Avoid calculation mistakes, strikeouts and draw diagrams with more visibility of notations..... Try to increase your font size of writting.... Overall your performance is good... Keep it up dear Hayat Ali...
All the best.

Review

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India's Best Institute for IES, GATE & PSUs

ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-1 : Network Theory + Control Systems [All Topics]

Name : ....

Roll No :

Test Centres	Student's Signature
Delhi ☐ Bhopal ☑ Jaipur ☐ Pune ☐ Kolkata ☐ Bhubaneswar ☐ Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	28
Q.2	28
Q.3	—
Q.4	—
Section-B	
Q.5	41
Q.6	30
Q.7	50
Q.8	—
Total Marks Obtained	177

Signature of Evaluator: Ch. Rafi

Cross Checked by ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

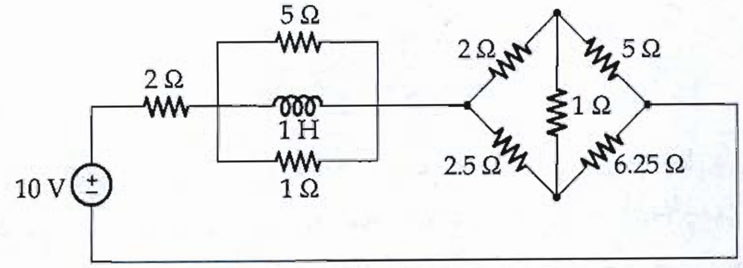
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Network Theory

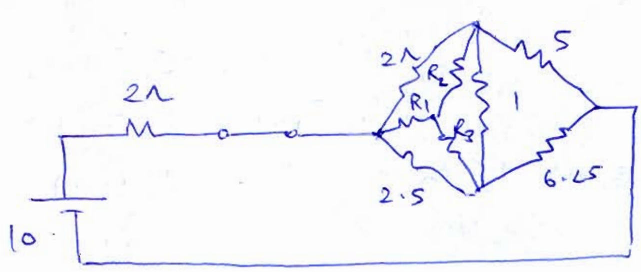
Q.1 (a) Consider the circuit shown below:



Calculate the power supplied by source.

[10 marks]

Sol^n Since source voltage is a DC source in steady state inductor will be short circuited the resultant network would be

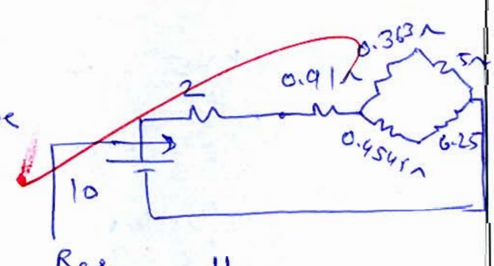


Star - Delta transformation

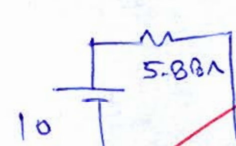
$$R_1 = \frac{2 \times 2.5}{2+2.5+1} = \frac{5}{5.5} = 0.91\,\Omega \qquad R_2 = \frac{2 \times 1}{2+2.5+1} = 0.363\,\Omega$$

$$R_3 = \frac{2.5 \times 1}{5.5} = 0.4545\,\Omega$$

Resultant circuit would be



$$R_{eq} = 2.91 + [\,5.363 \,\|\, 6.7045\,]$$
$$= 5.88\,\Omega$$

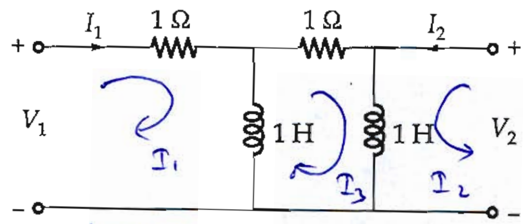


Power supplied by Source $= \dfrac{V^2}{R_{eq}}$

$$= \frac{100}{5.88\,\Omega}$$

$$\boxed{\text{Power} = 17.00 \text{ watt}}$$

Q.1 (b) Find the open circuit parameters for the two-port network shown below:



Loop 1 Loop 2 Loop 3

[10 marks]

Sol^n Open circuited parameters of a network is given by

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — (A)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (B)

KVL in loop 1

$V_1 = I_1 + s(I_1 - I_3)$

$V_1 = I_1[s+1] - sI_3$ — (1)

KVL in Loop 2

$I_3 + s(I_3 + I_2) + (I_3 - I_2) = 0$

$I_3[1+s+1] + I_2[s-1] = 0$

$I_3[s+2] + I_2[s-1] = 0$

$I_3 = \frac{-I_2(s-1)}{s+2}$

5

Putting value of I_3 in eq^n (1)

$V_1 = I_1(s+1) - s\left\{\frac{-I_2(s-1)}{s+2}\right\}$

$V_1 = I_1(s+1) + I_2\frac{s(s-1)}{s+2}$ — (3)

KVL in loop 3

$V_2 = s(I_2 + I_3)$ — (2)

Cal. error.

Putting I_3 value in eq^n (2)

$V_2 = sI_2 + s\frac{(-I_2)(s-1)}{s+2}$

$= I_2\left[s - \frac{s(s-1)}{s+2}\right]$

$= I_2\left[\frac{\cancel{s^2} + 2s - \cancel{s^2} + s}{s+2}\right]$

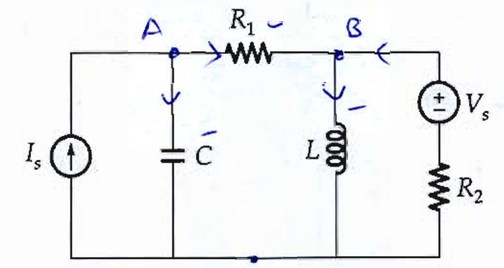
$V_2 = I_2\left[\frac{3s}{s+2}\right]$ — (4)

After comparing eq^n (3) and (4) with (A) & (B)

$Z_{11} = (s+1)\,\Omega$, $Z_{12} = \frac{s(s-1)}{s+2}\,\Omega$

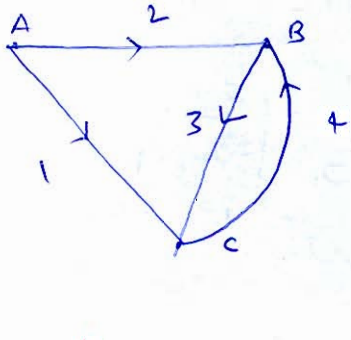
$Z_{21} = 0$ $\boxed{Z_{22} = \frac{3s}{s+2}\,\Omega}$

Q.1 (c) Draw the graph of the network shown in the figure below. How many trees are possible for this graph? Draw all the trees.



[10 marks]

Sol^n



current source and voltage sources are replaced by their internal impedance and three nodes are taken A,B,C as mentioned in network

Total number of trees possible for any graph is given by $[A][A^T]$ where $[A]$ matrix is reduced incidence matrix.

$$A' = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} +1 & +1 & 0 & 0 \\ 0 & -1 & +1 & -1 \\ -1 & 0 & -1 & +1 \end{bmatrix} \end{array}$$

for incidence matrix outgoing direction is taken as +ve and incoming direction is taken as negative.

→ To get "Reduced incidence matrix" from incidence matrix deleting last row.

7

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix}_{2\times 4}$$

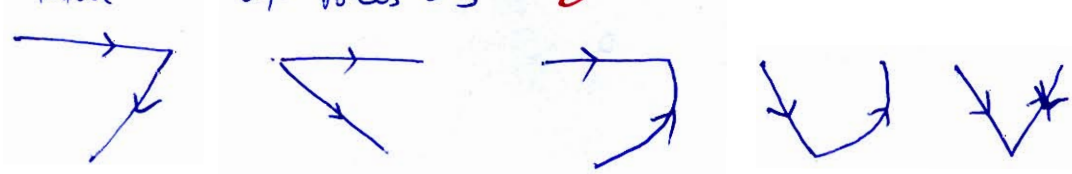
$$[A^T] = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}_{4\times 2}$$

$$[A][A^T] = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\det [A][A^T] = 6 - 1$$

$$= 5$$

total No. of trees = 5



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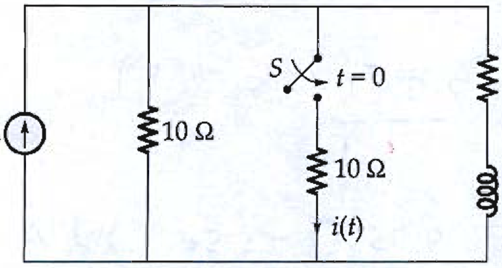
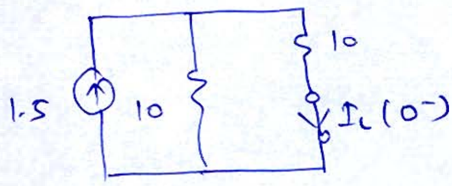
Q.1 (d) Consider the network shown below:



If switch S is closed at $t = 0$, calculate $i(t)$ for $t > 0$ by using Laplace transform approach.

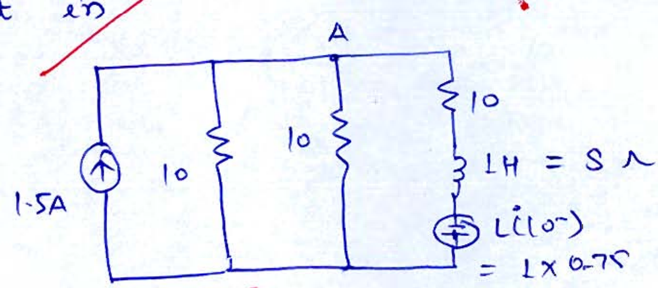
[10 marks]

Sol^n at $t = 0^-$ circuit is



$I_L(0^-) = \frac{1.5}{2} = 0.75A$

at $t = 0^+$ circuit is



$= 1 \times 0.75$

$= 0.75$ Volt

Let the voltage at node A is $V_1(s)$

Applying KVL at node A

$$1.5 = \frac{V_1(s)}{10} + \frac{V_1(s)}{10} + \frac{V_1(s) + 0.75}{10 + s}$$

$$= \frac{V_1}{5} + \frac{V_1(s)}{10+s} + \frac{0.75}{10+s}$$

$$1.5 - \frac{0.75}{10+s} = V_1\left[\frac{1}{5} + \frac{1}{s+10}\right]$$

$$\frac{15 + 1.5s - 0.75}{\cancel{s+10}} = V_1\left[\frac{s+10+5}{5\cancel{(s+10)}}\right]$$

$$1.5s + 14.25 = V_1\frac{(s+15)}{5}$$

$$\frac{7.5s + 71.25}{s+15} = V_1(s)$$

$$7.5\left[\frac{s + 9.5}{s+15}\right] = V_1(s)$$

~~$V_1(s) = \frac{7.5s}{s+15} + \frac{71.25}{s+15}$~~

~~Applying inverse LT~~

$$V_1(s) = 7.5\left[1 - \frac{5.5}{s+15}\right]$$

~~$V_1(s) = 7.$~~

Applying inverse LT

$$V_1(t) = 7.5\left[1 - 5.5e^{-15t}\right]$$

$$i'(t) = \frac{v_1(t)}{10}$$

$$= \frac{7.5\left[1 - 5.5e^{-15t}\right]}{10}$$

$$= 0.75\left[1 - 5.5e^{-15t}\right] A$$

$$= \left(0.75 - 4.125e^{-15t}\right) A$$

Q.1 (e) Realise Cauer II form of the function

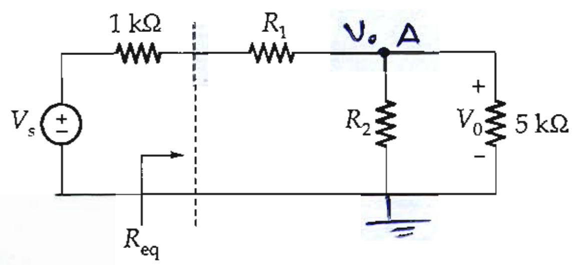
$$Z_{LC}(s) = \frac{s(s^4 + 3s^2 + 1)}{3s^4 + 4s^2 + 1}$$

[10 marks]

Q.1 (f) In a certain application, the circuit shown below must be designed to meet these two criteria:

(a) $\dfrac{V_0}{V_s} = 0.05$ (b) $R_{eq} = 39\ \text{k}\Omega$

If the load resistor 5 kΩ is fixed, find R_1 and R_2 to meet the criteria.



[10 marks]

Sol $R_{eq} = 39K$

$R_{eq} = R_1 + R_L \| 5K$

$39K = R_1 + \dfrac{(5K)R_L}{R_L + 5K}$

omitting K

$39 = R_1 + \dfrac{5R_L}{R_L+5}$

$39(R_L+5) = R_1R_L + 5R_1 + 5R_L$

$39R_L + 195 = R_1R_L + 5R_1 + 5R_L$

$34R_L + 195 = R_1[R_L+5]$

$\dfrac{34R_L+195}{R_L+5} = R_1$

2

Now Applying KCL at node A

$\dfrac{V_s - V_o}{1+R_1} = \dfrac{V_o}{R_L} + \dfrac{V_o}{5}$

$\dfrac{V_s}{1+R_1} = V_o\left[\dfrac{1}{R_L} + \dfrac{1}{5} + \dfrac{1}{R_1}\right]$

$\dfrac{V_s}{V_o} = (1+R_1)\left\{\dfrac{R_1+R_L}{R_1R_L} + \dfrac{1}{5}\right\}$

$\dfrac{1}{0.005} = (1+R_1)\left\{\dfrac{(R_1+R_L)5 + R_1R_L}{5R_1R_L}\right\}$

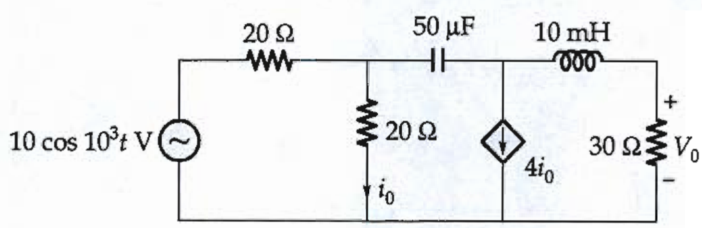
$200 \times 5R_1R_L = (1+R_1)\{5R_1 + 5R_L + R_1R_L\}$

Q.2 (a) Consider the circuit shown below and determine:

(i) V_0

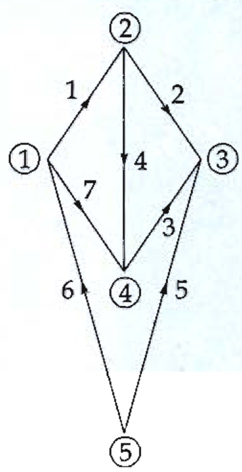
(ii) i_0

(iii) power factor between V_0 and i_0



[20 marks]

Q.2 (b) **(i)** Write the complete incidence matrix for the graph shown in the figure below. Find out how many trees are possible for the graph.



(ii) Draw the oriented graph corresponding to the reduced incidence matrix given by

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Also, find out how many tie sets are possible?

[10 + 10 marks]

For incidence matrix taking outgoing direction as positive

$$A' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & -1 & +1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

A' ↑ Incidence matrix

Possible no. of trees = det $[A A^T]$

where A = reduced incidence matrix deleting one row from A'

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$A A^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

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$$= \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

$$|A A^T| = 3\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 & -1 \\ 0 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} + 1\begin{vmatrix} -1 & 3 & -1 \\ 0 & -1 & 3 \\ -1 & -1 & -1 \end{vmatrix}$$

$$= 3 \times 16 + (-12) + (-12)$$

$$= 24$$

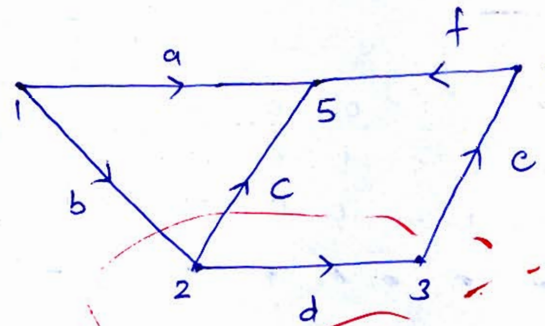
Total number of trees possible = 24

(ii) Reduced incidence matrix is given

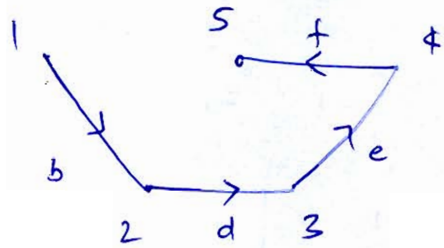
So incidence matrix is

$$A' = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Graph



tree



No. of nodes = 5

Number of branches in tree will be $= 5-1$
$= 4$

twigs = 4

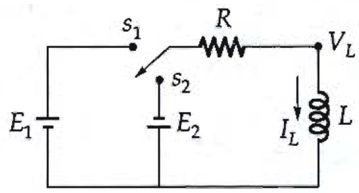
No. of links = b − twigs (b = branches)

= 6 − 4

= 2

So total tie sets possible = 2

Q.2 (c) For the initially relaxed circuit shown below, the switch is closed on to position s_1 at time $t = 0$ and changed to position s_2 at time $t = 0.5$ ms.

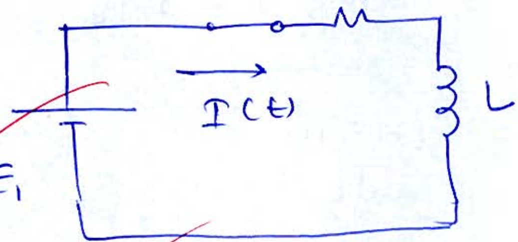


$E_1 = 100$ V; $E_2 = 50$ V
$R = 100\ \Omega$
$L = 0.2$ H

Obtain the equation for inductor current and voltage across the inductor in both the intervals and sketch the transients.

[20 marks]

at $t = 0^+$ ckt is



$$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

$$= \frac{E_1}{R} + \left[0 - \frac{E_1}{R}\right]e^{-tR/L}$$

$$= \frac{E_1}{R}\left[1 - e^{-Rt/L}\right]$$

$$= \frac{100}{100}\left[1 - e^{-100t/0.2}\right] \Rightarrow 1 - e^{-500t}\ A$$

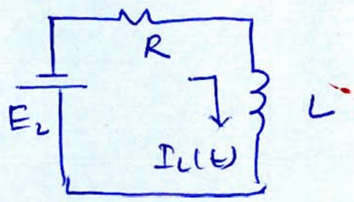
$$V_L(t) = 0.2 \times 500\, e^{-500t}$$
$$= 100\, e^{-500t}$$

at $t = 0.5$ ms $I(t)$ is

$$I(t) = 1 - e^{-500 \times 0.5 \times 10^{-3}}$$

$$I(0.5ms) = 0.221 \text{ A}$$

at $t = 0.5$ ms ckt changes to



$$I_L(t) = I_L(\infty) + [I_L(0.5ms) - I_L(\infty)]e^{-(t-0.5)/\tau}$$

$$\tau = L/R = \frac{0.2}{100}$$

$$I_L(t) = -\frac{E_L}{R} + \left[0.221 - \frac{(-E_L)}{R}\right]e^{-500(t-0.5)}$$

$$= -\frac{50}{100} + \left[0.221 + \frac{50}{100}\right]e^{-500(t-0.5)}$$

$$I_L(t) = -0.5 + 0.721\, e^{-500(t-0.5)} \text{ A}$$

where t is in millisec

$$\boxed{I_L(t) = -0.5 + 0.721 e^{-500t+250}}$$

$$V_L(t) = \frac{L\, dI_L(t)}{dt}$$

$$= (0.2)\left[0.721(-500)e^{-500t+250}\right]$$

$$\boxed{V_L(t) = -72.1\, e^{-500t+250} \text{ Volts}}$$

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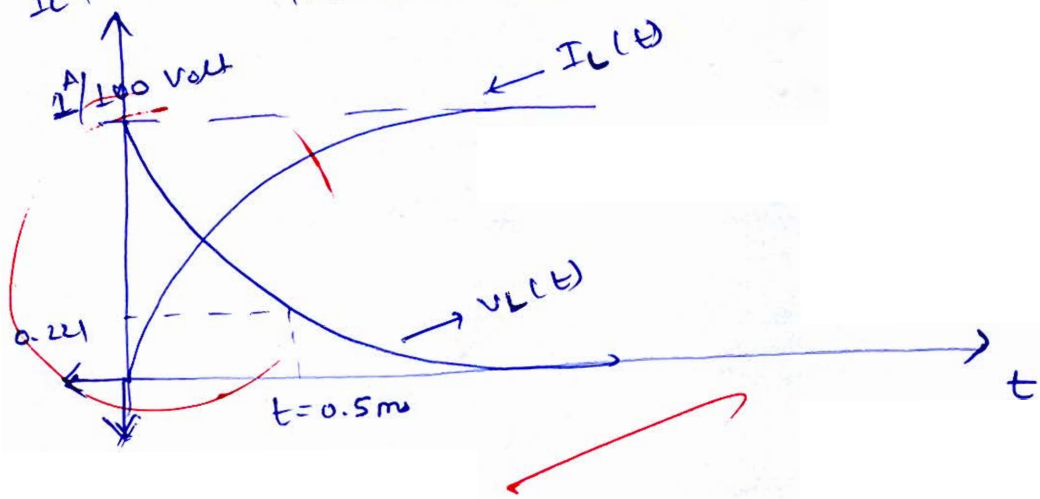
for $0 < t < 0.5$ ms



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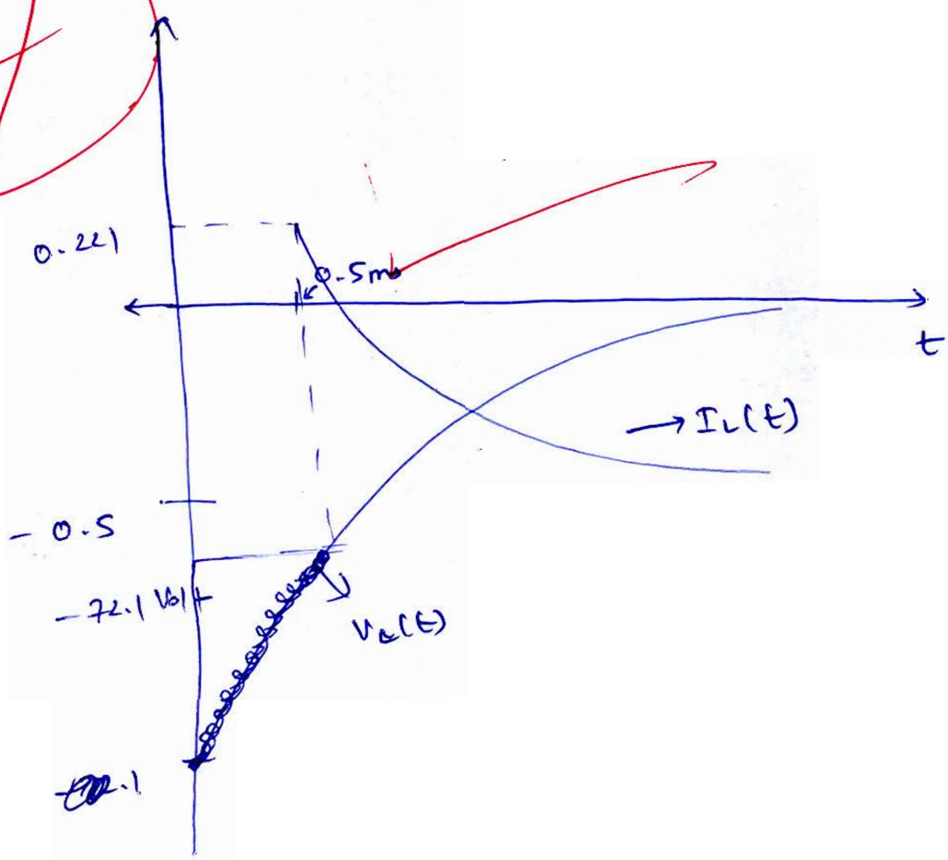
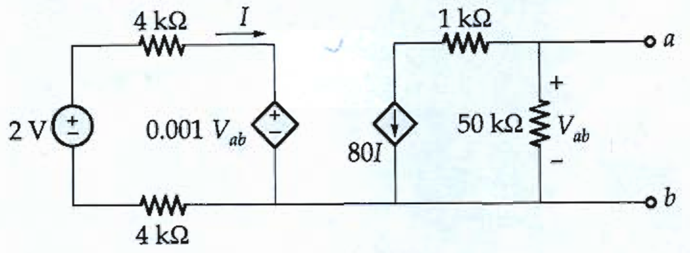
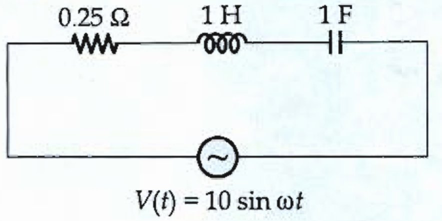
for $t \geq 0.5$



Q.3 (a) (i) Obtain Norton's equivalent at terminals *a*-*b* of the circuit.



(ii) For given RLC series circuit,



Find:

1. Resonant frequency 'f_0'.
2. Damping ratio 'ξ'.
3. Maximum possible voltage across the inductor.

[10 + 10 marks]

Q.3 (b) **(i)** Using Foster form I, synthesize the function

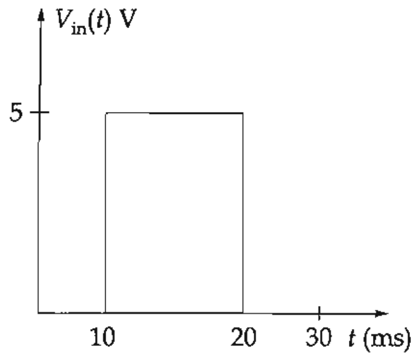
$$Z(s) = \frac{s(s^2+9)}{(s^2+5)(s^2+13)}$$

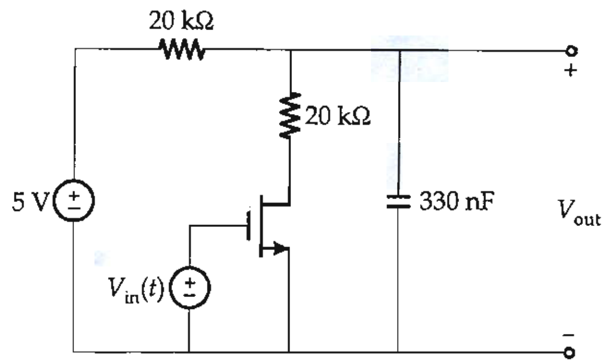
(ii) Using Foster form II, synthesize the function

$$Y(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)}$$

[10 + 10 marks]

Q.3 (c) Consider the network shown below:

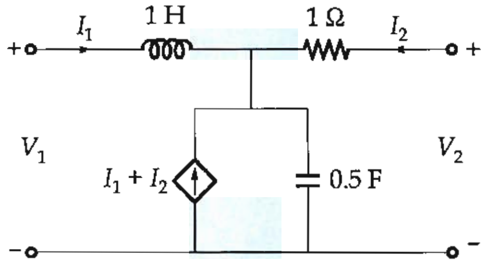




Draw the plot for $V_{out}(t)$ for $t > 0$.

[20 marks]

Q.4 (a) Determine the transmission parameters matrix for the two port network shown below.



[20 marks]

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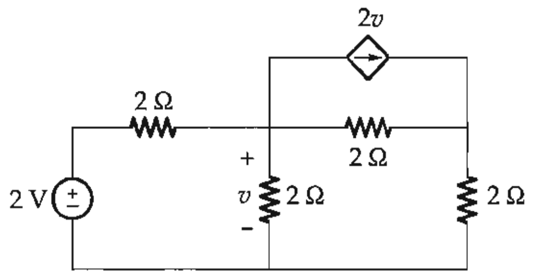
Q.4 (b) For the network shown in figure below, write down the f-cutset matrix and obtain the network equilibrium equation in matrix form using KCL and calculate v.



[20 marks]

Q.4 (c) **(i)** Derive expression for frequencies for maximum voltage across inductor in series RLC resonant circuit.

(ii) Calculate the maximum voltage across the inductor using result of **Q.4 (c) (i)** with constant voltage and variable frequency. Assume $R = 50\ \Omega$, $L = 0.05\text{H}$, $C = 20\ \mu\text{F}$ and $V = 100$ V.

[10 + 10 marks]

Section B : Control Systems

Q.5 (a) Consider a negative feedback system having the characteristic equation

$$1+\frac{K}{(1+s)(1.5+s)(2+s)}=0$$

It is desired that all the roots of the characteristic equation have real parts less than -1. Extend the Nyquist stability criterion to find the largest value of K satisfying the condition.

[10 marks]

Sol^n

$CE \Rightarrow (1+s)(1.5+s)(s+2) + k=0$

$(1.5+s+1.5s+s^2)(s+2)+k=0$

$(1.5+2.5s+s^2)(s+2)+k=0$

$s^3+2.5s^2+1.5s+2s^2+5s+3+k=0$

Putting $s = s-1$

$(s-1)^3 + 4.5(s-1)^2 + 6.5(s-1) + 3+k=0$

$s^3-1-3s^2+3s+4.5[s^2+1-2s]+6.5s-6.5+3+k=0$

$s^3-3s^2+3s-1+4.5s^2+4.5-9s+6.5s-3.5+k=0$

$s^3+1.5s^2+0.5s+k=0$ — (A)

~~Now for the roots of~~

From eq^n (A) finding open loop eq^n

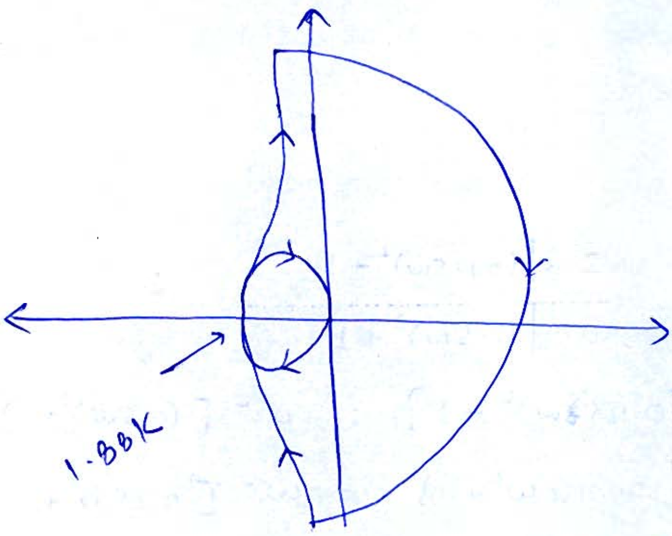
$$G' = \frac{k}{s(s^2+1.5s+0.5)}$$

Now drawing Nyquist

at $\omega=0$

$M=\infty$

Above system is type-1 and 3 order system so its ~~eq~~ Nyquist plot would be



~~Phase M~~

$$\phi = -90^\circ - \tan^{-1}\frac{1.5\omega}{0.5-\omega^2}$$

$$-180^\circ = -90 - \tan^{-1}\frac{1.5\omega}{0.5-\omega^2}$$

$$90 = \tan^{-1}\frac{1.5\omega}{0.5-\omega^2}$$

$$0.5-\omega^2 = 0$$

$$\omega^2 = 0.5$$

$$\omega_{pc} = \sqrt{0.5}$$

$$M = \frac{K}{\omega\sqrt{(1.5\omega)^2+(0.5-\omega^2)^2}}$$

at $\omega = \sqrt{0.5}$

$$M = \frac{K}{\sqrt{0.5}\sqrt{0.5625}}$$

$$|M| = 1.88K$$

Now in order for root be less than -1 Nyquist plot should encircle $(-1+0j)$ 0 times

For it to happen

$$|M| > -1$$

or

$$|M| < 1$$

$$1.88K < 1$$

$$\boxed{K < 0.53}$$

Q.5 (b) The loop transfer function of a negative feedback control system is given by

$$G(s)H(s) = \frac{2e^{-0.5s}(0.125s+1)}{s[0.5s+1]}$$

Determine the possible maximum phase margin and the frequency at which it occurs.

[10 marks]

Sol^n

$$M = \frac{2\sqrt{(0.125\omega)^2+1}}{\omega\sqrt{(0.5\omega)^2+1}} = 1$$

$$4\left[(0.125\omega)^2+1\right] = \omega^2\left[(0.5\omega)^2+1\right]$$

$$4\left[0.0156\omega^2+1\right] = \omega^2\left[0.25\omega^2+1\right]$$

let $\omega^2 = x$

$$0.0625x + 4 = 0.25x^2 + x$$

$$0.25x^2 + 0.9375x - 4 = 0$$

$$x^2 + 3.75x - 16 = 0$$

$$x = 2.54, \; -6.29$$

$$\omega^2 = 2.54$$

$$\boxed{\omega_{gc} = 1.6}$$

$$\phi = -0.5\omega + \tan^{-1}0.125\omega - 90^\circ - \tan^{-1}0.5\omega$$

$$= 0.8 + 0.1979 - 1.57 - 0.6742$$

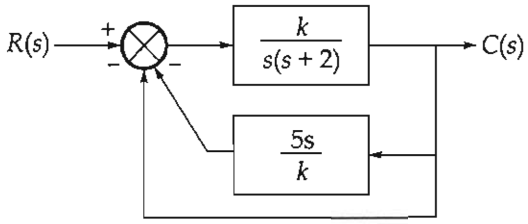
$$= -1.2463 \text{ rad}$$

$$\phi = -71.40^\circ$$

$$\boxed{PM = 180 - 71.40^\circ = 108^\circ}$$

3

Q.5 (c) Consider the control system shown below:



Design the value of k so that for an input of $100tu(t)$, there will be a 0.01 error in the steady state.

[10 marks]

Sol^n

$$e_{ss} = \frac{100}{K_V} = \frac{1}{100}$$

$$\boxed{K_V = 10^4}$$

$$K_V = \lim_{s \to 0} sG'(s)$$

Now $G_1(s) = \frac{K}{s(s+2)}$ $\quad H_1(s) = \frac{5s}{K}$

$$G' = \frac{G_1}{1+G_1H_1} = \frac{K/s(s+2)}{1+\frac{\cancel{K}}{\cancel{s}(s+2)}\cdot\frac{5\cancel{s}}{\cancel{K}}} = \frac{K}{s(s+2+5)} = \frac{K}{s(s+7)}$$

$$G' = \frac{K}{s(s+7)}$$

$$K_V = \lim_{s \to 0} \frac{\cancel{s}K}{\cancel{s}(s+7)} = \frac{K}{7}$$

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Now

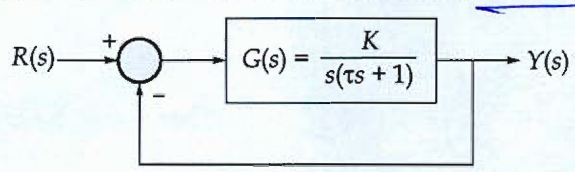
$$\frac{K}{7} = 10^4$$

$$\boxed{K = 7 \times 10^4}$$

Q.5 (d) For the feedback control system shown in figure, select the parameters K and τ so that the following time-domain specifications will be satisfied:

(i) Peak overshoot of the response to a step input is 5% and

(ii) The settling time to within 2% of the final value is 4 seconds.



[10 marks]

(i) $M_p = 5\%$

$e^{-\pi \cot\theta} = 0.05$

$\cot\theta = 0.9535$

$\theta = \cancel{1.31}\ 0.81$ rad

$\eta = \cos\theta$

$\boxed{\eta = 0.69}$

(ii) $T_s(2\%) = \dfrac{4}{\eta\omega_n}$

$4 = \dfrac{4}{\eta\omega_n}$

$\eta\omega_n = 1$

$\omega_n = 1/\eta$

$\boxed{\omega_n = 1.45}$

from (1)

$(1.45)^2 = K/\tau$ —(3)

from (2)

$(0.69)^2 = \dfrac{1}{4} \times \dfrac{1}{K\tau}$ —(4)

$CE = \tau s^2 + s + K$

$= \tau\left[s^2 + \dfrac{s}{\tau} + \dfrac{K}{\tau}\right]$

$\omega_n = \sqrt{K/\tau}$ —(1)

$2\eta\omega_n = 1/\tau$

$\eta = \dfrac{1}{2\omega_n\tau}$

$\eta = \dfrac{1}{2\tau}\sqrt{\dfrac{\tau}{K}}$ —(2)

$= \dfrac{1}{2\sqrt{K\tau}}$ —(2)

Multiply (3) and (4)

$1 = \dfrac{1}{4} \times \dfrac{K}{\tau} \times \dfrac{1}{K\tau}$

$\boxed{\tau = 0.5}$

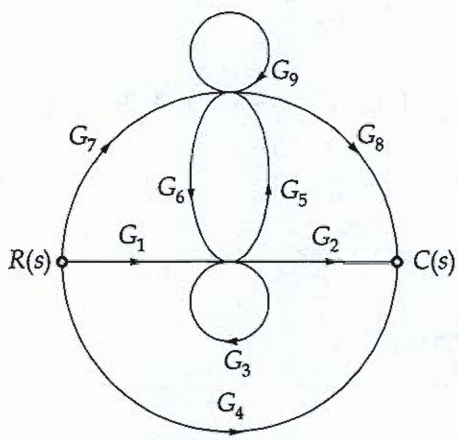
$K = \tau(1.45)^2$

$\boxed{K = 1.05128}$

5

4

Q.5 (e) Find $\frac{C(s)}{R(s)}$ using Mason's gain formula for the following system with the signal flow graph shown below:



[10 marks]

forward paths are

$P_1 = G_1 G_2$; $P_2 = G_7 G_8$; $P_3 = G_4$; $P_4 = G_7 G_6 G_2$

$P_5 = ?\ G_1 G_5 G_8$

Loops

$L_1 = G_9$ $\quad L_2 = G_5 G_6$ $\quad L_3 = G_3$

$\Delta = 1 -$ { summation of loop gain } + { summation of product of 2 non-touching loop gain }

$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$

$\Delta_k =$ Putting all loop gain to zero if they are touching k^{th} forward path

$$TF = \frac{\sum_{k=1}^{4} \Delta_k P_k}{\Delta}$$

5

$\Delta_1 = 1 - L_1$ $\quad\quad$ $\Delta_3 = 1 - (L_1 + L_2 + L_3) + L_1 L_3$

$\Delta_2 = 1 - L_3$ $\quad\quad$ $\Delta_4 = 1$

$$TF = \frac{\Delta_1 P_1 + \Delta_2 P_2 + \Delta_3 P_3 + \Delta_4 P_4}{\Delta}$$

$$= \frac{(1 - L_1) G_1 G_2 + (1 - L_3) G_7 G_8 + (1 - (L_1 + L_2 + L_3) + L_1 L_3) G_4 + G_7 G_6 G_2}{1 - (L_1 + L_2 + L_3) + L_1 L_3}$$

$$= \frac{(1 - G_9) G_1 G_2 + (1 - G_3) G_7 G_8 + (1 - G_9 - G_5 G_6 - G_3 + G_9 G_3) G_4 + G_7 G_6 G_2}{1 - G_9 - G_5 G_6 - G_3 + G_9 G_3}$$

$$TF = \frac{G_1 G_2 - G_1 G_2 G_9 + G_7 G_8 - G_3 G_7 G_8 + G_4 - G_4 G_9 - G_5 G_6 G_4 - G_3 G_4 + G_9 G_9 G_3 + G_7 G_6 G_2}{1 - G_9 - G_5 G_6 - G_3 + G_9 G_3}$$

Q.5 (f) A unity negative feedback system has open-loop transfer function

$$G(s) = \frac{K}{s(s+2)}$$

The system is modified to include a forward path zero at $s = -6$. What is the value of K, if steady state error for input $r(t) = 2t\,u(t)$ to the modified system is 0.1?

[10 marks]

Sol^n $G(s) = \dfrac{K}{s(s+2)}$

Modified system

$G'(s) = \dfrac{K(s+6)}{s(s+2)}$

e_{ss} for ramp with slope 2 $= \dfrac{2}{K_v}$

$K_v = \lim\limits_{s \to 0} sG'(s)$

$= \lim\limits_{s \to 0} \dfrac{\cancel{s}\,K(s+6)}{\cancel{s}(s+2)}$

$= \dfrac{K(6)}{2}$

$K_v = 3K$

$e_{ss} \Rightarrow \dfrac{2}{3K} = 0.1$

9

$20 = 3K$

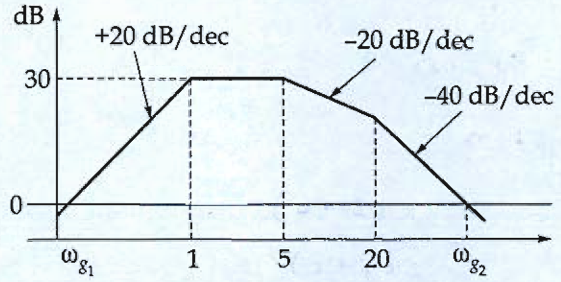
$\boxed{\dfrac{20}{3} = K}$

Q.6 (a) Consider a minimum phase system whose asymptotic amplitude frequency response is shown in figure below.

(i) Determine the transfer function $G(s)$ of the system.

(ii) Determine the two gain crossover frequencies ω_{g1} and ω_{g2}.

(iii) Determine the phase margin at ω_{g2}.



[10 + 5 + 5 marks]

Soln initial slope + 20dB/dec

→ Zero at origin

$\omega_1 = 1$ $\omega_2 = 5$ $\omega_3 = 20$

Pole Pole Pole

Because slope is decreasing by -20dB/dec

$$\boxed{TF = \frac{KS}{\left(\frac{S}{1}+1\right)\left(\frac{S}{5}+1\right)\left(\frac{S}{20}+1\right)}}$$

(ii) Slope = + 20dB/dec

$$\frac{30-0}{\log_{10} 1 - \log_{10} \omega_{g_1}} = 20 \text{ dB/dec}$$

$$\frac{30}{20} = \log_{10} \frac{1}{\omega_{g_1}}$$

$$10^{1.5} = \frac{1}{\omega_{g_1}}$$

$\boxed{\omega_{g_1} = 0.031}$ rad/sec.

~~Ans~~

$$\frac{30 - x}{\log_{10} 5 - \log_{10} 20} = -20$$

$$\frac{x - 30}{20} = \log_{10} 5/20$$

$$x - 30 = -12.0411$$

$\boxed{x = 17.95}$ at $\omega = 20$

$$\frac{17.95 - 0}{\log_{10} 20 - \log \omega_{g_2}} = -40$$

$$-0.4489 = \log_{10} \frac{20}{\omega_{g_2}}$$

$$0.355 = \frac{20}{\omega_{g_2}}$$

$\boxed{\omega_{g_2} = 56.338}$ rad/sec.

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(iii) $PM = 180 + \phi \Big|_{\text{at } \omega_{gc}}$

$\omega_{gc} = \omega_{g_1}$ and ω_{g_2}

$= 0.031$ and 56.338

For $\omega_{g_2} = 56.338$

$$\phi = 90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{5} - \tan^{-1}\frac{\omega}{20}$$

$$= 90° - \tan^{-1} 56.338 - \tan^{-1}\frac{56.338}{5} - \tan^{-1}\frac{56.338}{20}$$

$$= 90° - 88.98 - 84.928 - 70.455$$

$$= -154.363°$$

$PM = 180 - 154.363$

$= 25.637°$

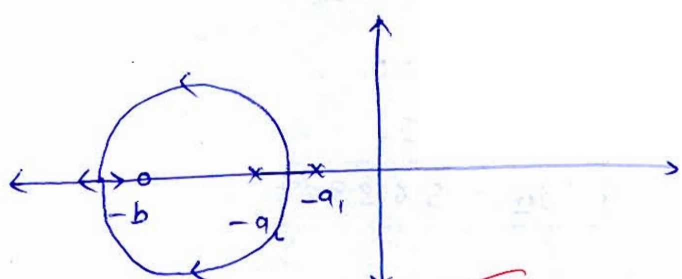
Q.6 (b) Prove that a combination of two poles $s = -a_1$ and $s = -a_2$ and one zero $s = -b$ to the left of both of the poles on the real axis, results in a root locus whose complex root branches form a circle centered at the zero with radius given by $\sqrt{(b-a_1)(b-a_2)}$. Sketch the root locus plot with the gain (K) varying from 0 to ∞.

[20 marks]

Sol^n

Poles $= -a_1, -a_2$

Zeros $= -b$



$$TF = \frac{K(s+b)}{(s+a_1)(s+a_2)}$$

Put $s = \sigma + j\omega$

$$= \frac{K(\sigma + j\omega + b)}{(\sigma + j\omega + a_1)(\sigma + j\omega + a_2)}$$

6

For $\sigma + j\omega$ to lie on Root locus it must satisfy magnitude cond^n

$$M = \frac{K\sqrt{(\sigma+b)^2+\omega^2}}{\sqrt{(\sigma+a_1)^2+\omega^2}\sqrt{(\sigma+a_2)^2+\omega^2}} = 1$$

$$(\sigma+b)^2 + \omega^2 = [(\sigma+a_1)^2+\omega^2][(\sigma+a_2)^2+\omega^2]$$

$$\sigma^2 + b^2 + 2\sigma b + \omega^2 = [\sigma^2 + a_1^2 + 2\sigma a_1 + \omega^2][\sigma^2 + a_2^2 + 2\sigma a_2 + \omega^2]$$

Do not write in this margin

$$\sigma^2 + b^2 + 2\sigma b + \omega^2 = \sigma$$

Q.6 (c) A control system is represented by the state equation given below,

$$\dot{x}(t) = Ax(t)$$

The response of the system is $x(t) = \begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ when $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $x(t) = \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$ when $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Calculate the system matrix A and the state transition matrix for the system.

[20 marks]

Sol^n

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

when $x(t) = \begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\cancel{\dot{x}(t) = Ax(t)}$

For $x(t) = \begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ eigen value $= 1$

For $x(t) = \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$ eigen value $= 2$

So $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

state transition matrix

$\phi(t) = L^{-1}[sI - A]^{-1}$

$= L^{-1}\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}\right]^{-1}$

$= L^{-1}\begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}^{-1}$

$= L^{-1}\left\{\frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}\right\}$

$= L^{-1}\begin{bmatrix} 1/s+1 & 0 \\ 0 & 1/s+2 \end{bmatrix}$

$$\boxed{\phi(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}}$$

Do not write in this margin

Q.7 (a) (i) The forward transfer function of a unity negative feedback type 1, second order system has a pole at –2 and zero at –2.5. If frequency of oscillation is $\sqrt{10}$ rad/s then determine:

1. Damping ratio
2. Damping factor
3. Steady state value, when input $r(t) = (5 + t)u(t)$ is applied.

(ii) For unit-step input to the second order system, define the following characteristics:

1. Rise time
2. Peak time
3. Peak overshoot
4. Settling time

[15 + 5 marks]

(i) Pole $= -2, 0$ zero $= -2.5$

$$G = \frac{s + 2.5}{s(s+2)}$$

$$CE = s^2 + 2s + s + 2.5$$

$$= s^2 + 3s + 2.5$$

$\omega_n^2 = 2.5$

$\boxed{\omega_n = 1.56}$

$2\eta\omega_n = 3$

$\eta = \dfrac{3}{3.16}$

$\boxed{\eta = 0.9486}$

(i) Pole = $-2, 0$ zero = -2.5

$$G = \frac{K(s+2.5)}{s(s+2)}$$

$$CE = s^2 + 2s + Ks + 2.5K$$
$$= s^2 + s[2+K] + 2.5K$$

$\omega_n = \sqrt{10}$ {given}

$\omega_n^2 = 10$

$2.5K = 10$

$$\boxed{K = 4}$$

$2\eta\omega_n = 2+K$

$2\eta \times \sqrt{10} = 6$

$\eta = \frac{3}{\sqrt{10}}$

$$\boxed{\eta = 0.9486}$$

$$TF = \frac{K(s+2.5)}{s^2 + s(2+K) + 2.5K} = \frac{4(s+2.5)}{s^2+6s+10}$$

For i/p 5 y(s) is

$$Y(s) = \left.\frac{20(s+2.5)}{s^2+6s+10}\right| \text{at } s=0$$

$$= \frac{50}{10} = 5$$

For i/p $x(t) = t\,u(t)$

$X(s) = 1/s^2$

$$Y(s) = \frac{4(s+2.5)}{s^2(s^2+6s+10)}$$

at $t = \infty$

$$y(t) = \lim_{s\to 0} \frac{s\,4(s+2.5)}{s^2(s^2+6s+10)}$$

$y(t) = \infty$

For i/p $(5+t)\,u(t)$ $\underline{y(t) = \infty}$

(ii) Rise time :- Time taken by system to reach from 0% to 100% in underdamped system ; 5% to 95% in critically damped system and 10% to 90% in over damped system is called rise time.

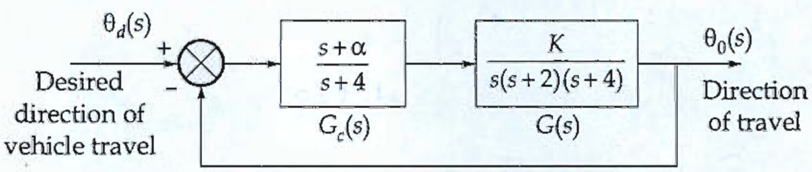
Do not write in this margin

(ii) Peak time :- Time taken by system for first overshoot to occur is called peak time.

Peak overshoot :- Difference between the peak value of system and steady state value of system called peak overshoot.

Settling time :- Time required by system to settle its output value within 2% tolerance is called settling time.

Q.7 (b) Consider the block diagram which form a scheme for controlling (automatically) the direction of travel of a road vehicle. The controller (a compensating network) is represented by $G_c(s)$ and engine-vehicle dynamics by $G(s)$.



(i) Determine the necessary conditions linking α, the controller parameter and gain K of the engine vehicle part for the overall system to be stable.

(ii) Also suggest suitable values of α and K, while assuring that steady-state error due to unit ramp direction input $\left[\theta_d(s) = \frac{1}{s^2}\right]$ is no more than 20%.

[20 marks]

$$G' = G_c(S)\,G(S)$$

$$= \frac{s+\alpha}{s+4} \times \frac{K}{s(s+2)(s+4)}$$

$$CE = s(s+4)(s+2)(s+4) + K(s+\alpha)$$

$$= s[s+4][s^2+6s+8] + Ks + \alpha K$$

$$= (s^2+4s)(s^2+6s+8) + Ks + \alpha K$$

$$= s^4 + 6s^3 + 8s^2 + 4s^3 + 24s^2 + 32s + Ks + \alpha K$$

$$= s^4 + 10s^3 + 32s^2 + (32+K)s + \alpha K$$

s^4 1 32 αK

s^3 10 $32+K$ 0

s^2 $\frac{320-32-K}{10}$ αK 0

s $\dfrac{\left(\frac{288-K}{10}\right)(32+K) - 10\alpha K}{\frac{288-K}{10}}$ 0

1 0

$\frac{288-K}{10} > 0$

$\boxed{K < 288}$

$288 \times 32 + 288K - 32K - K^2 - 100\alpha K > 0$

$9216 + 256K - K^2 - 100\alpha K > 0$

$$\boxed{\frac{9216 + 256K - K^2}{100K} > \alpha}$$

(ii) $e_{ss} \le \frac{1}{5}$

e_{ss} for ramp i/p $= \frac{1}{K_v}$

$K_v = \lim_{s \to 0} s\,G'(s)$

$= \lim_{s \to 0} \frac{\cancel{s}(s+\alpha)K}{\cancel{s}(s+2)(s+4)^2}$

$= \frac{\alpha K}{(2)(16)}$

$\Rightarrow \frac{1}{K_v} \le \frac{1}{5}$ $\rightarrow$ let $K = 150$ then

$K_v \geq 5$

$\frac{\alpha K}{32} \geq 5$

$\alpha K \geq 160$

$\alpha \geq \frac{160}{150}$

$\alpha \geq 1.066$

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For $k = 150$ for stability

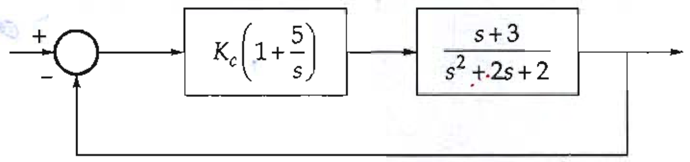
$$\alpha < \frac{9216 + 256k - k^2}{100k}$$

$$\alpha < 1.6744$$

hence $\alpha = 1.066$ satisfy the condition

So $\boxed{K = 150 \quad \alpha = 1.066}$

Q.7 (c) (i) Consider the closed loop system shown below. A PI controller controls a second order plant. Determine the range of K_c for which the closed-loop poles satisfy $\text{Re}(s) < -2$.



(ii) A unity-negative feedback control system has an open-loop transfer function,

$$G(s) = \frac{K}{s(s+1)(s+2)}; K \geq 0$$

Sketch the root locus plot of the system, explicitly identifying the centroid, the asymptotes, the breakaway points and $\pm j\omega$ axis crossover points.

[10 + 10 marks]

Sol^n $G_1 = K_c\left(1 + \frac{5}{s}\right)$ $\quad G_2 = \frac{s+3}{s^2+2s+2}$

$$G' = G_1 G_2$$
$$= K_c\left\{1 + \frac{5}{s}\right\}\left\{\frac{s+3}{s^2+2s+2}\right\}$$

$$CE = 1 + G'$$
$$= s(s^2+2s+2) + K_c[s+5][s+3]$$
$$CE = s^3 + 2s^2 + 2s + K_c[s^2 + 8s + 15]$$
$$= s^3 + s^2[2 + K_c] + s[2 + 8K_c] + 15K_c$$

Put $s = s-2$ in CE

$$(s-2)^3 + (s-2)^2[2+K_c] + (s-2)[2+8K_c] + 15K_c = 0$$

$$s^3 - 8 - 6s^2 + 12s + (s^2+4-4s)(2+K_c) + 2s + 8K_c s - 4 - 16K_c$$
$$+15K_c = 0$$

$$s^3 - 6s^2 + 12s - \cancel{8} + 2s^2 + \cancel{8} - 8s + K_c s^2 + 4K_c - 4K_c s + s[2+8K_c]$$
$$-4 - K_c = 0$$

$$s^3 + s^2[K_c - 4] + s[12 - 8 - 4K_c + 2 + 8K_c] + 4K_c - 4 - K_c = 0$$

$$s^3 + s^2(K_c - 4) + s[6 + 4K_c] + 3K_c - 4 = 0$$

s^3	1	$6+4K_c$
s^2	K_c-4	$3K_c-4$
s	$\dfrac{(K_c-4)(6+4K_c) - 3K_c + 4}{K_c-4}$	
1	$3K_c-4$	

$K_c - 4 > 0$ $\qquad$ $3K_c - 4 > 0$

$\boxed{K_c > 4}$ — (1) $\qquad$ $\boxed{K_c > 4/3}$ — (2)

$$6K_c - 24 + 4K_c^2 - 16K_c - 3K_c + 4 > 0$$

$$4K_c^2 - 13K_c - 20 > 0$$

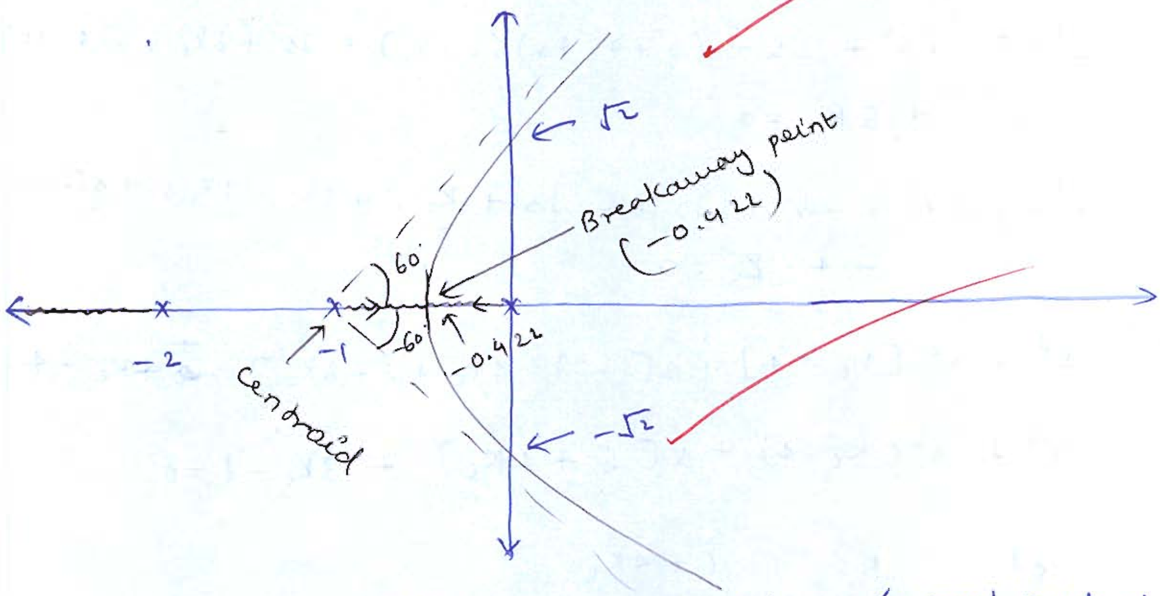
$K_c \quad 4.38 \;,\; -1.1$

So $K_c < -1.1$ or $K_c > 4.38$ — (3)

from eqn (1) (2) and (3)

$$\boxed{K_c > 4.38}$$

$$G(s) = \frac{K}{s(s+1)(s+2)} \qquad K \geq 0$$



$$\text{Centroid} = \frac{\Sigma \text{ Real part of poles} - \Sigma \text{ real part of zeros}}{P - Z}$$

$$= \frac{0 - 1 - 2}{3} = -1$$

→ asymptote angle

$$\phi = \pm \frac{(2q+1)180}{P - Z}$$

$$= 60, 180, 300°$$

→ RH table

s^3	1	2
s^2	3	k
s	$\frac{6-k}{3}$	0
1	k	

$\boxed{k = 6}$

$3s^2 + k = 0$

$3s^2 + 6 = 0$

$s^2 = -2$

$s = \pm j\sqrt{2}$

$\boxed{\omega = \sqrt{2}}$

→ Breakaway point

$CE = s[s^2 + 3s + 2] + k$

$\Rightarrow s^3 + 3s^2 + 2s + k = 0$

$$-k = s^3 + 3s^2 + 2s$$

$$-\frac{dk}{ds} = 3s^2 + 6s + 2 = 0$$

$$s = -0.422, \ -1.577$$

So Breakaway point $= -0.422$

doesn't lie on root locus

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Do not write in this margin

a) A unity-negative feedback system is characterized by the open-loop transfer function

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

(i) Determine the steady-state errors to $2u(t)$, $5t\,u(t)$ and $5t^2u(t)$ input [$u(t)$ is step input].

(ii) Determine rise time, peak time, peak overshoot and settling time ($\pm2\%$) of the unit-step response of the system.

[8 + 12 marks]

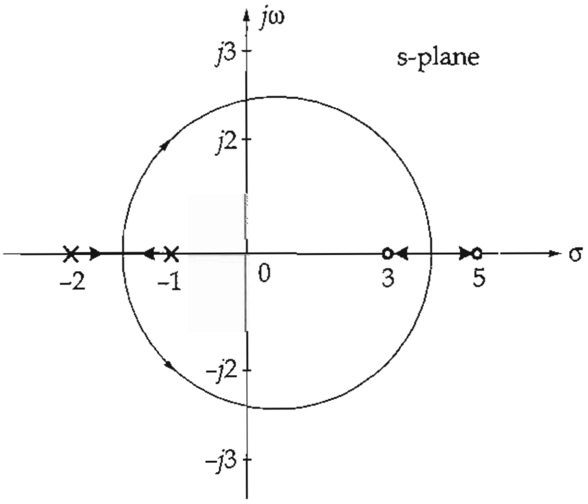
b) The open loop transfer function of a unity negative feedback control system is given by

$$G(s) = \frac{12}{s(s+1)}$$

The system is to have 6.5% first peak undershoot when compensated by tachometer feedback. Determine the tachometer feedback constant K_t.

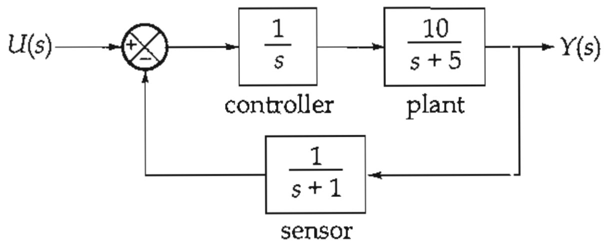
[20 marks]

Q.8 (c) **(i)** The root locus plot for the certain control system is shown below:



Find the break-away and break-in points for the above root locus plot.

(ii) Obtain a state-space model of the system shown in figure below:



[10 + 10 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

$$\begin{array}{r} 1 \\ s+15 \overline{) s+9.5} \\ \underline{s+15} \\ -5.5 \end{array}$$

$$1 - \frac{5.5}{s+15}$$

$180° \rightarrow \pi r$

$\frac{180}{\pi} x \rightarrow 1$