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Detailed Solutions

ESE-2023
Mains Test Series

Mechanical Engineering
Test No : 2

Section A : Heat Transfer + Refrigeration and Air Conditioning [All Topics]
Section B : Thermodynamics-1 + Strength of Materials & Mechanics-1 [Part Syllabus]

Section : A

1. (a)

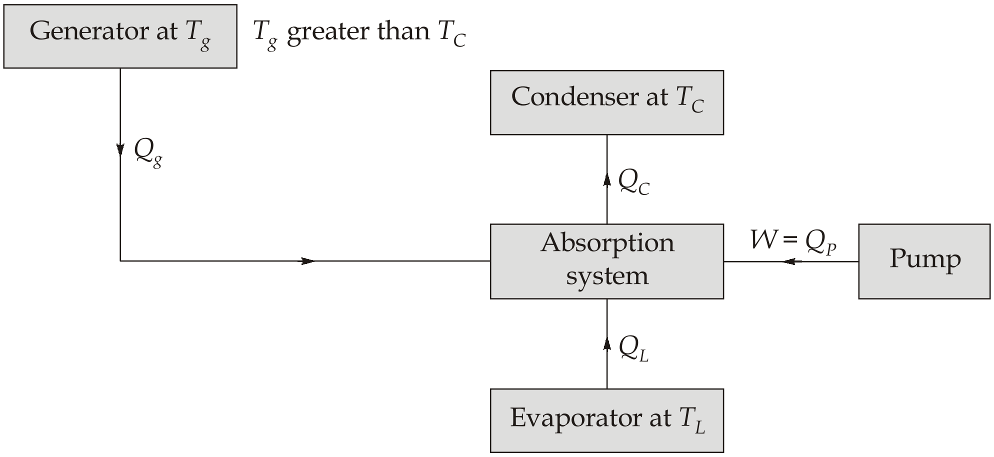
(i)

The objective of a refrigerator is to remove heat (Q_L) from the cold space. To accomplish this, it needs energy input. Therefore, the COP of a refrigerator is given by

$$\text{COP}_\text{R} = \frac{\text{Desired effect } (Q_L)}{\text{Energy input}}$$

The heat (Q_L) is absorbed by the refrigerant in the evaporator.

The energy input in an absorption refrigeration system includes: (1) The heat (Q_g) given to the refrigerant in the generator, and (2) The heat (Q_P) added to the refrigerant due to pump work as shown in figure below.



Theoretical COP of the vapour absorption system

Let Q_C be the heat dissipated to the atmosphere or cooling water from the condenser and absorber.

According to the first law of thermodynamics,

$$Q_C = Q_g + Q_L + Q_P$$

Since, the heat due to pump Q_P is very negligible,

$$Q_C = Q_g + Q_L \qquad \text{...(i)}$$

Let,

T_g be the temperature at which heat Q_g is supplied to the generator

T_C be the temperature at which Q_C is discharged to atmosphere or cooling water from the condenser and absorber.

T_L be the temperature at which heat Q_L is absorbed in the evaporator.

Since, the vapour absorption system can be considered as a perfectly reversible system, the initial entropy of the system must be equal to the entropy of the system after the change in its condition.

$$\therefore \qquad \frac{Q_g}{T_g} + \frac{Q_L}{T_L} = \frac{Q_C}{T_C} \qquad \text{...(ii)}$$

From equation (i), we can write

$$\frac{Q_C}{T_C} = \frac{Q_g + Q_L}{T_C}$$

Thus, equation (ii) becomes,

$$\Rightarrow \qquad \frac{Q_g}{T_g} - \frac{Q_g}{T_C} = \frac{Q_L}{T_C} - \frac{Q_L}{T_L}$$

$$\Rightarrow \qquad Q_g\left(\frac{T_C - T_g}{T_g \times T_C}\right) = Q_L\left(\frac{T_L - T_C}{T_C \times T_L}\right)$$

$$Q_g = Q_L\left(\frac{T_L - T_C}{T_C \times T_L}\right)\left(\frac{T_g \times T_C}{T_C - T_g}\right)$$

$$= Q_L\left(\frac{T_C - T_L}{T_C \times T_L}\right)\left(\frac{T_g \times T_C}{T_g - T_C}\right)$$

$$= Q_L\left(\frac{T_C - T_L}{T_L}\right)\left(\frac{T_g}{T_g - T_C}\right)$$

∴ The coefficient of performance (COP) of the ideal vapour absorption system is given by

$$\text{COP} = \frac{Q_L}{Q_g} = \left(\frac{T_L}{T_C - T_L}\right)\left(\frac{T_g - T_C}{T_g}\right) \qquad ...(iii)$$

In equation (iii),

1. The expression $\left(\frac{T_L}{T_C - T_L}\right)$ represents the COP of a Carnot refrigerator working between the temperature limits of T_L and T_C.

2. The expression $\left(\frac{T_g - T_C}{T_g}\right)$ represents the efficiency of a Carnot engine working between the temperature limits of T_g and T_C.

Thus, a theoretical or an ideal vapour absorption refrigeration system may be regarded as a combination of a Carnot engine and a Carnot refrigerator, and

$$\text{COP} = (\text{COP})_{\text{Carnot}} \times \eta_{\text{Carnot}}$$

(ii)

Given:

Evaporator temperature, T_E = –7°C = 266 K

Generator temperature, T_G = Saturation temperature of system

So, T_G = 113.3°C = 386.3 K

Atmospheric temperature, T_O = 30°C = 303 K

Maximum COP for a vapour absorption system is given by,

$$(\text{COP})_{\max} = \frac{T_E}{T_O - T_E} \times \frac{T_G - T_O}{T_G}$$

$$= \left(\frac{266}{303-266}\right) \times \left(\frac{386.3-303}{386.3}\right)$$

$$(\text{COP})_{\max} = 7.1892 \times 0.21563$$

$$(\text{COP})_{\max} = 1.55$$

1. (b)

(i)

Characteristic linear dimension for sphere,

$$l = \frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{10}{3} = 3.33 \text{ mm}$$

Biot number, $$B_i = \frac{hl}{k} = \frac{114 \times 3.33 \times 10^{-3}}{42.5} = 8.93 \times 10^{-3}$$

Since, the biot number, $B_i < 0.1$, so lumped analysis is valid here

$$\therefore \quad \frac{t - t_a}{t_i - t_a} = \exp\left(\frac{-hA\tau}{\rho Vc}\right)$$

$$\frac{120-27}{520-27} = \exp\left(\frac{-114 \times 3 \times \tau}{7850 \times 10 \times 10^{-3} \times 475}\right)$$

or $$0.1886 = \exp(-9.17 \times 10^{-3}\, \tau)$$

or $$1.668 = 9.17 \times 10^{-3} \times \tau$$

$$\therefore \quad \tau = 181.897 \text{ sec}$$

$$\tau = 3.03 \text{ minutes}$$

(ii)

The instantaneous heat transfer rate 2 minutes after the start of cooling is given by,

$$Q = -hA(t_i - t_a)\exp\left(\frac{-hA\tau}{\rho Vc}\right)$$

$$= -114 \times \left(4\pi \times 0.01^2\right) \times (520-27) \times \exp\left(\frac{-114 \times 3 \times 120}{7850 \times 10 \times 10^{-3} \times 475}\right)$$

$$Q = 23.49 \text{ W (Rejected)}$$

(iii)

The total energy transferred is,

$$Q_T = \rho Vc(t_i - t_o)\left[\exp\left(\frac{-hA\tau}{\rho Vc}\right) - 1\right]$$

$\because$
$$\frac{-hA\tau}{\rho Vc} = -\frac{114 \times 3 \times 120}{7850 \times 10^{-2} \times 475} = -1.1$$

$\therefore$
$$Q_T = 7850 \times 475\left(\frac{4}{3}\pi \times 0.01^3\right) \times (520 - 27) \times \left[e^{-1.1} - 1\right]$$

$$Q_T = 7700.14 \times (-0.667)$$

$$Q_T = 5136.98 \text{ W (Rejected)}$$

1. (c)

Thermodynamic Properties of Refrigerants:

(i) Latent heat of evaporation : High latent heat of evaporation (h_{fg}) per unit mass is desirable. A small quantity of refrigerant produces a large cooling effect which requires less power for its circulation. Among the commonly used refrigerants, the latent heat of evaporation for ammonia is the maximum.

(ii) Boiling Point : Refrigerants should have low boiling point temperature at atmospheric pressure. During operation, low temperature can be achieved with positive pressure of the refrigerant in evaporator coil. If the pressure inside the evaporator coil is less than the atmospheric pressure, then moisture may enter into the system through leakages. This would lead to the blockage of refrigerant flow at the expansion valve. It would not help in detecting leakage of refrigerants. Negative pressure in evaporator is undesirable.

(iii) Freezing Point : Freezing temperatures should be well below the evaporator temperatures to avoid freezing of the refrigerant.

(iv) Condensing Pressure : It depends on the temperature of the condenser cooling medium which is usually water or atmospheric air. A refrigerant should have low condensing pressure to avoid robust constructions and to reduce the tendency of leakages.

(v) Critical Temperature and Pressure : It is the temperature above which there will not be any phase change. The critical temperature of the refrigerant should be well above the condensing temperature for easy condensation of vapour.

(vi) Index of Compression Process : The work of compression per unit mass depends on the isentropic index $\gamma\left(\gamma = \frac{c_p}{c_v}\right)$. The smaller the index, the smaller will be the work of compression. The compression process can follow different curves on the *p-v* diagram based on isothermal, adiabatic and polytropic processes. The isothermal compression process ($pv = c$) requires the least work but it is not a practical process.

Chemical Properties of Refrigerants:

The refrigerants have to satisfy certain chemical properties for safe operation of refrigerants systems. Some of the required chemical properties for a refrigerant are as follows:

(i) Flammability : The refrigerant should not be inflammable in the presence of air or lubrication oils, CO_2 is the most inert while ethane and butane are high inflammable. Hydrocarbons such as methane, ethane, propane, butane, etc. are highly explosive and inflammable, Fluorocarbons are non-explosive and non-flammable too but have very high ODP. HFCs (R152, R143), which are alternative to the CFCs, are also flammable.

(ii) Toxicity : A refrigerant should not be toxic, poisonous or injurious. Ammonia is toxic as well as flammable, so it is not used in domestic refrigerators. R123 considered to be the replacement of R11 is found to be toxic. The freon group of refrigerants are safe from toxic point of view, Refrigerants like methyl chloride, NH_3 and SO_2 are toxic in nature.

(iii) Action of refrigerant with water : The presence of moisture is very critical in refrigeration systems. If it is carried along with the refrigerant, it will become ice at the expansion valve or capillary tube and result into choking of the valve. This is called moisture choking. It is avoided by proper dehydration of the system or by placing silica gel in the pipeline before the expansion valve. Ammonia is soluble in water. Its contact with water should therefore be avoided. R134a is also very hygroscopic in nature.

(iv) Corrosiveness : The refrigerant should not have any adverse effect on the materials used in the equipment either by itself or in the presence of moisture and lubricating oils.

(v) Leak Detection : The refrigerant should posses minimum tendency to leak and should lend itself to easy methods of leak detection.

(vi) Flash Point : Liquids with flash point below 21.2°C are regarded as highly inflammable.

(vii) **Miscibility with Oil :** Since refrigerants come in contact with lubricating oils in the compressor, it is desirable that the refrigerant does not harm the properties of lubrication. The freon group of refrigerants are highly miscible refrigerants. While ammonia, CO_2, sulphur dioxide and methyl chloride are relatively non-miscible.

(viii) **Stability :** Refrigerants should be capable of withstanding the effect of pressure and temperature inside the system without decomposition. It should not react with metals or lubricants inside the system. For example, NH_3 reacts with copper and brass in presence of moisture. Therefore, NH_3 as refrigerant should not be used in systems employing cuprous metals. NH_3 is safe to work with iron and steel. Trough the freon group of refrigerants do not react with steel, copper, brass, zinc and aluminimum, it is corrosive with aluminium alloys having more than 2% of magnesium. The freon group should not be used in systems employing natural rubber for packings and gaskets but these refrigerant do not react with synthetic rubber.

The oil required for lubrication of the compressor is contained in the crankcase of the compressor where it is subject to contact with the refrigerant. Therefore, the refrigerant must be chemically and physically stable at the operating conditions of the oil.

Physical Properties of Refrigerants:

(i) **Specific Volume :** The mass of refrigerant being pushed in every delivery stroke of the piston depends upon its density. If the specific volume is low, comparatively more mass can be pushed. It means greater cooling capacity can be achieved with a small flow rate of refrigerant. The specific volume of the refrigerant should be low in the vapour state so that the plant components can be small in size. It reduces the cost of the plant.

(ii) **Viscosity :** It is defined as a measure of fluid friction or as a measure of the resistance that a fluid offers to flow. Low viscosity is a desirable property to keep down the pressure losses in refrigeration piping and to obtain better heat transfer rates in evaporator and condenser.

(iii) **Thermal Conductivity :** The refrigerant must have high thermal conductivity in liquid phase as well as vapour phase.

(iv) **Dielectric Strength :** The refrigerant should have high electric strength, otherwise electrical short circuits are likely to occur in hermetically sealed compressors where the refrigerant vapour cools the motor winding. R12 has high dielectric strength while NH_3 has poor dielectric strength.

(v) **Handling and Maintenance :** The refrigerant should be safe to handle without any protective measures. The system should not pose any serious maintenance problems.

(vi) **Cost and Availability :** The refrigerant should be available at low cost, for example, NH_3 is cheap but R12 and R22 are more expensive.

1. (d)

(i)

From summation rule,

$$F_{11} + F_{12} = 1 \text{ or } F_{12} = 1 \qquad (\because \; F_{11} = 0)$$

By reciprocity,

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi d^2}{6l^2} \times 1 = \frac{\pi}{6} \qquad \because \; l = d$$

(ii)

For hemisphere and plane surface, by summation rule,

$$F_{21} + F_{22} = 1$$

or $$F_{21} = 1 \qquad [\because \; F_{22} = 0]$$

and $$F_{11} + F_{12} = 1$$

Now, $$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{\pi r^2}{2\pi r^2} \times 1 = 0.5$$

(iii)

From summation rule, $F_{11} + F_{12} + F_{13} = 1$

For flat surface, $F_{11} = 0$, that gives,

$$F_{12} = 1 - F_{13} = 1 - 0.17 = 0.83$$

Now, by reciprocity:

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\frac{\pi}{4} d^2}{\pi d l} \times 0.83 = \frac{0.83}{4} \qquad \because \; l = d$$

$\therefore$ $$F_{21} = 0.2075$$

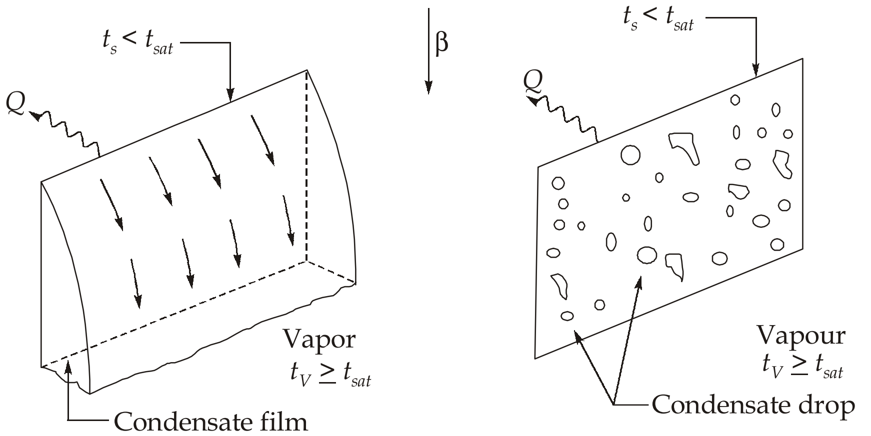
1. (e)

Depending upon the behaviour of condensate upon the cooled surface, the condensation process has been categorised into the following distinct modes:

(i) Film condensation : The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface. The liquid flows down the cooling surface under the action of gravity and the layer continuously grows in thickness because of newly condensing vapours. The continuous film offers thermal resistance and restricts further transfer of heat between the vapour and the surface.

Film condensation usually occurs when a vapour, relatively free from impurities, is allowed to condense on a clear surface.

(ii) Dropwise condensation : The liquid condensate collects in droplets and does not wet the solid cooling surface. The droplets develop in cracks and pits on the surface grow in size, break away from the surface, knock off other droplets and eventually run off the surface without forming a film. A part of the condensation surface is directly exposed to the vapour without an insulating film of condensate liquid. Evidently, there is no film barrier to heat flow and higher heat transfer rates are experienced. Heat transfer fluxes of the order of 750 kW/m^2 have been obtained with dropwise condensation.



(a) Film condensate

(b) Dropwise condensation

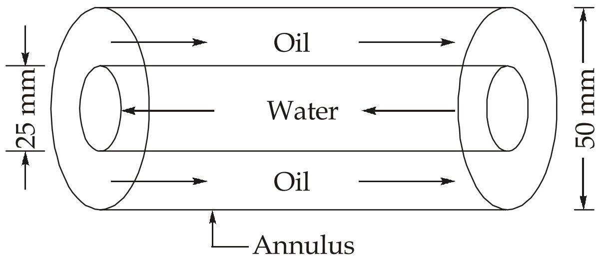
Film and dropwise condensation on a surface

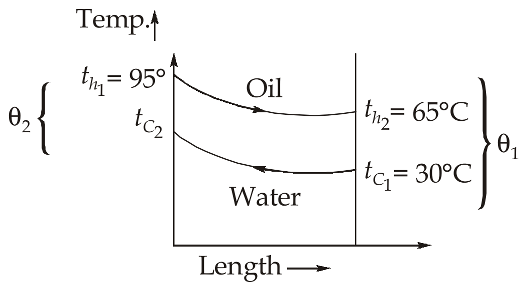
Dropwise condensation has been observed to occur either on highly polished surfaces, or on surfaces contaminated with impurities like fatty acids and organic compounds. Dropwise condensation gives coefficient of heat transfer generally five to ten times larger than with film condensation. Because of potential performance gain, dropwise condensation is provoked artificially by surface coatings called promoters that inhibit wetting. Silicons, teflons and an assortment of waxes and fatty acids are often used for this purpose. These substances are either applied to the heat transfer surface or introduced into the vapour. However, the phenomenon is highly unstable as these coatings gradually lose their effectiveness due to oxidation, fouling or outright removal and the surfaces become wetted when exposed to condensing vapour over an extended length of time. Consequently film condensation is generally encountered in industrial applications and is usually planned for condenser design calculations.

2. (a)

Given : $d_i = 25$ mm = 0.025; $d_o = 50$ mm = 0.05 m; $\dot{m}_w = \dot{m}_c = 0.2$ kg/s ;

$\dot{m}_{oil} = \dot{m}_h = 0.15$ kg/s; $t_{h_1} = 95°C$; $t_{h_2} = 65°C$; $t_{c_1} = 30°C$





The rate of heat transfer is given by,

$$Q = \dot{m}_h c_{ph}\left(t_{h_1} - t_{h_2}\right) = \dot{m}_c c_{pc}\left(t_{c_2} - t_{c_1}\right)$$

$$0.15 \times 2131 \times (95 - 65) = 0.2 \times 4174\left(t_{c_2} - 30\right)$$

$$\therefore \quad t_{c_2} = 41.48°\text{C}$$

LMTD is given by,

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln\left(\frac{\theta_2}{\theta_1}\right)} = \frac{(95 - 41.48) - (65 - 30)}{\ln\left(\frac{95 - 41.48}{65 - 30}\right)} = 43.6°\text{C}$$

Reynolds number for water,

$$Re = \frac{4\dot{m}_w}{\pi d_i \mu} = \frac{4 \times 0.2}{\pi \times 0.025 \times 725 \times 10^{-6}} = 1.4 \times 10^4$$

∴ Turbulent flow,

$$\therefore \quad \text{Nu} = \frac{h_i d_i}{k} = 0.023 \times \text{Re}^{0.8} \times \text{Pr}^{0.4}$$

$$= 0.023 \times (1.4 \times 104)^{0.8} \times (4.85)^{0.4}$$

$$\frac{h_i d_i}{k} = 89.72$$

$\therefore$ $$h_i = \frac{89.72 \times 0.625}{0.025} = 2243 \text{ W/m}^2\text{°C}$$

For Annulus part, $$D_h = d_o - d_i = 0.025 \text{ m}$$

$\therefore$ $$\text{Re} = \frac{\rho \cdot V \cdot D_h}{\mu} = \frac{\rho(d_o - d_i)}{\mu} \times \frac{\dot{m}_h}{\frac{\pi}{4}(d_o^2 - d_i^2) \cdot \rho}$$

or $$\text{Re} = \frac{4\dot{m}_h}{\pi(d_o + d_i)\mu} = \frac{4 \times 0.15}{\pi \times (0.075) \times 0.0325}$$

$$\text{Re} = 78.35 \qquad \text{(Laminar flow)}$$

Assuming uniform temperature along the inner surface of the annulus,

$$\text{Nu} = \frac{h_0 D_h}{k} = 3.65$$

$$h_0 = \frac{3.65 \times 0.138}{0.025} = 20.148 \text{ W/m}^2\text{°C}$$

The overall heat transfer coefficient,

$$\frac{1}{u} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{u} = \frac{1}{2243} + \frac{1}{20.148} = 19.96 \text{ W/m}^2\text{°C}$$

Also, $$Q = \dot{m}_h c_{ph} \left(t_{h_1} - t_{h_2}\right)$$

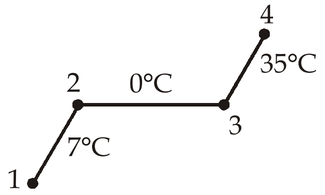
$$= uA\theta_m = u \cdot (\pi d_i L) \times \theta_m$$

$\therefore$ $$L = \frac{0.15 \times 2131 \times (95 - 65)}{19.96 \times \pi \times 0.025 \times 43.6}$$

$\therefore$ $$L = 140.3 \text{ m}$$

2. (b)

(a) Given: $\dot{m}_{ice} = 12$ tons/day ; $(c_p)_{\text{water}} = 4.1868$ kJ/kgK; $(c_p)_{\text{ice}} = 1.94$ kJ/kgK



$$\text{Refrigeration capacity } \left(\dot{Q}_0\right) = Q_{1\text{-}2} + Q_{2\text{-}3} + Q_{3\text{-}4}$$

$$\dot{Q}_0 = \dot{m}_{ice}\left[\left(c_p\right)_{ice}(0-(-7))+335+\left(c_p\right)_{water}\times(35-0)\right]$$

$$= \frac{12\times1000}{24\times3600}[1.94\times7+335+4.1868\times35]$$

$$= 68.76 \text{ kW}$$

(b) From the table of ammonia

$$h_1 = h_{g\,(-15^\circ\text{C})} = 1443.9 \text{ kJ/kg}$$

$$s_1 = s_{g\,(-15^\circ\text{C})} = 5.8223 \text{ kJ/kgK}$$

$$h_4 = h_3 = h_{f\,(35^\circ\text{C})} = 366.1 \text{ kJ/kg}$$

If $\dot{m}$ is the mass flow rate ammonia (refrigerant), then

$$R.C\left(\dot{Q}_0\right) = \dot{m}(h_1-h_4) = \dot{m}(1443.9-366.1)$$

$$\dot{m} = \frac{68.76}{(1443.9-366.1)} = 0.0638 \text{ kg/sec}$$

$$\dot{m} = 0.0638\times3600 = 229.689 \text{ kg/h}$$

(c) From the table, entropies and enthalpies of saturated vapour at 35°C and superheated vapour by 50 K and 100 K are, respectively

$$s_{g\,(35^\circ\text{C})} = 5.2086 \text{ kJ/kgK},\quad s_{(50\,\text{K})} = 5.648 \text{ kJ/kgK}$$

$$h_{g\,(35^\circ\text{C})} = 1488.6 \text{ kJ/kg},\quad h_{(50\,\text{K})} = 1634 \text{ kJ/kg}$$

$$s_{(100\text{ K})} = 5.982 \text{ kJ/kgK},\quad h_{(100\,\text{K})} = 1762 \text{ kJ/kg}$$

Now, $s_2 = s_1 = 5.8223 > 5.6466$

Hence, the vapour is superheated by more than 50 K.

Interpolating for discharge temperature,

$$t_2 = 35+50+\left(\frac{5.8223-5.648}{5.982-5.648}\right)(50)$$

$$t_2 = 35+50+26.093 = 111.093^\circ\text{C}$$

(d) Given : $\eta_v = 0.7$; $N = 150$ rpm; $\frac{L}{D} = 1.3$

From the table for ammonia,

$$v_1 = 0.508 \text{ m}^3/\text{kg}$$

We know,

$$\eta_v = \frac{\text{Actual entry to compressor}}{\text{Theoretical entry to compressor}}$$

$$0.7 \times \frac{\pi}{4} \times D^2 \times L \times \frac{N}{60} = \dot{m} v_1$$

$$\Rightarrow \quad 0.7 \times \frac{\pi}{4} D^3 \times 1.3 \times \frac{1500}{60} = 0.0638 \times 0.508$$

$$\Rightarrow \quad D = 0.122 \text{ m}$$

$$\Rightarrow \quad L = 0.158 \text{ m}$$

(e) Enthalpy at discharge, by interpolation,

$$h_2 = 1634 + \frac{26.093}{50}(1762 - 1634)$$

$$h_2 = 1634 + 66.798 = 1700.79 \text{ kJ/kg}$$

(Here, 1634 kJ/kg is the enthalpy of vapour superheated by 50 K)

$$\text{Power consumption, } P = \frac{\dot{m}(h_2 - h_1)}{\eta_a \eta_m}$$

$$= \frac{0.0638 \times (1700.79 - 1443.9)}{0.82 \times 0.95}$$

$$P = 21.04 \text{ kW}$$

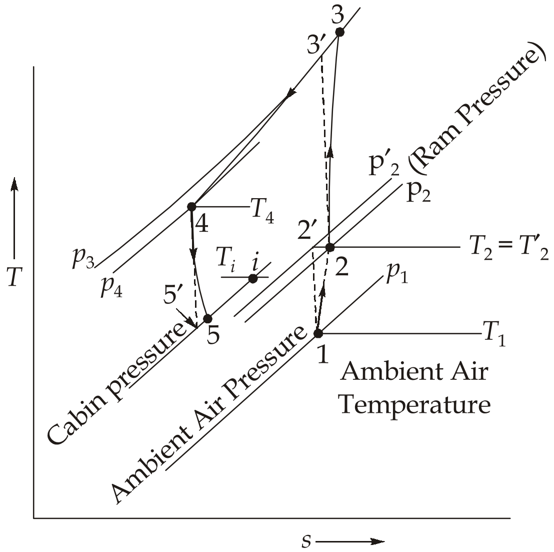
$$\text{Horse power, HP} = \frac{21.04 \times 10^3}{746} = 28.20 \text{ hp}$$

(f)

$$\text{Theoretical COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1443.9 - 366.1}{1700.79 - 1443.9} = 4.19$$

$$\text{Actual COP} = \frac{\dot{Q}_0}{\text{Power}} = \frac{68.76}{21.04} = 3.27$$

2. (c)



Simple aircraft refrigeration cycle

Given :

Cockpit temperature, (T_i) = 27°C = 300 K

Cockpit pressure, (P_i) = 1.01325 bar

Ambient temperature, (T_1) = –17°C = 256 K

Ambient pressure, (P_1) = 0.34 bar

Pressure ratio = 4

Compressor efficiency, η_c = 0.85

Plane speed (c) = 1200 km/hr = 333.33 m/s

Cooling load = 62.05 kW

Pressure drop through the cooler coil = 0.15 bar

Pressure of air leaving turbine (P_5) = 1.05 bar

(a) Stagnation temperature, $T_2 = T_1 + \dfrac{c^2}{2c_p}$

$$T_2 = 256 + \frac{333.33^2}{2 \times 1005} = 311.27 \text{ K}$$

Stagnation pressure, $P_2 = P_1 \left(\dfrac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

$$P_2 = 0.34\left(\frac{311.27}{256}\right)^{\frac{1.4}{1.4-1}} = 0.674 \text{ bar}$$

(b) Discharge pressure from the jet compressor

$$P_3 = 4 \times P_2 = 4 \times 0.674 = 2.696 \text{ bar}$$

So,
$$T_3' = T_2\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 311.27(4)^{0.286}$$

$$T_3' = 462.728 \text{ K}$$

Now, discharge temperature from the jet compressor (T_3)

$$\eta_c = \frac{T_3' - T_2}{T_3 - T_2}$$

$$T_3 = \frac{462.728 - 311.27}{0.85} + 311.27$$

$$= 489.455 \text{ K}$$

Other conditions are :
$$T_4 = T_2 = 311.27 \text{ K}$$

$$P_4 = 2.696 - 0.15 = 2.546 \text{ bar}$$

$$P_5 = 1.05 \text{ bar}$$

$$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{311.27}{\left(\frac{2.546}{1.05}\right)^{0.286}} = 241.631 \text{ K}$$

So, Refrigeration capacity $= \dot{m}c_p(T_i - T_5)$

$$\text{R.C.} = \dot{m} \times 1.005(300 - 241.631) = 62.05 \text{ kW}$$

$$\dot{m} = \frac{62.05}{(300 - 241.631) \times 1.005} = 1.0577 \text{ kg/sec}$$

(c) Volume handled by the compressor

$$\dot{V}_C = \frac{\dot{m}RT_2}{P_2} = \frac{1.0577 \times 0.287 \times 311.27}{0.674 \times 10^2}$$

$$\dot{V}_C = 1.402 \text{ m}^3\text{/sec}$$

Volume handled by expander,

$$\dot{V}_E = \frac{\dot{m}RT_5}{P_5} = \frac{1.0577 \times 0.287 \times 241.631}{1.05 \times 10^2}$$

$$\dot{V}_E = 0.6985 \text{ m}^3/\text{sec}$$

(d) Ram work (done by the engine in overcoming the drag of ram diffusion)

$$|w_R| = c_p(T_2 - T_1) = 1.005(311.27 - 256)$$

$$|w_R| = 55.546 \text{ kJ/kg}$$

$$\text{Compressor work } |w_C| = c_p(T_3 - T_2) = 1.005(489.455 - 311.27)$$

$$|w_C| = 179.075 \text{ kJ/kg}$$

$$\text{Expander work } |w_E| = c_p(T_4 - T_5) = 1.005(311.27 - 241.631) = 69.987 \text{ kJ/kg}$$

$$\text{Net work, } w = |(w_R + w_C)| - w_E$$

$$= (55.546 + 179.05) - 69.987$$

$$w = 164.609 \text{ kJ/kg}$$

$$\text{Net power} = \dot{m} \times w = 1.0577 \times 164.609$$

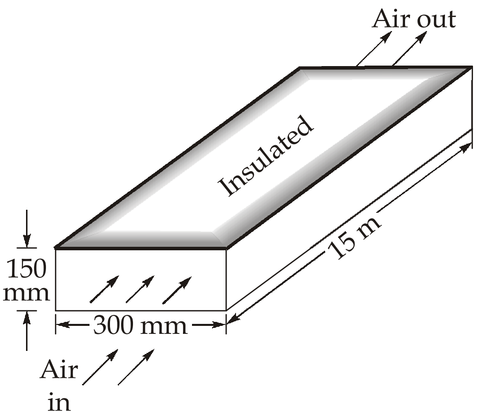
$$= 174.106 \text{ kW}$$

(e)

$$\text{COP} = \frac{R.C.}{\text{Net power}} = \frac{62.05}{174.106} = 0.356$$

3. (a)

There will be two vertical surfaces and one horizontal surface from which the heat will be gained by natural convection. There is no heat transfer from the top surface as it is insulated.



Now,

$$(\mathrm{Gr})_V = \frac{g\beta\Delta t\, H^3}{\nu^2}$$

$$= \frac{9.81\times\left(\frac{1}{273+20}\right)\times 0.15^3\times(30-10)}{(15.1\times10^{-6})^2}$$

$$(\mathrm{Gr})_V = 0.99\times10^7$$

$$(\mathrm{Gr\cdot Pr})_V = 0.99\times10^7\times0.71 = 7.03\times10^6$$

$$(\mathrm{Nu})_V = \frac{h_V H}{k} = 0.6(7.03\times10^6)^{0.25} = 30.89$$

$$\therefore \qquad h_v = \frac{h}{H}\times30.89 = \frac{0.256}{0.15}\times30.89 = 52.71\ \mathrm{W/m^2{}^\circ C}$$

$$Q_V = h_V\times A_V\times\Delta t$$

$$= 52.71\times(2\times15\times0.15)\times20$$

$$Q_V = 4743.9\ \mathrm{W}$$

For horizontal surface

$$(\mathrm{Gr})_h = \frac{9.81\times\frac{1}{(273+20)}\times0.3^3\times20}{(15.1\times10^{-6})^2} = 7.93\times10^7$$

$$(\mathrm{Gr\cdot Pr})_h = 7.93\times10^7\times0.71 = 5.63\times10^7$$

$$(\mathrm{Nu})_h = \frac{h_h\times B}{k} = 0.27(5.63\times10^7)^{0.25} = 23.38$$

$$h_h = \frac{23.38\times0.256}{0.3} = 19.95\ \mathrm{W/m^2{}^\circ C}$$

$$Q_h = h_h\times A_h\times\Delta t$$

$$Q_h = 19.95\times(15\times0.3)\times20$$

$$Q_h = 1795.6\ \mathrm{W}$$

$$\therefore \qquad \text{Total heat grain, } Q = Q_v + Q_h$$

$$= 4743.9 + 1795.6$$

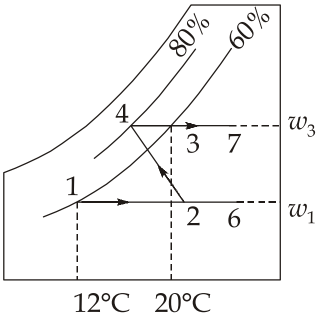
$$= 6539.5\ \mathrm{W}$$

or

$$Q \simeq 6.54\ \mathrm{kW}$$

3. (b)

The processes are shown in figure on Psychrometric chart.



Procedure for drawing process on Psychrometric chart.

Locate the points '1' and '3' on the Psychrometric chart as their conditions are known, then draw a constant specific humidity line through point '3' until it cuts the 80% RH line, then locate the point '4' as it represents the condition of air coming out of humidifier. Draw the constant enthalpy line through the point '4' and constant specific humidity line through point '1', the intersection of these two line locates the point '2' which is condition of air coming out of first heating coil.

From Psychrometric chart:

$$h_1 = 24.8 \text{ kJ/kg}; \qquad h_2 = h_4 = 37.4 \text{ kJ/kg}$$

$$h_3 = 42.4 \text{ kJ/kg}; \qquad w_1 = 4.8 \text{ grams/kg of dry air}$$

$$w_3 = 8.6 \text{ grams/kg of dry air};$$

$$T_2 = 24.6°\text{C}; \qquad T_4 = 15.5°\text{C};$$

$$v_{sa} = 0.814 \text{ m}^3/\text{kg}$$

$$\text{The mass of air supplied} = \frac{1600 \times 0.30}{v_{sa}} = \frac{1600 \times 0.30}{0.814} = 589.68 \text{ kg/min}$$

(a) $$\text{Capacity of first heating coil} = \frac{589.68}{60}(h_2 - h_1) = \frac{589.68}{60}(37.4 - 24.8)$$

$$\text{Capacity} = 123.832 \text{ kW}$$

The bypass factor (B.F) of the first-heating coil is given by

$$\text{B.F} = \frac{T_6 - T_2}{T_6 - T_1} = 0.35$$

$$\frac{T_6 - 24.6}{T_6 - 12} = 0.35$$

$$\Rightarrow \qquad T_6 = 31.384°\text{C}$$

(b) The capacity of the humidifier $= \frac{589.68}{60}(w_3 - w_1) \times \frac{1}{1000} = \frac{589.68}{60 \times 1000}(8.6 - 4.8)$

$$\text{Capacity of humidifier} = 0.0373 \text{ kg/sec}$$

$$= 0.0373 \times 3600 = 134.44 \text{ kg/hr}$$

(c) Heating capacity of second heater $= \frac{589.68}{60}(h_3 - h_4)$

$$= \frac{589.68}{60}(42.4 - 37.4)$$

$$\text{Capacity} = 49.14 \text{ kW}$$

The bypass factor of the second heating coil is given by

$$\text{B.F} = \frac{T_7 - T_3}{T_7 - T_4} = \frac{26 - 20}{26 - 15.5}$$

$$= 0.571$$

3. (c)

The general solution for temperature distribution is,

$$t = \frac{-q_g r^2}{4k} + c_1 \ln r + c_2 \qquad \text{...(i)}$$

$$\frac{dt}{dr} = \frac{-q_g r}{2k} + \frac{c_1}{r} \qquad \text{...(ii)}$$

Applying boundary conditions,

(i) At $r = r_1$, $t = t_1$

(ii) At $r = r_2$, $\frac{dt}{dr} = 0$ (Perfectly insulated)

∴ From equation (ii),

$$0 = \frac{-q_g r_2}{2k} + \frac{c_1}{r_2} \text{ or } c_1 = \frac{q_g r_2^2}{2k}$$

Also from equation (i),

$$t_1 = \frac{-q_g r_1^2}{4k} + \frac{-q_g r_2^2}{2k} \ln r_1 + c_2$$

or $$c_2 = t_1 + \frac{q_g}{2k}\left[\frac{r_1^2}{2} - r_2^2 \ln r_1\right]$$

Substituting 'c_1' and 'c_2' value in equation (i), we get

$$t = \frac{-q_g r^2}{4k} + \frac{q_g r^2}{2k}\ln r + t_1 + \frac{q_g}{2k}\left[\frac{r_1^2}{2} - r_2^2 \ln r_1\right]$$

$$t = t_1 + \frac{q_g}{2k}\left[\frac{r_1^2 - r^2}{2} - r_2^2 \ln\left(\frac{r}{r_1}\right)\right]$$

∴ Temperature distribution is parabolic.

Given: $r_2 = 5$ cm $= 0.05$ m; $r_1 = 2$ cm $= 0.02$ m; $\frac{I}{A} = 5000\ \text{amp/cm}^2$; $t_1 = 90°\text{C}$;

$k = 380$ W/mK; $\rho = 2 \times 10^{-8}\ \Omega\text{m}$

The maximum temperature occurs at the insulated surface i.e. at the outer radius and it equals,

$$t_{\max} = t_1 + \frac{q_g}{2k}\left[\frac{r_1^2 - r_2^2}{2} + r_2^2 \ln\left(\frac{r_2}{r_1}\right)\right]$$

$$t_{\max} = 90 + \frac{2 \times 10^{-8}(5000 \times 10^4)^2}{2 \times 380}\left[\frac{0.02^2 - 0.05^2}{2} + 0.05^2 \ln\left(\frac{0.05}{0.02}\right)\right]$$

$$t_{\max} = 90 + 81.62$$

$$t_{\max} = 171.62°\text{C}$$

Now, $$q_g = \frac{I^2 . \frac{\rho l}{A}}{A \cdot l} = \rho\left(\frac{I}{A}\right)^2$$

$$= 2 \times 10^{-8}\ (5000 \times 10^4)^2$$

$$= 50 \times 10^6\ \text{W/m}^3$$

Internal heat transfer rate, $$Q = -kA\left.\frac{dt}{dr}\right|_{r=r_1}$$

∵ $$\left.\frac{dt}{dr}\right|_{r=r_1} = \frac{-q_g r_1}{2k} + \frac{q_g r_2^2}{2k \cdot r_1}$$

$$= -\frac{50\times10^6\times0.02}{2\times380}+\frac{50\times10^6\times0.05^2}{2\times380\times0.02}=6907.89$$

$$\therefore \quad Q = -380 \times (2\pi \times 0.02 \times 1) \times 6907.89$$

$$Q = -329.62 \text{ kW/m}$$

Negative sign indicates that heat flow is radially inwards.

4. (a)

Given : $H = 1.5$ m; $D_o = 40$ mm $= 0.04$ m; $T_s = 180°\text{C}$; $T_a = 40°\text{C}$

$$h = \frac{80\times1000}{3600} = 22.22 \text{ W/m}^2\,°\text{C};\ n = 12;\ t = 0.75 \text{ mm};$$

$$L = 3.5 \text{ cm};\ k = \frac{260\times1000}{3600} = 72.22 \text{ W/m°C}$$

For fin,

$$\frac{P}{A_C} = \frac{2(b+t)}{b\cdot t} = \frac{2(b)}{b\cdot t} \qquad (\because\ b >>> t)$$

or

$$\frac{P}{A_C} = \frac{2}{t}$$

$$\therefore \quad m = \sqrt{\frac{hP}{kA_C}} = \sqrt{\frac{22.22\times2}{72.22\times0.75\times10^{-3}}}$$

$$m = 28.64 \text{ m}^{-1}$$

$$\therefore \quad ml = 28.64 \times 0.035 = 1.002$$

∴ For fin with insulated tip,

$$Q_{\text{fin}} = \sqrt{hpkA_C}\,\tanh ml \times n \times (t_s - t_a)$$

$$Q_{\text{fin}} = \sqrt{22.22\times2(1+0.75\times10^{-3})\times72.22\times(1\times0.75\times10^{-3})}\times(180-40)\times\tanh(1.002)\times12$$

$$Q_{\text{fin}} = 1.55 \times 140 \times 9.15$$

$$= 1.98 \text{ kW}$$

Surface area of cylinder head not occupied by the fins,

$$A_{\text{eff}} = (\pi \times 0.04 \times 1.5) - (12 \times 1 \times 0.75 \times 10^{-3})$$

$$A_{\text{eff}} = 0.18 \text{ m}^2$$

Heat dissipation from this surface is by convection

$$Q_{\text{cyl}} = hA_{\text{eff}}\,(t_s - t_a)$$

$$Q_{\text{cyl}} = 22.22 \times 0.18 \times (180 - 40)$$

$$Q_{\text{cyl}} = 560 = 0.56 \text{ kW}$$

Total heat transfer, with fins

$$Q_T = Q_{cyl} + Q_{fin} = 1.98 + 0.56$$

$$Q_T = 2.54 \text{ kW}$$

Heat transfer without fins,

$$Q_{\text{without fin}} = hA(t_s - t_a) = 22.22 \times (\pi \times 0.04 \times 1.5) \times (180 - 40)$$

$$= 0.568 \text{ kW}$$

$$\therefore \ \% \text{ increase in heat dissipation} = \frac{2.54 - 0.586}{0.586} \times 100 = 333.44\%$$

Now, temperature distribution along the fin length,

$$\frac{\theta}{\theta_0} = \frac{\cos hm(l - x)}{\cosh ml}$$

At centre, $x = 1.75$ cm

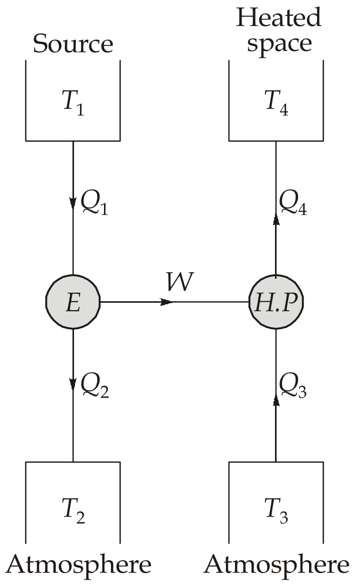
$$\therefore \quad \frac{T - 40}{180 - 40} = \frac{\cos h\, 28.64(0.035 - 0.0175)}{\cos h\,(28.64 \times 0.035)}$$

$$\therefore \quad T = 40 + 140 \times 0.73$$

$$T = 142.17°\text{C}$$

4. (b)

(i)



Given:

$$T_4 = 30°\text{C} = 303 \text{ K}$$

$$T_1 = 1000°\text{C} = 1273 \text{ K}$$

$$T_3 = T_2 = 7°\text{C} = 280 \text{ K}$$

$$Q_4 = 6500 \text{ kJ/min}$$

$$(\text{COP})_{\text{actual}} = 0.65(\text{COP})_{\text{ideal}}$$

$$\eta_{\text{actual}} = 0.5\, \eta_{\text{ideal}}$$

For the heatpump, $$(\text{COP})_{\text{ideal}} = \frac{T_4}{T_4 - T_3} = \frac{303}{23}$$

$$(\text{COP})_{\text{ideal}} = 13.1739$$

So, $$(\text{COP})_{\text{actual}} = 0.65 \times 13.1739 = 8.563$$

Ideal efficiency of engine $(\eta_{\text{ideal}}) = 1 - \dfrac{T_2}{T_1}$

$$\eta_{ideal} = 1 - \frac{280}{1273} = 0.78$$

∴ Actual efficiency of engine (η_{actual}) = 0.5 × 0.78 = 0.39

We know, $$(COP)_{actual} = \frac{Q_4}{W}$$

$$W = \frac{6500}{8.563 \times 60} = 12.651 \text{ kW}$$

and $$\eta_{actual} = \frac{W}{Q_1}$$

Heat supplied to the engine, $Q_1 = \dfrac{W}{\eta_{actual}} = \dfrac{12.651}{0.39} = 32.439$ kW

$$(COP)_{overall} = \frac{Q_4}{Q_1} = \frac{6500}{60 \times 32.439} = 3.339$$

(ii)

Given:

Refrigeration capacity, R.C = 12 tonnes

= 12 × 3.5 = 42 kW

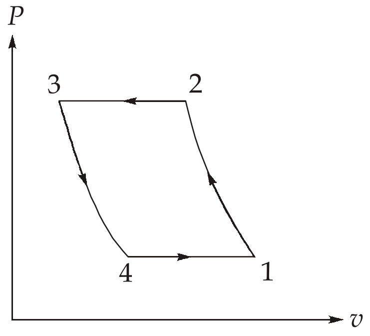
Cooler pressure = 3.9 bar

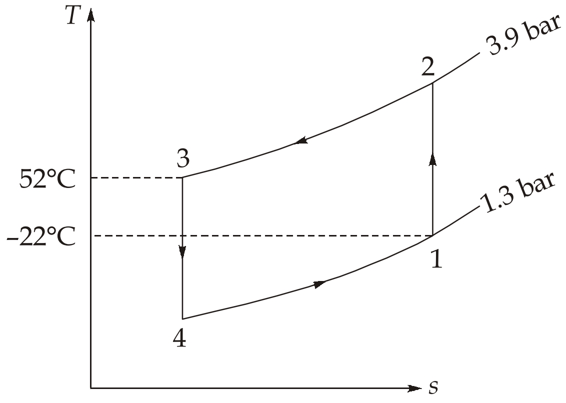
Refrigerator pressure = 1.3 bar

T_3 = 52°C = 325 K

T_1 = –22°C = 251 K

Pressure ratio, $r_p = \dfrac{P_2}{P_1} = \dfrac{P_3}{P_4} = 3$





Assumptions :

(i) Air is treated as ideal gas.

(ii) The ratio of specific heats for air is 1.4.

For process 1-2

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (3)^{0.286}$$

$$T_2 = 343.661 \text{ K}$$

For process 3-4

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = \frac{325}{(3)^{0.286}} = 237.3718 \text{ K}$$

(a)

$$\text{COP} = \frac{R.C}{\text{Net work input}}$$

$$\text{COP} = \frac{c_p(T_1 - T_4)}{c_p(T_2 - T_1) - c_p(T_3 - T_4)}$$

$$\text{COP} = \frac{(251 - 237.3718)}{(343.661 - 251) - (325 - 237.3718)}$$

$$\text{COP} = \frac{13.6282}{92.661 - 87.6282} = 2.707$$ Ans.

(b)

$$\text{R.C} = \dot{m}c_p(T_1 - T_4)$$

$$\dot{m} = \frac{R.C}{c_p(T_1 - T_4)} = \frac{42}{1.05(251 - 237.3718)}$$

$$\dot{m} = \frac{42}{14.30961} = 2.935 \text{ kg/sec}$$

$$\dot{m} = 2.935 \times 60 = 176.105 \text{ kg/min}$$

(c) Applying ideal gas equation to air at compressor inlet condition

$$P_1 v_1 = mRT_1$$

$$v_1 = \frac{176.105 \times 287 \times 251}{1.3 \times 10^5} = 97.585 \text{ m}^3/\text{min}$$

For theoretical piston displacement assume volumetric efficiency (η_v) = 100%

Theoretical piston displacement = $\eta_v \times v_1$

$$= 1 \times 97.585 = 97.585 \text{ m}^3/\text{min} \qquad \text{Ans.}$$

(d)

$$\text{Net power} = \dot{m}\left[c_p(T_2 - T_1) - c_p(T_3 - T_4)\right]$$

$$= \frac{176.105}{60}\left[1.05(343.661 - 251) - 1.05(325 - 237.3718)\right]$$

$$\text{Net power} = \frac{176.105}{60}[97.294 - 92.0096]$$

$$\text{Net power} = 15.510 \text{ kW}$$

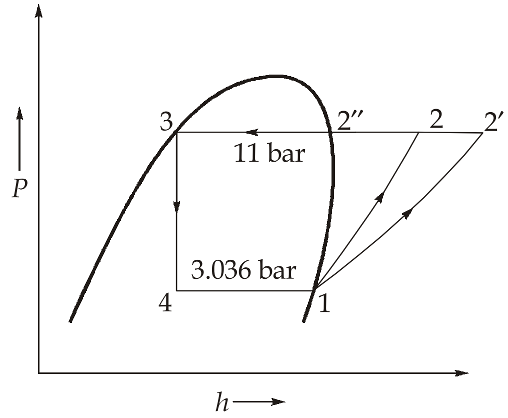
$$\frac{\text{Net power}}{\text{Ton of refrigeration}} = \frac{15.510}{12} = 1.2925 \qquad \text{Ans.}$$

4. (c)

Given : Cylinder size = 40 × 50 cm; RPM (N) = 500

$$\frac{\dot{V}_c}{\dot{V}_s} = c = 0.15$$

$$V_1 = 0.402 \text{ m}^3/\text{kg}$$



From the ammonia thermodynamic properties table,

$$h_1 = 1451.4 \text{ kJ/kg}$$

$$s_1 = 5.7409 \text{ kJ/kgK}$$

$$h_2'' = 1485 \text{ kJ/kg}$$

$$s_2'' = 5.2843 \text{ kg/kgK}$$

$$h_3 = h_4 = 332.1 \text{ kJ/kg}$$

$$T_2'' = T_{\text{sat}} = 28°\text{C} = 301 \text{ K}$$

$$T_1 = T_{\text{sat}} = -9°\text{C} = 264 \text{ K}$$

If the compression follows isentropic process and $n = 1.33$

then,
$$s_1 = s_2 = s_2'' + c_{pv} \ln \frac{T_2}{T_2''}$$

$$5.7409 = 5.2843 + 3.028 \ln \frac{T_2}{301}$$

$$T_2 = 349.989 \simeq 350 \text{ K}$$

So,
$$h_2 = h_2'' + c_{pv}(T_2 - T_2'')$$

$$h_2 = 1485 + 3.028\,(350 - 301)$$

$$h_2 = 1633.372 \text{ kJ/kg}$$

The volumetric efficiency of the compressor is given by,

$$\eta_v = 1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 1 + 0.15 - 0.15\left(\frac{11}{3.036}\right)^{\frac{1}{1.33}}$$

$$\eta_v = 1.15 - 0.39487 = 0.755 = 75.5\%$$

Displacement volume of the compressor

$$= \frac{\pi}{4} D^2 \times L \times N \times \eta_v$$

$$= \frac{\pi}{4} \times (0.4)^2 \times 0.5 \times 500 \times 0.755$$

$$= 23.722 \text{ m}^3/\text{min}$$

∴ Refrigerant flow through the system per minute

$$\dot{m}_r = \frac{23.722}{0.402} = 59.0122 \text{ kg/min} \qquad (\because \quad V_1 = 0.402 \text{ m}^3/\text{kg})$$

$$\dot{m}_r = \frac{59.0122}{60} = 0.9835 \text{ kg/sec}$$

Refrigerant capacity of the system

$$\text{R.C} = \dot{m}_r(h_1 - h_4)$$

$$\text{R.C} = 0.9835(1451.4 - 332.1)$$

$$\text{R.C} = 1100.873 \text{ kW} = \frac{1100.873}{3.5} = 314.53 \text{ TR}$$

Power required to run the compressor

$$= \dot{m}_r(h_2 - h_1) = 0.9835(1633.372 - 1451.4)$$

$$\text{Power} = 178.969 \text{ kW}$$

When compression is polytropic and n = 1.4

then

(i) Work input (W) $= P_1V_1 \times \frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1\right]$

$$W = \frac{3.036 \times 100 \times 0.402 \times 1.4}{0.4}\left[\left(\frac{11}{3.036}\right)^{0.286} - 1\right]$$

$$W = 189.907 \text{ kJ/kg}$$

So, Power required $= \dot{m} \times W = 0.9835 \times 189.907$

Power required = 186.78 kW

(ii) Temperature T_2' is given by

$$T_2' = T_1\left(\frac{P_2'}{P_1}\right)^{\frac{n-1}{n}} = (-9 + 273)\left(\frac{11}{3.036}\right)^{0.286}$$

$$T_2' = 381.508 \text{ K}$$

$$h_2' = h_2'' + c_{pv}(T_2' - T_2'') = 1485 + 3.028(381.508 - 301)$$

$$h_2' = 1728.778 \text{ kJ/kg}$$

Heat rejected in condensor $= \dot{m}_r(h_2' - h_3)$

$= 0.9835(1728.778 - 332.1)$

Heat rejected in condensor = 1373.632 kW

(iii) Apply the energy balance equation to the point 1 and 2′

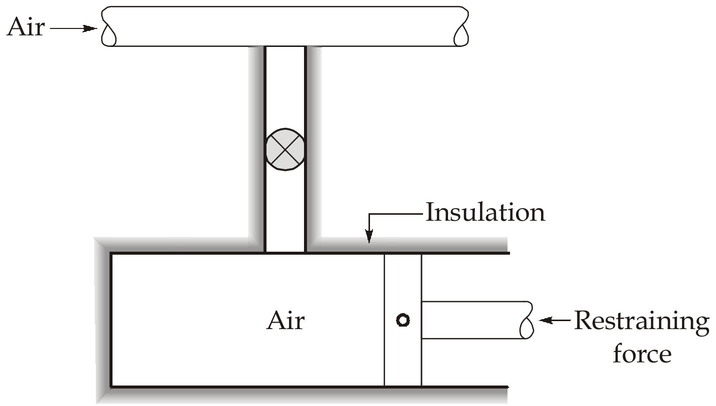
$$h_1 + W = h_2' - Q_r$$

$$-Q_r = W + (h_1 - h_2')$$

$$= 189.907 + (1451.4 - 1728.778)$$

$$Q_r = 87.471 \text{ kJ/kg}$$

5. **(a)**



Mass Balance:

$$m_2 - m_1 = m_i - m_e$$

Since, $m_e = 0$

$\therefore$ $m_2 - m_1 = m_i$ = Mass of air entered to the cylinder

Energy Balance:

$$U_2 - U_1 = m_i h_i + \overset{0}{\cancel{q}} - \overset{0}{\cancel{m_e h_e}} - W$$

$$\Rightarrow \quad m_2 u_2 - m_1 u_1 = (m_2 - m_1)h_p - W$$

Assuming air to be an ideal gas;

$$\frac{P_2 V_2}{R T_2} \times C_V T_2 - \frac{P_1 V_1}{R T_1} \times C_V T_1 = \left(\frac{P_2 V_2}{R T_2} - \frac{P_1 V_1}{R T_1}\right) \times C_P T_P - P_1 (V_2 - V_1)$$

$$(\because\ PV = mRT,\ u = C_V T,\ h = C_P T)$$

$$\Rightarrow \quad \frac{V_2 \times C_V}{R} - \frac{V_1 C_V}{R} = \left(\frac{V_2}{R T_2} - \frac{V_1}{R T_1}\right) C_P T_P - (V_2 - V_1) \quad (\because\ P_1 = P_2)$$

$$\Rightarrow (V_2 - V_1) - \frac{C_P T_P}{R}\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) + \frac{C_V}{R}(V_2 - V_1) = 0$$

$$\Rightarrow \quad (V_2 - V_1)\left\{1 + \frac{C_V}{R}\right\} - \frac{C_P T_P}{R}\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) = 0$$

$$\Rightarrow \quad (V_2 - V_1)\frac{C_P}{R} - \frac{C_P T_P}{R}\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) = 0$$

$$\Rightarrow \quad \frac{C_P}{R}\left\{(V_2 - V_1) - T_P\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right)\right\} = 0$$

$$\Rightarrow \quad (V_2 - V_1) - T_P\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) = 0$$

$$\Rightarrow \quad T_P\frac{V_2}{T_2} = (V_2 - V_1) + T_P\frac{V_1}{T_1}$$

$$\Rightarrow \quad T_2 = \frac{T_P V_2}{(V_2 - V_1) + \frac{T_P V_1}{T_1}}$$

$$\Rightarrow \quad T_2 = \frac{V_2}{\frac{(V_2 - V_1)}{T_P} + \frac{V_1}{T_1}}$$

$$\Rightarrow \quad T_2 = \frac{2V_1}{\frac{V_1}{T_P} + \frac{V_1}{T_1}} = \frac{2}{\frac{1}{T_P} + \frac{1}{T_1}} \qquad [\because \; V_2 = 2V_1]$$

$$\therefore \quad \text{Final temperature, } T_2 = \frac{2}{\frac{1}{T_1} + \frac{1}{T_P}}$$

Hence, it is proved.

5. (b)

Given : Diameter of copper bar, $d_c = 30$ mm; Inner diameter of steel tube, $(d_i)_s = 30$ mm; Outer diameter of steel tube, $(d_o)_s = 50$ mm; Diameter of pin, $d_p = 12$ mm; $\Delta T = 50$ K; $E_c = 1.2 \times 10^5$ N/mm^2; $E_s = 2.2 \times 10^5$ N/mm^2; $\alpha_c = 20 \times 10^{-6}$/K; $\alpha_s = 14 \times 10^{-6}$/K

$\therefore$ Area of cross-section of copper bar,

$$A_c = \frac{\pi}{4} \times (30)^2 = 706.858 \text{ mm}^2$$

Area of cross-section of steel tube,

$$A_s = \frac{\pi}{4} \times (50^2 - 30^2) = 1256.637 \text{ mm}^2$$

From compatibility equation;

Sum of axial deformations = Difference of thermal expansions

$$\Rightarrow \quad \frac{\sigma_c L}{E_c} + \frac{\sigma_s L}{E_s} = \alpha_c \Delta T L - \alpha_s \Delta T L$$

$$\Rightarrow \quad \frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = \Delta T(\alpha_c - \alpha_s)$$

$$\Rightarrow \quad \frac{\sigma_c}{1.2 \times 10^5} + \frac{\sigma_s}{2.2 \times 10^5} = 50 \times (20 - 14) \times 10^{-6}$$

$$\Rightarrow \quad \frac{\sigma_c}{1.2} + \frac{\sigma_s}{2.2} = 30 \quad \text{...(i)}$$

From statics:

$$\sigma_c A_c = \sigma_s A_s$$

$$\Rightarrow \quad \sigma_c \times 706.858 = \sigma_s \times 1256.637$$

$$\Rightarrow \quad \sigma_c = 1.78\sigma_s \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\sigma_c = 27.556 \text{ N/mm}^2$$

$$\sigma_s = 15.48 \text{ N/mm}^2$$

Force between steel tube and copper bar

$$F = \sigma_c A_c = 27.556 \times 706.858$$

$$= 19478.179 \text{ N}$$

Area of cross-section of the pin,

$$A_p = \frac{\pi}{4} \times (12)^2 = 113.097 \text{ mm}^2$$

Since the pin is fitted transversely and passes through the tube and rod, it will be in double shear.

$$\therefore \quad \text{Shear stress in pin} = \frac{19478.179}{2 \times 113.097}$$

$$= 86.11 \text{ N/mm}^2$$

5. **(c)**

Given: $N = 1440$ rpm; $T = 14$ Nm; $I = 15$A; $V = 220$V; $\dot{Q}_{out} = 20(T_s - T_o)$; $T_0 = 30°\text{C} = 303$ K

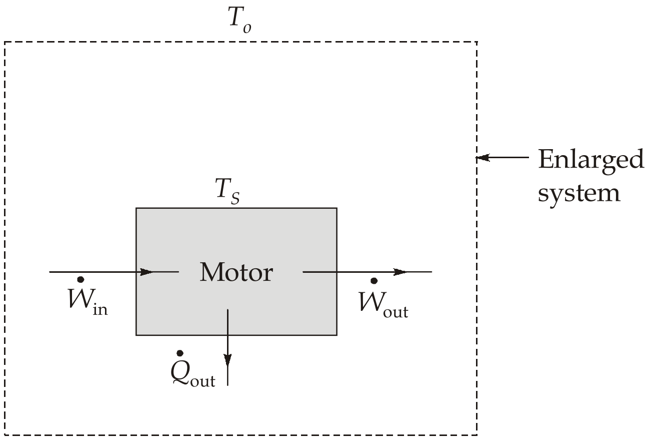
(i)

According to the first law of thermodynamics under steady state;

$$\frac{dE}{dt} = 0$$

$$\Rightarrow \quad \dot{W}_{in} - \dot{W}_{out} - \dot{Q}_{out} = 0$$

$$\Rightarrow \qquad \dot{Q}_{out} = \dot{W}_{in} - \dot{W}_{out} \qquad \text{...(i)}$$



$$\dot{W}_{in} = VI$$

$$= 220 \times 15 = 3300 \text{ W}$$

$$\dot{W}_{out} = T \times \omega$$

$$= 14 \times \left(\frac{2\pi \times 1440}{60}\right) = 2111.15 \text{W}$$

∴ From equation (i)

$$\dot{Q}_{out} = 3300 - 2111.15$$

$$= 1188.85 \text{ W}$$

Since, it is given that

$$\dot{Q}_{out} = 20(T_s - T_o)$$

$$\Rightarrow \qquad 1188.85 = 20 \times (T_s - 303)$$

$$\Rightarrow \qquad T_s = 362.44 \text{ K}$$

(ii)

If the system boundaries are defined such that to include the whole region of temperature gradient, the heat transfer crosses these boundaries at the ambient temperature of 303K. The entropy generation rate is, therefore

$$\dot{s}_{gen.} = \frac{\dot{Q}_{out}}{T_0} = \frac{1188.85}{303} = 3.92 \text{ W/K}$$

5. (d)

$$\text{Force, } F_1 = 120\cos 30°\hat{i} + 120\sin 30°\hat{j}$$

$$= 103.923\hat{i} + 60\hat{j} \text{ N}$$

$$F_2 = 220\cos 45°\hat{i} + 220\sin 45°\hat{j}$$

$$= 155.563\hat{i} + 155.563\hat{j} \text{ N}$$

Position vectors;

$$r_1 = 3\hat{i} + 5\hat{j}, \quad r_2 = 7\hat{i} + 4\hat{j}$$

Resultant force at origin O,

$$F_R = F_1 + F_2$$

$$= (103.923 + 155.563)\hat{i} + (60 + 155.563)\hat{j}$$

$$= 259.486\hat{i} + 215.563\hat{j} \text{ N}$$

$$\therefore \qquad |F_R| = 337.343 \text{ N}$$

Angle θ with x-axis

$$\theta = \tan^{-1}\frac{215.563}{259.486} = 39.717°$$

Resultant couple at origin,

$$C_R = r_1 \times F_1 + r_2 \times F_2 + c \qquad \text{...(i)}$$

$$r_1 \times F_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 0 \\ 103.923 & 60 & 0 \end{vmatrix}$$

$$= \hat{k}(180 - 519.615) = -339.615\hat{k}$$

$$r_2 \times F_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 4 & 0 \\ 155.563 & 155.563 & 0 \end{vmatrix}$$

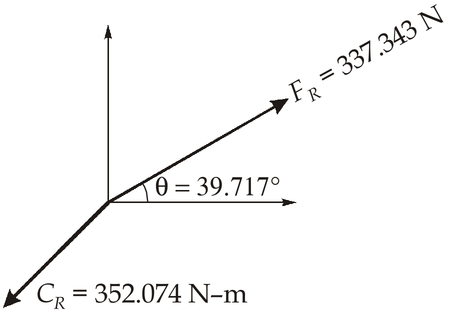
$$= \hat{k}(180.941 - 622.252) = 466.689\hat{k}$$

$$c = 1.5 \times 150\hat{k} = 225\hat{k} \qquad (\because \text{the couple is in the } xy \text{ plane})$$

From equation (i),

$$C_R = (-339.615 + 466.689 + 225)\hat{k}$$

$$= 352.074\hat{k} \text{ N-m}$$



5. (e)

Given : $\sigma_x = -80$ MPa; $\sigma_y = 120$ MPa; $\tau_{xy} = -60$ MPa

(i)

Calculation of principal planes:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times (-60)}{-80 - 120} = 0.6$$

$$\Rightarrow \quad 2\theta = 30.964°, 210.964°$$

$$\Rightarrow \quad \theta = 15.482°, 105.482°$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

At $\theta = 15.482°$

$$(\sigma_n)_1 = \frac{-80 + 120}{2} + \frac{(-80 - 120)}{2}\cos(2 \times 15.482) - 60\sin(2 \times 15.482)$$

$$= -96.619 \text{ MPa}$$

At $\theta = 105.482°$

$$(\sigma_n)_2 = \frac{-80 + 120}{2} + \frac{(-80 - 120)}{2}\cos(2 \times 105.482) - 60\sin(2 \times 105.482)$$

$$= 136.619 \text{ MPa}$$

∴ Maximum principal stress,

$$\sigma_1 = 136.619 \text{ MPa}$$

Minimum principal stress,

$$\sigma_2 = 96.619 \text{ MPa (C)}$$

(ii)

The maximum shear stress will occur at an angle of 45° from the planes of maximum and minimum principal planes.

$$\therefore \qquad \theta_3 = 15.482° + 45° = 60.482°$$

$$\theta_4 = 105.482° + 45° = 150.482°$$

Calculation of shear stresses:

At $\theta_3 = 60.482°$

$$(\tau_s)_\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}(-80-120)\sin(2\times 60.482) + (-60)\cos(2\times 60.482)$$

$$= 116.619 \text{ MPa}$$

At $\theta_4 = 150.482°$

$$(\tau_s)_\theta = -116.619 \text{ MPa}$$

($\because$ Maximum shear stress planes are complementy)

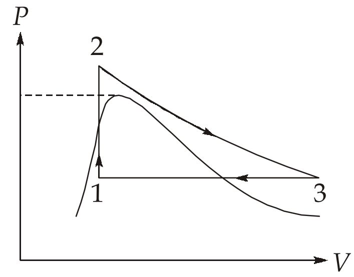
(iii)

Normal stress on maximum shear stress planes:

$$\sigma_n^* = \frac{\sigma_1 + \sigma_2}{2} = \frac{136.619 - 96.619}{2} = 20 \text{ MPa}$$

6. (a)

(i)



Constant volume heating : (1 - 2)

As the process is constant volume so, $\delta W = 0$

According to the first law:

$$\delta Q = \delta U + \delta W = U_f - U_i \qquad \text{...(i)}$$

At $P_i = 12$ bar and $x = 0.4$; using the steam table:

$$u_f = 796.96 \text{ kJ/kg}, u_g = 2587.8 \text{ kJ/kg}$$

$$v_f = 0.00113850 \text{ m}^3\text{/kg}, v_g = 0.16326 \text{ m}^3\text{/kg}$$

$$s_1 = 2.2159 \text{ kJ/kg-K}, s_{fg} = 4.3058 \text{ kJ/kg-K}$$

$$\therefore \quad u_i = 796.96 + 0.4 \times (2587.8 - 796.96)$$

$$= 1513.296 \text{ kJ/kg}$$

$$v_i = 0.00113850 + 0.4 \times (0.16326 - 0.00113850)$$

$$= 0.06598 \text{ m}^3\text{/kg}$$

$$s_i = 2.2159 + 0.4 \times 4.3058$$

$$= 3.9382 \text{ kJ/kg-K}$$

At the final state; $v_f = v_i = 0.06598 \text{ m}^3\text{/kg}$ and $P = 40$ bar

So, the final state is in the superheated region.

Using steam table and linear interpolation.

$$T_f = 340 + \frac{(350 - 340)(0.06598 - 0.0650190)}{(0.066473 - 0.0650190)}$$

$$T_2 = 346.609°\text{C}$$

$$u_f = 2808 + \frac{(2827.4 - 2808) \times 6.609}{10}$$

$$u_2 = 2820.82 \text{ kJ/kg}$$

$$s_f = 6.5435 + \frac{(6.5843 - 6.5435) \times 6.609}{10}$$

$$s_2 = 6.5722 \text{ kJ/kg-K}$$

From equation (i),

$$(\delta q)_{1\text{-}2} = u_f - u_i = 2820.82 - 1513.296$$

$$= 1307.524 \text{ kJ/kg}$$

Since mass, $m = 1$ kg

$$\therefore \quad (\delta Q)_{1\text{-}2} = 1307.524 \text{ kJ}$$

$$\Delta s = s_2 - s_1$$

$$= m(s_2 - s_1) = 1 \times (6.5722 - 3.9382)$$

$$= 2.634 \text{ kJ/K}$$

(ii)

Isothermal expansion : (2 - 3)

Since, the process is isothermal so, T = constant = 346.609°C and pressure, p = 12 bar

The steam is again superheated, so, using steam table and linear interpolation

$$s_3 = 7.1792 + \frac{(7.2139 - 7.1792) \times 6.609}{10}$$

$$= 7.2021 \text{ kJ/kg-K}$$

$$u_3 = 2856.2 + \frac{(2872.7 - 2856.2) \times 6.609}{10}$$

$$= 2867.104 \text{ kJ/kg}$$

$$v_3 = 0.23045 + \frac{(0.23455 - 0.23045) \times 6.609}{10}$$

$$= 0.23316 \text{ m}^3\text{/kg}$$

In a reversible isothermal process, heat transfer is related to entropy change as ;

$$q_{2\text{-}3} = T(s_3 - s_2)$$

$$= (346.609 + 273) \times (7.2021 - 6.5722)$$

$$= 390.29 \text{ kJ/kg}$$

Since, mass, m = 1 kg

$$\therefore \quad Q_{2\text{-}3} = 390.29 \text{ kJ}$$

According to first law;

$$\delta Q_{2\text{-}3} = \Delta U_{2\text{-}3} + \delta W_{2\text{-}3}$$

$$390.29 = 2867.104 - 2820.82 + \delta W_{2\text{-}3}$$

$$\Rightarrow \quad \delta W_{2\text{-}3} = 344.006 \text{ kJ}$$

(iii) Constant pressure cooling:

The work done on the steam during this process is given by:

$$\delta W_{3\text{-}1} = P_3(V_1 - V_3)$$

$$= 12 \times 100 \times (0.06598 - 0.23316)$$

$$= -200.616 \text{ kJ} \qquad [\because \quad \text{Mass} = 1 \text{ kg}]$$

$$\delta Q_{3\text{-}1} = \Delta U_{3\text{-}1} + \delta W_{3\text{-}1}$$

$$= (U_1 - U_3) + (-200.616)$$

$$= (1513.296 - 2867.104) - 200.616$$

$$= -1554.425 \text{ kJ} \qquad [\because \text{ mass} = 1 \text{ kg}]$$

$$(\Delta S)_{3\text{-}1} = S_1 - S_3 = (3.9382 - 7.2021) \times 1$$

$$= -3.2639 \text{ kJ/K}$$

Net heat input, $$\Sigma Q_{in} = 1307.524 + 390.29 = 1697.814 \text{ kJ}$$

Net work output, $$\Sigma W_{net} = 344.006 - 200.616 = 143.39 \text{ kJ}$$

$\therefore$ Efficiency, $$\eta = \frac{\Sigma W_{net}}{\Sigma Q_{in}} = \frac{143.39}{1697.814}$$

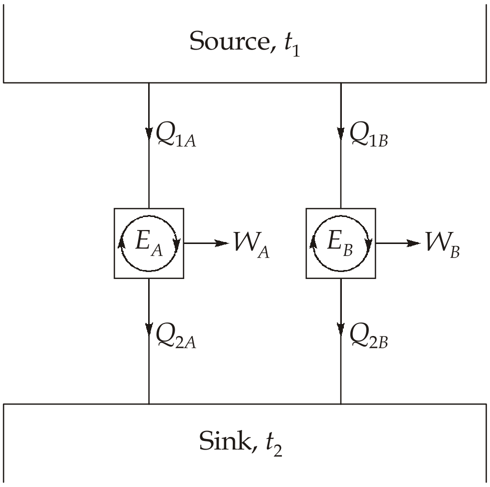
$$= 0.0844 = 8.44\%$$

6. (b)

(i)

Carnot's theorem states that of all heat engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine.

Let two heat engines E_A and E_B operate between the given source at temperature t_1 and the given sink at temperature t_2 as shown in figure below.



Let E_A be any heat engine and E_B be any reversible heat engine. We have to prove that the efficiency of E_B is more than that of E_A. Let us assume that this is not true and $\eta_A > \eta_B$. Let the rates of working of the engines be such that

$$Q_{1A} = Q_{1B} = Q_1$$

Since, $$\eta_A > \eta_B$$

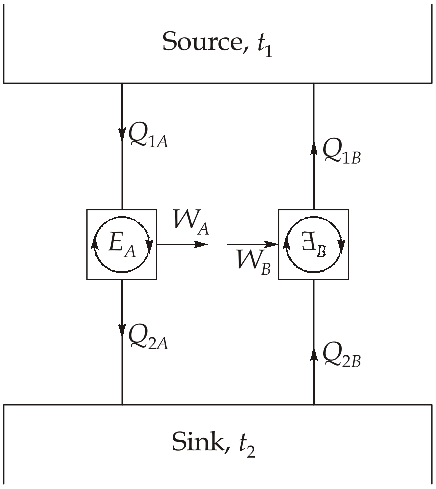
$$\frac{W_A}{Q_{1A}} > \frac{W_B}{Q_{1B}}$$

$\therefore$ $$W_A > W_B$$

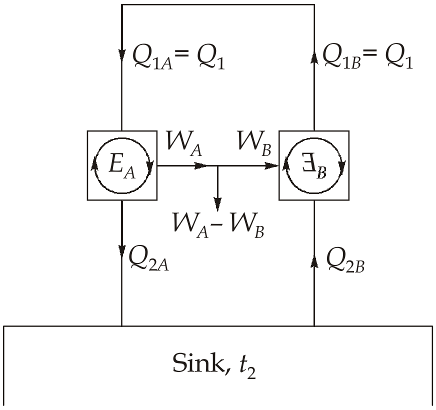
Now, let E_B be reversed. Since E_B is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same, but their directions will be reversed, as shown in figure (a). Since $W_A > W_B$, some part of W_A (equal to W_B) may be fed to drive the reversed heat engine $\exists_B$.

Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by $\exists_B$ may be supplied to E_A. The source may, therefore, be eliminated as shown in figure (b). The net result is that E_A and $\exists_B$ together constitute a heat engine which, operating in a cycle, produces net work $W_A - W_B$, while exchanging heat with a single reservoir at t_2. This violates the Kelvin-Planck statement of the second law. Hence the assumption that $\eta_A > \eta_B$ is wrong.

Therefore, $$\eta_B \geq \eta_A$$

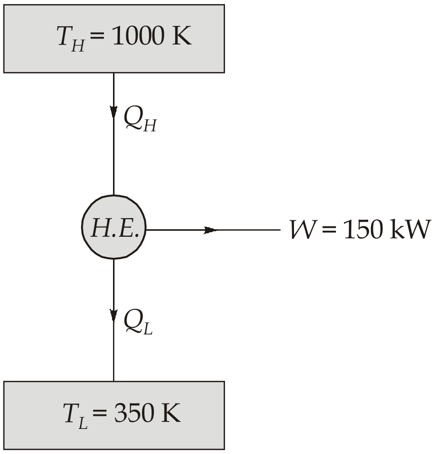


E_B is reversed
Fig. (a)



E_A and $\exists_B$ together violate the K-P statement
Fig. (b)

(ii)



(a) Given : $T_H = 1000$ K; $T_L = 350$ K; $W = 150$ kW

For completely reversible cycle, according to Claussius inequality:

$$\oint \frac{\delta Q}{T} = 0$$

$$\Rightarrow \quad \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

$$\Rightarrow \quad \frac{Q_H}{1000} = \frac{Q_L}{350}$$

$$\Rightarrow \quad Q_H = \frac{20}{7} Q_L \quad \text{...(i)}$$

Also, $\quad Q_H - Q_L = 150$

$$\Rightarrow \quad \frac{20}{7} Q_L - Q_L = 150$$

$$\Rightarrow \quad Q_L = 80.769 \text{ kW}$$

For completely reversible heat engine, entropy generation is zero.

(b)

When the engine is internally reversible but experiences external irreversibility during heat transfer.

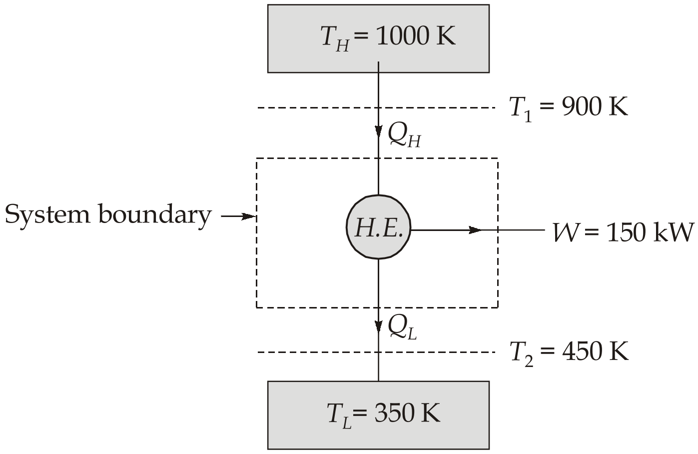
According to Claussius inequality for internally reversible heat engine:

$$\oint \frac{\delta Q}{T} = 0$$

$$\Rightarrow \quad \frac{Q_H}{900} - \frac{Q_L}{450} = 0$$

$$\Rightarrow \qquad Q_H = 2Q_L$$

According to the first law of thermodynamics,



$$Q_H - Q_L = 150$$

$$\Rightarrow \qquad 2Q_L - Q_L = 150$$

$$\Rightarrow \qquad Q_L = 150 \text{ kW}$$

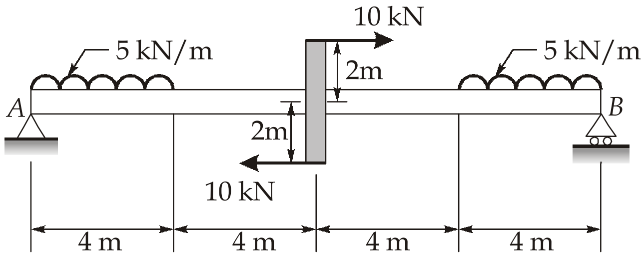
$$\therefore \qquad Q_H = 2 \times Q_L = 2 \times 150 = 300 \text{ kW}$$

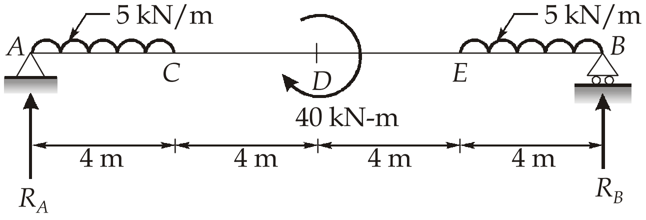
Entropy generation:

$$(\delta S)_{\text{gen.}} = -\frac{Q_H}{1000} + \frac{Q_L}{350}$$

$$= -\frac{300}{1000} + \frac{150}{350} = 0.1285 \text{ kW/K}$$

6. **(c)**





Reactions:

$$R_A + R_B = 20 + 20 = 40 \text{ kN}$$

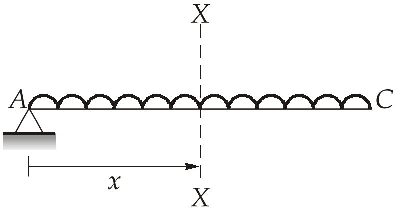
Taking moment about A:

$$20 \times 2 + 40 + 20 \times 14 = R_B \times 16$$

$\Rightarrow$
$$R_B = 22.5 \text{ kN}$$

and
$$R_A = 17.5 \text{ kN}$$

Shear force calculation:



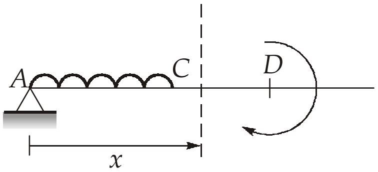
For AC:

$$(\text{SF})_x = R_A - 5 \times x$$

$\therefore$
$$(\text{S.F.})_A = R_A = 17.5 \text{ kN}$$

$$(\text{S.F.})_C = 17.5 - 5 \times 4 = -2.5 \text{ kN}$$

For CD:



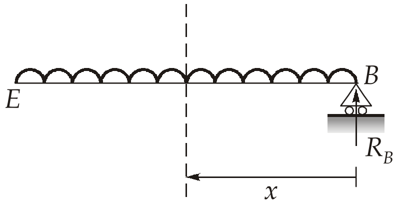
$$(\text{SF})_x = R_A - 5 \times 4$$

$$= 17.5 - 20 = -2.5 \text{ kN}$$

For DE:

$$\text{S.F.} = \text{constant} = -2.5 \text{ kN}$$

For EB :



$$(\text{SF})_x = -R_B + 5 \times x$$

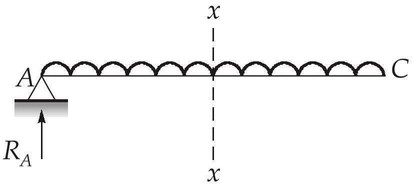
$$= -22.5 + 5x$$

$$\therefore \qquad (SF)_B = -22.5 \text{ kN}$$

$$(SF)_E = -22.5 + 20 = -2.5 \text{ kN}$$

Bending moment calculation:

For AC:

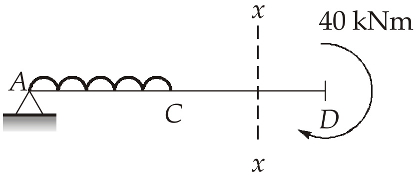


$$(BM)_x = R_A x - \frac{5x^2}{2}$$

$$= 17.5 \times x - \frac{5x^2}{2}$$

$$\therefore \qquad (BM)_A = 0 \text{ and } (BM)_C = 17.5 \times 4 - \frac{5 \times 4^2}{2} = 30 \text{ kN-m}$$

For CD:



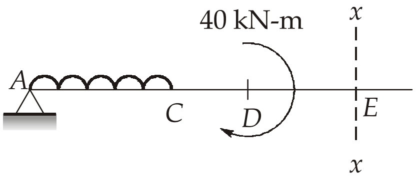
$$(BM)_C = 30 \text{ kNm}$$

$(BM)_D$ = Without considering concentrated moment

$$= 17.5 \times 8 - 5 \times 4 \times 6 = 20 \text{ kN-m}$$

$(BM)_D$ with considering concentrated moment = 60 kN-m.

For DE :

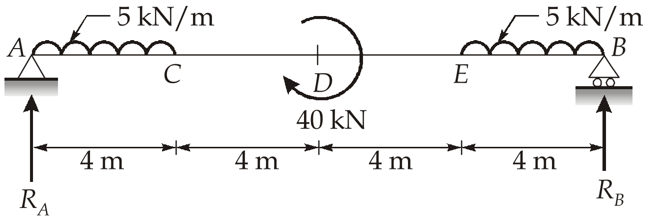


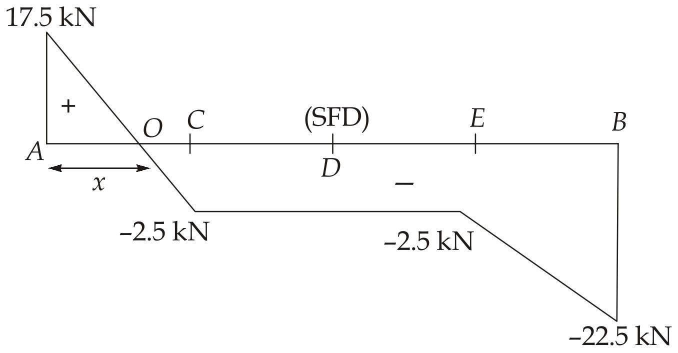
$$(BM)_E = 17.5 \times 12 - 5 \times 4 \times 10 + 40$$

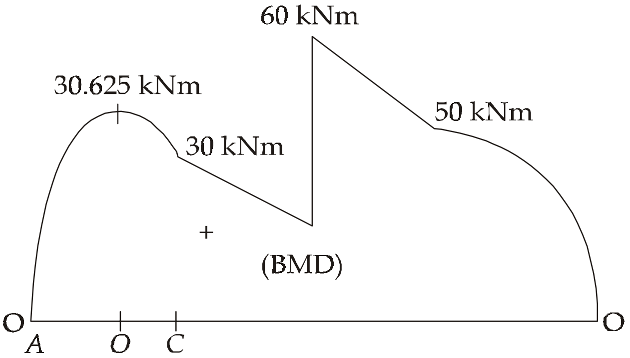
$$= 50 \text{ kN-m}$$

For EB :

$$(BM)_B = 0$$







At point O:

$$\frac{17.5}{x} = \frac{2.5}{4-x}$$

$$\Rightarrow \qquad 70 - 17.5x = 2x$$

$$\Rightarrow \qquad x = 3.5 \text{ m}$$

$$\therefore \qquad (\text{BM})_O = 17.5 \times 3.5 - \frac{5 \times (3.5)^2}{2} = 30.625 \text{ kNm}$$

There is no any point of contraflexure because bending moment does not change its sign anywhere.

7. (a)

(i)

Given : Inner radius, $r = \frac{600}{2} = 300 \text{ mm}$; Internal pressure, $p = 30 \text{ N/mm}^2$; Maximum permissible stress, $\sigma_{\max} = 80 \text{ N/mm}^2$

According to Lame's equations:

Radial pressure at any radius r is;

$$\sigma_r = \frac{2B}{r^3} - A \quad \text{...(i)}$$

Hoop stress at any radius r is

$$\sigma_h = \frac{B}{r^3} + A \quad \text{...(ii)}$$

At $r = 250$ mm

$$\sigma_r = 30 \text{ N/mm}^2 \text{ and } \sigma_h = 80 \text{ N/mm}^2$$

From equation (i) and (ii),

$$30 = \frac{2B}{(300)^3} - A \quad \text{...(iii)}$$

$$80 = \frac{B}{(300)^3} + A \quad \text{...(iv)}$$

On solving the above equations, we get

$$A = 43.33 \text{ and } B = 9.9 \times 10^8$$

At outer radius, i.e. at R

$$\sigma_R = 0$$

$$\Rightarrow \quad 0 = \frac{2B}{(R)^3} - A$$

$$\Rightarrow \quad A = \frac{2B}{(R)^3}$$

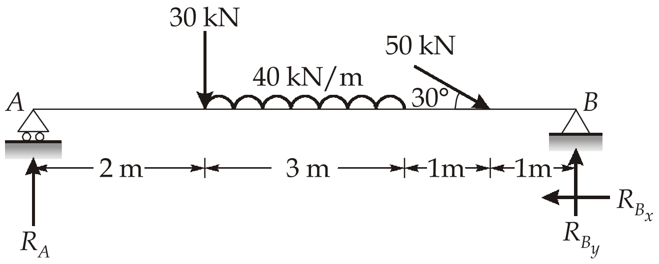
$$\Rightarrow \quad R = \left(\frac{2B}{A}\right)^{1/3} = 357.513 \text{ mm}$$

Now,

$$\text{Thickness} = R - r = 357.513 - 300$$

$$= 57.513 \text{ mm}$$

(ii)



$$\therefore \qquad R_A + R_{B_y} = 30 + 40 \times 3 + 50 \sin 30°$$

$$= 175 \text{ kN} \qquad \text{...(i)}$$

$$R_{B_x} = 50 \cos 30° = 43.3 \text{ kN} \qquad \text{...(ii)}$$

Taking moment about A:

$$30 \times 2 + 120 \times (2 + 1.5) + 50 \sin 30° \times 6 = R_{B_y} \times 7$$

$$\Rightarrow \qquad R_{B_y} = 90 \text{ kN}$$

$$\therefore \qquad R_A = 175 - 90 = 85 \text{ kN}$$

So, reaction at support A:

$$R_A = 85 \text{ kN}$$

Reaction at support B:

$$R_B = \sqrt{(R_{B_x})^2 + (R_{B_y})^2}$$

$$= \sqrt{(43.3)^2 + (90)^2} = 99.874 \text{ kN}$$

7. (b)

Given : Outer diameter of pole, $d_0 = 300$ mm; Inner diameter of pole, $d_i = 250$ mm; Height of pole, $h = 15$ m; Wind pressure, $P = 2$ kPa, Weight of pole, $W = 100$ kN

$$\text{Area of pole, } A_P = \frac{\pi}{4} \times \left(d_o^2 - d_i^2\right) = \frac{\pi}{4} \times (300^2 - 250^2)$$

$$= 21598.45 \text{ mm}^2$$

Area moment of inertia of pole about N.A.

$$I_{\text{NA}} = \frac{\pi}{64} \times (300^4 - 250^4) = 205860221.7 \text{ mm}^4$$

Polar moment of inertia of pole,

$$j = \frac{\pi}{32}\times(300^4 - 250^4) = 411720443.5 \text{ mm}^4$$

Area of sign board, $A_s = 4 \times 3 = 12 \text{ m}^2$

Height from the base of the pole to the centre of gravity of sign board

$$h' = (15 - 1.5) = 13.5 \text{ m}$$

Horizontal distance from axis of the pole to the centre of gravity of sign board

$$b = (2 + 0.15) = 2.15 \text{ m}$$

Stresses at point A:

$$\sigma = -\frac{W}{A} + \frac{M\times\frac{d_0}{2}}{I_{NA}}$$

$$= -\frac{100\times1000}{21598.45} + \frac{2\times1000\times12\times13.5\times\frac{300}{2}\times1000}{205860221.7}$$

$$= (-4.63 + 236.0825) \text{ N/mm}^2$$

$$= 231.452 \text{ N/mm}^2$$

$$\tau = \frac{T\times\frac{d_0}{2}}{j} = \frac{2\times1000\times12\times2.15\times1000\times150}{411720443.5}$$

$$= 18.8 \text{ N/mm}^2$$

Maximum tensile and compressive stresses:

$$\sigma_{1,\,2} = \frac{1}{2}\left[\sigma \pm \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[231.452 \pm \sqrt{(231.452)^2 + 4\times(18.8)^2}\right]$$

$\sigma_1 = 232.97 \text{ N/mm}^2$ (Maximum tensile stress)

$\sigma_2 = -1.517 \text{ N/mm}^2$ (Maximum compressive stress)

Maximum shear stress:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{232.97 + 1.517}{2}$$

$$= 117.24 \text{ N/mm}^2$$

7. (c)

(i)

Given : Mass of argon gas initially, $m_1 = 4$ kg; Initial pressure, $P_1 = 550$ kPa

Initial temperature, $T_1 = 30°\text{C} = 303$ K; Final pressure, $P_2 = 230$ kPa

Assumptions:

1. At specified conditions, argon can be treated as an ideal gas.
2. The process is given to be reversible and adiabatic, thus isentropic.
3. Isentropic relations can be applied.
4. The specific heat ratio of argon gas is 1.67.

From the isentropic relations for ideal gas:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \qquad T_2 = 303 \times \left(\frac{230}{550}\right)^{\frac{1.67-1}{1.67}} = 213.567 \text{ K}$$

Final mass in the tank:

$$P_2V_2 = m_2RT_2$$

$$\Rightarrow \qquad 230 \times V_2 = m_2 \times R \times 213.567 \qquad \text{...(i)}$$

Also, $$P_1V_1 = m_1RT_1$$

$$\Rightarrow \qquad \frac{V_1}{R} = \frac{m_1T_1}{P_1} \qquad \text{...(ii)}$$

Since, tank is rigid so, $V_1 = V_2$

From equation (i) and (ii),

$$230 \times \frac{m_1T_1}{P_1} = m_2 \times 213.567$$

$$\Rightarrow \qquad m_2 = \frac{230 \times 4 \times 303}{550 \times 213.567} = 2.373 \text{ kg}$$

Hence, the mass of argon escaped out:

$$= m_1 - m_2$$

$$= (4 - 2.373)\text{kg} = 1.626 \text{ kg}$$

(ii)

Mass of one raw food item, $m = 2.5$ kg

Specific heat of food item, $(c_p)_f = 3.75$ kJ/kg°C

Inlet temperature of water, $(T_w)_i = 0.6°C$

Outlet temperature of water $(T_w)_o = 3°C$

Inlet temperature of food item = 17°C

Outlet temperature of food item = 4°C

Number of food items per hour = 270 items/h

Surrounding temperature, $T_o = 27°C = 300$ K

Heat gain from surroundings = 150 kJ/h

Assumptions:

1. Steady state conditions exist.
2. Thermal properties of raw food items and water are constant.

Mass flow rate of raw food items;

$$\dot{m}_f = \frac{270}{3600} \times 2.5 = 0.1875 \text{ kg/s}$$

Heat loss from the food items;

$$\left(\dot{Q}_L\right)_f = 0.1875 \times 3.75 \times (17 - 4)$$

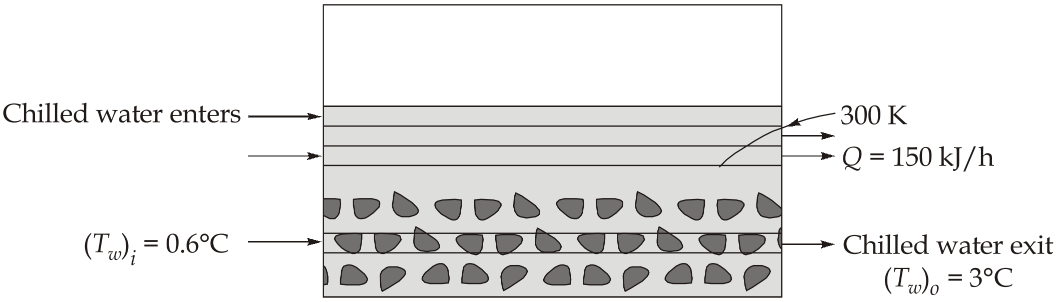
$$= 9.14 \text{ kW}$$

The chiller gains heat from the surroundings at the rate of 150 kJ/h. So, total rate of heat gain for the water;

$$\left(\dot{Q}_g\right)_w = 9.14 + \frac{150}{3600} = 9.1823 \text{ kW}$$

Rise of temperature of water is given by,

$$(\Delta T)_w = 3 - 0.6 = 2.4°C$$



∴ Mass flow rate of water;

$$9.1823 = \dot{m}_w \times 4.18 \times 2.4$$

$$\Rightarrow \quad \dot{m}_w = 0.915 \text{ kg/s}$$

Rate of entropy generation;

$$\dot{s}_{gen} = (\Delta s)_{\text{system}} + (\Delta s)_{\text{surr.}}$$

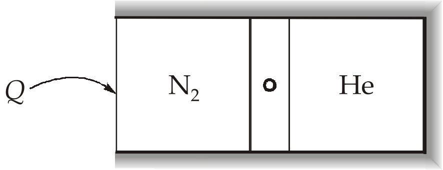
$$= \dot{m}_f (c_p)_f \ln\left(\frac{T_2}{T_1}\right)_f + \dot{m}_w c_w \times \ln\left(\frac{T_2}{T_1}\right)_w - \frac{150}{3600 \times 300}$$

$$= 0.1875 \times 3.75 \ln\frac{277}{290} + 0.915 \times 4.18 \ln\frac{276}{273.6} - \frac{150}{3600 \times 300}$$

$$= 1.017 \text{ W/K}$$

8. (a)

(i)



Given: Initial volume of N_2;

$$(V_1)_{N_2} = 0.3 \text{ m}^3$$

Mass of helium, m_{He} = 0.2 kg

Initial temperature, T_1 = 25°C = 298 K

Initial pressure, P_1 = 100 kPa

Final pressure of helium, P_2 = 130 kPa

Assumptions:

1. Kinetic energy and potential energy changes are negligible.
2. Nitrogen and helium are ideal gases with constant specific heats at room temperature.
3. The piston is adiabatic and frictionless.

(a)

Helium undergoes an isentropic compression process, and thus the final temperature of helium is determined by

$$(T_2)_{He} = (T_1)_{He} \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 298 \times \left(\frac{130}{100}\right)^{\frac{1.67-1}{1.67}} = 331.077 \text{ K}$$

(b)

Initial volume of helium;

$$(V_1)_{He} = \left(\frac{mRT}{P_1}\right)_{He} = \frac{0.2 \times \frac{8.314}{4} \times 298}{100} = 1.2387 \text{ m}^3$$

Final volume of helium;

$$(V_2)_{He} = \frac{0.2 \times \frac{8.314}{4} \times 331.077}{130} = 1.05868 \text{ m}^3$$

∴ Final volume of nitrogen,

$$(V_2)_{N_2} = 0.3 + (1.2387 - 1.05868) = 0.48 \text{ m}^3$$

(c)

Applying first law of thermodynamics to the whole system:

$$\delta Q = dU + \delta W$$

Since, the whole system boundary has zero work interaction with the surrounding, so,

$$\delta Q = dU$$

$$\Rightarrow \quad \delta Q = (dU)_{N_2} + (dU)_{He}$$

$$= \left[mc_v(T_2 - T_1)\right]_{N_2} + \left[mc_v(T_2 - T_1)\right]_{He} \quad \text{...(i)}$$

$$\text{Mass of nitrogen, } m_{N_2} = \left(\frac{P_1V_1}{RT_1}\right)_{N_2} = \frac{100 \times 0.3}{\frac{8.314}{28} \times 298} = 0.339 \text{ kg}$$

Final temperature of nitrogen;

$$(T_2)_{N_2} = \left(\frac{P_2V_2}{m \times R}\right)_{N_2} = \frac{130 \times 0.48}{0.339 \times \frac{8.314}{28}} = 619.916 \text{ K}$$

[∵ At equilibrium condition, pressure of helium and nitrogen will be equal]

∴ From equation (i)

$$\delta Q = 0.339 \times 0.742 \times (619.916 - 298) + 0.2 \times 3.102 \times (331.07 - 298) \qquad \left(\because c_v = \frac{R}{\gamma - 1}\right)$$

$$\delta Q = 101.49 \text{ kJ}$$

Entropy generation during the process;

$$s_{\text{gen.}} = (\Delta s)_{N_2} + (\Delta s)_{He} + (\Delta s)_{surr.}$$

Since, helium undergoes isentropic process, so, $(\Delta s)_{\text{He}} = 0$

$$\Rightarrow \quad s_{\text{gen.}} = \left[mc_p \ln\frac{T_2}{T_1} + mR\ln\frac{P_1}{P_2}\right]_{N_2} + \left(-\frac{101.49}{600+273}\right)$$

$$\Rightarrow \quad s_{\text{gen.}} = \left(0.339 \times 1.039 \times \ln\frac{619.916}{298} + 0.339 \times 0.2969 \times \ln\frac{100}{130} - \frac{101.49}{873}\right)$$

$$\left[\because c_p = \frac{R\gamma}{\gamma - 1};\ \gamma_{N_2} = 1.4,\ R_{N_2} = \frac{8.314}{28}\right]$$

$$= 0.1153 \text{ kJ/K}$$

(ii)

Given:

Mass of air in the cylinder initially, $m_1 = 12$ kg

Initial pressure, $P_1 = 45$ bar

Initial temperature, $T_1 = 30°\text{C} = 303$ K

Final pressure in the cylinder, $P_2 = 4.5$ bar

Ambient temperature, $T_0 = 30°\text{C} = 303$ K

$P_{\text{atm}} = 1$ bar

$(c_p)_{\text{air}} = 1.005$ kJ/kgK

Since air can be treated as an ideal gas, so we can use ideal gas equation,

$$Pv = mRT$$

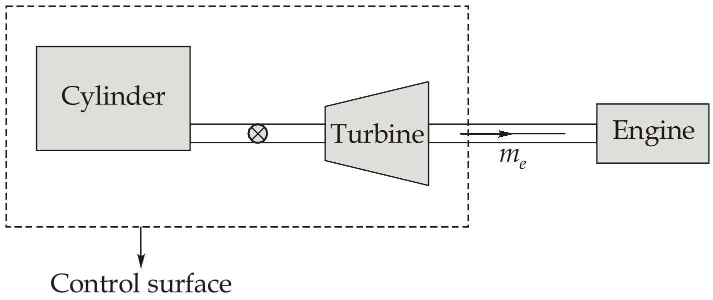
Since, volume and temperature are constant

So, $$P \propto m$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{m_1}{m_2}$$

$$\Rightarrow \qquad \frac{45}{4.5} = \frac{12}{m_2}$$

$$\Rightarrow \qquad m_2 = 1.2 \text{ kg}$$



Mass Balance:

$$(m_2 - m_1) = \cancelto{0}{m_{in}} - m_{out}$$

$$\Rightarrow \qquad m_{\text{out}} = (m_1 - m_2)_{\text{C.V}}$$

$$= 12 - 1.2 = 10.8 \text{ kg} = m_e$$

Energy Balance:

Neglecting K.E and P.E changes,

$$(U_2 - U_1)_{C.V} = \cancelto{0}{m_i h_i} + Q - m_e h_e - W_{C.V}$$

$$\Rightarrow \qquad (m_2 u_2 - m_1 u_1)_{C.V} = Q - (m_1 - m_2) \times c_p \times T_e - W_T \qquad \text{...(i)}$$

Control volume entropy analysis:

$$\left(\frac{ds}{dt}\right)_{\text{C.V}} = \dot{s}_i + \dot{s}_{gen} - \dot{s}_e$$

$$\text{or,} \qquad (ds)_{\text{C.V}} = \cancelto{0}{s_i} + s_{gen.} - s_e$$

$$\Rightarrow \qquad (m_2 s_2 - m_1 s_1) = s_{gen.} - m_e s_e - \frac{Q}{T_o}$$

$$\Rightarrow \qquad (m_2 s_2 - m_1 s_1) = s_{gen.} - (m_1 - m_2) s_e - \frac{Q}{T_o}$$

$\because$ The work output is maximum possible so, $s_{\text{gen}} = 0$

$$\therefore \qquad Q = T_0 \{ m_2 (s_e - s_2) + m_1 (s_1 - s_e) \}$$

$$= T_0 \{ -m_2 (s_2 - s_e) + m_1 (s_1 - s_e) \}$$

$$= 303\left\{-1.2 \times R\ln\frac{4.5}{1} + 12 \times R\ln\frac{45}{1}\right\}$$

$$= 3815.418 \text{ kJ}$$

From equation (i)

$$w_T = 3815.418 - (12 - 1.2) \times c_p \times 303 + (12 - 1.2) \times c_v \times 303 \text{ K}$$

$$(\because \quad T_1 = T_2 = T_e = 303 \text{ K})$$

$$= 3815.418 - 10.8 \times 303 \times (c_p - c_v)$$

$$= 3815.418 - 10.8 \times 303 \times 0.287$$

$$= 2876.24 \text{ kJ}$$

8. (b)

(i)

The general expression for entropy change in any process for an ideal gas is given by

$$ds = \frac{c_p}{T}dT - \frac{v}{T}dP$$

$$= \frac{c_p}{T}dT - \frac{R}{P}dP \qquad [\because \quad Pv = RT]$$

Integrating it over the process 1 - 2, we get

$$s_2 - s_1 = \int_1^2 \frac{c_p}{T}dT - \int_1^2 \frac{R}{P}dP$$

$$= c_p\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad [\because \quad c_p = \text{constant}]$$

Further, for any polytropic process of an ideal gas, we can substitute for $\frac{T_2}{T_1}$ in terms of $\frac{P_2}{P_1}$ form equation,

$$s_2 - s_1 = c_p\left(\frac{n-1}{n}\right)\ln\frac{P_2}{P_1} - R\ln\frac{P_2}{P_1}$$

$$= \left[c_p\left(\frac{n-1}{n} - R\right)\right]\ln\frac{P_2}{P_1}$$

$$\because \qquad c_p - c_v = R; \ \frac{c_p}{c_v} = \gamma; \ c_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \qquad s_2 - s_1 = \left(\frac{(n-1)\gamma}{n(\gamma-1)} - 1\right)R\ln\frac{P_2}{P_1}$$

$$s_2 - s_1 = \frac{(n-\gamma)R}{n(\gamma-1)}\ln\frac{P_2}{P_1}$$

Hence, it is proved.

(ii)

Given : Diameter of balloon, d = 20 m

Mass of the balloon along with cage, m_b = 150 kg

Mass of each person, m_p = 80 kg

P_{atm} = 95 kPa

T_o = 10°C = 283 K

$$\text{Volume of balloon, } V_b = \frac{4}{3}\times\pi\times r^3$$

$$= \frac{4}{3}\times\pi\times(10)^3 = 4188.79\ \text{m}^3$$

Density of air at the time of rising,

$$\rho = \frac{P}{RT} = \frac{95}{0.287\times 283} = 1.1696\ \text{kg/m}^3$$

Buoyancy force acting on the balloon;

$$F_B = (\rho g V)_{balloon} = 1.1696 \times 9.81 \times 4188.79$$

$$= 48063.22\ \text{N}$$

At equilibrium,

$$F_B = W_{hot\ air} + W_{cage} + W_{person}$$

$$48063.22 = (m_{hot\ air} + 150 + 2 \times 80) \times g$$

$$\Rightarrow \quad m_{hot\ air} = 4589.41\ \text{kg}$$

$$\therefore \quad \text{Average temperature, } T = \frac{PV}{mR} = \frac{95\times 4188.79}{4589.41\times 0.287}$$

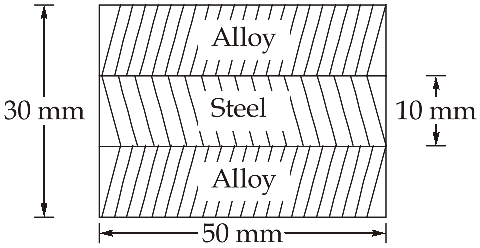
$$= 302.11\ \text{K}$$

8. **(c)**

(i)

$$\text{Area of steel, } A_s = (50 \times 10)\ \text{mm}^2$$

$$= 500\ \text{mm}^2$$



$$\text{Area of aluminium alloy, } A_a = 50 \times 20 = 1000 \text{ mm}^2$$

$$\therefore \quad \text{Total area, } A = A_s + A_s = (500 + 1000) \text{ mm}^2 = 1500 \text{ mm}^2$$

Modulus of elasticity of composite bar,

$$E = \frac{E_a A_a + E_s A_s}{A_a + A_s} = 113.33 \text{ GPa}$$

The elastic limit of the compound bar is the stress at which one of the members, either steel or aluminium alloy is stressed to its elastic limit.

Let us assume, P be the load at elastic limit of the compound bar and σ_a and σ_s be the corresponding stresses in the two materials at the elastic limit of the compound bar.

Now, since the strains in the two materials are equal,

$$\frac{\sigma_s}{E_s} = \frac{\sigma_a}{E_a}$$

$$\Rightarrow \quad \sigma_s = \frac{210}{65}\sigma_a = 3.23\sigma_a \qquad \text{...(i)}$$

$$\text{Also,} \quad \sigma_s A_s + \sigma_a A_a = P$$

$$\Rightarrow \quad 3.23\sigma_a \times 500 + \sigma_a \times 1000 = P$$

$$\Rightarrow \quad \sigma_a \times 2615.384 = P$$

$$\Rightarrow \quad \sigma_a = \frac{P}{2615.384} \qquad \text{...(i)}$$

$$\text{and} \quad \sigma_s = \frac{P}{809.716} \qquad \text{...(ii)}$$

Let σ_c be the elastic limit of the composite bar so,

$$\sigma_c \times (A_s + A_a) = P$$

$$\Rightarrow \quad \sigma_c \times 1500 = P \qquad \text{...(iii)}$$

From equation (i) and (iii)

$$\sigma_c \times 1500 = 2615.384 \times \sigma_a$$

$$\Rightarrow \quad \sigma_c \times 1500 = 2615.384 \times 45$$

$$\Rightarrow \quad \sigma_c = 78.46 \text{ MPa}$$

From equation (ii) and (iii),

$$\sigma_c \times 1500 = \sigma_s \times 809.716$$

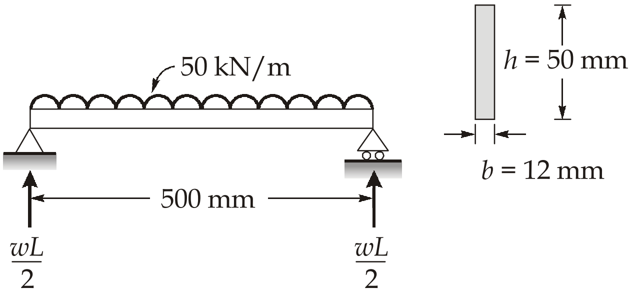
$$\Rightarrow \qquad \sigma_c = \frac{240 \times 809.716}{1500} = 129.55 \text{ MPa}$$

Hence, the elastic limit is the lower of the two values,

So, $\sigma_c = 78.46$ MPa

(ii)

Given : $w = 50$ kN/m; $b = 12$ mm; $h = 50$ mm; $L = 500$ mm



$$R_A = R_B = \frac{wL}{2} = \frac{50 \times 1000 \times 0.5}{2}$$

$$= 12500 \text{ N} = 12.5 \text{ kN}$$

So, maximum shear force, $P = 12.5$ kN

Shear stress:

For rectangular cross-section,

$$\tau_s = \frac{6P}{bh^3}\left[\left(\frac{h}{2}\right)^2 - (y)^2\right]$$

where, y is the distance of the fibre from the neutral axis.

At a distance of 10 mm from the top,

$$y = (25 - 10) = 15 \text{ mm}$$

So, $$\tau_s = \frac{6 \times 12500}{12 \times (50)^3}\left[(25)^2 - (15)^2\right] = 20 \text{ N/mm}^2$$

At a distance of 18.75 mm from the top,

$$y = 25 - 18.75 = 6.25 \text{ mm}$$

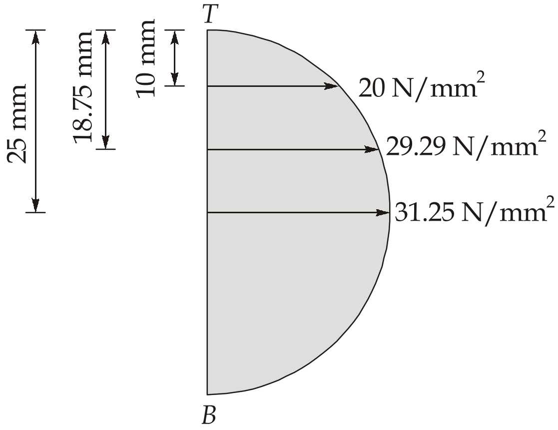
$$\therefore \qquad \tau_s = \frac{6 \times 12500}{12 \times (50)^3}\left[(25)^2 - (6.25)^2\right] = 29.29\ \text{N/mm}^2$$

At a distance of 25 mm from the top

$$y = 0$$

$$\therefore \qquad \tau_s = \frac{6 \times 12500}{12 \times (50)^3} \times (25^2 - 0) = 31.25\ \text{N/mm}^2$$

Shear stress distribution:



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