MADE EASY

India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2023 Mains Test Series

Mechanical Engineering Test No : 1

Section A : Thermodynamics [All Topics]

Section B : Strength of Materials and Mechanics [All Topics]

Section : A

1. (a) (i)

Given:

The piston-cylinder device is frictionless.

Initial pressure inside the cylinder,

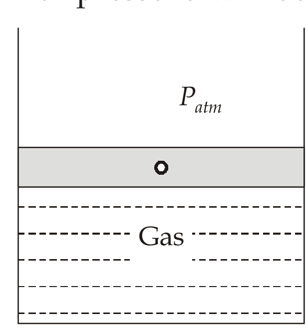
$$P_1 = 150 \text{ kPa}$$

Mass of piston, $m = 10$ kg

Diameter of piston, $d = 10$ cm $= 0.1$ m

Mass of one brick, $M = 16$ kg

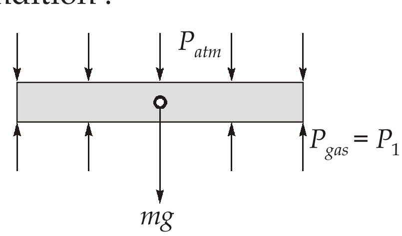
Final pressure inside the cylinder is increased by 200%, so the final pressure will be three times the initial pressure,



$$P_2 = 150 \times 3 = 450 \text{ kPa}$$

Mass of one brick, $M = 16$ kg

FBD of piston at initial condition :



**Updated Solutions on 12.04.2023*

At equilibrium condition,

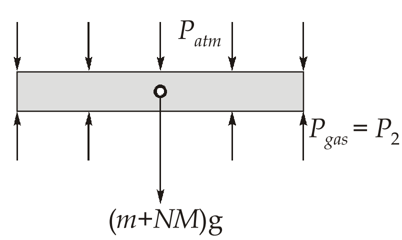
$$P_{atm} \cdot A + mg = P_1 A \qquad \text{where } A \text{ is area of piston}$$

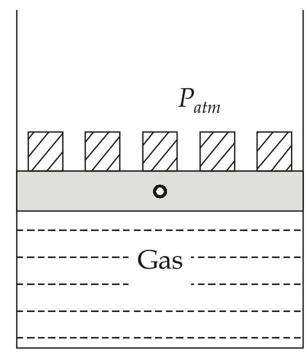
$$P_{atm} + \frac{mg}{A} = P_1$$

$$P_{atm} + \frac{mg}{\frac{\pi}{4} \times (0.1)^2 \times 1000} = P_1$$

$$\Rightarrow \qquad P_{atm} = 150 - \frac{10 \times 9.81}{\frac{\pi}{4} \times (0.1)^2 \times 1000} = 137.51 \text{ kPa}$$

Now, when the pressure is increased by 200%, by placing 'N' bricks, forces on the piston are:





At equilibrium condition :

$$P_{atm} A + (m + NM)g + = P_2 A$$

$$P_{atm} + \frac{(m + NM)g}{A} = P_2$$

$$P_{atm} + \frac{(m + NM)g}{\frac{\pi}{4} \times (0.1)^2 \times 1000} = P_2$$

$$\Rightarrow \quad 137.51 + \frac{(10 + N \times 16) \times 9.81}{\frac{\pi}{4} \times (0.1)^2 \times 1000} = 450$$

$$\Rightarrow \qquad N = 15.011 \simeq 15 \text{ bricks}$$

(ii)

Given:

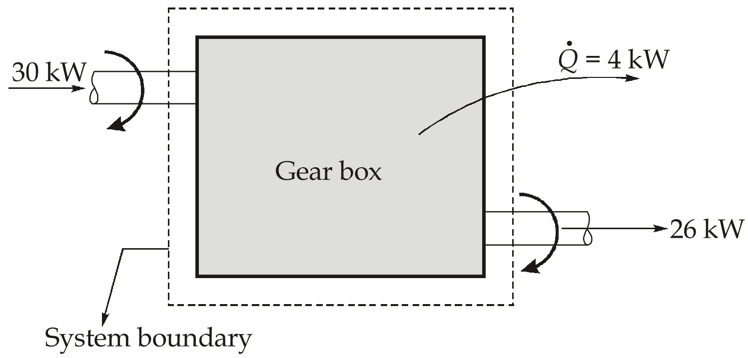
Input power = 30 kW

Output power = 26 kW

Surface temperature of gear box,

$$T_s = 55°C = 328 \text{ K}$$

Surrounding temperature, T_0 = 27°C = 300 K



From conservation of energy,

$$\frac{dE}{dt} = \dot{Q} + \dot{W}$$

$$\Rightarrow \qquad \dot{Q} = -\dot{W} \qquad \left[\text{Since, } \frac{dE}{dt} = 0\right]$$

$$\Rightarrow \qquad \dot{Q} = -(26 - 30) = 4 \text{ kW}$$

Since the gear box is operating under steady state conditions, so for a closed system the entropy rate balance would be

$$\frac{dS}{dt} = \Sigma\frac{\dot{Q}}{T} + \delta\dot{S}_{gen}$$

Here, heat transfer takes place only at temperature T_s so, we get

$$0 = \frac{-4}{328} + \delta\dot{S}_{gen}$$

$$\Rightarrow \qquad \delta\dot{S}_{gen} = \frac{4}{328} = 0.01219 \text{ kW/K} = 12.19 \text{ W/K}$$

Note : The value of entropy production rate calculated above signifies the irreversibility associated with friction and heat transfer within the gear box.

If we take immediate surrounding as the boundary i.e., at temperature 300 K an additional source of irreversibility is included in the enlarged system, namely the irreversibility associated with the heat transfer from the outer surface of the gear box at T_s to the surrounding at T_0.

1. (b)

Given :

Initial pressure, P_1 = 500 kPa

Initial temperature, T_1 = 727°C = 1000 K

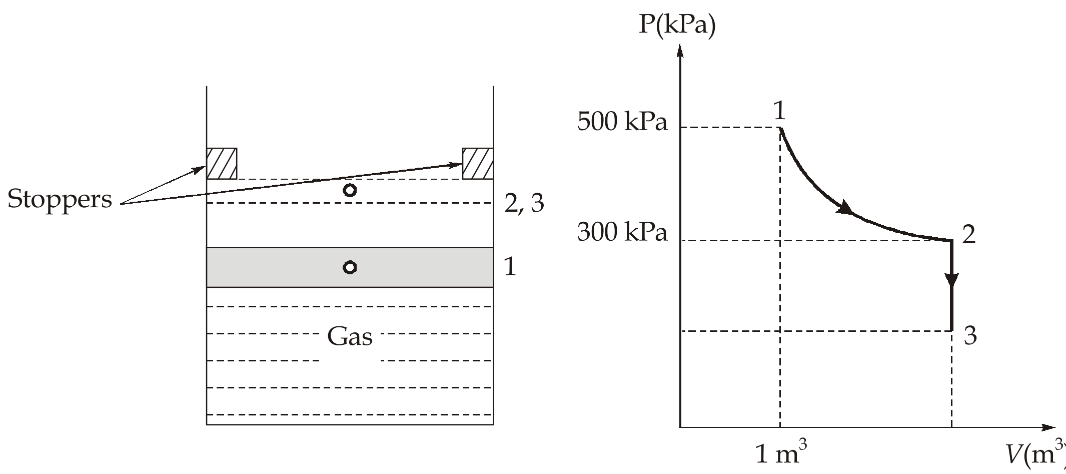
Initial volume, V_1 = 1 m^3

Process (1 - 2) is an isothermal process

Intermediate pressure, P_2 = 300 kPa

Final temperature, T_3 = 27°C = 300 K

The figure below shows the Schematic and *P-V* diagram for the given system,



Assumptions:

1. Air is assumed to be an ideal gas.
2. The kinetic energy and potential energy are neglected.
3. The piston-cylinder arrangement is frictionless.

Considering air inside the cylinder as system and applying energy balance equation for closed system:

$$\delta Q = \delta U + \delta W \qquad \text{...(i)}$$

For process 1 - 2 (Isothermal process)

$$T_1 = T_2$$

$$\Rightarrow \qquad \delta Q = mC_V(T_2 - T_1) + P_1V_1 \ln\left(\frac{P_1}{P_2}\right)$$

$$\Rightarrow \qquad \delta Q = 0 + 500 \times 1 \times \ln\left(\frac{500}{300}\right) = 255.41 \text{ kJ}$$

Mass of the air in the cylinder,

$$m = \frac{P_1 V_1}{R T_1} = \frac{500 \times 1}{0.287 \times 1000} = 1.742 \text{ kg}$$

For process (2 - 3), the process is isochoric i.e., constant volume.

$$(\delta Q)_V = mC_V (T_3 - T_2)$$

$$= 1.742 \times 0.718\,(300 - 1000)$$

$$(\delta Q)_V = -875.61 \text{ kJ}$$

Hence, for the combined process net heat transfer will be

$$\delta Q = 255.41 - 875.61 = -620.2 \text{ kJ}$$

Here, negative sign of the net heat transfer indicates that heat flows out of the system boundary,

or, $$(\delta Q)_{out} = 620.2 \text{ kJ}$$

1. (c)

Given :

Initial pressure, P_1 = 10 bar

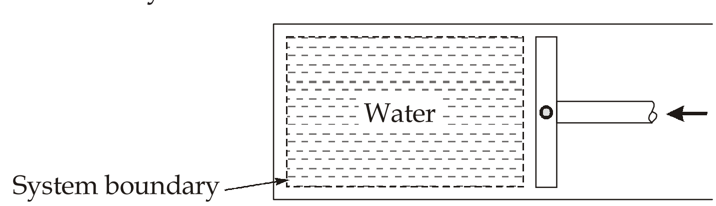
Initial temperature, T_1 = 500°C

Process (1 - 2) is isobaric

Process (2 - 3) is isochoric

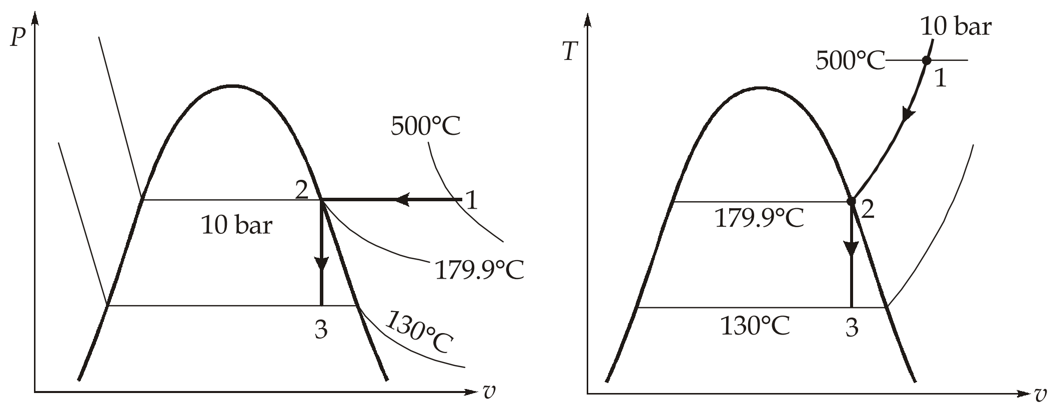
Assumptions:

1. The changes in kinetic energy and potential energy are negligible.
2. Water contained in a piston-cylinder device forms a closed system.
3. The piston is the only mode of work transfer.



(i)

The processes are shown in P-v and T-v diagrams as below:



Since at state 1, T_1 = 500°C which is greater than saturation temperature corresponding to P_1 = 10 bar i.e., 179.9°C so, state 1 is located in the superheated region.

(ii)

For the overall process;

$$\delta W = (\delta W)_{1\text{-}2} + (\delta W)_{2\text{-}3}$$

$$= \int_1^2 pdV + 0 \qquad (\because \text{ Process } (2-3) \text{ is isochoric})$$

$$\Rightarrow \qquad \delta W = P\int_1^2 dV = P(V_2 - V_1) \qquad (\because \text{ Process } (1-2) \text{ is isobaric})$$

$$\Rightarrow \qquad \frac{\delta W}{m} = P(v_2 - v_1)$$

From steam table; At P = 10 bar; T_1 = 500°C; v_1 = 0.3541 m^3/kg; u_1 = 3125 kJ/kg;
and At P = 10 bar; T_2 = 179.9°C; v_2 = 0.1944 m^3/kg

$$\therefore \qquad \frac{\delta W}{m} = 10 \times 100 \times (0.1944 - 0.3541)$$

$$= -159.7 \text{ kJ/kg}$$

The negative sign indicates that work is done on the system.

(iii)

For the overall process,

$$\delta Q = \delta U + \delta W$$

$$\Rightarrow \qquad \frac{\delta Q}{m} = (u_3 - u_1) + \frac{\delta W}{m} \qquad \text{...(i)}$$

Since state 3 is in the wet region so, to get specific internal energy u_3 we first need to find dryness fraction at this state.

At state 3; T_3 = 130°C and $v_3 = v_2$ = 0.1944 m^3/kg

So, at 130°C; v_f = 0.00106971 m^3/kg

v_g = 0.668 m^3/kg

$\because \quad v_3 = v_f + x_3(v_g - v_f)$

$\Rightarrow \quad 0.1944 = 0.00106971 + x_3 \times (0.668 - 0.00106971)$

$\Rightarrow \quad x_3 = 0.289 \simeq 0.29$

$\therefore \quad u_3 = u_f + x_3(u_g - u_f)$

$= 546.09 + 0.29 \times (2539.6 - 546.09)$

$= 1124.208$ kJ/kg

Substituting values in equation (i), we get

$$\frac{\delta Q}{m} = (1124.208 - 3125) + (-159.7)$$

$$= -2160.492 \text{ kJ/kg}$$

The negative sign shows that heat is transferred out from the system.

1. (d)

Given: Initial volume, V_1 = 2500 cm^3 = 2500 × 10^{-6} m^3

Initial pressure, P_1 = 6 bar

Initial temperature; T_1 = 1200 K

Final pressure, P_0 = 1.013 bar

Final temperature, T_0 = 27°C = 300 K

Assumptions:

1. The gaseous combustion products are assumed to be air i.e., ideal gas.
2. Closed system.
3. The effects of gravity and motion is neglected.

For non-flow gas (i.e., gas contained in a container or piston cylinder arrangement),

$$\text{A.E. or exergy} = (U - U_0) - T_0(S - S_0) + P_0(V - V_0)$$

$$\text{Specific exergy} = (u_1 - u_0) - T_0(s_1 - s_0) + P_0(v_1 - v_0) \qquad \text{...(i)}$$

$\therefore$ $$u_1 - u_0 = C_V(T_1 - T_0) = 0.718 \times (1200 - 300)$$
$$= 646.2 \text{ kJ/kg} \quad ...(ii)$$

$$s_1 - s_0 = C_P \ln\frac{T_1}{T_0} - R\ln\frac{P_1}{P_0}$$

$$= 1.005 \times \ln\left(\frac{1200}{300}\right) - \frac{8.314}{29}\ln\left(\frac{6}{1.013}\right) = 0.8832 \text{ kJ/kgK}$$

$\therefore$ $$T_0(s_1 - s_0) = 300 \times (0.8832)$$
$$= 264.974 \text{ kJ/kg} \quad ...(iii)$$

$$P_0(v_1 - v_0) = R\left(\frac{P_0 T_1}{P_1} - T_0\right) \qquad (\because\ Pv = RT)$$

$$= 0.287\left(\frac{1.013 \times 1200}{6} - 300\right) = -27.9538 \text{ kJ/kg} \quad ...(iv)$$

Substituting the values obtained in (ii), (iii) and (iv) in equation (i), we get

$$\frac{A.E.}{m} = 646.2 - 264.976 - 27.9538$$
$$= 353.27 \text{ kJ/kg}$$

Hence, the specific exergy of the gas is 353.27 kJ/kg.

1. (e)

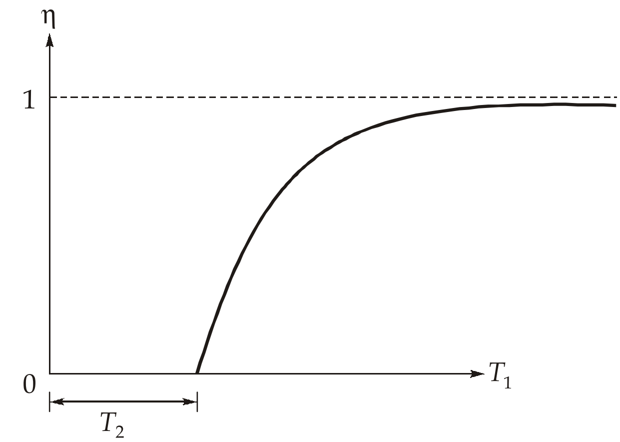
(i)

The efficiency of Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

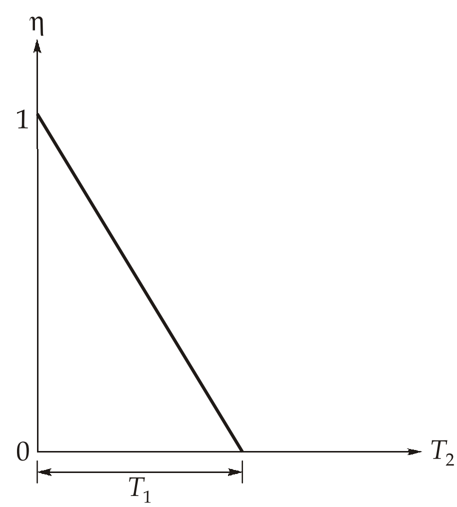
1. If T_2 is constant; $$\left(\frac{\partial \eta}{\partial T_1}\right)_{T_2} = \frac{T_2}{T_1^2} \quad ...(i)$$

As T_1 increases; η increases and the slope $\left(\frac{\partial \eta}{\partial T_1}\right)_{T_2}$ decreases, as shown in figure below,



2. If T_1 is constant; $\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} = \frac{-1}{T_1}$ ...(ii)

As T_2 decreases; η increases and slope $\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1}$ remains constant, as shown in figure below,



From equation (i) and (ii), it is clear that

$$\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} > \left(\frac{\partial \eta}{\partial T_1}\right)_{T_2} \qquad (\text{as } T_1 > T_2)$$

So, more effective way to increase the efficiency is to decrease T_2 keeping T_1 as constant.

Alternatively,

1. If we increase T_1 by ΔT and keep T_2 as constant,

$$\eta_I = 1 - \frac{T_2}{T_1 + \Delta T} = \frac{(T_1 - T_2) + \Delta T}{(T_1 + \Delta T)} \quad \text{...(iii)}$$

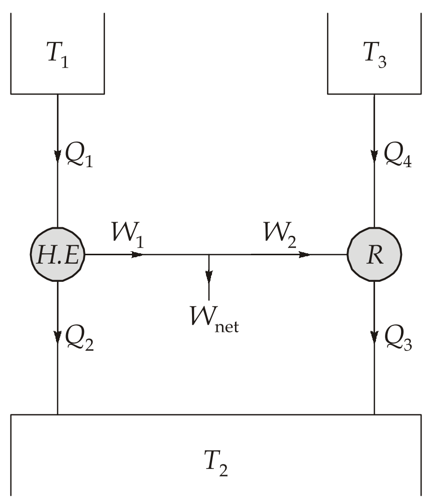
2. If we decrease T_2 by ΔT and keep T_1 as constant.

$$\eta_{II} = 1 - \frac{T_2 - \Delta T}{T_1} = \frac{(T_1 - T_2) + \Delta T}{T_1} \quad \text{...(iv)}$$

It can be clearly seen from equation (iii) and (iv) that $\eta_{II} > \eta_I$

Hence, the more effective way to increase the efficiency of Carnot engine is to decrease T_2 keeping T_1 as constant.

(ii)



Given: T_1 = 500°C = 773 K; T_2 = 30°C = 303 K; T_3 = –10°C = 263 K; Q_1 = 2200 kJ;

$W_{net} = W_1 - W_2$ = 400 kJ

We know that

$$\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{303}{773} = 0.608$$

Also, $$\eta_{carnot} = \frac{W_1}{Q_1} = 0.608$$

$\Rightarrow$ $$W_1 = 0.608 Q_1 = 0.608 \times 2200 = 1337.645 \text{ kJ}$$

Maximum COP of refrigerator:

$$(COP)_R = \frac{T_3}{T_2 - T_3} = \frac{263}{303 - 263} = 6.575$$

or, $$(COP)_R = \frac{Q_4}{W_2} = 6.575$$

$\Rightarrow$ $$Q_4 = 6.575\,W_2$$

Also, $$W_1 - W_2 = 400$$

$\Rightarrow$ $$W_2 = 1337.645 - 400 = 937.645 \text{ kJ}$$

$\therefore$ $$Q_4 = 937.645 \times 6.575 = 6165.016 \text{ kJ}$$

Now, $$Q_3 = Q_4 + W_2 = 6165.016 + 937.645$$

$$Q_3 = 7102.66 \text{ kJ}$$

and $$Q_2 = Q_1 - W_1 = 2200 - 1337.645$$

$$= 862.355 \text{ kJ}$$

Hence net heat transfer to the reservoir at 30°C is

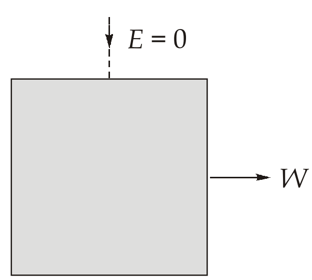
$$Q_3 + Q_2 = 7102.66 + 862.355$$

$$= 7965.015 \text{ kJ}$$

2. (a)

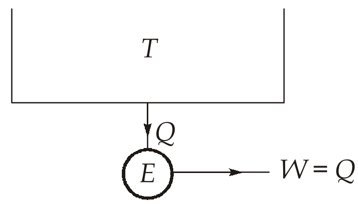
(i)

PMM1 (Perpetual motion machine of the first kind) : The concept of a machine which continuously supply work without some other form of energy disappearing simultaneously is called PMM1.



According to first law of thermodynamics, energy can neither be created nor destroyed. It only gets transformed from one form to another PMM1 violates the first law of thermodynamics in the sense that the internal energy could be used to produce some amount of work but there is a limit of internal energy and once it gets consumed then work production will stop. If further work is developed, it means there is energy creation which violates first law of thermodynamics. So, PMM1 is not possible.

PMM2 (Perpetual motion machine of the second kind): The concept of a machine which produces net work in a complete cycle by exchanging heat with only one reservoir.



$$\eta = \frac{W}{Q} = 1 = 100\%$$

The PMM2 is not possible because according to Kelvin-Planck statement of second law, it is impossible for a heat engine to produce net work in a cycle by exchanging heat with only one reservoir.

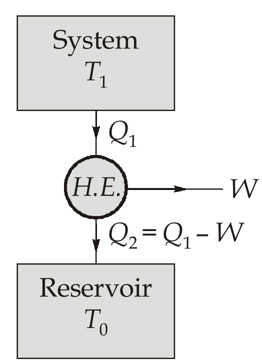
For PMM2 the efficiency would be 100% and it violates the Kelvin-Planck statement of second law.

Experience shows that $W < Q$, since heat Q transferred to a system cannot be completely converted to work in a cycle. Therefore, efficiency η is less than unity. A heat engine can never be 100% efficient. So, there is always a heat rejection. So, to produce net work in a cycle, a heat engine has to exchange heat with atleast two reservoirs.

(ii)

Given:

System is finite mass



$$\text{Heat capacity, } C_V = BT^3$$

$$B = 6 \times 10^{-5}\ \text{J/K}^4$$

$$T_1 = 300\ \text{K}$$

$$T_0 = 100\ \text{K}$$

$$Q_1 = \int_{T_1}^{T_2} C_V dT = \int_{T_1}^{T_2} BT^3 dT$$

$$= \int_{300}^{100} 6 \times 10^{-5} \times T^3 dT$$

$$= 6 \times 10^{-5} \left[\frac{T^4}{4}\right]_{300}^{100} = -120 \times 10^3\ \text{J}$$

Negative sign implies that heat is removed from the system.

Entropy change of the universe;

$$(\Delta S)_{universe} = (\Delta S)_{sys.} + (\Delta S)_{HE} + (\Delta S)_{resv.} \quad \text{...(i)}$$

$$(\Delta S)_{sys.} = \int_{300}^{100} C_V \frac{dT}{T} = \int_{300}^{100} 6 \times 10^{-5} \times T^3 \frac{dT}{T}$$

$$= 6 \times 10^{-5} \times \left[\frac{T^3}{3}\right]_{300}^{100} = -520 \text{ J/K}$$

$$(\Delta S)_{res.} = \frac{Q_2}{T_0} = \frac{Q_1 - W}{T_0} = \frac{120 \times 10^3 - W}{100}$$

$$(\Delta S)_{HE} = 0 \quad \text{(Cyclic device)}$$

∴ From equation (i),

$$(\Delta S)_{universe} = -520 + 0 + \frac{120 \times 10^3 - W}{100}$$

Since, the entropy of universe can never be less than zero. So, $(\Delta S)_{universe} \geq 0$

$$-520 + \frac{120 \times 10^3 - W}{100} \geq 0$$

$$\Rightarrow \quad \frac{120 \times 10^3 - W}{100} \geq 520$$

$$\Rightarrow \quad -W \geq 52 \times 10^3 - 120 \times 10^3$$

$$\Rightarrow \quad W \leq (120 - 52) \times 10^3$$

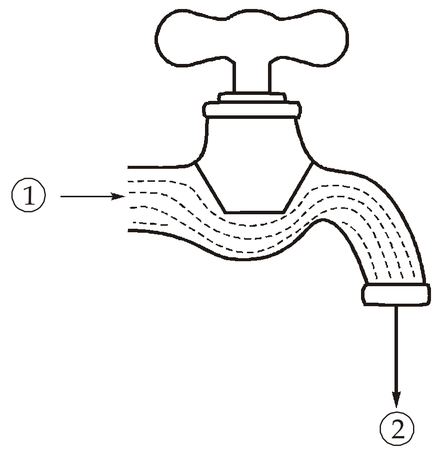
$$\Rightarrow \quad W_{max} = 68 \text{ kJ}$$

2. (b)

(i)

When a fluid flows through a constricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure and the flow said to be throttled.

Consider an insulated pipe containing a partially opened valve.



According to SFEE:

$$h_1 + \frac{V_1^2}{2} + gz_1 + q = h_2 + \frac{V_2^2}{2} + gz_2 + w \quad \text{...(i)}$$

Here, $q = 0$ ($\because$ the pipe is insulated)

Also, $w = 0$

The changes in potential energy are ignored and the fluid velocities in pipe in case of throttling is very low so can be neglected.

Hence, from equation (i)

$$h_1 = h_2$$

So, we can say the enthalpy of fluid before throttling is equal to the enthalpy of the fluid after throttling.

(ii)

Given:

Volume flow rate of water = 1 m^3/min = $\frac{1}{60} m^3/s$.

Inlet diameter, d_1 = 10 cm = 0.1 m

Outlet diameter, d_2 = 3 cm = 0.03 m

Potential head; $z_2 - z_1$ = 12 m

Change in temperature and pressure neglected,

$$\dot{Q} = 0.08\dot{W}_{C.V.}$$

Assumptions:

1. The pump is working in a steady state condition.
2. For water, density (ρ) = 1000 kg/m^3

Mass flow rate through the control volume i.e., pump,

$$\dot{m} = \rho Q = 1000 \times \frac{1}{60} = 16.67 \text{ kg/s}$$

$$\text{Inlet velocity; } V_1 = \frac{\dot{m}}{\rho A_1} = \frac{16.67}{1000 \times \frac{\pi}{4} \times (0.1)^2} = 2.122 \text{ m/s}$$

$$\text{Outlet velocity; } V_2 = \frac{\dot{m}}{\rho A_2} = \frac{16.67}{1000 \times \frac{\pi}{4} \times (0.03)^2} = 23.58 \text{ m/s}$$

Applying steady flow energy equation for the control volume;

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{C.V.}$$

$$\Rightarrow \dot{m}\left((h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right) = \dot{W}_{C.V.} - \dot{Q}$$

As both temperature and pressure are constant,

$$h_1 = h_2$$

$$\therefore 16.67\left\{0 + \frac{(2.122)^2 - (23.58)^2}{2} + 9.81 \times (-12)\right\} = \dot{W}_{C.V} - 0.08\dot{W}_{C.V}$$

$$\Rightarrow \frac{16.67}{0.92}\{-393.476\} = \dot{W}_{C.V.}$$

$$\Rightarrow \dot{W}_{C.V.} = -7129.627 = -7.129 \text{ kW}$$

The negative sign indicates that power is provided to the pump.

2. (c)

(i)

Given: $$P = \frac{RT}{v-b} - \frac{a}{Tv^2}$$

$$\Rightarrow \frac{RT}{v-b} = P + \frac{a}{Tv^2}$$

$$\Rightarrow RT = \left(P + \frac{a}{Tv^2}\right)(v-b)$$

$$\Rightarrow \quad \frac{RT}{P} = \left((v-b) + \frac{av}{PTv^2} - \frac{ab}{PTv^2}\right)$$

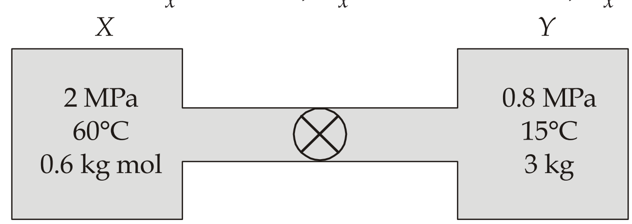
$$\Rightarrow \quad \frac{RT}{P} = (v-b) + \frac{a}{PTv} - \frac{ab}{PTv^2}$$

$$\Rightarrow \quad (RT - Pv) = -Pb + \frac{a}{vT} - \frac{ab}{v^2T}$$

$$\therefore \quad \lim_{\substack{P\to 0\\ T\to\infty}} (RT - Pv) = 0 + 0 + 0 = 0$$

(ii)

Given: $P_x = 2$ MPa; $T_x = 60°C = 333$ K; $n_x = 0.6$ kg mol



According to ideal gas equation:

$$P_x V_x = n_x \bar{R}_x T_x$$

$$\Rightarrow \quad 2000 \times V_x = 0.6 \times 8.314 \times 333$$

$$\Rightarrow \quad V_x = 0.83 \text{ m}^3$$

The mass present in vessel X;

$$m_x = n_x M_x$$

$$= 0.6 \times 32 = 19.2 \text{ kg}$$

Characteristic gas constant of oxygen;

$$R_y = \frac{8.314}{32} = 0.2598 \text{ kJ/kg-K}$$

For vessel Y:

$$P_y V_y = m_y R_y T_y$$

$$\Rightarrow \quad 0.8 \times 1000 \times V_y = 3 \times 0.2598 \times (15 + 273)$$

$$\Rightarrow \quad V_y = 0.281 \text{ m}^3$$

$\therefore$ Total volume of X and Y ;

$$V_T = V_x + V_y$$

$$= 0.83 + 0.28$$

$$= 1.111 \text{ m}^3$$

Total mass of oxygen gas = $m_x + m_y$

$\Rightarrow$ $m_T = 19.2 + 3 = 22.2$ kg

Final temperature; $T_f = 30°C = 303$ K

Applying perfect gas equation;

$$PV = mRT$$

$\Rightarrow$ $P \times 1.111 = 22.2 \times 0.2598 \times 303$

$\Rightarrow$ $P = 1572.97$ kPa $= 1.57$ MPa

Now, specific heat at constant volume;

$$C_V = \frac{R}{\gamma - 1} = \frac{0.2598}{1.4 - 1} = 0.6495 \text{ kJ/kg-K}$$

Since there is no work transfer, so,

$$\delta W = 0$$

From the first law of thermodynamics:

$$\delta Q = dU + \cancelto{0}{\delta W}$$

$\therefore$ $\delta Q = dU$

$$dU = \text{Change in internal energy} = (dU)_x + (dU)_y$$

$$= m_x C_V (T_f - T_x) + m_y C_V (T_f - T_y)$$

$$= 19.2 \times 0.6495 \times (303 - 333) + 3 \times 0.6495 \times (303 - 288)$$

$$= -344.88 \text{ kJ}$$

Hence, the heat transferred to the surrounding is –344.88 kJ.

3. (a)

(i)

Given:

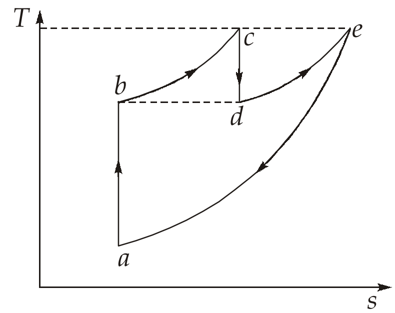
Process $(a - b)$: Reversible adiabatic compression

Process $(b - c)$: Constant volume heat addition

Process $(c - d)$: Reversible adiabatic expansion

Process $(d - e)$: Constant pressure heat addition

Process $(e - a)$: Constant volume heat rejection



For the complete cycle,

$$(\Delta s)_{\text{cycle}} = 0$$

$$(\Delta s)_{\text{cycle}} = (\Delta s)_{ab} + (\Delta s)_{bc} + (\Delta s)_{cd} + (\Delta s)_{de} + (\Delta s)_{ea}$$

or,
$$0 = 0 + C_V \ln\frac{T_c}{T_b} + 0 + C_P \ln\frac{T_e}{T_d} + C_V \ln\frac{T_a}{T_e}$$

or,
$$C_V \ln\frac{T_c}{T_b} + C_P \ln\frac{T_e}{T_d} + C_V \ln\frac{T_a}{T_e} = 0$$

or,
$$C_V \ln\frac{T_c}{T_b} + C_P \ln\frac{T_c}{T_b} + C_V \ln\frac{T_a}{T_c} = 0$$

or,
$$(C_P + C_V)\ln\frac{T_c}{T_b} + C_V \ln\frac{T_a}{T_c} = 0$$

Dividing the whole equation by C_V.

$$\left(\frac{C_P}{C_V} + 1\right)\ln\frac{T_c}{T_b} + \ln\frac{T_a}{T_c} = 0$$

or,
$$(\gamma + 1)\ln\frac{T_c}{T_b} + \ln\frac{T_a}{T_c} = 0$$

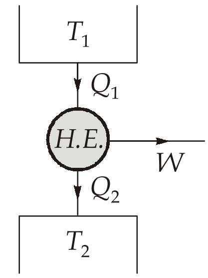
$\Rightarrow$
$$\left(\frac{T_c}{T_b}\right)^{\gamma+1} \cdot \frac{T_a}{T_c} = 1$$

$\Rightarrow$
$$T_a = \frac{T_b^{\gamma+1}}{T_c^{\gamma}}$$

If $T_b = 600$ K, $T_c = 800$ K and $\gamma = 1.4$ then

$$T_a = \frac{(600)^{1.4+1}}{(800)^{1.4}} = 401.085 \text{ K}$$

(ii)



Given:

For body 1:

$$\text{Initial temperature} = T_1$$
$$\text{Thermal capacity} = c_1$$

For body 2:

$$\text{Initial temperature} = T_2$$
$$\text{Thermal capacity} = c_2$$
$$\text{Final temperature of both bodies} = T$$

Entropy change for body 1:

$$(\Delta s)_1 = c_1 \ln \frac{T}{T_1}$$

Entropy change for body 2:

$$(\Delta s)_2 = c_2 \ln \frac{T}{T_2}$$

$$(\Delta s)_{\text{universe}} = (\Delta s)_1 + (\Delta s)_2 + (\Delta s)_{H.E} + (\Delta s)_{surr.}$$
$$= c_1 \ln \frac{T}{T_1} + c_2 \ln \frac{T}{T_2} + 0 + 0$$

[∵ Heat engine is a cyclic device and work is transferred to surrounding]

For a reversible process, $(\Delta s)_{\text{universe}} = 0$

$$\Rightarrow \quad c_1 \ln \frac{T}{T_1} + c_2 \ln \frac{T}{T_2} = 0$$

$$\Rightarrow \quad \left(\frac{T}{T_1}\right)^{c_1} \cdot \left(\frac{T}{T_2}\right)^{c_2} = 1$$

$$\Rightarrow \quad T^{c_1+c_2} = T_1^{c_1} \cdot T_2^{c_2}$$

Taking natural log on both sides:

$$(c_1 + c_2)\ln T = c_1 \ln T_1 + c_2 \ln T_2$$

$$\Rightarrow \quad \ln T = \frac{c_1 \ln T_1 + c_2 \ln T_2}{c_1 + c_2}$$

3. (b)

(i)

Given:

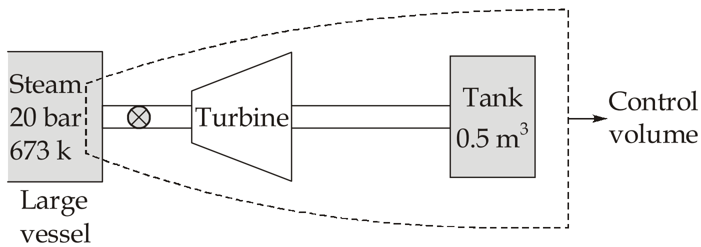
Pressure of large vessel = 20 bar

Temperature of large vessel = 400°C = 673 K

Volume of small tank = 0.5 m^3

Pressure of tank after filling = 20 bar

Temperature of tank after filling = 500°C = 773 K



Assumptions :

1. The state of the steam within the large vessel does not change.
2. The final state of steam in tank is an equilibrium state.
3. The amount of mass stored within the turbine and the interconnecting piping at the end of the filling process is negligible.

Mass balance:

Mass of the control volume = Mass entered – Mass exit

$$(m_2 - \cancelto{0}{m_1}) = (m_i - \cancelto{0}{m_0})$$

$$\Rightarrow \quad m_2 = m_i \quad \text{...(i)}$$

Now, $$m_2 = \frac{V}{v_2} = \frac{0.5}{0.17568} = 2.846 \text{ kg}$$

[From steam tables: At P = 20 bar and T = 500°C, v = 0.17568 m^3/kg]

Energy balance:

Energy of the control volume = Energy input – Energy output

$$(U_2 - \cancelto{0}{U_1}) = m_i h_i + \cancelto{0}{Q} - \cancelto{0}{m_e} h_e - W_{C.V.}$$

$$U_2 = m_i h_i - W_{C.V.}$$

$$m_2u_2 = m_2h_i - W_{C.V.}$$

$\Rightarrow$ $$W_{C.V.} = m_2(h_i - u_2)$$

From steam tables:

At P = 20 bar and T = 400°C; h_i = 3248.3 kJ/kg

At P = 20 bar and T = 500°C; u_2 = 3116.8 kJ/kg

$\therefore$ $$W_{CV} = 2.846 \times (3248.3 - 3116.8)$$
$$= 374.25 \text{ kJ}$$

So, work developed by turbine is 374.25 kJ.

(ii)

Given:

Volume of insulated tank = 10 m^3

Initial pressure of insulated tank P_1 = 500 kPa

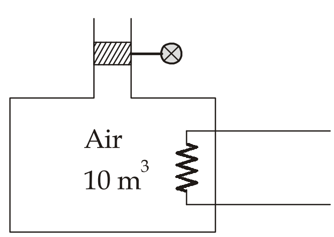
Initial temperature of insulated tank T_1 = 400 K

Final pressure of tank P_2 = 150 kPa

Final temperature of tank, T_2 = 400 K

Assumptions:

1. This is an unsteady process since the conditions within the device are changing during the process but it can be analyzed as a uniform flow process since the exit conditions remains constant.
2. Kinetic energy and potential energy are negligible.
3. Air is an ideal gas with constant specific heats.
4. Tank is insulated.



Mass balance:

$$m_2 - m_1 = (\cancel{m_i}^{\,0} - m_e)$$

$\Rightarrow$ $$m_e = m_1 - m_2$$

$$\Rightarrow \qquad m_e = \frac{P_1V_1}{RT_1} - \frac{P_2V_2}{RT_2}$$

$$= \frac{500 \times 10}{0.287 \times 400} - \frac{150 \times 10}{0.287 \times 400}$$

$$= 43.554 - 13.066$$

$$= 30.487 \text{ kg}$$

Energy balance:

$$U_2 - U_1 = \cancelto{0}{m_i h_i} + \cancelto{0}{Q} - m_e h_e - W_{C.V.}$$

$$m_2 u_2 - m_1 u_1 = -m_e h_e - W_{C.V.}$$

$$\Rightarrow \qquad W_{C.V.} = m_1 u_1 - m_2 u_2 - m_e h_e$$

$$= 43.554 \times 0.718 \times 400 - 13.066 \times 0.718 \times 400 - 30.487 \times 1.005 \times 400$$

$$W_{C.V.} = -3499.62 \text{ kJ}$$

So, the electrical energy supplied to air during this process is 3499.62 kJ.

In kWh, $W_{C.V.} = W_{\text{Electrical}}$

$$= \frac{3499.62}{3600} = 0.972 \text{ kWh}$$

3. (c)

(i) The Carnot principles are listed below:

(a) The efficiency of an irreversible engine can never be more than the efficiency of a reversible engine operating between same reservoirs.

(b) The efficiency of all reversible devices operating between same temperature limits is same.

(ii) Given,

Initial volume of air, $V_1 = 0.06 \text{ m}^3$

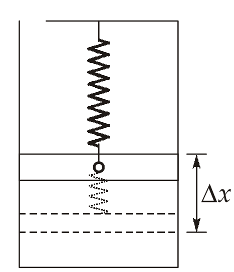
Initial pressure of air, $P_1 = 2300$ kPa

Initial temperature of air, $T_1 = 220°\text{C} = 493$ K

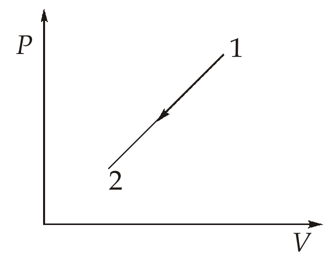
Piston diameter, $d = 22$ cm $= 0.22$ m

Spring constant, $k = 900$ N/m

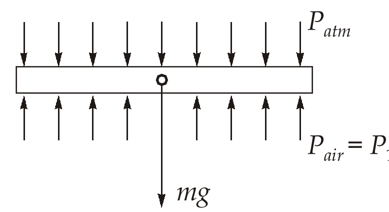
Final volume of air, $V_2 = 0.03 \text{ m}^3$



The process undergone by air is shown in P-v diagram below,

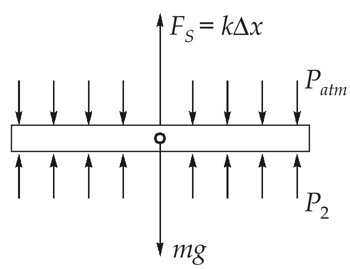


When no force is exerted on the piston by spring.



$$P_{atm} + \frac{mg}{A_p} = P_{air} = P_1 = 2300 \text{ kPa}$$

When volume becomes half of its initial volume then:



$$\frac{F_{spring}}{A_p} + P_2 = P_{atm} + \frac{mg}{A_p} = P_1$$

$$\Rightarrow \quad \frac{F_{spring}}{A_p} + P_2 = P_1 \quad \text{...(i)}$$

Distance moved by the piston,

$$\Delta x = \frac{V_1 - V_2}{A_p} = \frac{0.06 - 0.03}{\frac{\pi}{4} \times (0.22)^2} = 0.789 \text{ m}$$

$$\therefore \quad F_{spring} = k\Delta x = 900 \times 0.789 = 710.278 \text{ N}$$

So, from equation (i)

$$\frac{F_{spring}}{A_p} + P_2 = P_1$$

$$\Rightarrow \quad \frac{710.278}{\frac{\pi}{4} \times (0.22)^2} \times 10^{-3} + P_2 = P_1 = 2300$$

$\Rightarrow \quad P_2 = 2281.315 \text{ kPa}$

Since mass of air is constant, so:

$$\frac{P_2V_2}{RT_2} = \frac{P_1V_1}{RT_1}$$

or,
$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

or,
$$\frac{2281.315 \times 0.03}{T_2} = \frac{2300 \times 0.06}{493}$$

$\Rightarrow \quad T_2 = 244.497 \text{ K}$

So, change in internal energy,

$$\Delta U = mC_V(T_2 - T_1)$$

$$\text{mass, } m = \frac{P_1V_1}{RT_1} = \frac{2300 \times 0.06}{0.287 \times 493} = 0.975 \text{ kg}$$

$\therefore$
$$\Delta U = 0.975 \times 0.718 \times (244.497 - 493)$$
$$= -174.022 \text{ kJ}$$

Change in enthalpy,
$$\Delta H = mC_p\Delta T$$
$$= 0.975 \times 1.005 \times (244.497 - 493)$$
$$= -243.5 \text{ kJ}$$

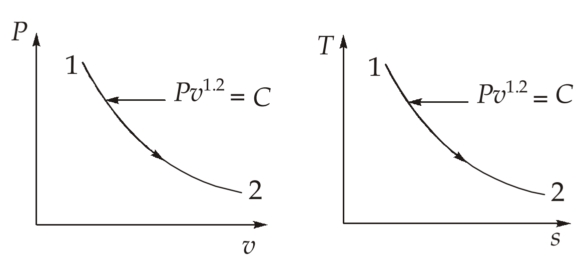
The negative sign implies that the heat is lost from the system.

4. (a)

Given:

$$P_1 = 15 \text{ bar}; T_1 = 300°\text{C} = 573 \text{ K}; n = 1.2; P_2 = 1.3 \text{ bar}$$

1. If the fluid is air:



The process is reversible polytropic

$\therefore$
$$P_1v_1^n = P_2v_2^n$$

Final volume:

$$\Rightarrow \qquad v_2 = v_1 \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = \frac{RT_1}{P_1} \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$= \frac{0.287 \times 573}{1500} \times \left(\frac{15}{1.3}\right)^{\frac{1}{1.2}} = 0.8415 \text{ m}^3/\text{kg}$$

Final temperature:

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

$$\Rightarrow \qquad T_2 = T_1 \times \frac{P_2 v_2}{P_1 v_1}$$

$$= 573 \times \frac{1.3 \times 0.8415}{15 \times 0.1096} = 381.298 \text{ K}$$

$$\left(v_1 = \frac{RT_1}{P_1} = 0.1096 \text{ m}^3/\text{kg}\right)$$

Heat transfer:

$$\delta Q = dU + \delta W$$

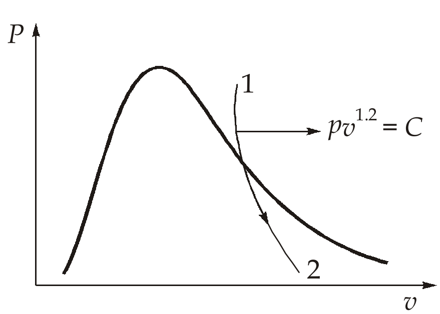
$$= C_V (T_2 - T_1) + \frac{P_2 v_2 - P_1 v_1}{1 - n}$$

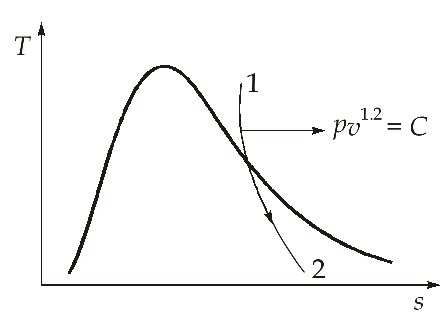
$$= 0.718 \times (318.298 - 573) + \frac{1.3 \times 100 \times 0.8415 - 15 \times 100 \times 0.1096}{1 - 1.2}$$

$$= -137.642 + 275.025$$

$$= 137.383 \text{ kJ/kg}$$

2. If the fluid is steam:





From steam table:

At 15 bar and 300°C

$$v_1 = 0.16971 \text{ m}^3/\text{kg}; \ u_1 = 2783.6 \text{ kJ/kg}$$

∴ **Final volume:**

$$v_2 = v_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 0.16971 \times \left(\frac{15}{1.3}\right)^{\frac{1}{1.2}} = 1.30266 \text{ m}^3/\text{kg}$$

At 1.3 bar;

$$v_g = 1.3253 \text{ m}^3/\text{kg}; \ u_f = 449.05 \text{ kJ/kg}; \ u_g = 2514.3 \text{ kJ/kg}$$

Since, $v_2 < v_g$ so the state 2 is in the wet region.

∴ $$v_2 = v_f + x_2(v_g - v_f)$$

⇒ $$1.30266 = 0.00104917 + x_2 \times (1.3253 - 0.00104917)$$

⇒ $$x_2 = 0.983$$

∴ $$u_2 = u_f + x_2(u_g - u_f)$$

$$= 449.05 + 0.983 \times (2514.3 - 449.05)$$

$$= 2478.99 \text{ kJ/kg}$$

Final temperature:

$$T_2 = (T_{sat})_{@1.3\text{ bar}} = 107.109°\text{C} = 380.109 \text{ K}$$

Heat transfer:

$$\delta q = du + \delta w$$

$$= (u_2 - u_1) + \frac{P_1 v_1 - P_2 v_2}{n-1}$$

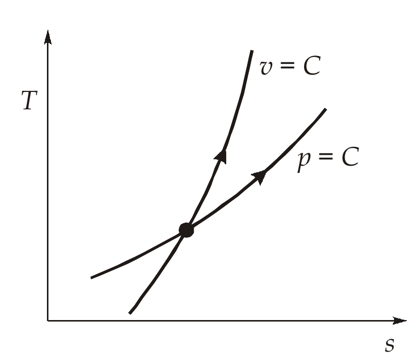
$$= (2478.99 - 2783.6) + \frac{15 \times 100 \times 0.16971 - 1.3 \times 100 \times 30266}{1.2 - 1}$$

$$= -304.61 + (426.096) = 121.486 \text{ kJ/kg}$$

4. (b)

(i)

Isochoric process:



$$Tds = du + Pdv$$

$$= C_V dT + Pdv$$

For isochoric / constant volume process;

$$dv = 0$$

$$Tds = C_V dT + 0$$

$$\Rightarrow \qquad \frac{dT}{ds} = \frac{T}{C_V} \qquad ...(i)$$

Isobaric process:

$$Tds = dh - vdp$$

or $$Tds = C_P dT - vdp$$

For constant pressure process;

$$dp = 0$$

$$\therefore \qquad Tds = C_P dT$$

$$\Rightarrow \qquad \frac{dT}{ds} = \frac{T}{C_P} \qquad ...(ii)$$

From equation (i) and (ii)

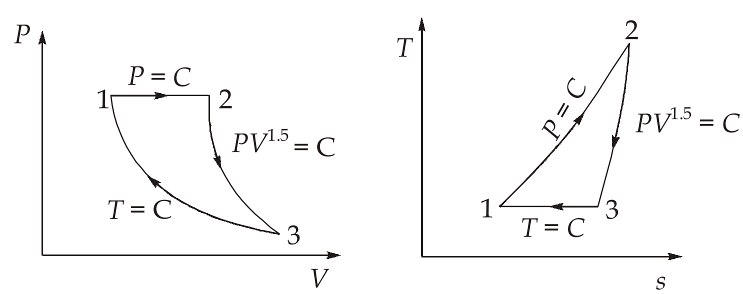
As $$C_P > C_V$$

So, $$\frac{T}{C_P} < \frac{T}{C_V}$$

$$\therefore \qquad \left(\frac{dT}{dS}\right)_{P=C} < \left(\frac{dT}{dS}\right)_{V=C}$$

Since $\frac{dT}{ds}$ denotes the slope of T-s diagram, it is proved that the slope of constant volume or isochoric curve is greater than the slope of isobaric curve on T-s diagram.

(ii)



Given: $P_1 = 600$ kPa; $V_1 = 0.04$ m^3; $T_1 = 230°C = 503$ K;
$V_2 = 0.12$ m^3; $n = 1.5$; $T_3 = T_1 = 503$ K

Assumptions:

1. All the processes are reversible.
2. Air is an ideal gas with $c_p = 1.005$ kJ/kg·K, $c_v = 0.718$ kJ/kg·K and $\gamma = 1.4$

From the ideal gas equation;

$$P_1V_1 = mRT_1$$

$$\Rightarrow \quad 600 \times 0.04 = m \times 0.287 \times 503$$

$$\Rightarrow \quad m = 0.166 \text{ kg}$$

For process 1 - 2; since $P = C$

$$\therefore \quad V \propto T$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{0.04}{0.12} = \frac{503}{T_2}$$

$$\Rightarrow \quad T_2 = 1509 \text{ K}$$

$$(\delta Q)_{1\text{-}2} = mC_P(T_2 - T_1)$$

$$= 0.166 \times 1.005 \times (1509 - 503)$$

$$= 167.83 \text{ kJ}$$

For process 2-3; polytropic process ($PV^{1.5} = C$)

$$(\delta Q)_{2\text{-}3} = \frac{\gamma - n}{\gamma - 1} \times \frac{mR(T_2 - T_3)}{n - 1}$$

$$= \frac{1.4 - 1.5}{1.4 - 1} \times \frac{0.166 \times 0.287 \times (1509 - 503)}{1.5 - 1}$$

$$= -23.964 \text{ kJ}$$

For process 3-1; isothermal process; $T = C$

$$(\delta Q)_{3\text{-}1} = (dU)_{3-1} + (\delta W)_{3-1}$$

$$(\delta Q)_{3\text{-}1} = 0 + mRT_3 \ln\left(\frac{P_3}{P_1}\right)$$

$$\frac{P_3}{P_2} = \left(\frac{T_3}{T_2}\right)^{\frac{n}{n-1}}$$

$$\Rightarrow \quad P_3 = 600 \times \left(\frac{503}{1509}\right)^{\frac{1.5}{0.5}} = 22.22 \text{ kPa}$$

$$\therefore \quad (\delta Q)_{3\text{-}1} = 0.166 \times 0.287 \times 503 \times \ln\left(\frac{22.22}{600}\right) = -78.983 \text{ kJ}$$

Now, for complete cycle,

$\therefore$ Total heat received $= (\delta Q)_{1\text{-}2} = 167.83$ kJ

Total heat rejected $= (\delta Q)_{2\text{-}3} + (\delta Q)_{3\text{-}1}$

$= 23.964 + 78.983$

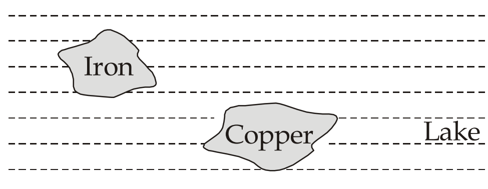
$= 102.947$ kJ

$$\text{Efficiency of the cycle; } \eta = 1 - \frac{Q_{rej.}}{Q_{add.}}$$

$$= 1 - \frac{102.947}{167.83} = 0.3867 = 38.67\%$$

4. (c)

(i)



Given : Mass of iron, $m_i = 40$ kg

Mass of copper, $m_c = 15$ kg

$T_i = T_c = 100°C = 373$ K

$T_L = 10°C = 283$ K

$(C_P)_{\text{iron}} = 0.5$ kJ/kg-K

$(C_P)_{\text{copper}} = 0.4$ kJ/kg-K

The thermal energy capacity of the lake is very large and thus the temperatures of both the iron and the copper blocks will drop to the lake temperature (10°C) when the thermal equilibrium is established.

Entropy change of iron:

$$(\Delta s)_{\text{iron}} = \left[mc\ln\frac{T_2}{T_1}\right]_{iron}$$

$$= 40 \times 0.5 \times \ln\frac{283}{373} = -5.522 \text{ kJ/K}$$

Entropy change of copper:

$$(\Delta s)_{\text{copper}} = \left[mc\ln\frac{T_2}{T_1}\right]_{copper}$$

$$= 15 \times 0.4 \times \ln \frac{283}{373} = -1.6567 \text{ kJ/K}$$

Heat lost by both iron and copper blocks

$$\delta Q = [mc(T_2 - T_1)]_{iron} + [mc(T_2 - T_1)]_{copper}$$

$$= 40 \times 0.5 \times (283 - 373) + 15 \times 0.4 \times (283 - 373)$$

$$= -2340 \text{ kJ}$$

So, heat lost = 2340 kJ

Entropy change of lake; $(\Delta s)_{\text{lake}} = \dfrac{\text{Heat lost}}{T_L} = \dfrac{2340}{283} = 8.268$ kJ/K

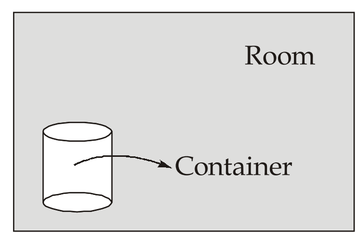
Hence, total entropy change for the process;

$$(\Delta s)_{\text{total}} = (\Delta s)_{\text{iron}} + (\Delta s)_{\text{copper}} + (\Delta s)_{\text{lake}}$$

$$= -5.522 - 1.6567 + 8.268$$

$$= 1.08985 \text{ kJ/K}$$

(ii)



Mass of liquid water = 50 kg = m_W

Initial temperature of water = $(T_W)_1$ = 90°C = 363 K

Volume of room = 100 m^3

Initial temperature of room, $(T_a)_1$ = 15°C = 288 K

R_{air} = 0.287 kJ/kg-K,

$(C_p)_a$ = 1.005 kJ/kg-K,

C_W = 4.18 kJ/kg-K

Assumptions:

1. Kinetic energy and potential energy changes are neglected.
2. Air is an ideal gas.

Mass of the air in the room :

$$m_a = \left(\frac{PV}{RT}\right)_a = \frac{1.0132 \times 100 \times 100}{0.287 \times 288} = 122.58 \text{ kg}$$

Since the room is well-sealed and insulated.

So, $\delta Q = 0$

Heat lost by water = Heat gain by air

$\Rightarrow \quad m_w c_w \left((T_w)_1 - T_2\right) = m_a c_v \left(T_2 - (T_a)_1\right)$

$\Rightarrow \quad 50 \times 4.18 \times (363 - T_2) = 122.58 \times 0.718 \times (T_2 - 288) \qquad \left[\because c_v = \frac{c_p}{\gamma}\right]$

$\Rightarrow \quad T_2 = 340.77 \text{ K}$

So, final equilibrium temperature = 340.77 K

Heat transfer between water and air:

$$\delta Q = \left[mc\left(T_{w_1} - T_2\right)\right]_w = \left[mc_v\left(T_2 - (T_a)_1\right)\right]_{air}$$

$\therefore \quad \delta Q = 50 \times 4.18 \times (363 - 340.77)$

$= 4646.07 \text{ kJ}$

So, amount of heat transfer between water and air = 4646.07 kJ.

Entropy generation:

$$(\Delta s)_{\text{water}} = \left[mc \ln \frac{T_2}{T_1}\right]_{water} = 50 \times 4.18 \times \ln \frac{340.77}{363}$$

$$= -13.207 \text{ kJ/K}$$

$$(\Delta s)_{\text{air}} = mc_v \ln \frac{T_2}{T_1} + \cancelto{0}{mR \ln \frac{V_2}{V_1}}$$

$$= 122.58 \times 0.718 \times \ln \frac{340.77}{288}$$

$$= 14.807 \text{ kJ/K}$$

$$(\Delta s)_{\text{total}} = (\Delta s)_{\text{gen.}} = 14.807 - 13.207 = 1.6 \text{ kJ/K}$$

So, entropy generation = 1.6 kJ/K

5. (a)

Given : Number of wires = 30

Diameter of each wire = 1.6 mm

Weight of cage = 1.5 kN

Weight of the rope = 4.6 N/m

Length of the rope = 40 m

$$E_{rope} = 70 \text{ GPa}$$
$$\sigma_{allowable} = 120 \text{ MPa}$$

Total area of cross-section, $A = \frac{\pi}{4} \times (1.6)^2 \times 4 \times 30$

$$= 241.27 \text{ mm}^2$$

The maximum stress occurs at the top of the wire rope where the weight of the rope is maximum.

So, maximum load = Weight of the cage + Weight of the rope

$$= 1500 + 4 \times 4.6 \times 40$$
$$= 2236 \text{ N}$$

Initial stress in the rope, $\sigma = \frac{2236}{241.27} = 9.267 \text{ MPa}$

So, equivalent static stress available for carrying the load = 120 – 9.267 = 110.73 MPa

The equivalent static load that can be carried,

$$P_e = 110.73 \times 241.27$$
$$= 26716.4 \text{ N}$$

Extension of the rope, $\Delta = \frac{110.73 \times 40 \times 1000}{70 \times 1000} = 63.27 \text{ mm}$

(i) With no drop, let W_1 be the weight which can be applied suddenly

So, $$W_1\Delta = \frac{1}{2}P_e\Delta$$

$\Rightarrow$ $$W_1 = \frac{26716.4}{2} = 13358.2 \text{ N}$$

(ii) With 100 mm drop, let W_2 be the weight so,

$$W_2 \cdot (h + \Delta) = \frac{1}{2} P_e\Delta$$

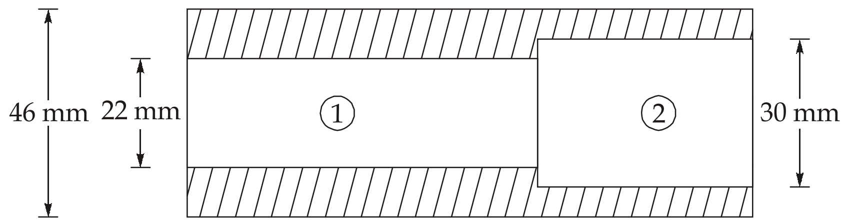
$\Rightarrow$ $$W_2(100 + 63.27) = \frac{1}{2} \times 26716.4 \times 63.27$$

$\Rightarrow$ $$W_2 = 5176.53 \text{ N}$$

5. (b)

(i)

Given : $N = 250$ rpm; $\tau_{perm} = 80$ MPa



Since, $$\tau_{perm} = \frac{16T}{\pi}\left(\frac{D}{D^4 - d^4}\right)$$

So, maximum shear stress will be at the portion where d is maximum.

$\therefore$ $$T_{max} = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)\tau_{perm}$$

$$= \frac{\pi}{16}\left(\frac{46^4 - 30^4}{46}\right) \times 80 = 1252.353 \text{ N-m}$$

$$(\text{Power})_{max} = \frac{2\pi N T_{max}}{60} = \frac{2\pi \times 250 \times 1252.353}{60} = 32.786 \text{ kW}$$

If the twist is to be same in the two portions then,

$$\theta = \frac{Tl_1}{GJ_1} = \frac{Tl_2}{GJ_2}$$

or $$\frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{46^4 - 22^4}{46^4 - 30^4} = 1.1569$$

$\Rightarrow$ $$l_1 = 1.1569 l_2$$

Also, $$l_1 + l_2 = 1000 \text{ mm}$$

$\therefore$ $$l_1 = 536.37 \text{ mm and } l_2 = 463.63 \text{ mm}$$

(ii)

Given: Modulus of rigidity, $G = 0.7 \times 10^5$ N/mm^2

Diameter of rod, $d = 10$ mm

Axial pull, $P = 15$ kN

Change in diameter, $\delta d = 4 \times 10^{-3}$ mm

$$\sigma = \frac{P}{A} = \frac{15 \times 10^3}{\frac{\pi}{4} \times (10)^2} = 190.98 \text{ N/mm}^2$$

$$\text{Poisson's ratio, } \mu = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}$$

$$\frac{\delta l}{l} = \frac{\sigma}{E} = \frac{190.98}{E}$$

$$\therefore \quad \mu = \frac{4 \times 10^{-3} \times E}{10 \times 190.98} = 2.094 \times 10^{-6} E$$

$$\therefore \quad \frac{\mu}{E} = 2.094 \times 10^{-6} \quad \text{...(i)}$$

Also, $E = 2G(1 + \mu)$

$= 2 \times 0.7 \times 10^5 \times (1 + \mu)$

$$\Rightarrow \quad \frac{1}{2 \times 0.7 \times 10^5} = \frac{1}{E} + \frac{\mu}{E} \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\Rightarrow \quad \frac{1}{1.4 \times 10^5} = \frac{1}{E} + 2.094 \times 10^{-6}$$

$$\Rightarrow \quad E = 198.08 \text{ GPa}$$

Substituting the value of 'E' in equation (i), we get

$$\therefore \quad \mu = 2.094 \times 10^{-6} \times 198.08 \times 10^3 = 0.41$$

5. (c)

Given : $L = 1000$ mm; $D = 500$ mm; $t = 5$ mm; $P = 10$ MPa; $E_C = 1 \times 10^5$ MPa; $\mu = 0.3$; $k = 2500$ MPa

Under the action of internal pressure, the cylinder expands and the oil is compressed.

Now, $$V = \frac{\pi}{4} D^2 \cdot L = \frac{\pi}{4} \times 500^2 \times 1000$$

$$V = 196.35 \times 10^6 \text{ mm}^3$$

$$\text{Expansion of cylinder, } \delta V_1 = \frac{PD}{4tE}(5 - 4\mu) \cdot V$$

$$= \frac{10 \times 500}{4 \times 5 \times 10^5}(5 - 4 \times 0.3) \times 196.35 \times 10^6$$

$$\delta V_1 = 1865325 \text{ mm}^3$$

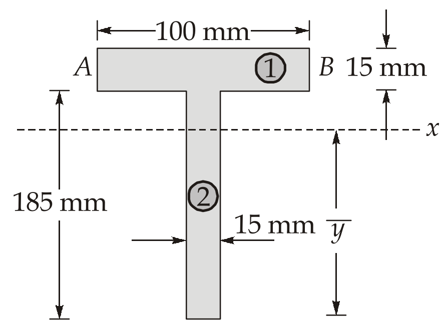
Compression of oil, $\delta V_2 = \frac{P}{K} V = \frac{10 \times 196.35 \times 10^6}{2500} = 785400 \text{ mm}^3$

$\therefore$ Volume of oil pumped $= \delta V_1 + \delta V_2$

$= 1865325 + 785400$

$= 2650.725 \text{ cm}^3$

5. (d)



Centroid of the cross-section is given by

$$\bar{y} = \frac{100 \times 15 \times 192.5 + 185 \times 15 \times 92.5}{100 \times 15 + 185 \times 15}$$

$$\bar{y} = 127.58 \text{ mm}$$

Now, from figure, $I_1 = \frac{100 \times 15^3}{12} + 100 \times 15 \times (64.92)^2 = 6350034.6 \text{ mm}^4$

$$I_2 = \frac{15 \times 185^3}{12} + 15 \times 185 \times 35.08^2 = 11329464.01 \text{ mm}^4$$

$\therefore$ $I_{xx} = I_1 + I_2$

$= 1767.95 \times 10^4 \text{ mm}^4$

At the junction of the web,

$$A\bar{y} = 100 \times 15 \times 64.92 = 97380 \text{ mm}^3$$

$\therefore$ $\tau_F = \frac{FA\bar{y}}{Ib} = \frac{250 \times 10^3 \times 97380}{1767.95 \times 10^4 \times 100}$

$\therefore$ $\tau_F = 13.77 \text{ N/mm}^2$

At the junction with the flange,

$$\tau_w = 13.77 \times \frac{100}{15} = 91.8 \text{ N/mm}^2$$

At the centroid,

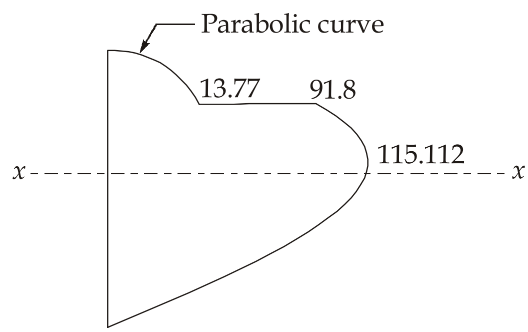
$$A\bar{y} = (A\bar{y})_F + (A\bar{y})_w$$

$$= (100 \times 15 \times 64.92) + (57.42 + 15 \times 28.71)$$

$$= 122107.923 \text{ mm}^3$$

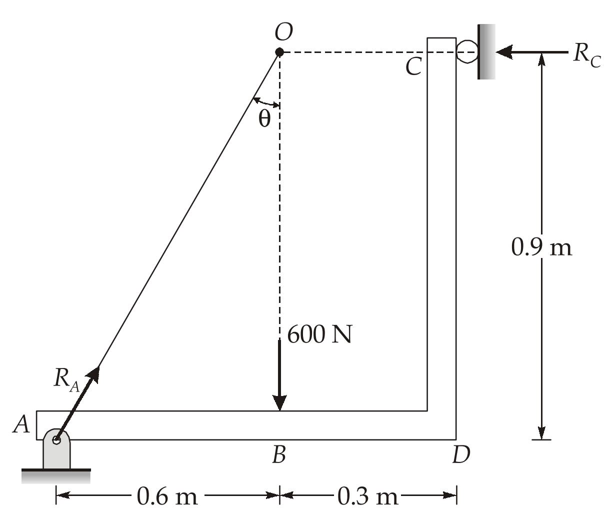
$$\therefore \quad \tau_{max} = \tau_{NA} = \frac{250 \times 10^3 \times 122107.923}{1767.95 \times 10^4 \times 15} = 115.112 \text{ N/mm}^2 \quad \text{Ans.}$$

The variation of shear stress is shown in figure below,



5. (e)

(i)



The reaction R_C will be horizontal in direction. The line of action of R_C and force 600 N meet at point *O*. So, for plate *ABCD* to remain in equilibrium, R_A will pass through *O*.

Therefore, $\tan\theta = \dfrac{0.6}{0.9} = 0.67$

$\Rightarrow \qquad \theta = 33.69°$

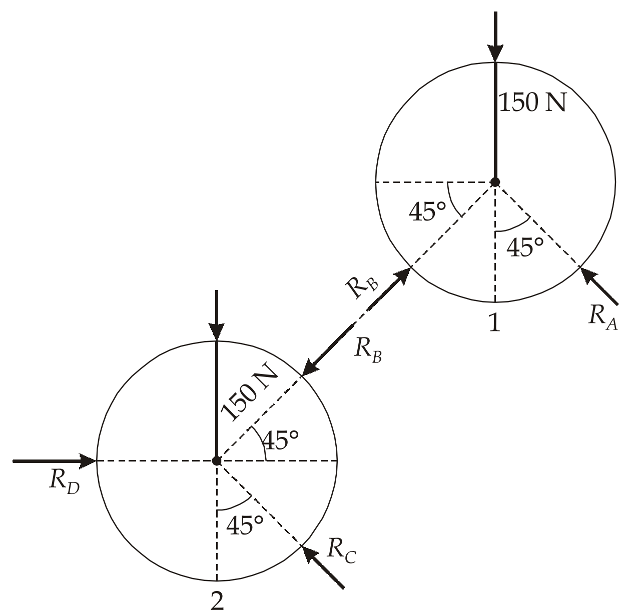
So, $\qquad R_A \sin\theta = R_C$

and $\qquad R_A \cos\theta = 600$

$\Rightarrow \qquad R_A = 721.11 \text{ N}$

and $\qquad R_C = 721.11 \sin 33.69° = 400 \text{ N}$

(ii)



FBD of two rollers

From the FBD of roller 1;

$\Sigma F_H = 0$

$\Rightarrow \qquad R_A \sin 45° = R_B \cos 45°$

$\therefore \qquad R_A = R_B \qquad$ ...(i)

Also, $\Sigma F_V = 0$

$$R_A \cos 45° + R_B \sin 45° = W$$

$\Rightarrow \qquad \dfrac{2 \times R_A}{\sqrt{2}} = 150$

$\Rightarrow \qquad R_A = \dfrac{150}{\sqrt{2}} = 106.07 \text{ N} \quad \text{or } 75\sqrt{2} \text{ N} \quad$ Ans.

From equation (i), $\qquad R_A = R_B = 106.07 \text{ N} \quad$ Ans.

From the FBD of roller 2 :

$\Sigma F_H = 0$

$$\Rightarrow \quad R_B \cos 45° + R_C \sin 45° = R_D \quad \text{...(i)}$$

$$\Rightarrow \quad 75\sqrt{2} \times \frac{1}{\sqrt{2}} + \frac{R_C}{\sqrt{2}} = R_D$$

$$75 + \frac{R_C}{\sqrt{2}} = R_D \quad \text{...(ii)}$$

$\Sigma F_V = 0$

$$\Rightarrow \quad R_B \sin 45° + W = R_C \cos 45°$$

$$75\sqrt{2} \times \frac{1}{\sqrt{2}} + 150 = R_C \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad R_C = 225\sqrt{2} \text{ N or } 318.19 \text{ N} \quad \text{Ans.}$$

From equation (ii),

$$75 + \frac{225\sqrt{2}}{\sqrt{2}} = R_D$$

$$\Rightarrow \quad R_D = 300 \text{ N} \quad \text{Ans.}$$

6. (a)

(a) Reactions:

Taking moment about 'A', we get

$$R_B = \frac{1}{8}[2 \times 2 \times 9 + 5 \times 5 + 10 \times 3 + 4 \times 3 \times 1.5]$$

$$= 13.625 \text{ kN}$$

$$R_A = (4 \times 3 + 10 + 5 + 2 \times 2) - 13.625$$

$$= 17.375 \text{ kN}$$

(b) SFD:

For section AD :

Taking section 'x' from 'A'

$$F_x = 17.375 - 4x$$

At $x = 0$, $F_A = 17.375$ kN

At $x = 3$, F_D (left) = 5.375 kN

For section DE:

$$F_x = 17.375 - 4 \times 3 - 10$$
$$= -4.625 \text{ kN (constant)}$$

$\therefore$ F_D (right) = –4.625 kN

F_E (left) = –4.625 kN

For section EB:

$$F_x = 17.375 - 4 \times 3 - 10 - 5$$
$$F_x = -9.625 \text{ kN}$$
$$F_E \text{ (right)} = -9.625 \text{ kN}$$
$$F_B \text{ (left)} = -9.625 \text{ kN}$$

For section BC:

$$F_x = 17.375 - 12 - 10 - 5 + 13.625 - 2(x - 8)$$

$\therefore$ $F_x = 20 - 2x$ (Linear)

$\therefore$ $x = 8$ m, F_B (right) $= 20 - 2 \times 8 = 4$ kN

$x = 10$ m, $F_C = 20 - 2 \times 10 = 0$

(c) B.M.D

For section AD, Taking 'x' from 'A'

$$M_x = 17.375x - \frac{4x^2}{2}$$
$$= 17.375x - 2x^2 \quad \text{(Parabolic)}$$

At, $x = 0$, $M_A = 0$; at $x = 3$ m, $M_D = 34.125$ kNm

For DE, $M_x = 17.375x - 12(x - 1.5) - 10(x - 3)$

$$= -4.625x + 48 \quad \text{(Linear)}$$

At $x = 3$, $M_D = 34.125$ kNm,

At $x = 5$, $M_E = 24.875$ kNm

For EB, $M_x = 17.375x - 12(x - 1.5) - 10(x - 3) - 5(x - 5)$

$$= -9.625x + 73 \quad \text{(Linear)}$$

At $x = 5$, $M_E = 24.875$ kNm,

At $x = 8$, $M_B = -4$ kNm

Hence, B.M. changes its sign in section *EB*, its value being zero at $x = \dfrac{73}{9.625} = 7.58$ m from *A*. Maximum sagging B.M. is at *D* and maximum hogging B.M. is at *B*, where S.F. changes its sign.

For section *BC*:

$$M_x = 17.375x - 12(x-1.5) - 10(x-3) - 5(x-5) + 13.625(x-8) - 2(x-8)\cdot\frac{(x-8)}{2}$$

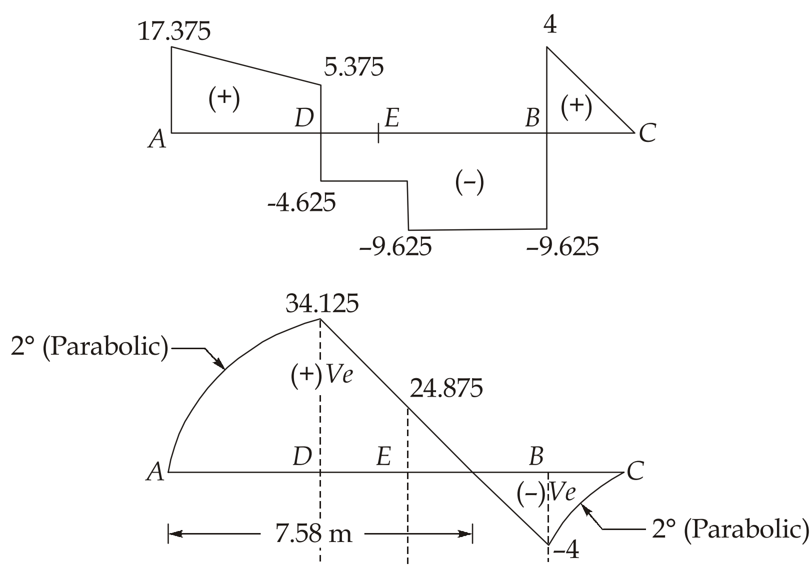
$$M_x = 4x - 36 - (x-8)^2 \qquad \text{(Parabolic)}$$

At $x = 8$ m, $M_B = -4$ kNm

At $x = 10$ m, $M_C = 4 \times 10 - 36 - (10-8)^2$

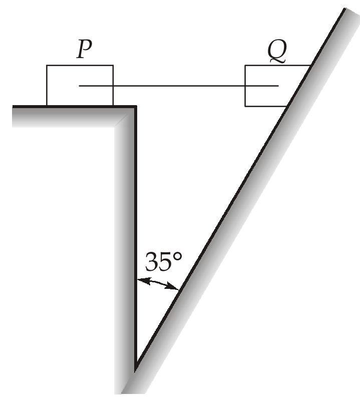
$$M_C = 0$$

The complete S.F.D. and B.M.D. is shown in figure.

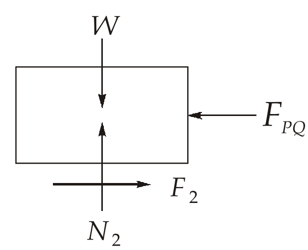


6. (b)

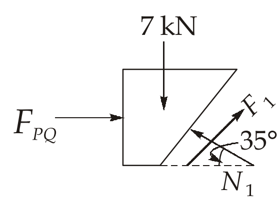
(i)



FBD of block P :



FBD of block Q :



Considering block Q:

$$F_1 = N_1 \tan 22° \qquad (\because \quad \mu = \tan 22°)$$

If $\Sigma V = 0$

$\Rightarrow \quad F_1 \sin 55° + N_1 \sin 35° = 7$

$\Rightarrow \quad N_1 = 7.74 \text{ kN}$

$\therefore \quad F_1 = 7.74 \tan 22° = 3.13 \text{ kN}$

If $\Sigma H = 0$

$$F_{PQ} + F_1 \cos 55° - N_1 \cos 35° = 0$$

$\therefore \ F_{PQ} + 3.13 \cos 55° - 7.74 \cos 35° = 0$

$\Rightarrow \quad F_{PQ} = 4.55 \text{ kN}$

Considering block P:

$\Sigma H = 0$

$$F_{PQ} = F_2 = 4.55 \text{ kN}$$

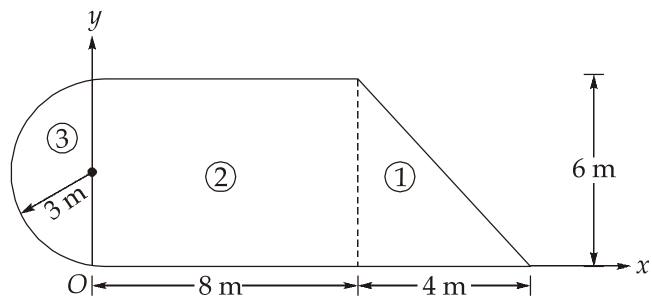
and $\quad F_2 = \mu N_2$

$\Rightarrow \quad N_2 = \dfrac{4.55}{0.3} = 15.17 \text{ kN}$

and $\sum V = 0$

$\Rightarrow$ $W = N_2 = 14.6$ kN

(ii)



The composite section can be divided into three parts i.e., semi-circle, rectangle and triangle.

$$\text{Area of semi-circle, } A_3 = \frac{\pi r^2}{2} = \frac{\pi \times (3)^2}{2} = 14.137 \text{ m}^2$$

$$\text{Area of rectangle, } A_2 = 8 \times 6 = 48 \text{ m}^2$$

$$\text{Area of triangle, } A_1 = \frac{1}{2} \times 4 \times 6 = 12 \text{ m}^2$$

(I) For triangle, the position of centroid;

$$x_1 = 8 + \frac{1}{3} \times 4 = 9.33 \text{ m}$$

$$y_1 = \frac{1}{3} \times 6 = 2 \text{ m}$$

(II) For rectangle, the position of centroid;

$$x_2 = 4 \text{ m}$$
$$y_2 = 3 \text{ m}$$

(III) For semi-circle, the position of centroid;

$$x_3 = \frac{-4r}{3\pi} = \frac{-4 \times 3}{3\pi} = -1.27 \text{ m}$$

$$y_3 = 3 \text{ m}$$

So, for the composite section;

$$\bar{x} = \frac{x_1 A_1 + x_2 A_2 + x_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{9.33 \times 12 + 4 \times 48 + (-1.27 \times 14.137)}{12 + 48 + 14.137} = 3.85 \text{ m}$$

$$\bar{y} = \frac{y_1A_1 + y_2A_2 + y_3A_3}{A_1 + A_2 + A_3}$$

$$= \frac{2\times 12 + 3\times 48 + 3\times 14.137}{12 + 48 + 14.137} = 2.838 \text{ m}$$

Hence, the co-ordinates of centroid of the composite section are (3.85, 2.838).

6. (c)

(i)

The following assumptions are made in this theory:

1. The material of the cylinder is homogeneous, isotropic and obeys Hooke's law. [The stresses are within proportionality limit]
2. Plane transverse sections of the cylinder remain plane under the action of pressure. This assumption is nearly true at a considerable distance from the ends of the cylinder. As a consequence of this assumption, the longitudinal strain remains constant at all points in the cylinder wall, i.e. it is independent of the cylinder radius.
3. All the fibres of the materials are stressed independently without being constrained by the adjacent fibres.

(ii)

Let us use suffix '1' for the shaft and '2' for the collar. Due to radial pressure 'P' at the junction, the shaft will have compressive stress throughout.

Let σ_{h_i} be the hoop stress at the junction.

$$\therefore \quad \sigma_{h_i} = \frac{P(R^2 + r^2)}{R^2 - r^2}$$

$$\sigma_{h_i} = \frac{P(150^2 + 75^2)}{150^2 - 75^2}$$

$$\sigma_{h_i} = 1.666P \quad \text{...(i)}$$

For steel collar,

$$\in_2 = \frac{1}{E}\left(\sigma_{h_i} + \mu P\right)$$

or

$$\frac{0.2}{150} = \frac{1}{2\times 10^5}(1.666P + 0.3P)$$

$$\therefore \quad P = \frac{2 \times 10^5 \times 0.2}{150 \times 1.96} = 136.05 \text{ MPa}$$

∴ The radial pressure between the collar and the shaft,

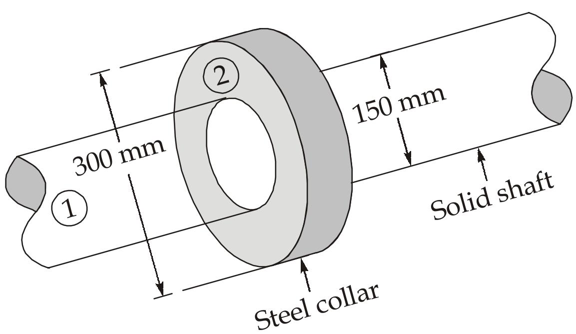
$$\sigma_r = -P = -136.05 \text{ MPa}$$

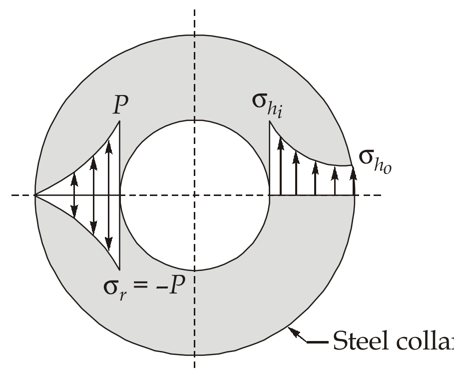
$$= 136.05 \text{ MPa (Compressive)}$$ **Ans. (a)**

From equation (i),

$$\sigma_{h_i} = 1.666 \times 136.05$$

$$= 226.66 \text{ MPa (Tension)}$$ **Ans. (b)**





For shaft, circumferential strain,

$$\in_1 = \frac{\delta d_1}{d_1} = \frac{P}{E} - \mu \frac{P}{E}$$

$$\frac{\delta d_1}{d_1} = \frac{P}{E}(1 - \mu)$$

$$\therefore \quad \delta d_1 = \frac{136.05 \times 150}{2 \times 10^5}(1 - 0.3)$$

$$\delta d_1 = 0.0714 \text{ mm}$$ **Ans. (c)**

7. (a)

(i)

For a plate in biaxial stress, to find the normal stresses σ_x and σ_y in the x and y directions, respectively, based upon the measured normal strains ε_x and ε_y :

$$\sigma_x = \frac{E}{1 - \mu^2}\left(\varepsilon_x + \mu \varepsilon_y\right)$$

$$= \frac{75 \times 10^3}{1-(0.33)^2}(-0.00060 + 0.33 \times 0.00130)$$

$$= -14.39 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\mu^2}\left(\varepsilon_y + \mu\varepsilon_x\right)$$

$$= \frac{75 \times 10^3}{1-(0.33)^2}(0.00130 + 0.33 \times (-0.00060))$$

$$= 92.75 \text{ MPa}$$

The normal strain in z-direction;

$$\varepsilon_z = -\frac{\mu}{E}(\sigma_x + \sigma_y)$$

$$\Rightarrow \quad \varepsilon_z = -\frac{0.33}{75 \times 10^3}(-14.39 + 92.75)$$

$$= -3.4478 \times 10^{-4}$$

Hence, the change in thickness,

$$\Delta t = \varepsilon_z \times t = -3.4478 \times 10^{-4} \times 5$$

$$= -1.7239 \times 10^{-3} \text{ mm}$$

Negative sign signifies that there is decrease in thickness of the aluminium plate for the given stress loading.

$$\text{Unit volume change, } \frac{\Delta V}{V} = \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y\right)$$

$$= \frac{1 - 2 \times 0.33}{75 \times 10^3}(-14.39 + 92.75)$$

$$= 3.552 \times 10^{-4}$$

The positive sign signifies that there is an increase in volume.

$$\text{Strain-energy density, } u = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\mu\sigma_x\sigma_y\right)$$

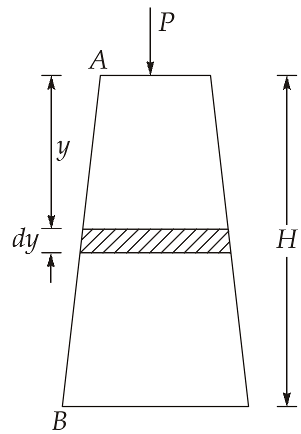
$$= \frac{1}{2 \times 75 \times 10^3}\left((-14.39)^2 + (92.75)^2 - 2 \times 0.33 \times (-14.39) \times 92.75\right)$$

$$= 64.6 \text{ kPa}$$

(ii)

Side of square at $A = b$

Side of square at $B = 1.5b$



Let us take an element dy at a distance of y from top.

$$\therefore \text{Side of the elemental square} = b + \frac{(1.5b - b)y}{H} = \frac{b}{H}(H + 0.5y)$$

$$\therefore \qquad \text{Area, } A_y = \left[\frac{b}{H}(H + 0.5y)\right]^2$$

Shortening of element dy;

$$d\delta = \frac{Pdy}{EA_y} = \frac{Pdy}{E\left(\frac{b^2}{H^2}(H + 0.5y)^2\right)}$$

Shortening of the entire post;

$$\delta = \int d\delta = \frac{PH^2}{Eb^2}\int_0^H \frac{dy}{(H + 0.5y)^2}$$

$$= \frac{PH^2}{Eb^2}\left[\frac{-1}{0.5(H + 0.5y)}\right]_0^H$$

$$= \frac{PH^2}{Eb^2}\left[\frac{-1}{0.5(1.5H)} + \frac{1}{0.5H}\right]$$

$$= \frac{PH^2}{Eb^2}\left[\frac{2}{3H}\right] = \frac{2PH}{3Eb^2}$$

Hence, the shortening of the post is given by $\frac{2PH}{3Eb^2}$ (Proved).

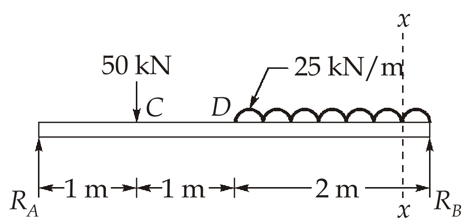
7. (b)

Taking moments about 'B' we have,

$$R_A \times 4 = 50 \times 3 + 25 \times 2 \times 1$$

$$R_A = 50 \text{ kN}$$

and $$R_B = 100 - 50 = 50 \text{ kN}$$



Takng a section x-x near the end B,

$$M_x = R_A \times x - 50(x-1) - \frac{25}{2}(x-2)^2$$

$$= 50x - 50(x-1) - 12.5(x-2)^2$$

Now, $$EI \cdot \frac{d^2y}{dx^2} = -M_x$$

$\therefore$ $$EI \cdot \frac{d^2y}{dx^2} = -50x + 50(x-1) + 12.5(x-2)^2$$

Integrating, we get

$$EI \cdot \frac{dy}{dx} = -50 \cdot \frac{x^2}{2} + 50\frac{(x-1)^2}{2} + 12.5\frac{(x-2)^3}{3} + c_1$$

or $$EI \cdot \frac{dy}{dx} = -25x^2 + 25(x-1)^2 + 4.166(x-2)^3 + c_1$$

Integrating again, we get

$$EIy = -25 \cdot \frac{x^3}{3} + 25\frac{(x-1)^3}{3} + 4.166\frac{(x-2)^4}{4} + c_1x + c_2$$

$\therefore$ $$EIy = \frac{-25x^3}{3} + \frac{25(x-1)^3}{3} + \frac{4.166(x-2)^4}{4} + c_1x + c_2$$

At $x = 0$, $y = 0$ and at $x = 4$, $y = 0$

$\therefore \quad c_2 = 0$

and
$$0 = \frac{-25(4)^3}{3} + \frac{25(3)^3}{3} + 1.04 \times 2^4 + c_1 \times 4$$

$$\therefore \quad c_1 = \frac{291.7}{4} = 72.92$$

$$\therefore \quad EIy = \frac{-25}{3}x^3 + \frac{25}{3}(x-1)^3 + 1.04(x-2)^4 + 72.92x$$

(a) at $x = 2$ m,

$$EIy = \frac{-25}{3}(2)^3 + \frac{25}{3}(2-1)^3 + 0 + 72.92 \times 2$$

$$y = \frac{87.5}{EI}$$

$$\therefore \quad y = \frac{87.5 \times 10^3}{25 \times 10^{-6} \times 210 \times 10^9}$$

$$y = 16.67 \text{ mm}$$

(b) For maximum deflection,

$$\frac{dy}{dx} = 0$$

$$\therefore \quad -25x^2 + 25(x-1)^2 + 4.166(x-2)^3 + 72.92 = 0$$

$$-25x^2 + 25(x^2 + 1 - 2x) + 4.166(x^3 - 8 - 6x^2 + 12x) + 72.92 = 0$$

$$25 - 50x + 4.166x^3 - 33.33 - 25x^2 + 50x + 72.92 = 0$$

$$4.166x^3 - 25x^2 + 64.6 = 0$$

On solving, we get

$$x = 1.96 \text{ m}$$

$$\therefore \quad EIy_{max} = -\frac{25}{3}(1.96)^3 + \frac{25}{3}(1.96-1)^3 + 72.92 \times 1.96$$

$$y_{max} = \frac{87.55}{EI}$$

$$\therefore \quad y_{max} = \frac{87.55 \times 10^3}{210 \times 10^9 \times 25 \times 10^{-6}}$$

$$y_{max} = 16.67 \text{ mm}$$

© Copyright: MADE EASY

(c) Slope at $x = 0$,

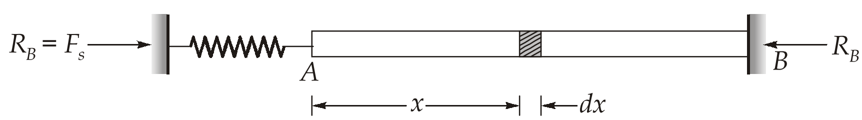
$$EI \cdot \frac{dy}{dx} = c_1$$

$$\therefore \qquad \theta_A = \frac{72.92 \times 10^3}{210 \times 10^9 \times 25 \times 10^{-6}}$$

$$\theta_A = 0.01389 \text{ rad.}$$

7. (c)

(i)



Consider an element dx at a distance x from end A.

$(d\delta)_1$ = Elongation of element dx due to temperature change

$$\therefore \qquad (d\delta)_1 = \alpha \Delta T(dx) = \alpha \Delta T_B \left(\frac{x^3}{L^3}\right) dx$$

$$\therefore \qquad \delta_1 = \int_0^L (d\delta)_1 = \int_0^L \alpha \Delta T_B \frac{x^3}{L^3} dx$$

$$= \frac{1}{4} \alpha (\Delta T_B) L$$

$$\text{Axial deformation, } \delta_2 = \frac{R_B L}{AE} + \frac{F_S}{K}$$

$$= \frac{R_B L}{AE} + \frac{R_B}{K} \qquad (\because F_S = R_B)$$

Using compatibility equation;

$$\delta_1 + \delta_2 = 0$$

$$\Rightarrow \qquad \frac{1}{4} \alpha (\Delta T_B) L + \frac{R_B L}{AE} + \frac{R_B}{K} = 0$$

$$\Rightarrow \qquad R_B \left(\frac{L}{AE} + \frac{1}{K}\right) = -\frac{\alpha (\Delta T_B) L}{4}$$

$$\Rightarrow \qquad R_B = \frac{-\alpha (\Delta T_B) L}{4\left(\frac{L}{AE} + \frac{1}{K}\right)} = \frac{-\alpha (T_B) \times L \times AEK}{4(KL + AE)}$$

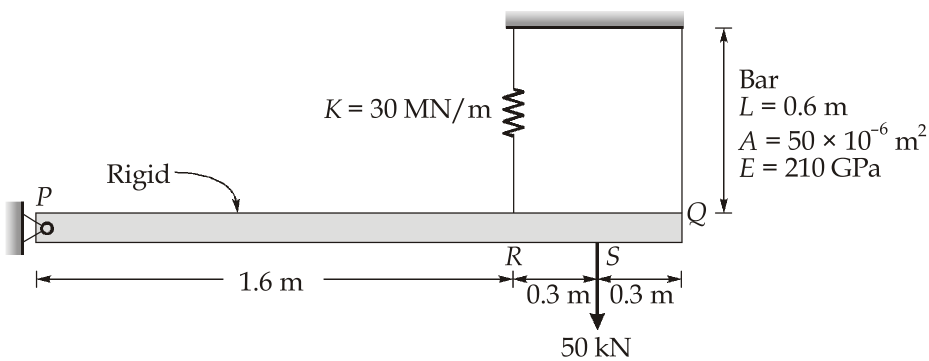
$$\Rightarrow \quad \frac{R_B}{A} = \frac{-\alpha \Delta T_B \times L \times EK}{4(EA + KL)}$$

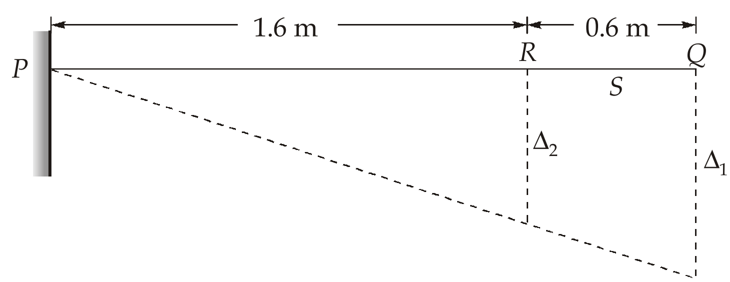
$$\Rightarrow \quad \sigma_c = \frac{\alpha(\Delta T_B) LKE}{4KL\left(\frac{EA}{KL} + 1\right)}$$

$$\Rightarrow \quad \sigma_c = \frac{\alpha(\Delta T_B) E}{4\left(\frac{EA}{KL} + 1\right)}$$

(ii)

Given : $K = 30$ MN/m^2; $A = 50 \times 10^{-6}$ m^2; $E = 210$ GPa; $P = 50$ kN; $L_{\text{steel rod}} = 0.6$ m





Taking moment about P:

$$P \times 1.9 = F_S \times 1.6 + T \times 2.2$$

$$\Rightarrow \quad F_S \times 1.6 + T \times 2.2 = 50 \times 1.9 \times 1000$$

$$\Rightarrow \quad F_S \times 1.6 + T \times 2.2 = 95000 \quad \text{...(i)}$$

According to deformation equation;

$$\frac{\Delta_1}{2.2} = \frac{\Delta_2}{1.6}$$

$$\Rightarrow \quad \frac{T \times L}{AE \times 2.2} = \frac{F_S}{K \times 1.6}$$

$$\Rightarrow \quad \frac{T \times 0.6}{50 \times 10^{-6} \times 210 \times 10^{9} \times 2.2} = \frac{F_S}{30 \times 10^{6} \times 1.6}$$

$$\Rightarrow \quad T = 0.802\, F_s \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{T}{0.802} \times 1.6 + T \times 2.2 = 95000$$

$$\Rightarrow \quad T = 22645.939 \text{ N}$$

∴ Vertical displacement of point Q;

$$\frac{TL}{AE} = \frac{22645.939 \times 0.6}{50 \times 10^{-6} \times 210 \times 10^{9}} \times 1000$$

$$= 1.29 \text{ mm}$$

8. (a)

Let D_s = Diameter of solid shaft; D_H = Outer diameter of hollow shaft; d_H = Inner diameter of hollow shaft

Now, for solid shaft torque, $T = \frac{\pi}{16} \tau D_s^3$

$$\because \quad T = \frac{60P}{2\pi N} = \frac{60 \times 300 \times 10^{3}}{2\pi \times 100}$$

$$T = 28647.89 \text{ Nm}$$

$$\therefore \quad 28647.89 \times 10^3 = \frac{\pi}{16} \times 70 \times D_s^3$$

$$\therefore \quad D_s^3 = \frac{16 \times 28647.89 \times 10^{3}}{\pi \times 70}$$

$$D_s = 127.74 \text{ mm}$$

For hollow shaft, since the weights are the same we have,

$$\frac{\pi}{4} D_s^2 \cdot L = \frac{\pi}{4}\left(D_H^2 - d_H^2\right) L$$

$$(127.74)^2 = D_H^2 \times 0.75$$

$$\therefore \quad D_H = 147.5 \text{ mm}$$

$$\therefore \quad d_H = \frac{D_H}{2} = 73.75 \text{ mm}$$

Now, for hollow shaft,

$$T = \frac{\pi}{16}\tau D_H^3(1-k^4)$$

$$T = \frac{\pi}{16}\times 70\times 147.5^3(1-0.5^4) = 41349.97 \text{ Nm}$$

$\therefore$ $$P = \frac{2\pi NT}{60}$$

$\therefore$ $$N = \frac{60P}{2\pi T} = \frac{60\times 300\times 10^3}{2\pi\times 41349.97} = 69.28 \text{ rpm}$$

$\therefore$ $$\text{Change in speed} = \frac{100-69.28}{100}\times 100 = 30.72\%$$

8. (b)

(i)

Velocity of coolar, $V = 8$ m/s

Time, $t = 3$s

Acceleration of collar,

$\therefore$ $$V = u + at$$

$\Rightarrow$ $$8 = 0 + a \times 3$$

$\Rightarrow$ $$a = \frac{8}{3} = 2.67 \text{ m/s}^2$$

FBD of collar

$\therefore$ $$F - 2T = m \times a$$

$\Rightarrow$ $$F - 2T = 8\times\frac{8}{3} = \frac{64}{3}\text{ N} \quad \text{...(i)}$$

If the acceleration of collar is a, then acceleration of mass A will be $2a$ because if collar moves a distance x in the horizontal direction then during same time, mass A goes up by $2x$ distance.

For mass A:

$$T - 3g = m_A \times a'$$

$$T - 3g = 3 \times 2 \times a = 6a = 6\times\frac{8}{3}$$

$\Rightarrow$ $$T - 3g = \frac{48}{3}$$

$\Rightarrow$ $$T = \frac{48}{3} + 3 \times 9.81 = 45.43 \text{ N}$$

So, from equation (i),

$$F = \frac{64}{3} + 2 \times 45.43 = 112.19 \text{ N}$$

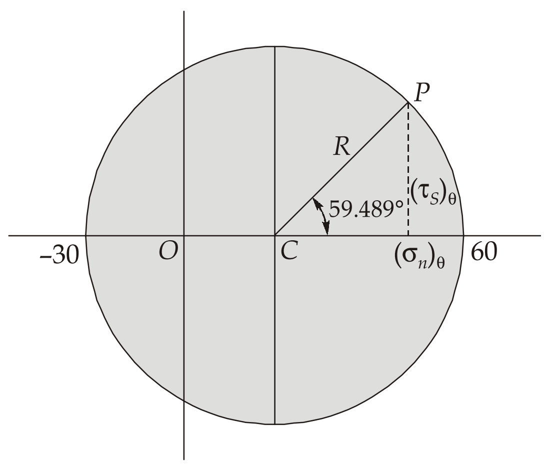
(ii)

Given :

$$\sigma_x = -30 \text{ MPa}$$
$$\sigma_y = 60 \text{ MPa}$$
$$\tau_{xy} = 0$$

$$\theta = \tan^{-1}\frac{2}{3.5} = 29.74°$$

$\Rightarrow$ $$2\theta = 59.489°$$



(a)

$$C = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{-30+60}{2}, 0\right)$$
$$= (15, 0)$$

Also, $$\text{Radius } (R) = \frac{|\sigma_x| + |\sigma_y|}{2} = \frac{30+60}{2} = 45$$

At point P:

$$(\sigma_n)_\theta = \sigma_c + R\cos 2\theta$$
$$= 15 + 45\cos 59.489°$$
$$= 37.846 \text{ MPa}$$

$$(\sigma_s)_\theta = R\sin 2\theta = 45\sin 59.489 = 38.768 \text{ MPa}$$

(b)

Maximum shear stress:

$$\tau_{max} = R = 45 \text{ MPa}$$

Associated normal stress:

$$\sigma_n^* = \sigma_c = \frac{\sigma_x + \sigma_y}{2} = \frac{-30+60}{2} = 15 \text{ MPa}$$

8. (c)

(i)

Given : σ_1 = 70 MPa; σ_2 = 50 MPa; σ_3 = –30 MPa; σ_{per} = 150 MPa; E = 200 GPa; ν = 0.3

(a) Total strain energy per unit volume

$$= \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$= \frac{10^{14}}{2 \times 200 \times 10^9}[49 + 25 + 9 - 0.6(35 - 15 - 21)]$$

$$= \frac{10^3}{4}[83 + 0.6] = 20.9 \text{ kNm/m}^3$$

(b) Volumetric strain energy per unit volume

$$= \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)^2\left(\frac{1-2\mu}{2E}\right)$$

$$= \frac{1}{3}(7+5-3)^2\left(\frac{1-2\times 0.3}{2\times 200\times 10^9}\right)\times 10^{14}$$

$$= \frac{0.4 \times 81 \times 10^3}{12} = 2.7 \text{ kNm/m}^3$$

(c) Shear strain energy per unit volume is

$$u = \frac{1}{12G}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)$$

Now,

$$G = \frac{E}{2(1+\mu)} = \frac{200\times 10^9}{2\times 1.3} = 76.923 \text{ GPa}$$

$\therefore$

$$U = \frac{10^{14}}{12 \times 76.923 \times 10^9}\left[(7-5)^2 + (5+3)^2 + (-3-7)^2\right]$$

$$= \frac{10^5}{923.07}[4 + 64 + 100] = 18.2 \text{ kNm/m}^3$$

Now, strain energy per unit volume under uniaxial loading is

$$= \frac{\sigma^2}{2E} = \frac{(150)^2 \times 10^{12}}{2 \times 200 \times 10^9} = 56.25 \text{ kNm/m}^3$$

$\therefore$ $$\text{Factor of safety} = \frac{56.25}{20.9} = 2.69$$

(ii)

Let the dimensions of the column at a distance x m below the top be d_x,

$\therefore$ $$d_x = 75 + \frac{x}{7.5}(150 - 75) = 75 + 10x \text{ mm}$$

$$I_x = \frac{d_x^4}{12} \text{ and } y_{max} = \frac{d_x}{\sqrt{2}} \text{mm}$$

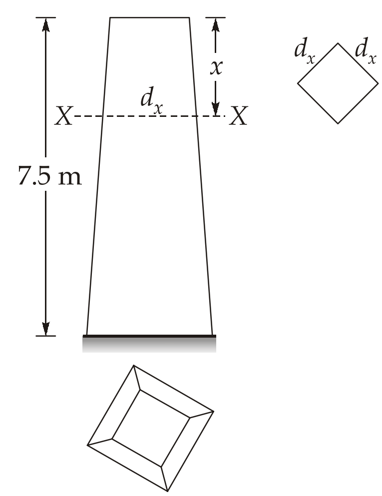
$\therefore$ $$z_x = \frac{I_x}{y_{max}} = \frac{d_x^3}{6\sqrt{2}}$$

or $$z_x = \frac{(75+10x)^3}{6\sqrt{2}}$$

Also, $$M_x = 5000x \text{ Nm} = 5 \times 10^6\, x \text{ N.mm}$$

$\therefore$ $$\sigma_x = \frac{M_x}{z_x} = \frac{5 \times 10^6 \times 6\sqrt{2} \cdot x}{(75+10x)^3}$$

$$\sigma_x = \frac{42.42x \times 10^6}{(75+10x)^3}$$



For maximum bending stress,

$$\frac{d\sigma_x}{dx} = 0$$

$\therefore$ $$42.42\left[\frac{(75+10x)^3 - x \cdot 3(75+10x)^2 \cdot 10}{(75+10x)^6}\right] = 0$$

or $(75 + 10x)^2(75 + 10x - 30x) = 0$

$\therefore$ $$75 - 20x = 0$$

$\Rightarrow$ $$x = 3.75 \text{ m}$$

$\therefore$ $$\sigma_{max} = \frac{42.42 \times 3.75 \times 10^6}{(75 + 10 \times 3.75)^3} = 111.72 \text{ N/mm}^2$$

❍❍❍❍