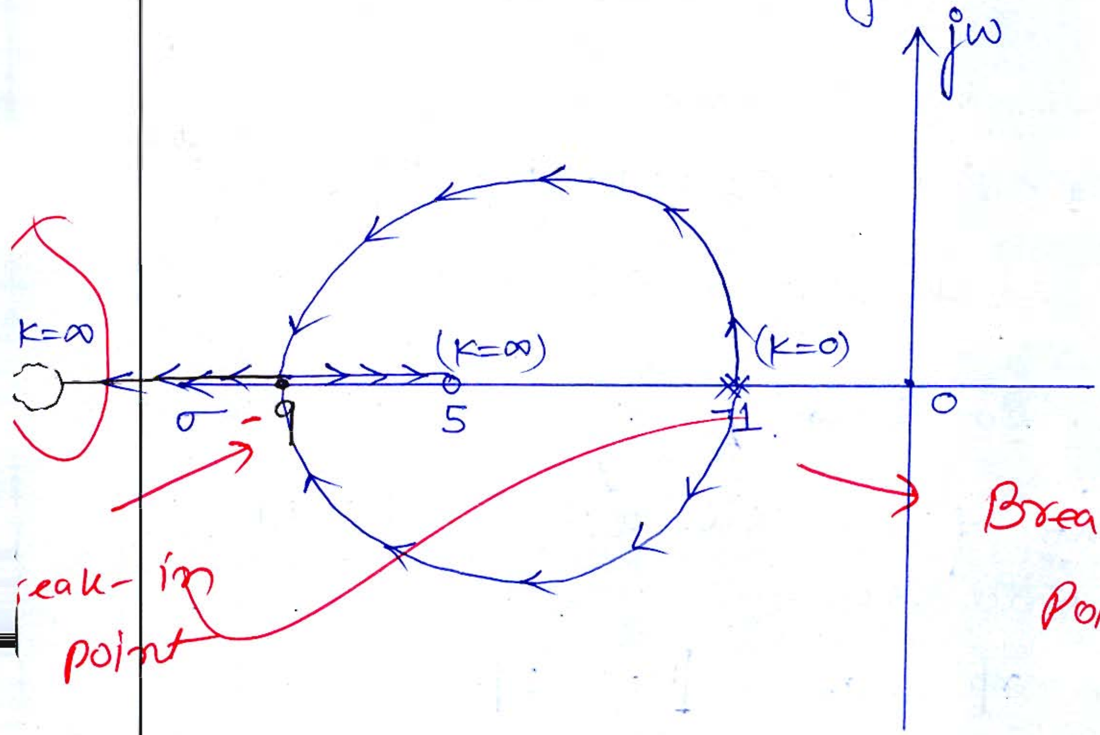
→ arrival of at simple zero = $180°/0°$

→ departure from multiple pole = $\frac{(2K+1)180°}{r}$

(r = No of multiple poles) $= 90°, 270°$

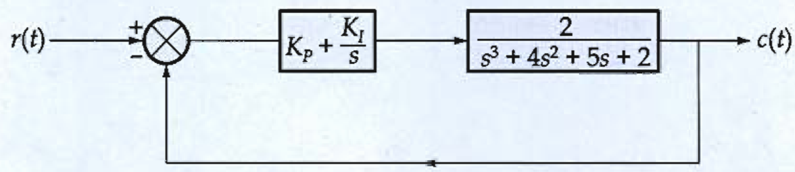
Do not write in this margin

Step-6 Root locus diagram –



17

Q.7 (a) The stability of overall system shown in figure is controlled by tuning the PI parameters K_p and K_I. Find the maximum value of K_I that can be selected so as to keep overall system stable or in worst case, marginally stable.



[20 marks]

Sol^n- Transfer function of given system will be-

$$T(S) = \frac{C(S)}{R(S)} = \frac{(K_p + K_I/s)\left(\frac{2}{s^3+4s^2+5s+2}\right)}{1 + \frac{(K_p+K_I/s)\,2}{s^3+4s^2+5s+2}}$$

$$T(S) = \frac{2K_p + 2K_I/s}{s^3+4s^2+5s+2 + 2K_p + 2K_I/s}$$

$$T(S) = \frac{2K_p s + 2K_I}{s^4 + 4s^3 + 5s^2 + 2s + 2K_p s + 2K_I}$$

characteristic equation -

$$q(S) = s^4 + 4s^3 + 5s^2 + (2+2K_p)s + 2K_I$$

By Routh stability criterian -

(i) Necessary condition -

→ coefficient of all the terms should have same sign -

$\boxed{K_I > 0}$ and $\boxed{K_p > -1}$

(ii) Sufficient condition -

→ first column in routh array must be positive (or there shouldn't be any sign change) -

s^4	1	~~4~~ 5	$2K_I$
s^3	4	$2+2K_p$	
s^2	$\frac{18-2K_p}{4}$	$2K_I$	
s^1	x		
s^0	$2K_I$		

→ $\frac{18-2K_p}{4} > 0 \Rightarrow \boxed{K_p < 9}$

where, $x = \dfrac{\left(\frac{18-2K_p}{4}\right)(2+2K_p) - 8K_I}{(18-2K_p)/4}$

$x = (9-K_p)(1+K_p) - 8K_I$

at Marginally stable condition - coefficient of $s^1 = 0$. so -

$K_I = \frac{1}{8}(-K_p^2 + 8K_p + 9)$

$\frac{dK_I}{dK_p} = 0 = -2K_p + 8 \Rightarrow \boxed{K_p = 4}$

at $K_p = 4 \Rightarrow \boxed{K_I = 3.125}$

→ so value of K_I for system to be marginally stable is -

$\boxed{K_I = 3.125}$

18

Good Approach

Q.7 (b) The open loop transfer function of a unity feedback system is $G_p(s) = \frac{K}{s(s+2)}$. Design a lead compensator to have a velocity-error constant of $20s^{-1}$ and phase margin of at least 50°. Indicate each step that you are using.

[20 marks]

Step-1

Soln- Let the transfer function of lead compensator be -

$$G_c(s) = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

then overall transfer function of system will be -

$$G(s) = G_p(s)\, G_c(s)$$

$$= \frac{K\alpha}{s(s+2)} \frac{(1+Ts)}{(1+\alpha Ts)} \quad \text{—— (1)}$$

Step-2

given, velocity error constant (K_v)

$$\boxed{K_v = 20}$$

→ we know,

$$K_v = \lim_{s\to 0} s\, G(s)$$

$$20 = \lim_{s\to 0} s \left(\frac{K\alpha(1+Ts)}{s(s+2)(1+\alpha Ts)}\right)$$

$$20 = \frac{K\alpha}{2} \Rightarrow \boxed{K\alpha = 40} \quad \text{—— (2)}$$

Step-3

Given, Phase Margin = 50°

we know,

Phase Margin $= 180^\circ + \angle GH$ at ω_{gc}.

$50^\circ = 180^\circ + \angle GH$ at ω_{gc}.

$\angle GH$ at $\omega_{gc} = -130^\circ$

from equation - (1) -

$$-90^\circ - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) - \tan^{-1}(T\omega_{gc}) - \tan^{-1}(\alpha T\omega_{gc}) = -130^\circ.$$

$$\tan^{-1}\left(\frac{\omega_{gc}}{2}\right) + \tan^{-1}(T\omega_{gc}) + \tan^{-1}(\alpha T\omega_{gc}) = 40^\circ. \quad (3)$$

where, ω_{gc} = gain cross over frequency

In complete solution

8

Q.7 (c) A negative unity feedback control system is expected to meet the following specifications : damping ratio is 0.5, natural frequency is $\sqrt{10}$ rad/sec and the steady-state error is 10%. The open-loop transfer function is $G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$. Find the values of K, α and β.

[20 marks]

Sol^n The characteristic equation of given system will be -

$$q(s) = 1 + GH = 0 \Rightarrow 1 + \frac{K(S+\alpha)}{(S+B)^2} = 0$$

$$(S+B)^2 + K(S+\alpha) = 0$$

$$S^2 + 2BS + B^2 + KS + K\alpha = 0$$

$$S^2 + (2B+K)S + (B^2+K\alpha) = 0 \quad \text{——} \; (1)$$

→ comparing the given equation with standard 2nd order equation's char. equation we get -

$$\omega_n^2 = B^2 + K\alpha = (\sqrt{10})^2$$

$$B^2 + K\alpha = 10 \quad \text{——} \; (2)$$

$$2\xi\omega_n = 2B + K$$

$$2 \times 0.5 \times \sqrt{10} = 2B + K \Rightarrow 2B + K = \sqrt{10} \quad \text{——} \; (3)$$

given -

Steady state error = 10% = e_{ss}

→ given system is type - 0 system thus the given e_{ss} is for step input -

Do not write in this margin

hence -

$$0{\cdot}1 \times A = \frac{A}{1+K_p} \Rightarrow \boxed{K_p = 9}$$

where,

K_p = positional error constant.

$$K_p = \lim_{S \to 0} G(S) = \lim_{S \to 0} \frac{K(S+\alpha)}{(S+\beta)^2}$$

$$9 = \frac{K\alpha}{\beta^2} \Rightarrow 9\beta^2 = K\alpha \text{ ——— } (4)$$

putting equation - (4) in eq^n (2) we get

$$10\beta^2 = 10 \Rightarrow \boxed{\beta = \pm 1}$$

→ as given system is stable thus its poles should be on right side

then -

$\boxed{\beta = +1}$ Answer

from equation - (3) -

$$2\beta + K = \sqrt{10} \quad \text{at } \beta = 1.$$

$\boxed{K = 1{\cdot}162}$ Answer

18

from equation - (2) - .

$$\beta^2 + K\alpha = 10$$

$$(1)^2 + 1{\cdot}162\alpha = 10$$

Good Approach

$\boxed{\alpha = 7{\cdot}745}$ Answer

Q.8 (a) Draw the log-magnitude asymptotic plot for the transfer function.

$$G(s)H(s) = \frac{1000s}{(s+10)(s+100)}$$

Also find the gain cross-over frequency and the frequencies at 3 dB attenuation.

[20 marks]

Do not write in this margin

Q.8 (b) Consider the system with state equation

$$\dot{X}(t) = AX(t) + BU(t)$$

where $$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

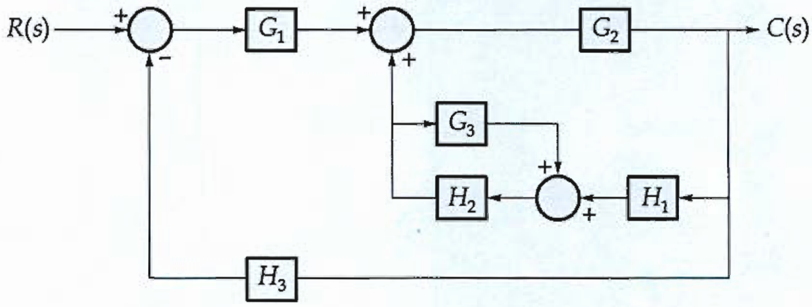
By using the state feedback control $u = -KX$, it is desired to have the closed-loop poles at

$$u_1 = -2 + j4, \; u_2 = -2 - j4, \; u_3 = -10$$

Determine the state feedback gain matrix K. Also check the validity of arbitrary pole placement.

[20 marks]

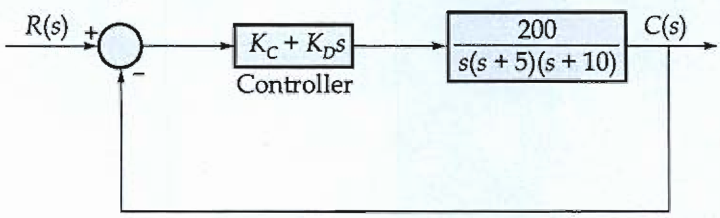
Q.8 (c) **(i)** For the block-diagram representation for a system shown below :



Draw the signal flow graph and determine the overall transfer function using Mason's gain formula.

(ii) A unity feedback system has plant transfer function $G_C(s) = \dfrac{200}{s(s+5)(s+10)}$.

The plant is controlled by a PD controller. Find the ranges of controller gains (K_C, K_D) for the system shown below to be stable. Also draw the region of stability.



[12 + 8 marks]

OOOO

Space for Rough Work

Space for Rough Work

$$\frac{10 \times j5}{10 + j5} - j4$$

$$\frac{10j}{2+j} - j4$$

$$10j - 8j + 4$$

$$\frac{4+2j}{2+j} = \frac{2 \times 4}{\cancel{6}3}$$

Space for Rough Work