MADE EASY

India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2023 Mains Test Series

Electrical Engineering Test No : 1

Section A : Electrical Circuits

Q.1 (a) Solution:

Let the voltages at nodes *a*, *b*, *c* and *d* are V_a, V_b, V_c and V_d respectively.

Now, KCL at node (c)

$$\frac{V_c - V_d}{10} + \frac{V_c}{5} + \frac{V_c - V_b}{4} = 0 \Rightarrow -5V_b + 11V_c - 2V_d = 0 \quad ...(1)$$

KCL at node (b)

$$\frac{V_b}{8} + \frac{V_b - V_c}{4} + \frac{V_b - 45 - V_a}{8} = 0$$

$$\Rightarrow \quad V_a - 4V_b + 2V_c = -45 \quad ...(2)$$

KCL at node (a)

$$\frac{V_a - 30 - V_d}{4} + \frac{V_a}{16} + \frac{V_a + 45 - V_b}{8} = 0$$

$$\Rightarrow \quad 7V_a - 2V_b - 4V_d = 30 \quad ...(3)$$

KCL at node (d)

$$\frac{V_d + 30 - V_a}{4} + \frac{V_d}{20} + \frac{V_d - V_c}{10} = 0$$

$$\Rightarrow \quad 5V_a + 2V_c - 8V_d = 150 \quad ...(4)$$

In matrix form,

$$\begin{bmatrix} 0 & -5 & 11 & -2 \\ 1 & -4 & 2 & 0 \\ 7 & -2 & 0 & -4 \\ 5 & 0 & 2 & -8 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 0 \\ -45 \\ 30 \\ 150 \end{bmatrix}$$

$$AV = B$$

$$V = A^{-1}B$$

Therefore,

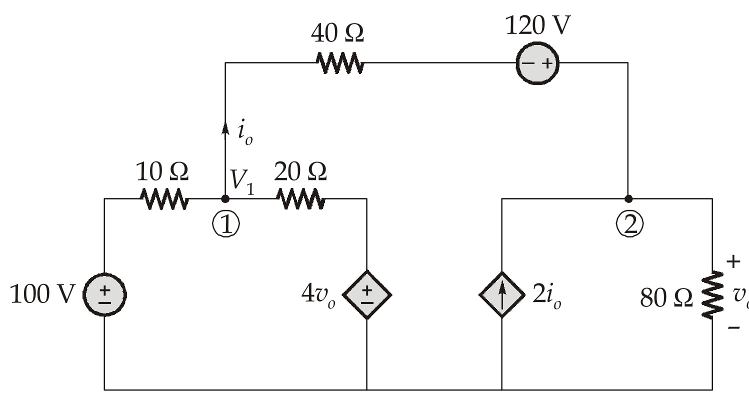
$$V_a = -5.28 \text{ V}$$

$$V_b = 10.28 \text{ V}$$

$$V_c = 0.69 \text{ V}$$

$$V_d = -21.88 \text{ V}$$

Q.1 (b) Solution:



KCL at node (1) :

$$\frac{V_1 - 100}{10} + \frac{V_1 - 4v_o}{20} + \frac{V_1 + 120 - v_o}{40} = 0$$

$$4V_1 - 400 + 2V_1 - 8v_o + V_1 + 120 - v_o = 0$$

$$7V_1 - 9v_o = 280 \quad \text{...(i)}$$

KCL at node (2)

$$i_o + 2i_o = \frac{v_o}{80}$$

$$3i_o = \frac{v_o}{80}$$

$$3\left[\frac{V_1 + 120 - v_o}{40}\right] = \frac{v_o}{80}$$

$$6V_1 - 7v_o = -720 \quad \text{...(4)}$$

On solving eqn. (3) and (4)

$$V_1 = -1688 \text{ V}$$
$$v_o = -1344 \text{ V}$$

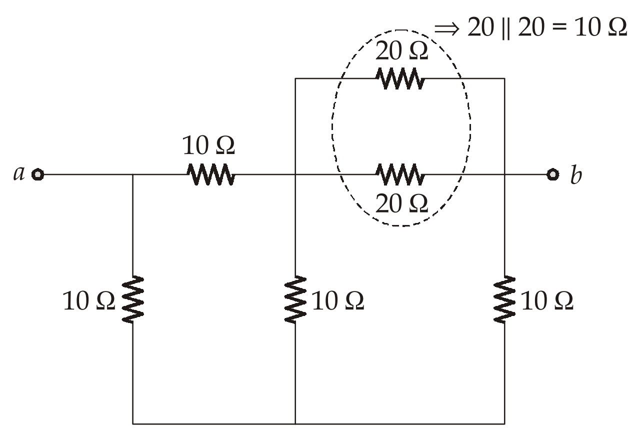
Therefore,

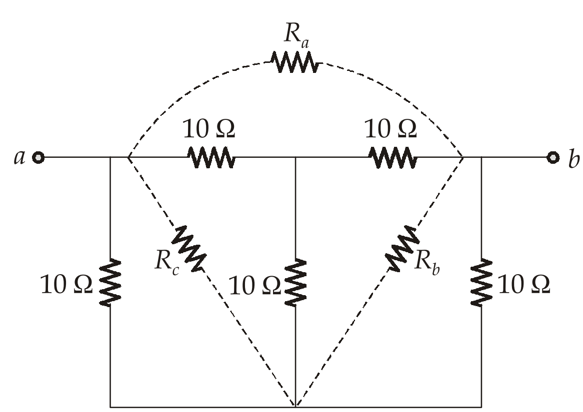
$$i_o = \frac{V_1 + 120 - v_o}{40} = \frac{-1688 + 120 + 1344}{40}$$
$$i_o = -5.60 \text{ A}$$

Hence,
$$v_o = -1344 \text{ V}$$
$$i_o = -5.60 \text{ A}$$

Q.1 (c) Solution:

To find R_{th} : Replacing the independent sources by their internal resistances.

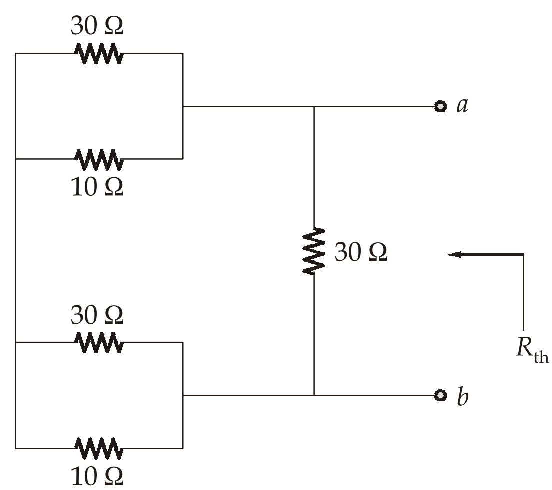




Using Y-Δ transformation

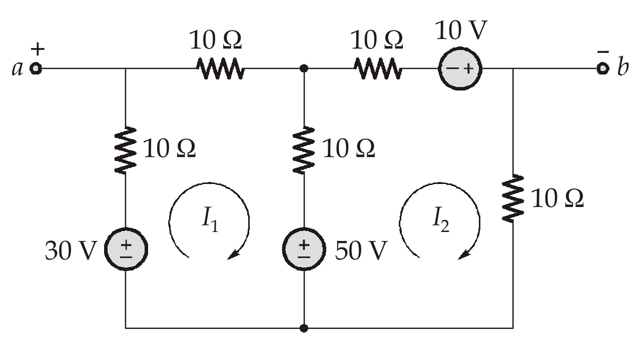
$\because$ Y is balanced.

$\therefore$ $$R_a = R_b = R_c = 10 \times 3 = 30\ \Omega$$



$$R_{th} = 30 \parallel (30 \parallel 10 + 30 \parallel 10)$$
$$= 30 \parallel (7.5 + 7.5) = 30 \parallel 15 = 10\ \Omega$$

To find V_{th} : We transform the 20 V and 5 V sources, we obtain the circuit shown in figure.



For loop 1 :

$$-30 + 10I_1 + 10I_1 + 10I_1 - 10I_2 + 50 = 0$$
$$3I_1 - I_2 = -2 \quad \text{...(1)}$$

For loop 2 :

$$-50 + 10I_2 - 10I_1 + 20I_2 - 10 = 0$$
$$-I_1 + 3I_2 = 6 \quad \text{...(2)}$$

On solving eqn. (1) and (2)

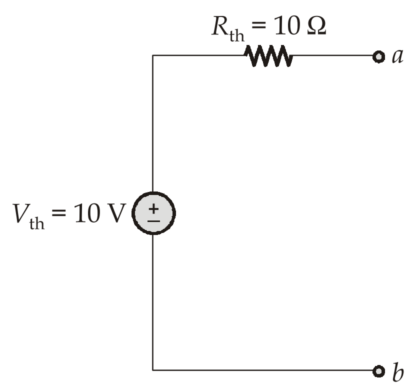
$$I_1 = 0\text{ A}, I_2 = 2\text{ A}$$

Applying KVL in outer loop,

$$-V_{ab} + 10I_1 + 10I_2 - 10 = 0$$

$$V_{ab} = 20 - 10 = 10 \text{ V}$$

Thevenin's equivalent network



Q.1 (d) Solution:

$$\frac{V_o}{V_g} = \frac{R_p}{(R_g + R_s + R_p)} = 0.125 \quad ...(1)$$

$$R_{eq} = R_p \parallel (R_g + R_s) = R_g$$

Since attenuator does not the output resistance.

$$R_g = \frac{R_p(R_g + R_s)}{R_p + R_g + R_s}$$

By (1), $R_g = 0.125(R_g + R_s)$

$\because$ $R_g = 100\ \Omega$

$\therefore$ $0.125R_s = (1 - 0.125) \times 100$

$$R_s = 700\ \Omega$$

By (1), $$0.125 = \frac{R_p}{R_p + 100 + 700}$$

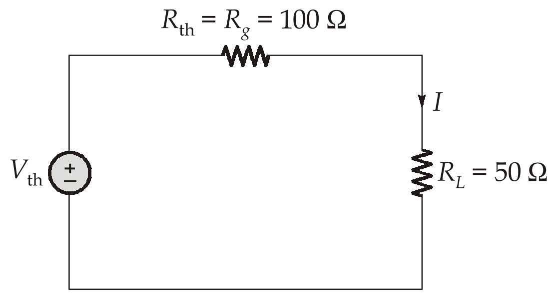
$$R_p = 114.286\ \Omega$$

Therefore, $R_p = 114.29\ \Omega$

$$R_s = 700\ \Omega$$

Now, $V_g = 12$ V, $R_L = 50\ \Omega$

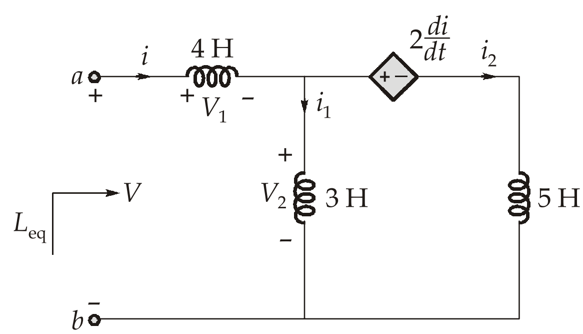
Thevenin equivalent



$$V_{th} = V_s = 0.125V_g = 1.5\text{ V}$$

$$I = \frac{V_{th}}{R_{th} + R_L} = \frac{1.5}{150} = 10\text{ mA}$$

Q.1 (e) Solution:



Let $$V = L_{eq}\frac{di}{dt} \quad ...(1)$$

$$V = V_1 + V_2 = 4\frac{di}{dt} + V_2 \quad ...(2)$$

$$i = i_1 + i_2 \Rightarrow i_2 = i - i_1 \quad ...(3)$$

$$V_2 = 3\frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{V_2}{3} \quad ...(4)$$

and $$-V_2 + 2\frac{di}{dt} + 5\frac{di_2}{dt} = 0$$

$$V_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \quad ...(5)$$

Incorporating eqn. (3), (4) and (5)

$$V_2 = 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt} = 7\frac{di}{dt} - \frac{5}{3}V_2$$

$$V_2\left[1+\frac{5}{3}\right] = 7\frac{di}{dt}$$

$$V_2 = \frac{21}{8}\frac{di}{dt} \qquad ...(6)$$

Substituting eqn. (6) in eqn. (2)

$$V = 4\frac{di}{dt}+\frac{21}{8}\frac{di}{dt}$$

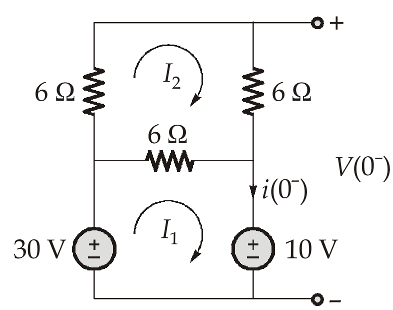
$$V = \frac{53}{8}\frac{di}{dt} \qquad ...(7)$$

Comparing eqn. (1) and (2)

$$L_{eq} = \frac{53}{8} = 6.625 \text{ H}$$

Q.2 (a) Solution:

For $t = 0^-$, the equivalent circuit is shown,



For loop (1)

$$-30 + 6I_1 - 6I_2 + 10 = 0$$

$$3I_1 - 3I_2 = 10 \qquad ...(1)$$

For loop (2)

$$18I_2 - 6I_1 = 0$$

$$I_1 = 3I_2 \qquad ...(2)$$

Putting eqn. (2) in eqn. (1)

$$2I_1 = 10$$

$$I_1 = 5 \text{ A}$$

and

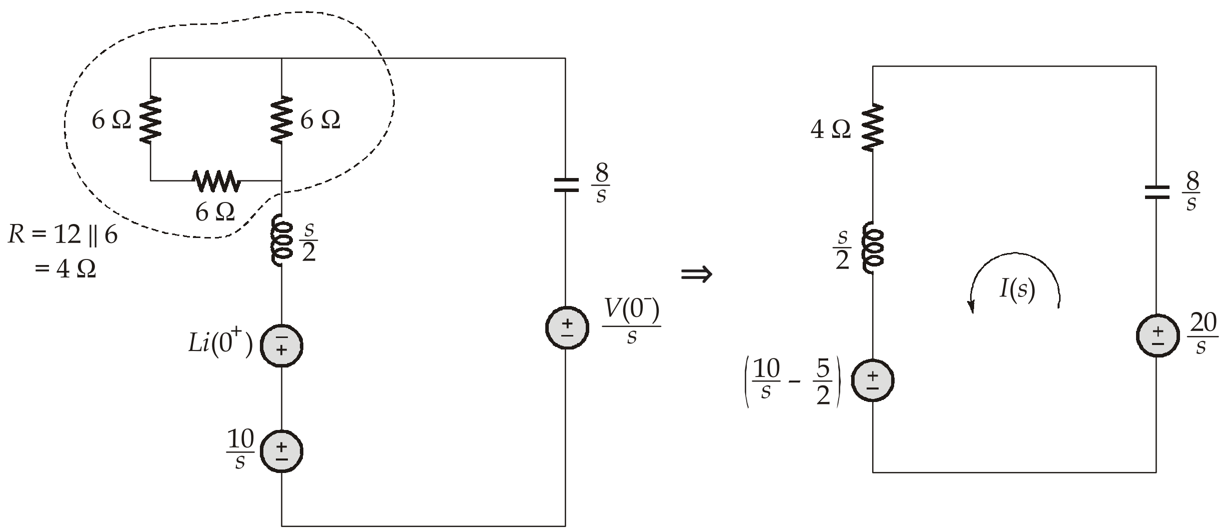
$$I_2 = \frac{I_1}{3} = \frac{5}{3}$$

$\because$ $$i(0^-) = I_1 = 5 \text{ A}$$

and $$V(0^-) = 6I_2 + 10$$

$$V(0^-) = 6 \times \frac{5}{3} + 10 = 20 \text{ V}$$

For $t > 0$:



By KVL; $$-\frac{20}{s} + \frac{8}{s}I(s) + 4I(s) + \frac{s}{2}I(s) + \left(\frac{10}{s} - \frac{5}{2}\right) = 0$$

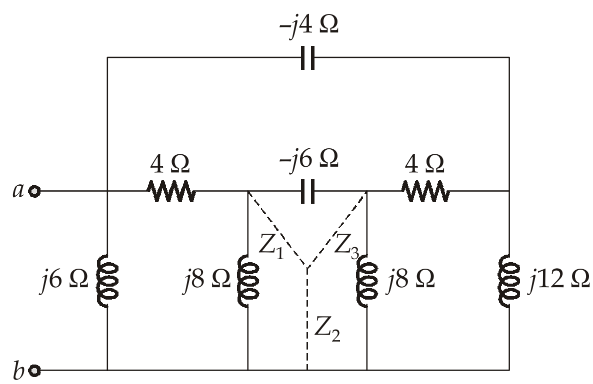
$$I(s) = \frac{\frac{10}{s} + 2.5}{\frac{8}{s} + 4 + \frac{s}{2}} = \frac{20 + 5s}{s^2 + 8s + 16} = \frac{5(s+4)}{(s+4)^2}$$

$$I(s) = \frac{5}{s+4}$$

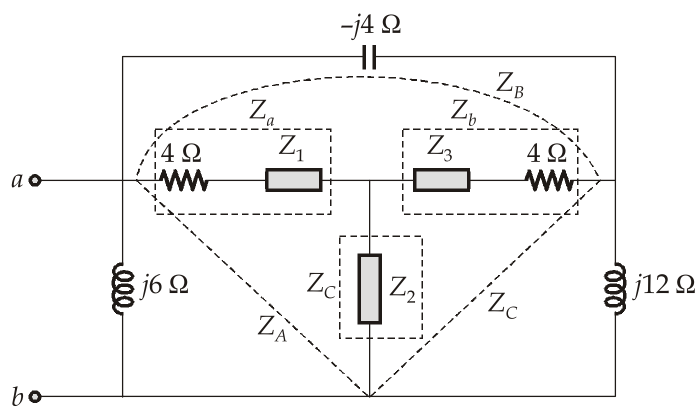
$\therefore$ $$i(t) = 5e^{-4t}u(t)$$

$\therefore$ for $t > 0$; $$i(t) = 5e^{-4t} \text{ A}$$

Q.2 (b) Solution:



Transform the delta connection to star connection as shown in figure.



Here,

$$Z_1 = \frac{(-j6)\times(j8)}{j8 + j8 - j6} = -j4.8\ \Omega$$

$$Z_2 = \frac{(j8)\times(j8)}{j10} = j6.4\ \Omega$$

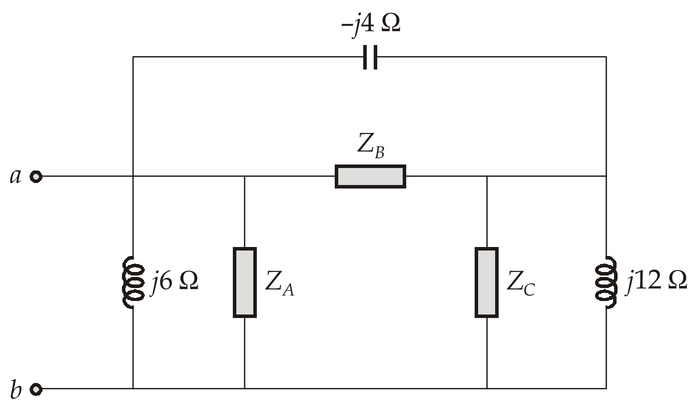
$$Z_3 = \frac{(-j6)\times(j8)}{j10} = -j4.8\ \Omega$$

$$Z_a = Z_1 + 2 = (4 - j4.8)\ \Omega$$

$$Z_b = 4 + Z_3 = (4 - j4.8)\ \Omega$$

$$Z_c = Z_2 = j6.4\ \Omega$$

Transform the star connection to delta connection as shown in figure.



$$Z_A = Z_a + Z_c + \frac{Z_a Z_c}{Z_b}$$

$$= 4 - j4.8 + j6.4 + \frac{(4 - j4.8)(j6.4)}{4 - j4.8} = (4 + j8)\ \Omega$$

$$Z_B = Z_a + Z_b + \frac{Z_a \cdot Z_b}{Z_c}$$

$$= 4 - j4.8 + 4 - j4.8 + \frac{(4 - j4.8)(4 - j4.8)}{j6.4}$$

$$Z_B = (2 - j8.5)\ \Omega$$

$$Z_C = Z_c + Z_b + \frac{Z_c \cdot Z_b}{Z_a}$$

$$= j6.4 + 4 - j4.8 + \frac{(j6.4)(4 - j4.8)}{(4 - j4.8)} = (4 + j8)\ \Omega$$

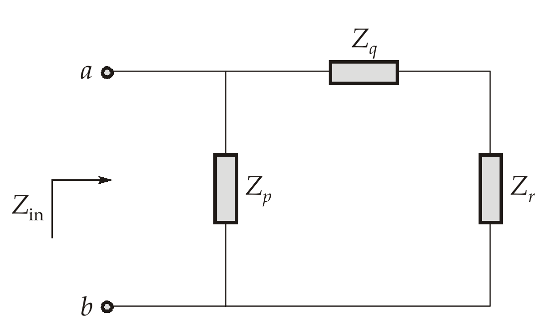
Again,

$$Z_p = (Z_A \parallel j6) = (0.68 + j3.62)\ \Omega$$

$$Z_q = (Z_B \parallel -j4) = (0.20 - j2.75)\ \Omega$$

$$Z_r = (Z_C \parallel j12) = (1.38 + j5.07)\ \Omega$$

Now,



$$Z_{in} = Z_p \parallel (Z_q + Z_r)$$
$$= (0.68 + j3.62) \parallel (0.20 - j2.75 + 1.38 + j5.07)$$
$$Z_{in} = (0.66 + j1.48)\ \Omega$$

Q.2 (c) Solution:

(i) Input impedance of given circuit is given as

$$Z_{in} = (j\omega L) \parallel \left(R + \frac{1}{j\omega C}\right)$$

$$Z_{in} = \frac{(j\omega L)\left[R + \frac{1}{j\omega C}\right]}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{\frac{L}{C} + j\omega LR}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z_{in} = \frac{\left(\frac{L}{C} + j\omega LR\right)\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

To have a resistive impedance, $I_m[Z_{in}] = 0$.

Hence, $\omega LR^2 - \left(\frac{L}{C}\right)\left(\omega L - \frac{1}{\omega C}\right) = 0$

$$\omega R^2 C = \omega L - \frac{1}{\omega C}$$
$$\omega^2 R^2 C^2 = \omega^2 LC - 1$$
$$L = \frac{1 + \omega^2 R^2 C^2}{\omega^2 C}$$

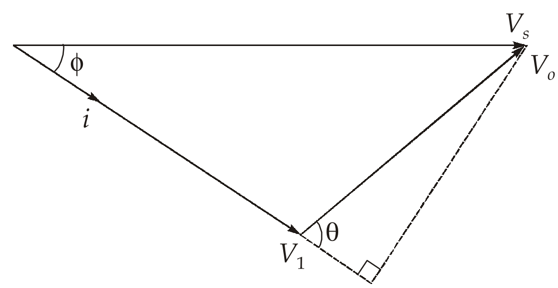
Now, we can solve for L.

$$L = R^2 C + \frac{1}{\omega^2 C}$$

$$= (2000)^2 \times (50 \times 10^{-9}) + \frac{1}{(2\pi \times 50000)^2 \times (50 \times 10^{-9})}$$

$$L = 200 \text{ mH}$$

(ii)



$$V_s^2 = V_1^2 + V_0^2 + 2V_0V_1 \cos\theta$$

$$\cos\theta = \frac{145^2 - 110^2 - 50^2}{2 \times 50 \times 110}$$

$$\theta = 54.26°$$

$$i = \frac{50}{80} = 0.625 \text{ A}$$

$$V_R = V_o \cos\theta = 110 \cos 54.26°$$

$$V_R = 64.25°$$

$$R = \frac{V_R}{i} = \frac{64.25}{0.625} = 102.8\ \Omega$$

$$R = 102.8\ \Omega$$

$$V_L = V_o \sin\theta = 110 \sin 54.26°$$

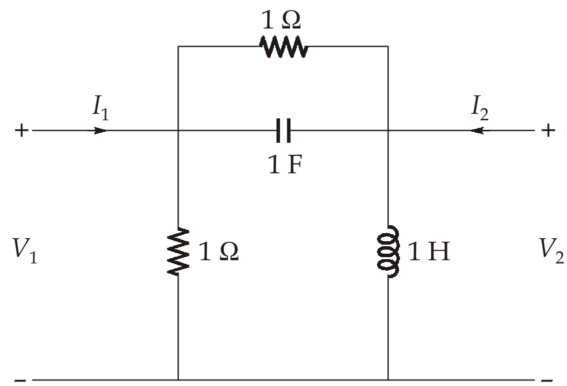
$$V_L = i.X_L = 89.284 \text{ V}$$

$$L = \frac{89.284}{0.625 \times 2 \times \pi \times 60}$$

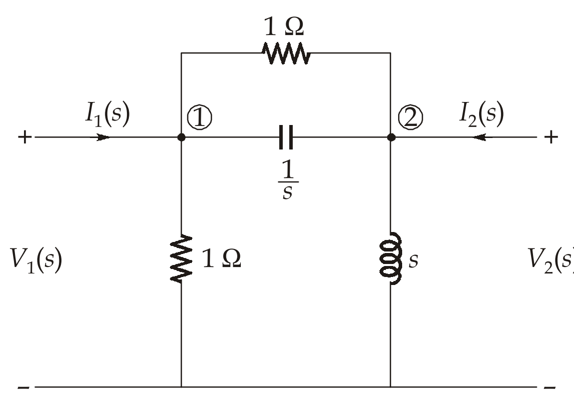
$$L = 0.3789 \text{ H}$$

Q.3 (a) Solution:

Considering the network (1)



Network in s-domain



KCL at node (1)

$$I_1(s) = \frac{V_1(s)}{1} + (V_1(s) - V_2(s))[s+1]$$

$$= V_1(s)[s+2] - V_2(s)[s+1] \quad ...(1)$$

Apply KCL at node (2)

$$I_2(s) = \frac{V_2(s)}{s} + (V_2(s) - V_1(s))[s+1]$$

$$= -(s+1)V_1(s) + \left(s+1+\frac{1}{s}\right)V_2(s) \quad ...(2)$$

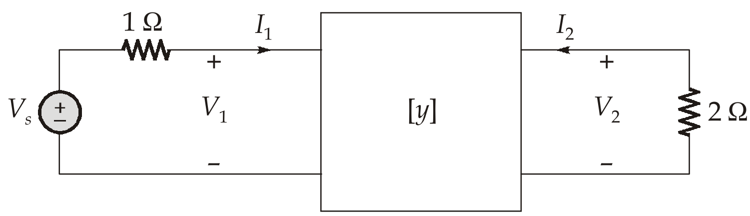
From eqn. (1) and (2)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \left(\dfrac{s^2+s+1}{s}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Therefore, y-parameters are

$$[y] = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \left(\dfrac{s^2+s+1}{s}\right) \end{bmatrix}$$

Now, consider the network



$$V_s = V_1 + I_1 \quad \text{or} \quad V_s - V_1 = I_1 \qquad ...(3)$$

Also, $$V_2 = -2I_2 \qquad ...(4)$$

$\Rightarrow$ $$I_1 = y_{11}V_1 + y_{12}V_2 \qquad ...(5)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad ...(6)$$

From (3) and (5)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2$$

$\Rightarrow$ $$V_s = (1 + y_{11})V_1 + y_{12}V_2 \qquad ...(7)$$

From (4) and (6)

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2$$

$\Rightarrow$ $$0 = y_{21}V_1 + (y_{22} + 0.5)V_2$$

$\Rightarrow$ $$V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \qquad ...(8)$$

Substituting (8) into (7)

$$V_s = -\left(\frac{1+y_{11}}{y_{21}}\right)(0.5 + y_{22})V_2 + y_{12}V_2$$

$$V_s = \left[\frac{-(3+s)(s^2+1.5s+1)}{-(s+1)\times s} - (s+1)\right]V_2$$

On solving, $$V_s = \frac{2.5s^2 + 4.5s + 3}{s(s+1)}V_2$$

$\because$ $$V_s = \frac{2}{s}$$

$$\frac{2}{s} = \frac{2.5s^2 + 4.5s + 3}{s(s+1)} V_2$$

$$V_2 = \frac{2(s+1)}{2.5s^2 + 4.5s + 3}$$

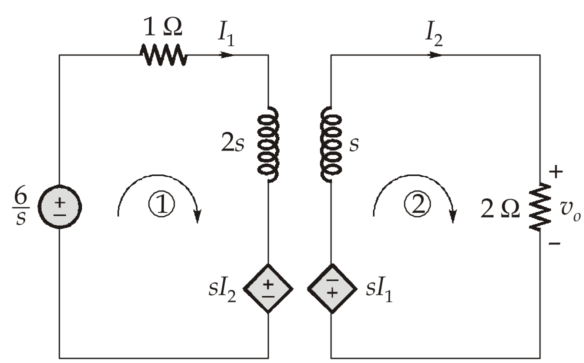
$$V_2 = \frac{0.8(s+1)}{s^2 + 1.8s + 1.2} \text{ Volts}$$

Q.3 (b) Solution:

For the given circuit, mutual inductance between the two coils is given as

$$M = K\sqrt{L_1 L_2} = \frac{1}{\sqrt{2}} \times \sqrt{1 \times 2} = 1 \text{ H}$$

Given circuit in s-domain can be drawn as



KVL in loop 1,

$$\frac{6}{s} = (1 + 2s)I_1 + sI_2 \quad \text{...(1)}$$

For loop 2,

$$sI_1 + sI_2 + 2I_2 = 0$$

$$sI_1 + (s+2)I_2 = 0 \quad \text{...(2)}$$

Substituting (2) in (1)

$$\frac{6}{s} = \left[\frac{-(1+2s)(s+2)}{s} + s\right] I_2$$

$$6 = [-(1 + 2s)(s + 2) + s^2]I_2$$

$$I_2 = \frac{6}{-(s^2 + 5s + 2)}$$

$$V_o = 2I_2$$

$$V_o = \frac{-12}{s^2 + 5s + 2} = \frac{-12}{(s + 0.438)(s + 4.561)}$$

$$V_o = \frac{A}{(s + 0.438)} + \frac{B}{s + 4.561}$$

$$A = \frac{-12}{4.123} = -2.91$$

$$B = \frac{-12}{-4.123} = 2.91$$

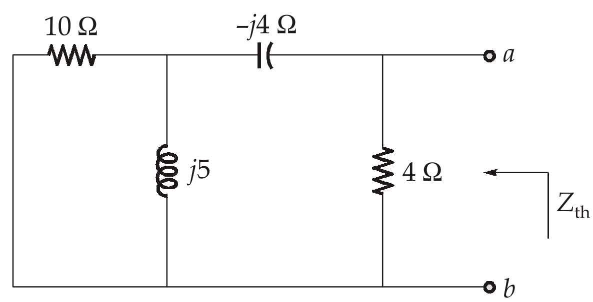
Therefore, $$V_o(s) = \frac{-2.91}{s + 0.438} + \frac{2.91}{s + 4.561}$$

On taking Inverse Laplace transform of above equation

$$V_o(t) = 2.91[e^{-4.561t} - e^{-0.438t}]u(t) \text{ Volts}$$

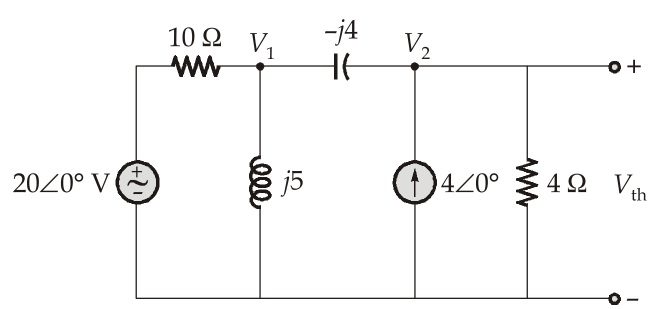
Q.3 (c) Solution:

(i) To find Z_{th}, consider the circuit



$$Z_{th} = 4 \parallel (-j4 + (10 \parallel j5))$$
$$= 4 \parallel (-j4 + 2 + j4)$$
$$Z_{th} = 4 \parallel 2 = 1.33\ \Omega$$

To find Thevenin voltage (V_{th}), consider the circuit



KCL at node 1,

$$\frac{V_1 - 20}{10} + \frac{V_1}{j5} + \frac{V_1 - V_2}{-j4} = 0$$

$$(1 + j0.5)V_1 - j2.5V_2 = 20 \qquad ...(1)$$

KCL at node 2,

$$4 + \frac{V_1 - V_2}{-j4} - \frac{V_2}{4} = 0$$

$$V_1 = (1 - j)V_2 + j16 \qquad ...(2)$$

Substituting (2) in (1) leads to

$$28 - j16 = (1.5 - j3)V_2$$

$$V_2 = \frac{28 - j16}{1.5 - j3} = (8 + j5.33) \text{ Volts}$$

Therefore, $V_{th} = V_2 = 9.615\angle 33.69$ V

Hence, $V_{th} = 9.615\angle 33.69°$ V

$Z_{th} = 1.33\ \Omega$

(ii) To find Z_{th}, consider the circuit,

$$Z_{th} = -j4 \parallel (4 + 10 \parallel j5)$$

$$= -j4 \parallel 6 + j4 = (2.667 - j4)\ \Omega$$

To find V_{th}, we will make use of result in part (1)

$$V_2 = 8 + j5.333 = \left(\frac{8}{3}\right)(3 + j2)$$

$$V_1 = (1 - j)V_2 + j16 = j16 + \left(\frac{8}{3}\right)(5 - j)$$

$$V_{th} = V_1 - V_2 = \frac{16}{3} + j8 = 9.614\angle 56.31° \text{ V}$$

Therefore, $V_{th} = 9.614\angle 56.31°$ V

$Z_{th} = (2.667 - j4)\ \Omega$

Q.4 (a) Solution:

When switch at position A,

$$V_c(0) = 12 \text{ V}, i_L(0) = 0 \text{ A}$$

At $t = 0$, switch moves from A to B. Since the circuit is parallel RLC type.

Characteristics equation of parallel RLC circuit is given as

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

The roots of characteristics equation can be given as

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 3 \times \frac{1}{30}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{30} \times 60 \times 10^{-3}}} = 22.36$$

$\because$ $\alpha < \omega_o$, circuit will produce underdamped response.

$$s_{1,2} = -5 \pm \sqrt{5^2 - 22.36^2}$$

$$s_{1,2} = -5 \pm j21.79$$

The voltage across igniter is $V_R = V_C$. Since parallel RLC circuit, therefore, equation of voltage across capacitor is given as

$$v_c(t) = e^{-5t}[A \cos 21.79t + B \sin 21.79t] \text{ Volts} \quad ...(1)$$

At $t = 0$,

$$V_c(0) = 12 \text{ V}$$

$$12 = (A + B \times 0) = A$$

$$A = 12$$

Also,

$$\frac{dV_c(t)}{dt} = -5e^{-5t}(A \cos 21.79t + B \sin 21.79t) + 21.79(-A \sin 21.79t + B \cos 21.7974t)e^{-5t} \quad ...(2)$$

By KCL;

$$-i_C = i_R + i_L$$

$$\frac{-CdV_C(0^-)}{dt} = \frac{V_C(0^-)}{R} + i_L(0^-)$$

$$\frac{dV_C(0^-)}{dt} = \frac{-[V_C(0^-) + Ri_L(0^-)]}{RC} = \frac{-12+0}{\left(\frac{1}{10}\right)}$$

$$= -120$$

Hence, $$-120 = -5A + 21.79B, \text{ leads to}$$

$$B = \frac{(5 \times 12 - 120)}{21.794} = -2.753$$

At the peak value of response,

$$\frac{dV_c(t_o)}{dt} = 0$$

i.e., $$0 = A + B\tan 21.79t_o + \left(\frac{21.79\text{ A}}{5}\right)\tan 21.79t_o - \frac{21.79B}{5}$$

$$\tan 21.79t_o\left[B + \frac{21.79A}{5}\right] = \left(\frac{21.79B}{5}\right) - A$$

$$\tan 21.79t_o = \frac{\left(\frac{21.79B}{5} - A\right)}{\left(B + \frac{A21.79}{5}\right)} = \frac{-24}{49.55} = -0.484$$

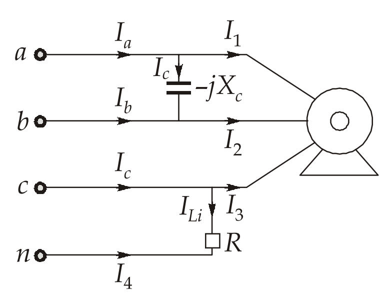
$$21.79t_o = \tan^{-1}(-0.484) = -0.451 = \pi - 0.451 = 2.69^r$$

Therefore, $$21.794t_o = 2.69^r$$

$$t_o = \frac{2.69}{21.794} = 123.45 \text{ msec}$$

Q.4 (b) Solution:

We will find the magnitude of various currents.



For motor,

$$I_1 = \frac{S}{\sqrt{3}V_L} = \frac{4000}{440\sqrt{3}} = 5.248 \text{ A}$$

For capacitor,

$$I_C = \frac{Q_C}{V_L} = \frac{1800}{440} = 4.091 \text{ A}$$

For lighting,

$$V_P = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_{Li} = \frac{P_{Li}}{V_P} = \frac{800}{254} = 3.15 \text{ A}$$

Now, if

$$V_{an} = V_p\angle 0°$$

$$V_{ab} = \sqrt{3}V_p\angle 30°$$

$$V_{cn} = V_p\angle 120°$$

$$I_c = \frac{V_{ab}}{-jX_c} = 4.091\angle 120° \text{ A}$$

$$I_1 = \frac{V_{ab}}{Z_D} = 5.249\angle(\theta + 30°)$$

where,

$$\theta = \cos^{-1}(0.72) = 43.95°$$

Therefore,

$$I_1 = 5.249\angle 73.95° \text{ A}$$

$$I_2 = 5.249\angle -46.05° \text{ A}$$

$$I_3 = 5.249\angle 193.95° \text{ A}$$

$$I_{Li} = \frac{V_{cn}}{R} = 3.15\angle 120° \text{ A}$$

Thus,

$$I_a = I_1 + I_C = 5.249\angle 73.95° + 4.091\angle 120°$$

$$= 8.608\angle 93.96° \text{ A}$$

$$I_b = I_2 - I_C = 5.249\angle -46.05° - 4.091\angle 120°$$

$$I_b = 9.272\angle -52.156° \text{ A}$$

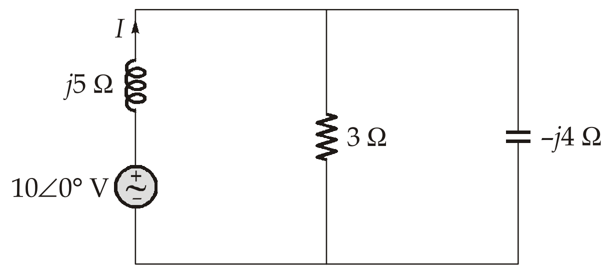
$$I_c = I_3 + I_{Li} = 5.249\angle 193.95° + 3.15\angle 120°$$

$$I_c = 6.827\angle 167.6° \text{ A}$$

$$I_n = I_{Li} = 3.15\angle -60° \text{ A}$$

Q.4 (c) Solution:

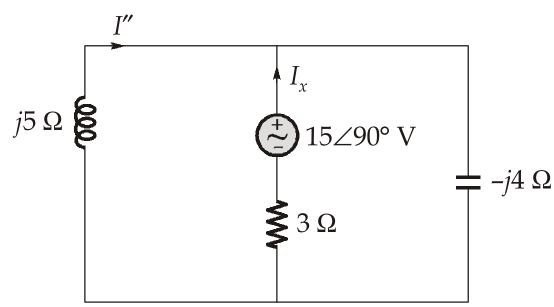
Let us consider 10∠0° V source acting alone in the circuit.



$$I' = \frac{10\angle 0^\circ}{j5 + (3 \,||\, -j4)} = \frac{10\angle 0^\circ}{j5 + 2.4\angle -36.86^\circ}$$

$$= 2.47\angle{-61.66^\circ}\text{ A}$$

Now, we will take 15∠90° volt source into consideration.



$$I_x = \frac{15\angle 90^\circ}{3 + (j5 \,||\, -j4)} = \frac{15\angle 90^\circ}{3 - j20}$$

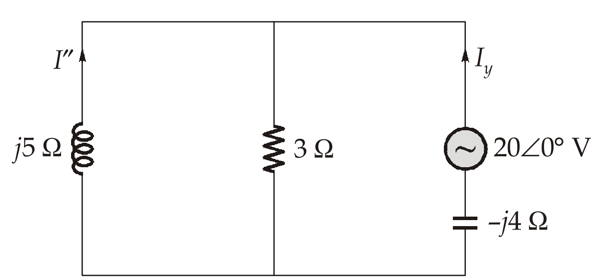
$$I_x = 0.7417\angle 171.46^\circ\text{ A}$$

Now, using current division rule,

$$I'' = -I_x \times \left[\frac{-j4}{j5 - j4}\right]$$

$$I'' = 2.96\angle 171.46^\circ\text{ A}$$

Finally, we will consider the 20∠0° volt source alone.



$$I_y = \frac{20\angle 0°}{-j4 + (3 \parallel j5)}$$

$$= \frac{20\angle 0°}{-j4 + 2.57\angle 30.96°}$$

$$I_y = 5.766\angle 50.50° \text{ A}$$

Now, again using current division rule.

$$I''' = -I_y \times \left[\frac{3}{3 + j5}\right]$$

$$I''' = 2.96\angle 171.46° \text{ A}$$

Finally, using the superposition principle,

$$I = I' + I'' + I'''$$

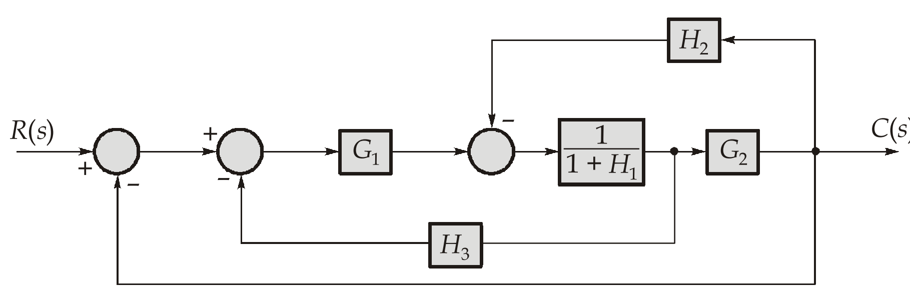
$$I = 2.47\angle{-61.66°} + 2.96\angle 171.46° + 2.96\angle 171.46°$$

$$I = 4.87\angle{-164.60°} \text{ A}$$

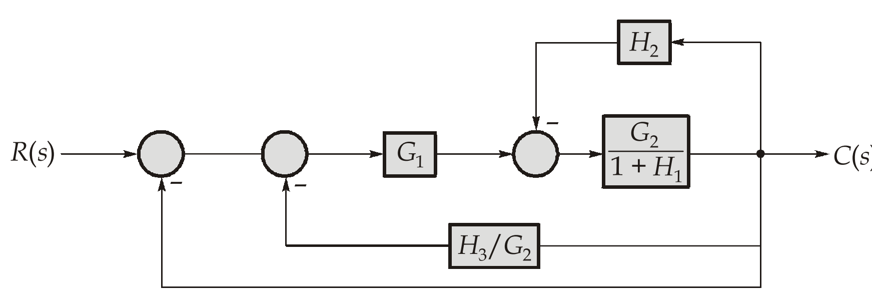
Section B : Control Systems

Q.5 (a) Solution:

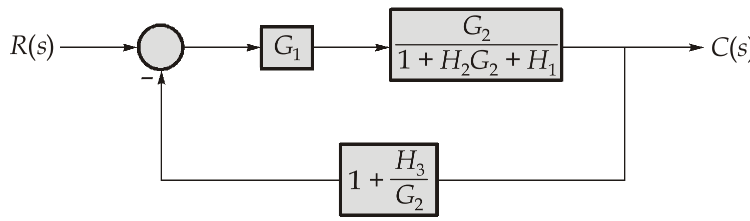
Step 1 : Eliminating the inner feedback path involving H_1 and split the first summing junction into two.



Step 2 : Shift the take-off point on the left of G_2 to its right.



Step 3 : Eliminating the feedback paths.



Step 4 : Combining the two blocks in cascade and eliminate the feedback path.

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2}{1 + H_1 + H_2 G_2}}{1 + \dfrac{G_1 G_2}{1 + H_1 + H_2 G_2}\left(1 + \dfrac{H_3}{G_2}\right)}$$

Transfer function,

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + H_1 + H_2 G_2 + G_1 G_2 + G_1 H_3}$$

Q.5 (b) Solution:

The given characteristic equation is

$s^6 + 3s^5 + 8s^4 + 18s^3 + 20s^2 + 24s + 16 = 0$

R-H Table :

s^6	1	8	20	16
s^5	(3)1	(18)6	(24)8	
s^4	(2)1	(12)6	(16)8	
s^3	0	0		

Here, s^3-row has all zero elements.

Auxiliary equations :

$$A(s) = s^4 + 6s^2 + 8$$

$$A'(s) = 4s^3 + 12s$$

Replacing zeros of s^3-row by the coefficients of $A'(s)$, the Routh array can be

s^6	1	8	20	16
s^5	(3)1	(18)6	(24)8	
s^4	(2)1	(12)6	(16)8	
s^3	(4)1	(12)3		
s^2	3	8		
s^1	$\frac{1}{3}$			
s^0	8			

Here, there is no sign changes in first-column of routh array.

Number of closed-loop poles on RHP = 0.

$$\text{Number of } j\omega\text{-axis poles} = 4$$

$$\text{Number of LHP poles} = 2$$

Hence, the system is marginal stable.

Q.5 (c) Solution:

Given : $$G(s) = \frac{K(s+\alpha)}{s(s^2+12s+32)}; \quad K_v = 6.25; \quad \omega_n = 5 \text{ rad/s}$$

Velocity error constant,

$$K_v = \lim_{s\to 0} s.G(s)$$

$$6.25 = \lim_{s\to 0} s \cdot \frac{K(s+\alpha)}{s(s^2+12s+32)}$$

$$= \frac{K\alpha}{32}$$

$$K\alpha = 200 \quad ...(1)$$

The characteristic equation,

$$s^3 + 12s^2 + (32 + K)s + K\alpha = 0 \quad ...(2)$$

The standard second-order characteristic equation is,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

Here, the system is a third order. So, assume that the third pole is at $s = -b$.

So, $$(s + b)(s^2 + 2\xi\omega_n s + \omega_n^2) = 0$$

$$s^3 + (2\xi\omega_n + b)s^2 + (2\xi\omega_n b + \omega_n^2)s + b\omega_n^2 = 0 \quad ...(3)$$

Comparing eqn. (2) and (3), we get

$$2\xi\omega_n + b = 12 \quad ...(4)$$

$$2\xi\omega_n b + \omega_n^2 = 32 + K \quad ...(5)$$

$$b\omega_n^2 = K\alpha \quad ...(6)$$

$$b = \frac{200}{(5)^2} = 8$$

From eqn. (4),

$$2\xi\omega_n = 12 - 8 = 4$$

$$\xi = \frac{4}{2 \times 5} = 0.4$$

From eqn. (5)

$$K = 2\xi\omega_n b + \omega_n^2 - 32$$

$$= 2 \times 0.4 \times 5 \times 8 + (5)^2 - 32$$

$$K = 25$$

$$K\alpha = 200$$

$$\alpha = \frac{200}{25} = 8$$

$$K = 25 \text{ and } \alpha = 8$$

Q.5 (d) Solution:

The transfer function,

$$G(s) = \frac{K\left(1 + \frac{s}{x}\right)}{\left(1 + \frac{s}{160}\right)}$$

The slope between x and 160 is +20 dB/dec.

$$20 = \frac{18 - (-6)}{\log 160 - \log(x)}$$

$$\log\left(\frac{160}{x}\right) = \frac{24}{20}$$

$$x = 10 \text{ rad/sec}$$

$$G(s) = \frac{K\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{160}\right)}$$

$$20 \log K = -6$$

$$\log K = \frac{-6}{20}$$

$$K = 0.5$$

Transfer function, $$G(s) = \frac{0.5\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{160}\right)}$$

$$G(s) = \frac{8(s+10)}{(s+160)}$$

The gain cross-over frequency is the frequency at which gain is 0 dB.

From figure, $$20 = \frac{18-0}{\log 160 - \log(\omega_{gs})}$$

$$20 = \frac{18}{\log\left(\frac{160}{\omega_{gc}}\right)}$$

$$\frac{160}{\omega_{gc}} = 10^{0.9}$$

$$\omega_{gc} = 20.143 \text{ rad/sec}$$

Q.5 (e) Solution:

Given :

$$A = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The zero input response,

$$y(t) = Ce^{At}X(0)$$

$$e^{At} = L^{-1}\left\{[sI - A]^{-1}\right\}$$

$$[sI - A]^{-1} = \begin{bmatrix} s & 0 \\ -3 & s+3 \end{bmatrix}^{-1}$$

$$= \frac{1}{s(s+3)}\begin{bmatrix} s+3 & 0 \\ 3 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{3}{s(s+3)} & \frac{1}{s+3} \end{bmatrix}$$

$$e^{At} = L^{-1}\begin{bmatrix} \frac{1}{s} & 0 \\ \frac{3}{s(s+3)} & \frac{1}{s+3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 - e^{-3t} & e^{-3t} \end{bmatrix}$$

Zero-input response

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 - e^{-3t} & e^{-3t} \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 + e^{-3t} \end{bmatrix}$$

$$y(t) = 3 + 2e^{-3t}$$

Q.6 (a) Solution:

The closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{K}{(s + a + K)}$$

For unit step, $R(s) = \frac{1}{s}$

$$C(s) = \frac{K}{s(s+a+K)}$$

$$= \frac{K}{a+K}\left[\frac{1}{s} - \frac{1}{(s+a+K)}\right]$$

Taking inverse laplace transform,

$$C(t) = \frac{K}{a+K}(1 - e^{-(a+K)t})$$

Time constant,

$$\tau_1 = \frac{1}{a+K} = \frac{1}{6}$$

$$a + K = 6 \quad \text{...(1)}$$

Open-loop pole is at $s_1 = -a$.

The new location of pole is at $s_2 = \frac{-a}{2}$.

The new time constant, $$\tau_2 = \frac{1}{\left(\frac{a}{2}\right) + K}$$

$$\frac{1}{4} = \frac{1}{\left(\frac{a}{2}\right) + K}$$

$$\frac{a}{2} + K = 4$$

$$\text{a} + 2K = 8 \quad \text{...(2)}$$

On solving eqn. (1) and (2), we get

$$K = 2 \text{ and } a = 4$$

Let $s = -a$, be the location of the open-loop pole for which time constant is $\frac{1}{8}$ sec.

Hence, $$\frac{1}{a_1 + K} = \frac{1}{8}$$

$$a_1 + K = 8$$

$$a_1 = 8 - 2 = 6$$

So, closed-loop poles for time constant of $\frac{1}{8}$ sec.

$$\frac{C(s)}{R(s)} = \frac{K}{(s + a_1 + K)} = \frac{2}{(s+2+6)}$$

Closed loop pole, $s_1 = -8$

Q.6 (b) Solution:

Given : $$G(s) = \frac{3(2-s)}{(s+1)(s+5)}$$

Here, there is no open-loop poles of $G(s)$ on the right half of s-plane, i.e.,

$$P = 0$$

$$G(j\omega) = \frac{3(2-j\omega)}{(j\omega+1)(j\omega+5)}$$

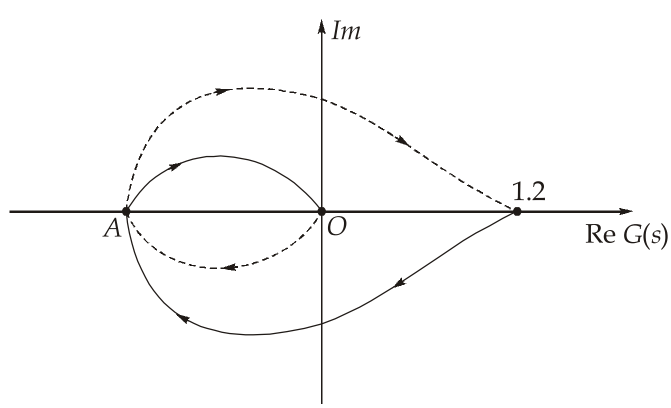
Magnitude, $$M = \frac{3\sqrt{4+\omega^2}}{\sqrt{\omega^2+1}\sqrt{\omega^2+25}}$$

Angle, $$\phi = -\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

When $\omega \to 0$; $G(j\omega) = 1.2\angle 0°$

When $\omega \to \infty$; $G(j\omega) = 0\angle -270°$

The Nyquist plot is



The intersection point A can be determined as,

$$\phi = -180°$$

$$-\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{5}\right) = -180^\circ$$

$$\tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{5}\right) = 180^\circ - \tan^{-1}(\omega)$$

$$\tan^{-1}\left(\frac{\frac{\omega}{2}+\frac{\omega}{5}}{1-\frac{\omega^2}{10}}\right) = 180^\circ - \tan^{-1}(\omega)$$

$$\frac{7\omega}{10-\omega^2} = -\omega$$

$$\omega^2 = 17$$

$$\omega = \sqrt{17}$$

The magnitude of $G(j\omega)$ at $\omega = \sqrt{17}$ rad/s.

$$|G(j\omega)|_{\omega=\sqrt{17}} = \frac{3\sqrt{21}}{\sqrt{18}\sqrt{42}} = 0.5$$

Since, the point (–1, j0) is left of polar plot and there is no encirclement of the Nyquist plot about (–1, j0), i.e., $N = 0$.

$$Z = P - N = 0$$

So, no roots of characteristic equation lies on the right half of the s-plane. Hence, the closed-loop system is stable.

Gain margin, $$\text{GM} = \frac{1}{0.5} = 2$$

$$\text{GM}_{\text{dB}} = 20\log(2) = 6\text{ dB}$$

The gain cross-over frequency,

$$|G(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{3\sqrt{\omega_{gc}^2+4}}{\sqrt{\omega_{gc}^2+1}\sqrt{\omega_{gc}^2+25}} = 1$$

$$(\omega_{gc}^2+1)(\omega_{gc}^2+25) = 9(\omega_{gc}^2+4)$$

$$\omega_{gc}^4 + 17\omega_{gc}^2 - 11 = 0$$

On solving, $\omega_{gc} = 0.79$ rad/sec

Phase Margin,

$$PM = 180° + \phi_{\omega_{gc}}$$

$$PM = 180° - \tan^{-1}\left(\frac{0.79}{2}\right) - \tan^{-1}(0.79) - \tan^{-1}\left(\frac{0.79}{5}\right)$$

$$= 180° - 68.8°$$

$$PM = 111.2°$$

Q.6 (c) Solution:

Given :

$$G(s)H(s) = \frac{K(s+5)}{(s+1)^2}$$

Open-loop poles, $s = -1, -1; P = 2$

Open-loop zeros, $s = -5; Z = 1$

Number of asymptotes $= P - Z = 2 - 1 = 1$

Angle of asymptotes, $\theta = \dfrac{(2q+1)\times 180°}{P-Z}$

$$\theta = \frac{(2q+1)\times 180°}{2-1}\text{; where q = 0, 1, ...}$$

$$\theta = 180°$$

Break-away and break-in points is given by the solution of $\dfrac{dK}{ds} = 0$.

$$K = \frac{-(s+1)^2}{s+5} = \frac{-(s^2+2s+1)}{s+5}$$

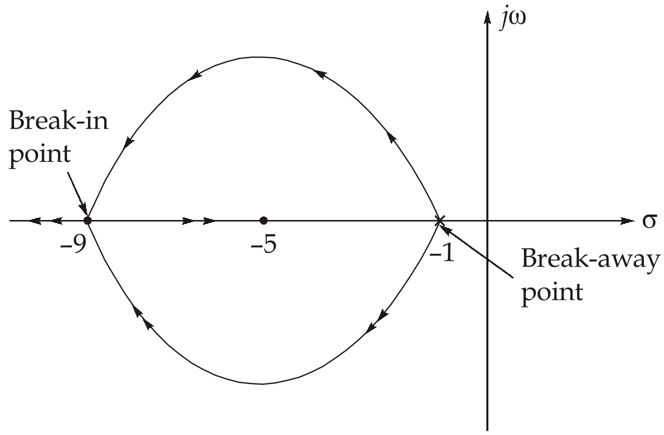
$$\frac{dK}{ds} = -\left[\frac{(s+5)(2s+2)-(s^2+2s+1)\cdot 1}{(s+5)^2}\right] = 0$$

$$s^2 + 10s + 9 = 0$$

$$s = -1, -9$$

Here, $s = -1$ is a break-away point and $s = -9$ is a break-in point.

The root locus plot is



Q.7 (a) Solution:

Open loop transfer function,

$$G(s)H(s) = \left[K_P + \frac{K_I}{s}\right] \cdot \left[\frac{2}{s^3 + 4s^2 + 5s + 2}\right]$$

$$= \frac{2(K_I + sK_P)}{s(s^3 + 4s^2 + 5s + 2)}$$

Characteristics equation of given system is,

$$1 + G(s)H(s) = 0$$

$$s^4 + 4s^3 + 5s^2 + (2K_p + 2)s + 2K_I = 0 \quad \text{...(1)}$$

Now, we proceed the answer using Routh-Hurwitz criteria.

From eqn. (1),

Routh Table :

s^4	1	5	$2K_I$
s^3	4	$2K_P + 2$	0
s^2	$\frac{18 - 2K_P}{4}$	$2K_I$	0
s^1	$\frac{(9 - K_P)(K_P + 1) - 8K_I}{\frac{18 - 2K_P}{4}}$	0	0
s^0	$2K_I$		

For stable system,

$$\frac{18 - 2K_P}{4} > 0$$

$$K_P < 9$$

and $$K_I > 0$$

Again, $$\frac{(9 - K_P)(1 + K_P) - 8K_I}{(4.5 - 0.5K_P)} > 0$$

$$\frac{(1 + K_P)}{0.5} - \frac{8K_I}{0.5(9 - K_P)} > 0$$

$$1 + K_P > \frac{8K_I}{9 - K_P}$$

$$(9 - K_P)(1 + K_P) > 8K_I$$

$$K_I < \frac{(9 - K_P)(1 + K_P)}{8}$$

$$K_I < \frac{9 + 8K_P - K_P^2}{8}$$

For maximum possible value of K_I,

$$\frac{d}{dK_P}(9 + 8K_P - K_P^2) = 0$$

$$-2K_P + 8 = 0$$

$$K_P = 4$$

For validity of maxima,

$$\frac{d^2}{dK_P}(9 + 8K_P - K_P^2) < 0$$

$$-2 < 0$$

Therefore, K_I is maximum at $K_P = 4$

and $$K_{I(\max)} = \frac{9 + 8 \times 4 - 16}{8}$$

$$K_{I(\max)} = \frac{25}{8} \text{ (for marginally stable system)}$$

Q.7 (b) Solution:

Given : $$G_P(s) = \frac{K}{s(s+2)}$$

Step 1 : Gain adjustment :

$$G_C(s) = \frac{1+Ts}{(1+\alpha Ts)} \quad \text{...lead compensator}$$

$$G_P(s)G_C(s) = \frac{K(1+Ts)}{s(1+\alpha Ts)(s+2)}$$

Velocity error constant,

$$K_v = \lim_{s\to 0} sG_C(s)\cdot G_P(s)$$

$$= \lim_{s\to 0} \frac{K(1+Ts)}{(1+\alpha Ts)(s+2)} = \frac{K}{2} = 20$$

Therefore, $$K = 40$$

Hence, $$G_P(s) = \frac{40}{s(s+2)} \text{ and } G_C(s) = \frac{1+Ts}{(1+\alpha Ts)}$$

Step 2 : Phase Margin

$$G_P(s) = \frac{40}{s(s+2)} = \frac{20}{s\left(1+\frac{s}{2}\right)}$$

$$G_P(j\omega) = \frac{20}{j\omega\left(\frac{j\omega}{2}+1\right)}$$

The gain cross-over frequency is obtained as,

$$\frac{20}{\omega_{gc}\sqrt{1+\frac{\omega_{gc}^2}{4}}} = 1$$

$$400 = \omega_{gc}^2\left(1+\frac{\omega_{gc}^2}{4}\right)$$

$$1600 = 4\omega_{gc}^2 + \omega_{gc}^4$$

Assume, $\omega_{gc}^2 = x$

$$x^2 + 4x - 1600 = 0$$

On solving, $x = 38.04$

Therefore, $\omega_{gc} = \sqrt{x} = 6.16$ rad/sec

Phase angle at $\omega_{gc} = 6.16$ rad/sec

$$\angle G(j\omega_{gc}) = -90° - \tan^{-1}\left(\frac{6.16}{2}\right) = 162°$$

Phase margin of the uncompensated system

$$\phi = 180° - 162° = 18°$$

Step 3 : Phase angle contribution required from the compensator.

Minimum phase margin specified = 50°

So, phase angle contribution from the compensator

$$\phi_m = 50° - 18° + \text{allowance}$$
$$= 50° - 18° + 5° = 37°$$

Step 4 : Gain of the compensator :

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 37°}{1+\sin 37°} = 0.25$$

Gain of the compensator at the frequency ω_m.

$$\Delta M = -10\log\alpha = -10\log 0.25 = 6 \text{ dB}$$

Step 5 : Pole and zero of the compensator :

The gain must be increased by 6 dB to find new gain cross-over frequency, ω_m.

$$\frac{20}{\omega_m\sqrt{1+\frac{\omega_m^2}{4}}} = \sqrt{\alpha} = 0.5$$

On solving,

$$\omega_m = 8.83 \text{ rad/sec}$$

Also, $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$

$$T = \frac{1}{\omega_m\sqrt{\alpha}}$$

$$T = \frac{1}{8.83 \times 0.5} = 0.226$$

and
$$\frac{1}{T\alpha} = 17.66$$

So,
$$G_C(s) = \frac{4(s+4.424)}{(s+17.66)}$$

$$K = 40 \text{ and } \frac{K}{\alpha} = 160$$

Therefore, open loop transfer function

$$G_C(s)G_P(s) = \frac{160(s+4.424)}{s(s+2)(s+17.66)}$$

Q.7 (c) Solution:

Given :

$$\xi = 0.5, \omega_n = \sqrt{10} \text{ rad/sec}, e_{ss} = 10\%$$

Open-loop transfer function,

$$G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$$

The characteristic equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+\alpha)}{(s+\beta)^2} = 0$$

$$s^2 + (2\beta + K)s + (\beta^2 + K\alpha) = 0 \quad ...(1)$$

The standard characteristic equation is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad ...(2)$$

On comparing, we get

$$2\beta + K = 2\xi\omega_n$$

$$2\beta + K = 2 \times 0.5 \times \sqrt{10}$$

$$2\beta + K = \sqrt{10} \quad ...(3)$$

$$\alpha K + \beta^2 = \omega_n^2$$

$$\alpha K + \beta^2 = 10 \quad ...(4)$$

From steady-state error,

The open-loop transfer function is a type-0 system. So, steady-state error

$$e_{ss} = \frac{1}{1+K_P}$$

$$\frac{10}{100} = \frac{1}{1+K_P}$$

$$K_p = 9$$

But $$K_p = \lim_{s\to 0} G(s)$$

$$9 = \lim_{s\to 0} \frac{K(s+\alpha)}{(s+\beta)^2}$$

$$9 = \frac{K\alpha}{\beta^2} \quad \text{...(5)}$$

From eqn. (4) and (5),

$$9\beta^2 + \beta^2 = 10$$

$$\beta = 1$$

From eqn. (3)

$$2\beta + K = \sqrt{10}$$

$$2 \times 1 + K = \sqrt{10}$$

$$K = 3.16 - 2 = 1.16$$

$$K = 1.16$$

From eqn. (4)

$$\alpha K + \beta^2 = 10$$

$$\alpha \times 1.16 + (1)^2 = 10$$

$$\alpha = \frac{9}{1.16}$$

$$\alpha = 7.76$$

Q.8 (a) Solution:

Given : $$G(s)H(s) = \frac{1000s}{(s+10)(s+100)}$$

$$G(j\omega)H(j\omega) = \frac{j1000\omega}{(j\omega+10)(j\omega+100)}$$

$$G(j\omega)H(j\omega) = \frac{j\omega}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}$$

The corner frequencies are

$$\omega_1 = 10 \text{ rad/sec}$$
$$\omega_2 = 100 \text{ rad/sec}$$

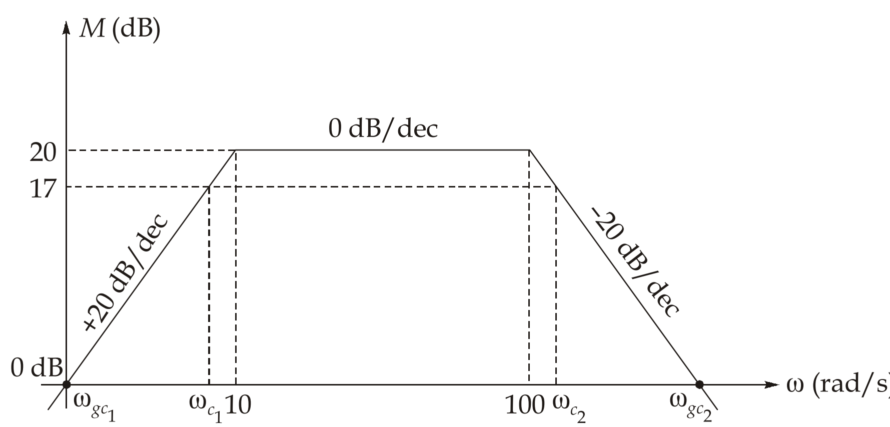
The transfer function has a zero at the origin. Hence, the initial slope of L-M asymptotic plot is +20 dB/dec.

At ω = 10 rad/sec, the LM is

$$20 \log \omega = 20 \log(10) = 20 \text{ dB.}$$

At ω = 100 rad/sec, the slope changes by –20 dB/dec and the slope between 10 rad/sec and 100 rad/sec is (20 – 20) = 0 dB/dec as at ω = 10 rad/sec, a pole is added.

The complete LM plot is



From LM plot, there are two gain cross-over frequencies.

$$20 = \frac{20-0}{\log 10 - \log(\omega_{gc_1})}$$

$$\log\left(\frac{10}{\omega_{gc_1}}\right) = 1$$

$$\omega_{gc_1} = 1 \text{ rad/sec}$$

Also, $$-20 = \frac{20-0}{\log 100 - \log(\omega_{gc_2})}$$

$$-1 = \log\left(\frac{100}{\omega_{gc_2}}\right)$$

$$\omega_{gc_2} = 1000 \text{ rad/sec}$$

The gain at 3 dB attenuation is

$$20 - 3 = 17 \text{ dB}$$

So, frequencies at 3 dB attenuation

$$20 = \frac{20-17}{\log 10 - \log(\omega_{C_1})}$$

$$\log\left(\frac{10}{\omega_{C_1}}\right) = \frac{3}{20}$$

$$\omega_{C_1} = 7.08 \text{ rad/sec}$$

Also, $$-20 = \frac{20-17}{\log 100 - \log(\omega_{C_2})}$$

$$\log\left(\frac{100}{\omega_{C_2}}\right) = \frac{-3}{20}$$

$$\omega_{C_2} = 141.25 \text{ rad/sec}$$

Q.8 (b) Solution:

Let us assume the state feedback gain matrix

$$K = [K_1 \quad K_2 \quad K_3]$$

Given system is, $$\dot{X}(t) = AX(t) + BU(t)$$

Also, $$U = -KX(t)$$

Therefore, $$\dot{X}(t) = AX(t) + B(-KX(t))$$

$$\dot{X}(t) = (A - BK)X(t) = A_{CL}X(t)$$

Therefore, gain matrix K is designed in such a way that

$$|sI - (A - BK)| = (s - u_1)(s - u_2)(s - u_3)$$

Now, $$BK = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K_1 & K_2 & K_3 \end{bmatrix}$$

$\Rightarrow$ $$A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K_1 & K_2 & K_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1-K_1 & -5-K_2 & -6-K_3 \end{bmatrix}$$

Now, $$[sI - (A - BK)] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1+K_1 & 5+K_2 & s+6+K_3 \end{bmatrix}$$

$$|sI - A + BK| = s(s^2 + 6s + sK_3 + 5 + K_2) + 1(0 + 1 + K_1) = 0$$

$$= s^3 + (6 + K_3)s^2 + (5 + K_2)s + 1(1 + K_1) = 0 \quad \text{...(1)}$$

Now, $$(s - u_1)(s - u_2)(s - u_3) = (s + 10)(s + 2 + j4)(s + 2 - j4) = 0$$

$\Rightarrow$ $$(s + 10)(s^2 + 4s + 10) = 0$$

$$s^3 + 14s^2 + 60s + 200 = 0 \quad \text{...(2)}$$

On comparing eqn. (1) and (2)

$$K_1 = 199$$
$$K_2 = 55$$
$$K_3 = 8$$

State feedback control matrix

$$K = \begin{bmatrix} 199 \\ 55 \\ 8 \end{bmatrix}$$

Necessary and Sufficient Condition for Arbitrary Pole Placement : The system should be completely controllable

For controllability :

$$[Q_C] = [B \quad AB \quad A^2B] \neq 0$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$A^2B = A.AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 31 \end{bmatrix}$$

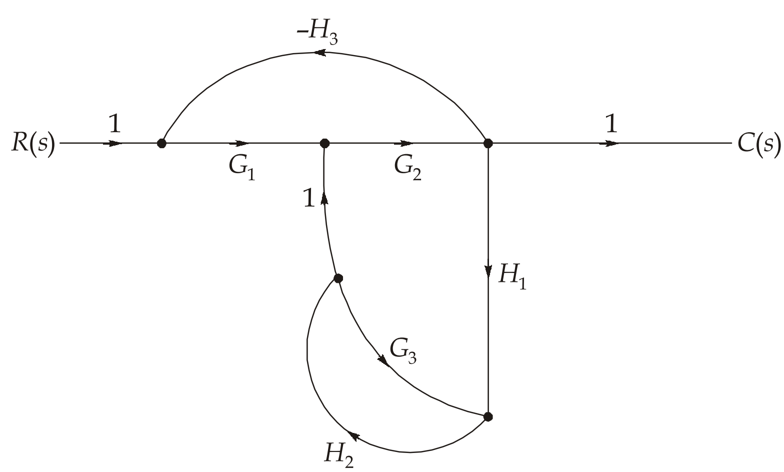
$$[Q_C] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

$$|Q_C| = -1 \neq 0$$

Therefore, we can place closed loop poles for given system any where in reasonable locations.

Q.8 (c) Solution:

(i) The signal flow graph of the above block diagram is shown as



Forward path gain, $P_1 = G_1G_2$

Δ_1 = The value of Δ with the forward path removed

Loop gains :

$$L_1 = -G_1G_2H_3,\ L_2 = G_2H_1H_2,\ L_3 = G_3H_2$$

Non-touching loops gain,

$$L_1L_3 = -G_1G_2G_3H_2H_3$$

$$\Delta = 1 - L_1 - L_2 - L_3 + L_1L_3$$

$$= 1 + G_1G_2H_3 - G_2H_1H_2 - G_3H_2 - G_1G_2G_3H_2H_3$$

The transfer function,

$$\frac{C(s)}{R(s)} = \frac{\Sigma P_K \Delta_K}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2(1-G_3H_2)}{1+G_1G_2H_3 - G_2H_1H_2 - G_3H_2 - G_1G_2G_3H_2H_3}$$

(ii) The characteristic equation of the system is

$$1+(K_C + K_D s)\frac{200}{s(s+5)(s+10)} = 0$$

$$s^3 + 15s^2 + (50 + 200K_D)s + 200K_C = 0$$

Forming Routh array,

s^3	1	$(50+200K_D)$
s^2	$(15)3$	$(200K_C)40K_C$
s^1	$\frac{150+600K_D-40K_C}{3}$	
s^0	$40K_C$	

For the system to be stable,

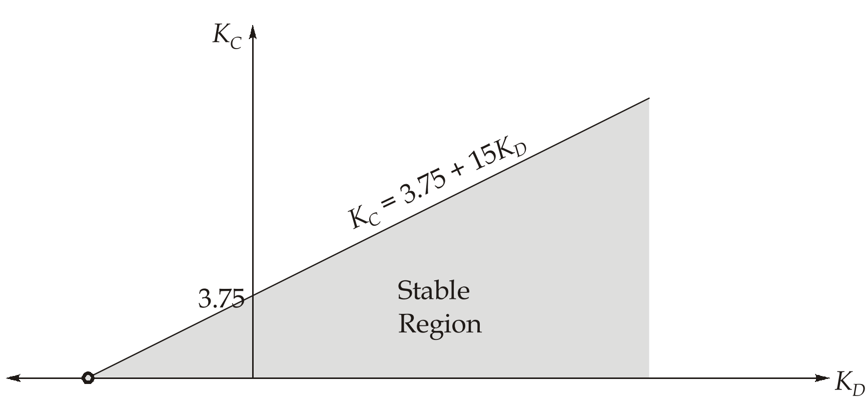
$$150 + 600K_D - 40K_C > 0$$

$$K_C < 3.75 + 15K_D$$

$$40K_C > 0$$

i.e., $K_C > 0$

The region of stability,



❍❍❍❍