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Detailed Solutions

ESE-2023 Mains Test Series

E & T Engineering Test No : 1

Section A : Network Theory

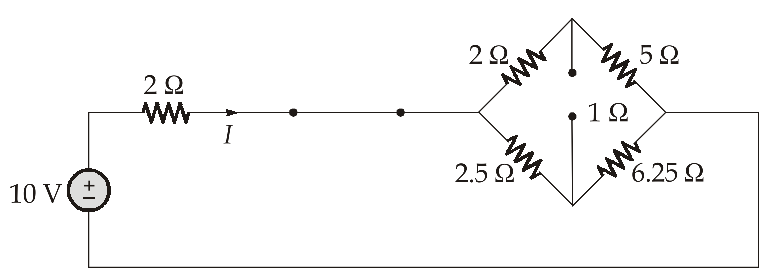
Q.1 (a) Solution:

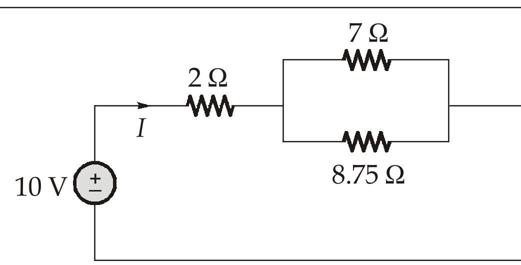
For given circuit:

- With D.C. source, inductor acts as short circuit.
- Since bridge network is in balanced situation i.e.,

$$Z_1 \times Z_4 = Z_2 \times Z_3$$

Now, the simplified circuit will be





$$I = \frac{10}{2+[7 \| 8.75]} = 1.698 \text{ Amp}$$

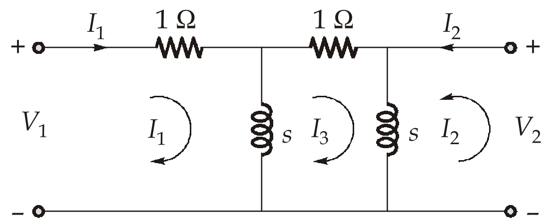
*Solutions updated on 14.04.2023

Now, power supplied by source is

$$P = VI = (10)(1.698)$$
$$= 16.98 \text{ Watt}$$

Q.1 (b) Solution:

Transform the given two port network into Laplace domain,



By applying KVL in Mesh 1;

$$V_1 - I_1 \times 1 - s(I_1 - I_3) = 0$$
$$V_1 = (s+1)I_1 - sI_3 \quad \text{...(i)}$$

By applying KVL in Mesh 3;

$$-s(I_3 - I_1) - I_3 \times 1 - s(I_3 + I_2) = 0$$
$$-sI_3 + sI_1 - I_3 - sI_3 - sI_2 = 0$$
$$I_3(2s+1) = -sI_2 + sI_1$$

$$\therefore \quad I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2 \quad \text{...(ii)}$$

By applying KVL in Mesh 2;

$$V_2 - s(I_2 + I_3) = 0$$
$$V_2 = sI_2 + sI_3 \quad \text{...(iii)}$$

Substituting equation (ii) in equation (i),

$$V_1 = (s+1)I_1 - s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$V_1 = \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I_2 \quad \text{...(iv)}$$

Substituting equation (ii) in equation (iii),

$$V_2 = sI_2 + s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

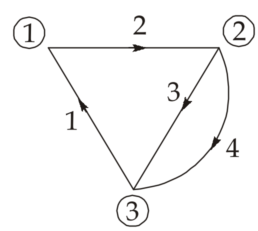
$$V_2 = \left(\frac{s^2}{2s+1}\right)I_1 + \left(\frac{s^2+s}{2s+1}\right)I_2 \quad \text{...(v)}$$

Comparing equation (iv) and (v),

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{2s+1} & \dfrac{s^2}{2s+1} \\ \dfrac{s^2}{2s+1} & \dfrac{s^2+s}{2s+1} \end{bmatrix}$$

Q.1 (c) Solution:

In constructing the graph of the network, the current source I_s is replaced by an open circuit, the voltage source V_s is replaced by a short circuit, and all other branches containing linear elements are shown by line segments. The network graph is shown below.



The graph has 3 nodes and 4 branches.

∴ The complete incidence matrix is given by

$$A_a = \begin{matrix} \\ ① \\ ② \\ ③ \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & -1 \end{bmatrix} \end{matrix}$$

The reduced incidence matrix is given by

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}_{2\times 4} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}_{4\times 2}$$

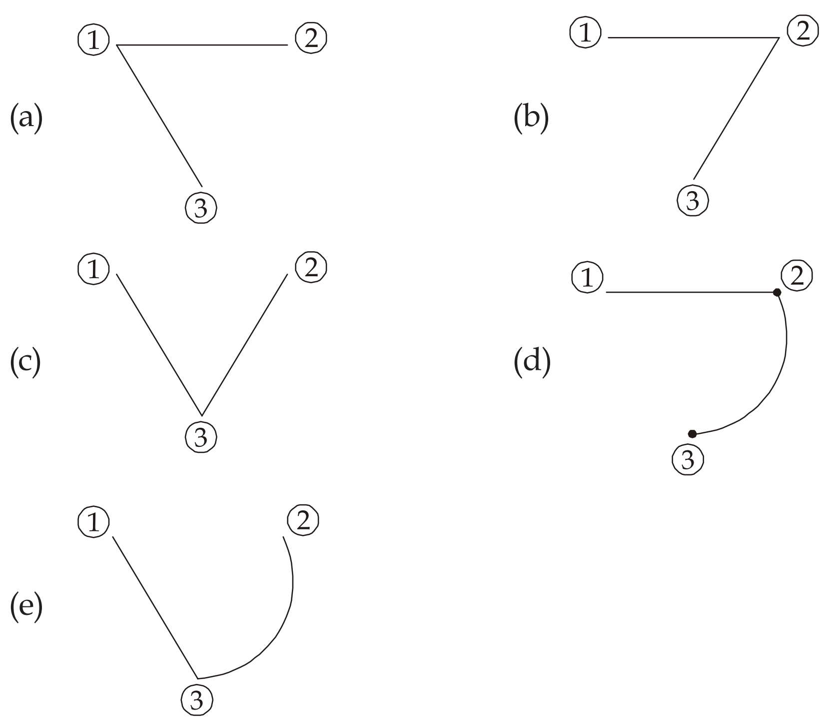
$$= \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\therefore \quad \text{No. of possible trees} = \det[AA^T]$$

$$= 6 - 1 = 5$$

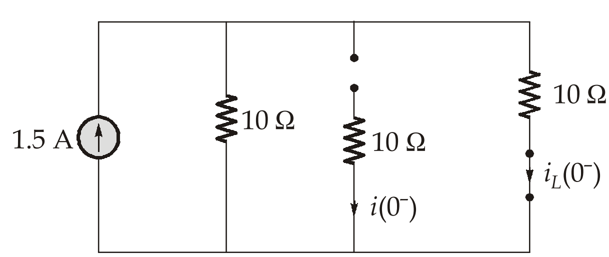
A tree is an undirected connected graph in which any two vertices are connected by only one path.

Now, drawing all the possible trees.



Q.1 (d) Solution:

At $t = 0^-$, switch is open and the circuit in steady state hence, inductor acts as short-circuit.

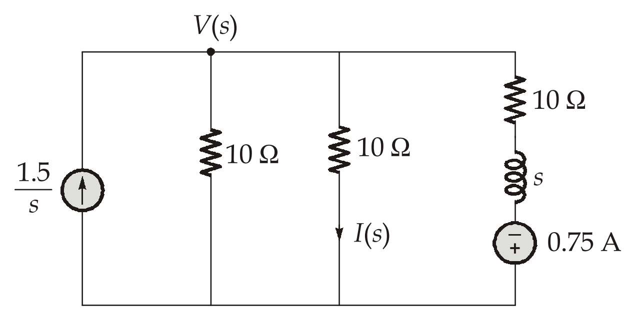


$$i_L(0^-) = \frac{1.5 \times 10}{10 + 10} = 0.75 \text{ A}$$

Since inductor current does not change instantly.

$$\therefore \quad i_L(0^-) = i_L(0^+) = 0.75 \text{ A}$$

For $t > 0$, the circuit is transformed into S-domain and is shown below,



∴ Applying KCL analysis at node $V(s)$,

$$\therefore \quad \frac{V(s)}{10}+\frac{V(s)}{10}+\frac{V(s)+0.75}{s+10} = \frac{1.5}{s}$$

$$V(s)\left[\frac{1}{10}+\frac{1}{10}+\frac{1}{s+10}\right] = \frac{1.5}{s}-\frac{0.75}{s+10}$$

$$V(s)\left[\frac{10+s+5}{5(s+10)}\right] = \frac{1.5(10+s)-0.75s}{s(s+10)}$$

$$\therefore \quad V(s) = \frac{5(0.75s+15)}{s(s+15)} \qquad \text{...(1)}$$

Applying partial fraction expansion on equation (1) we get,

$$V(s) = \frac{A}{s}+\frac{B}{s+10}$$

$$A = \left.\frac{5(0.75s+15)}{s+15}\right|_{s=0} = 5$$

$$B = \left.\frac{5(0.75s+15)}{s}\right|_{s=-15} = -1.25$$

$$\therefore \quad V(s) = \frac{5}{s}-\frac{1.25}{s+15}$$

Now, $$I(s) = \frac{V(s)}{10}$$

$$\therefore \quad I(s) = \frac{0.5}{s}-\frac{0.125}{s+15}$$

Now applying inverse Laplace transform, we get,

$$i(t) = (0.5 - 0.125e^{-15t}) \text{ for } t > 0$$

Q.1 (e) Solution:

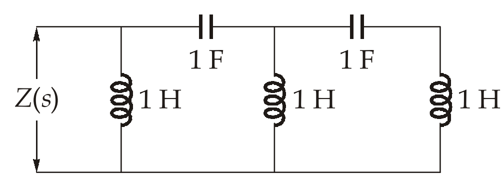
The Cauer II form is obtained by continued fraction expansion about the pole at the origin. The given function $Z(s)$ has a zero at the origin. The admittance function $Y(s)$ has a pole at origin. Hence, the continued fraction expansion of $Y(s)$ is carried out. Arranging the polynomials in ascending order of s, we have,

$$Y_{LC}(s) = \frac{3s^4 + 4s^2 + 1}{s^5 + 3s^3 + s} = \frac{1 + 4s^2 + 3s^4}{s + 3s^3 + s^5}$$

By continued fraction expansion of $Y(s)$, we have

$$\begin{array}{l}
s + 3s^3 + s^5 \,\big)\, 1 + 4s^2 + 3s^4 \,\big(\, \frac{1}{s} \leftarrow Y \\
\qquad\qquad\qquad\; \underline{1 + 3s^2 + s^4} \\
\qquad\qquad\qquad s^2 + 2s^4 \,\big)\, s + 3s^3 + s^5 \,\big(\, \frac{1}{s} \leftarrow Z \\
\qquad\qquad\qquad\qquad\qquad\quad \underline{s + 2s^3} \\
\qquad\qquad\qquad\qquad\qquad\qquad s^3 + s^5 \,\big)\, s^2 + 2s^4 \,\big(\, \frac{1}{s} \leftarrow Y \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\; \underline{s^2 + s^4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s^4 \,\big)\, s^3 + s^5 \,\big(\, \frac{1}{s} \leftarrow Z \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\;\; \underline{s^3} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s^5 \,\big)\, s^4 \,\big(\, \frac{1}{s} \leftarrow Y \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\;\; \underline{s^4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\;\; \times
\end{array}$$

The impedance are connected in series branches whereas the admittances are connected in parallel branches in a Cauer or Ladder realization.



Q.1 (f) Solution:

From circuit: $$R_{eq} = R_1 + [R_2 \| 5]$$

From voltage division rule:

$$V_0 = V_s \times \frac{R_2 \| 5}{1 + R_1 + R_2 \| 5}$$

$$\frac{V_0}{V_s} = \frac{R_2 \| 5}{1 + R_{eq}}$$

$$0.05 = \frac{\frac{R_2 \cdot 5}{R_2 + 5}}{1 + 39}$$

$$2 = \frac{5R_2}{R_2+5}$$

$$2R_2 + 10 = 5R_2$$

$$R_2 = \frac{10}{3} = 3.33\ \text{k}\Omega$$

$$\because \quad R_{eq} = R_1 + [R_2 \parallel 5]$$

$$39 = R_1 + 2$$

$$R_1 = 37\ \text{k}\Omega$$

Q.2 (a) Solution:

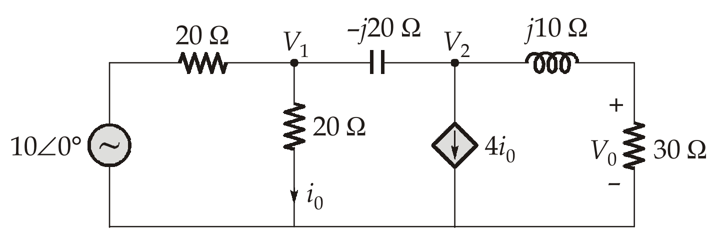
We have,

$$V_s = 10 \cos 10^3 t \Rightarrow 10\angle 0°, \omega = 10^3\ \text{rad/s}$$

$$C = 50\ \mu\text{F} \Rightarrow \frac{1}{j\omega C} = -j20\ \Omega$$

$$L = 10\ \text{mH} \Rightarrow j\omega L = j10\ \Omega$$

Now, the simplified circuit is



Apply nodal analysis at node V_1:

$$\frac{V_1-10}{20} + \frac{V_1}{20} + \frac{V_1-V_2}{-j20} = 0$$

$$V_1\left[\frac{1}{20} + \frac{1}{20} + \frac{1}{-j20}\right] = \frac{1}{2} - \frac{V_2}{j20}$$

$$V_1(0.1 + 0.05j) - j0.05V_2 = 0.5 \qquad \text{...(i)}$$

Apply Nodal analysis at node V_2:

$$\frac{V_2-V_1}{-j20} + 4i_0 + \frac{V_2}{30+j10} = 0$$

$$\frac{V_2-V_1}{-j20} + 4\left(\frac{V_1}{20}\right) + \frac{V_2}{30+j10} = 0 \qquad \left(\because\ i_0 = \frac{V_1}{20}\right)$$

$$V_1\left[\frac{1}{5} + \frac{1}{j20}\right] + j0.05V_2 + \frac{V_2}{30+j10} = 0$$

$$V_1(0.2 - j0.05) + V_2\left[0.05j + \frac{1}{30 + j10}\right] = 0$$

$$V_1(0.2 - j0.05) + V_2(0.03 + 0.04j) = 0$$

$$V_1 = \left(\frac{0.03 + 0.04j}{0.2 - j0.05}\right)(-V_2) \quad \text{...(ii)}$$

Put (ii) in (i), $V_2 = \dfrac{170}{0.6 - j26.2} = 0.148 + 6.48j$

Now; $V_0 = V_2 \cdot \left(\dfrac{30}{30 + j10}\right) = (0.148 + 6.48j)\left(\dfrac{30}{30 + j10}\right)$

$$= 2.07 + 5.78j$$

$$V_0 = 6.149 \angle 70.25° \text{ Volt}$$

Now, $i_0 = \dfrac{V_1}{20}$

where; $V_1 = \left(\dfrac{0.03 + 0.04j}{0.2 - j0.05}\right)(-V_2)$ ...(from eqn. (ii))

$$V_1 = [0.094 + 0.223j][-0.148 - 6.48j]$$

$$= 1.43 - 0.64j$$

$$V_1 = 1.56 \angle -24.16° \text{ volt}$$

$\therefore$ $i_0 = \dfrac{V_1}{20} = \dfrac{1.56\angle -24.16°}{20} = 0.078\angle -24.16° \text{ Amp}$

$$i_0 = 0.078 \cos[10^3 t - 24.16°] \text{ Amp}$$

Now; Power factor between V_0 and i_0 is

$$\text{P.F} = \cos[\theta_V - \theta_I] = \cos[70.25° - (-24.16°)]$$

$$\text{P.F} = \cos[94.41°]$$

$$= -0.076 \text{ (lagging)}$$

Therefore;

(i) $V_0 = 6.149 \angle 70.25°$ Volt

(ii) $i_0 = 0.078 \angle -24.16°$ Amp

(iii) Power factor (PF) = –0.076 (lagging)

Q.2 (b) Solution:

(i) Total no. of nodes = 5

Total no. of branches = 7

So, the complete incidence matrix A_a of the given graph will have 5 rows and 7 columns.

$$A_a = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \textcircled{1} & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ \textcircled{2} & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ \textcircled{3} & 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ \textcircled{4} & 0 & 0 & 1 & -1 & 0 & 0 & -1 \\ \textcircled{5} & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{matrix}$$

The reduced incidence matrix is given as:

$$A_a = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}$$

To find number of possible trees of a graph,

No. of possible trees of a graph = Det$[AA^T]$

$$[A^T] = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

Now, $$[AA^T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}_{4\times 7} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}_{7\times 4}$$

$$[AA^T] = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}_{4\times 4}$$

Now, no. of possible trees = $\det[AA^T]$

$$= \begin{vmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{vmatrix}$$

After solving, the determinant, we get

No. of possible trees of graph = 24

(ii) Given reduced incidence matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

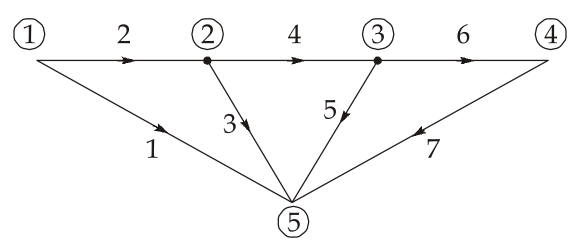
Now, complete incidence matrix A_a is given as

$$A_a = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

Now drawing the oriented graph corresponding to complete incidence matrix.

No. of nodes = 5

No. of branches = 7



No. of possible tie sets = $b - n + 1$

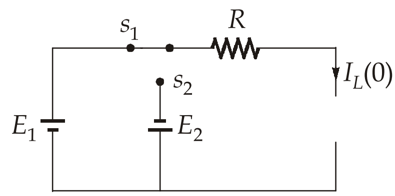
where $b \rightarrow$ No. of branches

$n \rightarrow$ No. of nodes

$\therefore$ Possible tie sets = $7 - 5 + 1 = 3$

Q.2 (c) Solution:

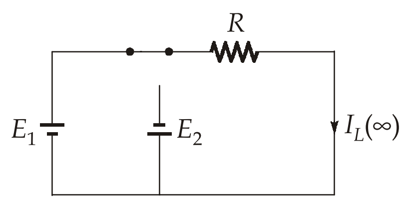
At $t = 0$, when the switch is in position s_1, the source is connected at $t = 0$, hence the network is in transient state.



$\therefore I_L(0) = 0$ A

At $t = \infty$,

The circuit is in steady state,



$$\therefore \quad I_L(\infty) = \frac{E_1}{R} = \frac{100}{100} = 1\text{ A}$$

time constant of the circuit,

$$\tau = \frac{L}{R} = \frac{0.2}{100} = \frac{1}{500}\text{sec.}$$

The inductor current,

$$I_L(t) = I_L(\infty) + (I_L(0) - I_L(\infty))\, e^{-t/\tau}$$

$$= 1 + (0 - 1)e^{-500t}$$

$$I_L(t) = 1 - e^{-500t}\text{ A}$$

Voltage across inductor,

$$V_L(t) = L\frac{dI_L(t)}{dt}$$

$$V_L(t) = 0.2\frac{d}{dt}\left[1 - e^{-500t}\right]$$

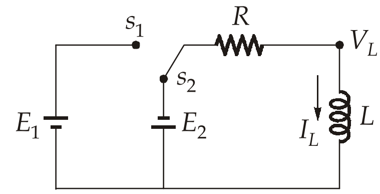
$$V_L(t) = 0.2 \times 500\, e^{-500t}\text{ V for } 0 < t \le 0.5\text{ ms}$$

$$\therefore \quad i_L(0.5 \times 10^{-3}) = 1 - e^{-0.25} = 0.2212\text{ A}$$

$$V_L(0.5 \times 10^{-3}) = 100e^{-0.25} = 77.88\text{ V}$$

Let the second switching occurs at time $t' = 0$

Then, $\quad t' = t - 0.5 \times 10^{-3}$



For time $t' > 0$, the mesh equation is

$$Ri_L(t') + L\frac{di_L}{dt'} = -E_2$$

i.e.,
$$\frac{di_L}{dt'} + \frac{R}{L}i_L(t') = \frac{-E_2}{L} \text{ with } i(0) = 0.2212 \text{ A}$$

$$\frac{di_L}{dt'} + \frac{R}{L}i_L(t') = -\frac{E_2}{L}$$

$\therefore$
$$i_L(t') = K e^{-500t'} - 0.5$$

where 'K' from initial condition can be obtained as

$$K - 0.5 = 0.2212$$

i.e.,
$$K = 0.7212$$

$$i_L(t') = 0.7212e^{-500t'} - 0.5 \text{ A}$$

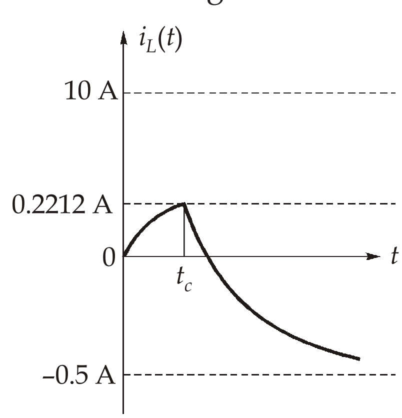
$$V_L(t') = L\frac{di_L(t')}{dt}$$

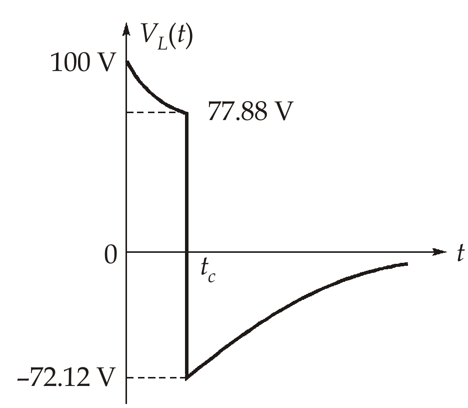
$$V_L(t') = 0.2 \times (-0.7212 \times 500)e^{-500t'}$$

$$V_L(t') = -72.12e^{-500t'} \text{ V}$$

When $t' = 0$, inductor voltage = –72.12 V

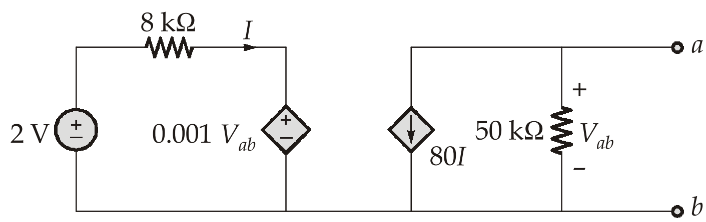
The current and voltage transients are,





Q.3 (a) Solution:

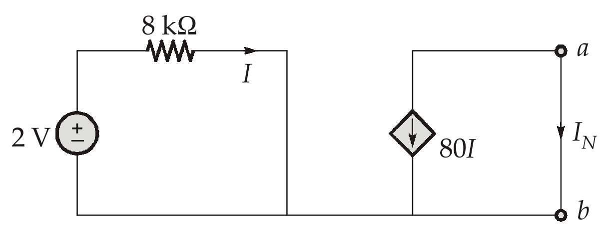
(i) When resistor is in series with current source, then resistor act as redundant.



Step I: To obtain short circuit current (I_N)

- Remove load (if present) and replace it with short circuit.

∴ Simplified network is given as:

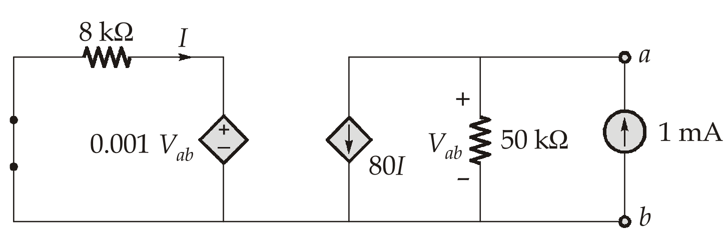


$$I = \frac{2}{8} = \frac{1}{4}\text{mA}$$

$$I_N = -80I = -80\left(\frac{1}{4}\right) = -20 \text{ mA}$$

Step 2: To obtain equivalent resistance across *ab*:

- Deactivate the dependent source i.e., current source replaced with open circuit and voltage source replaced with short circuit.
- Remove load (if present) and replace it with open circuit.
- Keep the dependent sources and place 1 mA independent current source across *ab* terminal.



∴ $$V_{ab} = 50[1 - 80I]$$

where, $$I = \frac{-0.001V_{ab}}{8}$$

$$V_{ab} = 50\left[1 + 80\left(\frac{0.001V_{ab}}{8}\right)\right]$$

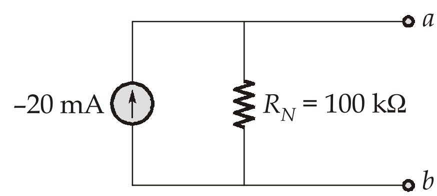
$$V_{ab} = 50 + 0.5\,V_{ab}$$

$$\therefore \quad V_{ab} = 100 \text{ Volt}$$

$$R_{eq} = R_N = \frac{V_{ab}}{1\text{ mA}} = 100\text{ k}\Omega$$

Step 3:

∴ Norton's equivalent circuit:



(ii) **1.** Resonant frequency, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$f_0 = \frac{1}{2\pi} = 0.1591\text{ Hz}$$

2. Damping ratio 'ξ'

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{1}{8} = 0.125$$

3. Maximum voltage across inductor 'V_L'.

$$f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1-\left(\frac{R^2C}{2L}\right)}}$$

$$f_L = \frac{1}{2\pi\sqrt{1\times 1}}\sqrt{\frac{1}{1-\left(\frac{(0.25)^2\times 1}{2\times 1}\right)}}$$

$$f_L = 0.1617\text{ Hz}$$

$$X_L = 2\pi f_L L = 1.0116\ \Omega$$

$$X_C = \frac{1}{2\pi f_L C} = 0.988\ \Omega$$

$$\text{Total impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad Z = 0.252\ \Omega$$

$$I = \frac{V}{Z} = \frac{\frac{10}{\sqrt{2}}}{0.252} = 28.059\text{ A}$$

$\therefore$ Maximum voltage across inductor V_L

$$V_L = IX_L = 28.51\text{ V}$$

Q.3 (b) Solution:

(i) For Foster-I form, function should be impedance function $Z(s)$ and apply the partial fraction method. In this case, numerator degree is lower than denominator.

$$Z(s) = \frac{s(s^2+9)}{(s^2+5)(s^2+13)}$$

$$Z(s) = \frac{s(s^2+9)}{(s^2+5)(s^2+13)} = \frac{As+B}{s^2+5} + \frac{Cs+D}{s^2+13}$$

After solving, we get,

$$A = \frac{1}{2}$$

$$B = 0$$

$$C = \frac{1}{2}$$

$$D = 0$$

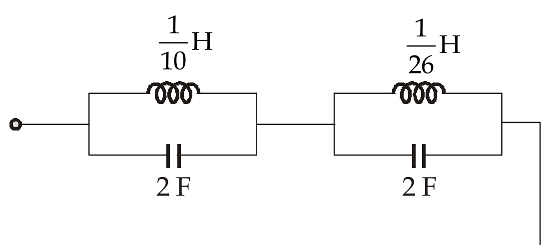
$$\therefore \quad Z(s) = \frac{1}{2s+\frac{10}{s}} + \frac{1}{2s+\frac{26}{s}} = Z_1(s) + Z_2(s)$$

where, $$Z_1(s) = \frac{1}{Y_1(s)} = \frac{1}{sC_1+\frac{1}{sL_1}} \text{ and } Z_2(s) = \frac{1}{Y_2(s)} = \frac{1}{sC_2+\frac{1}{sL_2}}$$

Therefore, $$C_1 = 2\text{F} \quad \text{and} \quad C_2 = 2\text{F}$$

$$L_1 = \frac{1}{10}\text{H} \qquad L_2 = \frac{1}{26}\text{H}$$

The network is shown as:



(ii) For Foster-II form, function should be admittance function Y(s) and apply the partial fraction method. For applying partial fraction, numerator degree should be lower than denominator.

$$Y(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)} = \frac{s(s^4+10s^2+24)}{s^4+8s^2+15}$$

$$Y(s) = \frac{s^5+10s^3+24s}{s^4+8s^2+15}$$

$$\begin{array}{r|l} s^4+8s^2+15 \,\big)\; s^5+10s^3+24s & s \\ s^5+\;8s^3+15s & \\ \hline 2s^3+\;9s & \end{array}$$

$$Y(s) = s+\frac{2s^3+9s}{s^4+8s^2+15} = s+\frac{2s^3+9s}{(s^2+3)(s^2+5)}$$

$$Y(s) = s+\frac{2K_2s}{s^2+3}+\frac{2K_4s}{s^2+5}$$

Now, $$2K_2s = (s^2+3)Y(s)\Big|_{s^2=-3} = \frac{s(s^2+4)(s^2+6)}{s^2+5}\Bigg|_{s^2=-3}$$

$$= \frac{s(-3+4)(-3+6)}{-3+5} = \frac{3}{2}s$$

or $$2K_2 = \frac{3}{2}$$

Again, $$2K_4s = (s^2+5)Y(s)\Big|_{s^2=-5}$$

$$= \frac{s(s^2+4)(s^2+6)}{s^2+3}\Bigg|_{s^2=-5} = \frac{s(-5+4)(-5+6)}{-5+3} = \frac{1}{2}s$$

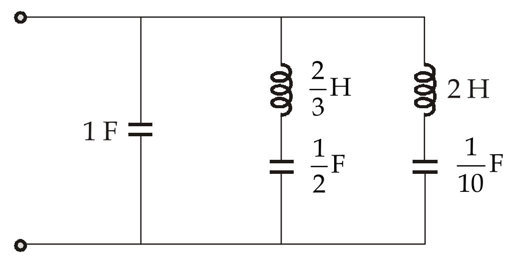
$\therefore$ $$Y(s) = s+\frac{1}{\frac{2}{3}s+\frac{2}{s}}+\frac{1}{2s+\frac{10}{s}} = Y_1(s)+Y_2(s)+Y_3(s)$$

where, $$Y_1(s) = sC_1,\; Y_2(s) = \frac{1}{Z_2(s)} = \frac{1}{sL_2+\frac{1}{sC_2}}$$

and $$Y_3(s) = \frac{1}{Z_3(s)} = \frac{1}{sL_3+\frac{1}{sC_3}}$$

Therefore, $$C_1 = 1\text{ F},\; L_2 = \frac{2}{3}\text{H},\; C_2 = \frac{1}{2}\text{F},\; L_3 = 2\text{ H and } C_3 = \frac{1}{10}\text{F}$$

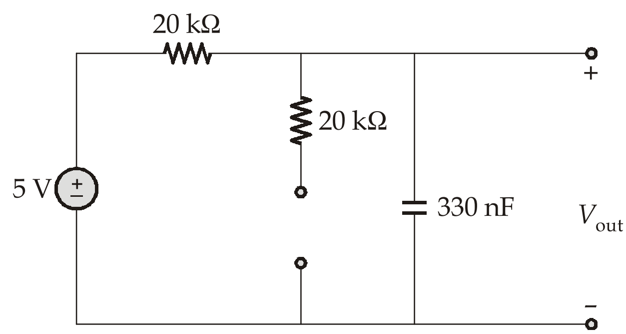
The network is shown as,



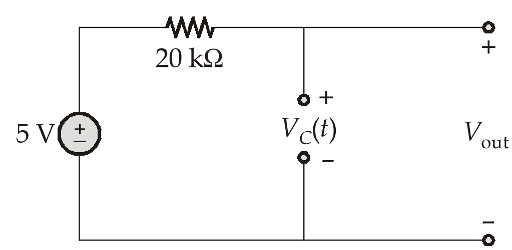
Q.3 (c) Solution:

1. When t < 10 ms

for $V_{in}(t) = 0$, the NMOS transistor is off.



Since the 5V DC source is connected to the network, it is said to be in steady state, hence capacitor is treated as open circuit.

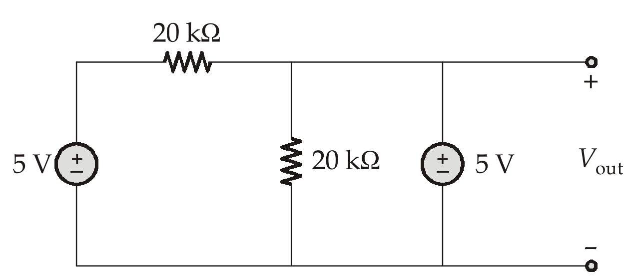


$$V_C(t) = V_{out}(t) = 5 \text{ V for } t < 10 \text{ msec}$$

2. When 10 ms < t < 20 ms

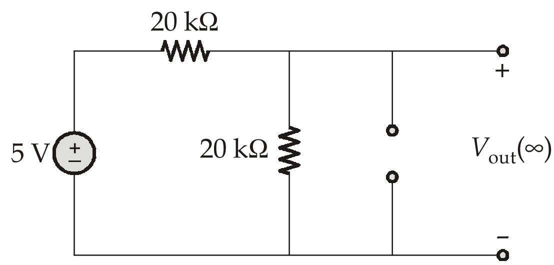
During this period, $V_{in}(t) = 5$ V, so the transistor is ON. $(\because V_{GS} = 5 \text{ V})$

The resultant network at $t = 10$ ms can be drawn as



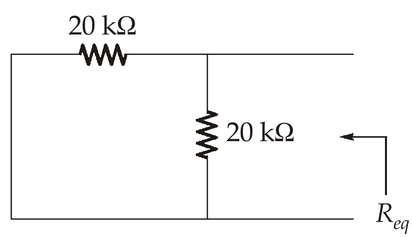
$\therefore \quad V_{out}(10\ ms^+) = V_{out}(10\ ms^-) = V_C(t) = 5\ V$

The steady state value of the circuit when the transistor is in ON is equal to $V_{out}(\infty)$.



$$V_{out}(\infty) = \frac{5 \times 20\ K}{40\ K} = 2.5\ V$$

Time constant of circuit,



$$R_{eq} = \frac{20\ K \times 20\ K}{40\ K} = 10\ k\Omega$$

$$\tau = RC = 10 \times 10^3 \times 330 \times 10^{-9} = 3.3\ ms$$

for 10 ms < t < 20 ms

$$V_{out}(t) = V_{out}(\infty) + \left[V_{out}(10\ ms^+) - V_{out}(\infty)\right]e^{-t/\tau}$$

$$V_{out}(t) = 2.5 + (5 - 2.5)e^{-\frac{(t-10)\ ms}{3.3\ ms}}$$

$$V_{out}(t) = 2.5 + 2.5e^{-\frac{(t-10)\ ms}{3.3\ ms}}\ V$$

3. **When t > 20 ms:**

The transistor is turned off again. Because the voltage across the capacitor can't change instantaneously, $V_{out}(20\ ms^+) = V_{out}(20\ ms^-)$ and $V_{out}(20\ ms^-)$ can be written as

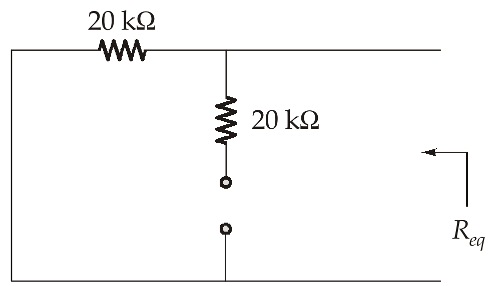
$$V_{out}(20\ ms^-) = 2.5 + 2.5e^{-\frac{20\ ms - 10\ ms}{3.3\ ms}}$$

$$V_{out}(20\ ms^-) = 2.62\ V$$

The $V_{out}(\infty)$ is same as the steady state value when the transistor is off,

$\therefore \quad V_{out}(\infty) = 5\text{ V}$

The time constant,



$$\therefore \quad R_{eq} = 20\text{ k}\Omega$$

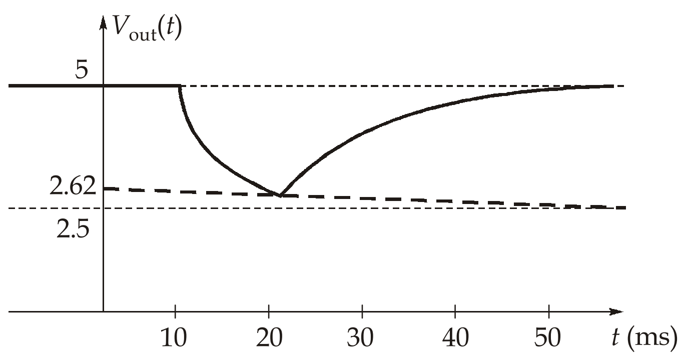
$$\tau = RC = 20\text{ K} \times 330 \times 10^{-9} = 6.6\text{ ms}$$

$\therefore \quad V_{out}(t)$ for $t > 20$ ms,

$$V_{out}(t) = V_{out}(\infty) + \left[V_{out}(20\text{ ms}^+) - V_{out}(\infty)\right]e^{-t/\tau}$$

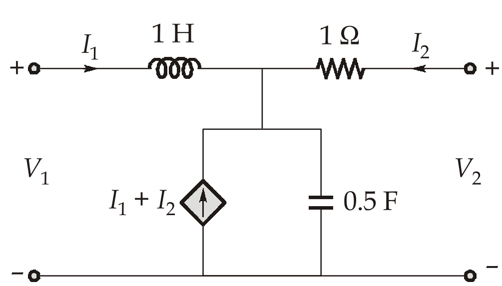
$$V_{out}(t) = 5 + (2.62 - 5)e^{-\frac{t-20\text{ ms}}{6.6\text{ ms}}}\ V$$

$$V_{out}(t) = 5 - 2.38e^{-\frac{t-20\text{ ms}}{6.6\text{ ms}}}\ \text{V for } t > 20\text{ ms}$$

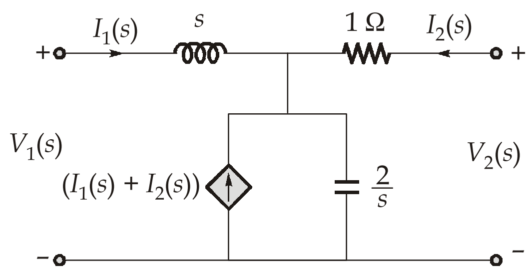


Q.4 (a) Solution:

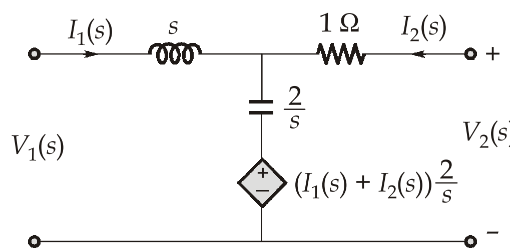
Given two port network,



By taking Laplace transform,



By using source transformation theorem, we can redraw the given two port network as



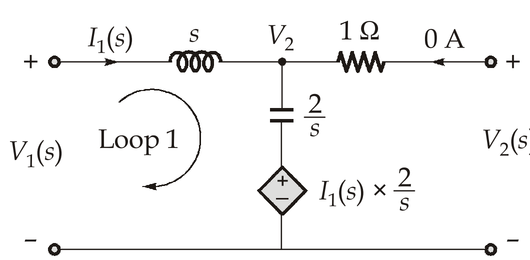
we know that,

transmission parameters can be written as,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

For A:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$



By writing KVL in loop 1,

$$V_1(s) - I_1(s)\cdot s - V_2(s) = 0 \quad \text{...(i)}$$

where,
$$V_2(s) = \frac{2}{s}\cdot I_1(s) + \frac{2}{s}\cdot I_1(s)$$

$$V_2(s) = \frac{4}{s} I_1(s)$$

$$V_2(s)\cdot\frac{s}{4} = I_1(s) \quad \text{...(ii)}$$

Substituting equation (ii) in equation (i),

$$V_1(s) - \frac{s}{4}\cdot V_2(s)\cdot s - V_2(s) = 0$$

$$V_1(s) = \left(1+\frac{s^2}{4}\right)V_2(s)$$

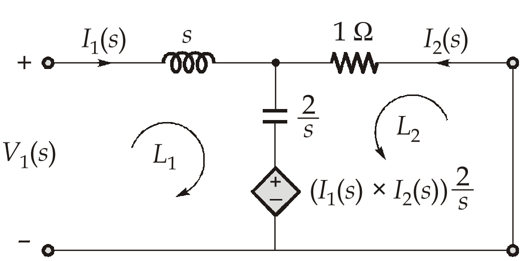
$$\therefore \qquad A = \left.\frac{V_1(s)}{V_2(s)}\right|_{I_2=0} = 1+\frac{s^2}{4}$$

For C:

From equation (ii), $$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{s}{4}$$

For B:

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$$



By writing KVL in loop L_1:

$$V_1(s) - I_1(s)\cdot s - \frac{2}{s}(I_1(s)+I_2(s)) - (I_1(s)+I_2(s))\frac{2}{s} = 0$$

$$V_1(s) - I_1(s)\cdot s - \frac{4}{s}I_1(s) - \frac{4}{s}I_2(s) = 0$$

$$V_1(s) - \left(s+\frac{4}{s}\right)I_1(s) - \frac{4}{s}I_2(s) = 0 \qquad \text{...(iii)}$$

by writing KVL in loop L_2:

$$0 - I_2(s)\cdot 1 - (I_1(s)+I_2(s))\frac{2}{s} - (I_1(s)+I_2(s))\frac{2}{s} = 0$$

$$-I_1(s)\left(\frac{4}{s}\right) - I_2(s)\left[1+\frac{4}{s}\right] = 0$$

$$\frac{4}{s}\cdot I_1(s) = -I_2(s)\left[\frac{s+4}{s}\right]$$

$$I_1(s) = -I_2(s)\left[\frac{s+4}{4}\right] \quad \text{...(iv)}$$

Substituting equation (iv) in equation (iii),

$$V_1(s)+\left(\frac{s^2+4}{s}\right)\left(\frac{s+4}{4}\right)I_2(s)-\frac{4}{s}I_2(s) = 0$$

$$V_1(s)+\left(\frac{s^3+4s^2+4s+16}{4s}-\frac{4}{s}\right)I_2(s) = 0$$

$$V_1(s)+\frac{s^3+4s^2+4s+16-16}{4s}I_2(s) = 0$$

$$V_1(s) = -\left(\frac{s^3+4s^2+4s}{4s}\right)I_2(s) = -\left(\frac{s^2+4s+4}{4}\right)I_2(s)$$

$\therefore$
$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} = \left(\frac{s^2+4s+4}{4}\right)$$

For D:

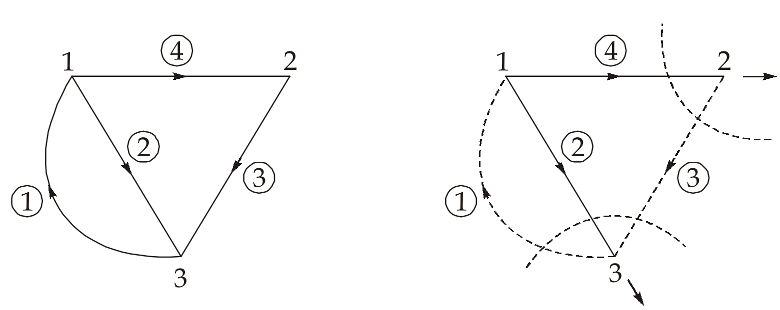
From equation (iv)
$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} = \left(\frac{s+4}{4}\right)$$

$\therefore$ The transmission parameters matrix is

$$[T] = \begin{bmatrix} 1+\dfrac{s^2}{4} & \left(\dfrac{s^2+4s+4}{4}\right) \\ \dfrac{s}{4} & \left(\dfrac{s+4}{4}\right) \end{bmatrix}$$

Q.4 (b) Solution:

The oriented graph and its selected tree are shown in figure. Since voltage v is to be determined, branch 2 is chosen as twig.



Twigs : {2, 4},

f-cutset 2 : {2, 1, 3}, f-cutset 4 : {4, 3}

Cut set matrix $[Q]$

$$[Q] = \begin{matrix} \text{f-cutset 2} \\ \text{f-cutset 3} \end{matrix} \overset{\begin{matrix} ① & ② & ③ & ④ \end{matrix}}{\begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}}$$

The KCL equation in matrix form is given by

$$[Q]\,[Y_b]\,[Q^T]\,[V_t] = [Q]\,[I_s] - [Q]\,[Y_b]\,[V_s] \qquad ...(1)$$

where, $[V_t]$ = Twig voltage column matrix

$[Q]$ = Cutset Matrix

$[Y_b]$ = Branch Admittance matrix

$[I_s]$ = Source current column matrix

$[V_s]$ = Source voltage column matrix

$$[Y_b] = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2v \end{bmatrix}$$

$$[V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad [V_t] = \begin{bmatrix} v_{t_2} \\ v_{t_4} \end{bmatrix}$$

$$[Q^T] = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Now,
$$[Q][Y_b] = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[Q][Y_b] = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix}$$

$$[Q][Y_b][Q^T] = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$[Q][Y_b][Q^T] = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

$$[Q][I_s] = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -2v \end{bmatrix}$$

$$[Q][I_s] = \begin{bmatrix} 0 \\ -2v \end{bmatrix}$$

$$[Q][Y_b][V_s] = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[Q][Y_b][V_s] = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Now put all the values in equation (1),

$$\begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}\begin{bmatrix} v_{t_2} \\ v_{t_4} \end{bmatrix} = \begin{bmatrix} 0 \\ -2v \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}\begin{bmatrix} v_{t_2} \\ v_{t_4} \end{bmatrix} = \begin{bmatrix} 1 \\ -2v \end{bmatrix}$$

From the figure, $v_{t_2} = v$

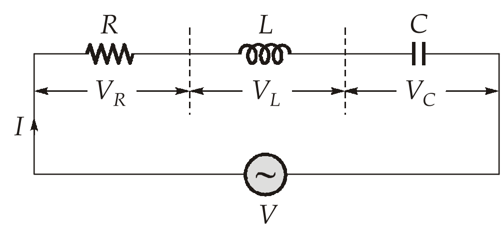
Now, $1.5v - 0.5v_{t_4} = 1$ ...(2)

$-0.5v + v_{t_4} = -2v$ ...(3)

$$v_{t_4} = -1.5\, v$$
$$v = 0.44 \text{ Volt}$$
$$v_{t_4} = -0.666 \text{ Volt}$$

Q.4 (c) Solution:

(i) Consider series RLC resonant circuit,



The voltage across inductor is V_L and is given by,

$$V_L = I\cdot(\omega L)$$

but $$I = \frac{V}{Z}$$

$$V_L = \frac{V(\omega L)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad ...(1)$$

Squaring equation (1),

$$V_L^2 = \frac{V^2\omega^2L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$V_L^2 = \frac{V^2\omega^2L^2}{\frac{R^2C^2\omega^2 + (\omega^2LC-1)^2}{\omega^2C^2}} = \frac{V^2\cdot\omega^4L^2C^2}{\omega^2R^2C^2 + (\omega^2LC-1)^2}$$

By differentiating V_L^2 with respect to ω and equating only numerator term to zero.

We have,

$$2\omega^2LC - \omega^2R^2C^2 - 2 = 0$$
$$\omega^2(2LC - R^2C^2) = 2$$
$$\omega^2 = \frac{2}{2LC - R^2C^2}$$
$$\omega = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}} \text{ rad/sec}$$

Therefore, the frequency 'f_L' at which inductor voltage V_L is maximum is given by

$$f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}}$$

$$\therefore \qquad f_L = \frac{1}{2\pi\sqrt{LC}\sqrt{1-\frac{R^2C}{2L}}}\text{Hz}$$

(ii) Given, $R = 50\ \Omega$, $L = 0.05$ H, $C = 20\ \mu$F, $V = 100$ V

Maximum voltage across inductor in series resonant circuit occurs at frequency given by

$$f_L = \frac{1}{2\pi\sqrt{LC}\sqrt{1-\frac{R^2C}{2L}}}$$

$$f_L = 225.07\text{ Hz}$$

The impedance of series resonant circuit at f_L is given by,

$$Z = R + j(X_L - X_C)$$

where,

$$X_L = (2\pi f_L)L = (2 \times \pi \times 225.07)(0.05) = 70.7\ \Omega$$

$$X_C = \frac{1}{(2\pi f_L C)} = \frac{1}{(2\pi\times 225.07)(20\times 10^{-6})} = 35.35\ \Omega$$

$$Z = 50 + j(70.7 - 35.35)$$

$$Z = 50 + j35.35\ \Omega$$

Total current in series circuit is,

$$I = \frac{V}{Z} = \frac{100}{50+j35.35} = 1.633\angle -35.26°\text{ Amp}$$

$$\therefore \text{ Voltage across inductor } = V_L = I(jX_L)$$

$$= [1.633\angle -35.26°][70.7\angle 90°]$$

$$V_L = 115.458\angle 54.73\text{ V}$$

Section B : Control Systems

Q.5 (a) Solution:

Given: From the characteristic equation

$$\therefore \qquad G(s)H(s) = \frac{K}{(1+s)(1.5+s)(2+s)}$$

Put $\qquad s = -1 + j\omega$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(1.5-1+j\omega)(2-1+j\omega)}$$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(0.5+j\omega)(1+j\omega)}$$

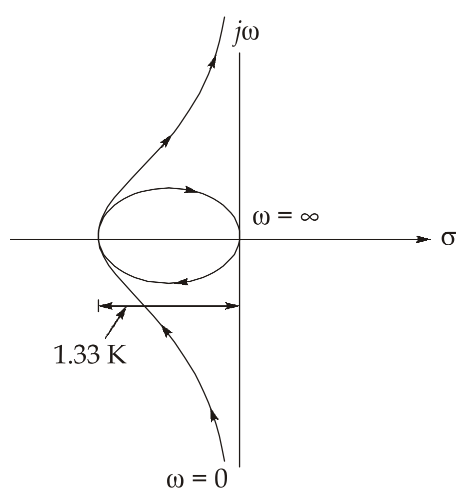
$$M = |GH(-1 + j\omega)| = \frac{K}{\omega\left(\sqrt{1+\omega^2}\right)\left(\sqrt{0.25+\omega^2}\right)}$$

$$M = \frac{2K}{\omega\left(\sqrt{1+4\omega^2}\right)\left(\sqrt{1+\omega^2}\right)}$$

Phase angle $\angle GH(-1 + j\omega) = \phi = -90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

ω	M	ϕ
0	∞	–90°
∞	0	–270°
0.5	$2.53K$	–161.56°
0.1	$19.5K$	–107°
1	$0.63K$	–198°
10	$9.94\times10^{-4}K$	–261°
0.707	$1.33K$	–180°

Nyquist plot for the system is given as



Now, the frequency at which phase is –180° is given by

$$-180° = -90° - \tan^{-1}\omega - \tan^{-1}(2\omega)$$

$$\frac{\omega+2\omega}{1-2\omega^2} = \frac{1}{0}$$

$$\omega = \frac{1}{\sqrt{2}} \text{rad/sec}$$

At $\omega = \frac{1}{\sqrt{2}}$ rad/sec, the value of M is given as,

$$M = \frac{2K}{\frac{1}{\sqrt{2}} \times \sqrt{1 + 4 \times \frac{1}{2}} \times \sqrt{1 + \frac{1}{2}}} = \frac{4}{3}\text{K}$$

From the Nyqusit criterion, $Z = P - N$

Here, Z = No. of closed-loop poles inside the right half of S = –1 plane.

P = No. of open-loop poles inside the right half of S = –1 plane.

N = No. of encirclement around S = –1.

As $P = 0$,

∴ For system to be stable, N should be zero.

For N to be zero, $1.33K < 1$

$$K < \frac{1}{1.33}$$

$$K < 0.75$$

Hence, K = 0.75 is the largest value of K.

Q.5 (b) Solution:

Given, $$G(s)H(s) = \frac{2e^{-0.5s}(0.125s + 1)}{s(0.5s + 1)}$$

We know that,

Phase margin, $$\text{PM} = 180° + \angle G(j\omega)H(j\omega)\Big|_{\omega = \omega_{gc}}$$

∴ $$\text{PM} = \frac{180° \times \pi}{180} + \left[-0.5\omega_{gc} + \tan^{-1}\left[0.125\omega_{gc}\right] - \frac{90° \times \pi}{180} - \tan^{-1}(0.5\omega_{gc})\right]$$

$$\text{PM} = \frac{\pi}{2} - 0.5\omega_{gc} + \tan^{-1}(0.125\omega_{gc}) - \tan^{-1}(0.5\omega_{gc})$$

In order to find maximum phase margin,

$$\frac{dPM}{d\omega_{gc}} = 0$$

$$0 - 0.5 + \frac{0.125}{1 + (0.125\omega_{gc})^2} - \frac{0.5}{1 + (0.5\omega_{gc})^2} = 0$$

Let $\omega_{gc}^2 = t$

$$\frac{8}{64+t} - \frac{2}{4+t} = 0$$

$$t^2 + 56t + 448 = 0$$

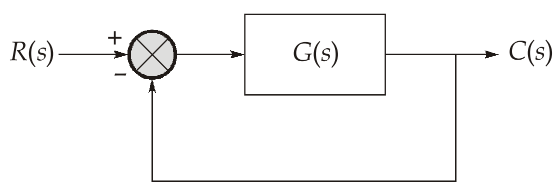
$$t = -9.66, -46.33$$

Here, ω_{gc} comes out to be imaginary for maximum phase margin.

∴ Maximum phase margin is NOT defined for given system.

Q.5 (c) Solution:

The given control system can be redrawn as,



where,
$$G(s) = \frac{\frac{k}{s(s+2)}}{1+\frac{k}{s(s+2)}\cdot\frac{5s}{k}} = \frac{\frac{k}{s(s+2)}}{1+\frac{5}{(s+2)}}$$

$$G(s) = \frac{k}{s(s+7)}$$

The steady state error due to input $tu(t)$ for type 1 system,

$$e_{ss} = \frac{1}{k_v}$$

where,

velocity error constant, $k_v = \underset{s\to 0}{lt}\ sG(s)$

Here, input given is $100t\,u(t)$.

∴ steady state error, $e_{ss} = \frac{100}{k_v}$

$$k_v = \underset{s\to 0}{lt}\ s\cdot\frac{k}{s(s+7)}$$

∴ $$k_v = \frac{k}{7}$$

steady state error due to $100t\,u(t)$, is 0.01

$$e_{ss} = \frac{100}{k_v} = 0.01$$

$$\therefore \quad \frac{100}{\left(\frac{k}{7}\right)} = 0.01$$

$$\Rightarrow \quad \frac{k}{7} = 10^4$$

$$\therefore \quad k = 7 \times 10^4$$

Q.5 (d) Solution:

We have peak overshoot with step input given as 5%.

i.e.,
$$\%M_p = 5\%$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.05$$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = \ln(0.05)$$

$$-3 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

$$\xi = 0.69$$

We have settling time for ±2% of tolerance as $T_s = 4$ sec.

i.e.,
$$T_s = \frac{4}{\xi\omega_n}$$

$$\xi\omega_n = \frac{4}{4} = 1$$

$$\omega_n = \frac{1}{\xi} = \frac{1}{0.69} = 1.44 \text{ rad/s}$$

Now, characteristic equation for given feedback system is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(\tau s + 1)} = 0$$

$$s(\tau s + 1) + K = 0$$

$$s^2\tau + s + K = 0$$

$$s^2 + \frac{1}{\tau}s + \frac{K}{\tau} = 0$$

On comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{\frac{K}{\tau}}$$

$$2\xi\omega_n = \frac{1}{\tau} \Rightarrow \tau = 0.5 \qquad \ldots\xi\omega_n = 1$$

$$\because \quad \omega_n = 1.44 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{K}{\tau}}$$

$$\Rightarrow \quad \omega_n^2 = \frac{K}{\tau}$$

$$K = \tau \cdot \omega_n^2$$

$$K = 0.5(1.44)^2$$

$$K = 1.03$$

$$K = 1.03 \cong 1$$

Q.5 (e) Solution:

Number of forward paths,

$$P_1 = G_1G_2; \quad P_2 = G_4; \quad P_3 = G_7G_8$$

$$P_4 = G_1G_5G_8; \quad P_5 = G_7G_6G_2$$

Number of individual loops,

$$L_1 = G_9; \quad L_2 = G_3; \quad L_3 = G_5G_6$$

There is one pair of two non touching loops.

$$L_1L_2 = G_9G_3$$

By Mason's Gain formula,

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{k} P_i\Delta_i}{\Delta}, \text{ where } k = \text{Number of forward paths.}$$

where,

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1L_2)$$

$$= 1 - (G_9 + G_3 + G_5G_6) + (G_3G_9)$$

$$= 1 - G_9 - G_3 - G_5G_6 + G_3G_9$$

$$\Delta_1 = 1 - L_1 = 1 - G_9$$

$$\Delta_2 = 1 - (L_1 + L_2 + L_3) + L_1L_2$$

$$= 1 - G_9 - G_3 - G_5G_6 + G_3G_9$$

$$\Delta_3 = 1 - L_2 = 1 - G_3$$

$$\Delta_4 = 1$$

$$\Delta_5 = 1$$

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{(G_1G_2)(1-G_9) + G_4[(1-G_9-G_3-G_5G_6)+(G_3G_9)] + (G_7G_8)(1-G_3) + (G_1G_5G_8)(1) + (G_7G_6G_2)(1)}{1-G_9-G_3-G_5G_6+(G_3G_9)}$$

Q.5 (f) Solution:

For type-1 system; $K_v = \lim_{s\to 0} s\cdot G(s)$

So, $G(s) = \lim_{s\to 0} s\cdot\frac{K(s+6)}{s(s+2)}$

$= 3K$

For input $r(t) = 2tu(t)$, the steady state error is given as:

$$e_{ss} = \frac{A}{K_v} \quad \text{...'}A\text{' is amplitude of input}$$

$$e_{ss} = \frac{2}{3K}$$

$\because$ $e_{ss} = 0.1$

$$0.1 = \frac{2}{3K}$$

$$3K = 20$$

$$K = \frac{20}{3} = 6.66$$

Q.6 (a) Solution:

(i) From the given figure,

It is clear, that there is a zero at origin due to which there is +20 dB/dec slope before ω = 1 rad/sec.

$\Rightarrow$ $G(s) = Ks$

Now, At ω = 1 rad/sec, slope changes to 0 dB/dec from +20 dB/dec. Hence, there is a pole at ω = 1 rad/sec.

$\therefore$ $G(s) = \frac{Ks}{(s+1)}$

⇒ At ω = 5 rad/sec, slope changes to –20 dB/dec from 0 dB/dec. Hence, there is a pole at ω = 5 rad/sec

$$\therefore \qquad G(s) = \frac{Ks}{(s+1)\left(\frac{s}{5}+1\right)}$$

⇒ At ω = 20 rad/sec, slope changes to –40 dB/dec from –20 dB/dec. Hence, there is a pole at ω = 20 rad/sec.

$$\therefore \qquad G(s) = \frac{Ks}{(s+1)\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

Now from the figure $30 = 20\log_{10}K$

$$K = 31.623$$

∴ Transfer function of the system

$$G(s) = \frac{31.623s}{(s+1)\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

(ii) For ω_{g1} ⇒

$$20 = \frac{30-0}{\log_{10}1-\log_{10}\omega_{g1}} = \frac{30}{-\log_{10}(\omega_{g1})}$$

$$\omega_{g1} = 0.03162 \text{ rad/sec}$$

For ω_{g2} ⇒

$$-40 = \frac{0-M_2}{\log_{10}(\omega_{g2})-\log_{10}20}$$

Where M_2 is the magnitude in dB at ω = 20 rad/sec.

The magnitude, M_2 at 20 rad/sec is obtained as

$$-20 = \frac{30-M_2}{\log_{10}(5)-\log_{10}(20)}$$

$$\therefore \qquad M_2 = 18 \text{ dB}$$

$$\therefore \qquad -40 = \frac{-18}{\log_{10}(\omega_{g2})-\log_{10}20}$$

$$\omega_{g2} = 56.234 \text{ rad/sec}$$

(iii) Gain crossover frequency $\omega_{gc_2} = 56.234$ rad/sec

$$\therefore \quad \text{Phase at } \omega_{gc_2} = \phi = +90° - \tan^{-1}(\omega_{gc}) - \tan^{-1}\left(\frac{\omega_{gc}}{5}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{20}\right)$$

$$\text{At} \quad \omega_{gc_2} = 56.234 \text{ rad/sec}$$

$$\phi = 90° - \tan^{-1}[56.234] - \tan^{-1}\left[\frac{56.234}{5}\right] - \tan^{-1}\left[\frac{56.234}{20}\right]$$

$$\phi = -154.32°$$

$$\text{Now,} \quad \text{phase margin PM} = 180° + \phi$$

$$= 180° - 154.32°$$

$$\text{PM} = 25.68°$$

Q.6 (b) Solution:

Consider the given open loop transfer function,

$$F(s) = \frac{K(s+b)}{(s+a_1)(s+a_2)}; K \geq 0$$

The pole-zero configuration is shown as

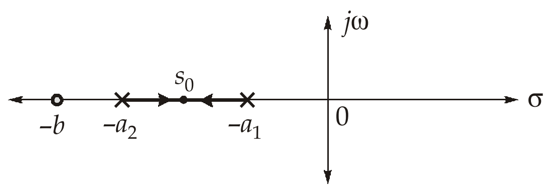


Fig. (a)

Now check for all the points in the s-plane for which the angle criterion is satisfied.

Let us examine any point s_0 in between the two poles. As this point is joined by phasors to the poles at $s = -a_1$ and $s = -a_2$, and the zero at $s = -b$, it is easily seen that

(i) the pole at $s = -a_1$ contributes an angle of 180°.

(ii) the pole at $s = -a_2$ contributes an angle of 0° and

(iii) the zero at $s = -b$ contributes an angle of 0°.

Therefore, the angle criterion is satisfied and the point s_0 is on the root locus.

To examine s-plane points not on the real axis, we consider a representative point $s = \sigma + j\omega$.

At this point,
$$F(s) = \frac{K(\sigma + j\omega + b)}{(\sigma + j\omega + a_1)(\sigma + j\omega + a_2)}$$

$$= \frac{K(\sigma + b + j\omega)}{(\sigma + a_1)(\sigma + a_2) + j\omega(\sigma + a_1 + \sigma + a_2) - \omega^2}$$

$$= \frac{K(\sigma + b + j\omega)}{(\sigma + a_1)(\sigma + a_2) - \omega^2 + j\omega(2\sigma + a_1 + a_2)}$$

$$\angle F(s) = \tan^{-1}\left(\frac{\omega}{\sigma + b}\right) - \tan^{-1}\left[\frac{\omega(2\sigma + a_1 + a_2)}{(\sigma + a_1)(\sigma + a_2) - \omega^2}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\omega}{\sigma + b} - \frac{\omega(2\sigma + a_1 + a_2)}{(\sigma + a_1)(\sigma + a_2) - \omega^2}}{1 + \frac{\omega}{\sigma + b}\left[\frac{\omega(2\sigma + a_1 + a_2)}{(\sigma + a_1)(\sigma + a_2) - \omega^2}\right]}\right]$$

$\angle F(s)$ is a multiple of 180° if,

$$\frac{\frac{\omega}{\sigma + b} - \frac{\omega(2\sigma + a_1 + a_2)}{(\sigma + a_1)(\sigma + a_2) - \omega^2}}{1 + \frac{\omega}{\sigma + b}\left[\frac{\omega(2\sigma + a_1 + a_2)}{(\sigma + a_1)(\sigma + a_2) - \omega^2}\right]} = 0$$

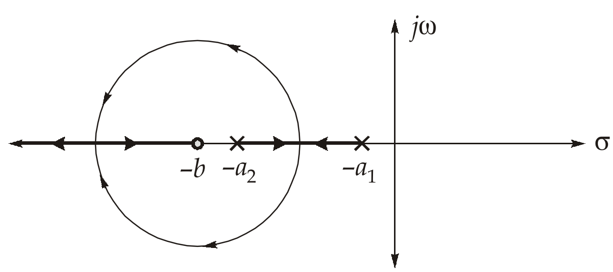
After manipulation of this equation, we get

$$(\sigma + b)^2 + \omega^2 = (b - a_1)(b - a_2)$$

This is the equation of a circle with centre at (–b, 0) and radius = $\sqrt{(b - a_1)(b - a_2)}$.

It can easily be verified that at every point on this circle in the s plane, $\angle F(s)$ is ±180°. Every point on the circle, therefore, satisfies the angle criterion.

The root locus plot is shown as



Q.6 (c) Solution:

For the given system assuming state transition matrix,

$$\phi(t) = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix}$$

We know, $$x(t) = \phi(t) \cdot x(0) \quad \text{...(i)}$$

Using above equation (i), we can write

$$\begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix} = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$e^{-t} = \phi_{11}(t) - 2\,\phi_{12}(t) \quad \text{...(ii)}$$

$$-2e^{-t} = \phi_{21}(t) - 2\,\phi_{22}(t) \quad \text{...(iii)}$$

and

$$\begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix} = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$e^{-2t} = \phi_{11}(t) - \phi_{12}(t) \quad \text{...(iv)}$$

$$-e^{-2t} = \phi_{21}(t) - \phi_{22}(t) \quad \text{...(v)}$$

Subtracting equation (iv) from equation (ii), we get

$$e^{-t} - e^{-2t} = -\phi_{12}(t)$$

$$\phi_{12}(t) = e^{-2t} - e^{-t}$$

Now using obtained value of $\phi_{12}(t)$ in equation (ii), we get

$$\phi_{11}(t) = e^{-t} + 2\left[e^{-2t} - e^{-t}\right]$$

$$= 2e^{-2t} - e^{-t}$$

Subtracting equation (v) from equation (iii), we get

$$-2e^{-t} + e^{-2t} = -\phi_{22}(t)$$

(or)

$$\phi_{22}(t) = 2e^{-t} - e^{-2t}$$

Now using obtained value of $\phi_{22}(t)$ in equation (v), we get

$$\phi_{21}(t) = \phi_{22}(t) - e^{-2t}$$

$$= 2e^{-t} - e^{-2t} - e^{-2t}$$

$$= 2e^{-t} - 2e^{-2t}$$

State-transition matrix, $\phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-t} & e^{-2t} - e^{-t} \\ 2e^{-t} - 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$

$$\phi(s) = \begin{bmatrix} \dfrac{2}{s+2} - \dfrac{1}{s+1} & \dfrac{1}{s+2} - \dfrac{1}{s+1} \\ \dfrac{2}{s+1} - \dfrac{2}{s+2} & \dfrac{2}{s+1} - \dfrac{1}{s+2} \end{bmatrix}$$

$$\phi(s) = \frac{1}{(s+2)(s+1)}\begin{bmatrix} 2(s+1)-(s+2) & (s+1)-(s+2) \\ 2(s+2)-2(s+1) & 2(s+2)-(s+1) \end{bmatrix}$$

$$= \frac{1}{(s+2)(s+1)}\begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} = [sI - A]^{-1}$$

$$= \frac{1}{\Delta}\begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

Matrix $[sI - A]$ can be identified as shown below using above result,

$$sI - A = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

Q.7 (a) Solution:

(i) We have; $$G(s)H(s) = \frac{K(s+2.5)}{s(s+2)}$$

The characteristic eqn. $\Rightarrow 1 + G(s).H(s) = 0$

$$s(s+2) + K(s+2.5) = 0$$

$$s^2 + 2s + Ks + 2.5K = 0$$

$$s^2 + s(K+2) + 2.5K = 0 \quad \text{...(1)}$$

Compare with standard 2nd order system's characteristic equation:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 2.5K, \qquad 2\xi\omega_n = K + 2$$

$\because$ $$\omega_n = \sqrt{10} \text{ rad/s} \qquad \xi = \frac{K+2}{2\omega_n} = \frac{4+2}{2\sqrt{10}}$$

$$10 = 2.5K \qquad \xi = 0.948$$

$$K = 4$$

1. Damping ratio $(\xi) = 0.948$

2. Damping factor $(\alpha) = \xi\omega_n = 3$

3. $r(t) = (5 + t)u(t)$

$$R(s) = \frac{5}{s} + \frac{1}{s^2} = \frac{5s+1}{s^2}$$

Now, steady state error;

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1 + G(s)\cdot H(s)} = \lim_{s\to 0} \frac{s\left[\frac{5s+1}{s^2}\right]}{1 + \frac{K(s+2.5)}{s(s+2)}}$$

$$e_{ss} = 0.2 \quad \text{... when } K = 4$$

(ii) **1.** **Rise time (t_r):** For underdamped systems, the rise time is normally defined as the time required for the step response to rise from 0 to 100% of its final value for the first time. For overdamped systems, the 10 to 90% rise time is commonly used.

2. **Peak time (t_p):** It is the time required for the response to reach to peak value or maximum value of the overshoot.

3. **Peak overshoot (m_p):** It is the maximum amount by which the response overshoots the steady-state value. It is common to use per cent peak overshoot given as:

$$\%m_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100\%$$

4. **Settling time (t_s):** The time required for the response to reach and stay within specified tolerance band usually 2%.

Q.7 (b) Solution:

(i) The system's characteristic equation can be written as,

$$1 + \left(\frac{s+\alpha}{s+4}\right)\left(\frac{K}{s(s+2)(s+4)}\right) = 0$$

$\therefore \quad s^4 + 10s^3 + 32s^2 + (K + 32)s + K\alpha = 0$

Routh array is formed below:

s^4	1	32	$K\alpha$
s^3	10	$K+32$	
s^2	$\frac{288-K}{10}$	$K\alpha$	
s^1	$\frac{(288-K)(K+32)-100K\alpha}{288-K}$		
s^0	$K\alpha$		

For overall system to be stable, following conditions should be satisfied.

$$K < 288$$

$$K\alpha > 0$$

$$(288 - K)(K + 32) - 100\,K\alpha > 0$$

(ii) Now we need to choose K and α to meet the required conditions.

So, if we choose $K = 200$

then, $$\alpha = \frac{88 \times 232}{100 \times 200} \cong 1$$

This result gives system's velocity error constant as

$$K_v = \lim_{s \to 0} sG_c(s) \cdot G(s)$$

$$= \frac{\alpha \cdot K}{4 \times 2 \times 4} = \frac{K\alpha}{32} = \frac{200 \times 1}{32} = \frac{25}{4}$$

$\therefore$ % velocity error for unit-ramp input,

$$= \frac{100}{K_v} = \frac{4}{25} \times 100 = 16$$ ... which is acceptable

$\therefore$ $K = 200$ and $\alpha = 1$ are suitable values.

Q.7 (c) Solution:

(i) The characteristic equation of the system is,

$$s^2 + 2s + 2 + K_c\left(1 + \frac{5}{s}\right)(s+3) = 0$$

or $s^3 + (2 + K_c)s^2 + (2 + 8K_c)s + 15K_c = 0$

Letting $s = \hat{s} - 2$, we have

$$(\hat{s} - 2)^3 + (2 + K_c)(\hat{s} - 2)^2 + (2 + 8K_c)(\hat{s} - 2) + 15K_c = 0$$

$$\hat{s}^3 + (K_c - 4)\hat{s}^2 + (4K_c + 6)\hat{s} + (3K_c - 4) = 0$$

Applying the Routh's criterion,

$\hat{s}^3$	1	$4K_c + 6$
$\hat{s}^2$	$K_c - 4$	$3K_c - 4$
$\hat{s}^1$	$\frac{4K_c^2 - 13K_c - 20}{K_c - 4}$	
$\hat{s}^0$	$3K_c - 4$	

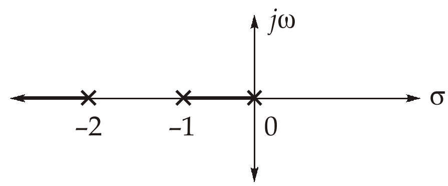
If there is no sign changes in the first column of the array, then all roots satisfy $\text{Re}(\hat{s}) < 0$, that is the roots of the original characteristic equation satisfy Re(s) < –2. Thus, we require that K_c satisfy all the following condition,

$$K_c > 4; \; K_c > 4.3892 \quad \text{or} \quad K_c < -1.1392; \; K_c > \frac{4}{3}$$

The requirement $K_c < -1.1392$ is disregarded since K_c cannot be negative. Therefore, we have $R_c(s) < -2$ for all closed-loop poles provided that $K_c > 4.3892$.

(ii) **Step 1:** There are three open loop poles i.e., $s = 0$, $s = -1$ and $s = -2$ and no open loop zero.

So pole-zero plot is as shown below,



Step 2: Number of asymptotes = $p - z = 3 - 0 = 3$

$$\text{Angle of asymptotes } \phi_A = \frac{(2K+1)180°}{p-z} \quad \text{where } K = 0, 1, ...p - z - 1$$

$$\therefore \qquad \phi_A = 60°, 180°, 300°$$

Step 3:
$$\text{Centroid } \sigma = \frac{\Sigma\,\text{poles} - \Sigma\,\text{zeros}}{p-z} = \frac{-1-2-0}{3} = -1$$

Step 4: The breakaway point can be found as solution of the equation $\frac{dK}{ds} = 0$

where, $$K = -[s(s+1)(s+2)]$$

∴ These solution are $s = -0.422$ and $s = -1.577$

but $s = -1.577$ do not lies on valid root loci so $s = -1.577$ is eliminated and $s = -0.422$ is valid breakaway point. At $s = -0.422$, $K = 0.38$

Step 5: The $j\omega$-axis crossover points is founded by RH array criteria.

$$1 + GH = 0 \quad \text{...characteristic equation}$$

$$s^3 + 3s^2 + 2s + K = 0$$

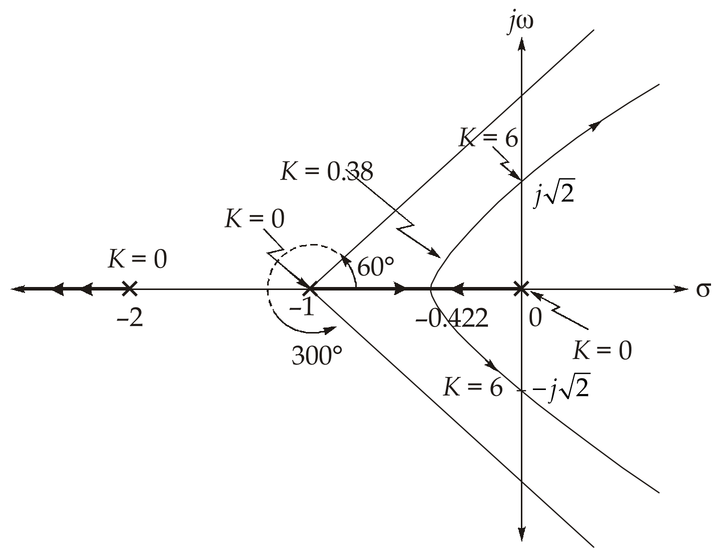
s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	0
s^0	K	

At $K = 6$, system is marginally stable and the intersection point of root locus with $j\omega$ axis is given by the auxiliary equation.

$$3s^2 + K = 0 \quad \Rightarrow \quad 3s2 = -6$$

$$\Rightarrow \qquad s = \pm j\sqrt{2}$$

Step 6: The root locus plot:



Q.8 (a) Solution:

(i) ∵ Given system is type-1 system. The error constants are obtained as follow:

$$K_p = \lim_{s\to 0} G(s) = \infty$$

$$K_v = \lim_{s\to 0} s.G(s) = 1$$

$$K_a = \lim_{s\to 0} s^2.G(s) = 0$$

Now, steady state error (e_{ss}):

For $r(t) = 2u(t) \quad \Rightarrow \quad e_{ss} = \dfrac{A}{1+K_p}$ ...'A' is amplitude of input.

$$e_{ss} = \frac{2}{1+\infty} = 0$$

For $r(t) = 5t\cdot u(t) \quad \Rightarrow \quad e_{ss} = \dfrac{A}{K_v} = \dfrac{5}{1} = 5$

For $r(t) = 5t^2\cdot u(t) = 10\left(\dfrac{t^2}{2}\right)u(t)$

$$\Rightarrow \qquad e_{ss} = \frac{A}{K_a} = \frac{10}{0} = \infty$$

(ii) The characteristic equation is given as:

$$1 + G(s)\times H(s) = 0$$
$$s(0.5s + 1)(0.2s + 1) + 1 = 0$$
$$s^3 + 7s^2 + 10s + 10 = 0$$
$$(s + 5.52)(s^2 + 1.484s + 1.81) = 0$$

$\because$ The magnitude of the real root is more than 7 times the magnitude of the real part of the complex roots. Therefore, the complex-conjugate root pair gives the dominant closed loop poles with

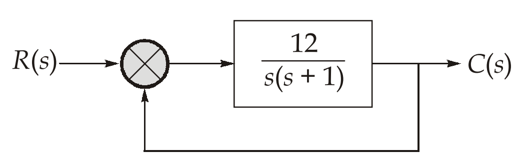
$$\omega_n = \sqrt{1.81} = 1.35 \text{ rad/s}$$
$$\xi = \frac{1.484}{2\times 1.34} = 0.55$$
$$\omega_d = \omega_n\sqrt{1-\xi^2} = 1.35\sqrt{1-(0.55)^2} = 1.12 \text{ rad/s}$$

Now, Rise time $(t_r) = \dfrac{\pi - \cos^{-1}(\xi)}{\omega_d} = \dfrac{\pi - 0.988}{1.12} = 1.92$ sec

- Peak time $(t_p) = \dfrac{\pi}{\omega_d} = 2.79$ sec
- Peak overshoot $(m_p) = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.1265$
- Settling time $(\pm 2\%) \Rightarrow T_s = \dfrac{4}{\xi\omega_n} = \dfrac{4\times 2}{1.484} = 5.39$ sec

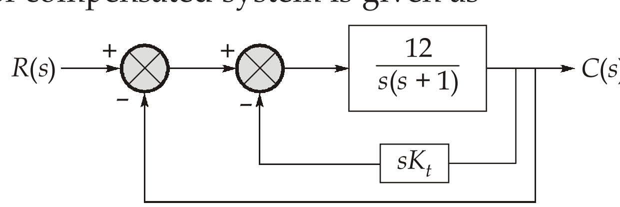
Q.8 (b) Solution:

The block diagram of uncompensated system is shown below:



Now the system is to be compensated by tachometer feedback.

$\therefore$ Block diagram of compensated system is given as



Now, the overall transfer function of the compensated system is given as

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\dfrac{12}{s(s+1)}}{1+\dfrac{12sK_t}{s(s+1)}}}{1+\dfrac{\dfrac{12}{s(s+1)}}{1+\dfrac{12sK_t}{s(s+1)}}} = \frac{\dfrac{12}{s^2+s+12sK_t}}{1+\dfrac{12}{s^2+s+12sK_t}}$$

$$\frac{C(s)}{R(s)} = \frac{12}{s^2+s+12sK_t+12}$$

The characteristic equation for the compensated system is

$$s^2 + s(1 + 12K_t) + 12 = 0$$

Comparing it with standard 2nd order characteristic equation, we get,

$$\omega_n = \sqrt{12} = 3.46 \text{ rad/sec}$$

$$2\xi\omega_n = 1 + 12\,K_t \qquad \text{...(1)}$$

Now given, first peak undershoot is 6.5%.

$$\frac{6.5}{100} = e^{\frac{-2\xi\pi}{\sqrt{1-\xi^2}}}$$

After solving we get, $\xi = 0.4$

Put in equation (1),

$$2 \times 0.4 \times 3.46 = 1 + 12\,K_t$$

$$K_t = 0.147$$

Q.8 (c) Solution:

(i) Using the open-loop poles and zeros, we represent the open-loop system whose root locus is given as

$$G(s)H(s) = \frac{k(s-3)(s-5)}{(s+1)(s+2)} = \frac{k(s^2-8s+15)}{(s^2+3s+2)}$$

But for all points along the root locus,

$kG(s)H(s) = -1$, and along the real axis, $s = \sigma$

Hence, $$\frac{k(\sigma^2-8\sigma+15)}{(\sigma^2+3\sigma+2)} = -1$$

$$k = \frac{-(\sigma^2+3\sigma+2)}{(\sigma^2-8\sigma+15)}$$

for break points, $\frac{dk}{d\sigma} = 0$

$$\frac{dk}{d\sigma} = \frac{-\left[(2\sigma+3)(\sigma^2-8\sigma+15)-(\sigma^2+3\sigma+2)(2\sigma-8)\right]}{(\sigma^2-8\sigma+15)^2} = 0$$

$$\frac{dk}{d\sigma} = \frac{11\sigma^2-26\sigma-61}{(\sigma^2-8\sigma+15)^2} = 0$$

$$11\sigma^2 - 26\sigma - 61 = 0$$

$$\sigma = \frac{26 \pm \sqrt{26^2+4\times 61\times 11}}{2\times 11}$$

$\therefore$ $\sigma = -1.45;\ 3.82$

$\therefore$ The breakaway point is,

$$\sigma_p = -1.45$$

The break-in point, $\sigma_z = 3.82$

(ii) Given system involves one integrator and two delay integrators. The output of each integrator or delayed integrator can be a state variable. Let us define the output of the plant as x_1, the output of the controller as x_2 and output of the sensor as x_3. Then we obtain,

$$\frac{X_1(s)}{X_2(s)} = \frac{10}{s+5}$$

$$\frac{X_2(s)}{U(s)-X_3(s)} = \frac{1}{s}$$

$$\frac{X_3(s)}{X_1(s)} = \frac{1}{s+1}$$

$$Y(s) = X_1(s)$$

which can be rewritten as

$$sX_1(s) = -5X_1(s) + 10X_2(s) \quad \text{...(i)}$$

$$sX_2(s) = -X_3(s) + U(s) \quad \text{...(ii)}$$

$$sX_3(s) = X_1(s) - X_3(s) \quad \text{...(iii)}$$

$$Y(s) = X_1(s) \quad \text{...(iv)}$$

by taking the inverse Laplace transform of the above equations (i), (ii), (iii), (iv)

$$\dot{x}_1 = -5x_1 + 10x_2$$

$$\dot{x}_2 = -x_3 + u$$

$$\dot{x}_3 = x_1 - x_3$$

$$y = x_1$$

Thus, the state space model of the system in the standard form is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 10 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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