MADE EASY

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Detailed Solutions

ESE-2023 Mains Test Series

Civil Engineering Test No : 1

Section A : Strength of Materials

Q.1 (a) Solution:

$$\text{Modulus of elasticity of copper } (E) = 2G(1+\mu)$$

$$\Rightarrow \quad E = 2 \times 40000\,(1 + 0.35)$$

$$\Rightarrow \quad E = 108000 \text{ N/mm}^2$$

(i) When the ends do not yield

Due to fall in temperature, bar will shorten in length,

$$\delta l = \alpha \Delta T\, l$$

$$= 0.0000175 \times (80 - 20) \times 5000$$

$$= 5.25 \text{ mm}$$

As this bar does not yield, tensile stress gets generated, i.e.,

$$\sigma = \frac{\delta l}{l} \times E = \frac{5.25}{5000} \times 108000 = 113.4 \text{ N/mm}^2$$

So, Tensile force in bar $= \sigma \times A = 113.4 \times 305$

$= 34587 \text{ N} = 34.587 \text{ kN}$

(ii) When the ends yield by 2.5 mm

Restrained change in length due to fall in temperature,

$$\delta l = (\alpha\, \Delta T l - 2.5)$$

$$= 0.0000175 \times (80 - 20) \times 5000 - 2.5$$

$$= 2.75 \text{ mm}$$

Tensile stress developed, $\sigma = \frac{\delta l}{l} \times E = \frac{2.75}{5000} \times 108000 = 59.4 \text{ N/mm}^2$

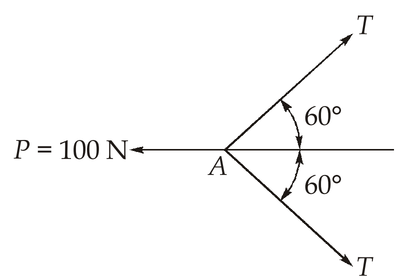
$\therefore$ Force in bar $= 59.4 \times 305 = 18117 \text{ N} = 18.117 \text{ kN}$

Q.1 (b) Solution:

Suppose,

$$P = 100 \text{ N}$$
$$\alpha = 60°$$
$$H = 1500 \text{ mm} = 1.5 \text{ m}$$
$$b = 300 \text{ mm} = 0.3 \text{ m}$$

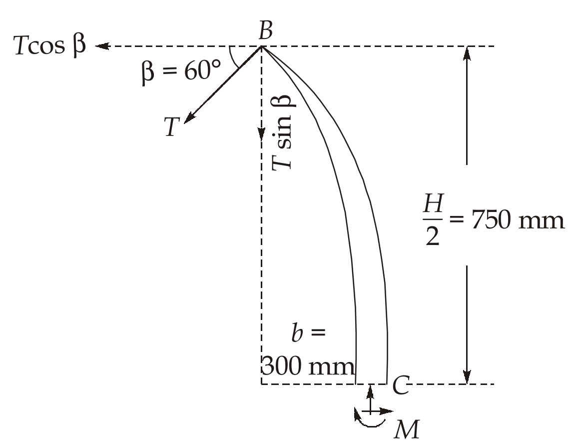
Free body diagram of point-A:



Let T be the tensile force in the bow string

$$\because \quad \Sigma F_H = 0$$
$$\Rightarrow \quad 2T \cos 60° = 100 \text{ N}$$
$$\Rightarrow \quad T = 100 \text{ N}$$

Now, free body diagram of segment BC is as shown below,



$$\Sigma M_C = 0$$

$$\Rightarrow \quad -T\cos\beta \times \frac{H}{2} - T\sin\beta \times b + M = 0$$

$$\Rightarrow \quad M = 100 \times \cos 60° \times 0.75 + 100 \times \sin 60° \times 0.3$$

$$\Rightarrow \quad M = 63.48 \text{ Nm} \approx 63.5 \text{ N-m}$$

So, bending moment at midpoint of the bow is 63.5 N-m

Q.1 (c) Solution:

(i) Stress in x-direction,

$$\sigma_x = \frac{E}{(1-\mu^2)}(\epsilon_x + \mu\,\epsilon_y) = \frac{0.8\times10^5}{(1-0.3^2)}(0.00088 + 0.3\times0.00022)$$

$$= 83.16 \text{ MPa}$$

Stress in y-direction,

$$\sigma_y = \frac{E}{(1-\mu^2)}(\epsilon_y + \mu\epsilon_x) = \frac{0.8\times10^5}{(1-0.3^2)}(0.00022 + 0.3\times0.00088)$$

$$= 42.55 \text{ MPa}$$

(ii) $\theta = 30°$ to x-direction

$$\therefore \quad \epsilon_{30} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{0.00088 + 0.00022}{2}\right) + \left(\frac{0.00088 - 0.00022}{2}\right)\cos(2\times30°)$$

$$= 0.00055 + 0.000165$$

$$= 0.000715 = 7.15 \times 10^{-4}$$

(iii) Normal stress on the inclined plane,

$$\sigma_{30} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$

$$= 83.16 \cos^2 30° + 42.55 \sin^2 30°$$

$$= 62.37 \text{ MPa} + 10.6375 \text{ MPa}$$

$$= 73 \text{ MPa}$$

Shear stress on the inclined plane,

$$\tau_{30} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$= \left(\frac{83.16 - 42.55}{2}\right)\sin(2\times30°) = 17.58 \text{ MPa}$$

Q.1 (d) Solution:

Outer diameter, $D = 70$ mm

Inner diameter, $d = D - 2t = (70 - 2t)$

Critical load, $P_{cr} = FOS \times P = 2 \times 35 = 70$ kN

Moment of Inertia, $I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(70^4 - (70-2t)^4)$

We know that, $P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{L^2}$ [$\because L_e = L = 3$ m since pin jointed]

$$\Rightarrow \quad 70 \times 10^3 = \frac{\pi^2 \times 210 \times 10^3 \times \frac{\pi}{64}(70^4 - (70-2t)^4)}{3000^2}$$

$$\Rightarrow \quad 70^4 - (70 - 2t)^4 = 6192294.611$$

$$\Rightarrow \quad (70 - 2t)^4 = 17817705.39$$

$$\Rightarrow \quad 70 - 2t = 64.97$$

$$\Rightarrow \quad 2t = 70 - 64.97 = 5.03 \text{ mm}$$

$$\Rightarrow \quad t = 2.515 \text{ mm} \approx 2.52 \text{ mm}$$

So, the minimum required thickness of the tube is 2.52 mm

Q.1 (e) Solution:

(i) Given, $f = 15$ Hz $= 15$ rps

$P = 25$ kW

We know that, $P = \frac{2\pi NT}{60} = 2\pi f\, T$

$$\Rightarrow \quad 25 \times 10^3 = 2\pi \times 15 \times T$$

$$\Rightarrow \quad T = 265.258 \text{ N-m}$$

(*a*) Maximum shear stress developed,

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16 \times 265.258}{\pi \times (0.030)^3} \text{N/m}^2 = 50.035 \text{ MPa}$$

(*b*) Minimum permissible diameter of the shaft,

$$\tau_{allowable} = 40 \text{ MPa}$$

$$\Rightarrow \quad \frac{16T}{\pi d^3} = 40 \times 10^6$$

$$\Rightarrow \qquad \frac{16 \times 265.258}{\pi d^3} = 40 \times 10^6$$

$$\Rightarrow \qquad d = 0.03232 \text{ m} = 32.32 \text{ mm}$$

(ii) (*a*) **St. Venant's theory or Maximum principal strain theory:**

According to this theory, a ductile material begins to yield when the maximum principle strain reaches the strain at which yielding occurs in uniaxial loading test.

Mathematically for no failure,

$$\in_1 \le \frac{\sigma_y}{E}$$

For 3-dimensional case, $\in_1 = \dfrac{\sigma_1}{E} - \dfrac{\mu\sigma_2}{E} - \dfrac{\mu\sigma_3}{E}$

Some limitations of this theory are:

- It is not suitable for hydrostatic loading.
- It can not be applied for pure shear.
- It overestimates the elastic strength of ductile material.
- This theory is not applicable for brittle matieral.

(*b*) **Guest's theory or Maximum shear stress theory:**

According to this theory, for no failure, maximum shear stress developed in a strained body should be less than or equal to maximum shear stress in uniaxial loading.

Mathematically, $\tau_{max} \le \dfrac{\sigma_y}{2}$

$$\Rightarrow \qquad \left|\frac{\sigma_1 - \sigma_2}{2}\right| \le \frac{\sigma_y}{2}$$

$$\Rightarrow \qquad |\sigma_1 - \sigma_2| \le \sigma_y$$

Some limitations of this theory are:

- It is not applicable when principle stresses are nearly equal and alike.
- It is not applicable for brittle materials.
- It results in oversafe result for ductile material.

Q.2 (a) Solution:

Determination of loading diagram:

$$V_A = +14 \text{ kN}$$

$\Rightarrow$ $$R_A = 14 \text{ kN}$$

$$V_B^- = -6 \text{ kN}$$

$\because$ Inclined shear force diagram means UDL in loading diagram

$\because$ $$w = \frac{dV}{dx}$$

$\Rightarrow$ $$w = \frac{14-(-6)}{2} = 10 \text{ kN/m}$$

Now, vertical downward SF means concentrated load at joint

$\therefore$ $$V_B^+ = -18 \text{ kN}$$

$\Rightarrow$ $$-6 + P_B = -18$$

$\Rightarrow$ $$P_B = -18 + 6 = -12 \text{ kN}$$

$$= 12 \text{ kN (downward)}$$

Now, $$V_C^- = -18 \text{ kN}$$

$$V_C^+ = 10 \text{ kN}$$

Reaction force at support C;

$$-18 + R_C = 10 \text{ kN}$$

$\Rightarrow$ $$R_C = 10 + 18 = 28 \text{ kN}$$

Now, for portion CD it is given that,

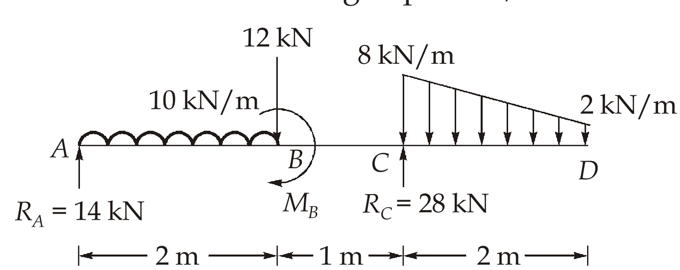
$$V = \frac{3x^2}{2} - 8x + 10$$

$\Rightarrow$ $$w = \frac{dV}{dx} = 3x - 8$$

At point C ($x = 0$) ; $F = -8$ kN

At point D ($x = 2$ m) ; $F = 3 \times 2 - 8 = -2$ kN

Now, given a concentrated moment is acting at point B;



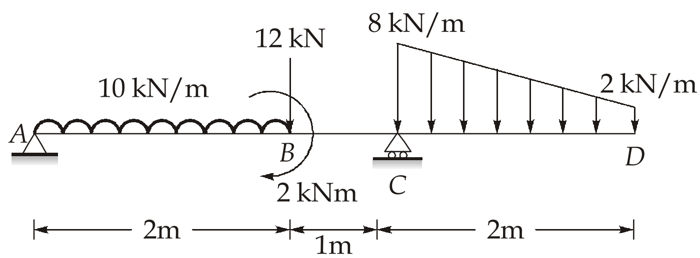
$$\because \qquad \Sigma M_A = 0$$

$$\Rightarrow 10\times2\times1+12\times2+M_B-28\times3+\frac{1}{2}\times6\times2\times\left(3+\frac{2}{3}\right)+(2\times2)(3+1) = 0$$

$$\Rightarrow \qquad M_B - 2 = 0$$

$$\Rightarrow \qquad M_B = 2\text{ kNm}$$

Required loading diagram is:

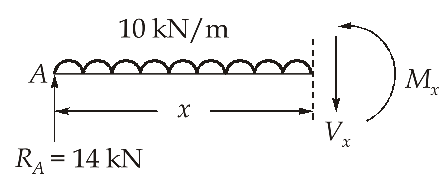


Determination of bending moment diagram:

- Section AB ($0 \le x \le 2$ m, x from A)

$$\Sigma M_x = 0$$

$$\Rightarrow \qquad R_A\times x - 10\times\frac{x^2}{2} - M_x = 0$$



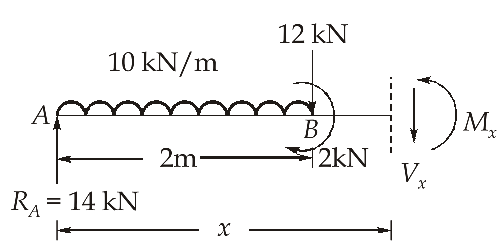
$$\Rightarrow \qquad M_x = 14x - \frac{10x^2}{2}$$

At $x = 0$; $\qquad M_A = 0$ kNm

At $x = 2$ m; $\qquad M_B^- = 14\times2 - \frac{10\times2^2}{2}$

$$= 8\text{ kNm}$$

- Section BC ($2\text{ m} \le x \le 3$ m, x from A)



$$\Sigma M_x = 0$$

$$\Rightarrow R_A\cdot x - 10\times2(x-1) - 12(x-2) + 2 - M_x = 0$$

$$\Rightarrow \qquad M_x = -18x + 46$$

At $x = 2$ m ; $\quad M_B^+ = -18(2) + 46$

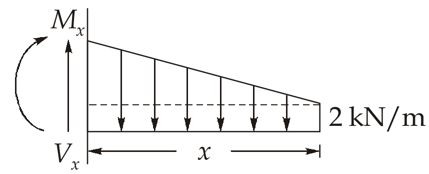
$= 10$ kNm

At $x = 3$ m ; $\quad M_C = -18(3) + 46 = -8$ kNm

- Section DC ($0 \le x \le 2$ m, x from D)

At $x = 2$ m ; change in load $= (8 - 2) = 6$ kN/m

At $x = x$ m ; change in load $= \frac{6}{2}x = 3x$ kN/m



$\therefore$ Load at section x-x $= (2 + 3x)$ kN/m

$$\Sigma M_x = 0$$

$$\Rightarrow \quad M_x + \left(\frac{3x}{2}\right) \times \frac{x^2}{3} + 2x \times \frac{x}{2} = 0$$

$$\therefore \quad M_x = \frac{-x^3}{2} - x^2$$

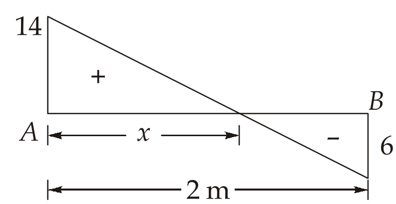
$\therefore$ At $x = 0$, $\quad M_D = 0$ kNm

At $x = 2$ m ; $\quad M_C = \frac{-2^3}{2} - 2^2 = -8$ kNm

Now, since, shear force is changing sign in section AB ;

So, bending moment in section AB is,

$$M_x = 14 \times x - 10 \times \frac{x^2}{2}$$



From similar triangles, $\quad \frac{14}{x} = \frac{6}{(2 - x)}$

$$\Rightarrow \quad x = 1.4 \text{ m}$$

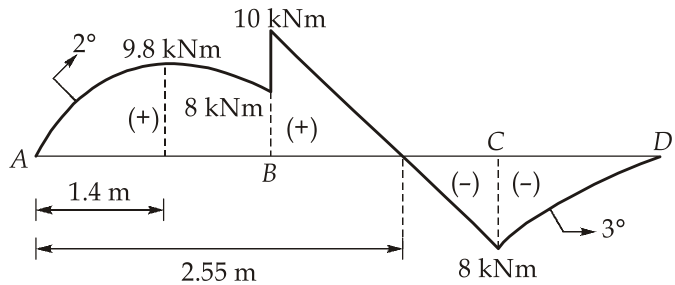
$$\therefore \quad M_{1.4\text{ m}} = 14 \times 1.4 - 10 \times \frac{1.4^2}{2} = 9.8 \text{ kN-m}$$

Now, bending moment changes sign at a distance x from point A in section BC, such that,

$$M_x = -18x + 46 = 0$$

$$\Rightarrow \quad x = \frac{46}{18} \text{m} = 2.55 \text{ m}$$

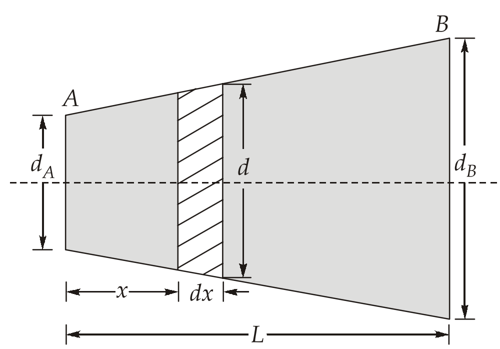
Required bending moment diagram is shown below,



∴ Maximum binding moment for given beam is 10 kNm and point of contraflexure is 2.55 m from point A.

Q.2 (b) Solution:

(i) Consider a shaft which tapers uniformly from d_A to d_B from end A to end B and is subjected to a twisting moment T.



The diameter d at a distance x from left end A is given by,

$$d = d_A + \left(\frac{d_B - d_A}{L}\right)x$$

For a small length dx , the angle of twist is $d\theta$.

Hence, $$\frac{T}{J_x} = \frac{G \times d\theta}{dx}$$

where, J_x = Polar moment of inertia corresponding to diameter, $d = \frac{\pi}{32}d^4$

G = Modulus of rigidity

$$d\theta = \frac{Tdx}{G \times \frac{\pi}{32}d^4}$$

$$\Rightarrow \qquad d\theta = \frac{32T\,dx}{G\times\pi\times(d_A + K\cdot x)^4} \qquad \left[\text{Where, } K = \frac{d_B - d_A}{L}\right]$$

$\therefore$ The total angle of twist for the total length of shaft is obtained by integrating the above equation

$$\therefore \qquad \theta = \int d\theta = \int_0^L \frac{32T\,dx}{\pi G(d_A + K\cdot x)^4}$$

$$= \frac{32T}{\pi G}\int_0^L \frac{dx}{(d_A + K\cdot x)^4} = \frac{32T}{\pi G}\frac{\left[(d_A + K\cdot x)^{-3}\right]_0^L}{(-3)(K)}$$

$$= (-)\frac{32T}{\pi G}\times\frac{1}{3K}\left[(d_A + K\cdot L)^{-3} - d_A^{-3}\right]$$

$$= (-)\frac{32T}{\pi G(3K)}\left[\frac{1}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)L\right]^3} - \frac{1}{d_A^3}\right]$$

$$= (-)\frac{32T}{\pi G(3K)}\left[\frac{1}{d_B^3} - \frac{1}{d_A^3}\right] = \frac{32TL}{\pi G\times 3(d_B - d_A)}\left[\frac{1}{d_A^3} - \frac{1}{d_B^3}\right]$$

$$= \frac{32TL}{\pi G\times 3d_A^4\left(\frac{d_B}{d_A} - 1\right)}\left[1 - \frac{1}{\left(\frac{d_B}{d_A}\right)^3}\right]$$

$$= \frac{TL}{\left(\frac{\pi}{32}d_A^4\right)G\times 3(\beta - 1)}\left[1 - \frac{1}{\beta^3}\right] \qquad \left[\because \beta = \frac{d_B}{d_A}\right]$$

$$= \frac{TL(\beta^3 - 1)}{G(I_P)_A(\beta - 1)3\beta^3}$$

$$= \frac{TL(\beta - 1)(\beta^2 + 1 + \beta)}{G(I_P)_A(\beta - 1)3\beta^3} \qquad \{\because \beta^3 - 1^3 = (\beta - 1)(\beta^2 + 1 + \beta)\}$$

$$\therefore \qquad \theta = \frac{TL}{G(I_P)_A}\left(\frac{\beta^2 + \beta + 1}{3\beta^3}\right)$$

(ii) For tapered bar,

Angle of twist, $$(\theta_1) = \frac{TL}{G(I_P)_A}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right)$$

For prismatic bar,

Angle of twist, $$(\theta_2) = \frac{TL}{G(I_P)_A}$$

According to question,

$$\theta_1 = \frac{1}{4}\theta_2$$

$$\Rightarrow \quad \frac{TL}{G(I_P)_A}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right) = \frac{1}{4}\left(\frac{TL}{G(I_P)_A}\right)$$

$$\Rightarrow \quad 4(\beta^2+\beta+1) = 3\beta^3$$

$$\Rightarrow \quad 3\beta^3 - 4\beta^2 - 4\beta - 4 = 0$$

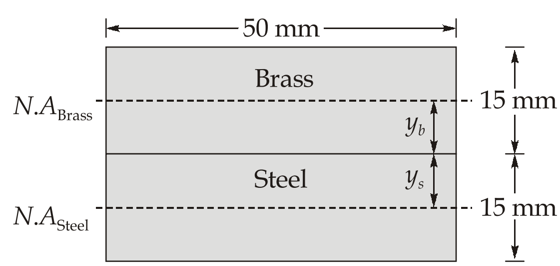
On solving,

$$\beta = 2.2098 \text{ (other roots are imaginary)}$$

Hence, $$\frac{d_B}{d_A} = 2.2098 \approx 2.21$$

Q.2 (c) Solution:

(i) **1st case :** The plates are separate and can bend independently,



Since, two plates bend independently, each will have its own neutral axis, which will lie at the mid-depth of each section.

Assumption : Radius of curvature R is the same for both the plates,

$$\because \quad \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \quad R = \frac{E\cdot y}{\sigma} = \frac{E_s \times y_s}{\sigma_s} = \frac{E_b \times y_b}{\sigma_b}$$

$$\therefore \quad \frac{\sigma_s}{\sigma_b} = \frac{E_s \times y_s}{E_b \times y_b}$$

$$\because \quad y_s = y_b = \frac{15}{2}\text{mm}$$

$$\therefore \quad \frac{\sigma_s}{\sigma_b} = \frac{E_s}{E_b} = \frac{2\times 10^5}{8\times 10^4} = 2.5$$

Given $(\sigma_{all})_{steel} = 115\ \text{N/mm}^2$

$$\therefore \quad \sigma_b = \frac{115}{2.5} = 46\ \text{N/mm}^2$$

If $(\sigma_{all})_{brass} = 80\ \text{N/mm}^2$

then, $\sigma_s = 2.5 \times 80 = 200\ \text{N/mm}^2$

$\sigma_s = 200\ \text{N/mm}^2$ is more than allowable stress in steel

So, take; $\sigma_s = 115\ \text{N/mm}^2$ and $\sigma_b = 46\ \text{N/mm}^2$

$\therefore$ The total moment of resistance $(M) = M_S + M_b$

$$= \frac{\sigma_s}{y_s}\times I_s + \frac{\sigma_b}{y_b}\times I_b$$

$$= \frac{115}{7.5}\times\frac{50\times 15^3}{12} + \frac{46}{7.5}\times\frac{50\times 15^3}{12}$$

$$\therefore \quad M = 301875\ \text{Nmm} = 301.875\ \text{Nm}$$

Let, W = Maximum load applied at centre of a simply supported beam

$$\therefore \quad M = \frac{WL}{4}$$

$$\Rightarrow \quad 301.875 = \frac{W\times 3.0}{4}$$

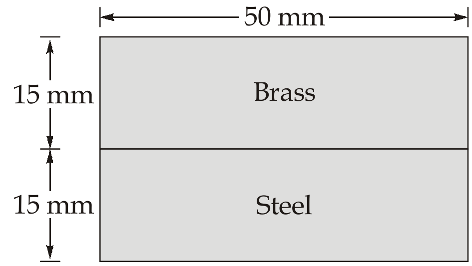
$$\Rightarrow \quad W = 402.5\ \text{N}$$

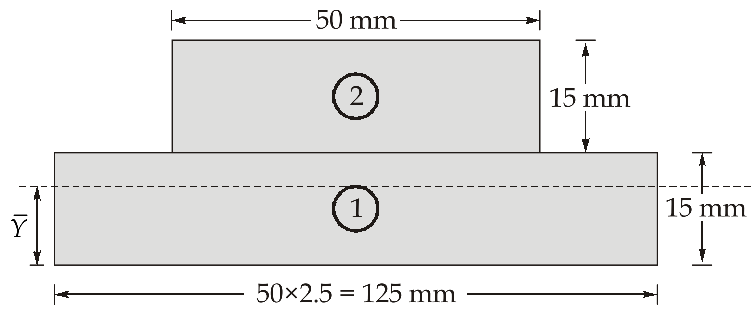
(ii) 2nd case : The plates are firmly secured throughout their length

Here the unit has a single N.A. So, transforming the composite section into an equivalent brass section,

$$\therefore \quad m = \frac{E_s}{E_b} = 2.5$$

So, the equivalent brass width for steel plate will be 50 × 2.5 = 125 mm





Let, $\quad \bar{Y}$ = Distance of C.G of equivalent brass section from bottom face

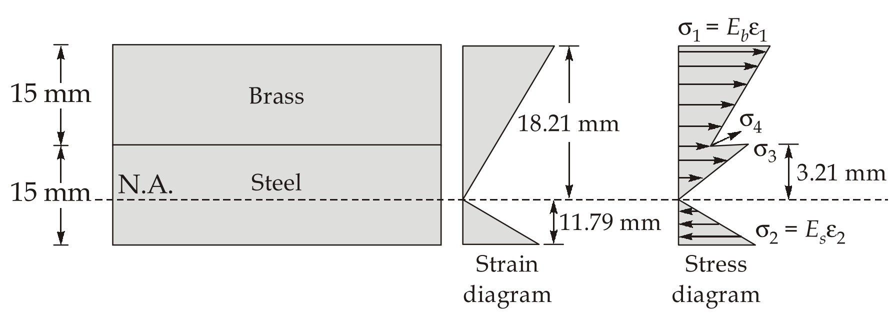
$$\therefore \quad \bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{125 \times 15 \times 7.5 + 50 \times 15 \times (15 + 7.5)}{125 \times 15 + 50 \times 15}$$

$$\Rightarrow \quad \bar{Y} = 11.79 \text{ mm}$$

Now, moment of inertia about NA is given by

$$I = \frac{125 \times 15^3}{12} + 125 \times 15 \times (11.79 - 7.5)^2 + \frac{50 \times 15^3}{12} + 50 \times 15 \times (22.5 - 11.79)^2$$

$$\Rightarrow \quad I = 100090.58 + 69663.94 = 169754.52 \text{ mm}^4$$



$$(\sigma_{all})_{brass} = \sigma_1 = 80 \text{ N/mm}^2$$

$$\because \quad \frac{\sigma_1}{\sigma_4} = \frac{18.21}{3.21} \qquad \text{[within brass]}$$

$$\Rightarrow \quad \sigma_4 = \frac{80 \times 3.21}{18.21} = 14.102 \text{ N/mm}^2$$

$\because \qquad \dfrac{\sigma_4}{\sigma_3} = \dfrac{E_b}{E_s}$ [at junctin of brass and steel]

$\Rightarrow \qquad \sigma_3 = \dfrac{14.102 \times 2 \times 10^5}{8 \times 10^4} = 35.255 \text{ N/mm}^2$

$\dfrac{\sigma_3}{\sigma_2} = \dfrac{3.21}{11.79}$ [within steel]

$\Rightarrow \qquad \sigma_2 = \dfrac{35.255 \times 11.79}{3.21} = 129.49 \text{ N/mm}^2 > (\sigma_{all})_{steel}$

Hence, brass cannot be fully stressed

$\therefore$ Take, $(\sigma_{all})_{steel} = 115 \text{ N/mm}^2 = \sigma_2$

So, $\qquad \dfrac{\sigma_3}{\sigma_2} = \dfrac{3.21}{11.79}$ [within steel]

$\Rightarrow \qquad \sigma_3 = \dfrac{115 \times 3.21}{11.79} = 31.31 \text{ N/mm}^2$

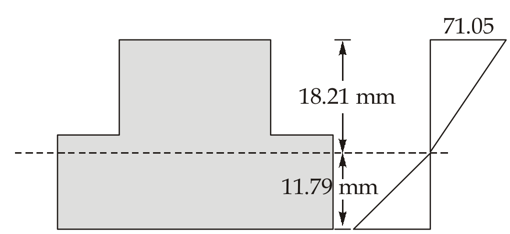
$\dfrac{\sigma_4}{\sigma_3} = \dfrac{E_b}{E_s}$ [at junctin of brass and steel]

$\Rightarrow \qquad \sigma_4 = \dfrac{31.31 \times 8 \times 10^4}{2 \times 10^5} = 12.524 \text{ N/mm}^2$

$\dfrac{\sigma_1}{\sigma_4} = \dfrac{18.21}{3.21}$ [within brass]

$\Rightarrow \qquad \sigma_1 = \dfrac{12.524 \times 18.21}{3.21} = 71.05 \text{ N/mm}^2 < (\sigma_{all})_{brass}$

So, Take : $\sigma_s = 115 \text{ N/mm}^2$ and $\sigma_b = 71.05 \text{ N/mm}^2$



Now, $\qquad M = \dfrac{\sigma}{y} \times I$

$= \dfrac{71.05 \times 169754.52}{18.21}$

$= 662331.61 \text{ N-mm} = 622.33 \text{ Nm}$

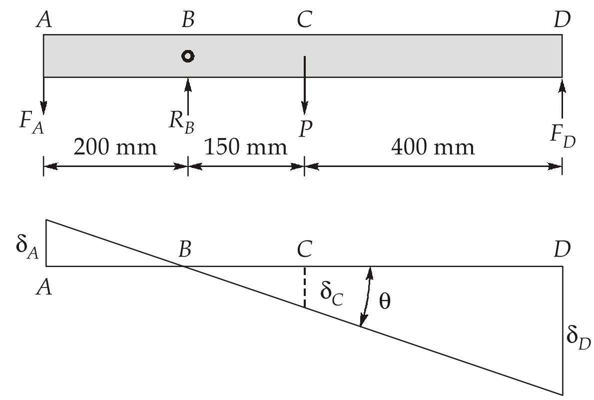
$\Rightarrow \qquad \dfrac{WL}{4} = 662.33 \text{ Nm}$

$\Rightarrow \qquad \dfrac{W \times 3}{4} = 662.33$

$\Rightarrow \qquad W = 883.11 \text{ N}$

Q.3 (a) Solution:

Free body diagram and displacement diagram are shown below,



Equation of equilibrium:

$\because$ $$\Sigma M_B = 0$$

$$\Rightarrow \quad F_A \times (200) - P \times 150 + F_D \times 550 = 0 \quad \text{...(i)}$$

Compatibility equation, $$\frac{\delta_A}{200} = \frac{\delta_D}{(400+150)} = \frac{\delta_D}{550} \quad \text{...(ii)}$$

Also, $$\delta_A = \frac{F_A}{K_A}$$

and $$\delta_D = \frac{F_D}{K_D} \quad \text{...(iii)}$$

From equation (ii) and (iii),

$$\Rightarrow \quad \frac{F_A}{200\,K_A} = \frac{F_D}{550\,K_D} \quad \text{...(iv)}$$

Solving equation (i) and (iv),

$$\Rightarrow \quad \frac{200 F_D K_A}{550\,K_D} \times 200 - 150P + 550F_D = 0$$

$$\Rightarrow \quad F_D = \frac{150P}{\left(\dfrac{200^2 K_A}{550\,K_D} + 550\right)} \quad \text{...(v)}$$

and $$F_A = \frac{150P}{\left(200 + \dfrac{550^2 K_D}{200\,K_A}\right)} \quad \text{...(vi)}$$

Now, $$\theta = \frac{\delta_D}{550} = \frac{F_D}{550K_D}$$

$\Rightarrow$ $$\theta = \frac{150P(550K_D)}{(200^2 K_A + 550^2 K_D)(550K_D)}$$

$$= \frac{150P}{(200^2 K_A + 550^2 K_D)}$$

Maximum load, $$P_{max} = \frac{\theta_{max}}{150}(200^2 K_A + 550^2 K_D)$$

$$= \frac{2 \times \frac{\pi}{180}}{150}(200^2 \times 15(\text{kN/m}) + 550^2(25\,\text{kN/m}))$$

$$= 1899.5 \text{ N} \approx 1900 \text{ N}$$

Now, forces in spring A and D are

$$F_A = \frac{150 \times 1900}{\left(200 + \frac{550^2 \times 25}{200 \times 15}\right)} = 104.747 \text{ N} \quad \{\text{From eq. (v)}\}$$

$$F_D = \frac{150 \times 1900}{\left(\frac{200^2 \times 15}{550 \times 25} + 550\right)} = 480.092 \text{ N} \quad \{\text{From eq. (vi)}\}$$

Now, reaction force at B:

$$F_A + P = R_B + F_D$$

$\Rightarrow$ $$104.747 + 1900 = R_B + 480.092$$

$\Rightarrow$ $$R_B = 1524.655 \text{ N}$$

Q.3 (b) Solution:

Moment of inertia about z-z axis :

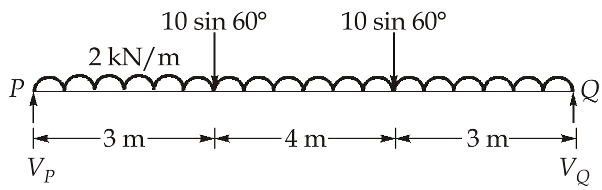
$$I_{ZZ} = \frac{100 \times 140^3}{12} - \frac{80 \times 100^3}{12}$$

$\Rightarrow$ $$I_{ZZ} = 1.62 \times 10^7 \text{ mm}^4$$

Moment of inertia about y-y axis :

$$I_{YY} = \frac{2 \times 20 \times 100^3}{12} + \frac{100 \times 20^3}{12} = 0.34 \times 10^7 \text{ mm}^4$$

Vertical loading diagram:

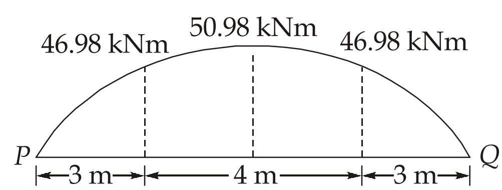


Since, loading is symmetric

So, Vertical reactions, $V_P = V_Q = \frac{1}{2}(2 \times 10 + 2 \times 10 \sin 60°)$

$\Rightarrow$ $V_P = V_Q = 18.66$ kN

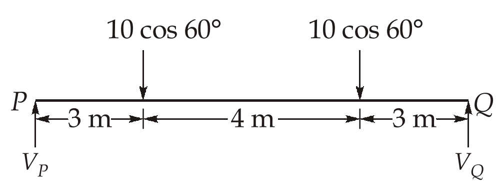
Bending moment diagram corresponding to vertical loading



$\therefore$ Maximum bending moment along Z-Z axis is,

$$M_Z = 50.98 \text{ kNm}$$

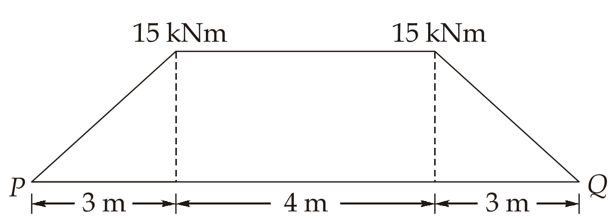
Horizontal loading diagram:



Since, loadings are symmetric,

So, horizontal support reactions, $V_P = V_Q = \frac{1}{2}(2 \times 10 \cos 60°) = 5$ kN

Maximum bending moment diagram corresponding to horizontal loading,



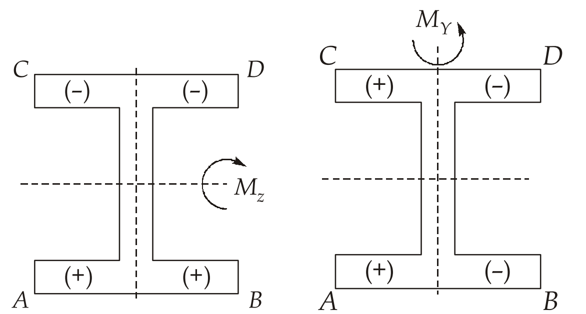
$\therefore$ Bending moment along y-y axis

$$M_y = 15 \text{ kNm}$$

Sign conventions used are as follows,

Tension (+ve) and compression (–ve)

Due to moment M_z **Due to moment M_y**



Now, stresses at any point is given by

$$\sigma = \pm\frac{M_z \cdot y}{I_{zz}} \pm \frac{M_y \cdot z}{I_{yy}}$$

Stresses at point A, B, C and D are as follows:

At point A ;
$$\sigma_A = \frac{(50.98\times10^6)\times70}{162\times10^5} + \frac{(15\times10^6)\times50}{34\times10^5}$$
$$= 440.872 \text{ MPa (Tension)}$$

At point B ;
$$\sigma_B = \frac{(50.98\times10^6)\times70}{162\times10^5} - \frac{(15\times10^6)\times50}{34\times10^5}$$
$$= -0.304 \text{ MPa (Compression)}$$

At point C ;
$$\sigma_C = \frac{-(50.98\times10^6)\times70}{162\times10^5} + \frac{(15\times10^6)\times50}{34\times10^5}$$
$$= 0.304 \text{ MPa (Tension)}$$

At point D ;
$$\sigma_D = \frac{-(50.98\times10^6)\times70}{162\times10^5} - \frac{(15\times10^6)\times50}{34\times10^5}$$
$$= -440.872 \text{ MPa (Compression)}$$

Now location of neutral axis (NA):

$$\sigma = \frac{-M_z \cdot y}{I_{zz}} + \frac{M_y \cdot z}{I_{yy}} = 0$$

$$\Rightarrow \quad \frac{-50.98\times10^6\times y}{162\times10^5} + \frac{(15\times10^6)\times z}{34\times10^5} = 0$$

$$\Rightarrow \quad -3.1469y + 4.41176z = 0$$

$$\Rightarrow \quad y = 1.4019z$$

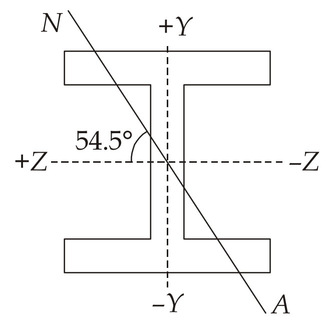
$\because$ $y = mz$

So, $m = \tan\theta = 1.4019$

$\Rightarrow$ $\theta = \tan^{-1}(1.4019) = 54.5°$

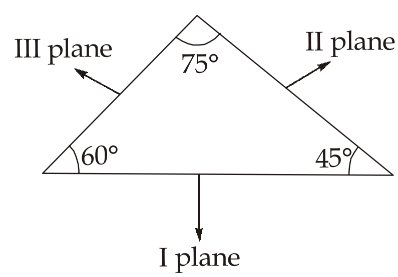
Since line of action of load passes through I and III quadrant. Hence NA will pass through II and IV quadrant.

$\therefore$ NA will pass at an angle 54.5° from Z-Z axis.

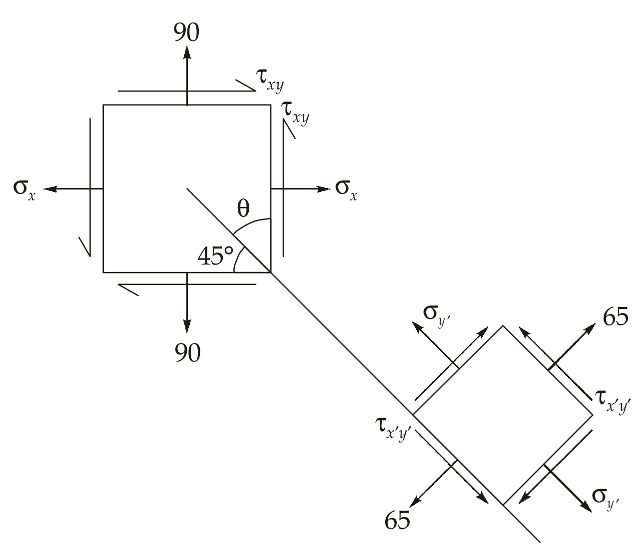


Q.3 (c) Solution:

Let us assume the plane arrangement as shown in figure,



For II plane:



Now, $\sigma_y = 90$ MPa

$$\theta = 90° - 45° = 45°$$

$$\sigma_x' = 65 \text{ MPa}$$

As we know, $\sigma_x' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

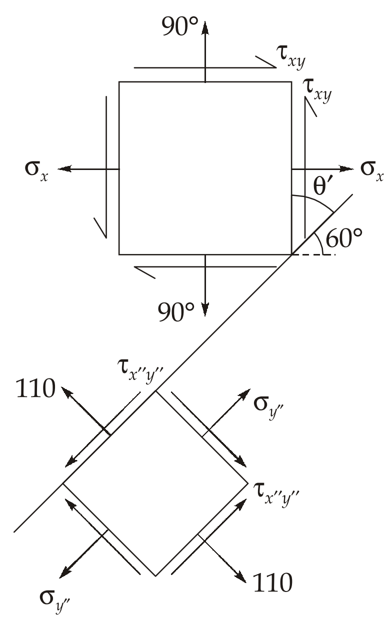
$$\Rightarrow \quad 65 = \cos^2 45° + 90\sin^2 45° + 2\tau_{xy}\sin 45° \cdot \cos 45°$$

$$\Rightarrow \quad 65 = \frac{\sigma_x}{2} + \frac{90}{2} + \tau_{xy} \times 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 2 \times 65 = \sigma_x + 90 + 2\tau_{xy}$$

$$\Rightarrow \quad 40 = \sigma_x + 2\tau_{xy} \quad \text{...(i)}$$

For III plane:



Here, $\sigma_x'' = 110$ MPa

$$\theta' = -30°$$

Now, $\sigma_x'' = \sigma_x \cos^2\theta' + \sigma_y \sin^2\theta' + 2\tau_{xy}\sin\theta' \cdot \cos\theta'$

$$\Rightarrow \quad 110 = \sigma_x \cos^2(-30°) + 90(\sin^2(-30°)) + 2\tau_{xy}\sin(-30°)\cos(-30°)$$

$$\Rightarrow \quad 110 = \frac{3}{4} \cdot \sigma_x + \frac{90}{4} - \frac{\sqrt{3}}{2} \cdot \tau_{xy}$$

$$\Rightarrow \quad 440 = 3\sigma_x + 90 - 2\sqrt{3}\tau_{xy}$$

$$\Rightarrow \qquad 350 = 3\sigma_x - 2\sqrt{3}\tau_{xy} \qquad \text{...(ii)}$$

On solving (i) and (ii), we get

$$\sigma_x = 88.6 \text{ MPa}$$

$$\tau_{xy} = -24.30 \text{ MPa}$$

[–ve sign indicates that assumed direction is wrong]

Similarly for II plane, $\theta = 45°$

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\cos\theta \cdot \sin\theta + \tau_{xy}\left[\cos^2\theta - \sin^2\theta\right]$$

$$= [90 - 88.6]\cos 45° \cdot \sin 45° + (-24.3)\left[\cos^2 45° - \sin^2 45°\right]$$

$$= 0.7 \text{ MPa}$$

Similarly for III plane,

$$\theta = -30°$$

$$\tau_{x''y''} = [90 - 88.6]\cos(-30°)\sin(-30°) + (-24.3)\left[\cos^2(-30°) - \sin^2(-30°)\right]$$

$$= -12.756 \text{ MPa}$$

Now, $\sigma_x = 88.6$ MPa

$\sigma_y = 90$ MPa

$\tau_{xy} = -24.3$ MPa

Principal stresses are,

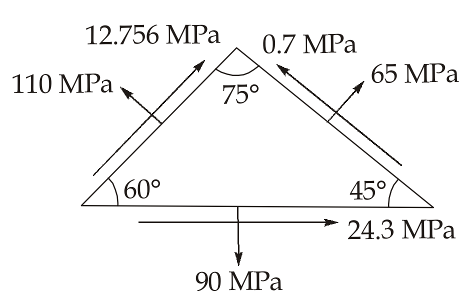
$$p_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau xy^2}$$

$$= \frac{88.6 + 90}{2} \pm \frac{1}{2}\sqrt{[88.6 - 90]^2 + 4 \times (-24.3)^2} = 89.3 \pm 24.31$$

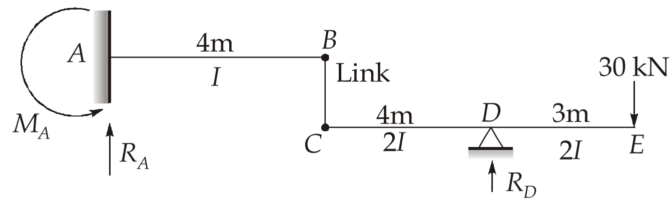
Now, $p_1 = 113.61$ MPa

and $p_2 = 64.99$ MPa

The final stress element is as shown below,



Q.4 (a) Solution:



Step-1 : Reactions at *A* and *D*:

$$\Sigma F_y = 0 \quad \Rightarrow \quad R_A + R_D = 30 \qquad \text{...(i)}$$

$$\Sigma M_A = 0 \quad \Rightarrow \quad R_D \times 8 - 30 \times 11 + M_A = 0 \qquad \text{..(ii)}$$

$$\Sigma M_C = 0 \quad \Rightarrow \quad R_D \times 4 - 30 \times 7 = 0$$

$$\Rightarrow \quad R_D = 52.5 \text{ kN}$$

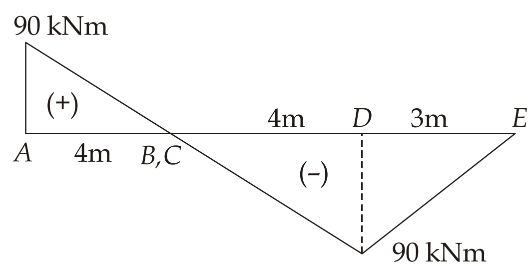
$\therefore$ From equation (i), we get

$$R_A = -22.5 \text{ kN i.e. } 22.5 \text{ kN}(\downarrow)$$

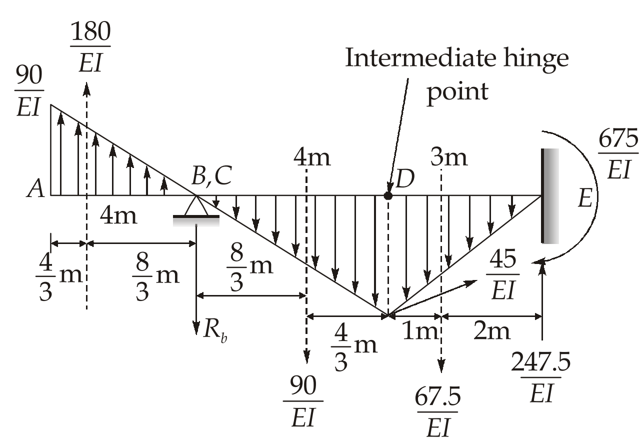
and from equation (ii), we get

$$M_A = -90 \text{ kN-m i.e. } 90 \text{ kNm (clockwise)}$$

Step-2 : Bending Moment Diagram:



Step-3 : Conjugate Beam:



Corresponding to the position of the link in the given beam, an intermediate support is considered for the conjugate beam and corresponding to the intermediate support at *D*, a hinge is provided in the conjugate beam. Further in the conjugate beam, end A is free and end E is fixed.

Loading on the conjugate beam consists of the following:

1. Upward load on $AB = \frac{1}{2}\times 4\times\frac{90}{EI} = \frac{180}{EI}\uparrow$ at $\frac{4}{3}$ m from A

2. Downward load on $CD = \frac{1}{2}\times 4\times\frac{45}{EI} = \frac{90}{EI}\downarrow$ at $\frac{4}{3}$ m from D

3. Downward load on $DE = \frac{1}{2}\times 3\times\frac{45}{EI} = \frac{67.5}{EI}\downarrow$ at 1 m from D

Taking moments about the hinge D from the left end

$$R_B\times 4+\frac{180}{EI}\left(4+\frac{8}{3}\right) = \frac{90}{EI}\left(\frac{4}{3}\right)$$

$$\Rightarrow \quad R_B = \frac{-270}{EI} = \frac{270}{EI}\text{(downwards)}$$

From $\Sigma F_y = 0$, reaction at E, $R_E = \frac{90}{EI}+\frac{67.5}{EI}+\frac{270}{EI}-\frac{180}{EI} = \frac{247.5}{EI}$

From $\Sigma M_D = 0$, Moment at $E = \frac{-67.5}{EI}\times 1+\frac{247.5}{EI}\times 3 = \frac{675}{EI}$(Clockwise)

(i) Deflection at E for the beam = B.M at E for the conjugate beam

$$= \frac{675}{EI} = \frac{675\times 10^9}{200\times 8.3\times 10^7} = 40.66\text{ mm}$$

(ii) Deflection at B for the beam = B.M at B for the conjugate beam

$$= \frac{180}{EI}\times\frac{8}{3} = \frac{480}{EI} = \frac{480\times 10^9}{200\times 8.3\times 10^7} = 28.92\text{ mm}$$

(iii) Slope at B for the beam = Shear force on LHS of B for the conjugate beam

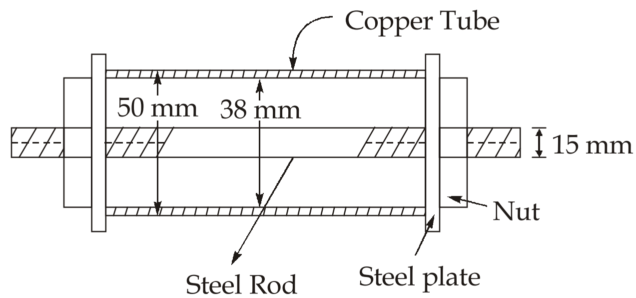
$$= \frac{180}{EI} = \frac{180\times 10^6}{200\times 8.3\times 10^7}\text{radian} = -0.01084\text{ radian}$$

(iv) Slope at C for the beam = S.F. on RHS of B for the conjugate beam

$$= \frac{180}{EI}-\frac{270}{EI} = \frac{-90}{EI}$$

$$= \frac{-90\times 10^6}{200\times 8.3\times 10^7}\text{radian} = -0.005422\text{ radian}$$

Q.4 (b) Solution:



Given, diameter of steel rod, $D_S = 15$ mm

External diameter of copper tube,

$$D_o = 50 \text{ mm}$$

Internal diameter of copper tube,

$$D_i = 38 \text{ mm}$$

Length of tube, $L = 3$ m

Rise in temperature, $\Delta T = 65°$ C

Change in length, $\Delta L = 0.635$ mm

Now, area of steel rod, $A_S = \frac{\pi}{4} \times D_S^2 = \frac{\pi}{4} \times 15^2 = 176.715 \text{ mm}^2$

Area of copper tube, $A_C = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(50^2 - 38^2) = 829.38 \text{ mm}^2$

Case-I : Stresses due to tightening of nuts:

When nuts are tightened, the steel rod will be subjected to tensile stress and the copper tube will be subjected to compressive stress. Let p_C and p_S be the stresses in copper and steel respectively.

$$\text{Total compression in copper} = \text{Total tension in steel}$$

$$\Rightarrow \quad p_C \cdot A_C = p_S \cdot A_S$$

$$\Rightarrow \quad p_S = \frac{A_C}{A_S} \cdot p_C = \frac{829.38}{176.715} \times p_C = 4.7\, p_C$$

Strain in copper, $\in_C = \frac{\text{Change in Length}}{\text{Original Length}} = \frac{0.635}{3000}$

$\therefore$ Stress in copper, $p_C = \in_C \times E_C$

$$= \frac{0.635}{3000} \times 1.05 \times 10^5 = 22.225 \text{ N/mm}^2$$

$\therefore$ Stress in steel, $p_S = 4.7 \times p_C = 4.7 \times 22.225$

$= 104.46 \text{ N/mm}^2$

Case-II: Stresses due to rise in temperature :

If these two members had been free to expand fully then expansion of steel rod,

$$= \alpha_S \cdot \Delta T \cdot L_S$$

$$\text{Free expansion of copper} = \alpha_C \cdot \Delta T \cdot L_C$$

$$\because \quad \alpha_C > \alpha_S$$

The free expansion of copper is greater than the free expansion of steel. But since the ends of the rod are provided with washers and nuts, the members are not free to expand fully. Final expansion of copper is less than $\alpha_C \cdot \Delta T \cdot L_C$ while free expansion of steel is more than $\alpha_S \cdot \Delta T \cdot L_S$. Hence the steel rod will be subjected to a tensile stress while the copper tube will be subjected to a compressive stress. Let f_S and f_C be the stresses in steel and copper respectively. For the equilibrium of whole system,

$$\text{Total tension in steel} = \text{Total compression in copper}$$

$$\Rightarrow \quad f_S \cdot A_S = f_C \cdot A_C$$

But, Final expansion of steel = Final expansion of copper

$$\alpha_S \cdot \Delta T \cdot L_S + \frac{f_S}{E_S} \cdot L_S = \alpha_C \cdot \Delta T \cdot L_C - \frac{f_C}{E_C} \cdot L_C$$

But, $\Delta T = 65° \text{ C},$

$$L_S = 3000 + 2 \times 25 = 3050 \text{ mm}$$

$$L_C = 3000 \text{ mm}$$

$$\therefore 1.2 \times 10^{-5} \times 65 \times 3050 + \frac{4.7 \cdot f_C \times 3050}{2.1 \times 10^5} = 1.75 \times 10^{-5} \times 65 \times 3000 - \frac{f_C \times 3000}{1.05 \times 10^5}$$

$$2.379 + 0.068 f_C = 3.4125 - 0.029 f_C$$

$$\Rightarrow \quad f_C = 10.65 \text{ N/mm}^2 \text{ (Compression)}$$

and $f_S = 4.7 \times f_C$

$= 4.7 \times 10.65 = 50.06 \text{ N/mm}^2$ (Tensile)

The final stresses due to tighening of the nuts and rise of temperature will be as follows:

$$\text{Stress in copper} = p_C + f_C = 22.225 + 10.65 = 32.875 \text{ N/mm}^2$$

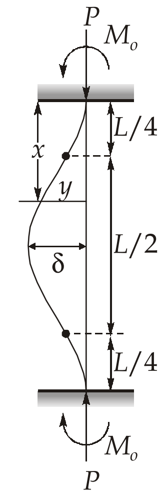
(Compression)

$$\text{Stress in steel} = p_S + f_S = 104.46 + 50.06 = 154.52 \text{ N/mm}^2$$

(Tensile)

Q.4 (c) Solution:

General differential equation is given by,



$$EI\frac{d^2y}{dx^2} = M_x$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = M_o - Py$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}+\left(\frac{P}{EI}\right)y = \frac{M_o}{EI} \quad \text{...(i)}$$

Let, $$K^2 = \frac{P}{EI}$$

$$\therefore \quad \frac{d^2y}{dx^2}+K^2y = \frac{M_o}{EI}$$

General solution of the above differential equation is

$$y = C_1\sin Kx + C_2\cos Kx + \frac{M_o}{P} \quad \text{...(ii)}$$

Differentiating equation (ii),

$$y' = C_1K\cos Kx - C_2K\sin Kx \quad \text{...(iii)}$$

Boundary conditions:

At $x = 0$; $y = 0$

$\therefore$ From equation (ii),

$$C_2 = \frac{-M_o}{P}$$

and, at $x = 0$; $y' = 0$

From equation (iii),

$$0 = C_1K - 0$$

$$\Rightarrow \quad C_1 = 0$$

On substituting values of C_1 and C_2 in equation (ii);

$$y = \frac{-M_o}{P}\cos Kx + \frac{M_o}{P}$$

$$\Rightarrow \quad y = \frac{M_o}{P}(1-\cos Kx)$$

Also, at, $x = L\,;\, y = 0$

So, $$0 = \frac{M_o}{P}(1 - \cos KL)$$

$\Rightarrow$ $$\cos KL = 1$$

$\Rightarrow$ $$KL = 2n\pi = 0, 2\pi, 4\pi, ...$$

$\Rightarrow$ $$KL = 2\pi \qquad (n = 1 \text{ for single bow condition})$$

$\Rightarrow$ $$K^2 = \frac{4\pi^2}{L^2} = \frac{P}{EI}$$

$\therefore$ $$\text{Critical load } (P_{cr}) = \frac{4\pi^2 EI}{L^2}$$

Now, since δ is the deflection at midpoint (i.e, $x = L/2$)

So, $$\delta = \frac{M_o}{P}\left(1 - \cos\frac{KL}{2}\right)$$

$\because$ $$KL = 2\pi$$

$\therefore$ $$\delta = \frac{M_o}{P}(1 - \cos\pi)$$

$\Rightarrow$ $$\delta = \frac{2M_o}{P}$$

$\Rightarrow$ $$\frac{M_o}{P} = \frac{\delta}{2}$$

Hence, equation of buckled shape of column is

$$y = \frac{\delta}{2}(1 - \cos Kx)$$

$\Rightarrow$ $$y = \frac{\delta}{2}\left(1 - \cos\frac{2\pi x}{L}\right)$$

Section B : Environmental Engineering

Q.5 (a) Solution:

Per Capita demand

It is annual average amount of daily water required by one person and includes the domestic use, industrial use, commercial use, public use, waste, thefts etc.

Mathematically,

$$\text{Per capita demand (in lpcd)} = \frac{\text{Total yearly requirement of city}}{365 \times \text{Design population}}$$

Estimation of total yearly water required

Given,

$$P_0 = 60,000$$
$$P_1 = 1,20,000$$
$$P_2 = 1,70,000$$

Now,

$$\text{Saturation population, } P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2}$$

$$= \frac{2 \times 60000 \times 120000 \times 170000 - 120000^2(60000 + 170000)}{60000 \times 170000 - 120000^2}$$

$$= 205715$$

$$m = \frac{P_s - P_0}{P_0} = \frac{205715 - 60000}{60000} = 2.43$$

$$n = \frac{1}{t_1}\ln\left[\frac{P_0(P_s - P_1)}{P_1(P_s - P_0)}\right]$$

$$= \frac{1}{20}\ln\left(\frac{60000(205715 - 120000)}{120000(205715 - 60000)}\right) = -0.0612 \text{ year}^{-1}$$

Now, $$P_{2025} = \frac{P_s}{1 + m\log_e^{-1}(nt)} \qquad \text{(where } t = 60 \text{ years)}$$

$$= \frac{205715}{1 + 2.43\log_e^{-1}(-0.0612 \times 60)} = 193745$$

Now, total yearly water requirement of city

$$= \text{(Population in 2025)} \times \text{Per capita demand} \times 365$$

$$= 193745 \times 250 \times 365$$
$$= 17679.2 \times 10^6 \text{ litres}$$
$$= 17679.2 \text{ million litres}$$

Q.5 (b) Solution:

Given, Population $= 10000$ persons

Average per capita sewage $= 135$ lpcd

Average sewage discharge $= 135 \times 10000\ l/\text{day}$

$= 1.35$ MLD

$\therefore$ Maximum sewage discharge $= 2 \times$ Average sewage discharge

$= 2 \times 1.35 = 2.7$ MLD

So, BOD content in sewage $= 2.7 \times 10^6 \times 250 \times 10^{-6}$

$= 675$ kg/day

Now, assuming the organic loading rate in hot climate as 250 kg/hec/day

$$\therefore \quad \text{Surface area required} = \frac{675 \text{ kg/day}}{250 \text{ kg/hec/day}} = 2.7 \text{ hectares}$$

Since, area of each pond should be in range of 0.5 to 1 hectare.

So, Adopt three units of oxidation pond

$$\therefore \quad \text{Area of each unit} = \frac{2.7 \times 10^4}{3} = 9000 \text{ m}^2 = 0.9 \text{ hec}$$

Assuming, length to width ratio be 3

$$L \times B = 9000$$
$$\Rightarrow \quad 3B^2 = 9000$$
$$\Rightarrow \quad B = 54.77 \text{ m}$$
$$\therefore \quad L = 3B = 3 \times 54.77 = 164.31 \text{ m}$$

So, take length and width of pond as 165 m and 55 m respectively

Taking depth as 1.2 m

$$\text{Provided volume} = 1.2 \times 55 \times 165 = 10890 \text{ m}^3$$

$$\text{Detention period} = \frac{10890}{\left(\dfrac{2.7 \times 10^3}{3}\right)} = 12.1 \text{ days}$$

Usually detention time is between 2-6 weeks

However, the given waste water is treated in less time,

Hence, use three units of oxidation pond, each having length as 165 m, width as 55 m and overall depth as 1.5 m including 0.3 m as freeboard and detention period of 12 days.

Design of inlet pipe:

Assuming an average velocity of sewage as 1 m/sec and daily flow for 8 hours then,

$$\text{Sewage discharge in pipe} = \frac{2.7\times 10^3}{8\times 3600} = 0.09375 \text{ m}^3/\text{sec}$$

So,
$$\text{area of pipe required} = \left(\frac{\text{Dicharge}}{\text{Velocity}}\right) = \frac{0.09375}{1} = 0.09375 \text{ m}^2 = 937.5 \text{ cm}^2$$

$\therefore$
$$\text{diameter of inlet pipe} = \sqrt{\frac{4\times 937.5}{\pi}} = 34.549 \text{ cm} = 35 \text{ cm (say)}$$

Q.5 (c) Solution:

(i) Power of sound, $P = 0.0018$ W

At distance of 10 m from source,

$$\text{Intensity of sound, } I_{10} = \frac{P}{4\pi r^2} \quad \text{where } r \text{ is distance from source}$$

$$= \frac{0.0018}{4\pi\times 10^2} = 1.43 \times 10^{-6} \text{ Watt/m}^2$$

Now,
$$\text{sound intensity level, } L_{10} = 10\log_{10}\left(\frac{I_{10}}{10^{-12}}\right)$$

$$= 10\log_{10}\left(\frac{1.43\times 10^{-6}}{10^{-12}}\right) = 61.55 \text{ dB}$$

(ii) Intensity of sound at 15 m,
$$I_{15} = \frac{P}{4\pi r^2} = \frac{0.0018}{4\pi\times 15^2} = 6.37 \times 10^{-7}$$

Therefore,
$$\text{sound intensity level, } L_{15} = 10\log_{10}\left(\frac{I_{15}}{10^{-12}}\right)$$

$$= 10\log_{16}\left(\frac{6.37\times 10^{-7}}{10^{-12}}\right) = 58.04 \text{ dB}$$

Now, $$L_{15} = 20\log_{10}\left(\frac{P_{rms}}{P_0}\right)$$

where P_{rms} is sound pressure and P_0 is 20 μPa

So, $$58.04 = 20\log_{10}\left(\frac{P_{rms}}{20\,\mu Pa}\right)$$

$\Rightarrow$ $$P_{rms} = 15959.9\ \mu\text{Pa}$$

(iii) As we know,

$$I_{15} = \frac{P_{rms}^2}{\rho C}$$

where, P_{rms} is sound pressure at 15 m from source (in Pa)

ρ is density of air in kg/m^3

C is speed of sound in air in m/s

So, $$6.37 \times 10^{-7} = \frac{(15959.9 \times 10^{-6})^2}{1.16 \times C}$$

$\Rightarrow$ $$C = 344.72\ \text{m/s}$$

(iv) At 15 m, there are two sounds of 58.04 dB and 50 dB

Now, sound pressure corresponding to 58.04 dB

$$= 15959.9\ \mu Pa$$

Also, $$50 = 20\log_{10}\left(\frac{P_{rms}}{20\,\mu Pa}\right)$$

$\Rightarrow$ $$P_{rms} = 6324.56\ \mu Pa$$

Hence, equivalent sound pressure

$$= \sqrt{(15959.9)^2 + (6324.56)^2} = 17167.37\ \mu Pa$$

Now, sound pressure level $= 20\log_{10}\left(\frac{P_{rms}}{20\,\mu Pa}\right) = 20\log_{10}\left(\frac{17167.37}{20}\right) = 58.67$ dB

Q.5 (d) Solution:

(i) Organic matter present in water can be assessed in terms of oxygen required to completely oxidise the organic matter to CO_2, H_2O and other organic species. The oxygen required to oxidise the organic matter present in a given waste water is known as its 'Chemical Oxygen Demand'. It can be theoretically computed, if the organics present in waste-water are known. However, it is determined by mixing the sample of waste water with a standard solution of potassium dichromate in an acid solution and titrating the mix. It is to be noted that organic matter in sewage water can be of two types, i.e. biologically active and biologically inactive. Chemical oxygen demand is a measure of total organic matter (biodegradable as well as non-biodegradable) while biochemical oxygen demand is a measure of biodegradable organic matter only.

(ii) BOD_5 of seeded water, $BOD_{sw} = (B_i - B_f) \times (D \cdot F_1)$

Where, B_i = Initial DO of seeded water only = 8 mg/l

B_f = Final DO of seeded water only = 7 mg/l

$D \cdot F_1$ = Dilution factor when only seeded water is incubated

$$= \frac{300 \text{ ml}}{300 \text{ ml}} = 1$$

So, $BOD_{sw} = (8 - 7) \times 1 = 1$ mg/l ...(i)

Now, when sample is diluted with seeded water,

then, $$BOD_{mix} = \frac{BOD_{ww} \times V_{ww} + BOD_{sw} \times V_{sw}}{V_{ww} + V_{sw}} \quad ...(ii)$$

Where, BOD_{mix} = BOD of mix of 5 ml sample and 295 ml seeded water

BOD_{ww} = BOD of sample

V_{ww} = Volume of sample

BOD_{sw} = BOD of seeded water

= 1 mg/l [from equation (i)]

V_{sw} = Volume of seeded water

Now, $BOD_{mix} = DO_i - DO_f$

Where DO_i = Initial D.O of mix = 8 mg/l

DO_f = Final D.O. of mix = 4.5 mg/l

= (8 – 4.5) mg/l

= 3.5 mg/l ...(iii)

Putting values from equation (iii) and (i) in equation (ii),

$$3.5 = \frac{BOD_{ww} \times 5 + 1 \times 295}{5 + 295}$$

$\Rightarrow$ $BOD_{ww} = 151$ mg/l

Q.5 (e) Solution:

Indore method of composting uses manual turning of piled up mass (refuse + night soil), for its decomposition under aerobic conditions. In this method, layers of vegetable wastes and night soil are alternatively piled in depth of about 7.5 to 10 cm, each to a total depth of about 1.5 m in a trench, or above, the ground to form a mound called 'windrow'.

A 'windrow' is a long mound or stack of the organic MSW dumped on land in a height of about 1.5 m to 2 m, usually about 2.5 m to 3 m wide at base. Most windrows are conical in cross-section and about 50 m in length. The composting waste is aerated by periodically turning the waste mix in the windrow, or in trench, as the case may be. The manual turning with a pitchfork can be adopted at smaller installations, while at larger plants, mechanical devices like self propelled overcab loaders, rotary ploughs etc. may be used to turn the refuse once or twice per week, which serves to introduce oxygen and to control the temperature. The moisture content of turning mass is maintained at about 55% for getting optimal decomposition of waste mass. This process of turning is continued for about 4 to 5 weeks, during which the readily biodegradable organics are consumed. The waste compost mass is finally allowed to cure for another 2 to 8 weeks without any turning. The entire process, thus takes about 3-4 months of time to complete, after which compost becomes ready for being taken out for use.

The 'Bangalore method of composting' involves anaerobic decomposition of waste and does not involve any turning or handling of mass and is clearance than Indore method. This method is therefore, widely adopted by municipal authorities throughout India. The refuse and night soil in this method are piled up in layers and in an underground earthen trench (about 10 m × 1.5 m × 1.5 m). This mass is covered at its top by layer of earth of about 15 cm depth, and is finally left over for decomposition.

Within 2 to 3 days of burial, intensive biological action starts taking place and organic matter begins to be destroyed. Considerable amount of heat also gets evolved in the process, which raises the temperature of decomposing mass to about 75° C. This heat prevents the breeding of flies by destroying the larvae. After about 4 to 5 months, refuse gets fully stabilized and changes into a brown colored odourless innocuous powdery mass, called 'humus'. The humus is removed from trenches, sieved on 12.5 mm sieves to remove stones, broken glass, brickbats etc and then sold out in market as manure.

Q.6 (a) Solution:

(i) 1. Formation of mud balls:

In a rapid sand filter, mud from the atmosphere usually accumulates on sand surface, so as to form a dense mat. If the washing of filter is inadequate, then this mud may sink down into sand bed. This mud then sticks to sand grains and other arrested impurities, thereby forming mud balls. These mud balls slowly and steadily go on increasing in size and weight. They may then sink down into gravel, thus interfering with upward movement of wash water during cleaning. They cause turbulence around them and thus hinder with uniform application of wash water. The high velocities created around the edges of these balls, also displace the gravel and thereby forming mounds above the balls and thus the sand is improperly washed and mud gets accumulated.

2. Cracking of filters:

The fine sand contained in top layers of filter bed, shrinks and causes the development of shrinkage cracks in sand bed. These cracks are more prominent near wall junctions. With use of filter, the loss of head, and therefore, pressure on sand bed goes on increasing, which further goes on widening these cracks. The floc, mud and other impurities arrested in filter, penetrate deep into filter through these cracks and thus impairing both the washing of filter and efficiency of filtration.

(ii) 1. Population of town = 3 lakh

Average daily water demand of town = 175 lpcd

So, maximum water demand per day = 1.8 × 3 lakh × 175 lpcd

= 94.5 Million litres

As 5% of filtered water is to be used in backwashing,

$$\text{Total daily water requirement} = \frac{94.5}{0.95} = 99.47 \text{ Milion litres}$$

Since 30 minutes is lost daily in backwashing the filter, effective time left for working of filter units

$$= 24 - 0.5 = 23.5 \text{ hours}$$

So,
$$\text{total hourly water requirement} = \frac{99.47}{23.5} = 4.23 \text{ Million litres}$$

$$\text{Filtration rate} = 24 \text{ m}^3/\text{m}^2/\text{hr}$$

Therefore, area of filter required $= \dfrac{4.23 \times 10^6 \times 10^{-3}}{24} = 176.25 \text{ m}^2$

So, number of filters $= \dfrac{\text{Area of filter required}}{\text{Area of one filter}}$

$$= \frac{176.25}{50} = 3.5 \approx 4 \text{ (say)}$$

As one filter is to be provided as stand by, therefore 5 filters will be required.

2. As we know,

$$D(1 - \eta) = D'(1 - \eta') \quad \text{...(i)}$$

where, D and D' are depth of original and expanded bed respectively

η and η' are porosity of original and expanded bed respectively

Putting values in (i),

$$0.6(1 - 0.5) = 0.66(1 - \eta')$$

$$\Rightarrow \quad \eta' = 0.545$$

Now, $$\eta' = \left(\frac{V_B}{V_S}\right)^{0.22} \quad \text{...(ii)}$$

where V_B is backwash velocity and V_S is settling velocity of particle

$$\Rightarrow \quad 0.545 = \left(\frac{0.18}{60 \times V_S}\right)^{0.22} \qquad [\because V_B = 18 \text{ cm/min}]$$

$$\Rightarrow \quad V_S = 0.047 \text{ m/s}$$

Now, $$V_s^2 = \frac{4}{3} g \frac{d(G-1)}{C_D} \quad \text{...(iii)}$$

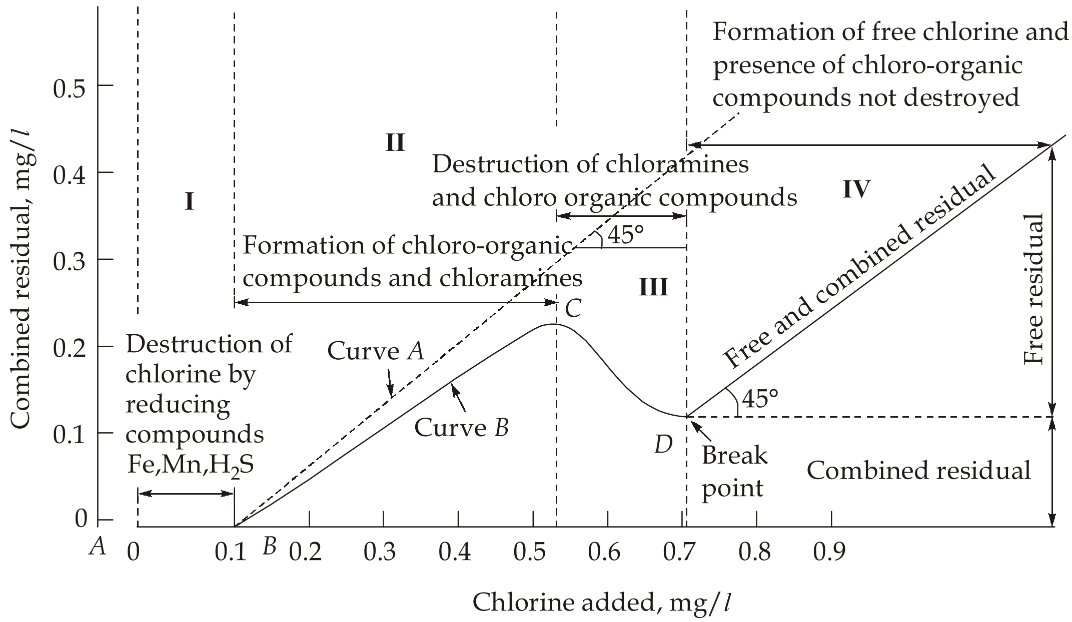
Putting values in (iii), we get

$$0.047^2 = \frac{4}{3} \times \frac{9.8 \times d \times (2.5 - 1)}{5}$$

$$\Rightarrow \quad d = 0.563 \text{ mm}$$

Q.6 (b) (i) Solution:

Break point chlorination is a term which gives us an idea of the extent of chlorine added to water. Infact, it represents that much dose of chlorination, beyond which any further additional chlorine will appear as free residual chlorine.



When chlorine is added to pure water which has no chlorine demand, a curve, such as curve *A* in figure above is obtained between applied chlorine and residual chlorine relationship.

(i) As raw water has a chlorine demand therefore curve *B* is generally obtained between residual chlorine and applied chlorine. During the disinfection process, the amount of residual chlorine will be very less in beginning (marked by stage I) in figure, during which various chemicals such as ions of ferrous, sulphides or nitrites present in water, will be oxidised.

(ii) Stage-II reflects the forming of combined residuals as the ammonia or amines react with HOCl that has formed. During stage II, as the demand for disinfection is satisfied, combined residual chlorine goes on increasing till a point C is reached where amount of combined residual is maximum. The stage of point C is accompanied by bad taste and odour indicating the starting of stage III.

(iii) During stage-III, oxidation of organic matter present in water starts. Therefore, after point C, with increase in applied chlorine, residual chlorine goes on decreasing as shown by curve CD in figure. In this stage, free chlorine breaks down chloramines into nitrogen compounds. At point *D*, bad smell and taste disappear showing that oxidation of organic matter is completed. At point *D*, residual chlorine has minimum value and all the organic matter is oxidized. This point is called breakpoint chlorimation.

(iv) Further addition of applied chlorine results in increase in residual chlorine as represented by line *DE*, the slope of which is 45° as shown in figure during stage IV.

Point *D* on curve represents break point since further addition of chlorine breaks through water and residual chlorine.

When chlorine is added to water, HOCl is formed which dissociates as expressed by the chemical equation.

$$\text{HOCl} \rightleftharpoons \text{H}^+ + \text{OCl}^-$$

$$K_i = \frac{[\text{H}^+][\text{OCl}^-]}{[\text{HOCl}]}$$

$$\Rightarrow \qquad \frac{[\text{OCl}^-]}{[\text{HOCl}]} = \frac{K_i}{[\text{H}^+]} \qquad \text{...(i)}$$

Now, percentage distribution of HOCl in freely available chlorine

$$= \left(\frac{[\text{HOCl}]}{[\text{HOCl}]+[\text{OCl}^-]}\right) \times 100 = \left(\frac{1}{1+\frac{[\text{OCl}^-]}{[\text{HOCl}]}}\right) \times 100$$

$$= \left[\frac{1}{1+\frac{K_i}{[\text{H}^+]}}\right] \times 100 \qquad \text{...(ii)}$$

Given, 0.8 mg/litre of total chlorine is required at pH = 7

So, at pH = 7,

$$\%\ \text{HOCl} = 100\left[\frac{1}{1+\frac{2.7\times10^{-8}}{10^{-7}}}\right] = \frac{100}{1+0.27} = 78.7\%$$

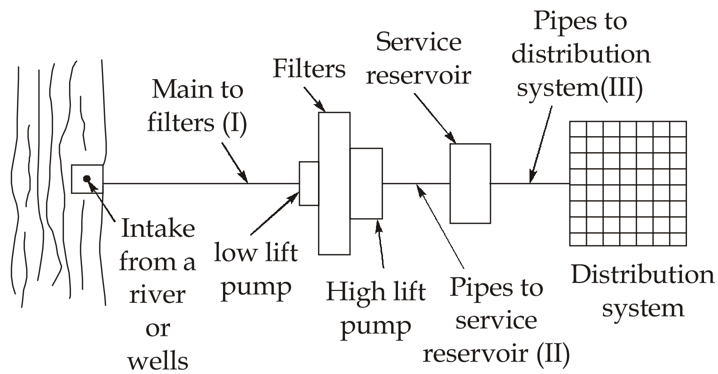
At pH = 8,

$$\%\text{HOCl} = 100\left[\frac{1}{1+\frac{2.7\times10^{-8}}{10^{-8}}}\right] = \frac{100}{1+2.7} = 27\%$$

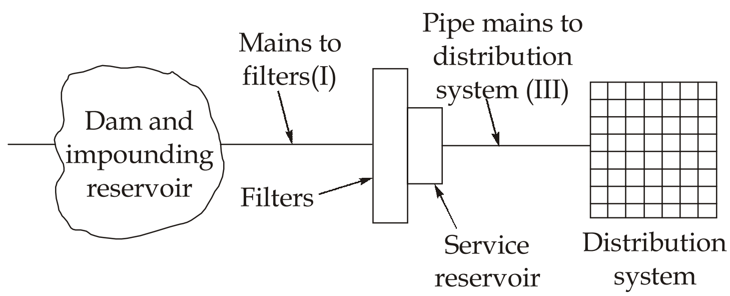
So, dose required at pH of 8 = $0.8 \times \frac{78.7}{27}$ = 2.33 mg/l

Q.6 (b) (ii) Solution:

Figure below shows a possible layout of water scheme when a well or river is used as source of water, needing pumping equipments.



Similarly, if a dam and reservoir is used, then possible layout of water supply scheme is shown in figure below.



As dam is used in later case, therefore pumping of water is not required.

The various units involved in these supply schemes should be designed to serve the variations in demand along with average daily demand and maximum daily demand when it arises. The following recommendations may be adopted for designs of different components.

(a) **Source of supply** : It may be designed for **maximum daily demand** or **average daily demand** depending upon other conditions.

(b) **Pipe mains**: The pipes (I and II in figure above) taking water from source upto service reservoir may be designed for **maximum daily demand**.

(c) **Filter and Other units**: They are also generally designed for maximum daily demand. However, sometimes, due to break-downs and repairs, they are designed for twice the average daily demand.

(d) **Pumps**: Pumps lifting the water may be designed for **maximum daily demand**. For safety against repairs and break-downs, maximum daily demand is taken as twice the average daily demand for designing of these pumps.

(e) **Distribution system**: It also includes the pipes carrying water from service reservoir to distribution system i.e. Type III pipes shown in figure above.

These should be designed for maximum hourly draft of **maximum day or coincident draft** whichever is more.

(f) **Service reservoir**: It is designed to take care of hourly fluctuations, fire demands, emergency reserves and provision required when pumps have to pump the entire's day water in fewer than 24 hours.

Q.6 (c) Solution:

(i) Total wet mass of MSW = 25 + 60 + 10 + 5 = 100 kg

Total dry mss of MSW = 7.5 + 56.4 + 9.8 +4 = 77.7 kg

So, total moisture present in 100 kg MSW

= 100 – 77.7 = 22.3 kg

Total Carbon in MSW = 3.6 + 24.54 + 5.88 + 1.98 = 36 kg

Total Hydrogen in MSW = 0.48 + 3.38 + 0.7 + 0.24 = 4.8 kg

Total Oxygen in MSW = 2.82 + 24.82 + 2.24 + 1.71 = 31.59 kg

Total Nitrogen in MSW = 0.19 + 0.17 + 0.008 = 0.368 kg

Total Sulphur in MSW = 0.03 + 0.11 + 0.004 = 0.144 kg

Total ash in MSW = 0.38 + 3.38 + 0.98 + 0.058 = 4.798 kg

It is to be noted that moisture present in waste will also contribute to Hydrogen and Oxygen

So, Total Hydrogen in MSW = $4.8 + \frac{2}{18} \times 22.3$ = 7.28 kg

Total Oxygen in MSW = $31.59 + \frac{16}{18} \times 22.3$ = 51.41 kg

Now,

Number of moles of Carbon = $\frac{36 \times 10^3}{12} = 3 \times 10^3$

Number of moles of Hydrogen = $\frac{7.28 \times 10^3}{1} = 7.28 \times 10^3$

Number of moles of Oxygen = $\frac{51.41 \times 10^3}{16} = 3.21 \times 10^3$

Number of moles of Nitrogen = $\frac{0.368 \times 10^3}{14} = 0.026 \times 10^3$

$$\text{Number of moles of Sulphur} = \frac{0.144 \times 10^3}{32} = 4.5$$

As sulphur in chemical formula is given as 1,

So normalized moles of other components have to be computed as:

$$\text{Normalized moles of Carbon} = \frac{3 \times 10^3}{4.5} = 666.67$$

$$\text{Normalized moles of Hydrogen} = \frac{7.28 \times 10^3}{4.5} = 1617.78$$

$$\text{Normalized moles of Oxygen} = \frac{3.21 \times 10^3}{4.5} = 713.33$$

$$\text{Normalized moles of Nitrogen} = \frac{0.026 \times 10^3}{4.5} = 5.78$$

$$\text{Normalized moles of Sulphur} = 1$$

So, chemical formula of solid waste is

$$C_{666.7}H_{1617.78}O_{713.33}N_{5.78}S$$

(ii) Energy content on wet basis (in kJ/kg) = $337C + 1428\left(H - \frac{O}{8}\right) + 9S$

Now,

Component	**Percent by mass**
Carbon	36
Hydrogen	7.28
Oxygen	51.41
Nitrogen	0.368
Suphur	0.144
Ash	4.798

So, Energy content (in kJ/kg) $= 337 \times 36 + 1428\left(7.28 - \frac{51.41}{8}\right) + 9 \times 0.144$

$= 13352.45$ kJ/kg

Also, energy content on dry mass basis (in kJ/kg)

$$= 13352.45 \times \frac{100}{77.7} = 17184.62 \text{ kJ/kg}$$

Q.7 (a) (i) Solution:

In a gravitational settling chamber,

$$\frac{V_f}{L} = \frac{V_t}{h} \quad \text{...(i)}$$

Where, V_f = Horizontal velocity in chamber,

V_t = Settling velocity in chamber

L = Length of chamber

h = Height of chamber

By Stoke's law

$$V_t = \frac{g}{18}\frac{(\rho_s - \rho_a)d^2}{\mu} \quad \text{...(ii)}$$

where ρ_s and ρ_a are mass density of particle and air respectively and μ is viscosity of air

Combining equation (i) and (ii), we get

$$\frac{V_f h}{L} = \frac{g}{18}\frac{(\rho_s - \rho_a)d^2}{\mu} \quad \text{...(iii)}$$

Putting values in equation (iii), we get

$$V_f \times \frac{1.5}{7.5} = \frac{9.81}{18} \times \frac{(2000-1.2)\times(48\times10^{-6})^2}{2.1\times10^{-5}}$$

$$\Rightarrow \quad V_f = 0.598 \approx 0.6 \text{ m/s}$$

Q.7 (a) (ii) Solution:

As we know,

$$\text{Efficiency} = 1 - e^{-\left(\frac{\omega A}{Q}\right)} \quad \text{...(i)}$$

where ω is drift velocity in m/s, Q is flow rate of gas stream in m^3/s,

A is area of collection plates in m^2

So, putting values we get

$$0.9 = 1 - e^{-\left(\frac{0.2\times A}{15}\right)}$$

$$\Rightarrow \quad A = 172.7 \text{ m}^2$$

Q.7 (a) (iii) Solution:

$$\text{Required area of cloth} = \frac{12 \times 60}{2} = 360 \text{ m}^2$$

Now, Area of one bag $= \pi DH = \pi \times 0.35 \times 5.8 = 6.38 \text{ m}^2$

Therefore, number of bags required

$$= \frac{360}{6.38} = 56.43 \approx 57 \text{ (say)}$$

Q.7 (b) (i) Solution:

1. **Sludge age:**

It is defined as average time for which particles of suspended solids remain under suspension. It, thus indicates the residence time of biological solids in the system. It is also known as solids retention time (SRT) or mean cell residence time (MCRT).

It is generally represented by θ_C (in days).

It is expressed as ratio of mass of MLSS in aeration tank to mass to suspended solids leaving the system per day.

Matematically,

$$\text{Sludge age } (\theta_C) = \frac{\text{Mass of suspended solids in system}}{\text{Mass of solids leaving the system per day}} \quad ...(i)$$

Now,

Mass of solids in reactor, $M = VX_t$ ...(ii)

where, V is volume of aeration tank, X_t is MLSS in tank (in mg/l)

Solids removed from system per day

$$= Q_w X_R + (Q - Q_w) X_e \quad ...(iii)$$

where,

Q_W = Volume of wasted sludge per day,

X_R = Concentration of solids in returned sludge (in mg/l)

Q = Sewage inflow per day

X_e = Concentration of solids in effluent in mg/l

Putting equation (ii) and (iii) in (i), we get

$$\text{Sludge age, } \theta_C = \frac{VX_t}{Q_w X_R + (Q - Q_w) X_e}$$

2. Sludge-volume index:

It is used to indicate physical state of sludge produced in a biological aeration system. It represents degree of concentration of sludge in system and hence decides the rate of recycle of sludge (Q_R) required to maintain desired MLSS and F/M ratio in aeration tank to achieve the desired degree of purification. It is defined as volume occupied (in ml) by one gram of solids in mixed liquor after settling for 30 minutes and is determined experimentally as explained below.

Its measurement involves collection of one litre sample of mixed liquor from aeration tank from near its discharge end in a graduated cylinder. This 1 litre sample of mixed liquor is allowed to settle for 30 minutes and settled sludge volume (V_{ob}) in millilitres is recorded so as to represent sludge volume. This volume V_{ob} in ml per litre of mixed liquor will represent the quantity of sludge in liquor in ml/l.

The above sample of mixed liquor, after remixing settled solids is analysed in laboratory for MLSS by standard procedure adopted for measuring suspended solids in a sewage. Let, concentration of suspended solids in mixed liquor in mg/l be X_{ob}.

Then, sludge volume index,

$$\text{SVI} = \frac{V_{ob(\text{ml/l})}}{X_{ob(\text{mg/l})}} = \frac{V_{ob}}{X_{ob}} \times 1000\ (\text{ml/g})$$

3. Food to biomass ratio :

It is a manner of expressing BOD loading with regard to microbial mass in system. The BOD load applied to system in kg or *gm* is represented as Food (F), and the total microbial suspended solids in mixed liquor of aeration tank is represented by *M*.

So, $$\text{F/M ratio} = \frac{\text{Daily BOD load applied to system}}{\text{Total microbial mass in system}} \quad \text{...(i)}$$

Now,

Microbial mass in system, $M = VX_t$ ...(ii)

where V is volume of tank in m^3

X_t is MLSS in mg/l

BOD load applied to system = $Q \cdot S_0$ ...(iii)

where Q is flow rate in m^3/day

S_0 is BOD applied to system in mg/l

Putting equation (iii) and (ii) in (i),

$$\text{F/M} = \frac{QS_0}{VX_t}$$

Q.7 (b) (ii) Solution:

Advantages of ASP :

1. Area requirement in this process is less as compared to other methods.
2. Effluent of high quality is obtained.
3. Although conventional process is difficult to operate, but modifications in this process have made the process easy to operate.
4. Loss of head through the plant is quite less.
5. There is no fly or odour nuisance in this type of plant as in the case of a trickling filter.

Disadvantages of ASP :

1. Its operating cost is high because of high power consumption for operating air compressors and sludge circulation pumps.
2. If there is variation in volume or character of sewage, then adverse effects are produced on working of process, producing inferior effluent.
3. Quantity of sludge obtained is high and needs suitable thickening and disposal.

Q.7 (c) (i) Solution:

The possible four mechanisms of coagulation are as below:

1. **Ionic layer compression:**

   The quantity of ions in water surrounding a colloid has an effect on decay function of electrostatic potential. A high ionic concentration compresses the layer that is composed predominantly of counter ions towards the surface of celluloid. If this layer is sufficiently compressed, then the Van-der Waals forces will be predominant across the entire area of influence, so that net force will be attractive and no energy barriers will exist. An example of ionic layer compression occurs in nature when a turbid stream flows into the ocean. There, the ion content of water increases drastically and coagulation and settling occur. Eventually, deposits are formed from material which was originally so small that it could not have settled without coagulation.

2. **Adsorption and charge neutralization:**

   Unlike the ionic layer compression, nature rather than quantity of ions is of prime importance in theory of adsorption and charge neutralization. For instance when alum is used as coagulant in water its ionization produces sulphate anions and aluminium cations. Sulphate ions which may remain in the form of ion or combine with other cation. However, the Al^{3+} cations react immediately with water to form a variety of aquometallic ions and hydrogen. These aquometallic ions become part of ionic cloud surrounding colloid, and are adsorbed on the surface of colloid where they neutralize surface charge. Once the surface charge has been neutralized, ionic cloud dissipates and electrostatic potential disappears so that contact occurs freely.

3. **Sweep coagulation:**

The last product in hydrolysis of alum when used as a coagulant is aluminium hydroxide. The $Al(OH)_3$ forms in amorphous, gelatinous flocs that are heavier than water and settle by gravity. Colloids may become entrapped in a floc as it is formed, or they become stricky as the flocs settle. The process by which colloids are swept from suspension in this manner is known as sweep coagulation.

4. **Inter-particle bridging:**

Large molecules may be formed when aluminium or ferric salts dissociate in water. Synthetic polymers also may be used instead of , or in addition to , metallic salts. There polymers may be linear or branched and are highly surface reactive. Thus, several colloids may become attached to one polymer or series of polymer, colloid groups may become enmeshed resulting in a settleable mass. In addition to adsorption forces, charges on polymer may assist in coagulation process. Metallic polymers formed by addition of aluminium or ferric salts are positively charged, while synthetic polymers may carry positive or negative charge or may be neutral.

Q.7 (c) (ii) Solution:

Surface overflow rate, V_0 = 32 m/day = 0.37 mm/s

Now, weight fraction less than or equal to the stated size is calculated below in table by subtracting from 100

Particle size, mm	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
Settling velocity in mm/s	1.06	0.88	0.72	0.65	0.37	0.25	0.17	0.10	0.04	0.01
Wt. fraction less than stated size (%) or stated settling velocity	90	85	65	35	25	20	15	10	5	0

Now, from the table, it can be seen that particles having settling velocities less than 0.37 m/s is 25 percent. So 75% of particles will be removed completely in basin. As we know that partial removal of smaller particles will also occur in case of settling basin.

The percentage of smaller particles is given as

$$= \frac{1}{V_0}\Sigma V_s \cdot \Delta X$$

where, V_δ = settling velocity corresponding to a size,

V_0 = surface overflow rate

Let us divide values between 0 to 25% into five intervals of 5%, each giving ΔX as 5.

Now, the value of $V_S \cdot \Delta X$ can be computed from table as:

$$\Sigma V_S \cdot \Delta X = 0.04 \times 5 + 0.1 \times 5 + 0.17 \times 5 + 0.25 \times 5 + 0.37 \times 5$$

$$= 4.65$$

So, $$\text{Efficiency} = 75 + \frac{4.65}{0.37} = 87.568\%$$

Q.8 (a) (i) Solution:

Given,

$$\text{Discharge of wastewater, } Q_w = 1250 \text{ m}^3/\text{d} = 0.0145 \text{ m}^3/\text{s}$$

$$\text{Discharge of stream, } Q_s = 20000 \text{ m}^3/\text{d} = 0.23 \text{ m}^3/\text{s}$$

Now, $$\text{DO of mix} = \frac{\text{DO of wastewater } \times Q_w + DO \text{ of stream} \times Q_s}{Q_w + Q_s}$$

$$= \frac{0 \times 0.0145 + 8 \times 0.23}{0.0145 + 0.23} = 7.52 \text{ mg/l}$$

BOD_5 of mixture (i.e 5 day BOD at 20° C)

$$= \frac{\text{BOD}_5 \text{ of wastewater } \times Q_w + \text{BOD}_5 \text{ of stream} \times Q_s}{Q_w + Q_s}$$

$$= \frac{200 \times 0.0145 + 2 \times 0.23}{0.0145 + 0.23} = 13.74 \text{ mg/l}$$

$$\text{Temperature of mix} = \frac{\text{Temp. of wastewater} \times Q_w \times \text{Temp. of stream} \times Q_s}{Q_w + Q_s}$$

$$= \frac{26 \times 0.0145 + 22 \times 0.23}{0.0145 + 0.23} = 22.24° \text{ C}$$

Now,

Ultimate BOD of mixture, $$L_o = \frac{(BOD_5)_{\text{mix}}}{1 - e^{-K_1 \times 5}} = \frac{13.74}{1 - e^{-0.35 \times 5}} = 16.63 \text{ mg/l}$$

Equilibrium concentration of DO of mixture at 22.24° C,

$$C_s = 8.99 + \left(\frac{8.83 - 8.99}{23 - 22}\right) \times (22.24 - 22) = 8.95 \text{ mg/l}$$

Therefore, initial oxygen deficit,

$$D_o = C_s - C$$

$$= 8.95 - 7.52 = 1.43 \text{ mg/l}$$

Also,
$$(K_1)_{22.24} = K_{1\,(20°)}\,(1.04)^{T-20°}$$
$$= 0.35 \times (1.04)^{(22.24\,-\,20)} = 0.38 \text{ day}^{-1}$$
$$(K_2)_{22.24} = (K_2)\cdot(1.02)^{T-20}$$
$$= 0.55 \times (1.02)$$
$$= 0.57 \text{ day}^{-1}$$

Now, time of critical oxygen deficit,

$$t_c = \frac{1}{K_1(f-1)}\ln\left[\left\{1-(f-1)\frac{D_o}{L_o}\right\}f\right]$$

where,
$$f = \frac{K_2}{K_1} = \frac{0.57}{0.38} = 1.5$$

Putting values, we get

$$t_c = \frac{1}{0.38\times(1.5-1)}\ln\left[\left\{1-(1.5-1)\times\frac{1.43}{16.63}\right\}1.5\right]$$
$$= 1.9 \text{ days}$$

Critical oxygen deficit,
$$D_c = \frac{L_o}{f}e^{-K_1t_c} = \frac{16.63}{1.5}e^{-0.38\times1.9} = 5.39 \text{ mg/l}$$

Therefore,

$$\text{Minimum oxygen level} = (8.95 - 5.39) \text{ mg/l}$$
$$= 3.56 \text{ mg/l}$$

Q.8 (a) (ii) Solution:

Population equivalent:

Industrial wastewaters are generally compared with per capita normal domestic wastewaters using the concept of population equivalent.

Mathematically,

$$\begin{bmatrix}\text{Standard BOD (5 days)}\\ \text{of industrial sewage}\end{bmatrix} = \begin{bmatrix}\text{Standard BOD (5 days) of domestic}\\ \text{sewage per person per day}\end{bmatrix}\times\begin{pmatrix}\text{Population}\\ \text{equivalent}\end{pmatrix}$$

The population equivalent, thus indicates the strength of industrial wastewaters for estimating the treatment required at municiplal sewage treatment plant.

Relative stability:

It may be defined as ratio of oxygen available in effluent (as DO, nitrite) to total oxygen required to satisfy its first stage BOD demand. It is expressed as percentage of total oxygen required and can be expressed by equation.

Relative stability, $$S = 100\left[1-(0.794)^{t_{20}}\right]$$
$$= 100\left[1-(0.630)^{t_{37}}\right]$$

where, t_{20} and t_{37} represent the time in days for a sewage sample to decolourise a standard volume of methylene blue solution, when incubated at 20° C or 37° C respectively. The decolorization caused by enzymes produced by anaerobic bacteria is an indication of available oxygen used in oxidizing the unstable organic matter.

Q.8 (b) (i) Solution:

L_{eq} is that statistical value of sound pressure level that can be equated to any fluctuating noise levels. It can be defined as a constant noise level, which over a given time, expends the same amount of energy, as is expended by fluctuating noise levels over same time.

Mathematically, $$L_{eq} = 10\log\sum_{i=1}^{n} 10^{Li/10} \times t_i$$

where, n = Total number of samples,

L_i = Noise level of any i^{th} sample,

t_i = Time duration of i^{th} sample, expressed as fraction of total sample time

From the given data:

Time (in s)	10	20	30	40	50	60	70	80	90	100
Noise (dBA)	72	74	73	77	75	80	82	84	81	78
$t_i = \frac{t}{100}$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$\frac{L_i}{10}$	7.2	7.4	7.3	7.7	7.5	8.0	8.2	8.4	8.1	7.8

$$\sum 10^{L_i/10} \times t_i = 84132815.74$$

Now, $$L_{eq} = 10\log_{10}\sum_{1}^{10} 10^{L_i/10} \times t_i$$
$$= 10 \times \log_{10}(84132815.74)$$
$$= 79.25 \text{ dBA}$$

Q.8 (b) (ii) Solution:

Primary sludge

$$\text{Area of primary clarifier, } A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 25^2 = 490.87 \text{ m}^2$$

$$\text{Overflow rate} = \frac{14350}{490.87} = 29.23 \text{ m/day}$$

Now, mass of solids removed in primary treatment $= \eta \times SS \times Q$

$= 0.6 \times 240 \times 10^{-3} \times 14350$

$= 2066.4$ kg/day

Now, Solids content in primary sludge = 4%

So, sludge formed from 4 kg primary solids = 100 kg

Hence, sludge formed from 2066.4 kg primary solids $= \frac{100}{4} \times 2066.4 = 51660$ kg/day

Specific gravity of primary sludge = 1.02

So, volume of primary sludge $= \frac{51660}{1.02 \times 1000} = 50.647 \text{ m}^3\text{/day}$

Secondary sludge

BOD entering in aerator = (1 – 0.3) × 210 = 147 mg/l

Effluent BOD = 10 mg/l

So, BOD consumed in aerator = 137 mg/l $= 137 \times 10^{-3} \times 14350$

= 1965.95 kg/day

Therefore, Mass of secondary solids = 0.34 × 1965.95 = 668.42 kg/day

Now, solid content in secondary sludge = 1%

So, sludge formed from 1 kg secondary solid = 100 kg

Sludge formed from 668.42 kg secondary solids = 100 × 668.42 = 66842 kg/day

Now, specific gravity of secondary sludge = 1.03

So, volume of secondary sludge $= \frac{66842}{1000 \times 1.03} = 64.89 \text{ m}^3\text{/day}$

Therefore, total mass of sludge = 51660 + 66842 = 118502 kg/day

Also, total mass of solids in sludge = 2066.4 + 668.42 = 2734.82 kg/day

Total, volume of sludge $= 50.65 + 64.89 = 115.54 \text{ m}^3\text{/day}$

Now, assuming negligible solids in thickened supernatant, the total mass of solids in thickened sludge is 2734.82 kg/day

Solids content in thickened sludge = 3%

So, thickened sludge formed from 3 kg solids = 100 kg

Thickened sludge formed from 2734.82 kg solids = $\frac{100}{3} \times 2734.82 = 91160.67$ kg/day

So, volume of thickened sludge = $\frac{91160.67}{1300} = 70.12\,m^3/\text{day}$

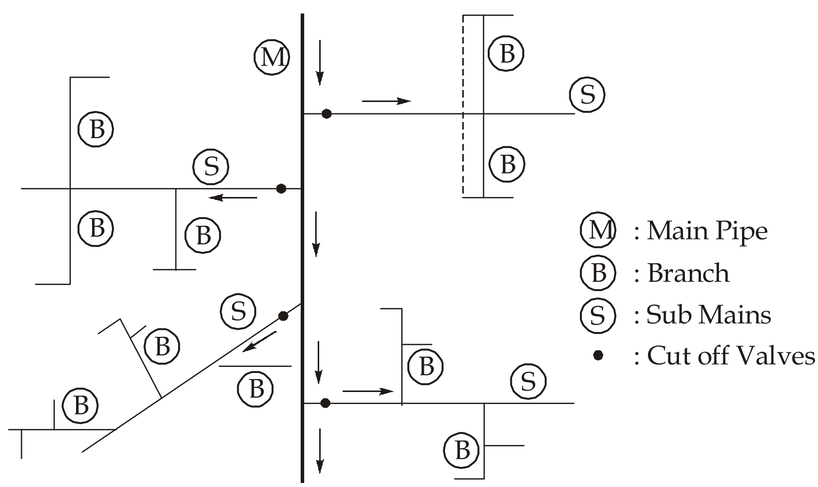
Therefore, percent volume reduction = $\left(\frac{115.54 - 70.12}{115.54}\right) \times 100 = 39.31\%$

Q.8 (c) (i) Solution:

In general there are four types of water distribution network which are as follows:

(i) **Dead end system:** This network system is adopted for cities which have developed in a haphazard manner with no definite pattern of growth. Ex- old cities.

Dead End or Tree System



Advantages:

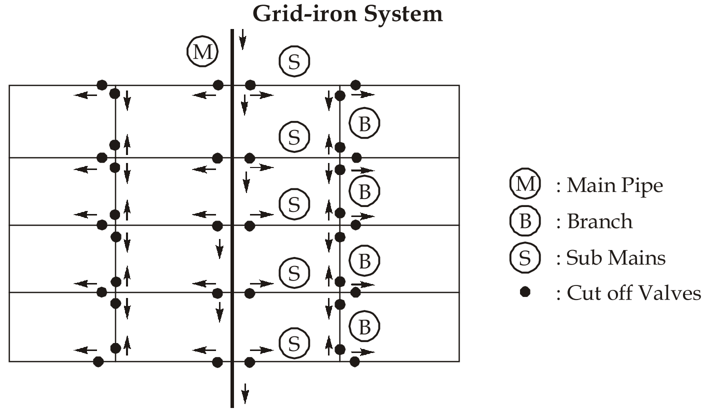
- It is relatively a cheap system.
- Determination of discharge and pressure is easy due to less number of valves being employed in the network.
- Laying of pipes is easy.
- Pipes used are of lesser diameter and hence economical.

Disadvantages:

- Owing to a large number of dead ends, stagnation of water occurs which cannot be utilized at locations where demand is high.
- During repairs and maintenance, entire downstream portion below the valve has to be cut-off.

(ii) Grid iron system: It is suitable for cities with rectangular layout where water mains and branches are laid in a rectangular fashion.

Grid-iron System



Advantages:

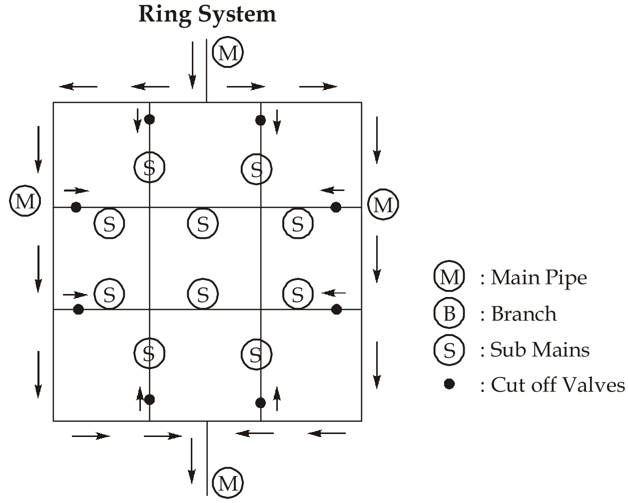
- Due to the absence of dead ends, whole water remains in circulation.
- Due to continuously circulating water, there are less chances of water getting contaminated as compared to still water.
- In case of repairs and breakdown, water is available from other locations and only a part of network needs to be isolated.

Disadvantages:

- Due to large number of valves required to be provided at all branches, it is difficult to compute discharge and pressure and thus exact calculation of pipe size required is difficult.

(iii) Circular/Ring system: In this system of water distribution, the main line is laid all along the periphery roads and sub-mains branch out from the main system of pipes.

Ring System



Advantages:

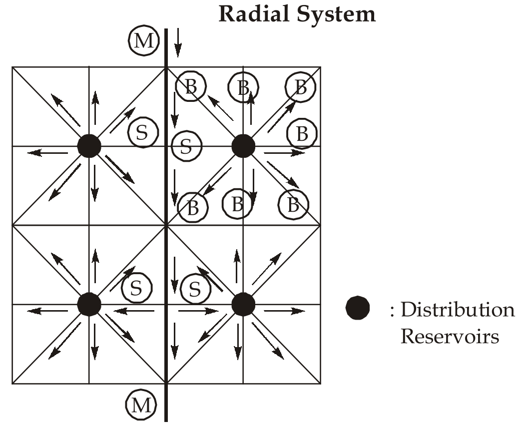
- The biggest advantage of this system is that water can be supplied to any point from at least two directions thereby ensuring availability of water for longer time per day.

Disadvantages:

- More pipe material is required.
- It involves high maintenance cost.

(iv) Radial system: Here the area is divided into different zones and water is kept in the distribution reservoir provided in the middle of each zone. Supply pipes are laid radially towards the periphery of the system.

Radial System



Advantages:

- Here calculation of pipe size is very easy.
- Water is made available in a short time as compared to other distribution systems.
- It is the most economical system if combined pumping and gravity flow are adopted.

Disadvantages:

- House at the distance end faces fluctuations.
- Design is very complex.

Q.8 (c) (ii) Solution:

(i) Air valve:

- It is a special kind of valve which is generally placed along the pipe line at "summits" on both sides of the sluice valves and also on the downstream side of all other sluice valves.

- When placed on summits which are very near to the HGL or sometimes above the HGL (especially during negative water hammers) it ensures the safety of the pipe against collapse.
- Similarly when placed below the ordinary sluice valves, it will protect the pipe against the negative pressures.
- A valve which will open out automatically as soon as the pressure in the pipe falls below a certain fixed predetermined value, and thus allowing air to enter the pipe is known as an air inlet valve.
- Sometimes air gets accumulated at high points when the supply is restored and the pipe is refilled after repairs. The accumulated air obstruct the free flow of water and the pipe may get air locked. Air relief valves are therefore required to be provided at all summits to remove the accumulated air under such circumstances.

(ii) Reflux valve:

- It is sometimes called as non-return valve because it prevents water to flow back in the opposite direction.
- It may be installed on the delivery side of the pumping set, so as to prevent the back flow of stored or pumped water, when the pump is stopped.
- It is also installed on pump discharges to reduce water hammer pressures on the pump.
- It is also required at inter-connections between a polluted water system and a potable water system, so as to prevent the entry of pollution into the pure water.
- The simplest type of reflux valve or check valve is a flap shutter hinged at the outlet end of pipe, which opens out in the direction of flow but closes the valve due to its own weight when the flow in the permissible direction ceases.

(iii) Flanged joint:

- It is used for pumping stations, filter plants, and at other locations where it may be necessary to occasionally disjoint the pipe.
- Two flanged pipes are brought together, keeping a rubber washer in between so as to make them water tight. They are then fixed by means of nuts and bolts.
- These joints are strong but rigid and hence cannot be used where deflections or vibrations are expected.
- They are expensive and mostly used for indoor works such as pumping stations, filter plants, etc.

(iv) Expansion joint:

- It is provided at suitable intervals in the pipelines, so as to counteract the thermal stresses produced due to temperature variations.
- The socket end is cast flanged and the spigot end is kept plain to provide an expansion joint in cast iron pipe.
- While making this joint, a small space is kept between the face of the spigot and the inner face of the socket and the spigot is filled up by means of a rubber gasket.
- The flanges are then tightened by means of nuts and bolts.
- When the pipe expands, the socket end moves forward and the gap left just closes.
- Similarly, when the pipes contract, the socket moves backward creating the gap.

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