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Detailed Solutions

ESE-2023 Mains Test Series

E & T Engineering Test No : 2

Section A : Signals and Systems + Microprocessors and Microcontroller

Q.1 (a) Solution:

Since $X(s)$ has 4 poles and no zeros in the finite s-plane, we may assume that $X(s)$ is of the form,

$$X(s) = \frac{A}{(s-a)(s-b)(s-c)(s-d)}$$

Since $x(t)$ is real, the poles of $X(s)$ must occur in conjugate reciprocal pairs.

Therefore, we may assume that $b = a^*$ and $d = c^*$.

$$X(s) = \frac{A}{(s-a)(s-a^*)(s-c)(s-c^*)}$$

Since, $x(t)$ is even, $X(s)$ must also be even. Hence, the poles must be symmetric about the $j\Omega$-axis. Therefore, $c = -a^*$

Therefore, $$X(s) = \frac{A}{(s-a)(s-a^*)(s+a^*)(s+a)}$$

It is given that the location of one of the poles is $\left(\frac{1}{2}\right)e^{j\frac{\pi}{4}}$. If we assume that this pole is a, we have

$$X(s) = \frac{A}{\left(s-\frac{1}{2}e^{j\frac{\pi}{4}}\right)\left(s-\frac{1}{2}e^{-j\frac{\pi}{4}}\right)\left(s+\frac{1}{2}e^{-j\frac{\pi}{4}}\right)\left(s+\frac{1}{2}e^{j\frac{\pi}{4}}\right)}$$

$$X(s) = \frac{A}{\left(s^2-\frac{s}{\sqrt{2}}+\frac{1}{4}\right)\left(s^2+\frac{s}{\sqrt{2}}+\frac{1}{4}\right)}$$

Also, we are given that

$$\int_{-\infty}^{\infty} x(t)dt = X(0) = 4$$

Substituting in the above expression for $X(s)$, we have $A = \frac{1}{4}$.

Therefore,
$$X(s) = \frac{\left(\frac{1}{4}\right)}{\left(s^2-\frac{s}{\sqrt{2}}+\frac{1}{4}\right)\left(s^2+\frac{s}{\sqrt{2}}+\frac{1}{4}\right)}$$

Q.1 (b) Solution:

(i) SBI: Substract Immediate with Borrow.
SBI 8-bit data e.g. SBI 45H
The 8-bit data and the borrow are substracted from the contents of the accumulator, and the results are placed in the accumulator. All flags are affected to reflect the result of the operation.

(ii) SHLD: Store H and L Register Direct
SHLD 16-bit (address) e.g. SHLD 3000 H
The contents of register L are stored in the memory location specified by the 16-bit address in the operand, and the contents of H register are stored in the next memory location by incrementing the operand. The content of registers HL are not altered. This is a 3-byte instruction. No flags are affected.

(iii) RRC: Rotate Accumulator Right
Each binary bit of the accumulator is rotated right by one position. Bit D_0 is placed in the position of D_7 as well as in the carry flag. CY is modified according to bit D_0. S, Z, P, AC flags are not affected.

(iv) SPHL: Copy H and L Registers to the Stack Pointer.
The instruction loads the contents of H and L registers into the stack pointer register; the content of H register provide the high order address, and the content of the L register provide the low-order address. The contents of the H and L registers are not altered.

(v) DAD: Add Register pair to H and L Registers.

DAD Reg. Pair e.g., DAD B, DAD D, DAD H

The 16-bit contents of the specified register pair are added to the contents of the HL register and the sum is stored in the HL register. The contents of the source register pair are not altered. If the result is larger than 16 bits, the CY flag is set. No other flags are affected.

Q.1 (c) Solution:

Let a_k is the Fourier series coefficeint of the signal $x(t)$.

The fundamental frequency of $x(t)$ is $\omega_f = 100$ rad/sec.

We know that,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \longleftrightarrow X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - 100k)$$

Let given, $$g_1(t) = y_1(t) * h(t)$$

where, $$y_1(t) = x(t)\cos(\omega_0 t)$$

$$Y_1(j\omega) = \frac{1}{2}\{X(j(\omega-\omega_0)) + X(j(\omega+\omega_0))\}$$

$$Y_1(j\omega) = \pi \sum_{k=-\infty}^{\infty} [a_k\delta(\omega - 100k - \omega_0) + a_k\delta(\omega - 100k + \omega_0)]$$

$$= \pi \sum_{k=-\infty}^{\infty} [a_{-k}\delta(\omega + 100k - \omega_0) + a_k\delta(\omega - 100k + \omega_0)]$$

If $\omega_0 = 500$, then the term in the above summation with $k = 5$ becomes

$$\pi a_{-5}\delta(\omega) + \pi a_5\delta(\omega)$$

Since $x(t)$ is real, $a_{-k} = a_k^*$. Therefore, the above expression becomes,

$$2\pi \text{Re}\{a_5\}\delta(\omega)$$

Which is an impulse at $\omega = 0$.

Hence, we get $Y_1(j\omega) = 2\pi\ \text{Re}\{a_5\}\ \delta(\omega)$

Taking inverse Fourier transform,

$$y_1(t) = \text{Re}\{a_5\}$$

$\therefore$ $$g_1(t) = \text{Re}\{a_5\} * h(t) = \text{Re}\{a_5\}$$ [Given]

Therefore, $$h(t) = \delta(t) \Rightarrow H(j\omega) = 1$$

Q.1 (d) Solution:

8255 Control Word:

IO/BSR	Mode	of A	Port A	Port C_U	Mode of B	Port B	Port C_L
D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0

Since we are using I/O mode, $D_7 = 1$. Since, both groups (Group A and Group B) are to be in Mode 0.

Hence, $D_6D_5 = 00$ and $D_2 = 0$

For input, the corresponding bit is to be 1, and for output, it is to be 0.

Since port A is to be an o/p port, $D_4 = 0$

Since port C_U is to be an i/p port, $D_3 = 1$

Since port B is to be an i/p port, $D_1 = 1$

Since port C_L is to be an o/p port, $D_0 = 0$

Thus, the control word can be written as

1	0	0	0	1	0	1	0
D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0
8				A			

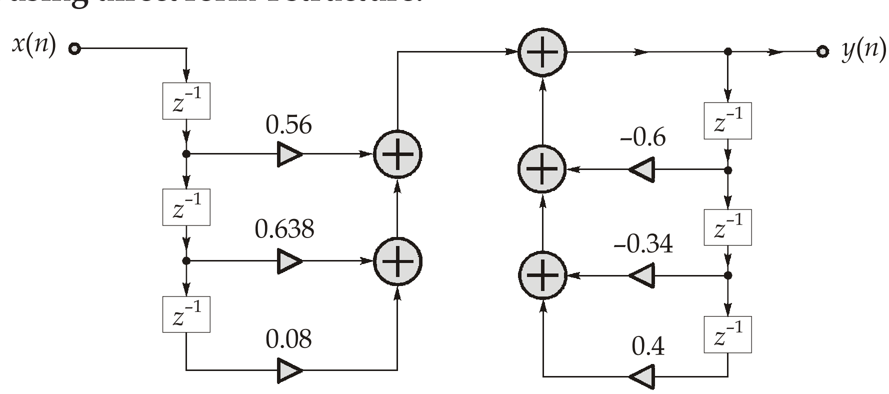
Q.1 (e) Solution:

The given transfer function can also be written as,

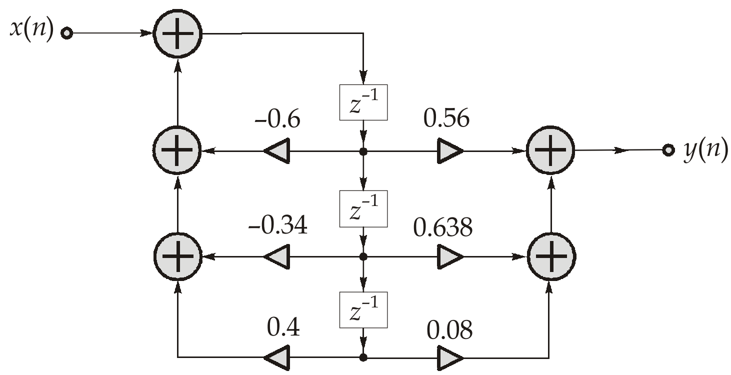
$$H(z) = \frac{0.28z^{-1} + 0.319z^{-2} + 0.04z^{-3}}{0.5 + 0.3z^{-1} + 0.17z^{-2} - 0.2z^{-3}}$$

$$= \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$

Realization using direct form-I structure:



Realization using direct form-II structure:



Q.1 (f) Solution:

The program to transfer a string of data in 8085 microprocessor can be written as follows:

```
         LXI H, 4000H
         LXI D, 5000H
         MVI C, 10H
AGAIN :  MOV A, M
         STAX D
         INX H
         INX D
         DCR C
         JNZ AGAIN
         HLT
```

The program to transfer a string of data in 8086 microprocessor can be written as follows:

```
         MOV SI, 4000H
         MOV DI, 5000H
         MOV CX, 10H
AGAIN :  MOV AX, [SI]
         MOV [DI], AX
         INC SI
         INC DI
         DEC CX
         JNZ AGAIN
         HLT
```

Q.2 (a) Solution:

(i) Given, $x_1(t) = e^{-|t|}\cos(2t)$

Let $x(t) = e^{-|t|}$

We know that, $e^{-t}u(t) \xleftrightarrow{F.T} \dfrac{1}{1+j\omega}$

$$x(t) = e^{-|t|} \xleftrightarrow{F.T} X(\omega) = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{2}{1+\omega^2}$$

$$\therefore \quad e^{-|t|} \xleftrightarrow{F.T} \frac{2}{1+\omega^2}$$

We know that,

$$\cos(2t) \xleftrightarrow{F.T} \pi[\delta(\omega-2)+\delta(\omega+2)]$$

$$x(t)\cos(2t) \longleftrightarrow \frac{\pi}{2\pi}[X(\omega-2)+X(\omega+2)]$$

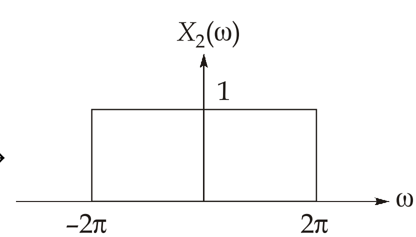
Therefore, from the above multiplying property

$$e^{-|t|}\cos(2t) \xleftrightarrow{F.T} \frac{\pi}{2\pi}\left[\frac{2}{1+(\omega-2)^2} + \frac{2}{1+(\omega+2)^2}\right] \qquad \left[\because X(\omega) = \frac{2}{1+\omega^2}\right]$$

$$\therefore \quad e^{-|t|}\cos(2t) \xleftrightarrow{F.T} \frac{1}{1+(\omega-2)^2} + \frac{1}{1+(\omega+2)^2}$$

(ii) Given, $x_2(t) = \dfrac{\sin(2\pi t)}{\pi(t-1)}$

We know that, $x_2(t) = \dfrac{\sin(2\pi t)}{\pi t} \longleftrightarrow$



$$x_2(t) = \frac{\sin(2\pi t)}{\pi t} \longleftrightarrow u(\omega+2\pi) - u(\omega-2\pi)$$

$$\frac{\sin(2\pi t)}{\pi(t-1)} = \frac{\sin(2\pi t - 2\pi)}{\pi(t-1)} = \frac{\sin(2\pi(t-1))}{\pi(t-1)}$$

$$= x_2(t-1)$$

We know that, $x(t) \xleftrightarrow{F.T} X(\omega)$

$$x(t-1) \xleftrightarrow{F.T} e^{-j\omega} X(\omega)$$

Therefore, $$\frac{\sin(2\pi t)}{\pi t} \xleftrightarrow{F.T} u(\omega+2\pi) - u(\omega-2\pi)$$

$$\frac{\sin(2\pi(t-1))}{\pi(t-1)} \xleftrightarrow{F.T} e^{-j\omega}\left[u(\omega+2\pi)-u(\omega-2\pi)\right]$$

(iii) Given, $$x_3(t) = \begin{cases} t^2 \ ; \ 0 < t < 1 \\ 0 \ ; \ \text{otherwise} \end{cases}$$

We may write, $$x_3(t) = t^2[u(t) - u(t-1)]$$

We know that, $$u(t) \xleftrightarrow{F.T} \frac{1}{j\omega} + \pi\delta(\omega)$$

$$u(t)-u(t-1) \xleftrightarrow{F.T} \frac{1}{j\omega} + \pi\delta(\omega) - e^{-j\omega}\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

$$u(t)-u(t-1) \xleftrightarrow{F.T} \frac{1}{j\omega} + \pi\delta(\omega) - e^{-j\omega} \times \frac{1}{j\omega} - \pi\delta(\omega)$$

$$u(t)-u(t-1) \xleftrightarrow{F.T} \frac{1-e^{-j\omega}}{j\omega}$$

We know that, $$t\, f(t) \longleftrightarrow j\frac{d}{d\omega}F(j\omega)$$

$$t^2 f(t) \longleftrightarrow \frac{-d^2}{d\omega^2}F(j\omega)$$

$$t^2\left[u(t)-u(t-1)\right] \longleftrightarrow -\frac{d^2}{d\omega^2}\left[\frac{1-e^{-j\omega}}{j\omega}\right]$$

$$t^2\left[u(t)-u(t-1)\right] \longleftrightarrow -\frac{d}{d\omega}\left[\frac{d}{d\omega}\left[\frac{1-e^{-j\omega}}{j\omega}\right]\right]$$

$$t^2\left[u(t)-u(t-1)\right] \longleftrightarrow -\frac{d}{d\omega}\frac{1}{j}\left[\frac{\omega\left[je^{-j\omega}\right]-\left[1-e^{-j\omega}\right](1)}{\omega^2}\right]$$

$$t^2\left[u(t)-u(t-1)\right] \longleftrightarrow \frac{j\omega^2 e^{-j\omega} + 2\omega e^{-j\omega} + 2j(1-e^{-j\omega})}{\omega^3}$$

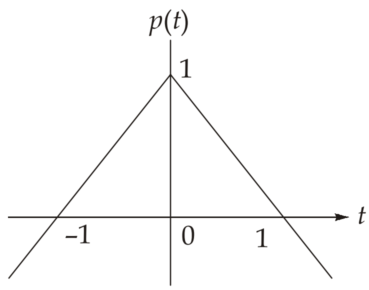
$$x_3(t) \xleftrightarrow{F.T} \frac{j}{\omega}e^{-j\omega} + \frac{2}{\omega^2}e^{-j\omega} + \frac{2j}{\omega^3}(1 - e^{-j\omega})$$

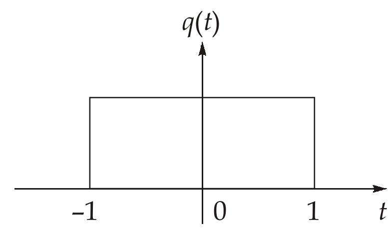
(iv) Given, $x_4(t) = (1 - |t|)u(t+1)u(1-t)$

Let, $p(t) = (1 - |t|)$

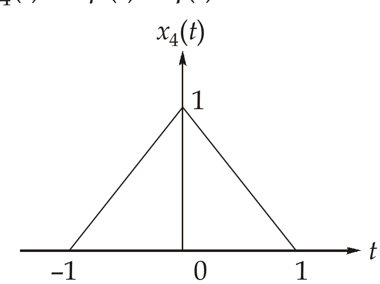
$q(t) = u(t+1)u(1-t)$

Drawing the signals for $p(t)$ and $q(t)$,

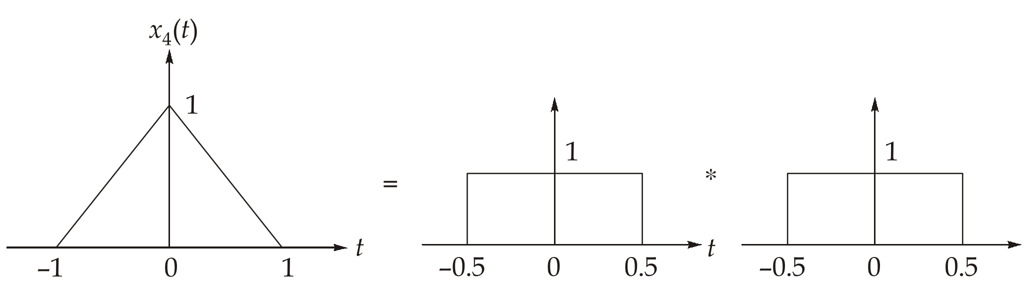




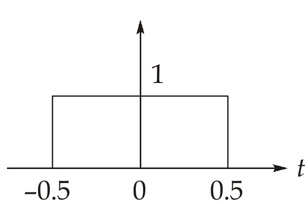
$\therefore$ $x_4(t) = p(t) \times q(t)$



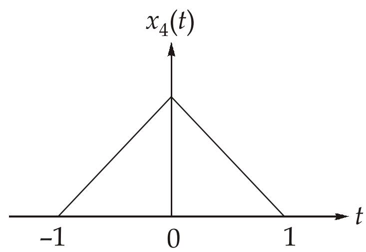
We may write $x_4(t)$ as



We know that,



$$\longleftrightarrow Sa\left(\frac{\omega}{2}\right) = \frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$$

$\therefore$ 

$$\longleftrightarrow \frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)} \cdot \frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$$

$$\longleftrightarrow \frac{4}{\omega^2}\left[\frac{1-\cos 2\left(\frac{\omega}{2}\right)}{2}\right]$$

$$\longleftrightarrow \frac{2(1-\cos\omega)}{\omega^2}$$

Q.2 (b) Solution:

(i) Programmed data transfer schemes: In programmed data transfer schemes, the data transfer is controlled by CPU. Programmed data transfer schemes can be of three types:

1. **Synchronous Transfer :** In synchronous data transfer, the device which sends the data and device which receives the data are synchronised with same clock. It is suitable when both microprocessor and I/O device matches in speed. The data transfer takes place through IN and OUT instruction in I/O mapped I/O devices and with instruction related to memory read/write in case of memory mapped I/O devices.
2. **Asynchronous Transfer :** This technique of data transfer is used when speed of microprocessor and that of I/O device is not matching. The microprocessor continuously checks the status of I/O device to see whether device is ready to send data or not. When I/O devices becomes ready, it sends signals called handshake signals. READY signal is handshake signal used by 8085 for asynchronous data transfer.
3. **Interrupt driven data transfer :** In this mode of data transfer, the microprocessor initiates the I/O device to get ready and then it executes its main programmes, instead of remaining in loop to check the status of I/O device. When I/O device is ready to transfer data, it sends a high signal to microprocessor through interrupt line. On receiving an interrupt signal through interrupt line, the microprocessor complete the current instruction being executed and then attend I/O device. After completing the data transfer, the microprocessor return back to main program. Interrupt data transfer is used for slow I/O devices. It is efficient than asynchronous data transfer mode because time of microprocessor is not wasted in waiting while I/O device is getting ready.

(ii) Direct Memory Access (DMA): It is a process of high speed direct data transfer between memory and external peripherals (I/O device) without programming intervention. DMA involves communication or data transfer controlled by an external peripheral. Wherever the microprocessor-controlled data transfer is too slow, DMA is generally used; e.g. data transfer between hard disk and read/write memory of the system. 8237/8257 ICs are DMA controllers. DMA controllers takes control of the buses and transfer data directly between memory and external peripherals. When large amount of data is to be printed out from the memory of a computer, DMA is used because it avoids using MPU, thus, allowing the MPU to attend to another job.

DMA transfer is faster than either interrupt initiated I/O or polling based I/O for large data transfer.

Q.2 (c) Solution:

The overall impulse response of the system can be given by,

$$h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$$

$$h_3(n) * h_4(n) = [(n+1)\,u(n)] * [\delta(n-2)] = (n-1)\,u(n-2)$$

$$h_2(n) - [h_3(n) * h_4(n)] = (n+1)\,u(n) - (n-1)\,u(n-2) = [nu(n) + u(n)]$$
$$- [nu(n-2) - u(n-2)]$$
$$= \delta(n-1) + u(n) + u(n-2) = 2u(n) - \delta(n)$$

$$h_1(n) = \left\{\underset{\uparrow}{\frac{1}{2}}, \frac{1}{4}, \frac{1}{2}\right\} = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$$

Hence,
$$h(n) = [2u(n) - \delta(n)] * \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right]$$

$$= \left[u(n) + \frac{1}{2}u(n-1) + u(n-2)\right] - \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right]$$

$$h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$$

Given that,
$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3) = \left\{1, 0, \underset{\uparrow}{0}, 3, 0, -4\right\}$$

$h(n)$:	$\underset{\uparrow}{\frac{1}{2}}$	$\frac{5}{4}$	2	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$			......
$x(n)$:	1	0	$\underset{\uparrow}{0}$	3	0	-4					
	$\frac{1}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	......
		0	0	0	0	0	0	0	0	0	......
			$\underset{\uparrow}{0}$	0	0	0	0	0	0	0	......
				$\frac{3}{2}$	$\frac{15}{4}$	6	$\frac{15}{2}$	$\frac{15}{2}$	$\frac{15}{2}$	$\frac{15}{2}$	......
					0	0	0	0	0	0	......
						-2	-5	-8	-10	-10	......
$y(n)$:	$\frac{1}{2}$	$\frac{5}{4}$	$\underset{\uparrow}{2}$	4	$\frac{25}{4}$	$\frac{13}{2}$	5	2	0	0	......

So,
$$y(n) = \left\{\frac{1}{2}, \frac{5}{4}, \underset{\uparrow}{2}, 4, \frac{25}{4}, \frac{13}{2}, 5, 2\right\}$$

Q.3 (a) Solution:

```
        LXI H, 6000 H  ; Initialize memory pointer
        MVI C, 00 H    ; Initialize number counter
        MVI D, 00 H    ; Initialize positive number counter
        MVI B, 00 H    ; Initialize negative number counter
        MVI E, 00 H    ; Initialize zero number counter
BEGIN : MOV A, M       ; Get the number
        CPI 00 H       ; If number = 0
        JZ ZERONUM     ; Go to ZERONUM
        ANI 80 H       ; If MSB of number = 1
        JNZ NEGNUM     ; If number is negative goto NEGNUM
        INR D          ; Otherwise increment positive number counter
        JMP LAST
```

```
ZERONUM:  INR E          ; Increment zero number counter
          JMP LAST
NEGNUM:   INR B          ; Increment negative number counter
LAST :    INX H          ; Increment memory pointer
          INR C          ; Increment number counter
          MOV A, C
          CPI 32 H       ; If number counter = 50₁₀
          JNZ BEGIN      ; Then store otherwise check next number
          LXI H, 7000 H  ; Initialize memory pointer
          MOV M, B       ; Store negative number count
          INX H
          MOV M, E       ; Store zero number count
          INX H
          MOV M, D       ; Store positive number count
          HLT
```

Q.3 (b) Solution:

(i) Given,
$$x(n) = \left(\frac{1}{2}\right)^n \{u[n]-u[n-10]\}$$

by the definition of z-transform,

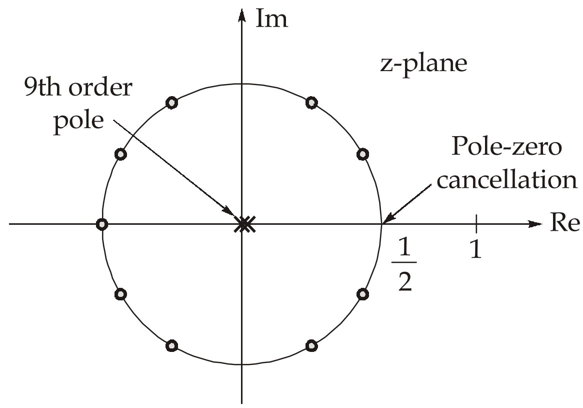
$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$= \sum_{n=0}^{9}\left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{9}(2z)^{-n} = \frac{1-(2z)^{-10}}{1-(2z)^{-1}} = \frac{1-\dfrac{1}{(2z)^{10}}}{1-\dfrac{1}{2z}}$$

$$X(z) = \frac{z^{10}-\left(\dfrac{1}{2}\right)^{10}}{z^9\left(z-\dfrac{1}{2}\right)};\ \text{ROC}: |z|>0$$

Pole-zero plot for $X(z)$



(ii) $$x[n] = 7\left(\frac{1}{3}\right)^n \cos\left[\frac{2\pi n}{6} + \frac{\pi}{4}\right]u[n]$$

by the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{3}\right)^n \cos\left[\frac{2\pi n}{6} + \frac{\pi}{4}\right]z^{-n}$$

$$= 7\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n \left[\frac{e^{j\left[\frac{2\pi}{6}n+\frac{\pi}{4}\right]} + e^{-j\left[\frac{2\pi}{6}n+\frac{\pi}{4}\right]}}{2}\right]z^{-n}$$

$$= \frac{7}{2}\left[e^{j\frac{\pi}{4}}\sum_{n=0}^{\infty}\left(\frac{1}{3}e^{j\frac{2\pi}{6}}\cdot z^{-1}\right)^n + e^{-j\frac{\pi}{4}}\sum_{n=0}^{\infty}\left(\frac{1}{3}e^{-j\left(\frac{2\pi}{6}\right)}z^{-1}\right)^n\right]$$

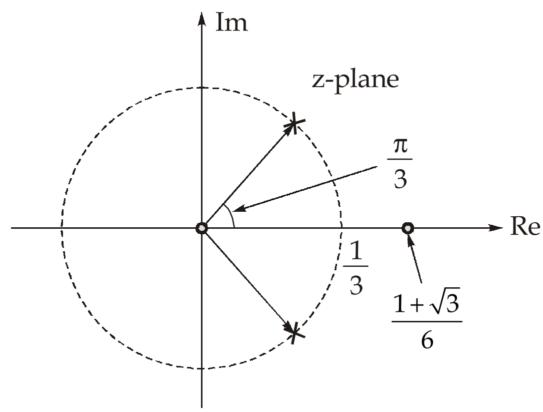
$$= \frac{7}{2}\left[\frac{e^{j\frac{\pi}{4}}}{1-\frac{1}{3}e^{j\left(\frac{2\pi}{6}\right)}z^{-1}} + \frac{e^{-j\frac{\pi}{4}}}{1-\frac{1}{3}e^{-j\left(\frac{2\pi}{6}\right)}z^{-1}}\right]$$

$$= \frac{7z}{2}\left[\frac{e^{j\frac{\pi}{4}}}{z-\frac{1}{3}e^{j\left(\frac{2\pi}{6}\right)}} + \frac{e^{-j\frac{\pi}{4}}}{z-\frac{1}{3}e^{-j\left(\frac{2\pi}{6}\right)}}\right]$$

$$X(z) = \frac{7z}{2}\left[\frac{2z\cos\left(\frac{\pi}{4}\right)-\frac{2}{3}\cos\left(\frac{2\pi}{6}-\frac{\pi}{4}\right)}{\left(z-\frac{1}{3}e^{j\left(\frac{2\pi}{6}\right)}\right)\left(z-\frac{1}{3}e^{-j\left(\frac{2\pi}{6}\right)}\right)}\right],$$

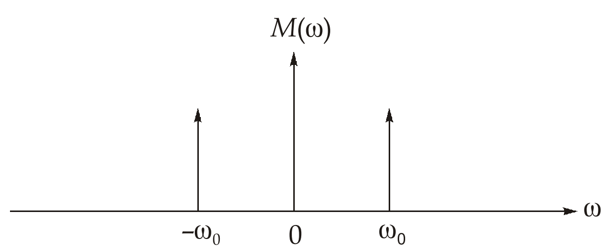
where ROC : $|z| > \frac{1}{3}$

The pole-zero plot,



Q.3 (c) Solution:

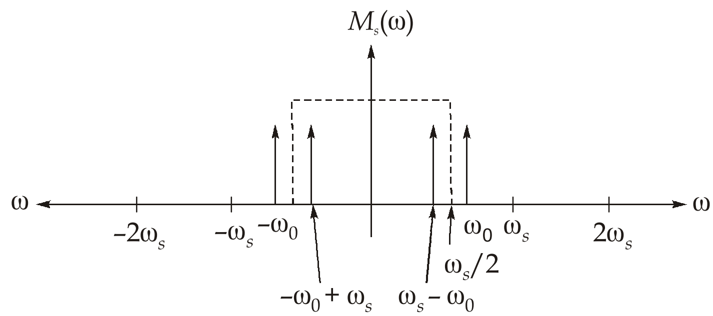
(i) The spectrum of $m(t) = \cos\omega_0 t$ is given by $F[\cos\omega_0 t] = M(\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$. The spectrum of $m(t)$ is shown below:



The spectrum of the sampled signal is given as

$$M_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)$$

Figure below shows the spectrum of $M_s(\omega)$ of the ideal sampled signal $m_s(t)$ with $\omega_s = \frac{3}{2}\omega_0$, $\omega_s = 2\pi f_s$.



Also indicated by a dashed line is the passband of low pass filter with $\omega_c = \omega_s/2$. It may be seen that aliasing does occur , and the low pass filter output $x_r(t)$ is given by

$$x_r(t) = \cos(\omega_s - \omega_0)t = \cos\frac{1}{2}\omega_0 t \neq m(t)$$

In the figure given below is depicted $m(t)$, its samples and reconstructed signal $x_r(t)$.

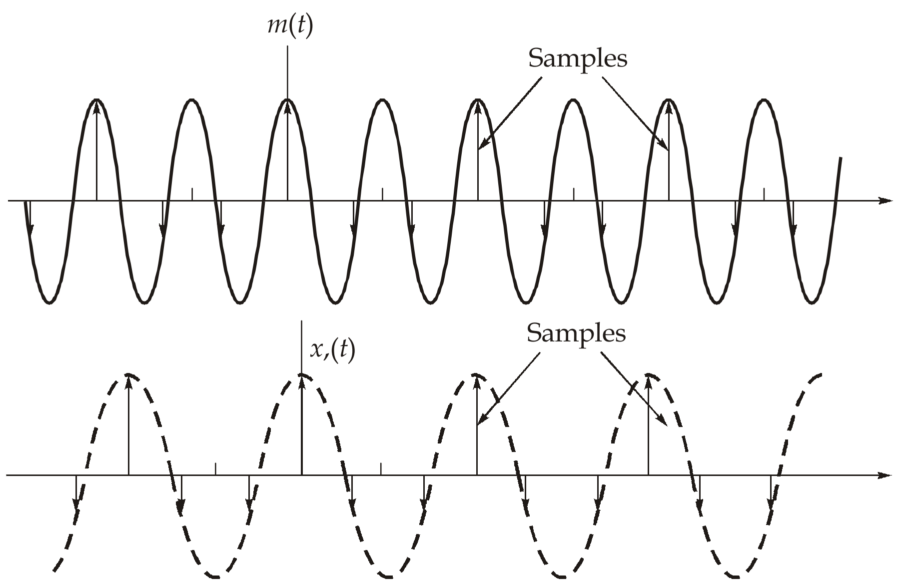


Fig. Effect of aliasing on a sinusoidal signal

(ii) Following are the three instructions, which use stack memory for their execution:

1. PUSH
2. POP
3. XTHL

1. PUSH R_P $R_P \rightarrow$ BC, DE, HL and PSW	(i) Stores the contents of register pairs R_P on two top locations of stack. (ii) 1-byte instruction (iii) Register indirect addressing mode (iv) 3 machines cycles and 12-T states (v) No flag is affected.

2.	POP R_P $R_P \rightarrow$ BC, DE, HL and PSW	(i)	Receives the contents of two top locations of stack to register pair R_P.
		(ii)	1-byte instruction.
		(iii)	Register indirect addressing mode
		(iv)	3 machine cycles and 10-T states
		(v)	No flag is affected.
3.	XTHL $R_P \rightarrow$ BC, DE, HL and PSW	(i)	Exchanges the contents of two top locations of stack with contents of HL pair.
		(ii)	1-byte instruction.
		(iii)	Register indirect addressing mode
		(iv)	5 machine cycles and 16-T states
		(v)	No flag is affected.

Q.4 (a) Solution:

(i) Algorithm:

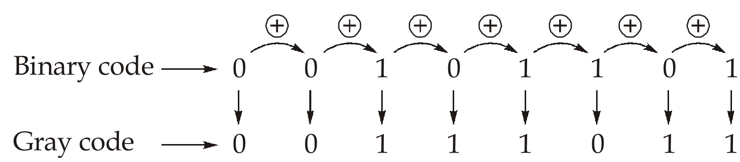
1. Load the BCD number in the accumulator.
2. Unpack the 2 digit BCD number into two separate digits. Let the left digit be BCD2 and the right one BCD1.
3. Multiply BCD2 by 10 and add BCD1 to it.

Program:

```
LDA 2200 H   ;  Get the BCD number
MOV B, A     ;  Save it in B register
ANI 0F H     ;  Mask most significant four bits
MOV C, A     ;  Save unpacked BCD1 in C register
MOV A, B     ;  Get BCD again
ANI F0 H     ;  Mask least significant four bits
RRC          ;  Convert most significant four bits into unpacked BCD2
RRC          ;  Convert most significant four bits into unpacked BCD2
RRC          ;  Convert most significant four bits into unpacked BCD2
RRC          ;  Convert most significant four bits into unpacked BCD2
MOV B, A     ;  Save unpacked BCD 2 in B register
XRA A        ;  Clear accumulator
MVI D, 0AH   ;  Set D as a multiplier of 10
```

```
SUM : ADD D        ;  Add 10 until (B) = 0
      DCR B        ;  Decrement BCD2 by one
      JNZ SUM      ;  Is multiplication complete?
                      If not, go back and add again
      ADD C        ;  Add BCD1
      STA 2300 H   ;  Store the result
      HLT          ;  Terminate program execution
```

(ii) Consider an 8-bit binary number 00101101



Program logic: To XOR each bit with it's adjacent bit, we right shift the contents of original number and then XOR the result with the original number.

i.e.,

	0 0 1 0 1 1 0 1	← Binary number
$\oplus$	0 0 0 1 0 1 1 0	← Right shift binary number
	0 0 1 1 1 0 1 1	← Gray code number

Program:

```
MOV A, 52 H   ;  Load binary number
MOV R0, A     ;  Save binary number into register R0
CLR C         ;  Clear carry flag
RRC A         ;  Right shift Accumulator content (The value of carry
                 flag moves into MSB)
XRL A, R0     ;  XOR the shifted content with original number
HLT
```

Q.4 (b) Solution:

Given input to an LTI system,

$$x_1(t) = \sum_{k=-\infty}^{\infty} \alpha^{|k|} c^{jk\frac{\pi}{4}t}$$

From the definition of Fourier series,

$$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \text{ ; where, } \omega_0 = \frac{2\pi}{T_0}$$

where, T_0 is the fundamental time period.

On comparing,

$$C_k = \alpha^{|k|}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{4}$$

$$\therefore \quad T_0 = 8$$

Therefore, the given signal $x_1(t)$ is periodic with period $T = 8$ and has Fourier series coefficients $C_k = \alpha^{|k|}$.

The average energy in the input signal is

$$\frac{1}{T}\int_T |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |C_k|^2$$

$$= \sum_{k=-\infty}^{\infty} \left[\alpha^{|k|}\right]^2$$

$$= \sum_{k=-\infty}^{\infty} \alpha^{2|k|}$$

$$= \sum_{k=-\infty}^{-1} (\alpha^{-2})^k + \sum_{k=0}^{\infty} (\alpha^2)^k$$

$$= \frac{1}{1-\alpha^2} - 1 + \frac{1}{1-\alpha^2} = \frac{1-1+\alpha^2+1}{1-\alpha^2} = \frac{1+\alpha^2}{1-\alpha^2}$$

Let the low pass filter passes some number, say m of the harmonic components of $x_1(t)$ so that the average energy in the output signal is

$$\frac{1}{T}\int_T |y(t)|^2\, dt = \sum_{k=-m}^{m} \alpha^{2|k|}$$

$$= \frac{1-\alpha^{2m+2}}{1-\alpha^2} + \frac{1-\alpha^{2m+2}}{1-\alpha^2} - 1 = \frac{1+\alpha^2-2\alpha^{2m+2}}{1-\alpha^2}$$

To make the energy in the output at least 90% of that in the input,

$$\frac{1+\alpha^2-2\alpha^{2m+2}}{1-\alpha^2} \geq 0.9\left(\frac{1+\alpha^2}{1-\alpha^2}\right)$$

$$1 + \alpha^2 - 2\alpha^{2m+2} \geq 0.9(1+\alpha^2)$$

$$\alpha^{2m+2} \leq \frac{1+\alpha^2}{20}$$

by taking 'log' on both sides,

$$(2m + 2)\log \alpha \leq \log\left[\frac{1+\alpha^2}{20}\right]$$

$$(m + 1)\log \alpha^2 \leq \log\frac{1+\alpha^2}{20}$$

$$\therefore \qquad m > \frac{\log\left(\frac{1+\alpha^2}{20}\right)}{\log\alpha^2} - 1 \qquad [\because |\alpha| < 1]$$

Because the harmonics are spaced at $\frac{2\pi}{T}$ intervals in frequency,

Therefore, the minimum value of W is

$$W > \left(\frac{\log\left(\frac{1+\alpha^2}{20}\right)}{\log\alpha^2} - 1\right)\frac{2\pi}{8}$$

$$\therefore \qquad W_{\min} = \left(\frac{\log\left(\frac{1+\alpha^2}{20}\right)}{\log\alpha^2} - 1\right)\frac{\pi}{4}$$

Q.4 (c) Solution:

(i) From the given pole-zero plot,

we get,
$$H(z) = A'\cdot\frac{\left(z-\frac{1}{a}\right)}{(z-a)} = A'\frac{z\left(1-\frac{1}{a}z^{-1}\right)}{z(1-az^{-1})} = A\frac{(z^{-1}-a)}{1-az^{-1}}$$

where, $A = A'/a$

put, $z = e^{j\omega}$

$$H(e^{j\omega}) = A\frac{e^{-j\omega}-a}{1-ae^{-j\omega}}$$

and
$$\left|H(e^{j\omega})\right|^2 = H(e^{j\omega})\,H^*(e^{j\omega})$$

$$= |A|^2\left[\frac{e^{-j\omega}-a}{1-ae^{-j\omega}}\right]\left[\frac{e^{j\omega}-a}{1-ae^{j\omega}}\right]$$

Therefore, $$|H(e^{j\omega})|^2 = |A|^2 \frac{1 - ae^{-j\omega} - ae^{j\omega} + a^2}{1 - ae^{-j\omega} - ae^{j\omega} + a^2}$$

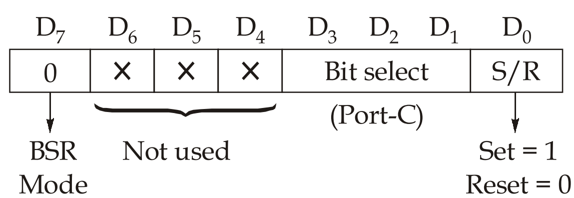
$\therefore$ $$|H(e^{j\omega})|^2 = |A|^2$$

This implies that, $$|H(e^{j\omega})| = |A| = \text{constant}$$

(ii) BSR is a special mode and is applicable only for the bits at port C. In the control word format, if the MSB is mode 0 (D_7 = 0), the BSR (bit set/reset) mode takes effect.

In this mode, any bit at port C can be set or reset by specifying the bit which has to be set or reset. However, at a time, only one bit can be addressed and that bit is to be either set or reset. The corresponding control word has to be decided and moved to the control word register (CWR)

Control Word Format:



		D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0		
1.	PC_0 to be set:	0	X	X	X	0	0	0	1	=	01 H
				treat 'X' as '0'							
2.	PC_7 to be reset:	0	0	0	0	1	1	1	0	=	0E H
3.	PC_1 to be set:	0	0	0	0	0	0	1	1	=	03 H
4.	PC_7 to be set:	0	0	0	0	1	1	1	1	=	0F H

Here D_0 = 1 means set;

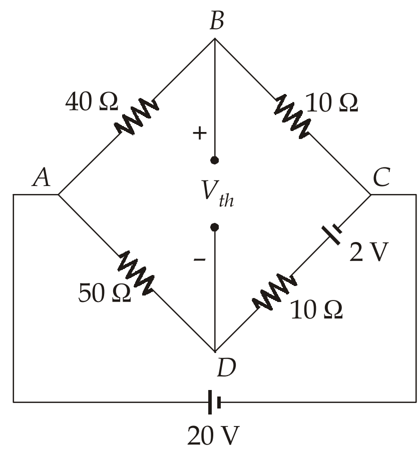
D_0 = 0 means reset

In the BSR mode, port C should be in o/p mode.

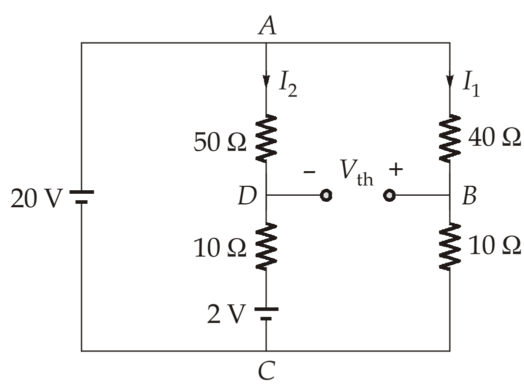
Section B : Network Theory-1 + Control Systems-1

Q.5 (a) Solution:

For calculating Thevenin's voltage, open the resistor *R*,



The above circuit can be redrawn as:



The branch current, $$I_1 = \frac{20}{40+10} = 0.4\text{ A}$$

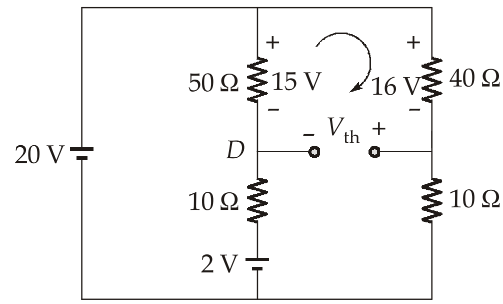
the voltage drop across 40 Ω resistor is,

$$V_{40\,\Omega} = I_1 \times 40 = 0.4 \times 40 = 16\text{ V}$$

The branch current, $$I_2 = \frac{20-2}{50+10} = \frac{18}{60} = 0.3\text{ A}$$

The voltage drop across 50 Ω resistor is,

$$V_{50\,\Omega} = I_2 \times 50 = 0.3 \times 50 = 15\text{ V}$$

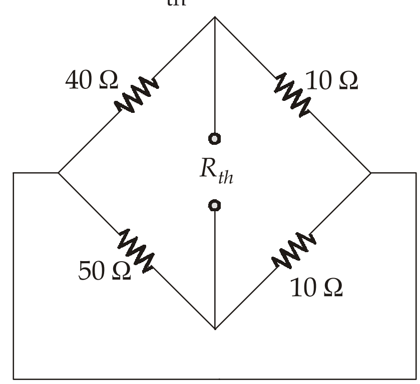


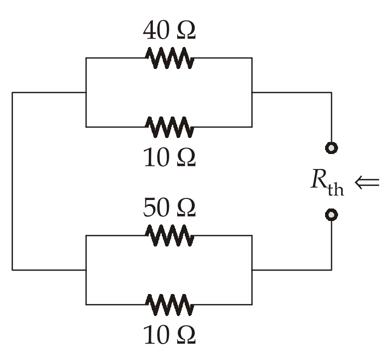
by using KVL in the above loop,

$$15 - 16 - V_{th} = 0$$

$$\therefore \quad V_{th} = -1 \text{ V}$$

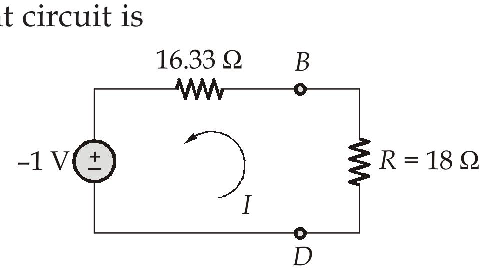
For calculating Thevenin's resistance, R_{th} deactivate all active sources.





$$R_{th} = (40 \,||\, 10) + (50 \,||\, 10) = 16.33\ \Omega$$

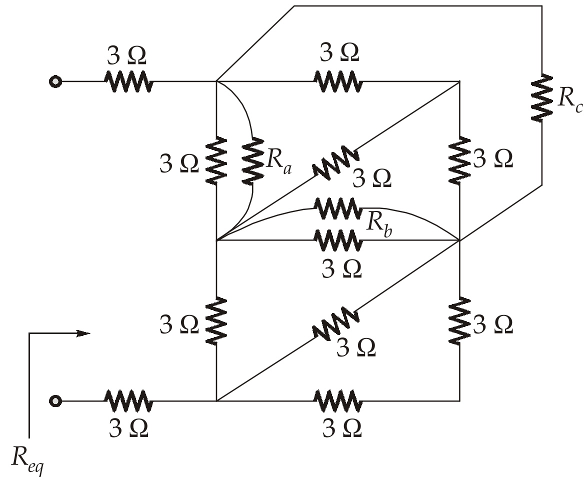
The Thevenin's equivalent circuit is



$$\therefore \quad \text{The current } I = \frac{1}{16.33 + 18} = 29.12 \text{ mA}$$

Q.5 (b) Solution:

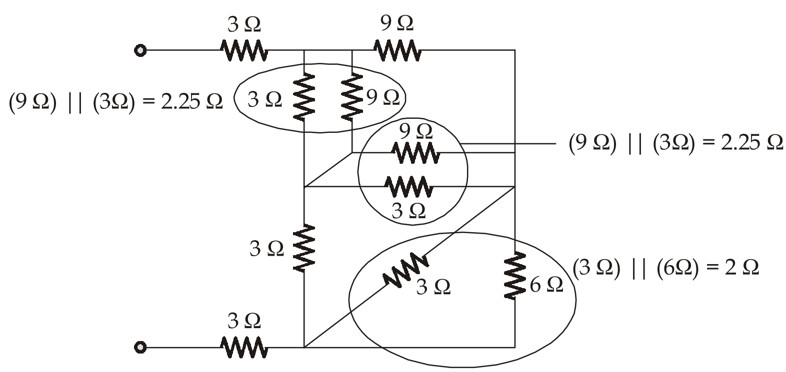
Convert Y network of 3Ω, 3Ω, 3Ω into Δ



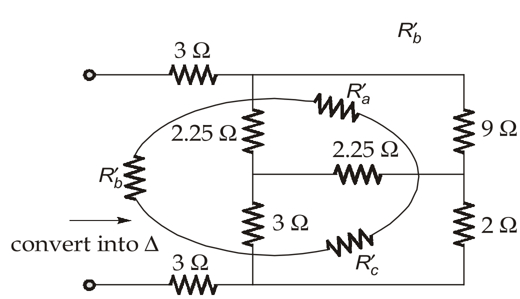
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_a = R_b = R_c = 9\ \Omega$$

The circuit is reduced to,



The circuit is reduced to,



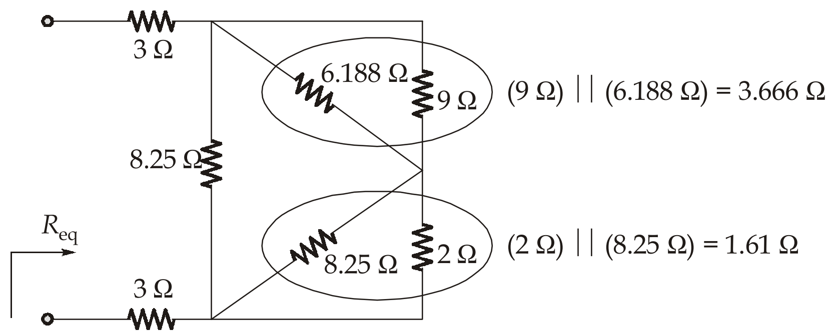
$$R_1 = 3\ \Omega,\ R_2 = 2.25\ \Omega,\ R_3 = 2.25\ \Omega$$

$$R'_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = 6.188\ \Omega$$

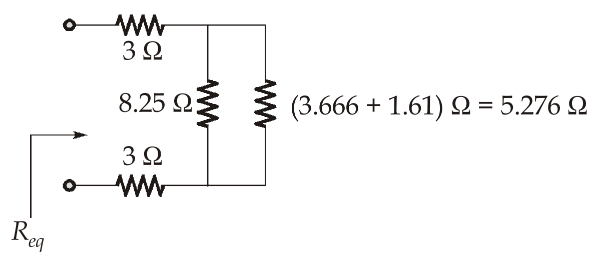
Similarly, $R'_b = 8.25\ \Omega$

$R'_c = 8.25\ \Omega$

The circuit is reduced to,



The circuit is reduced to,



$$R_{eq} = 3 + [(8.25) \,||\, (5.276)] + 3$$
$$R_{eq} = 3 + 3.218 + 3$$
$$R_{eq} = 9.218\ \Omega$$

Q.5 (c) Solution:

Gain Margin: It is the reciprocal of the magnitude $|G(j\omega)|$ at the frequency at which the phase angle is –180°.

$$K_g = \frac{1}{|G(j\omega_c)|}$$

For a stable minimum phase system, the gain margin indicates how much the gain can be increased before the sytem become unstable. For an unstable system, gain margin is indicative of how much the gain must be decreased to make the system stable.

$\omega_c \rightarrow$ Phase crossover frequency at which the phase angle of $G(j\omega)$ is –180°.

$$K_g\,(\text{dB}) = 20 \log K_g = -20 \log |G(j\omega_c)|$$

Phase Margin : The phase margin is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability. The gain crossover frequency is the frequency at which $|G(j\omega)| = 1$.

Phase margin, $\gamma = 180° + \phi$

ϕ – Phase angle of open loop transfer function at gain crossover frequency, i.e., $\phi = \angle G(j\omega_g)$.

Importance of GM and PM in Control System design:

- PM and GM of a control system are a measure of closeness of the polar plot to the $-1 + j0$ point. These margins are used as design criteria.
- Either GM alone or PM alone does not give sufficient indication of relative stability. Both should be given in the determination of relative stability.
- For minimum-phase system, both PM and GM must be positive for the system to be stable. Negative margins indicates instability.
- For satisfactory performance, the PM should be between 30°-60° and GM should be greater than 6 dB. With these values, a minimum phase system has guaranteed stability, even if open loop system gain and time constants vary to a certain extent.

Q.5 (d) Solution:

- The Routh table for the given characteristic equation can be formed as follows:

s^4	1	2	10
s^3	K	$K+1$	0
s^2	$\dfrac{2K-(K+1)}{K} = \left(1-\dfrac{1}{K}\right)$	10	0
s^1	$(K+1)-\dfrac{10K^2}{(K-1)}$	0	0
s^0	10	0	0

- For the system to be stable, all the elements in the first column of the Routh table should have same sign. For this, the following conditions should be satisfied:

From s^3 row, $K > 0$ ...(i)

From s^2 row, $1-\dfrac{1}{K} > 0$

$K > 1$ ...(ii)

From s^1 row, $(K+1)-\dfrac{10K^2}{(K-1)} > 0$

$K^2 - 1 - 10K^2 > 0$

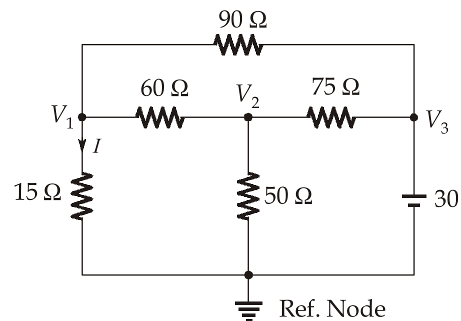
$-9K^2 - 1 > 0$ ...(iii)

- No real value of "K" satisfies all the equations (i), (ii) and (iii) simultaneously. So, the given system is unstable for any real value of "K".

Q.5 (e) Solution:

In any passive linear bilateral network, if the single voltage source V_x in branch x produces the current response I_y in branch y, then the removal of the voltage source from branch x and its insertion in branch y will produce the current response I_y in the branch x.

The current I can be calculated with nodal analysis



$$V_3 = 30 \text{ V} \quad \text{...(i)}$$

Nodal equations

$$\frac{V_1 - V_2}{60} + \frac{V_1}{15} + \frac{V_1 - V_3}{90} = 0$$

Putting $V_3 = 30$ V in above equation gives,

$$0.094V_1 - 0.017V_2 = 0.33 \quad \text{...(ii)}$$

At node V_2,

$$\frac{V_2 - V_1}{60} + \frac{V_2}{50} + \frac{V_2 - V_3}{75} = 0$$

It reduces to,

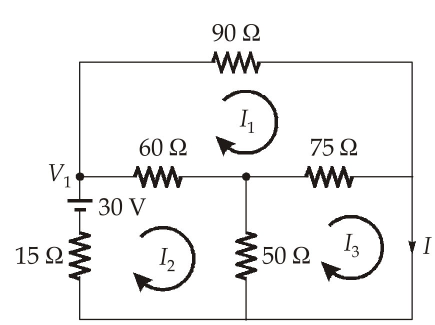
$$-0.017V_1 + 0.05V_2 = 0.4 \quad \text{...(iii)}$$

Solving (ii) and (iii) gives,

$$V_1 = 5.28 \text{ V}; \ V_2 = 9.8$$

$$I = \frac{V_1}{15} = \frac{5.28}{15} = 0.35 \text{ A}$$

Let us now interchange source $V = 30$ V and measure current in branch where source is shorted



Mesh analysis: Mesh equations for loop 1,

$$-90I_1 - 75(I_1 - I_3) - 60\,(I_1 - I_2) = 0$$

$$225\,I_1 - 60\,I_2 - 75\,I_3 = 0 \quad \text{...(iv)}$$

Mesh equations for loop 2,

$$-50(I_2 - I_3) - 60\,(I_2 - I_1) - 15I_2 + 30 = 0$$

$$-60\,I_1 + 125\,I_2 - 50\,I_3 = 30 \quad \text{...(v)}$$

Mesh equations for loop 3,

$$-75(I_3 - I_1) - 50\,(I_3 - I_2) = 0$$

$$-75\,I_1 - 50\,I_2 + 125\,I_3 = 0 \quad \text{...(vi)}$$

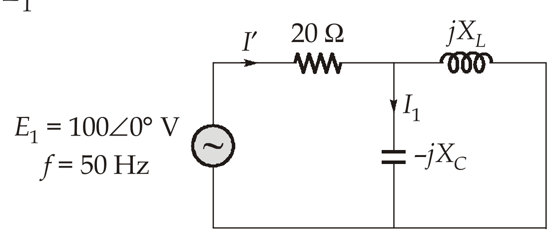
Solving (iv), (v) and (vi) we get

$$I_3 = I = 0.35 \text{ A proved}$$

Q.5 (f) Solution:

The given circuit can be broken into two constituent circuits by considering individual source.

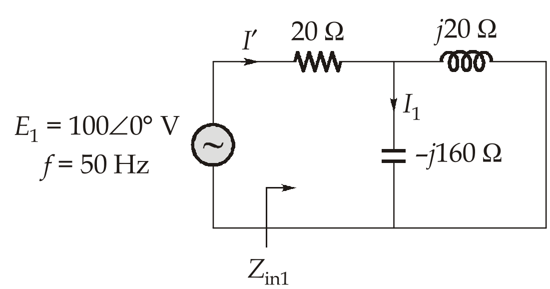
By considering source, E_1



where,

$$X_L = \omega L = 2\pi \times 50 \times 63.33 \times 10^{-3} \simeq 20\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 19.89 \times 10^{-6}} \simeq 160\ \Omega$$



impedance seen from source E_1, Z_{in1}

$$Z_{in1} = 20 + (-j160 \parallel j20)$$

$$= 20 + \frac{-j160 \times j20}{-j140} = 20 + j22.857$$

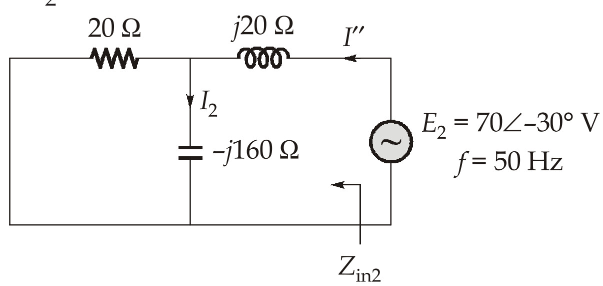
$$Z_{in1} = 30.372\ \angle 48.81°\ \Omega$$

$$\text{Current, } I' = \frac{100\angle 0°}{Z_{in1}} = \frac{100\angle 0°}{30.372\angle 48.81°} = 3.293\angle{-48.81°}\text{A}$$

$$I_1 = I' \times \frac{j20}{-j160 + j20} = 3.293\angle -48.81° \times \frac{j20}{-j140}$$

$$I_1 = 0.47\angle 131.19°\text{A}$$

By considering source, E_2



impedance seen from source E_2,

$$Z_{in2} = j20 + [20 \| (-j160)]$$

$$= j20 + \frac{20 \times (-j160)}{20 - j160}$$

$$= j20 + 19.692 - j2.462$$

$$= 19.692 + j17.538$$

$$Z_{in2} = 26.37\angle 41.69°\ \Omega$$

$$\text{Current, } I'' = \frac{70\angle -30°}{Z_{in2}} = \frac{70\angle -30°}{26.37\angle 41.69°}$$

$$I'' = 2.655\angle{-71.69°}\text{ A}$$

$$I_2 = \frac{I'' \times 20}{20 - j160} = \frac{53.10\angle -71.69°}{161.25\angle -82.87°}$$

$$I_2 = 0.329\angle 11.18°\text{ A}$$

Using superposition theorem,

the total current, $I = I_1 + I_2$

$$= 0.47\angle 131.19° + 0.329\angle 11.18°$$

$$I = 0.42\angle 88.2°\text{A}$$

Q.6 (a) Solution:

(i) From the given Bode plot, it is clear that, there is one pole at origin, one pole at ω_1 and one zero at ω_2.

So, the transfer function can be given as,

$$G(s) = \frac{K\left(1+\frac{s}{\omega_2}\right)}{s\left(1+\frac{s}{\omega_1}\right)}$$

Finding the value of *K*:

With starting slope,

$$-20\log_{10}(\omega) + 20\log_{10}K = M_{dB}$$

At $\omega = 1$ rad/sec,

$$-20\log_{10}(1) + 20\log_{10}K = 0$$

$$20\log_{10}K = 20\log_{10}(1)$$

So, $K = 1$

Finding the value of ω_1 :

From the slope between ω_1 and $\omega = 1$ rad/sec,

$$\frac{-20-0}{\log_{10}(\omega_1)-\log_{10}(1)} = -20$$

$$\log_{10}(\omega_1) - 0 = 1$$

$$\omega_1 = 10 \text{ rad/sec}$$

Finding the value of ω_2 :

From the slope between ω_1 and ω_2,

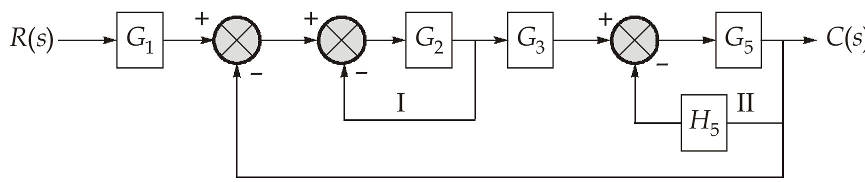
$$\frac{(-60)-(-20)}{\log_{10}(\omega_2)-\log_{10}(\omega_1)} = -40$$

$$\log_{10}\left(\frac{\omega_2}{\omega_1}\right) = 1$$

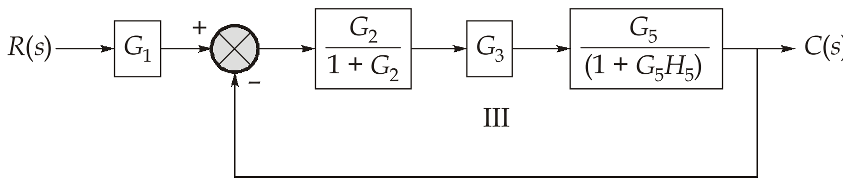
$$\frac{\omega_2}{\omega_1} = 10 \Rightarrow \omega_2 = 10\omega_1 = 100 \text{ rad/sec}$$

So, the transfer function, $G(s) = \dfrac{(1)\left(1+\frac{s}{100}\right)}{s\left(1+\frac{s}{10}\right)} = \dfrac{(s+100)}{10s(s+10)}$

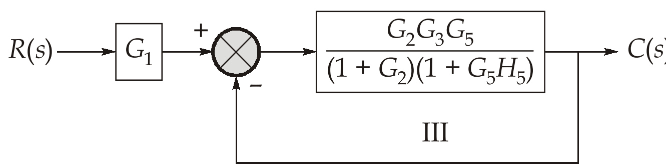
(ii) With $X(s) = 0$, the given block diagram reduces as,



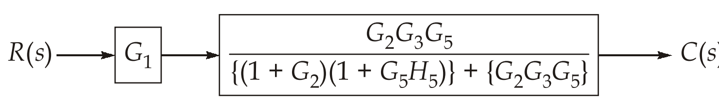
By eliminating feedback loops I, II



Combining the blocks in the loop III,

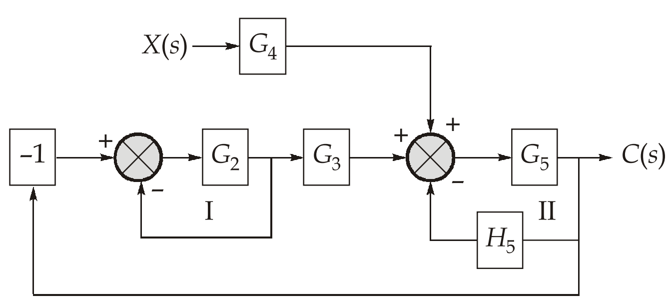


eliminating the feedback loop III,

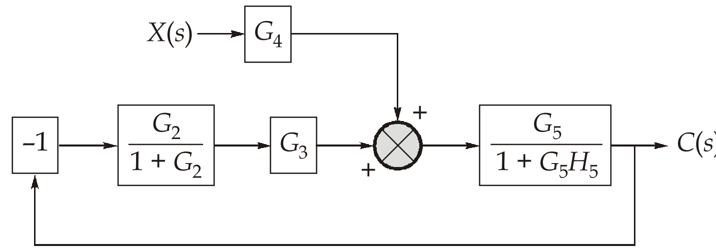


$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5}{\{(1+G_2)(1+G_5H_5)\}+\{G_2G_3G_5\}}$$

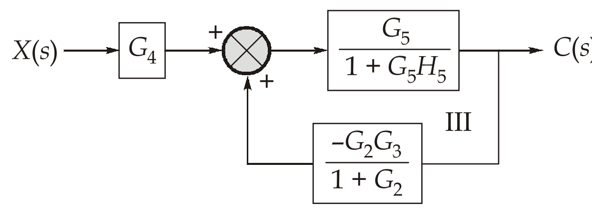
With $R(s) = 0$, G_1, Vanishes, but minus sign at summing point must be considered by introducing block of –1 as shown,

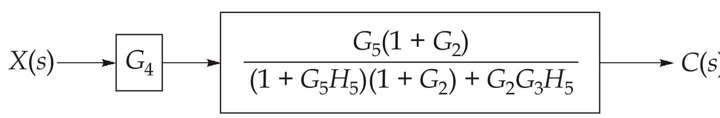


By eliminating feedback loops I, II.



Combine the blocks in series,





$$\therefore \qquad \frac{C(s)}{X(s)} = \frac{G_4 G_5 (1+G_2)}{(1+G_5H_5)(1+G_2)+G_2G_3H_5}$$

Q.6 (b) Solution:

- The Routh table for the given characteristic equation can be formed as follows:

s^6	1	-2	-7	-4
s^5	1	-3	-4	0
s^4	$\frac{-2+3}{1}=1$	$\frac{-7+4}{1}=-3$	-4	0
s^3	0	0	0	0
s^2				
s^1				
s^0				

- All the elements in s^3 row are zeros. If we continue to formulate the Routh table further, then all the elements in the subsequent rows will be infinity. So, this row of zeros causes a difficulty in formulating the complete Routh table.
- The existence of the row of zeros in the Routh table indicates that, some roots of the given characteristic equation are symmetrically located with respect to origin. The number of roots that are symmetrical about the origin is equal to the order of the

auxiliary equation, which can be formed by using the coefficients of the row just above the row of zeros in the Routh table as follows:

$$A(s) = s^4 - 3s^2 - 4 = 0 \quad \text{...(i)}$$

Method-1 to overcome the difficulty:

- The row of zeros can be replaced with the coefficients of the first derivative of the auxiliary equation and the Routh table can be formulated as follows:

$$\frac{dA(s)}{ds} = 4s^3 - 6s + 0 = 0$$

s^6	1	−2	−7	−4
s^5	1	−3	−4	0
s^4	1	−3	−4	0
s^3	4	−6	0	0
s^2	$\frac{-12+6}{4} = -1.5$	−4	0	0
s^1	$\frac{9+16}{-1.5} = -16.67$	0	0	0
s^0	−4	0	0	0

- There is one sign change in the elements of the first column. So, there will be one root of the characteristic equation which lies in the right-half of s-plane. Hence, the given system is unstable.

Method-2 to overcome the difficulty:

- The roots of the auxiliary equation also satisfy the characteristic equation. So, the given characteristic equation can be factorized as follows:

$$\begin{array}{r|l} & s^2 + s + 1 \\ \hline s^4 - 3s^2 - 4 & s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 \\ & s^6 - 3s^4 - 4s^2 \\ \hline & s^5 + s^4 - 3s^3 - 3s^2 - 4s - 4 \\ & s^5 - 3s^3 - 4s \\ \hline & s^4 - 3s^2 - 4 \\ & s^4 - 3s^2 - 4 \\ \hline & 0 \end{array}$$

- So, the given characteristic equation can be written as,

$$(s^4 - 3s^2 - 4)(s^2 + s + 1) = 0 \quad \text{...(ii)}$$

- The roots of auxiliary equation are,

$$s^4 - 3s^2 - 4 = 0$$
$$(s^2 + 1)(s^2 - 4) = 0$$
$$s^2 = -1 \Rightarrow s = -j1, +j1$$
$$s^2 = 4 \Rightarrow s = -2, +2$$

So, one root of auxiliary equation lies in the right-half of s-plane.

- The nature of the roots of "$s^2 + s + 1 = 0$" can be determined as follows:

s^2	1	1
s^1	1	0
s^0	1	0

There is no sign change in the elements of the first column of the Routh table of "$s^2 + s + 1 = 0$". So, all the roots of "$s^2 + s + 1 = 0$" lie in the left-half of s-plane.

- From the above two points, it is clear that, one root of the given characteristic equation lies in the right-half of s-plane. So, the given system is unstable.

To determine all the roots of given characteristic equation:

- The roots of auxiliary equation will also satisfy the given characteristic equation. So, the four possible roots of the given characteristic equation are : $s = -2, +2, -j1, +j1$.
- The remaining two roots of the given characteristic equation can be solved as follows:

$$s^2 + s + 1 = 0$$

$$s = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = -0.5 \pm j\sqrt{0.75}$$

- So, all the six roots of the given characteristic equation are :

$s = -2, +2, -j1, +j1, -0.5 + j\sqrt{0.75}, -0.5 - j\sqrt{0.75}$.

Q.6 (c) Solution:

The open loop transfer function is

$$G(s)H(s) = \frac{10}{(s + 1 + 10k)s}$$

As k is not a multiplying factor, we modify the equation such that k appears as the multiplying factor. Since the characteristic equation is

$$s^2 + s + 10\,ks + 10 = 0$$

We rewrite this equation as follows:–

$$1+\frac{10ks}{s^2+s+10} = 0$$

Let $10k = K$

The above equation becomes,

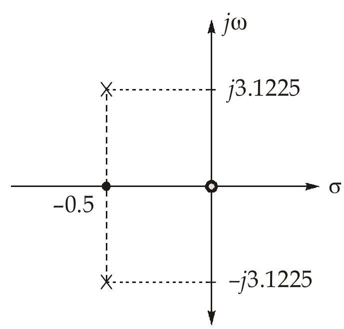
$$1+\frac{Ks}{s^2+s+10} = 0$$

$$\therefore \quad G(s) = \frac{Ks}{s^2+s+10}$$

Step 1: The system has a open-loop zero at $s = 0$, and two open loop poles at

$$s = \frac{-1\pm\sqrt{1-40}}{2} = -0.5\pm j3.1225$$

Pole-zero plot



Step 2: angle of asymptote $\phi_A = \frac{(2k+1)180}{p-z}$

where $k = 0, 1, 2 ..... p\text{-}z\text{–}1$

since $p = 2$, $z = 1$, therefore $k = 0$

$$\therefore \quad \phi_A = 180°$$

$$\text{Centroid} = \sigma = \frac{\Sigma\text{Real part of pole} - \Sigma\text{Real part of zero}}{p-z}$$

$$= \frac{-0.5-0.5-0}{2-1} = -1$$

Step 3: For breakaway point

$$\frac{dK}{ds} = 0$$

$$K = -\left[\frac{s^2+s+10}{s}\right]$$

$$\frac{dK}{ds} = -\left[\frac{s(2s+1)-(s^2+s+10)(1)}{s^2}\right] = 0$$

$$s^2 - 10 = 0$$

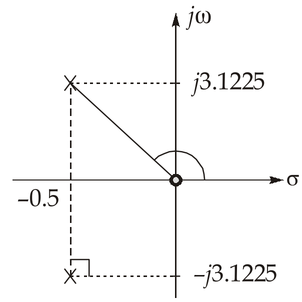
$$\therefore \quad s = \pm\sqrt{10} = \pm 3.16$$

Here, only s = –3.16 is valid breakaway point.

Step 4: angle of departure

$$\phi_d = \pm[180° + \phi]$$

ϕ = Σ angle made by zero at that point – Σ angle made by pole at that point



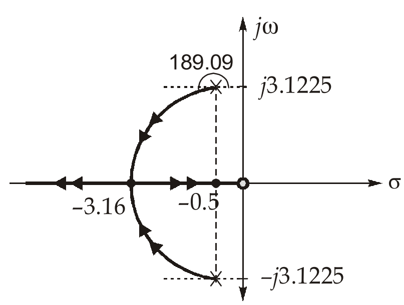
$$\phi_z = -\tan^{-1}\left(\frac{3.1225}{0.5}\right) = 99.1°$$

$$\phi_p = 90°$$

$$\therefore \quad \phi = 99.1° - 90° = 9.097°$$

$$\phi_d = \pm[180° + 9.097°] = \pm 189.09°$$

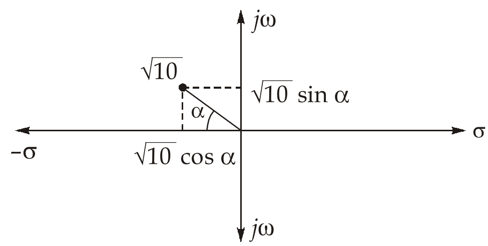
Step 5: Root locus



As we require ξ = 0.7 for the closed loop poles, we find the intersection of the circular root locus and a line having angle

$$\cos^{-1}\xi = \cos^{-1}0.7 = 45.57°$$

This angle is with negative real axis as ξ = 0.7 is an underdamped system. The intersection can be found as:



$$\sigma = -\sqrt{10}\cos\alpha$$

$$\cos\alpha = \cos 45.57° = 0.7$$

$$\sigma = -\sqrt{10}\times 0.7 = -2.214$$

$$\omega = \sqrt{10}\sin\alpha = \sqrt{10}\cdot\sin 45.57° = 2.258$$

The intersection of $\xi = 0.7$ with root locus is at

$$s = -2.214 + j2.258$$

The gain K for this $\quad s = -2.214 + j2.258$

$$K = -\left.\frac{(s^2+s+10)}{s}\right|_{s=(-2.214+j2.258)}$$

$$K = 3.427$$

Hence, desired value of velocity feedback gain

$$k = \frac{K}{10} = 0.3427$$

Q.7 (a) Solution:

$$G(s) = \frac{(s+1)(s+2)}{s^3(s+10)(s+20)}$$

$$G(j\omega) = \frac{(1+j\omega)(2+j\omega)}{(j\omega)^3(10+j\omega)(20+j\omega)}$$

Magnitude, $\quad M = \dfrac{1}{\omega^3}\sqrt{\dfrac{(1+\omega^2)(4+\omega^2)}{(100+\omega^2)(400+\omega^2)}}$

Phase angle, $\phi = -270° + \tan^{-1}(\omega) + \tan^{-1}\left(\dfrac{\omega}{2}\right) - \tan^{-1}\left(\dfrac{\omega}{10}\right) - \tan^{-1}\left(\dfrac{\omega}{20}\right)$

At $\omega = 0$, $\quad M = \infty$ and $\phi = -270°$ (or) $90°$

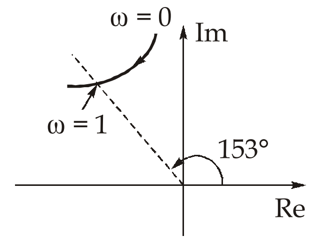
At $\omega = \infty$, $\quad M = 0$ and $\phi = -270°$ (or) $90°$

So, starting angle and ending angle both are "–270°".

At $\omega = 1$, $\phi = -270° + \tan^{-1}(1) + \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{10}\right) - \tan^{-1}\left(\frac{1}{20}\right)$

$= -207°$ (or) $153°$

Finding intersections with 180° line:



$$-270° + \tan^{-1}\left(\frac{\frac{3\omega}{2}}{1-\frac{\omega^2}{2}}\right) - \tan^{-1}\left(\frac{\frac{3\omega}{20}}{1-\frac{\omega^2}{200}}\right) = -180°$$

$$\tan^{-1}\left(\frac{3\omega}{2-\omega^2}\right) - \tan^{-1}\left(\frac{30\omega}{200-\omega^2}\right) = 90°$$

$$\frac{\frac{3\omega}{(2-\omega^2)} - \frac{30\omega}{(200-\omega^2)}}{1+\frac{90\omega^2}{(2-\omega^2)(200-\omega^2)}} = \infty$$

So, $(2 - \omega^2)(200 - \omega^2) + 90\omega^2 = 0$

$\omega^4 - 202\omega^2 + 400 + 90\omega^2 = 0$

$\omega^4 - 112\omega^2 + 400 = 0$

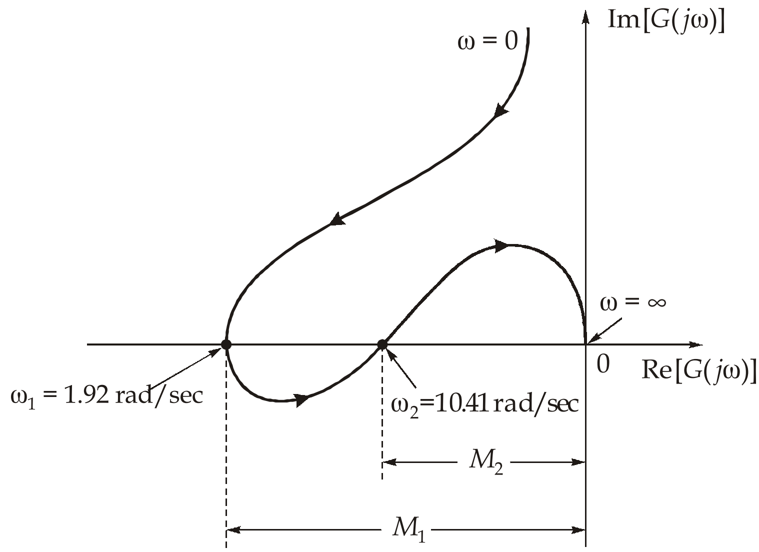
$$\omega^2 = \frac{112 \pm \sqrt{(112)^2 - 4(400)}}{2} = 56 \pm \sqrt{2736} = 3.7, 108.3$$

$$\omega_1 = \sqrt{3.7} = 1.92 \text{ rad/sec}$$

$$\omega_2 = \sqrt{108.3} = 10.41 \text{ rad/sec}$$

So, the polar plot intersects 180° line two times.

The resultant plot can be given as,



The value of M_1 is,

$$M_1 = \frac{1}{\omega_1^3}\sqrt{\frac{(1+\omega_1^2)(4+\omega_1^2)}{(100+\omega_1^2)(400+\omega_1^2)}}$$

$$= \frac{1}{(1.92)^3}\sqrt{\frac{(1+3.7)(4+3.7)}{(100+3.7)(400+3.7)}} = 4.15 \times 10^{-3}$$

The value of M_2 is,

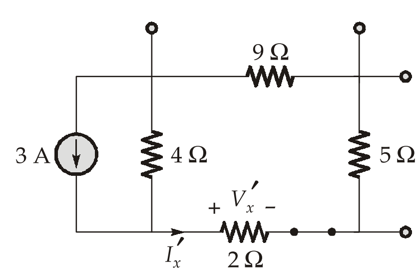
$$M_2 = \frac{1}{\omega_2^3}\sqrt{\frac{(1+\omega_2^2)(4+\omega_2^2)}{(100+\omega_2^2)(400+\omega_2^2)}}$$

$$= \frac{1}{(10.41)^3}\sqrt{\frac{(1+108.3)(4+108.3)}{(100+108.3)(400+108.3)}} = 3 \times 10^{-4}$$

Q.7 (b) Solution:

(i) To determine V_x using superposition theorem :

- When 3 A current source is acting alone,

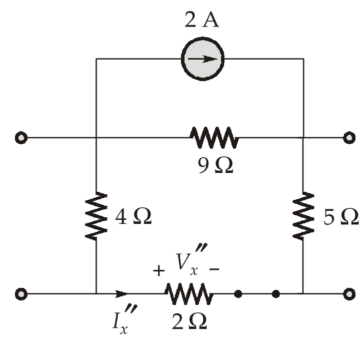


Using current division rule,

$$I_x' = 3\text{ A} \times \frac{4}{9+4+5+2} = 3 \times \frac{4}{20}\text{ A} = \frac{3}{5}\text{ A}$$

$$V_x' = I_x' \times 2\,\Omega = \frac{6}{5}\text{ V} = 1.2\text{ V}$$

- When 2 A current source is acting alone,

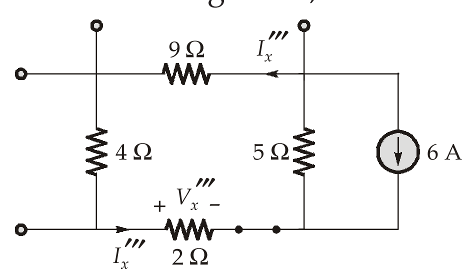


Using current division rule,

$$I_x'' = -2\text{ A} \times \frac{9}{9+4+2+5} = -\frac{18}{20}\text{ A}$$

$$V_x'' = I_x'' \times 2\,\Omega = -1.8\text{ V}$$

- When 6 A current source is acting alone,

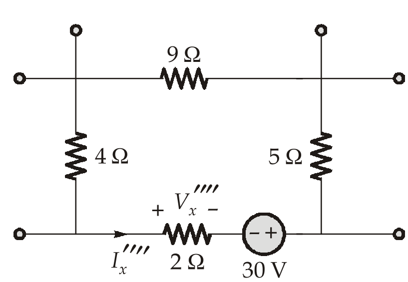


Using current division rule,

$$I_x''' = -6\text{ A} \times \frac{5}{9+4+2+5} = -\frac{3}{2}\text{ A}$$

$$V_x''' = I_x''' \times 2\,\Omega = -3\text{ V}$$

- When 30 V voltage source is acting alone,



$$I_x'''' = \frac{30}{9+5+4+2}\text{ A} = \frac{3}{2}\text{ A}$$

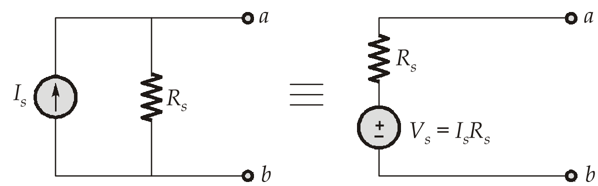
$$V_x'''' = I_x'''' \times 2\,\Omega = 3\text{ V}$$

- When all the 4 sources are acting simultaneously, using superposition theorem,

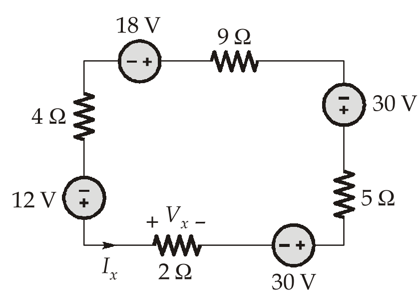
$$V_x = V_x' + V_x'' + V_x''' + V_x'''' = 1.2 - 1.8 - 3 + 3 = -0.6\text{ V}$$

(ii) To determine V_x using source transformation technique:

A practical current source can be transformed into a practical voltage source as shown below.



Using the above property, the given circuit can be simplified as,

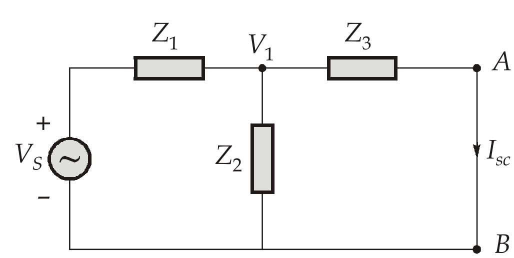


$$I_x = \frac{12+30-30-18}{9+5+4+2}\text{ A} = -\frac{6}{20}\text{ A}$$

$$V_x = I_x \times 2\,\Omega = -\frac{6}{10}\text{ V} = -0.6\text{ V}$$

Q.7 (c) Solution:

To find the current through Z_L by Norton's theorem, let us short the terminals A-B for calculating Norton's current. The network is redrawn and shown in below figure.



Let the voltage at node be V_1. Using nodal analysis,

$$\frac{V_S - V_1}{Z_1} = \frac{V_1}{Z_2} + \frac{V_1}{Z_3}$$

or $$V_1\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) = \frac{V_S}{Z_1}$$

or $$V_1(Y_1 + Y_2 + Y_3) = V_S Y_1$$

or $$V_1 = \frac{Y_1 V_S}{Y_1 + Y_2 + Y_3}$$

Now, $$Y_1 = \frac{1}{Z_1} = \frac{1}{1+j3} = 0.1 - j0.3\ \mho$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{2+j3} = 0.154 - j0.230\ \mho$$

and $$Y_3 = \frac{1}{Z_3} = \frac{1}{3+j2} = 0.230 - j0.154\ \mho$$

∴ $$V_1 = \left(\frac{0.1 - j0.3}{0.484 - j0.684}\right) \times (40 + j0)$$

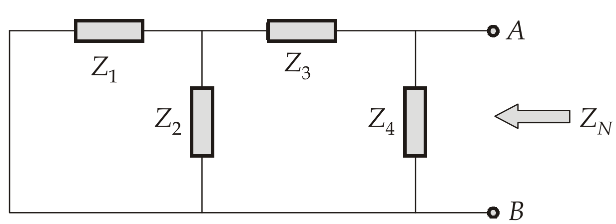
$$= 14.45 - j4.375 = 15.09 \angle -16.85° \text{ V}$$

Now, $$I_{sc} = \frac{V_1}{Z_3} = V_1 Y_3$$

$$= (14.45 - j4.375) \times (0.230 - j0.154) = 2.65 - j3.23$$

$$= 4.18 \angle -50.63° \text{ A}$$

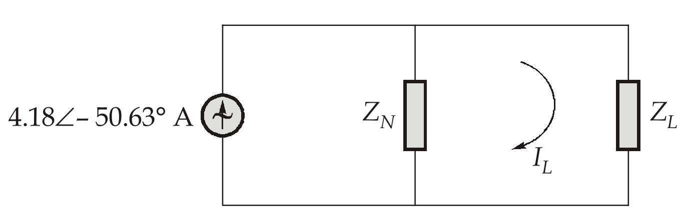
To find Norton impedance, the circuit is redrawn as shown:



From the above figure, $$Z_N = [(Z_1 \parallel Z_2) + Z_3] \parallel Z_4 = (3.74 + j3.54) \parallel (1 + j2)$$

$$= 0.850 + j1.33 = 1.578 \angle 57.41° \ \Omega$$

The Norton equivalent circuit is shown in figure below.



From the above figure, we get,

$$I_L = I_{sc}\left(\frac{Z_N}{Z_N + Z_L}\right)$$

$$= (2.65 - j3.23)\left(\frac{0.850 + j1.33}{0.850 + j1.33 + 1 - j3}\right)$$

$$= 1.74 + j2 = 2.65 \angle 48.97° \text{ A}$$

Q.8 (a) Solution:

Put $s = j\omega$

$$G(j\omega) = \frac{K(j\omega)^3}{(j\omega + 1)(j\omega + 2)}$$

or $$G(j\omega) = \frac{-jK\omega^3}{(2-\omega^2) + j3\omega}$$

or $$G(j\omega) = \frac{-jK\omega^3\left((2-\omega^2) - j3\omega\right)}{(2-\omega^2)^2 + (3\omega)^2}$$

$$G(j\omega) = \frac{-3K\omega^4}{(2-\omega^2)^2 + (3\omega)^2} - \frac{jK\omega^3(2-\omega^2)}{(2-\omega^2)^2 + (3\omega)^2}$$

(i) $\lim\limits_{\omega \to 0} G(j\omega)$: Re$(G(j0)) \to -0$, Img $(Gj0) \to +j0$

(ii) The intersection of $G(j\omega)$ with positive real axis.

$$\text{Img}\, G(j\omega) = 0$$

$$\frac{-K\omega^3(2-\omega^2)}{(2-\omega^2)^2 + (3\omega)^2} = 0$$

$$2 - \omega^2 = 0$$

$\Rightarrow$ $$\omega = \pm\sqrt{2} \text{ rad/sec}$$

The intersection is obtained by putting $\omega = \sqrt{2}$ in real part of $G(j\omega)$ i.e.,

$$G(j\sqrt{2}) = \frac{-3K(\sqrt{2})^4}{\left(2-\sqrt{2^2}\right)^2 + (3\sqrt{2})^2} = \frac{-3K \times (2)(2)}{(2-2)^2 + 3 \times 3 \times 2}$$

or $$G(j\sqrt{2}) = -\frac{2}{3}K$$

(iii) $\underset{\omega \to \infty}{G(j\omega)}$: Re $G(j\infty) = -3K$

$$\text{Img}\, G(j\infty) = +j\infty$$

As per step (i), (ii) and (iii), general shape of the Nyquist plot is drawn as shown below:

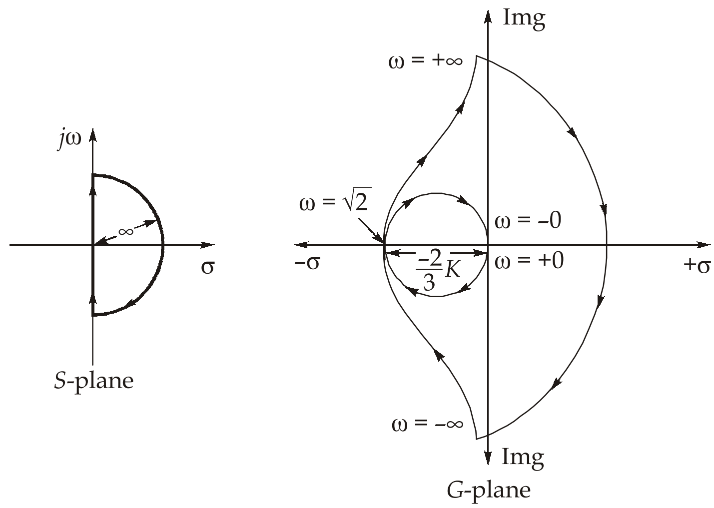


Fig: Nyquist plot for the system

$$\underset{\omega\to\infty}{G(j\omega)} = \frac{K(j\omega)^3}{(j\omega+1)(j\omega+2)} = Kj\omega = Kj(\infty)$$

$$\underset{\omega\to\infty}{G(j\omega)} = |\infty|\underline{|90^\circ}$$

The closing of Nyquist plot from $\omega = \infty$ to $\omega = -\infty$ is as explained below:

In s-plane, the RHS region is closed from $s = j\infty$ to $s = -j\infty$ through a semi circle of infinite radius in clockwise direction, hence the corresponding points in $G(s)$-plane i.e., $G(j\infty)$ to $G(-j\infty)$ are closed through a semi circle of infinite radius to account for effective numerator term i.e., $Kj\infty$ of $G(j\omega)$ as $\omega \to \infty$.

It is seen that number of poles of $G(s)$ having positive real part is nil i.e., $P_+ = 0$. The encirclement of critical point $(-1 + j0)$ are determined below:

1. If $K < \frac{3}{2}$

The critical point $(-1 + j0)$ lies outside the Nyquist plot, hence $N = 0$.

$\therefore$
$$N = P_+ - Z_+$$
$$0 = 0 - Z_+$$
$$Z_+ = \text{Nil}$$

The system is stable for $K < \frac{3}{2}$

2. $K > \frac{3}{2}$

The critical point $(-1 + j0)$ is encircled twice in clockwise direction of Nyquist plot, hence

$$N = -2$$
$$N = P_+ - Z_+$$
$$-2 = 0 - Z+$$
$$\therefore \quad Z_+ = 2$$

System is unstable for $K > \dfrac{3}{2}$

For stability $K \le \dfrac{3}{2}$

Q.8 (b) Solution:

When terminals of network are open circuited, the circuit takes the form shown below in fig. (b). By KVL in the left-hand mesh,

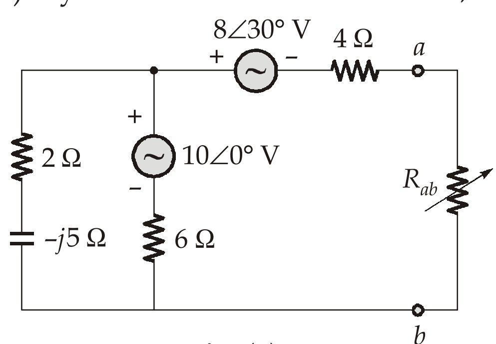


Fig. (a)

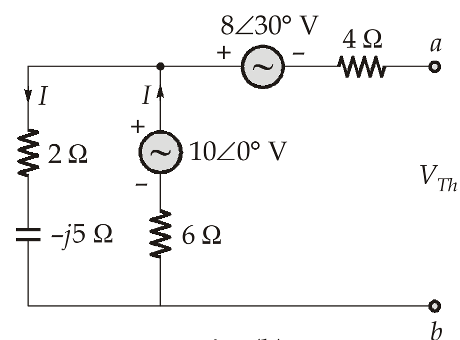


Fig. (b)

$$(2 - j5)I - 10\angle 0° + 6I = 0$$

$$I = \frac{10\angle 0°}{8 - j5} = \frac{10\angle 0°}{9.434\angle -32°} = 1.06\angle 32°\text{A}$$

By KVL in the right-hand mesh in fig. (b)

$$-6I + 10\angle 0° - 8\angle 30° - 0 \times 4 - V_{Th} = 0$$

$$V_{Th} = -6 \times 1.06\angle 32° + 10\angle 0° - 8\angle 30°$$
$$= -5.39 - j3.37 + 10 - 6.928 - j4$$
$$= -2.318 - j7.37 = 7.726\angle -107.46° \text{ V}$$

Determination of Z_{th}:

The voltage sources are short circuited as shown in figure (c).

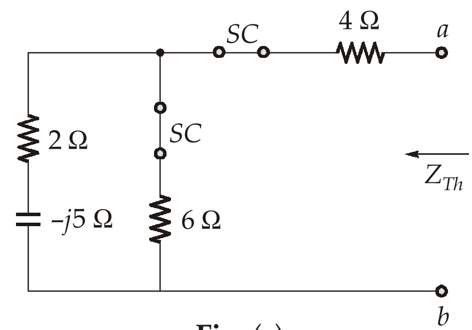


Fig. (c)

The Thevenin impedance Z_{Th} is the impedance between the open-circuited terminals *a*-*b* given as

$$
\begin{aligned}
Z_{\text{Th}} &= 4 + [6 \,||\, (2 - j5)] \\
&= 4 + \frac{6(2 - j5)}{6 + 2 - j5} \\
&= 4 + \frac{32.31\angle -68.2^\circ}{9.434\angle -32^\circ} = 4 + 3.4248\angle -36.2^\circ \\
&= 4 + 2.7637 - j2.02 \\
&= (6.7637 - j2.02)\Omega
\end{aligned}
$$

Determination of current through R_{ab}

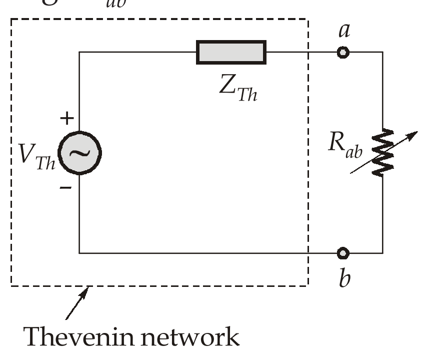


Fig. (d)

Figure (d) shows the Thevenin equivalent network in which R_{ab} is reconnected at terminals *a*-*b*. The current through R_{ab} is given by

$$I_{ab} = \frac{V_{Th}}{Z_{Th} + R_{ab}}$$

Since the resistance of R_{ab} is varied from 2 Ω to 10 Ω, maximum current through R_{ab} is obtained when R_{ab} = 2 Ω.

$$(I_{ab})_{\max} = \frac{7.726\angle -107.46^\circ}{6.7637 - j2.02 + 2}$$

$$= \frac{7.726\angle -107.46°}{8.993\angle -12.98°} = 0.859\angle -94.48° \text{A}$$

Minimum current through R_{ab} is obtained when R_{ab} = 10 Ω.

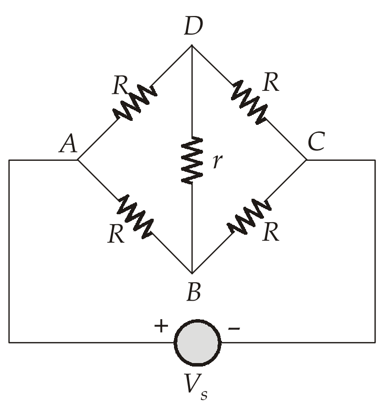
$$(I_{ab})_{\text{min}} = \frac{7.726\angle -107.46°}{6.7637 - j2.02 + 10} = 0.457\angle -100.59° \text{ A}$$

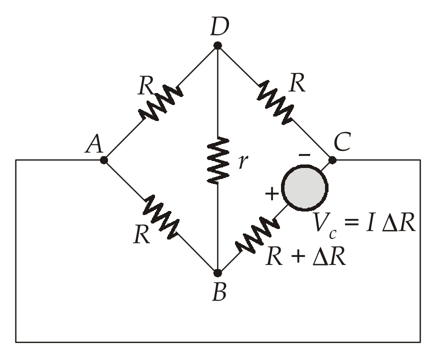
Hence, the current through R_{ab} varies from 0.859 ∠–94.48° A to 0.457 ∠–100.59° A

Q.8 (c) Solution:

The given circuit is balanced. Hence, V_{BD} = 0. Current in each resistor $I = (V_s/2R)$. When the resistor in branch BC is changed from R to $(R + \Delta R)$, the change in current in any branch may be calculated by assuming that an ideal voltage source of voltage $(I\Delta R)$ is connected in series with $(R + \Delta R)$ and the voltage source V_s is replaced by a short circuit. Such an arrangement is shown in below figure. From these figures,

$$R_1 = R \| \left(r + \frac{R}{2}\right)$$





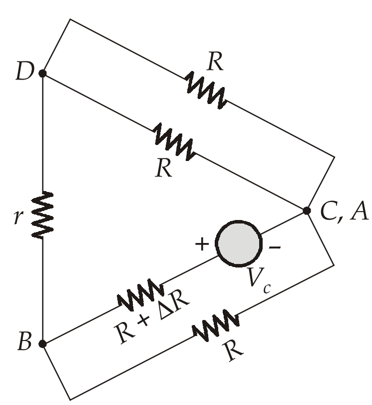


Fig. (a)

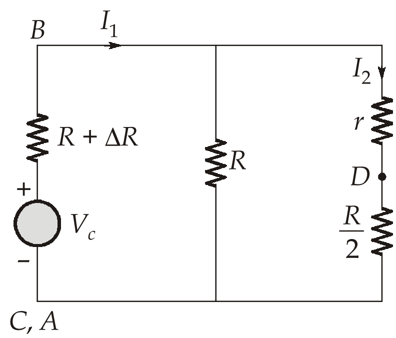


Fig. (b)

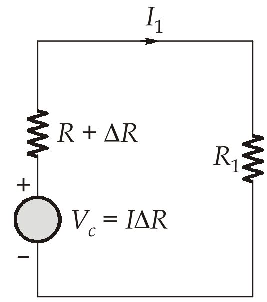


Fig. (c)

From fig. (b)
$$R_1 = \frac{R(r+R/2)}{R+r+R/2} = \frac{R(R+2r)}{3R+2r}$$

From fig. (c),
$$I_1 = \frac{I\Delta R}{R+\Delta R+R_1}$$

From fig. (b),
$$I_2 = \frac{R}{R+r+R/2}I_1 = \frac{2R}{3R+2r}I_1$$

Voltage drop across BD is

$$V_{BD} = rI_2 = \frac{2RrI\,\Delta R}{(3R+2r)(R+\Delta R+R_1)}$$

$$= \frac{2Rr(V_s/2R)\Delta R}{(3R+2r)\left[R+\Delta R+\dfrac{R(R+2r)}{3R+2r}\right]}$$

$$= \frac{V_s r\,\Delta R}{(R+\Delta R)(3R+2r)+R(R+2r)}$$

$$= \frac{V_s r\,\Delta R}{4R(R+r)+\Delta R(3R+2r)}$$

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