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Detailed Solutions

ESE-2023 Mains Test Series

Electrical Engineering Test No : 2

Section A : Systems and Signal Processing + Microprocessors

Q.1 (a) Solution:

Given :

$$x(n) = \alpha^n u(n)$$

$$h(n) = \alpha^{-n} u(-n); \quad 0 < \alpha < 1$$

Taking z-transform,

$$X(z) = \frac{z}{z-\alpha}; \quad |z| > |\alpha|$$

$$H(z) = \frac{1}{(1-\alpha z)} = \frac{-1}{\alpha\left(z - \frac{1}{\alpha}\right)}; \quad |z| < \frac{1}{|\alpha|}$$

The response,

$$Y(z) = X(z).H(z)$$

$$Y(z) = -\frac{1}{\alpha}\cdot\frac{z}{(z-\alpha)\left(z-\frac{1}{\alpha}\right)}; \quad \alpha < |z| < \frac{1}{\alpha}$$

By using partial fraction

$$\frac{Y(z)}{z} = \frac{-1}{\alpha}\cdot\frac{1}{(z-\alpha)\left(z-\frac{1}{\alpha}\right)} = \frac{-1}{\alpha}\left[\frac{A}{z-\alpha}+\frac{B}{z-\frac{1}{\alpha}}\right]$$

where
$$A = \left.\frac{1}{z-\frac{1}{\alpha}}\right|_{z=\alpha} \Rightarrow A = \frac{-\alpha}{1-\alpha^2}$$

$$B = \left.\frac{1}{z-\alpha}\right|_{z=\frac{1}{\alpha}} \Rightarrow B = \frac{\alpha}{1-\alpha^2}$$

$$\frac{Y(z)}{z} = \frac{-1}{\alpha}\left[\frac{\frac{-\alpha}{(1-\alpha^2)}}{z-\alpha}+\frac{\frac{\alpha}{(1-\alpha^2)}}{z-\frac{1}{\alpha}}\right]$$

$$Y(z) = \frac{1}{1-\alpha^2}\left[\frac{z}{z-\alpha}-\frac{z}{z-\frac{1}{\alpha}}\right]; \quad \alpha < |z| < \frac{1}{\alpha}$$

Taking inverse z-transform,

$$y(n) = \frac{1}{1-\alpha^2}\left[(\alpha)^n u(n) - \left\{-\left(\frac{1}{\alpha}\right)^n u(-n-1)\right\}\right]$$

$$y(n) = \frac{1}{1-\alpha^2}\left[(\alpha)^n u(n) + (\alpha)^{-n} u(-n-1)\right]$$

$$y(n) = \frac{1}{1-\alpha^2}\cdot\alpha^{|n|}$$

Q.1 (b) Solution:

(i) **Logical Operations :** These instructions perform various logical operations with the content of the accumulator.

- **AND, OR, Exclusive-OR :** Any 8-bit number, or the contents of a register, or of a memory location can be logically ANDed, ORed or Exclusive-ORed with the contents of the accumulator. The results are stored in the accumulator. e.g., ORA H.

- **Rotate :** Each bit in the accumulator can be shifted either left or right to the next position. e.g., RLC.
- **Compare :** Any 8-bit number, or the content of a register, or a memory location can be compared for equality, greater than or less than, with the contents of the accumualtor. e.g., CMP.
- **Complement :** The contents of the accumulator can be complemented; all 0s are replaced by 1s and all 1s are replaced by 0s. e.g., CMC.

(ii) **Branching Operations :** The group of instructions that alter the sequence of program execution either conditionally or unconditionally.

- **Jump :** Conditional jumps are an important aspect of the decision-making process in programming. These instructions test for a certain condition (e.g., zero or carry flag) and alter the program sequence when the condition is met. In addition, the instruction set includes an instruction called unconditional jump. e.g., JMP.
- **Call, Return and Restart :** These instructions change the sequence of a program either by calling a subroutine or returning from a subroutine. The conditional call and return instructions also can test condition flags. e.g., CALL 2023H

Q.1 (c) Solution:

(i)
$$X(s) = \frac{s^2+6s+7}{s^2+3s+2} = 1 + \frac{3s+5}{(s^2+3s+2)}$$

$$X_1(s) = \frac{3s+5}{(s^2+3s+2)} = \frac{3s+5}{(s+1)(s+2)}$$

Using partial fraction,

$$X_1(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

where,
$$A = \left.\frac{3s+5}{s+2}\right|_{s=-1} \Rightarrow A = 2$$

$$B = \left.\frac{3s+5}{s+1}\right|_{s=-2} \Rightarrow B = 1$$

$$X(s) = 1 + \frac{2}{s+1} + \frac{1}{s+2}; \quad \text{Re}\{s\} > -1$$

Taking inverse laplace transform,

$$x(t) = \delta(t) + [2e^{-t} + e^{-2t}]u(t)$$

(ii)
$$X(s) = \frac{5s+13}{s(s^2+4s+13)}$$

Using partial fraction,

$$X(s) = \frac{A}{s} + \frac{Bs+C}{s^2+4s+13}$$

$$5s + 13 = (A + B)s^2 + (4A + C)s + 13A$$

On comparing, we get,

$$13A = 13 \Rightarrow A = 1$$

$$A + B = 0 \Rightarrow B = -A \Rightarrow B = -1$$

$$4A + C = 5 \Rightarrow C = 5 - 4A \Rightarrow C = 1$$

$$X(s) = \frac{1}{s} + \frac{-s+1}{s^2+4s+13}$$

$$X(s) = \frac{1}{s} - \frac{s-1}{(s+2)^2+(3)^2}$$

$$= \frac{1}{s} - \frac{s+2-2-1}{(s+2)^2+(3)^2}$$

$$= \frac{1}{s} - \frac{(s+2)}{(s+2)^2+(3)^2} + \frac{3}{(s+2)^2+(3)^2}; \quad \text{Re}\{s\} > 0$$

Taking inverse laplace transform,

$$x(t) = u(t) - e^{-2t}\cos 3t\, u(t) + e^{-2t}\sin 3t\, u(t)$$

$$x(t) = [1 - e^{-2t}(\cos 3t - \sin 3t)]u(t)$$

Q.1 (d) Solution:

8086 has 16-bit flag register, and there are 9 valid flag bits. The flag bits are changed to 0 or 1 depending upon the value of result after arithmetic and logical operations. The format of flag register is

D_{15}	D_{14}	D_{13}	D_{12}	D_{11}	D_{10}	D_9	D_8	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0
×	×	×	×	O	D	I	T	S	Z	×	AC	×	P	×	CY

These flag bits are divided into two sections :

(i) Status flags

(ii) Control flags

(i) **Status Flags :** In 8086, there are 6 different flags which are set or reset after 8-bit or 16-bit operations.

- **Sign Flag (S) :** After any operation if MSB is 1, then it indicates that the number is negative and this flag is set to 1 else reset to 0.
- **Zero Flag (Z) :** If the result is 0 after any arithmetical or logical operation, this flag is set to 1 otherwise it becomes reset, i.e., 0.
- **Auxiliary Carry (AC) :** When some arithmetic operations generate carry after the lower half and sends it to upper half, the AC will be 1, otherwise it will be 0.
- **Parity Flag (P) :** This is even parity flag. When result has even number of 1, it will be set to 1; otherwise 0.
- **Carry Flag (CY) :** This is a carry bit. If a carry is generating after any operation this flag is set to 1; otherwise it will be 0.
- **Overflow Flag (O) :** This flag is set to 1 when the result of a signed operation is too large to fit in the number of bits available to represent it, otherwise reset, i.e., 0.

(ii) **Control Flags :** In 8086, there are 3 different flags which are used to enable or disable some basic operations of the microprocessor.

- **Directional Flag (D) :** This flag is used in string related operations, i.e., if $D = 1$, then the string will be accessed from higher memory address to lower memory address, and if $D = 0$, it will do the reverse.
- **Interrupt Flag (I) :** If $I = 1$, then MPU will recognize the interrupts from peripherals. For $I = 0$, the interrupts will be ignored.
- **Trap Flag (T) :** This flag is used for on-chip debugging. When $T = 1$, it will work in a single step mode. After each instruction, one internal interrupt is generated. It helps to execute some program instruction by instruction.

Q.1 (e) Solution:

(i) Static or Dynamic :

$$y(n) = |x(n)|$$

1. The output is the magnitude of present input samples. Hence, the system is static.
2. **Linear or Non-Linear :**

$$y(n) = T\{x(n)\} = |x(n)|$$

$$y_1(n) = T\{x_1(n)\} = |x_1(n)|$$

$$y_2(n) = T\{x_2(n)\} = |x_2(n)|$$

The response of the system to linear combination of two inputs $x_1(n)$ and $x_2(n)$ will be,

$$y_3(n) = T\{a_1x_1(n) + a_2x_2(n)\} = |a_1x_1(n) + a_2x_2(n)|$$

Now, the linear combination of two output is

$$y'_3(n) = a_1y_1(n) + a_2y_2(n) = a_1|x_1(n)| + a_2\,|\,x_2(n)|$$

Here, $y_3(n) \neq y'_3(n)$

Hence, the system is non-linear.

3. **Time Varying or Time-Invariant :** Delaying the input by 'k' samples output will be

$$y(n, k) = T\{x(n-k)\} = |x(n-k)|$$

And the delayed output will be

$$y(n-k) = |x(n-k)|$$

Since, $y(n, k) = y(n-k)$

Hence, the system is time-invariant.

4. **Causal and Non-Causal :** The output depends upon present input. Hence, the system is causal.

5. **Stable or Unstable :** For the given equation, it is clear that as long as $x(n)$ is bounded, $y(n)$ will be bounded. Hence, the system is stable.

(ii) $y(n) = \text{sgn}[x(n)]$

1. **Static or Dynamic :** Since the output depends on present input only, then this system is static.

2. **Linear or Non-linear :** The given system equation is

$$y(n) = T\{x(n)\} = \text{sgn}[x(n)]$$

The response due to two different inputs

$$y_1(n) = T\{x_1(n)\} = \text{sgn}[x_1(n)]$$

$$y_2(n) = T\{x_2(n)\} = \text{sgn}[x_2(n)]$$

The response of the system to linear combination of inputs.

$$y_3(n) = T\{a_1x_1(n) + a_2x_2(n)\} = \text{sgn}[a_1x_1(n) + a_2x_2(n)]$$

The linear combination of two outputs given by equations

$$y'_3(n) = a_1y_1(n) + a_2y_2(n) = a_1\,\text{sgn}[x_1(n)] + a_2\,\text{sgn}[x_2(n)]$$

Here, $y_3(n) \neq y'_3(n)$

Hence, the system is non-linear.

3. **Time-Varying or Time-Invariant :** The resposne due to delayed input

$$y(n, k) = T\{x(n_s - k)\} = \text{sgn}[x(n-k)]$$

And the delayed output will be

$$y(n-k) = \text{sgn}[x(n-k)]$$

Here, $y(n, k) = y(n-k)$

Hence, the given system is time-invariant.

4. **Causal or Non-Causal :** Since $y(n)$ depends upon the present input. Hence, the system is causal.

5. **Stable or Unstable :** Since $y(n) = \text{sgn}[x(n)]$ has a value of ±1 depending upon 'n', this is a stable system.

Q.2 (a) Solution:

We know that, for the N-point DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N};\ k = 0, 1, ..., N-1$$

Here, $N = 8$

$$X(k) = \sum_{n=0}^{7} x(n)e^{-\frac{j2\pi nk}{8}} = \sum_{n=0}^{7} x(n)e^{\frac{-j\pi nk}{4}}$$

$k = 0$:

$$X(0) = \sum_{n=0}^{7} x(n) = 1+1+2+2+3+3+0+0$$

$$= 12$$

$k = 1$:

$$X(1) = \sum_{n=0}^{7} x(n)e^{\frac{-j\pi n}{4}}$$

$$= 1 + e^{\frac{-j\pi}{4}} + 2e^{\frac{-j2\pi}{4}} + 2e^{\frac{-j3\pi}{4}} + 3e^{\frac{-j4\pi}{4}} + 3e^{\frac{-j5\pi}{4}}$$

$$= 1 + \left(\frac{1-j}{\sqrt{2}}\right) + 2(-j) + 2\left(\frac{-1-j}{\sqrt{2}}\right) + 3(-1) + 3\left(\frac{-1+j}{\sqrt{2}}\right)$$

$$= -2 - 2\sqrt{2} - j2 = -4.828 - j2$$

$k = 2:$

$$X(2) = \sum_{n=0}^{7} x(n)e^{\frac{-j\pi n}{2}}$$

$$= 1 + e^{\frac{-j\pi}{2}} + 2e^{-j\pi} + 2e^{\frac{-j3\pi}{2}} + 3e^{-j2\pi} + 3e^{\frac{-j5\pi}{2}}$$

$$= 1 + (-j) + 2(-1) + 2(j) + 3(1) + 3(-j)$$

$$X(2) = 2 - j2$$

$k = 3:$

$$X(3) = \sum_{n=0}^{7} x(n)e^{\frac{-j3\pi n}{4}}$$

$$= 1 + e^{\frac{-j3\pi}{4}} + 2e^{\frac{-j3\pi}{2}} + 2e^{\frac{-j9\pi}{4}} + 3e^{-j3\pi} + 3e^{\frac{-j15\pi}{4}}$$

$$= 1 + \frac{-(1+j)}{\sqrt{2}} + 2(j) + 2\left(\frac{1-j}{\sqrt{2}}\right) + 3(-1) + 3\left(\frac{1+j}{\sqrt{2}}\right)$$

$$X(3) = -2 + 2\sqrt{2} + j2 = 0.828 + j2$$

$k = 4:$

$$X(4) = \sum_{n=0}^{7} x(n)e^{-j\pi n}$$

$$= 1 + e^{-j\pi} + 2e^{-j2\pi} + 2e^{-j3\pi} + 3e^{-j4\pi} + 3e^{-j5\pi}$$

$$= 1 - 1 + 2(1) + 2(-1) + 3(1) + 3(-1)$$

$$X(4) = 0$$

$k = 5:$

$$X(5) = \sum_{n=0}^{7} x(n)e^{\frac{-j5\pi n}{4}}$$

$$= 1 + e^{\frac{-j5\pi}{4}} + 2e^{\frac{-j5\pi}{2}} + 2e^{\frac{-j15\pi}{4}} + 3e^{-j5\pi} + 3e^{\frac{-j25\pi}{4}}$$

$$= 1 + \left(\frac{1-j}{\sqrt{2}}\right) + 2(-j) + 2\left(\frac{1+j}{\sqrt{2}}\right) + 3(-1) + 3\left(\frac{1-j}{\sqrt{2}}\right)$$

$$X(5) = -2 + 2\sqrt{2} - j2 = 0.828 - j2$$

$k = 6:$

$$X(6) = \sum_{n=0}^{7} x(n) e^{\frac{-j6\pi n}{4}}$$

$$= 1 + e^{\frac{-j3\pi}{2}} + 2e^{-j3\pi} + 2e^{\frac{-j9\pi}{2}} + 3e^{-j6\pi} + 3e^{\frac{-j15\pi}{2}}$$

$$X(6) = 1 + j + 2(-1) + 2(-j) + 3(1) + 3(j)$$

$$= 2 + j2$$

$k = 7:$

$$X(7) = \sum_{n=0}^{7} x(n) e^{\frac{-j7\pi n}{4}}$$

$$= 1 + e^{\frac{-j7\pi}{4}} + 2e^{\frac{-j7\pi}{2}} + 2e^{\frac{-j21\pi}{4}} + 3e^{-j7\pi} + 3e^{\frac{-j35\pi}{4}}$$

$$= 1 + \left(\frac{1+j}{\sqrt{2}}\right) + 2(j) + 2\left(\frac{-1+j}{\sqrt{2}}\right) + 3(-1) + 3\left(\frac{-1-j}{\sqrt{2}}\right)$$

$$X(7) = -2 - 2\sqrt{2} + j2 = -4.828 + j2$$

Therefore, $X(k) = \{12, -4.828 - j2, 2 - j2, 0.828 + j2, 0, 0.828 - j2, 2 + j2, -4.828 + j2\}$

Q.2 (b) Solution:

Algorithm :

1. Load a 16-bit number from memory 2040 H into a register pair (H–L).
2. Move content of register L to the accumulator.
3. Complement the content of the accumulator.
4. Move the content of accumulator to the register L.
5. Move the content of register H to the accumulator.
6. Complement the content of the accumulator.
7. Move the content of accumulator to the register H.
8. Store the content of register pair in memory 2050 H (1's complement).
9. Increment the content of register pair by 1.
10. Store the content of register pair in memory 2052 H (2's complement).
11. Stop

Program :

LHLD 2040H;	Load H-L register pair with the content present at 2040H.
MOV A, L;	Move the content of register L into the accumulator.
CMA;	Complement the content of accumulator.
MOV L, A;	Move the content of accumulator into register L.
MOV A, H;	Move the content of register H into the accumulator.
CMA;	Complement the content of accumulator.
MOV H, A;	Move the content of accumulator into the register H.
SHLD 2050H;	Store the data from register pair H-L into memory 2050H (1's complement).
INX H;	Increase the H-L register pair by 1.
SHLD 2052H;	Store the data from register pair H-L into memory 2052H (2's complement)
HLT;	Stop

Q.2 (c) Solution:

(i) Given :

$$H(\omega) = \frac{4 + j\omega}{6 - \omega^2 + 5j\omega}$$

(a)

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{4 + j\omega}{6 + (j\omega)^2 + 5j\omega}$$

$$(j\omega)^2 Y(\omega) + 5j\omega Y(\omega) + 6Y(\omega) = j\omega X(\omega) + 4X(\omega)$$

Taking inverse fourier transform to obtain the differential equation relating the input $x(t)$ and output $y(t)$ of system S :

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

(b) Given :

$$H(\omega) = \frac{4 + j\omega}{6 + (j\omega)^2 + 5j\omega}$$

$$H(\omega) = \frac{4 + j\omega}{(2 + j\omega)(3 + j\omega)}$$

Using partial fraction

$$H(\omega) = \frac{A}{2 + j\omega} + \frac{B}{3 + j\omega}$$

where, $$A = \left.\frac{4+j\omega}{3+j\omega}\right|_{j\omega=-2} \Rightarrow A = 2$$

$$B = \left.\frac{4+j\omega}{2+j\omega}\right|_{j\omega=-3} \Rightarrow B = -1$$

$$H(\omega) = \frac{2}{2+j\omega} - \frac{1}{3+j\omega}$$

Taking inverse fourier transform,

$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$$

$$h(t) = (2e^{-2t} - e^{-3t})u(t)$$

(c) Given : $$x(t) = e^{-4t}u(t) - te^{-4t}u(t)$$

Taking fourier transform,

$$X(\omega) = \frac{1}{4+j\omega} - \frac{1}{(4+j\omega)^2} = \frac{3+j\omega}{(4+j\omega)^2}$$

The response, $$Y(\omega) = X(\omega).H(\omega)$$

$$Y(\omega) = \frac{3+j\omega}{(4+j\omega)^2}\cdot\frac{4+j\omega}{(3+j\omega)(2+j\omega)} = \frac{1}{(4+j\omega)(2+j\omega)}$$

Using partial fraction,

$$Y(\omega) = \frac{A}{2+j\omega} + \frac{B}{4+j\omega}$$

where, $$A = \left.\frac{1}{4+j\omega}\right|_{j\omega=-2} \Rightarrow A = \frac{1}{2}$$

$$B = \left.\frac{1}{2+j\omega}\right|_{j\omega=-4} \Rightarrow B = \frac{-1}{2}$$

$$Y(\omega) = \frac{\frac{1}{2}}{2+j\omega} + \frac{-\frac{1}{2}}{4+j\omega}$$

Taking inverse fourier transform,

$$y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t)$$

$$y(t) = \frac{1}{2}[e^{-2t} - e^{-4t}]u(t)$$

(ii) Given :

$$x(n) = \{1, \underset{\uparrow}{3}, 0, 4, -2\}$$

$$h(n) = \{2, 4, \underset{\uparrow}{-1}, -3\}$$

$$y(n) = x(n) * h(n)$$

$$x(n) = \delta(n+1) + 3\delta(n) + 4\delta(n-2)\delta - 2\delta(n-3)$$

$$h(n) = 2\delta(n+2) + 4\delta(n+1) - \delta(n) - 3\delta(n-1)$$

$$y(n) = [\delta(n+1) + 3\delta(n) + 4\delta(n-2) - 2\delta(n-3)] * [2\delta(n+2) + 4\delta(n+1) - \delta(n) - 3\delta(n-1)]$$

$$= 2\delta(n+3) + 4\delta(n+2) - \delta(n+1) - 3\delta(n) + 6\delta(n+2) + 12\delta(n+1) - 3\delta(n) - 9\delta(n-1) + 8\delta(n) + 16\delta(n-1) - 4\delta(n-2) - 12\delta(n-3) - 4\delta(n-1) - 8\delta(n-2) + 2\delta(n-3) + 6\delta(n-4)$$

$$y(n) = 2\delta(n+3) + 10\delta(n+2) + 11\delta(n+1) + 2\delta(n) + 3\delta(n-1) - 12\delta(n-2) - 10\delta(n-3) + 6\delta(\mathrm{n}-4)$$

$$y(n) = \{2, 10, 11, \underset{\uparrow}{2}, 3, -12, -10, 6\}$$

Q.3 (a) Solution:

(i) Status Pins (S_2, S_1, S_0) : The status pins $\bar{S}_0, \bar{S}_1, \bar{S}_2$ of 8086 are available at pin 26, 27 and 28 respectively. These pins are used by the 8288 bus controller for generating (all the memory and I/O operations) access control signals :

S_2	S_1	S_0	Status
0	0	0	Interrupt acknowledgement
0	0	1	I/O Read
0	1	0	I/O Write
0	1	1	Halt
1	0	0	Opcode fetch
1	0	1	Memory Read
1	1	0	Memory Write
1	1	1	Passive State

Queue Status Pins (Q_{S0}, Q_{S1}) : These are queue status signals and are available at pin 24 and 25. These signals provide the status of instruction queue.

Q_{s0}	Q_{s1}	Status
0	0	No operation
0	1	First byte of opcode from the queue
1	0	Empty the queue
1	1	Subsequent byte from the queue

(ii) The pointer registers are SP and BP while index registers are SI and DI.

All four are 16-bit registers and are used to store offset address of memory locations relative to sequent registers. They act as Memory Pointers. As an example, MOV AH, [SI] imples "move the byte whose address is contained in SI into AH". If now, SI = 2000H, then execution of above instruction will put the value of FFH in register AH as shown in table, [SI + 1 : SI] = ABFFH, where obviously SI + 1 points to memory location 2001H and [SI + 1] = ABH.

2005 H	0A	
2004 H	07	
2003 H	85	
2002 H	90	
2001 H	AB	
2000 H	FF	← SI

SI is pointing to memory location 2000 H

SI and DI are used as general purpose registers. Again in certain string (block move) instrctions, SI and DI are used as source and destination index registers respectively. For such cases, contents of SI are added to content of DS register to get the actual source address of data while the contents of DI are added to the contents of ES to get the actual destination address of data.

SP and BP stands for Stack Pointer and Base Pointer with SP containing the offset address or the stack top address. The actual stack address is computed by adding the contents of SP and SS.

Data area(s) may exist in stack, to access such data area in stack segment, BP register is used which may contains the offset address. BP register is also used as a general purpose register.

Instruction Pointer (IP) is also included in index pointer group. IP points to the offset instead of the actual address of the next instruction to be fetched (from current code segment) in BIU. IP resides in BIU but cannot be programmed by the programmer.

Q.3 (b) Solution:

The desired response of filter is

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-2\pi/3} e^{j\omega n}d\omega + \int_{-\pi/3}^{\pi/3} e^{j\omega n}d\omega + \int_{2\pi/3}^{\pi} e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi n}\left[\frac{e^{j\omega n}}{j}\Bigg|_{-\pi}^{-2\pi/3} + \frac{e^{j\omega n}}{j}\Bigg|_{-\pi/3}^{\pi/3} + \frac{e^{j\omega n}}{j}\Bigg|_{2\pi/3}^{\pi}\right]$$

$$= \frac{1}{\pi n}\left[\frac{e^{jn\pi} - e^{-jn\pi}}{2j} + \frac{e^{j\frac{\pi}{3}n} - e^{-j\frac{\pi}{3}n}}{2j} + \frac{e^{-j\frac{2\pi}{3}n} - e^{j\frac{2\pi}{3}n}}{2j}\right]$$

$$h_d(n) = \frac{1}{\pi n}\left[\sin \pi n + \sin\frac{\pi}{3}n - \sin\frac{2\pi}{3}n\right]$$

$\because$ $h_d(n) = h_d(-n)$

Therefore, filter coefficients are symmetrical about $n = 0$.

Now, for $n = 0$

$$h_d(0) = \lim_{n\to 0}\frac{1}{n\pi}\left[\sin \pi n + \sin\frac{\pi}{3}n - \sin\frac{2\pi}{3}n\right]$$

$$= \lim_{n\to 0}\left[\frac{\sin \pi n}{\pi n} + \frac{\sin\left(\frac{\pi}{3}\right)n}{3\left(\frac{\pi}{3}\right)n} - \frac{2}{3}\frac{\sin\left(\frac{2\pi}{3}\right)n}{\left(\frac{2\pi}{3}n\right)}\right]$$

$$h_d(0) = 1 + \frac{1}{3} - \frac{2}{3} = \frac{2}{3} = 0.667$$

For $n = 1$,

$$h_d(1) = h_d(-1) = \frac{1}{\pi}\left[\sin\pi + \sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right)\right] = 0$$

For $n = 2$,

$$h_d(2) = h_d(-2) = \frac{1}{2\pi}\left[\sin 2\pi + \sin\left(\frac{2\pi}{3}\right) - \sin\left(\frac{4\pi}{3}\right)\right] = 0.2757$$

For $n = 3$,

$$h_d(3) = h_d(-3) = \frac{1}{3\pi}[\sin 3\pi + \sin\pi - \sin(2\pi)] = 0$$

For $n = 4$,

$$h_d(4) = h_d(-4) = \frac{1}{4\pi}\left[\sin 4\pi + \sin\frac{4\pi}{3} - \sin\frac{8\pi}{3}\right] = -0.1378$$

For $n = 5$,

$$h_d(5) = h_d(-5) = \frac{1}{5\pi}\left[\sin 5\pi + \sin\left(\frac{5\pi}{3}\right) - \sin\left(\frac{10\pi}{3}\right)\right] = 0$$

Since,
$$h(n) = \begin{cases} h_d(n), & |n| \le 5 \\ 0, & \text{otherwise} \end{cases}$$

The transfer function of the filter is

$$H(z) = \sum_{n=-(M-1)/z}^{(M-1)/z} h(n)z^{-n}$$

$$H(z) = -0.1378z^{-4} + 0.2757z^{2} + 0.667 + 0.2757z^{-2} - 0.1378z^{-4}$$

Since, $h(n) = h_d(n)$ for $|n| \le 5$. This system is not practically reliable because $h(n) \neq 0$ for $n < 0$, i.e., non-causal system. And as we know that non-causal system are not practically reliable.

Now, our purpose is to make it a causal system. Therefore, impulse response of causal-type hand reject filter will be

$$h_c(n) = h(n-5) = h_d(n-5)$$

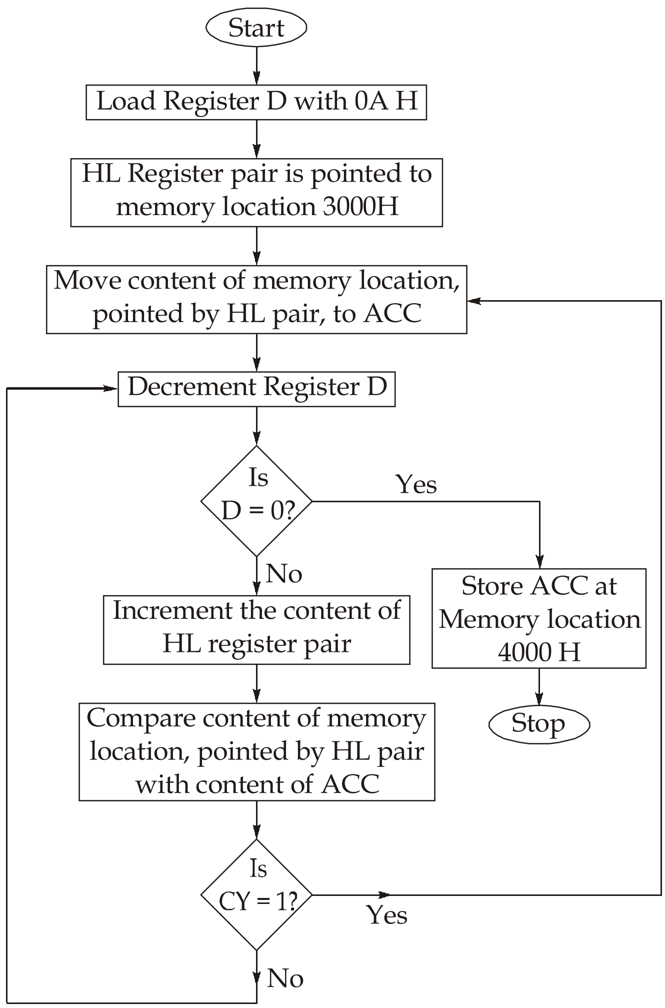
$$H_c(z) = z^{-5}H(z)$$

$$H_c(z) = z^{-5}[-0.1378z^{4} + 0.2757z^{2} + 0.667 + 0.2757z^{-2} - 0.1378z^{-4}]$$

$$= -0.1378z^{-1} + 0.2757z^{-3} + 0.667z^{-5} + 0.2757z^{-7} - 0.1378z^{-9}$$

Therefore,
$$H_c^{(ej\omega)} = e^{-j5\omega}[0.667 + 0.5514\cos 2\omega - 0.2756\cos 4\omega]$$

Q.3 (c) Solution:



8085 Assembly Language Program :

```
        MOV D, 0AH
        LXI H, 3000H
SAVE :  MOV A, M
NEXT :  DCR D
        JZ STOP
        INX H
        CMP M
        JC SAVE
```

JNC NEXT

STOP : STA 4000H

HLT

Q.4 (a) Solution:

(i) This problem, we will proceed using the eigen function property of LSI system.

If the input to an LSI system is $x(n) = \cos \omega_o n$, then the response will be

$$y(n) = \left|H(e^{j\omega_o})\right|\cos(n\omega_o + \phi_n(\omega_o))$$

Therefore, we need to find the frequency response of the system we know the unit sample response of ideal low pass filter,

$$H_1(e^{j\omega}) = \begin{cases} 1; & |\omega| \le \omega_c \\ 0; & \omega_c < |\omega| \le \pi \end{cases}$$

Therefore, $$h_1(n) = \frac{\sin n\omega_c}{\pi n}$$

Because, $h(n) = 2h_1(n - 1)$ with $\omega_c = \frac{\pi}{2}$, an expression may be derived for $H(e^{j\omega})$ in terms of $H_1(e^{j\omega})$ as follows :

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-jn\omega} = \sum_{n=-\infty}^{\infty} h_1(n-1)\cdot e^{-j\omega n}$$

$$= 2\sum_{n=-\infty}^{\infty} h_1(n)e^{-j(n+1)\omega} = 2e^{-j\omega}\sum_{n=-\infty}^{\infty} h_1(n)e^{-j\omega n}$$

$$= 2e^{-j\omega}H_1(e^{j\omega})$$

Therefore, $$H(e^{j\omega}) = \begin{cases} 2e^{-j\omega}, & |\omega| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\omega| \le \pi \end{cases}$$

Because $|H(e^{j\omega})| = 0$ at $\omega = \frac{3\pi}{4}$, the sinusoid in x(n) is filtered out and the output is simply.

$$y(n) = 2\left|H(e^{j\pi/4})\right|\cos\left(\frac{n\pi}{4} + \phi_n\left(\frac{\pi}{4}\right)\right)$$

$$y(n) = 4\cos\left(\frac{n\pi}{4} - \frac{\pi}{4}\right) = 4\cos\left((n-1)\frac{\pi}{4}\right)$$

(ii) First we take the one-sided z-transform of each term in the difference equation.

$$Y(z) = z^{-1}Y(z) + Y(-1) - [z^{-2}Y(z) + z^{-1}Y(-1) + Y(-2)] + \frac{1}{2}X(z) + \frac{1}{2}z^{-1}X(z)$$

Substituting the given values for the initial condition, we have

$$Y(z) = z^{-1}Y(z) + \frac{3}{4} - z^{-2}Y(z) - \frac{3}{4}z^{-1} - \frac{1}{4} + \frac{1}{2}X(z) + \frac{1}{2}z^{-1}X(z)$$

Because
$$x(n) = \left(\frac{1}{2}\right)^n u(n) \Rightarrow X(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)}; \quad |z| > \frac{1}{2}$$

which gives
$$Y(z) = \frac{\frac{1}{2} - \frac{3}{4}z^{-1}}{1 - z^{-1} + z^{-2}} + \frac{\frac{1}{2} + \frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)(1 - z^{-1} + z^{-2})}$$

Expanding the second term using partial fraction expansion, we have

$$Y(z) = \frac{\frac{1}{2} - \frac{3}{4}z^{-1}}{1 - z^{-1} + z^{-2}} + \frac{\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{z^{-1}}{1 - z^{-1} + z^{-2}}$$

$$Y(z) = \frac{\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{1}{2} + \frac{1}{4}z^{-1}}{1 - z^{-1} + z^{-2}}$$

Therefore,
$$y(n) = \left(\frac{1}{2}\right)^{n+1} u(n) + \left[\frac{\sqrt{3}}{6}\sin\left(\frac{4\pi}{3}\right) + \frac{\sqrt{3}}{3}\sin(n-1)\frac{\pi}{3}\right]u(n)$$

Q.4 (b) Solution:

Step 1 :

$x(0) = 1$	$x(4) = 4$
$x(1) = 2$	$x(5) = 3$
$x(2) = 3$	$x(6) = 2$
$x(3) = 4$	$x(7) = 1$

Step 2 :

$$W_N^k = e^{j\left(\frac{2\pi}{N}\right)k} \quad \text{where } N = 8 \text{ and } k = 0, 1, 2, 3$$

for $k = 0$:

$$W_8^0 = e^{-j\left(\frac{2\pi}{8}\right)\times 0} = e^0 = 1$$

for $k = 1$:

$$W_8^1 = e^{-j\left(\frac{2\pi}{8}\right)\times 1} = e^{\frac{-j\pi}{4}} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} = 0.707 - j0.707$$

for $k = 2$:

$$W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)\times 2} = e^{\frac{-j\pi}{2}} = \frac{\cos\pi}{2} - \frac{j\sin\pi}{2} = -j$$

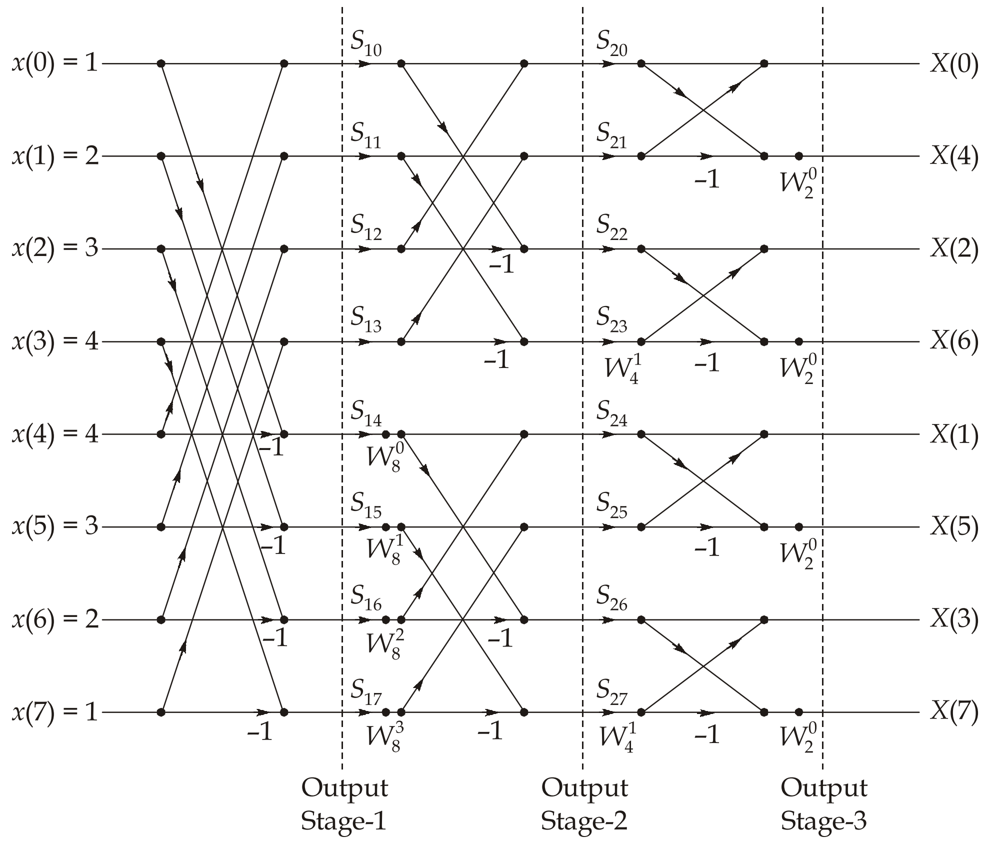
for $k = 3$:

$$W_8^3 = e^{-j\left(\frac{2\pi}{8}\right)\times 3} = e^{\frac{-j3\pi}{4}} = \cos\left(\frac{3\pi}{4}\right) - j\sin\left(\frac{3\pi}{4}\right)$$

$$= -0.707 - j0.707$$

Using DIF FFT algorithm, we can compute $X(k)$ from the given sequence $x(n)$ as shown below :

Step 3 :



Step 4 : Stage-wise calculations

Output of Stage-1 :

$$S_{10} = x(0) + x(4) = 1 + 4 = 5$$
$$S_{11} = x(1) + x(5) = 2 + 3 = 5$$
$$S_{12} = x(2) + x(6) = 3 + 2 = 5$$
$$S_{13} = x(3) + x(7) = 4 + 1 = 5$$
$$S_{14} = x(0) - x(4) = 1 - 4 = -3$$
$$S_{15} = x(1) - x(5) = 2 - 3 = -1$$
$$S_{16} = x(2) - x(0) = 3 - 2 = 1$$
$$S_{17} = x(3) - x(7) = 4 - 1 = 3$$

Output of Stage-2 :

$$S_{20} = S_{10} + S_{12} = 10$$
$$S_{21} = S_{11} + S_{13} = 10$$
$$S_{22} = S_{10} - S_{12} = 5 - 5 = 0$$

$$S_{23} = S_{11} - S_{13} = 5 - 5 = 0$$
$$S_{24} = S_{14} + W_8^2.S_{16} = -3 - j$$
$$S_{25} = S_{15}.W_8^1 + S_{17}.W_8^3 = -2.828 - j1.414$$
$$S_{26} = S_{14}.W_8^0 + S_{16}.W_8^2 = -3 + j$$
$$S_{27} = S_{15}.W_8^1 - S_{17}.W_8^3 = 1.414 + j2.828$$

Output of Stage-3 :

$$X(0) = S_{20} + S_{21} = 20$$
$$X(4) = S_{21} - S_{21} = 10 - 10 = 0$$
$$X(2) = S_{22} + S_{23}W_4^1 = 0 + 0 = 0$$
$$X(6) = S_{22} - W_4^1 S_{23} = 0 - 0 = 0$$
$$X(1) = S_{24} + S_{25} = -5.828 - j2.414$$
$$X(5) = S_{24} - S_{25} = -0.172 + j0.414$$
$$X(6) = S_{26} + W^1{}_4 S_{27}$$
$$= -3 + j - j(1.414 + j2.828) = -0.172 - j0.414$$
$$X(7) = S_{26} - W^1{}_4 S_{27}$$
$$= -3 + j + j(1.414 + j2.828) = -5.828 + j2.414$$

Therefore, $X(k) = \{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)\}$

$$X(k) = \{20, 0, 0, 0, -5.828 - j2.414, -0.172 + j0.414, -0.172 - j0.414, -5.828 + j2.414\}$$

Q.4 (c) Solution:

(i) To implement this system using a lattice filter structure, we must find the reflection coefficients that generate the polynomial $H(z)$.

First, however, it is necessary to normalize $H(z)$ so that the first coefficient is unity.

$$H(z) = 8[1 + 0.5z^{-1} + 0.25z^{-2} + 0.125z^{-3}]$$

Now, with

$$H_3(z) = 1 + 0.5z^{-1} + 0.25z^{-2} + 0.125z^{-3}$$

We see that $k_3 = 0.125$

Next, we generate the second order system $H_2(z)$ using the step down recursion.

$$H_2(z) = \frac{1}{1-k_3^2}[H_3(z) - k_3 z^{-3} H_3(z^{-1})]$$

$$= \frac{1}{1-(0.125)^2}(1 + 0.5z^{-1} + 0.25z^{-2} + 0.125z^{-3} - 0.125z^{-3}(1 + 0.5z^{1} + 0.25z^{2} + 0.125z^{3})]$$

$$H_2(z) = 1 + 0.4762z^{-1} + 0.1905z^{-2}$$

Therefore, $k_2 = 0.1905$

Finally, we have

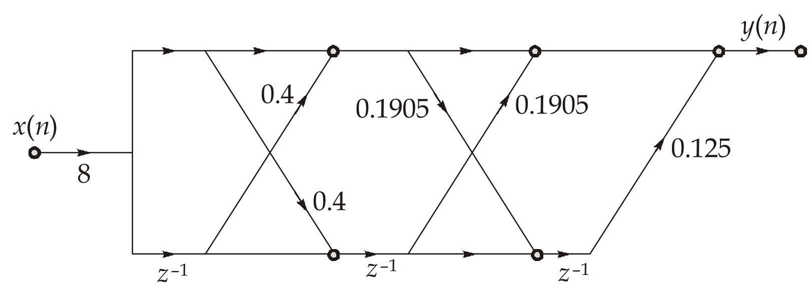
$$H_1(z) = \frac{1}{1-k_2^2}[H(z) - k_2 z^{-2} H(z^{-1})]$$

$$= \frac{1}{1-(0.1905)^2}[1 + 0.4762z^{-1} + 0.1905z^{-2} - 0.1905z^{-2}(1 + 0.4762z + 0.1905z^{2})]$$

$$H_1(z) = 1 + 0.4z^{-1}$$

Therefore, $k_1 = 0.4$

Thus, the lattice filter structure is shown in figure below.



(ii) The program to transfer a string of data microprocessor can be written as follows :

```
        LXI H, 4000 H
        LXI D, 5000 H
        MVI C, 10H
AGAIN   MOV A, M
        STAX D
        INX H
        INX D
        DCR C
        JNZ AGAIN
        HLT
```

Section B : Electrical Circuits - 1 + Control Systems - 1

Q.5 (a) Solution:

Given : The characteristic equation is

$$1 + G(s) = 0$$

$$1+\frac{K(s+1)}{s(s-1)(s^2+4s+20)} = 0$$

$$s^4 + 3s^3 + 16s^2 + (K-20)s + K = 0$$

The Routh array is

$$\begin{array}{c|ccc} s^4 & 1 & 16 & K \\ s^3 & 3 & K-20 & \\ s^2 & \dfrac{68-K}{3} & K & \\ s^1 & \dfrac{\left(\dfrac{68-K}{3}\right)(K-20)-3K}{(68-K)/3} & & \\ s^0 & K & & \end{array}$$

For the system to be stable,

$$68 - K > 0 \Rightarrow K < 68$$

$$\frac{79K-K^2-1360}{68-K} > 0$$

$$K^2 - 79K + 1360 < 0$$

$$(K - 53.65)(K - 25.35) < 0$$

and $K > 0$, $K > 25.35$ and $K < 53.65$ combining these conditions, the range of K is

$$25.35 < K < 53.65$$

Two critical value of K are

$$K = 25.35 \text{ and } K = 53.65$$

From s^1 row, for $K = 25.35$

For oscillation frequency,

$$\left(\frac{68-K}{3}\right)s^2 + K = 0$$

$$\frac{(68-25.35)}{3}s^2 + 25.35 = 0$$

$$s = \pm j1.335$$

The oscillation frequency, $\omega_1 = 1.335$ rad/sec.

For $k = 53.65$

$$\frac{68-53.65}{3}s^2 + 53.65 = 0$$

$$s = \pm j3.349$$

The oscillation frequency, $\omega_2 = 3.349$ rad/sec

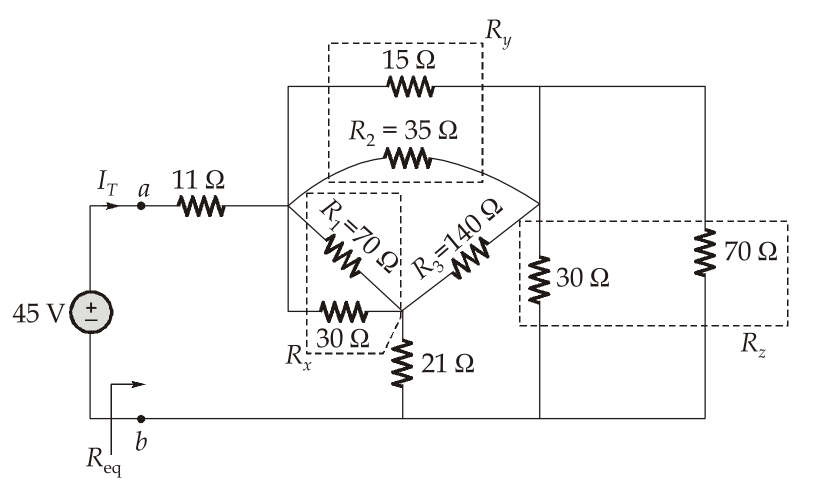
Q.5 (b) Solution:

Convert Y to Δ, we get

$$R_1 = \frac{10\times20+20\times40+10\times40}{20} = 70\ \Omega$$

$$R_2 = \frac{10\times20+20\times40+10\times40}{40} = 35\ \Omega$$

$$R_3 = \frac{10\times20+20\times40+10\times40}{10} = 140\ \Omega$$

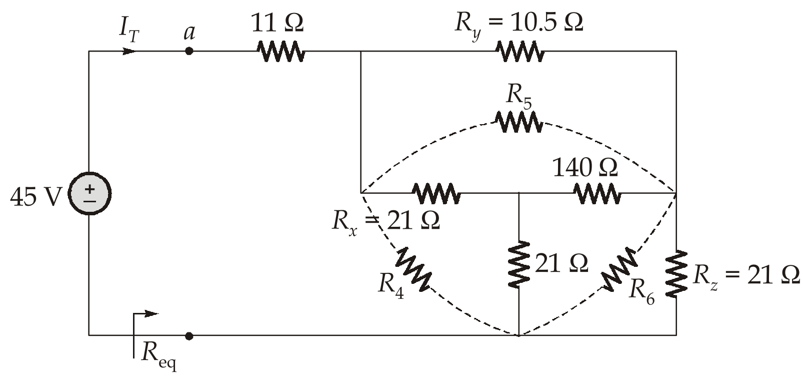


$$R_x = 30 \,||\, 70 = \frac{30\times70}{30+70} = 21\ \Omega$$

$$R_y = 15 \,||\, 35 = \frac{15\times35}{15+35} = 10.5\ \Omega$$

$$R_z = 30 \parallel 70 = \frac{30 \times 70}{30 + 70} = 21\ \Omega$$

The circuit reduces to

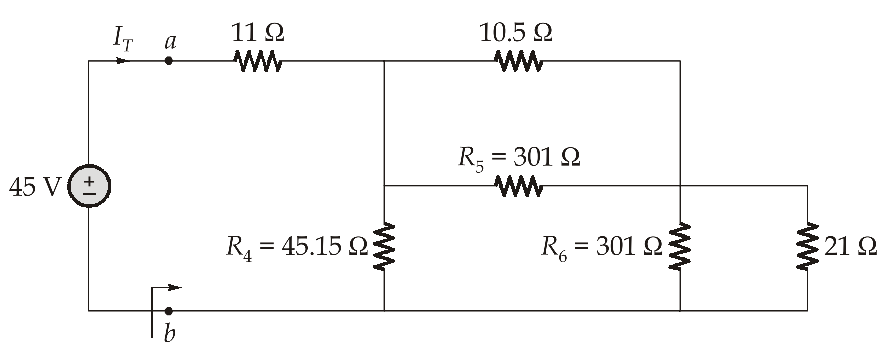


Convert Y to Δ, we obtain :

$$R_4 = \frac{21 \times 21 + 21 \times 140 + 21 \times 140}{140} = 45.15\ \Omega$$

$$R_5 = R_6 = \frac{21 \times 21 + 21 \times 140 + 21 \times 140}{21} = 301\ \Omega$$

The circuit reduces to



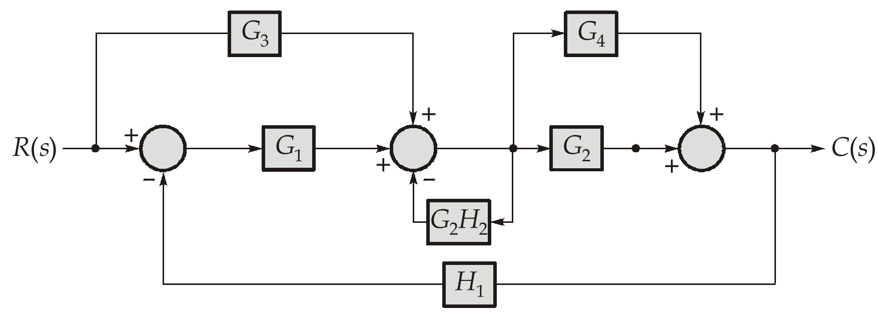
$$R_{eq} = 11 + 45.15 \parallel [(10.5 \parallel 301) + (21 \parallel 301)]$$

$$R_{eq} = 28.943\ \Omega$$

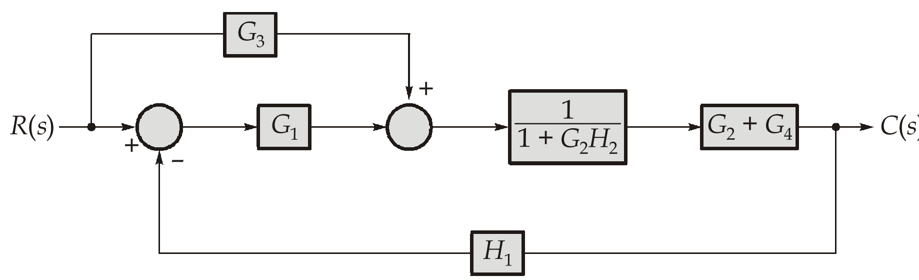
Total current, $$I_T = \frac{45}{R_{eq}} = \frac{45}{28.943} = 1.555\ \text{A}$$

Q.5 (c) Solution:

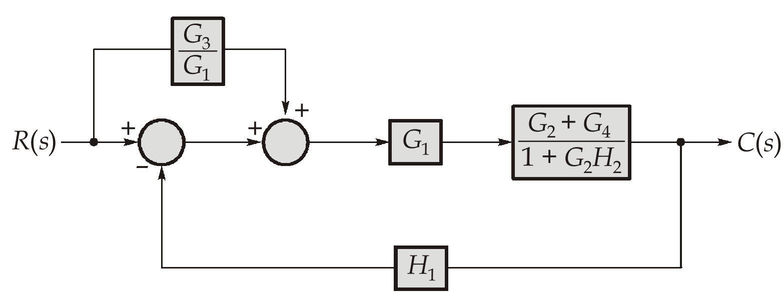
Step-1 : Shift the take-off point on the right of G_2 to its left.



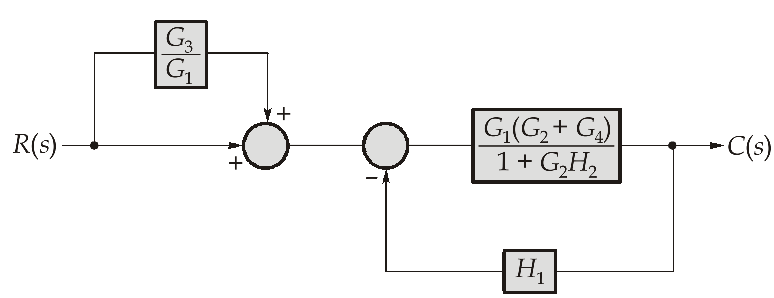
Step-2 : Eliminate the inner feedback path and combine the blocks in parallel.



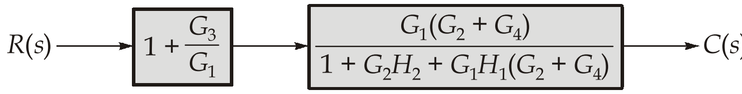
Step-3 : Move the summing junction on the right of the block G_1 to its left and combine the blocks in cascade.



Step-4 : Interchange the summing junction and combine the blocks in cascade.



Step-5 : Combine the blocks in parallel and in cascade.



Transfer function :

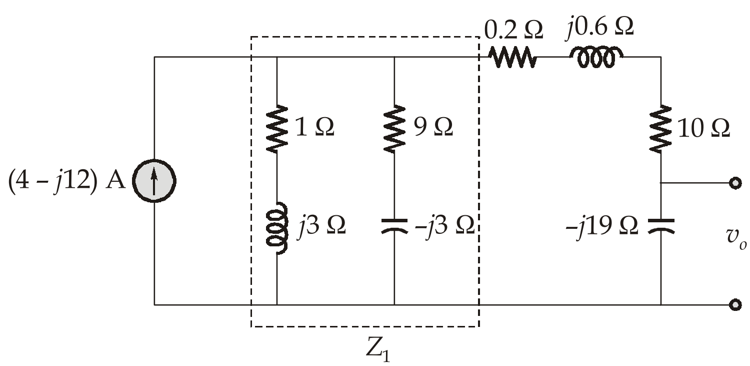
$$\frac{C(s)}{R(s)} = \frac{(G_1 + G_3)(G_2 + G_4)}{1 + G_2H_2 + G_1G_2H_1 + G_1G_4H_1}$$

Q.5 (d) Solution:

Step-1 : Replace the $40\angle 0°$ V voltage source and the impedance $(1 + j3)$ Ω with parallel combination of a current source and the impedance.

$$I = \frac{40\angle 0°}{1 + j3} = (4 - j12) \text{ A}$$

The circuit reduces to



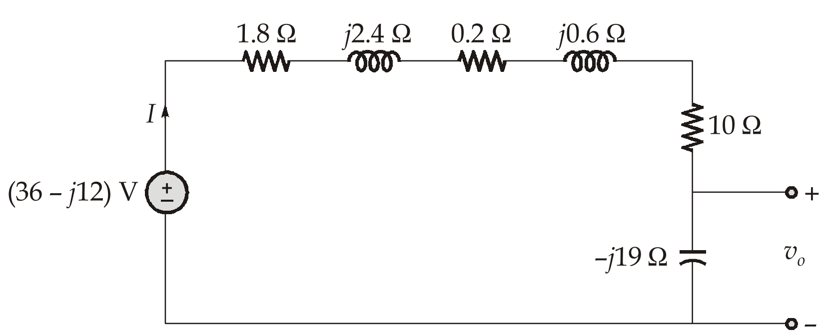
$$z_1 = (1 + j3) \parallel (9 - j3) = \frac{(1 + j3)(9 - j3)}{1 + j3 + 9 - j3}$$

$$z_1 = (1.8 + j2.4) \ \Omega$$

Apply source transformation for the current source and impedance z_1

$$V = (4 - j12)(1.8 + j2.4) = (36 - j12) \text{ V}$$

The circuit reduces to



Total current,

$$I = \frac{36 - j12}{1.8 + j2.4 + 0.2 + j0.6 + 10 - j19}$$

$$I = 1.897\angle 34.7° \text{ Amp}$$

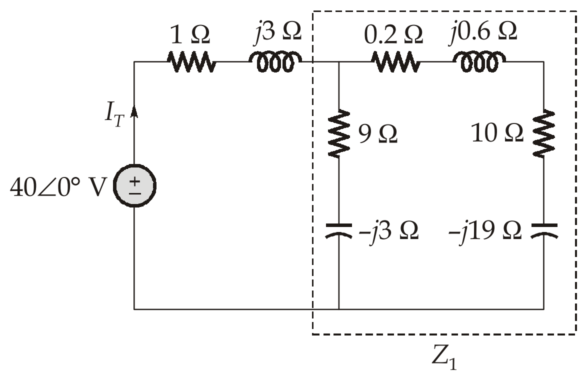
The voltage,

$$v_o = I \times (-j19)$$

$$= 1.897 \times (-j19)$$

$$v_o = 36.05\angle -55.3° \text{ Volt}$$

For I_T,



$$Z_1 = (9 - 3j) \parallel (10.2 - 18.4j)$$

$$Z_2 = (6.93 - 0.6097j)\ \Omega$$

$$I_T = \frac{40\angle 0°}{1 + 3j + Z_1} = 5.75\angle 5.03 \text{ A}$$

Q.5 (e) Solution:

Given :

$$G(s) = \frac{K}{s(s+4)},\ e_{ss} < 0.4 \text{ degree}$$

$$\text{Input} = 3 \text{ rpm, i.e., } N = 3 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3}{60} = \frac{\pi}{10} \text{ rad/s}$$

Hence, input will be $r(t) = \omega t = \dfrac{\pi t}{10}$.

Velocity error constant, for ramp input

$$K_v = \lim_{s \to 0} s \cdot G(s)$$

$$K_v = \lim_{s\to 0} s\cdot\frac{K}{s(s+4)} = \frac{K}{4}$$

Steady state error, for input At

$$e_{ss} = \frac{A}{K_v} < 0.4 \text{ degree}$$

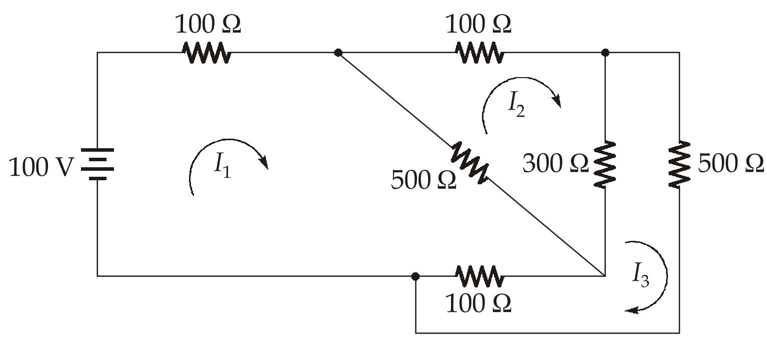
$$\frac{\frac{\pi}{10}}{(K/4)} < 0.4\times\frac{\pi}{180°}$$

$$K > \frac{4\pi}{10}\times\frac{180°}{0.4\pi}$$

$$K > 180°$$

Q.6 (a) Solution:

(i) The circuit can be



Apply KVL in mesh (1),

$$100 = 100I_1 + 500(I_1 - I_2) + 100(I_1 - I_3)$$

$$7I_1 - 5I_2 - I_3 = 1 \qquad \text{...(1)}$$

Apply KVL in mesh (2),

$$100I_2 + 300(I_2 - I_3) + 500(I_2 - I_1) = 0$$

$$5I_1 - 9I_2 + 3I_3 = 0 \qquad \text{...(2)}$$

Apply KVL in mesh (3)

$$500I_3 + 100(I_3 - I_1) + 300(I_3 - I_2) = 0$$

$$I_1 + 3I_2 - 9I_3 = 0 \qquad \text{...(3)}$$

On solving eqn. (1), (2) and (3)

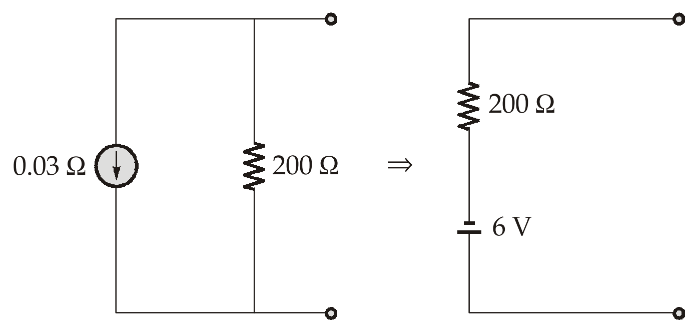
$$I_1 = 0.3 \text{ A}$$

$$I_2 = 0.2 \text{ A}$$
$$I_3 = 0.1 \text{ A}$$

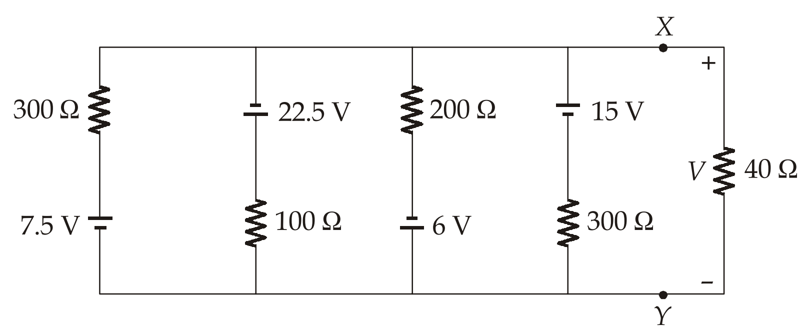
The current supplied by the battery is

$$I_1 = 0.3 \text{ Amp}$$

(ii) By applying source transformation,



The circuit reduces to



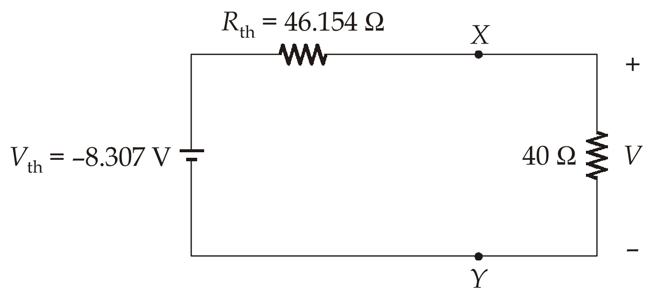
$$V_{\text{th}} = V_{XY} = \frac{\frac{7.5}{300} - \frac{22.5}{100} - \frac{6}{200} + \frac{15}{300}}{\frac{1}{300} + \frac{1}{100} + \frac{1}{200} + \frac{1}{300}}$$

$$V_{th} = V_{XY} = -8.307 \text{ V}$$

$$R_{th} = \frac{1}{\frac{1}{300} + \frac{1}{100} + \frac{1}{200} + \frac{1}{300}}$$

$$R_{th} = 46.154 \ \Omega$$

The equivalent circuit becomes



The voltage across 40 Ω resistor,

$$V_{40\ \Omega} = \frac{40}{40 + 46.154} \times (-8.307)$$

$$V_{40\ \Omega} = -3.857 \text{ Volt}$$

Q.6 (b) Solution:

Given :
$$G(s) = \frac{K(s+4)}{(s+10)^2},\ \omega_{gc} = 30 \text{ rad/s}$$

$$G(j\omega) = \frac{K(j\omega+4)}{(j\omega+10)^2}$$

Magnitude,
$$|G(j\omega)| = \frac{K\sqrt{\omega^2+16}}{\left(\sqrt{\omega^2+100}\right)^2}$$

At $\omega_{gc} = 30$ rad/s,
$$|G(j\omega)|\big|_{\omega_{gc}} = 1$$

$$1 = \frac{K\sqrt{(30)^2+16}}{[(30)^2+100]}$$

$$K = \frac{900+100}{\sqrt{916}} = 33.041$$

$$K = 33.041$$

Phase margin,
$$PM = 180 + \phi|_{\omega_{gc}}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{4}\right) - 2\tan^{-1}\left(\frac{\omega}{10}\right)$$

At $\omega = \omega_{gc}$,

$$\phi = \tan^{-1}\left(\frac{30}{4}\right) - 2\tan^{-1}\left(\frac{30}{10}\right)$$

$$\phi = -60.725°$$
$$PM = 180 - 60.725°$$
$$PM = 119.275°$$

Calculation of phase-crossover frequency.

At $\omega = \omega_{pc}$, $\phi = -180°$

$$-180° = \tan^{-1}\left(\frac{\omega_{pc}}{4}\right) - 2\tan^{-1}\left(\frac{\omega_{pc}}{10}\right)$$

Since ω_{pc} is undefined, hence GM is also undefined. The system is said to be stable based on positive phase margin.

Q.6 (c) Solution:

KVL in loop-1, 2 and 3 simultaneously (supermesh):

$$-28 + 200I_1 + 100I_2 + 150I_3 = 0$$
$$200I_1 + 100I_2 + 150I_3 = 28 \quad ...(i)$$

By KCL :

$$I_1 - 0.02\ V_x = I_2 \quad ...(ii)$$
$$V_x = (I_3 - I_2) \times 200 \text{ (by ohm's law)} \quad ...(iii)$$

Substituting equation (iii) in (ii), we get

$$I_1 + 3I_2 - 4I_3 = 0 \quad ...(iv)$$

By KCL :

$$0.02\ V_x = 0.06 + (I_3 - I_2) \quad ...(v)$$

Substituting equation (iii) in (v), we get

$$-3I_2 + 3I_3 = 0.06 \quad ...(vi)$$

On solving equation (i), (iv) and (vi), we get

$$I_1 = 0.1 \text{ A},$$
$$I_2 = 0.02 \text{ A}$$

and $I_3 = 0.04$ A

Power delivered by the independent voltage source

$$P_1 = 28I_1 = 2.8 \text{ W}$$
$$V_2 = -350I_3 + 200I_2 = -10 \text{ V}$$

Power delivered by the independent current source

$$P_2 = 0.06\ V_2 = -0.6 \text{ W}$$

Q.7 (a) Solution:

(i) (a) Given : $r(t) = u(t)$ and $C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$

Transfer function of closed loop system is given as

$$T(s) = \frac{C(s)}{R(s)}$$

Now,
$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$= \frac{1}{s} + \frac{0.2(s+10) - 1.2(s+60)}{s^2 + 70s + 600}$$

$$= \frac{1}{s} + \frac{(-s-70)}{s^2 + 70s + 600}$$

$$C(s) = \frac{600}{s(s^2 + 70s + 600)} \quad \text{...(1)}$$

Also,
$$R(s) = \frac{1}{s} \quad \text{...(2)}$$

From equation (1) and (2)

$$T(s) = \frac{C(s)}{R(s)} = \frac{600}{s^2 + 70s + 600} \quad \text{...(3)}$$

(b) This is a second order system. For this type standard form of

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{...(4)}$$

On comparing eqn. (3) and (4)

$$\omega_n^2 = 600 \Rightarrow \omega_n = \sqrt{600} = 24.5 \text{ rad/sec}$$

$$\xi = \frac{70}{2\omega_n} = \frac{70}{2 \times 24.5} = 1.43$$

(ii) Closed loop transfer function is given as,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{4}{s(s+5)+4} = \frac{4}{s^2+5s+4}$$

Now input to the system is $R(s) = \frac{1}{s}$, then output is given as

$$C(s) = T(s).R(s) = \frac{4}{s(s^2+5s+4)}$$

$$= \frac{4}{s(s+1)(s+4)}$$

Using partial fraction,

$$C(s) = \frac{4}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = sC(s)|_{s=0} = 1$$

$$B = (s+1)C(s)|_{s=-1} = \frac{4}{-1\times(-1+4)} = \frac{-4}{3}$$

$$C = (s+4)C(s)|_{s=-9} = \frac{4}{-4(-4+1)} = \frac{1}{3}$$

Now, $$C(s) = \frac{1}{s} - \frac{4}{3(s+1)} + \frac{1}{3(s+4)}$$

On taking inverse laplace of above expression, the output response is

$$C(t) = \left(1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}\right)u(t)$$

$$= \left[1 + \frac{1}{3}(e^{-4t} - 4e^{-t})\right]u(t)$$

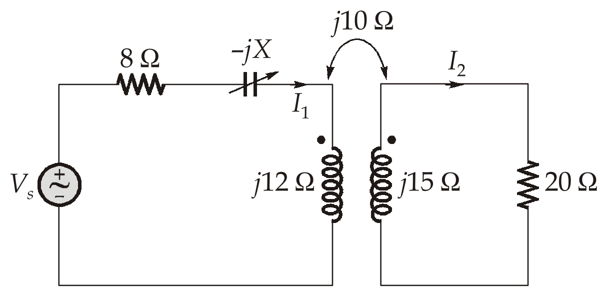
From the transfer function, $T(s)$,

$$2 \times 2\xi = 5 \Rightarrow \xi = 1.25$$

Hence, the system is overdamped system. So, it does not exhibit peak overshoot.

Q.7 (b) Solution:

(i)



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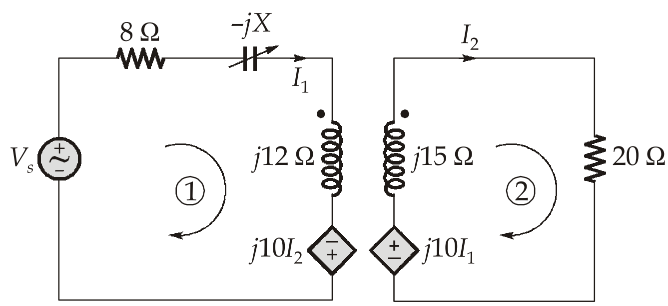


KVL in loop (2)

$$j10I_1 = (20 + j15)I_2$$

⇒ $$I_2 = \left(\frac{j10}{20 + j15}\right)I_1 \qquad ...(1)$$

KVL in loop (1)

$$V_s = (8 - jX + j12)I_1 - j10I_2$$

Putting I_2 from eqn. (1) in above expression

$$V_s = (8 - jX + j12)I_1 - \frac{j10 \times j10}{20 + j15}I_1$$

$$V_s = \left(8 - jX + j12 + \frac{100}{20 + j15}\right)I_1$$

$$V_s = [11.2 + j(9.6 - X)]I_1$$

Here to transfer maximum power to load, we need to maximize the current I_1.

So, $X = 9.6$ (for maximum power to 20 Ω load)

Now, current I_1 can be computed as

$$I_1 = \frac{V_s}{11.2} = \frac{220}{11.2} = 19.64 \text{ A}$$

Now, current I_2 can be computed as,

$$I_2 = \left[\frac{j10}{20 + j15}\right] \times 19.64$$

$$I_2 = \frac{55}{7}\angle 53.13° \text{A}$$

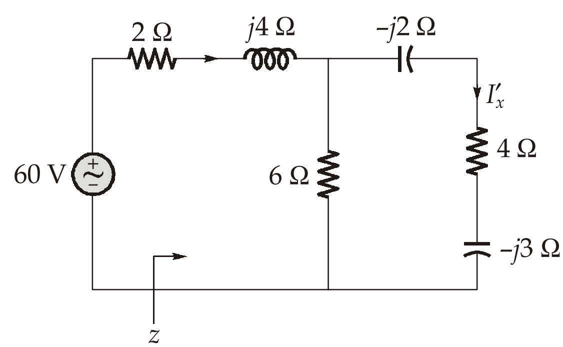
Maximum power to load,

$$P_{max} = |I_2|^2 \times 20 = 1234.70 \text{ Watts}$$

(ii) We will calculate the 'I_X' using superposition principle.

Considering the 60 V source alone.

Let the current in branch (4 – j3) Ω due to 60 V is I'_X.



$$Z = [(4 - j3 - j2) \parallel 6] + (2 + j4)$$

$$Z = 5.72\angle 26.56° \ \Omega$$

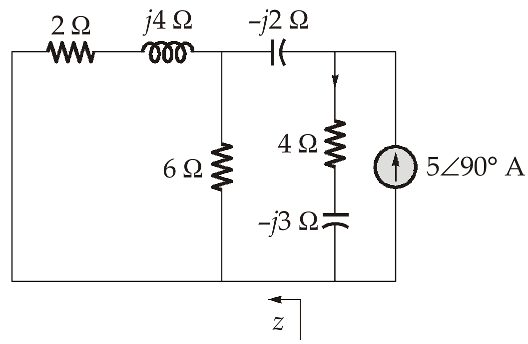
$$I = \frac{60}{5.72\angle 26.56°} = 10.48\angle -26.56° \text{ A}$$

Now apply the current division rule

$$I'_X = \frac{6}{10 - j5} \times 10.48\angle - 26.56°$$

$$I'_X = 5.625$$

Now, consider the $5\angle 90°$ A source,



$$Z = [(2 + j4) \parallel 6 + (-j2)]$$
$$Z = 2.40\angle{-4.76}$$

Now, apply the current division rule.

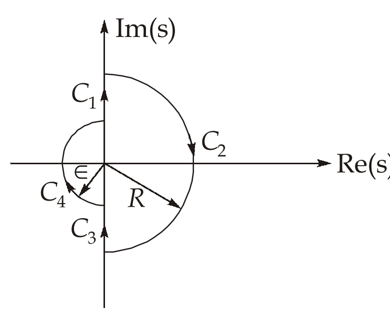
$$I''_X = \frac{Z}{Z + 4 - j3} \times 5\angle 90^\circ$$
$$= \frac{2.40\angle -4.76}{7.15\angle -26.56} \times 5\angle 90^\circ$$
$$I''_X = 1.677\angle 112^\circ \text{ A}$$

Now current,

$$I_X = I'_X + I''_X$$
$$= 5.625\angle{-9.86^\circ} + 1.677\angle 112^\circ$$
$$I_X = 5.238\angle 17.35^\circ \text{ A}$$

Q.7 (c) Solution:

Nyquist contour :



C_1 : $s_1 = j\omega$ where $\omega : 0^+$ to $+\infty$

C_2 : $s = Re^{j\theta}$, where $R \to \infty$ and $\theta : +90^\circ$ to 0° to -90°

C_3 : $s = -j\omega$, where $\omega : -\infty$ to 0^+

C_4 : $s = \in e^{j\phi}$, where $\in \to 0$ and $\phi : -90^\circ$ to 0° to 90°

Mapping of C_1 :

$$G(j\omega)H(j\omega) = \frac{1+j4\omega}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

$$|GH| = \frac{\sqrt{1+16\omega^2}}{\omega^2\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$\angle G(j\omega) = -180° + \tan^{-1}(4\omega) - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

At $\omega = 0 \Rightarrow$ $|GH| = \infty$ and $\angle GH = -180°$

At $\omega = \infty \Rightarrow$ $|GH| = 0$ and $\angle GH = -180° + 90° - 90° - 90° = -270°$

At $\omega = 1 \Rightarrow$ $|GH| = 1.30$ and $\angle GH = -212.47°$

Need to check real axis crossing point.

$\Rightarrow -180° + \tan^{-1} 4\omega - \tan^{-1} \omega - \tan^{-1} 2\omega = -180°$

$$\tan^{-1}(4\omega) - \tan^{-1}\left[\frac{\omega+2\omega}{1-2\omega^2}\right] = 0$$

$$\tan^{-1}\left[\frac{4\omega - \dfrac{\omega+2\omega}{1-2\omega^2}}{1+\dfrac{12\omega^2}{1-2\omega^2}}\right] = 0$$

$$\tan^{-1}\left[\frac{4\omega - 8\omega^3 - 3\omega}{1-2\omega^2+12\omega^2}\right] = 0$$

$$\Rightarrow \quad \omega - 8\omega^3 = 0$$

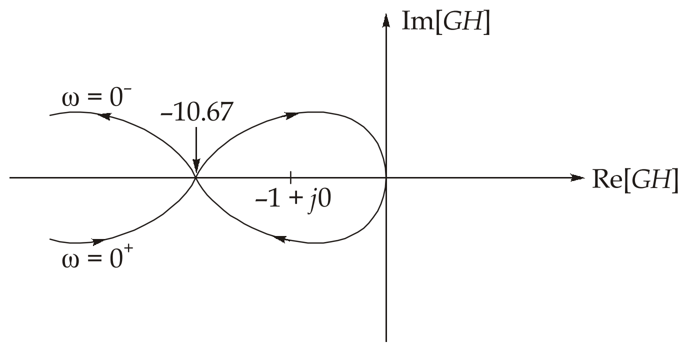
$$\omega[1 - 8\omega^3] = 0$$

$$\Rightarrow \quad \omega = 0, 0.35 \text{ rad/sec}$$

For $\omega = 0.35$

$$|GH| = \frac{\sqrt{2.96}}{0.1225\times\sqrt{1.1225}\times\sqrt{1.50}} = 10.67$$

$\therefore$ Mapping of C_1 and C_3 (mirror image) is shown in figure.



Mapping of C_2 :

$$G(Re^{j\theta})H(Re^{j\theta}) = \lim_{R\to\infty} \frac{1+4Re^{j\theta}}{(Re^{j\theta})^2(1+Re^{j\theta})(1+2Re^{j\theta})} \simeq \frac{1}{(Re^{j\theta})^3}$$

$$= 0 \text{ (maps to origin itself)}$$

Mapping of C_4 :

$$G(\in e^{j\phi})H(\in e^{j\phi}) = \lim_{\in\to 0} \frac{1+4\in e^{j\phi}}{\in^2 e^{j2\phi}(1+\in e^{j\phi})(1+2\in e^{j\phi})} \simeq \frac{1}{\in^2 e^{j2\phi}}$$

$$G(\in e^{j\phi})H(\in e^{j\phi}) = \frac{1}{\in^2 e^{j2\phi}}$$

$$|GH| = \infty \qquad [\because \in \to 0]$$

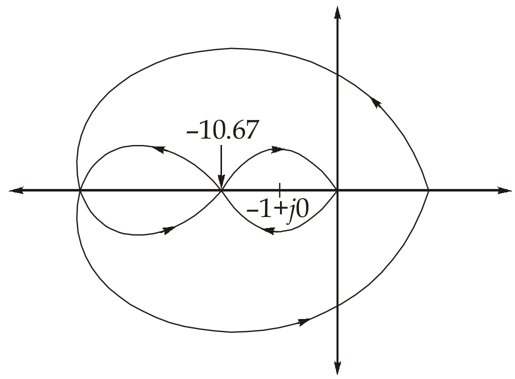
$$\angle GH = -2\phi$$

When $\phi = \frac{-\pi}{2}$: $\angle GH = \pi, \phi = \frac{-5\pi}{4} \to \angle GH = 2.5\pi$

When $\phi = 0°$: $\angle GH = 0$

When $\phi = \frac{\pi}{2}$: $\angle GH = -\pi, \phi = \frac{3\pi}{4} \to \angle GH = -1.5\pi$

Complete Nyquist plot is shown in figure.



Now stability : from Nyquist plot

$$N = 0 \text{ and } P = 0$$

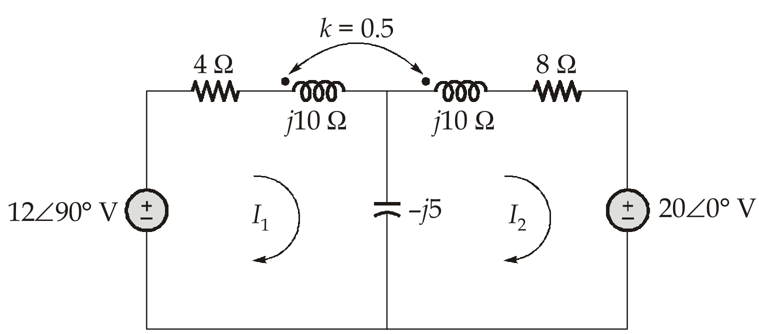
$$N = P - Z$$

$$Z = 0$$

Therefore, closed loop system have no poles in R.H.S. of *s*-plane. Hence, closed loop system is stable.

Q.8 (a) Solution:

(i) Transfer the current source to a voltage source as shown below :



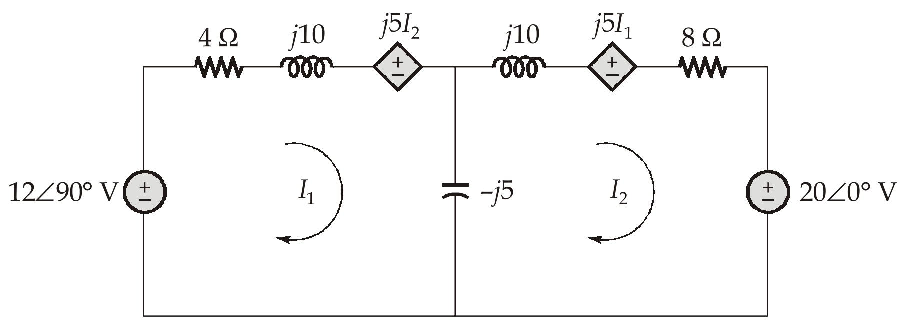
$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = K\sqrt{L_1 L_2}$$

$$\omega M = K\sqrt{\omega L_1 \omega L_2}$$

$$\omega M = 0.5 \times \sqrt{10 \times 10} = 5\ \Omega$$

Now above circuit can be converted as,



For Mesh 1,

$$j12 = (4 + j10)I_1 + j5I_2 - j5(I_1 - I_2)$$

$$j12 = (4 + j5)I_1 + j10I_2 \quad \text{...(1)}$$

For Mesh 2,

$$-j5(I_2 - I_1) + j10I_2 + 8I_2 + j5I_1 = -20$$

$$j10I_1 + (8 + j5)I_2 = -20 \quad \text{...(2)}$$

From eqn. (1) and (2)

$$\begin{bmatrix} 4 + j5 & j10 \\ j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j12 \\ -20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 4 + j5 & j10 \\ j10 & 8 + j5 \end{vmatrix} = 122.67\angle 29.28°$$

Using Cramer's rule

$$I_1 = \frac{\Delta_1}{\Delta}$$

where, $$\Delta_1 = \begin{vmatrix} j12 & j10 \\ -20 & 8 + j5 \end{vmatrix} = 302\angle 101.45°$$

Therefore, $$I_1 = \frac{302\angle 101.45°}{122.67\angle 29.28} = 2.4620\angle 72.17°$$

And $$I_2 = \frac{\Delta_2}{\Delta}$$

where, $$\Delta_2 = \begin{vmatrix} 4 + j5 & j12 \\ j10 & -20 \end{vmatrix} = 107.70\angle -68.19°$$

Therefore, $$I_2 = \frac{107.70\angle -68.18°}{122.67\angle 29.28°} = 0.878\angle -97.48° \text{ A}$$

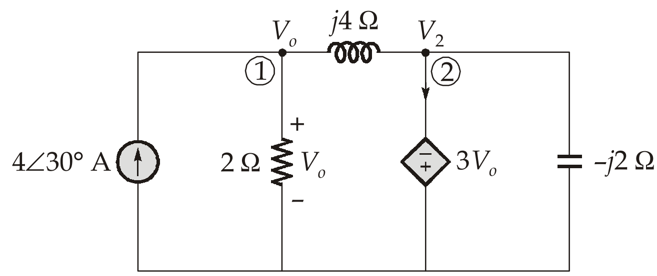
and $$\vec{I}_3 = \vec{I}_1 - \vec{I}_2 = 2.4620\angle 72.17° - 0.878\angle -97.48°$$

$$\vec{I}_3 = 3.329\angle 74.89° \text{ A}$$

Therefore,

$$I_1 = 2.4620\angle 72.19° \text{ A}$$
$$I_2 = 0.878\angle -97.48° \text{ A}$$
$$I_3 = 3.329\angle 74.89° \text{ A}$$

(ii)



KCL at node (1)

$$\frac{V_o}{2} + \frac{V_o - V_2}{j4} = 4\angle -30°$$

$$j4V_o + 2V_o - 2V_2 = 32\angle 60°$$

$$(2 + j4)V_o - 2V_2 = 32\angle 60° \quad ...(1)$$

Also, $$V_2 = -3V_o$$

Now, from eqn. (1)

$$(2 + j4)V_o - 2 \times (-3\ V_o) = 32\angle 60°$$

$$(8 + j4)V_o = 32\angle 60°$$

$$V_o = 3.57\angle 33.43° \text{ Volts}$$

Q.8 (b) Solution:

(i) Characteristic equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s-1)}{(s+2)(s+3)} = 0$$

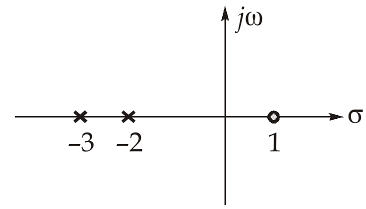
$$s^2 + (5 + K)s + (6 - K) = 0$$

Routh array,

$$\begin{array}{c|cc} s^2 & 1 & 6-K \\ s^1 & 5+K & \\ s^0 & 6-K & \end{array}$$

For stability,

$$5 + K > 0$$
$$K > -5$$
$$6 - K > 0$$
$$K < 6$$
$$-5 < K < 6$$



Break away point, $K = \dfrac{s^2 + 5s + 6}{(1-s)}$

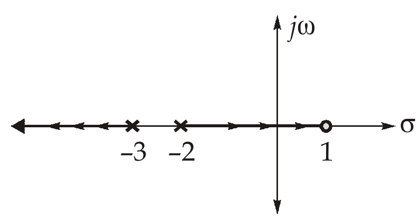
$$\frac{dK}{ds} = \frac{(1-s)(2s+5) + (s^2+5s+6)}{(1-s)^2} = 0$$

or, $-2s^2 - 3s + 5 + s^2 + 5s + 6 = 0$

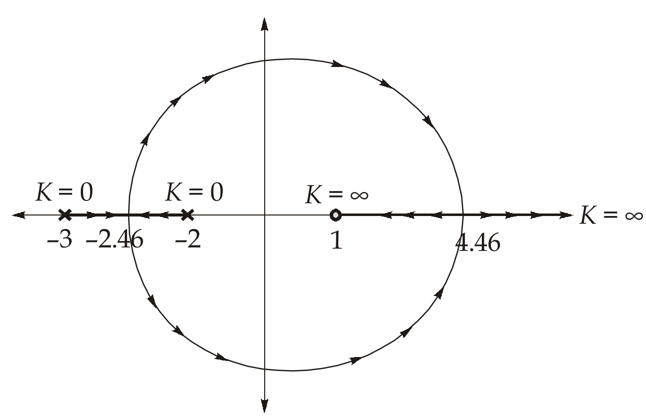
or, $-s^2 + 2s + 11 = 0$

$\Rightarrow$ $s = 4.46, -2.46$

(ii) For $0 \le K \le 6$



$-5 \le K < 0$



(iii) Smallest settling time will occur when both roots are as far away as possible to the left of $j\omega$-axis. Using magnitude condition,

$$= \left|\frac{K(s-1)}{(s+2)(s+3)}\right|_{s=-2.46+j0} = 1$$

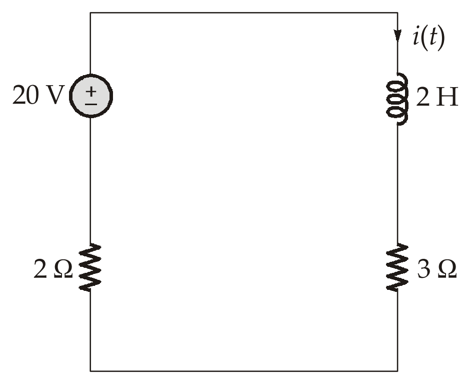
$$\left|\frac{K(-2.46-1)}{(-2.46+2)(-2.46+3)}\right| = 1$$

$$\frac{K \times 3.46}{0.46 \times 0.54} = 1$$

$$K = 0.0718$$

Q.8 (c) Solution:

(i) Circuit for $t < 0$:



$$\tau = \frac{L}{R_{eq}} = \frac{2}{5} = 0.4 \text{ sec}$$

Inductor current equation can be given as

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

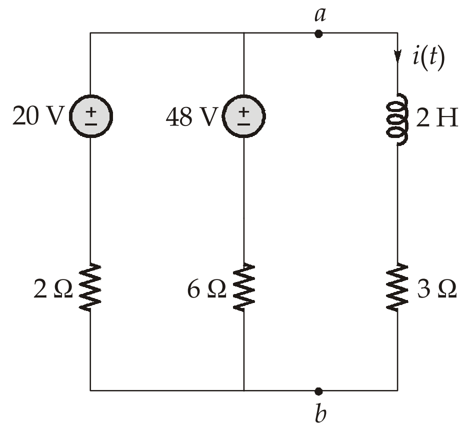
$$\because \qquad i(\infty) = \frac{20}{5} = 4 \text{ A}$$

Therefore, $\quad i(t) = 4 + (0 - 4)e^{-t/0.4} = 4(1 - e^{-2.5t}) \text{ A}; t < 0$

for $t > 0$:

Now, assume that before switching at $t = 0$, inductor current attained the steady state, therefore,

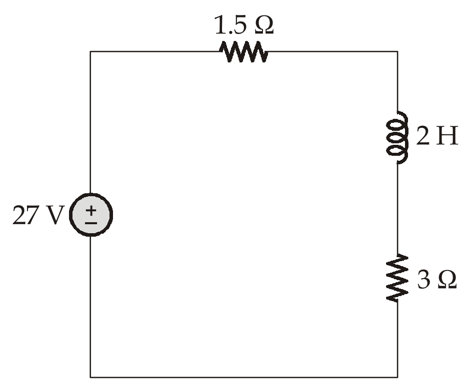
$$i(0) = 4 \text{ A}$$



Taking Thevenins equivalent across terminals $a - b$.

$$V_{th} = \frac{48 \times 2 + 6 \times 20}{6 + 2} = 27 \text{ V}$$

$$R_{th} = 6 \,||\, 2 = 1.5 \, \Omega$$



$$\tau = \frac{L}{R_{eq}} = \frac{2}{3 + 1.5}$$

At steady state, $$i(\infty) = \frac{27}{1.5 + 3} = 6 \text{ A}$$

Now, inductor current equation can be given as

$$i(t) = i(\infty) + [i(0^-) - i(\infty)]e^{-t/\tau}$$

$$= 6 + [4 - 6)e^{-t/\tau}$$

$$i(t) = (6 - 2e^{-2.25t}) \text{ A}$$

Therefore, $$i(t) = \begin{cases} 4(1 - e^{-2.25t})\text{A}, & t < 0 \\ (6 - 2e^{-2.25t})\text{A}, & t > 0 \end{cases}$$

(ii) Light is ON 75 Volts until 30 V. During that time we essentially have a 120 Ω resistor in parallel with 6 μF capacitor.

During discharging :

$$V(0) = 75\text{ V};\ V(\infty) = 0,\ \tau = RC = 120 \times 6 \times 10^{-6} = 0.72\text{ msec}$$

$$V(t_1) = 75e^{-t_1/\tau} = 30$$

which leads to

$$t_1 = -0.72\ \ln(0.4)\text{ msec} = 659.72\ \mu\text{sec of lamp on time}$$

Now, during capacitor charging

$$\tau = RC = 4 \times 6 = 24\text{ sec}$$

Since

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$V(t_1) - V(\infty) = [V(0) - V(\infty)]e^{-t_1/\tau} \quad \text{...(1)}$$

$$V(t_2) - V(\infty) = [V(0) - V(\infty)]e^{-t_2/\tau} \quad \text{...(2)}$$

Dividing eqn. (1) by (2)

$$\frac{V(t_1) - V(\infty)}{V(t_2) - V(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_o = t_2 - t_1 = \tau \ln\left[\frac{V(t_1) - V(\infty)}{V(t_2) - V(\infty)}\right]$$

$$t_o = 24 \ln\left[\frac{30 - 120}{75 - 120}\right] = 24\ \ln 2$$

$$t_o = 16.636\text{ sec}$$

So, time interval between light flashes is 16.636 sec.

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