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**ESE 2022**

**Main Exam  
Detailed Solutions**

**Mechanical  
Engineering**

**PAPER-II**

**EXAM DATE : 26-06-2022 | 2:00 PM to 5:00 PM**

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# ANALYSIS

## Mechanical Engineering ESE 2022 Main Examination

**Paper-II**

Sl.	Subjects	Marks
1.	Engineering Mechanics	32
2.	Strength of Materials	52
3.	Engineering Materials	12
4.	Mechanisms and Machines	84
5.	Design of Machine Elements	72
6.	Manufacturing Engineering	72
7.	Industrial & Maintenance Engineering	64
8.	Mechatronics	52
9.	Robotics	40
		<b>Total 480</b>

**Scroll down for  
detailed solutions**



**Section-A**

**Q.1 (a)** Find the centroidal coordinates of an area bounded by the following curves:

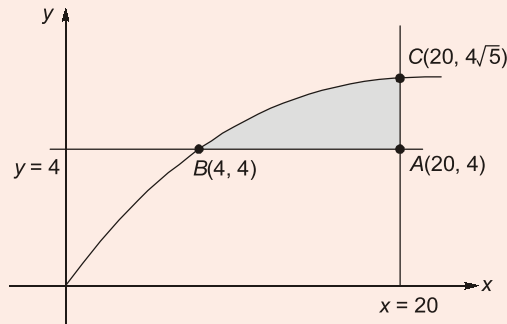
$$y = 4$$

$$x = 20$$

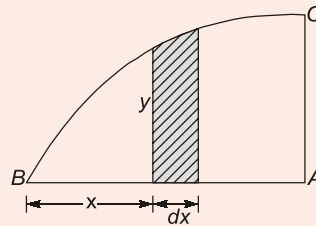
$$y^2 = 4x$$

[12 marks : 2022]

**Solution:**



Points of intersection are given as A (20, 4), B(4, 4) and C(20,  $4\sqrt{5}$ )



Area of the bounded figure is given as

$$A = \int y dx = \int_4^{20} (\sqrt{4x} - 4) dx = 2 \left[ \frac{2}{3} x^{3/2} \right]_4^{20} - 4|x|_4^{20}$$

$$= \frac{4}{3} [20^{3/2} - 4^{3/2}] - 4[16]$$

$$= 44.59 \text{ m}^2$$

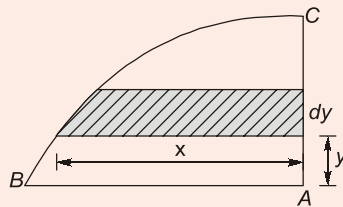
x-coordinate of the centroid is given by

$$\bar{x} = \frac{\int x dA}{\int dA} = \frac{\int xy dx}{44.59}$$

$$= \frac{\int_4^{20} x(\sqrt{4x} - 4) dx}{44.59} = \frac{2 \int_4^{20} x^{3/2} dx - 4 \int_4^{20} x dx}{44.59}$$

$$= \frac{2 \times \frac{2}{5} [x^{5/2}]_4^{20} - 2[x^2]_4^{20}}{44.59}$$

$$= \frac{0.8 \times (1756.85) - 768}{44.59} = 14.296 \text{ meters}$$



$$\bar{y} = \frac{\int y dy}{A} = \frac{\int yx dy}{44.59} = \frac{\int_4^{4\sqrt{5}} y \left(20 - \frac{y^2}{4}\right) dy}{44.59}$$

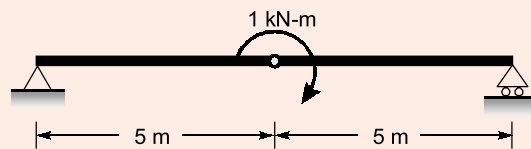
$$= \frac{\frac{20}{2} [y^2]_4^{4\sqrt{5}} - \frac{1}{4 \times 4} [y^4]_4^{4\sqrt{5}}}{44.59}$$

$$= \frac{640 - 384}{44.59} = 5.74 \text{ meters}$$

Centroidal co-ordinates are:  
(14.296 m, 5.74 m)

**End of Solution**

**Q.1 (b)** Draw the shear force and bending moment diagrams for the following beam :



[12 marks : 2022]



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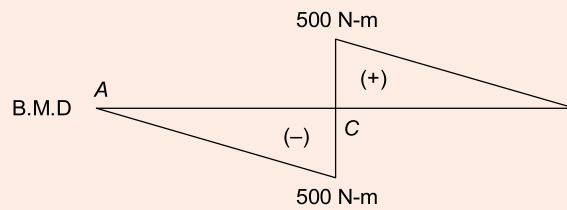
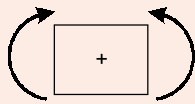
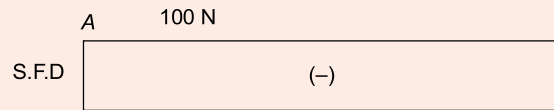
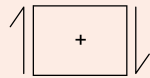
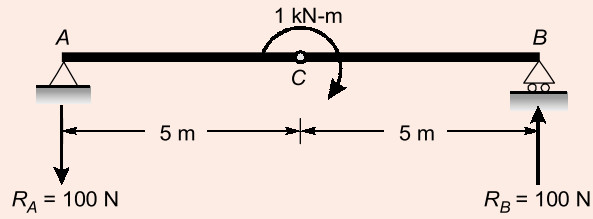


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**Solution:**



BM in AC ( $x$  from A),

$$M_x = -100x$$

$$M_A(x=0) = 0$$

$$M_C(x=5) = -500 \text{ Nm}$$

BM in CB ( $x$  from B)

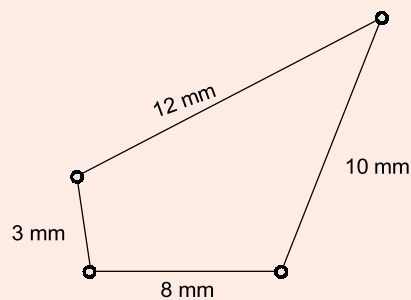
$$M_x = 100x$$

$$M_B(x=0) = 0$$

$$M_C(x=5) = 500 \text{ Nm}$$

**End of Solution**

**Q.1 (c)** Define kinematic chain. Find all the inversions of the chain shown in the figure.



[12 marks : 2022]

**Solution:**Shortest length,  $s = 3$  mmLongest length,  $l = 12$  mmOther two link length,  $p = 10$  mm and  $q = 8$  mm

$$(s + l) = 3 + 12 = 15 \text{ mm}$$

$$(p + q) = 10 + 8 = 18 \text{ mm}$$

Here,  $(s + l) < (p + q)$  $\Rightarrow$  Grashof's law is satisfied.

When:

3 mm link is fixed  $\rightarrow$  Double-crank mechanism

When

8 mm link is fixed  $\rightarrow$  3 mm link will be adjacent to fixed $\Rightarrow$  Crank – Rocker mechanism

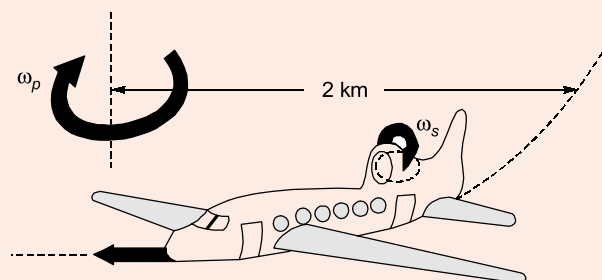
When

12 mm link is fixed  $\rightarrow$  3 mm link will be adjacent to fixed $\Rightarrow$  Crank – Rocker mechanism

When, 10 mm link length is fixed

 $\rightarrow$  3 mm shortest link will be coupler $\Rightarrow$  Double-rocker mechanism.**End of Solution**

- Q.1 (d)** The rotor of a turbojet engine has a mass of 250 kg and a radius of gyration of 25 cm. The engine rotates at a speed of 12000 rpm in the clockwise direction if viewed from the front of the aeroplane. The plane while flying at 1500 km/hr turns with a radius of 2 km to the right. Compute the gyroscopic moment the rotor exerts on the plane structure. Also, determine whether the nose of the plane tends to rise or fall when the plane turns.

**[12 marks : 2022]**

**Solution:**

$$m = 250 \text{ kg}; R = 25 \text{ cm} = 0.25 \text{ m}$$

$$I = mR^2 = 250 \times (0.25)^2 = 15.625 \text{ kg-m}^2$$

$$N = 12000 \text{ rpm}$$

$$\Rightarrow \omega = \frac{2\pi \times 12000}{60} = 400\pi \text{ rad/s}$$

$$V = 1500 \text{ km/hr}$$

$$= 1500 \times \frac{5}{18} \text{ m/s} = 416.6666 \text{ m/s}$$

$$R = 2 \text{ km} = 2000 \text{ m}$$

$$(\omega_p)_{\text{turning}} = \frac{V}{R} = 0.208333 \text{ rad/s}$$

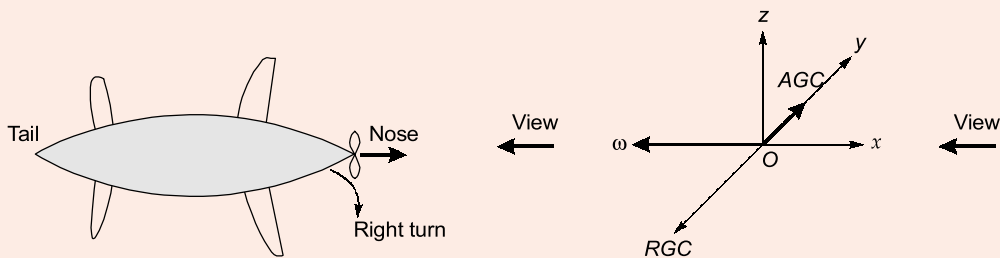
**Gyroscopic couple:**

$$C = I\omega\omega_p = 15.625 \times 400\pi \times 0.208333$$

$$C = 4090.61478 \text{ Nm}$$

$$C = 4.0906 \text{ kNm}$$

Ans.



AGC : Active gyroscopic couple

RGC : Reactive gyroscopic couple

Effect : Nose will go little bit up and tail will go little bit down.

**End of Solution**

**Q.1 (e)** A bolt is loaded with an axial pull of 12 kN along with a transverse shear force of 6 kN. Find the diameter of the bolt required, according to

(i) the maximum principal stress theory.

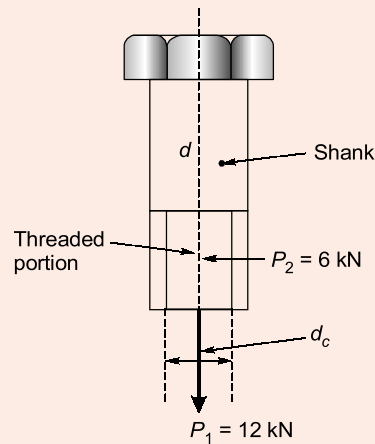
(ii) the maximum shear stress theory.

Take permissible tensile stress at elastic point = 100 MPa, Poisson's ratio = 0.3.

[12 marks : 2022]



**Solution:**



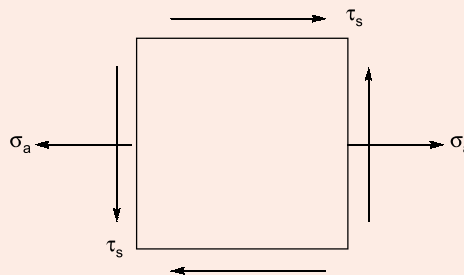
$$(\sigma_t)_{\text{per}} = 100 \text{ MPa}$$

$$\mu = 0.3$$

Let  $d$  = Major or nominal or shank diameter of bolt

$d_c$  = core or minor diameter of bolt

Critical cross-section is the cross-section of threaded portion of bolt. State of stress at the critical point is shown in figure below



$$\sigma_a = \frac{4P_1}{\pi d_c^2} = \frac{4 \times 12 \times 10^3}{\pi (d_c)^2}$$

$$\sigma_a = \frac{15278.875}{d_c^2} \text{ MPa}$$

$$\tau_s = \frac{4P_2}{\pi d_c^2} = \frac{4 \times 6000}{\pi (d_c)^2} = \frac{7639.437}{d_c^2} \text{ MPa}$$

By using M.P.S.T.

$$\frac{1}{2} \left[ \sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2} \right] \leq (\sigma_t)_{\text{per}}$$

$$\frac{1}{2} \left[ \frac{15278.875}{d_c^2} + \sqrt{\left( \frac{15278.875}{d_c^2} \right)^2 + 4 \left( \frac{7639.437}{d_c^2} \right)^2} \right] \leq 100$$

$$d_c \geq 13.58 \text{ mm}$$

$$\text{Diameter bolt } (d) = \frac{d_c}{0.84} = 16.167 \text{ mm}$$

∴ M18 bolts should be selected w.r.t. M.P.S.T

By using M.S.S.T.

$$\sqrt{\sigma_x^2 + 4\tau_{xy}^2} \leq (\sigma_t)_{per}$$

$$\sqrt{\left(\frac{15278.875}{d_c^2}\right)^2 + 4\left(\frac{7639.437}{d_c^2}\right)^2} \leq 100$$

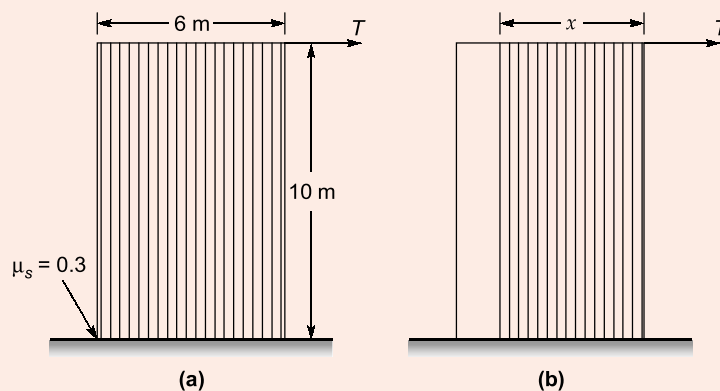
$$d_c \geq 14.698 \text{ mm}$$

$$\text{diameter of bolt } (d) = \frac{d_c}{0.84} = 17.498 \text{ mm}$$

∴ M18 bolts should be selected w.r.t. M.S.S.T.

**End of Solution**

- Q.2 (a)** A rectangular case is loaded with uniform vertical thin rods such that when it is full, as shown in the figure (a), the case has a total weight of 1000 N. The case weighs 100 N when empty and has a coefficient of static friction 0.3 with the floor as shown in the diagram. A force T of 200 N is maintained on the case. If the rods are unloaded from the left end as shown in the figure (b), what is the limiting value of x for equilibrium to be maintained?



[20 marks : 2022]

**Solution:**

When the case is full its weight is 1000 N and when the case is empty, its weight is 100 N. Therefore, when the rods completely fill 6 m, their mass is 900 N.

∴ When 6 m is filled, mass of rods is 900 N

When x m is filled, mass of rods is 150x N



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Free-body diagram in the limiting case

Calculating net moment about the point A

$$T \times 10 = 100 \times 3 + 150x \times \frac{x}{2}$$

$$T \times 10 = 300 + 75x^2$$

$$\frac{2000 - 300}{75} = x^2$$

$$x = 4.761 \text{ meters}$$

From equilibrium of forces,

$$T = \mu N \dots \dots \dots \text{in horizontal direction}$$

$$200 = 0.3 N$$

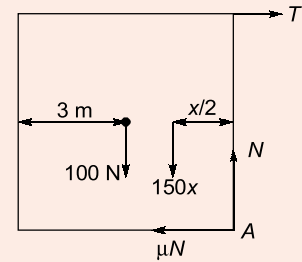
$$N = 666.67 \text{ N}$$

$$N = 150x + 100 \dots \dots \text{in vertical direction}$$

$$666.67 = 150x + 100$$

$$x = 3.777 \text{ meters}$$

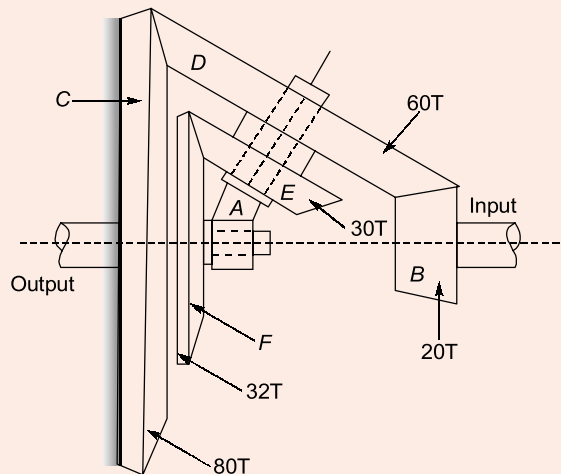
∴ Therefore, limiting value of x is 4.761 meters.



**End of Solution**

**Q.2 (b) What is planetary or epicyclic gear train?**

In the gear train shown in the figure, the wheel gear C is fixed. The gear B is connected to the input shaft, and gear F is connected to the output shaft. The arm A, carrying the compound gears D and E, turns freely on the output shaft. If the input shaft speed is 1000 rpm in the counter clockwise (ccw) direction when seen from the right, determine the speed of the output shaft. The number of teeth on each gear is indicated in the figure.



[20 marks : 2022]

**Solution:**

Motions	Arm (A)	B ( $T_B=20$ )	D/E ( $\frac{T_D=60}{T_E=30}$ )	F ( $T_F=32$ )	C ( $T_C=80$ )
1. Let arm fixed (w.r.t. Arm) and gear B speed = $+x$ (clockwise)	0	$+x$	$\pm x \cdot \frac{20}{60}$	$-x \cdot \frac{20}{60} \times \frac{30}{32}$	$-x \cdot \frac{20}{60} \times \frac{60}{80}$
2. Arm effect consideration	$y$	$(y+x)$	$(y \pm \frac{x}{3})$	$(y - \frac{5x}{16})$	$(y - \frac{x}{4})$

Given:  $N_c = 0$

Sign convention



$$y - \frac{x}{4} = 0 \quad \dots(i)$$

Input speed = 1000 rpm (Anticlockwise)

$\Rightarrow$

$$N_B = -1000$$

$$y + x = -1000 \quad \dots(ii)$$

By equation (ii) – equation (i)

$$\frac{5x}{4} = -1000$$

$$x = -\frac{4000}{5} = -800$$

$$y - x = -1000$$

$$y - 800 = -1000$$

$$y = -200$$

$$N_{\text{output}} = N_F$$

$$= y - \frac{5x}{16}$$

$$= -200 - 5 \times \frac{(-800)}{16}$$

$$= -200 - 5 \times (-50)$$

$$= -200 + 250$$

$$= 50 \text{ rpm}$$

$$N_{\text{output}} = 50 \text{ rpm (clockwise)}$$

Ans.

**End of Solution**

**Q2 (c)** An internal expanding shoe brake has a diameter of 320 mm and a width of 30 mm. The actuating forces are equal. The maximum pressure is not to exceed  $80 \text{ kN/m}^2$ . In reference to the figure given below, the values of various parameters are as follows:

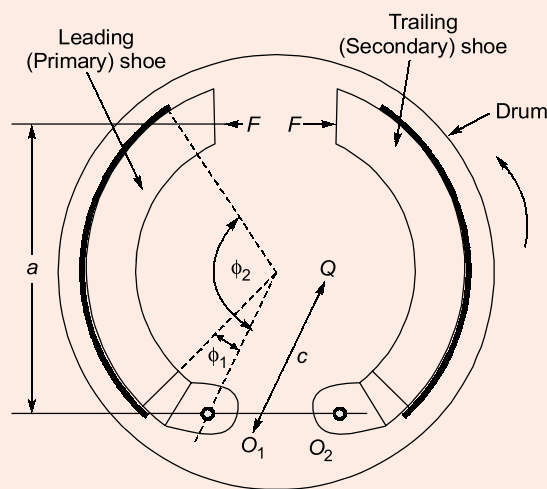
$$\phi_1 = 15^\circ$$

$$\phi_2 = 145^\circ$$

$$a = 220 \text{ mm}$$

$$c = 125 \text{ mm}$$

Determine the actuating force and the braking torque if the value of the coefficient of friction is 0.32.



[20 marks : 2022]

**Solution:**

Show brake diameter = 320 mm;  $a = 220 \text{ mm}$ ;  $r =$  Shoe brake radius = 160 mm;

$c = 125 \text{ mm}$ ; face width =  $w = 30 \text{ mm}$ ;  $\phi_1 = 15^\circ$ ;  $\phi_2 = 145^\circ$

Maximum pressure =  $p_{\max} = 80 \text{ kN/m}^2 = 0.08 \text{ MPa}$ ;  $\mu =$  coefficient of friction = 0.32

$p'_{\max}$  = Maximum pressure for trailing shoe

$M_n$  = Moment due to normal force

$$M_n = p_{\max} w c r \left[ \frac{\phi_2 - \phi_1}{2} - \frac{\sin 2\phi_2}{4} + \frac{\sin 2\phi_1}{4} \right]$$

$$\phi_2 = 145^\circ \times \frac{\pi}{180} = 2.53 \text{ rad};$$

$$M_n = (0.08)(320)(125)(160) \left[ \frac{2.53}{2} - 0.235 + 0.125 \right]$$

$$M_n = 591.136 \text{ Nm}$$

$M_f$  = Moment due to friction force

$$M_f = \rho_{\max} \mu wr \left( r - \cos \phi_2 + r \cos \phi - \frac{a}{2} (\cos^2 \phi_2 - \cos^2 \phi_1) \right)$$

$$M_f = (0.08)(0.32)(160) [160.82 + 154.55 - 110(-0.262)]$$

$$M_f = 42.294 \text{ Nm}$$

$$F = \text{Actuating force} = \frac{M_n - M_f}{a} = 2494.74 \text{ N}$$

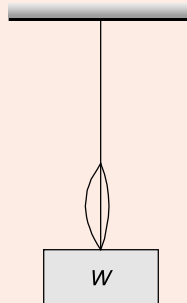
$$\rho'_{\max} = (\rho_{\max}) \frac{Fa}{M_n + M_f} = 0.0693 \text{ MPa}$$

$$\begin{aligned} \text{Braking torque} &= T_f = \mu wr^2 (\cos \phi_1 - \cos \phi_2) (\rho_{\max} + \rho'_{\max}) \\ &= (0.32)(30)(160)^2 [\cos 15^\circ - \cos 145^\circ] [0.08 + 0.0693] \\ T_f &= 589.657 \text{ Nm} \end{aligned}$$

**End of Solution**

**Q.3 (a)** Three wires each of 5 mm diameter are used to lift a load of  $W = 7500 \text{ N}$ . An indicative diagram is shown below. The unstressed lengths of the three wires are 18 m, 17.997 m and 17.994 m. Find

- (i) the stress in the longest wire.
  - (ii) the stress in the shortest wire if the load is reduced to 2000 N.
- Take  $E = 2.1 \times 10^5 \text{ N/mm}^2$



[20 marks : 2022]

**Solution:**

Cross-section area of each wire,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (5)^2 = 19.634 \text{ mm}^2$$

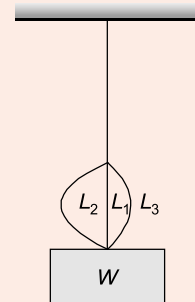
$$A = 19.6349 \text{ mm}^2$$

(i)

$$W = 7500 \text{ N}$$

$$L_1 = 17.994 \text{ m}$$

$$L_2 = 17.997 \text{ m}$$



$$L_3 = 18 \text{ m}$$

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

Initially shortest wire will bear the whole load.

Difference in length between  $L_1$  and  $L_2$

$$L_2 - 4 = 0.003 \text{ m}$$

As the  $L_1$  is stretched by 0.003 m,  $L_2$  will share some part of total load.

Load shared by the first wire,

$$\Rightarrow P_1 = \frac{\delta AE}{L_1} = \frac{3 \times 19.6349 \times 2.1 \times 10^5}{17994}$$

$$P_1 = 687.4506 \text{ N} < 7500 \text{ N}$$

Hence, the second wire will also be stretched  $L_3 - L_2 = 3 \text{ mm} \rightarrow$  Stretching of second wire then first wire will stretched by  $3 + 3 = 6 \text{ mm}$

$$\text{Now, } P_2 = \frac{\delta AE}{L_2} = \frac{3 \times 19.634 \times 2.1 \times 10^5}{17.997 \times 10^3}$$

$$= 687.304 \text{ N}$$

$$P_1 = \frac{\delta AE}{L_1} = \frac{6 \times 19.634 \times 2.1 \times 10^5}{17.994 \times 10^3}$$

$$= 1374.839 \text{ N}$$

$$P_1 + P_2 = 2062.142 \text{ N} < 7500 \text{ N}$$

Hence, third wire will also get stretched.

At equilibrium,

$$P_1 + P_2 + P_3 = 7500 \quad \dots(i)$$

Let the deflection in third wire is  $\delta_3$

$$\Rightarrow \delta_1 = \delta_3 + 6$$

$$\frac{P_1 L_1}{AE} = \frac{P_3 L_3}{AE} + 6$$

$$\Rightarrow \frac{P_1 \times 17994}{19.634 \times 2.1 \times 10^5} = \frac{P_3 \times 18000}{19.634 \times 2.1 \times 10^5} + 6$$

$$\Rightarrow 4.364 \times 10^{-3} P_1 = 4.3656 \times 10^{-3} P_3 + 6$$

$$P_1 = P_3 + 1374.885 \quad \dots(ii)$$

$$\delta_2 = \delta_3 + 3$$

$$\frac{P_2 L_2}{AE} = \frac{P_3 L_3}{AE} + 3$$

$$\frac{P_2 \times 17997}{19.634 \times 2.1 \times 10^5} = \frac{P_3 \times 18000}{19.634 \times 2.1 \times 10^5} + 3$$



$$P_2 = P_3 + 687.316 \quad \dots(iii)$$

From (i), (ii) and (iii)

$$P_3 + 1374.885 + P_3 + 687.316 + P_3 = 7500$$

$$P_3 = 1812.599 \text{ N}$$

$$P_2 = 2499.9154 \text{ N}$$

$$P_1 = 3187.484$$

The stress in the longest wire,

$$\sigma_3 = \frac{P_3}{A} = \frac{1812.599}{19.6349}$$

$$\sigma_3 = 92.315 \text{ MPa}$$

(ii) If load W is reduced to 2000 N, then the second wire will not stretched completely since,

$$W_1 + W_2 = 2062.142 > 2000$$

Let it deflect by  $\delta_2$

$$\Rightarrow \delta_1 = (\delta_2 + 3) \text{ mm}$$

$$\frac{P_1 L_1}{AE} = \frac{P_2 L_2}{AE} + 3$$

$$\Rightarrow \frac{P_1(17.994 \times 1000)}{19.6349 \times 2.1 \times 10^5} = \frac{P_2(17.997 \times 1000)}{19.6349 \times 2.1 \times 10^5} + 3$$

$$4.3639 \times 10^{-3} P_1 = (4.3646 \times 10^{-3}) P_2 + 3$$

$$P_1 = P_2 + 687.458 \quad \dots(iv)$$

From equilibrium equation,

$$P_1 + P_2 = 2000 \quad \dots(v)$$

Solving (iv) and (v),

$$P_1 = 1343.729 \text{ N}$$

$$P_2 = 656.27 \text{ N}$$

Stress in the shortest wire, 
$$\sigma_1 = \frac{P_1}{A} = \frac{1343.729}{19.6349}$$

$$\sigma_1 = 68.4357 \text{ MPa}$$

**End of Solution**

**Q3 (b)** Free vibration amplitude of a 500 kg machine mounted on an isolator consisting of a spring and damping element (viscous) is shown in the figure. The time period is also shown in the figure. Find out the characteristics of the isolator i.e.,

- (i) natural frequency of the system.
- (ii) stiffness of the spring.
- (iii) damping coefficient of the isolator.



**Recorded Classes**

## **General Studies & Engineering Aptitude for ESE 2023 Prelims (Paper-I)**

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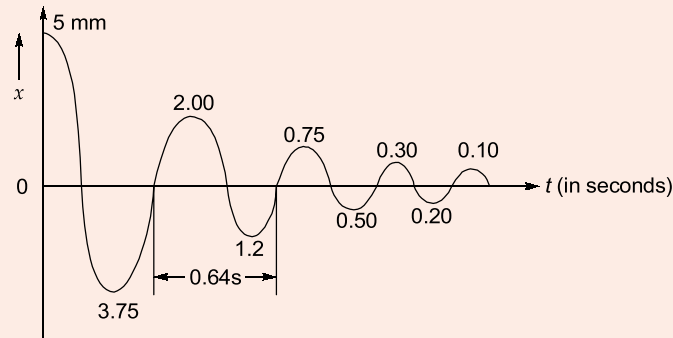
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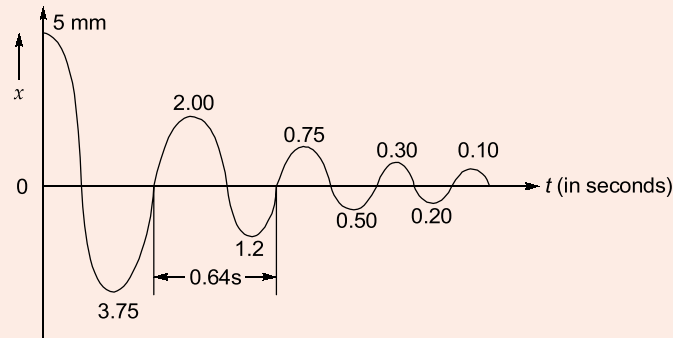


iOS



[20 marks : 2022]

**Solution:**



The logarithmic decrement is calculated as,

$$\delta = \log_e \left( \frac{5}{1.2} \right) = 0.9163$$

decrement ratio is given as,

$$\frac{x_0}{x_1} = \frac{5}{1.2} = 2.5$$

logarithmic decrement ratio is given by,

$$= \log_e \frac{x_0}{x_1} = \log_e 2.5 = 0.916$$

We know that,

$$\text{logarithmic decrement} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.916 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$(1-\xi^2)(0.916)^2 = 4\pi^2\xi^2$$

$$0.839 - 0.839\xi^2 = 4\pi^2\xi^2$$

$$\xi = 0.144$$

(i) Natural frequency of the system is given as,

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} \quad \left\{ \omega_d = \frac{2\pi}{T_d} = 9.817 \text{ rad/s} \right\}$$

$$\omega_n = 9.920 \text{ rad/s}$$

$$c = \xi m \omega_n = 2 \times 0.1443 \times 500 \times 9.920$$

$$c = 1431.456 \text{ Ns/m}$$

(ii) Stiffness of spring is given as:

$$s = m \omega_n^2 = 500 \times (9.920)^2 = 49203.2 \text{ N/m}$$

(iii) Damping coefficient is given as,

$$\frac{c}{m} = 2 \xi \omega_n$$

$$c = 2 m \xi \omega_n$$

$$= 2 \times 500 \times 0.144 \times 9.920$$

$$= 1428.48 \text{ Ns/m}$$

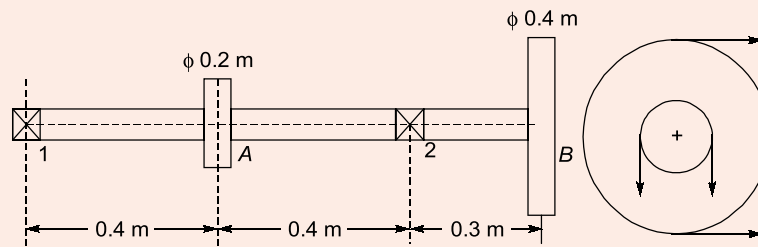
**End of Solution**

**Q3 (c)** A shaft is supported on two bearings 0.8 m apart as shown in the figure. A pulley of diameter 0.2 m is mounted on the shaft which takes the power from vertical belt drive. The shaft also carries another pulley, which transmits power to a machine. The diameter of the pulley is 0.4 m and it is placed to the right hand side of the right bearing at a distance of 0.3 m from the bearing. Both the pulleys contain flat belt at right angles to each other having tension ratio of 2.5 : 1. If the maximum tension in the belt is limited to 3000 N, find the shaft diameter.

Given : Allowable shear stress = 44 MN/m<sup>2</sup>

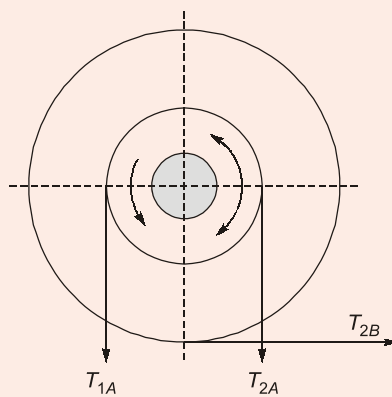
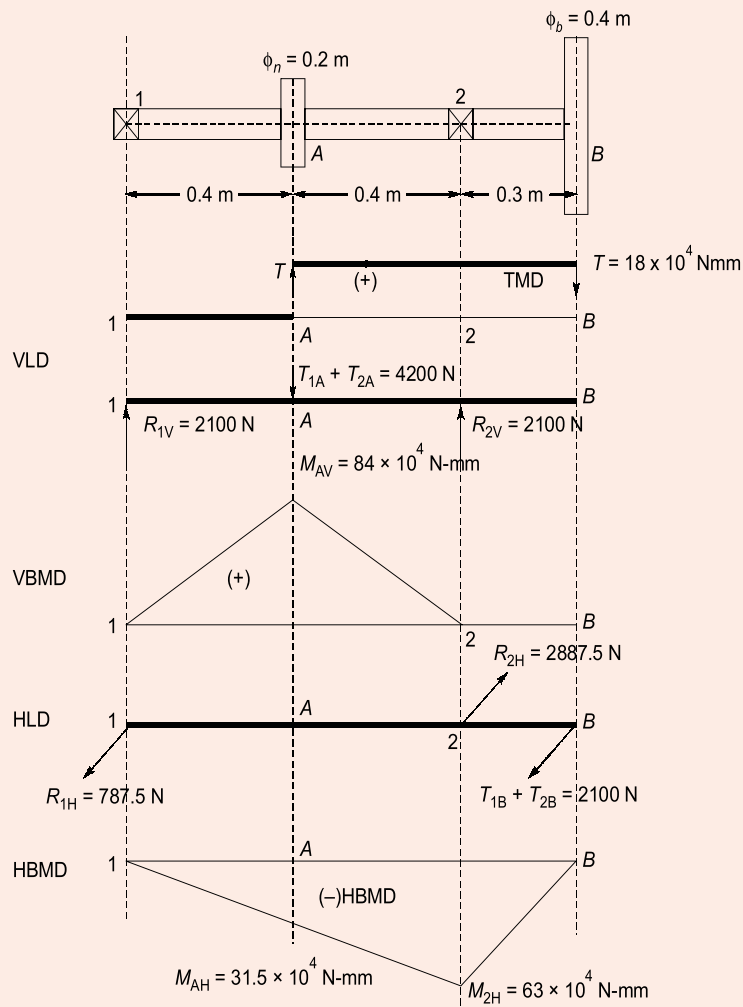
$k_b$  = Combined shock and fatigue factor applied to bending = 1.4

$k_t$  = Combined shock and fatigue factor applied to torsional moment = 1.2



[20 marks : 2022]

**Solution:**



$$\frac{T_1}{T_2} = 2.5$$

$$T_{\max} = 3000 \text{ N}$$

$$\tau_{\text{permissible}} = 44 \text{ MPa}$$

$$k_t = 1.2$$

$$k_b = 1.4$$

Calculation:

$$T_A = T_B = T$$

$$(T_{1A} - T_{2A})R_A = (T_{1B} - T_{2B})R_B$$

$$(1.5T_{1A})(100) = (1.5T_{1B})(200)$$

$$T_{1A} = 2T_{1B}$$

$$T_{1A} = T_{\text{max}} = 3000 \text{ N}$$

$$T_{2A} = 1200 \text{ N}$$

$$T_{1B} = \frac{T_{1A}}{2} = 1500 \text{ N}$$

$$T_{2B} = 600 \text{ N}$$

$$T = 18 \times 10^4 \text{ N-mm}$$

$$R_{1V} = R_{2V} = \frac{1}{2}(T_{1A} + T_{2A}) = 2100 \text{ N}(\uparrow)$$

$$M_{AV} = R_{1V} \times 400 = 84 \times 10^4 \text{ N-mm}$$

$$R_{2H} = 2887.5 \text{ N}; R_{1H} = 787.5 \text{ N}$$

$$M_{AR} = \sqrt{(M_{AV})^2 + (M_{AH})^2} = 89.71 \times 10^4 \text{ N-mm}$$

$$M_{2R} = M_{2H} = 63 \times 10^4 \text{ N-mm}$$

Hence, critical cross-section is the cross-section where bearing (2) is mounted by using MSST

$$(T_e)_{\text{max}} = (T_e)_2 = \sqrt{(k_b M_{2R})^2 + (k_t T_2)^2}$$

$$= 1274406.378 \text{ N-mm}$$

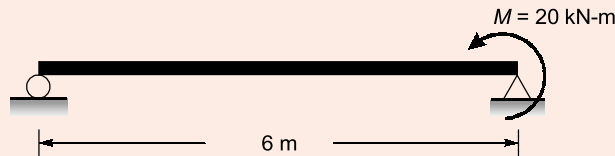
$$= \frac{\pi}{16} d^3 \tau_{\text{per}}$$

$$d = \sqrt[3]{\frac{16(T_e)_{\text{max}}}{\pi(44)}} = 52.83 \text{ mm}$$

Dia. of shaft = 50 mm

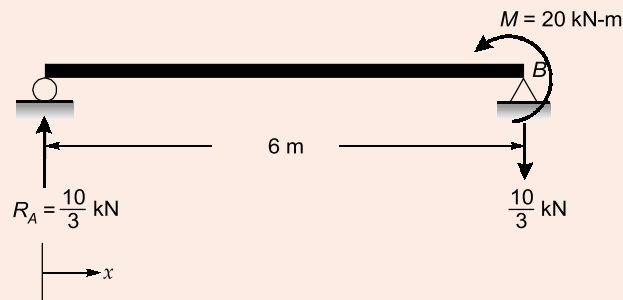
**End of Solution**

- Q.4 (a)** A simply supported beam of length 6 m is loaded with a couple-moment of 20 kN-m at the right end as shown in the figure. Determine
- the equation for elastic deflection.
  - the maximum deflection.
  - the slope at the ends.
- Take  $E = 2.1 \times 10^5 \text{ N/mm}^2$   $I = 3000 \text{ cm}^4$



[20 marks : 2022]

**Solution:**



$$R_A = R_B = \frac{20}{6} \text{ kN}$$

$$= \frac{10}{3} \text{ kN} = \frac{10^4}{3} \text{ N}$$

$$M_{xx} = \frac{10^4}{3} x; \text{ Nm}$$

where  $x$  is from left end,

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{10^4 x}{3EI};$$

$$EI = 2.1 \times 10^5 \times 10^6 \times 3000 \times 10^{-8}$$

$$= 2.1 \times 3 \times 10^6$$

$$EI = 63 \times 10^5 \text{ Nm}^2$$

$$\frac{d^2y}{dx^2} = \frac{10^4 x}{3 \times 63 \times 10^5} = \frac{x}{1890}$$

On integrating, 
$$\frac{dy}{dx} = \frac{x^2}{3780} + C_1 \quad \dots(i)$$

Again integrating,

$$y = \frac{x^3}{11340} + C_1x + C_2 \quad \dots(ii)$$

At  $x = 0$ ;  $y = 0$

⇒

$$C_2 = 0; \text{ from equation (ii)}$$

Also,

$$x = 6; y = 0$$

⇒

$$0 = \frac{(6)^3}{11340} + C_1(6) + 0$$

⇒

$$C_1 = \frac{-36}{11340} = \frac{-1}{315}$$

From equation (ii),

(i) 
$$y = \frac{x^3}{11340} - \frac{1}{315}x; \quad [\text{Equation for elastic deflection}]$$

(ii) For maximum deflection

$$\frac{dy}{dx} = 0$$

⇒ 
$$\frac{x^2}{3780} + C_1 = 0$$

⇒ 
$$\frac{x^2}{3780} = \frac{1}{315}$$

⇒ 
$$x = 2\sqrt{3} \text{ m}$$

$$y_{\max} = \frac{(2\sqrt{3})^3}{11340} - \frac{2\sqrt{3}}{315}$$

$$y_{\max} = -7.3314 \times 10^{-3} \text{ m}$$

$$y_{\max} = 7.3314 \text{ mm } (\downarrow)$$

(iii) Slope at the left end,

$$\left(\frac{dy}{dx}\right)_{x=0} = \left[\frac{x^2}{3780} - \frac{1}{315}\right]_{x=0}$$

$$\left(\frac{dy}{dx}\right)_{x=0} = -3.1746 \times 10^{-3} \text{ rad (c.w.)}$$





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Slope at the right end,

$$\left(\frac{dy}{dx}\right)_{x=6m} = \left(\frac{x^2}{3780} - \frac{1}{315}\right)_{x=6}$$

$$= \left(\frac{(6)^2}{3780} - \frac{1}{315}\right) = \frac{2}{315}$$

$$\left(\frac{dy}{dx}\right)_{x=6m} = 6.349 \times 10^{-3} \text{ rad (ccw)}$$

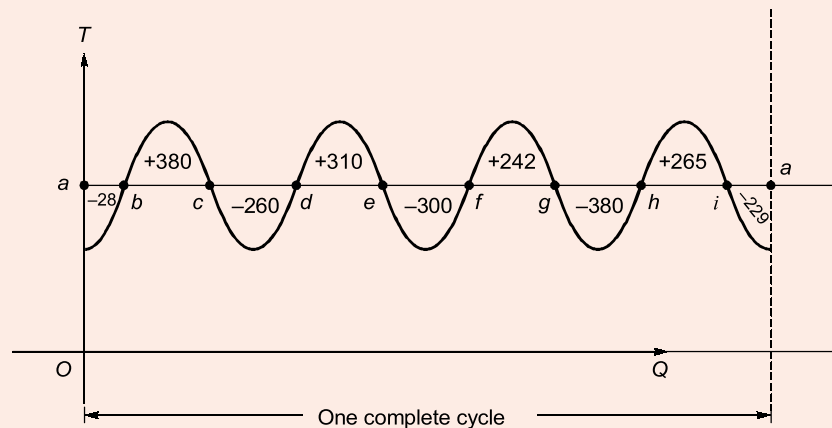
**End of Solution**

**Q.4 (b)** The turning moment diagram for a multicylinder engine is to be drawn to a vertical scale of 1 mm = 650 Nm and a horizontal scale of 1 mm = 4.5°. The areas above and below the mean torque line -28, +380, -260, +310, -300, +242, -380, +265 and -229 mm<sup>2</sup>.

The fluctuation of speed is limited to ± 1.8% of the mean speed which is 400 rpm. The density of the rim material is 7000 kg/m<sup>3</sup> and the width of the rim is 4.5 times its thickness. The centrifugal stress in the rim material is limited to 6 N/mm<sup>2</sup>. Neglecting the effect of boss and arm, determine the diameter and cross-section of the flywheel rim. The turning moment diagram may be drawn free-hand.

[20 marks : 2022]

**Solution:**



Let energy at a:

$$E_a = E$$

$$E_b = E - 28$$

$$E_c = E - 28 + 380 = E + 352$$

$$E_d = E + 352 - 260 = E + 92$$

$$E_e = E + 92 + 310 = E + 402$$

$$E_f = E + 402 - 300 = E + 102$$

$$E_g = E + 102 + 242 = E + 344$$

$$E_h = E + 344 - 380 = E - 36$$

$$E_i = E - 36 + 265 = E + 229$$

$$E_a = E + 229 - 229 = E$$

$$E_{\max} = E + 402$$

$$E_{\min} = E - 36$$

Maximum fluctuation of energy

$$(\Delta E)_{\max} = E_{\max} - E_{\min} = (E + 402) - (E - 36)$$

$$= 402 + 36 = 438 \text{ mm}^2$$

$$= 438 \times 51.0508 \text{ Joules}$$

$$= 22360.2504 \text{ Joules}$$

$$C_8 = \pm 1.8\% = 3.6\% = \frac{3.6}{100} = 0.036$$

$$N = 400 \text{ rpm (Mean speed)}$$

$$\rho = 7000 \text{ kg/m}^3$$

$$\text{Width : } W = (4.5)t$$

$$\sigma_{\text{bearing}} = 6 \times 10^6 \text{ N/m}^2$$

$$(\Delta E)_{\max} = 22360.2504 = I\omega^2 c_s$$

$$22360.2504 = I \times \left( \frac{2\pi \times 400}{60} \right)^2 \times 0.036$$

$$I = 353.9948 \text{ kg/m}^2$$

$$C_s = \frac{N_{\max} - N_{\min}}{N} = 0.036$$

$$N_{\max} - N_{\min} = 0.036 \times 400 = 14.4 \quad \dots(i)$$

$$\frac{N_{\max} + N_{\min}}{2} = 400$$

$$N_{\max} + N_{\min} = 800 \quad \dots(ii)$$

By equation (i) and equation (ii),

$$N_{\max} = 407.2 \text{ rpm}$$

$$V_{\max} = (R_w)_{\max} = \sqrt{\frac{\sigma_b}{\rho}}$$

$$R \times \omega_{\max} = \sqrt{\frac{6 \times 10^6}{7000}}$$

$$R \times 2\pi \times \frac{407.2}{60} = 29.2770$$

$$R = 0.68627 \text{ m} \quad \text{Ans.}$$

$$\text{Diameter, } D = 2R = 1.37315 \text{ m} \quad \text{Ans.}$$

$$I = mR^2$$

$$353.9948 = m \times (0.68627)^2$$

$$m = 751.6347 \text{ kg} \quad (\text{mass of flywheel})$$

$$m = (\text{width} \times \text{thickness} \times 2\pi R) \times \rho$$

$$751.6347 = Wt \times 2\pi R \times \rho$$

$$= (4.5)t \times t \times 2\pi R \times \rho$$

$$751.6347 = (4.5)t^2 \times 2\pi \times 0.68627 \times 7000$$

$$t = 0.0743 \text{ meter}$$

$$\text{Width, } W = 4.5 \times t$$

$$W = 0.3347 \text{ meter}$$

Cross-section area,

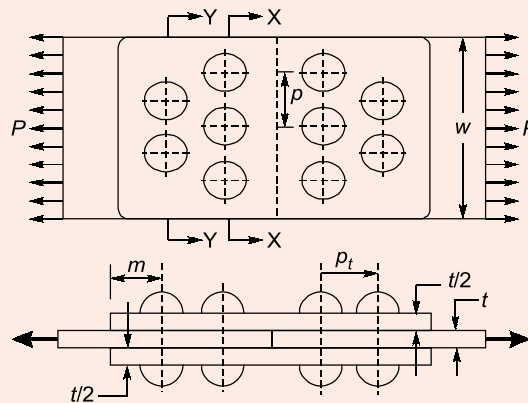
$$A = W \times t$$

$$A = 0.024787 \text{ m}^2$$

**End of Solution**

**Q.4 (c)** Two flat plates subjected to a tensile force  $P$  are connected together by means of double-strap butt joint as shown in the figure. The force  $P$  is 250 kN and the width of the plate  $w$  is 200 mm. The rivets and plates are made of same steel and the permissible stresses in tension, compression and shear are 70, 100 and 60 N/mm<sup>2</sup>, respectively. Calculate

- (i) the diameter of the rivets,
- (ii) the thickness of the plates
- (iii) the dimension of the seam viz.  $p$ ,  $p_t$  and  $m$ ,
- (iv) the efficiency of the joint.



[20 marks : 2022]

**Solution:**

(i) diameter of rivet ( $d$ ),  $P_s = P$

$$(N)(K) \frac{\pi}{4} d^2 \tau_{per} = P$$

$$(5)(1.875) \frac{\pi}{4} (d)^2 (60) = 250 \times 10^3$$

$$d = 23.788 \approx 24 \text{ mm}$$

2. Thickness of plate ( $t$ ):

$$P \leq \text{Tearing st. of joint [i.e. } (P_t)_I \text{ \& } (P_t)_{II}]$$

$$P \leq (P_t)_I [\because (P_t)_{II} > (P_t)_I]$$

$$250 \times 10^3 \leq (W - 2d) (t) (\sigma_t)_{\text{per}}$$

$$t \geq 23.496, t = 24 \text{ mm}$$

3.

$$P_C = Ndt(\sigma_c)_{\text{per}} = 288 \text{ kW}$$

4.

$$\text{St. of each rivet} = \frac{1}{5} [\text{Min of } P_s \text{ and } P_C]$$

$$P_s = (N)(K) \frac{\pi}{4} d^2 \tau_{\text{per}} = 254.57 \text{ kN}$$

$$\text{St. of each rivet} = 50.914 \text{ kN}$$

5.

$$(P_t)_I = (W - 2d) (t) (\sigma_t)_{\text{per}} = 255.36 \text{ kN}$$

6.

$$\begin{aligned} (P_t)_{II} &= (W - 3d) (t) (\sigma_t)_{\text{per}} + (N_r) \text{St. of each rivet} \\ &= (200 - 72)(24)(70) + (2) (50.914 \times 1000) \\ &= 316.868 \text{ kN} \end{aligned}$$

7.

$$\text{St. of solid plate} = P_{\text{solid}} = (W)(t) (\sigma_t)_{\text{per}} = 336 \text{ kN}$$

8.

$$\text{Efficiency of RJ} = \frac{\text{min of } [P_s, P_C, (P_t)_I \text{ \& } (P_t)_{II}]}{P_{\text{solid}}} \times 100$$

$$\eta = \frac{P_s}{P_{\text{solid}}} \times 100 = 75.765\%$$

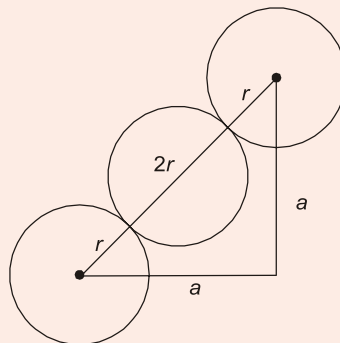
**End of Solution**

### Section-B

**Q5** (a) What do you mean by Atomic packing Factor? Find the volume of unit cell and atomic packing factor for face-centered cubic structure having atomic radius ' $R$ '.  
[12 marks : 2022]

**Solution:**

Atomic packing factor is defined as the percentage volume of unit cell covered by the atoms.



For a face centred cubic structure,  $4R = \sqrt{2}a$

$$\therefore \text{Volume of the unit cell} = a^3 = \left( \frac{4R}{\sqrt{2}} \right)^3 = 16\sqrt{2}R^3$$

Atomic packing factor for FCC:

$$n = 4$$

$$\therefore \text{APF} = \frac{4 \times \frac{4}{3} \times \pi R^3}{a^3} = \frac{4 \times \frac{4}{3} \times \pi R^3}{16\sqrt{2}R^3} = 0.74 = 74\%$$

**End of Solution**

**Q.5 (b)** A pipe of annealed steel with internal diameter of 60 mm and wall thickness of 3.0 mm is to be reduced down to the internal diameter of 54 mm and wall thickness 2.0 mm. Die angle used for pipe drawing is  $30^\circ$ ,  $\mu = 0.1$  and draft = 3.1. Compare the drawing force on the plug and the movable mandrels. Given :  $\sigma_0 = 240 \text{ N/mm}^2$ .

[12 marks : 2022]

**Solution:**

Given:  $D_i = 60 \text{ mm}$ ,  $D_f = 54 \text{ mm}$ ,  $h_i = 3 \text{ mm}$ ,  $h_f = 2 \text{ mm}$ ,  $2\alpha = 30^\circ$ ,  $\mu = 0.1$ ,  $\alpha = 15^\circ$

Assumption,  $\mu = \mu_1 = \mu_2$

$$\beta = 3.1^\circ, \mu_o = 240 \text{ N/mm}^2$$

(i) Tube drawing with plug or floating mandrel

$$\sigma_d = \left( \frac{2\sigma_o}{\sqrt{3}} \right) \left( \frac{1+B_1}{B_1} \right) \left[ 1 - \left( \frac{h_2}{h_1} \right)^{B_1} \right] + \sigma_b \left( \frac{h_2}{h_1} \right)^{B_1}$$

$$\sigma_b = 0 \text{ [Back stresses]}$$

$$B_1 = \frac{\mu_1 + \mu_2}{\tan \alpha - \tan \beta} = \frac{0.1 + 0.1}{\tan 15^\circ - \tan 3.1^\circ} = 0.9355$$

$$\sigma_d = \frac{2 \times 240}{\sqrt{3}} \left( \frac{1 + 0.9355}{0.9355} \right) \left[ 1 - \left( \frac{2}{3} \right)^{0.9355} \right]$$

$$= 573.363 \times 0.3157$$

$$\sigma_d = 180.992 \text{ MPa}$$

$$\text{Drawing force, } F_d = \sigma_d \times A_f$$

$$= \sigma_d \times \pi D_f h_f = 180.992 \times \pi \times 54 \times 2$$

$$F_d = 61.409 \text{ kN}$$

(ii) Moving mandrel case

**Assumption:**

- Moving cylindrical mandrel
- $B = 0$ ,  $\mu_1 = \mu_2 = \mu$

$$B = \frac{\mu}{\tan \alpha} = \frac{0.1}{\tan 15^\circ} = 0.3732$$

$$\sigma_d = \frac{F_d}{\pi D_f h_f} = \sigma_o (1+B) \ln \frac{h_i}{h_f}$$

$$\sigma_d = \frac{F_d}{\pi D_f h_f} = 240 (1 + 0.3732) \ln \frac{3}{2}$$

$$\sigma_d = 133.628 \text{ MPa}$$

$$F_d = \sigma_d \times \pi D_f h_f = 133.628 \times \pi \times 54 \times 2$$

$$F_d = 45.339 \text{ kN}$$

Drawing force comparison,

- Drawing force in case of plug = 61.409 kN ... (i)

- Drawing force in case of moving mandrel = 45.339 kN ... (ii)

$$\text{Comparison} = \frac{(F_d)_{\text{plug}}}{(F_d)_{\text{moving mandrel}}} = \frac{61.409}{45.339} = 1.354$$

**End of Solution**

- Q.5 (c) (i)** Write the basic assumptions of Linear Programming Problem.
- (ii)** There are two suppliers X and Y producing a product used in automobiles. The suppliers X and Y can produce maximum 100 and 60 units of the product per day, respectively. Buyers A and B consume 50 and 110 units of the product per day, respectively. The transportation costs from supplier X to buyers A and B are ₹30 and ₹50 per unit of the product, respectively. Similarly, the transportation costs from supplier Y to buyers A and B are ₹20 and ₹80 per unit of the product, respectively. Formulate the Linear Programming Problem to minimize the transportation cost.

[4+8 marks : 2022]

**Solution:**

(i)

**Assumption in Linear Programming:**

1. There is a well defined objective function such as maximizing profit or minimizing cost.
2. There are a number of restrictions or constraints (on the amount and extent of available resources for satisfying the objective function) which can be expressed in quantitative terms. These may refer to man-hours, machine hours, raw material, storage space, capital etc.
3. The parameters are subject to variation in magnitude.
4. The relationship expressed by constraints and the objective function are linear.
5. The objective function is to be optimised w.r.t. the decision variables involved in the phenomenon. The decision variables are non-negative and represent real life situation.

(ii)

Transportation matrix

	X	Y	
A	30	20	50
B	50	80	110
	100	60	160
			160

Minimize,

$$Z = 30X_A + 20Y_A + 50X_B + 80Y_B$$



# ESE 2023









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Demand = Supply  $\Rightarrow$  Problem is balanced

Applying Vogel's approximation method,

	X	Y	
A	30	20	50/0 10
B	50	80	110/10/0 (30)
	100/0	60/10/0	
	20	(60)	

$$Z = 50 \times 20 + 50 \times 100 + 80 \times 10 = ₹6800$$

Applying u-v method to check optimality:

Cost matrix for allocated cells

	0	30
-10		20
50	50	80

$u_i + v_j$  matrix for unallocated cell

	0	30
-10	-10	
50		

Subtracting  $u_i + v_j$  cell values from original matrix.

20	

Since no cell value is -ve, the solution is optimum.

**End of Solution**

**Q5 (d)** List down the requirements that govern the selection of actuator sizing for a motion axis in a mechatronic equipment. Will the requirements remain same for different types of actuators? Justify the answer.

[12 marks : 2022]

**Solution:**

There are majorly 4 parameters to be known for sizing any actuator for a mechatronic system, they are:

1. Maximum and minimum Supply pressure
2. Type of Actuator used
3. Modes of Failure
4. Operating torque

Actuator sizing is generally done after gathering information on

1. the maximum and minimum rated pressure
2. the type of actuator desired
3. the torque requirements as calculated

Actuator sizing is usually done using a manufacturer's sizing chart. Additional specifications to consider are

- (a) **the required speed of operation:** since speed has a direct relationship to the power requirement.
- (b) the required operating load.
- (c) **Fail mode:** actuator must be made such that it can hold, close and open position without any failure.

If the actuator is undersized, it will be unable to overcome the forces against it. This will cause slow and erratic stroking.

If the actuator is not stiff enough to hold the close position, the closure element will slam into the seat, causing a pressure surge.

If the actuator is oversized, it will cost more, weigh more, and be more sluggish in terms of speed and response.

Larger actuators may also provide a higher thrust that will damage internal valve parts. Actuators tend to be oversized because of safety factors but smaller sizes function just as well when the built-in safety factors are considered.

To ensure the best fit for different types of actuators, one should take into account following considerations when making their selections:

How much control will the end user have over motion profiles?

How clean is the operating environment? What are the potential hazards?

What is the stated life cycle? How much maintenance is required?

How quiet is the system?

How energy-efficient is it?

How much space is required for operation?

How easy is it to integrate with other applications?

What is the impact on total cost of operating

so for different types of actuators, we have to check all above parameters and have to ensure optimum actuator size for any system

**End of Solution**

**Q.5 (e) Discuss the importance of Fault Tree Analysis (FTA). Construct a FTA for a windowless room that contains one switch and two light-bulbs. Assume that the top fault event is dark room.**

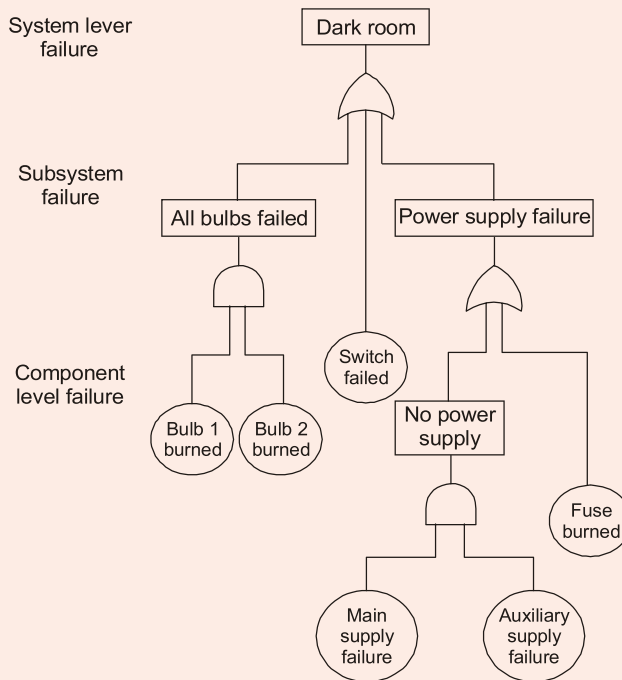
**[12 marks : 2022]**

**Solution:**

Fault Tree Analysis (FTA) is a graphical mathematical tool to explore the causes of system level failures. It is a structural approach to identify failure modes in a system operation that can be used to perform both qualitative and quantitative analysis.

Top down approach is used to identify component level failure that can cause system failures. It is used by several industries like nuclear, aerospace industries for failure analysis in complex systems.

FTA for dark room with 2 bulbs and one switch:



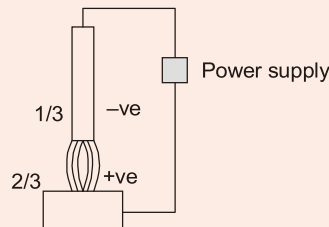
End of Solution

- Q.6 (a)** (i) Discuss the applications of straight polarity and reverse polarity in welding.  
(ii) Write the functions of flux used in welding process.  
(iii) Calculate the melting efficiency in the case of arc welding of steel with a potential 25 V and current 180 Amp. The travel speed is 4 mm/sec and cross-sectional area of the joint is 16 mm<sup>2</sup>. Heat required to melt steel may be taken as 12 Joule/mm<sup>3</sup> and heat transfer efficiency as 0.80.

[5+5+10 marks : 2022]

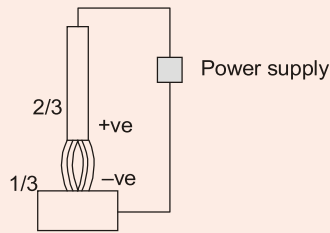
**Solution:**

- (i)  
Direct current straight polarity (DCSP, DCEN)



- (a) Electrode is -ve and workpiece is +ve.  
(b) More heat on the workpiece compared to electrode.  
(c) Used for high thickness and higher MP materials.  
(d) Depth of penetration is more.

**Direct current reverse polarity (DCRP, DCEP)**



- Electrode is +ve and workpiece is -ve.
- More heat on the electrode when compared to workpiece.
- Used for less thickness and low MP materials.
- Depth of penetration is less.

**(ii)**

- Flux coatings will act as de-oxidizers.
- By forming the slag liquid metal can be protected from the atmospheric gases and slag will control the heat transfer losses from liquid metal in turn properties of weld head.
- Flux coatings will increase the stability of the Arc.
- By reducing heat transfer losses from electric arc heat concentration on the workpiece is increased.
- By forming CO<sub>2</sub> gases, oxide formation will reduce.

**(iii)**

Given:  $V = 25$  volts,  $I = 180$  A,  $v = 4$  mm/s,  $A_b = 16$  mm<sup>2</sup>,  $H_m = 12$  J/mm<sup>3</sup>,  $\eta_h = 0.8$

$$\therefore \text{Melting efficiency, } \eta_m = \frac{H_m}{\frac{VI}{A_b \times v} \times \eta_h} = \frac{12 \times 16 \times 4}{25 \times 180 \times 0.8}$$

$$\eta_m = 0.2133 = 21.33\%$$

**End of Solution**

**Q.6 (b) (i)** Differentiate between True stress-strain and Engineering stress-strain, using a stress-strain diagram for mild steel.

(ii) A cylindrical specimen of alloy steel having an original diameter of 12 mm is subjected to a tensile load of 60 kN. If the instantaneous cross-sectional diameter of 10 mm and elongation observed is 10%, determine the true stress and strain hardening exponent 'n' in true stress-strain diagram. The value of tensile strength (K) is given as 1035 MPa

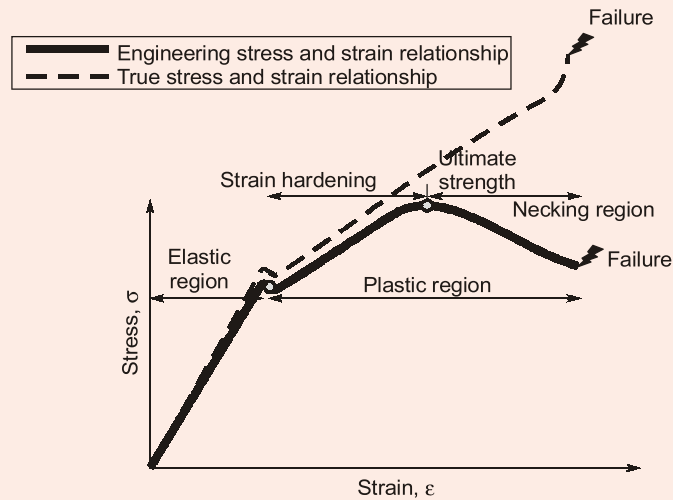
[8+12 marks : 2022]

**Solution:**

**(i)**

The curve based on the original cross-section and gauge length is called the engineering stress-strain curve, while the curve based on instantaneous cross-section area and length is called the true stress-strain curve.

For engineering stress, we assume the length and diameter of the sample remain constant throughout the whole experiment.



**Stress/strain graph of a tension test experiment**

Engineering stress is calculated by

$$\sigma = \frac{F}{A_0}$$

Engineering strain is calculated by

$$\epsilon = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}$$

True stress and strain are different from engineering stress and strain. In a tensile test, true stress is larger than engineering stress and true strain is less than engineering strain. The difference between the true and engineering stresses and strains will decrease with plastic deformation. At low strains (in elastic region), the differences between the two are negligible.

True stress is determined by instantaneous load acting on the instantaneous cross-sectional area.

$$\sigma_T = \frac{F_i}{A_i} = \sigma(1 + \epsilon)$$

True strain is logarithmic, it is the natural log of the instantaneous length over the original length is calculated by the formula

$$\epsilon_T = \ln\left(\frac{L_f}{L_i}\right) = \ln(1 + \epsilon)$$

[ $L_f$  = Final length,  $L_i$  = Initial length]

(ii)

Given:  $d_o = 12$  mm,  $P = 60$  kN,  $d_i = 10$  mm,  $\epsilon_E = \frac{10}{100} = 0.1$ ,  $k = 1035$  MPa

$$\text{True stress, } \sigma_T = \frac{P}{A_i} = \frac{60 \times 10^3}{\frac{\pi}{4} \times (0.01)^2} = 763.94 \text{ MPa}$$

Strain hardening exponent:

$$\text{True strain, } \epsilon_T = \ln(1 + \epsilon_E) = \ln(1 + 0.1) = 0.0953$$

∴

$$\sigma_T = k\epsilon_T^n$$

⇒

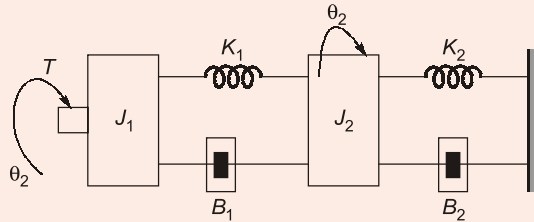
$$763.94 = 1035 \times (0.0953)^n$$

⇒

$$n = 0.129 \approx 0.13$$

**End of Solution**

**Q.6 (c)** A rotational mechanical system representing motor trailer is shown below. Derive the system of differential equation and the transfer function.



where,  $T$  is a torque applied to the free end

$\theta_1$  is an angular displacement of rotor with moment of inertia  $J_1$

$B_1$  is the damping coefficient of torsional damper

$K_1$  is the torsional spring stiffness

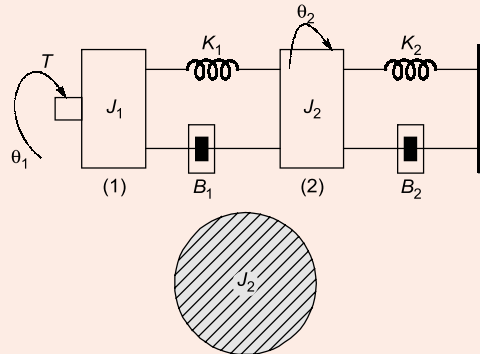
$K_2$  is the torsional spring stiffness

$B_2$  is the torsional damping coefficient

$\theta_2$  is the angular displacement of rotor with moment of inertia  $J_2$ .

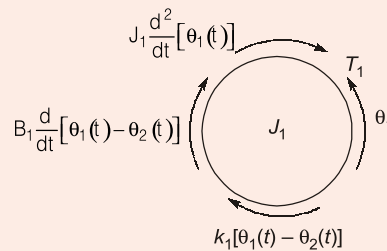
[20 marks : 2022]

**Solution:**



We have to find the transfer function of above mechanical system i.e.  $\frac{\theta(s)}{T(s)}$

By making free body diagram of rotor 1:



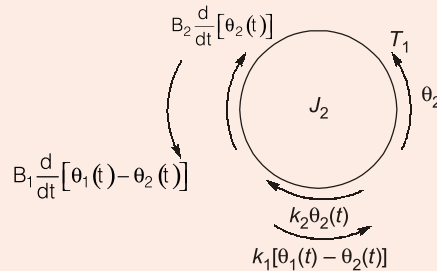
Governing differential equation will be

$$T = J_1 \frac{d^2}{dt^2} [\theta_1(t)] + B_1 \frac{d}{dt} [\theta_1(t) - \theta_2(t)] + k_1 [\theta_1(t) - \theta_2(t)]$$

By taking Laplace transform of above equation:

$$T(s) = J_1 S^2 \theta_1(s) + B_1 s [\theta_1(s) - \theta_2(s)] + k_1 [\theta_1(s) - \theta_2(s)]$$

By making free body diagram of rotor 2:



Governing differential equation for rotor 2:

$$\frac{J_2 d^2 [\theta_2(t)]}{dt^2} + k_2 \theta_2(t) + B_2 \frac{d}{dt} [\theta_2(t)] = k_1 [\theta_1(t) - \theta_2(t)] + B_1 \left[ \frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt} \right]$$

By taking Laplace transform of above equation:

$$J_2 S^2 \theta_2(s) + k_2 \theta_2(s) + B_2 S \theta_2(s) = k_1 [\theta_1(s) - \theta_2(s)] + B_1 s [\theta_1(s) - \theta_2(s)]$$

$$\theta_2(s) J_2 S^2 (B_1 + B_2) S \theta_2(s) + (k_1 + k_2) \theta_2(s) = B_1 S \theta_1(s) + k_1 \theta_1(s)$$

$$\frac{\theta_2(s)}{\theta_1(s)} = \frac{B_1(s) + k_1}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \quad \dots \text{(iii)}$$

Similarly by using equation 1:

$$\theta_2(s) = \left[ \frac{B_1(s) + k_1}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \right] \theta_1(s)$$

Replacing above term in equation 1 :

$$T(s) = J_1 S^2 \theta_1(s) + B_1 s [\theta_1(s)] - \left[ \frac{B_1 S + k_1}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \right] \theta_1(s) + k_1 - \left[ \theta_1(s) - \left[ \frac{B_1(s) + k_1}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \right] \right] \theta_1(s)$$

$$\frac{T(s)}{\theta_1(s)} = J_1 S^2 + B_1(s) \left[ \frac{J_2 S^2 + B_2(s) + k_2}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \right] + k_1 \left[ \frac{J_2 S^2 + B_2(s) + k_2}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \right]$$

$$\frac{T(s)}{\theta_1(s)} = J_1 S^2 + [B_1(s) + k_1] \left[ \frac{J_2 S^2 + B_2(s) + k_2}{J_2 S^2 + (B_1 + B_2) S + (k_1 + k_2)} \right]$$

$$\frac{T(s)}{\theta_1(s)} = \frac{J_1 S^2 [J_2 S^2 + (B_1 + B_2)s + (k_1 + k_2)] + [B_1(s) + k_1][J_2 S^2 + B_2(s) + k_2]}{J_2 S^2 + (B_1 + B_2)s + (k_1 + k_2)}$$

$$\text{or } \frac{\theta_1(s)}{T(s)} = \frac{J_2 S^2 + (B_1 + B_2)s + (k_1 + k_2)}{J_1 S^2 [J_2 S^2 + (B_1 + B_2)s + (k_1 + k_2)] + [B_1(s) + k_1][J_2 S^2 + B_2(s) + k_2]}$$

End of Solution

**Q.7 (a) (i)** The demand for an item in a company is observed as 15000 units per year and the production capacity of the plant is 2000 units per month. The set-up cost is ₹800 and the inventory holding cost is ₹25 per unit per year. The shortage cost of one unit is given as ₹250 per year. Determine the economic batch quantity and the number of shortages. Also, determine the cycle time, production time, and utilization percentage.

(ii) Discuss the classification of inventory items such as ABC analysis, SDE, VED and FSN.

[10+10 marks : 2022]

**Solution:**

(i)

Given:  $D = 15000$  units/year,  $d = \frac{15000}{12} = 1250$  units/month,  $C_o = ₹800$ ;  $C_h = ₹25$  per unit per year,  $C_b = ₹250$ /unit/year,  $P = 2000$  units/month

$$\begin{aligned} \text{Economic batch quantity, } Q^* &= \sqrt{\frac{2DC_o}{C_h}} \times \sqrt{\frac{P}{P-d}} \times \sqrt{\frac{C_b + C_h}{C_b}} \\ &= \sqrt{\frac{2 \times 15000 \times 800}{25}} \times \frac{2000}{750} \times \frac{275}{250} \\ &= 1678.094 \text{ units} \end{aligned}$$

$$S^* = \frac{Q^* C_h (P - d)}{(C_b + C_h) P} = \frac{1678.094 \times 25 \times (2000 - 1250)}{(250 + 25) \times 2000}$$

$$S^* = 57.207 \text{ units}$$

$$\text{Cycle time, } T_C = \frac{Q^*}{D} = \frac{1678.094}{15000} = 0.11187 \text{ yrs.}$$

$$T_C = 1.342 \text{ months}$$

Production time ( $t_p$ );

$$Q^* = t_p(P)$$

$$t_p = \frac{1678.094}{2000} = 0.839047 \text{ months}$$

$$\text{Utilization percentage} = \frac{Q^*}{P} = \frac{1678.094}{2000} = 83.9\%$$



(ii)

**ABC analysis:** ABC analysis divides inventories into three groupings in terms of percentage of number of items and percentage of total value. It is based on Pareto Analysis. In ABC analysis important items (high usage valued items) are grouped in A, while trivial items (low usage valued items) are grouped in C and the remaining middle level items are considered B items.

**VED analysis:** VED analysis specially pertains to the classification of maintenance spare parts. The spares are split into three categories of importance from the viewpoint of functional utility.

V stands for vital items, without which production would come to halt. These items would render the equipment or whole line operation in the process totally and immediately inoperative, unsafe. If these items go out of stock or are not readily available there will be loss of production.

E – E is for essential items, without which the performance or efficiency of the equipment will be reduced. Non-availability of these items may result in temporary loss of productivity. D - D stands for desirable items. The remaining items which do not cause any immediate loss in production fall under this category.

**SDE analysis:** This analysis is based on availability position of each item. In this analysis:

S refers to scarce items, which are short in supply and their availability is scarce. This includes imported items.

D refers to difficult items, which cannot be procured easily. These items may not be available in local market and have to be procured from far off cities or these are items for which there are a limited number of suppliers, or items for which quality suppliers are difficult to get.

E refers to easily available items.

The SDE analysis proves to be very useful in industrial situations where certain materials are scarce in supply and gives proper guidelines for deciding inventory policies.

**FSN analysis:** Here the items are classified into fast-moving (F), slow-moving (S) and non-moving (N) items on the basis of quantity and rate of consumption. The non-moving items (usually, not consumed over a period of two years) are of great importance. It is found that many companies maintain huge stocks of non-moving items blocking quite a lot of capital.

Moreover, there are thousands of such items. Scrutiny of these items is made to determine whether they could be used or to be disposed off. The classification of fast and slow moving items helps in arrangement of stocks in stores and their distribution and handling methods.

**End of Solution**



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✓ <b>Patna</b> : 21 <sup>st</sup> June, 2022	✓ <b>Lucknow</b> : 23 <sup>rd</sup> June, 2022	✓ <b>Hyderabad</b> : 27 <sup>th</sup> June, 2022	✓ <b>Bhopal</b> : 7 <sup>th</sup> June, 2022
✓ <b>Bhubaneswar</b> : 26 <sup>th</sup> May, 2022	✓ <b>Jaipur</b> : 15 <sup>th</sup> June, 2022	✓ <b>Kolkata</b> : 2 <sup>nd</sup> July, 2022	✓ <b>Pune</b> : 2 <sup>nd</sup> July, 2022

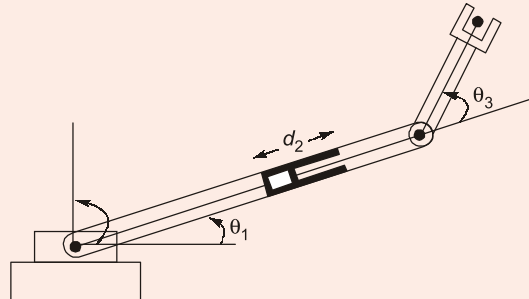
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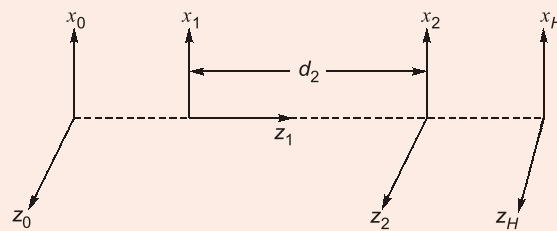
**Q.7 (b)** For the below given 3 DOF (Degree of Freedom) arm, determine the forward kinematic model, using D-H algorithm. Clearly indicate the assumptions, if any, to derive the kinematic model.



[20 marks : 2022]

**Solution:**

In this solution, the locations of the origins of some of the frames are arbitrary. Therefore, intermediate matrices might be different for each case. However, the final answer should be the same.



Link	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
0-1	0	$90^\circ$	0	$\theta_1$
1-2	0	$-90^\circ$	$d_2$	0
2-H	0	0	0	$\theta_3$

Individual transformation matrices are given below,

$${}^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 c90^\circ & s\theta_1 s90^\circ & 0 \\ s\theta_1 & c\theta_1 c90^\circ & -c\theta_1 s90^\circ & 0 \\ 0 & s90^\circ & c90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly,

$${}^1T_2 = \begin{bmatrix} c(0) & -s(0)c(-90^\circ) & s(0)s(-90^\circ) & 0 \\ s(0) & c(0)c(-90^\circ) & -c(0)s(-90^\circ) & 0 \\ 0 & s(-90^\circ) & c(-90^\circ) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$${}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_H = \begin{bmatrix} c\theta_3 & -s\theta_3c(0) & s\theta_3s(0) & 0 \\ s\theta_3 & c\theta_3c(0) & -c\theta_3s(0) & 0 \\ 0 & s(0) & c(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_H = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall transformation matrix for robot is given by,

$${}^0T_H = {}^0T_1 \times {}^1T_2 \times {}^2T_H$$

$${}^0T_H = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_3 & -c\theta_3 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

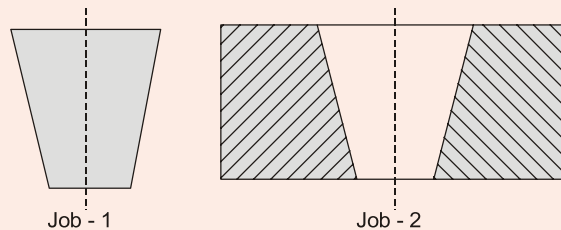
∴

$${}^0T_H = \begin{bmatrix} c\theta_1c\theta_3 - s\theta_1s\theta_3 & -(s\theta_1c\theta_3 + c\theta_1s\theta_3) & 0 & d_2s\theta_1 \\ s\theta_1c\theta_3 - c\theta_1s\theta_3 & (c\theta_1c\theta_3 - s\theta_1s\theta_3) & 0 & -d_2c\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = \begin{bmatrix} c(\theta_1 + \theta_3) & -s(\theta_1 + \theta_3) & 0 & d_2s\theta_1 \\ s(\theta_1 - \theta_3) & c(\theta_1 + \theta_3) & 0 & -d_2c\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**End of Solution**

**Q.7 (c)** As an engineer, you were supplied with the two tapered parts as shown below whose taper is to be measured using standard balls and rollers.



Elaborate the steps necessary to measure the taper angles of these parts. Provide the list of instruments and materials required, with detailed mathematical derivation for the measurement. Specify the precautions you need to take while measuring the angles.

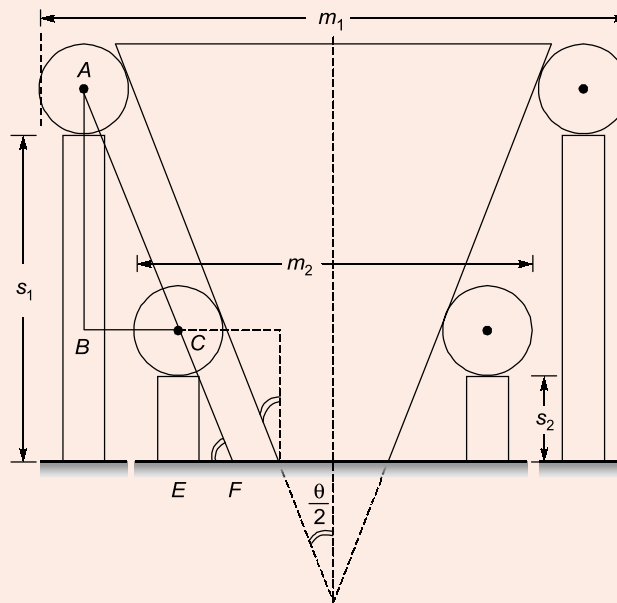
[20 marks : 2022]

**Solution:**

The steps necessary to measure the taper angles of these parts are:

1. First a small ball of radius  $R_2$  is inserted in the hole in lower position and depth  $H_2$  from upper surface of tapered hole to the top of the ball is measured. Small ball must be of such size as to be seated somewhere in between hole.
2. Then bigger ball of radius  $R_1$  is placed in the hole and distance  $H_1$  from the upper surface of tapered hole to the top of the ball is measured with height gauge.
3.  $O_1$  and  $O_2$  represent the centres of the two balls.
4. Draw  $O_2B$  parallel to  $AC$  and  $O_1B$  perpendicular to  $O_2B$ , then angle  $O_2 = \frac{\theta}{2}$ , where  $\theta$  is the angle of the tapered hole.

$\therefore$  Taper angle of job 1 =  $\theta$



$s_1 s_2 =$  Height of slips

$m_1, m_2 =$  micrometer readings

All rollers are of same size, i.e.  $r$

Now, in  $\Delta BAC$  and  $\Delta ECF$

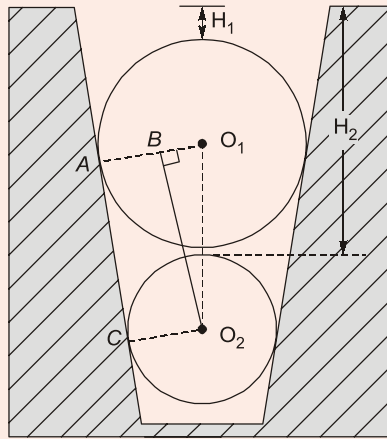
$$\angle BAC = \angle ECF = \frac{\theta}{2}$$

$$\therefore \tan \frac{\theta}{2} = \frac{BC}{AB} = \frac{EF}{EC}$$

$$\therefore \tan \frac{\theta}{2} = \frac{\left(\frac{m_1}{2} - r\right) - \left(\frac{m}{2} - r\right)}{(s_1 + r) - (s_2 + r)} = \frac{(m_1 - m_2)}{s_1 - s_2} = \frac{m_1 - m_2}{2(s_1 - s_2)}$$

$$\theta = 2 \tan^{-1} \left( \frac{m_1 - m_2}{2(s_1 - s_2)} \right)$$

- In this method, we require two balls of different sizes, depth gauge, height gauge etc.
- A word of caution in measurement of small taper measurement by balls should always be remembered. If the ball is allowed to drop in tapered hole, it will wedge in the taper and may be several thousands of mm lower. The weight of the ball and the measuring pressure exerted over it can cause appreciable axial movement to give inaccurate readings.



Let us use two rollers of radius  $R_1$  and  $R_2$ . The depth from top to bottom is mentioned in the above diagram.

$H_1$  = Distance from top to upper roller.

$H_2$  = Distance from top to lower roller.

$R_1$  = Radius of the larger roller.

$R_2$  = Radius of lower roller.

$$\text{In } \triangle O_1BO_2, \angle B = 90^\circ, \angle O_2 = \frac{\theta}{2}$$

$$\therefore \sin \frac{\theta}{2} = \frac{O_1B}{O_1O_2} = \frac{(R_1 - R_2)}{(H_2 - H_1) - (R_1 - R_2)}$$

$$O_1O_2 = (H_2 + R_2) - (H_1 + R_1)$$

$$= (H_2 - H_1) - (R_1 - R_2)$$

$$O_1B = R_1 - R_2$$

$$\therefore \theta = 2 \sin^{-1} \frac{(R_1 - R_2)}{(H_2 - H_1) - (R_1 - R_2)}$$

**End of Solution**

**Q.8 (a) (i)** The position of a point P of a rigid body B is located at  $B_P = [1, 2, 3]^T$ . Determine the global position after rotation  $30^\circ$  about X-axis and then  $45^\circ$  about Y-axis.

(ii) If a homogeneous transformation matrix T is given by

$$\begin{bmatrix} c\theta & -c\alpha s\theta & s\alpha s\theta & ac\theta \\ s\theta & c\alpha c\theta & -s\alpha c\theta & as\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine its inverse, using concepts of rigid body motion.

[10+10 marks : 2022]

**Solution:**

(i)

Rotation about X-axis by  $30^\circ$

$$[B_P]_{X,30^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \sqrt{3}-1 \\ \frac{3}{2} + \frac{3\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.732 \\ 4.0980 \end{bmatrix}$$

and, rotation about Y-axis by  $45^\circ$

$$[B_P]_{Y,45^\circ} = \begin{bmatrix} \cos 45^\circ & 0 & \sin 45^\circ \\ 0 & 1 & 0 \\ -\sin 45^\circ & 0 & \cos 45^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 0.732 \\ 4.0980 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0.732 \\ 4.0980 \end{bmatrix} = \begin{bmatrix} 3.604 \\ 0.732 \\ 2.1906 \end{bmatrix}$$

So, global position of  $B_P$  after both the rotations is  $\begin{bmatrix} 3.604 \\ 0.732 \\ 2.1906 \end{bmatrix}$

(ii)

Consider two frames, frame (1) and frame (2), rotated and translated relative to each other as shown in figure knowing  ${}^1T_2$  its inverse  ${}^2T_1$  to be found.

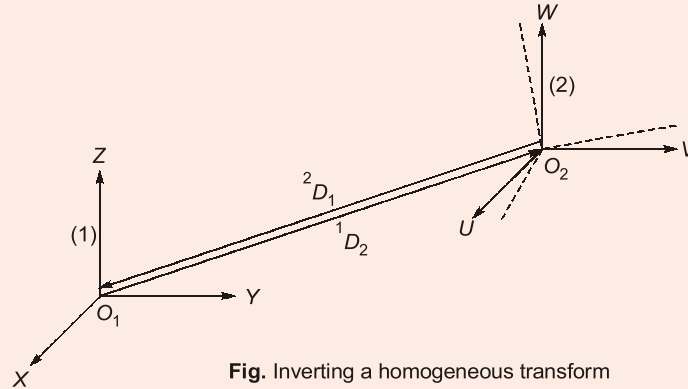


Fig. Inverting a homogeneous transform

Now, 
$${}^1T_2 = \left[ \begin{array}{ccc|c} {}^1R_2 & & & {}^1D_2 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

and 
$${}^2T_1 = \left[ \begin{array}{ccc|c} {}^2R_1 & & & {}^2D_1 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

$\therefore {}^2R_1 = {}^1R_2^T$

Therefore, mapping of a point P from frame (2) to frame (1), is

$${}^1P = {}^1D_2 + {}^1R_2 {}^2P$$

Premultiplying both sides by  ${}^2R_1$  gives,

$${}^2R_1 {}^1P = {}^2R_1 {}^1D_2 + {}^2R_1 {}^1R_2 {}^2P$$

$\therefore {}^2R_1 {}^1R_2 = I$

$\therefore {}^2R_1 {}^1P = {}^2R_1 {}^1D_2 + {}^2P$

or 
$${}^2P = {}^2R_1 {}^1P - {}^2R_1 {}^1D_2 \quad \dots (a)$$

The mapping of a point P from frame (1) to frame (2) is

$${}^2P = {}^2R_1 {}^1P + {}^2D_1 \quad \dots (b)$$

Comparing equation (a) and (b), gives

$${}^2D_1 = -{}^2R_1 + {}^1D_2$$

or, 
$${}^2D_1 = -{}^1R_2^T {}^1D_2 \quad \dots (c)$$

Substituting equation (c) in matrix  ${}^2T_1$ , gives

$${}^2T_1 = \left[ {}^1T_2 \right]^{-1} = \left[ \begin{array}{ccc|c} {}^1R_2^T & & & -{}^1R_2^T {}^1D_2 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

Now, it is given that, matrix

$$T = \begin{bmatrix} c\theta & -c\alpha s\theta & s\alpha s\theta & ac\theta \\ s\theta & c\alpha c\theta & -s\alpha c\theta & as\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



We know,

$$T = \left[ \begin{array}{c|c} R_{3 \times 3} & P_{3 \times 1} \\ \hline 0 & 1 \end{array} \right]$$

So,

$$R = \begin{bmatrix} c\theta & -c\alpha s\theta & s\alpha s\theta \\ s\theta & c\alpha c\theta & -s\alpha c\theta \\ 0 & s\alpha & c\alpha \end{bmatrix}$$

and,

$$P = \begin{bmatrix} ac\theta \\ as\theta \\ d \end{bmatrix}$$

So, inverse of T,

$$T^{-1} = \left[ \begin{array}{c|c} R^T & -R^T P \\ \hline 0 & 1 \end{array} \right]$$

∴

$$R^T = \begin{bmatrix} c\theta & s\theta & 0 \\ -c\alpha s\theta & c\alpha c\theta & s\theta \\ s\alpha s\theta & -s\alpha c\theta & c\alpha \end{bmatrix}$$

and,

$$-R^T P = \begin{bmatrix} -c\theta & -s\theta & 0 \\ c\alpha s\theta & -c\alpha c\theta & -s\alpha \\ -s\alpha s\theta & s\alpha c\theta & -c\alpha \end{bmatrix} \begin{bmatrix} ac\theta \\ as\theta \\ d \end{bmatrix}$$

$$= \begin{bmatrix} -ac^2\theta - as^2\theta \\ ac\alpha s\theta c\theta - ac\alpha s\theta c\theta - ds\alpha \\ -as\alpha c\theta s\theta + as\alpha c\theta s\theta - dc\alpha \end{bmatrix} = \begin{bmatrix} -a \\ -ds\alpha \\ -dc\alpha \end{bmatrix}$$

then,

$$T^{-1} = \begin{bmatrix} c\theta & -s\theta & 0 & -a \\ c\alpha s\theta & -c\alpha c\theta & -s\alpha & -ds\alpha \\ -s\alpha s\theta & s\alpha c\theta & -c\alpha & -dc\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**End of Solution**

- Q.8 (b) (i)** Discuss the failure rate model having Burn-in period, Useful life period and Wear-out period.
- (ii) Show the relationship between failure rate ( $\lambda$ ), reliability (R), pdf(f) and cdf(F).
- (iii) Twenty machines have been operated for 100 hours. One machine fails in 70 hours and another in 80 hours. What is the mean time between failure and reliability at 500 hours? Assume the constant failure rate for the above machines.

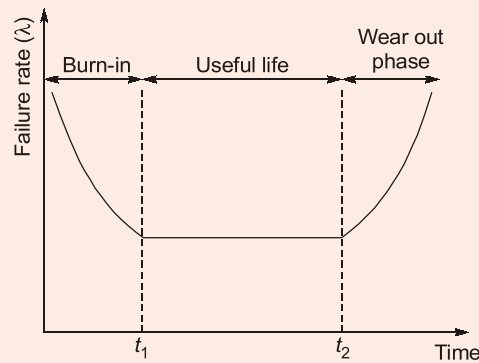
[5+5+10 marks : 2022]

**Solution:**

(i)

Most products go through three distinct phases from product inception to wear-out. Life cycle curve for failure rate ' $\lambda$ ' can be represented as a function of time. This model is known as bath tub curve which has three phases:

- **Burn in period:** Also known as infant mortality phase or debugging phase. The failure rate decrease in this phase as initial problems identified are ironed out.
- **Useful life period:** In this phase failures occur randomly and independently. Failure rate is constant.
- **Wear out phase:** At the end of useful life components age and wear out. An increase in failure rate is observed.



(ii)

pdf describes the shape and the area of failure distribution.

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$R(t) = \int_t^{\infty} f(t) dt$$

$$f(t) = \int_0^t f(t) dt$$

For useful life of the product, when failure rate is constant, exponential distribution is used to describe time to failure of the product.

Probability density function (f):

$$f = \lambda e^{-\lambda t}, \quad t \geq 0 \text{ and } \lambda \text{ is the failure rate}$$

Cumulative density function (F),  $F = \int_0^t e^{-\lambda t} dt$

Reliability at a time  $t$  is given by  $R(t) = 1 - F(t) = 1 - \int_0^t e^{-\lambda t} dt = e^{-\lambda t}$

(iii)

Number of failures = 2

Total time =  $100 \times 20 = 2000$  hrs.

$$MTBF = \frac{2000}{2} = 1000 \text{ hrs.}$$

$$\lambda = \frac{1}{MTBF} = 0.001/\text{hr.}$$

$$R(t) = e^{-\lambda t}$$

$$R(500) = e^{-0.001 \times 500} = 0.6065$$

End of Solution



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**Q.8 (c) (i)** The armature controlled DC motor has the following ratings:

$$K_T = 0.08 \text{ Nm/A, Maximum current} = 3 \text{ A}$$

$$K_e = 0.06 \text{ V rad/s, Maximum speed} = 600 \text{ rad/s}$$

$$\text{Armature resistance} = 1.5 \Omega$$

Determine the maximum output torque, maximum output power, maximum armature voltage and no-load motor speed. Assume that the frictional torque is zero.

(ii) What are the important components of Programmable Logic Controllers (PLCs)? List down the differences between PLCs and computers in terms of program and memory used, power supply and its disruption, and flexibility in operation.

[10 + 10 marks : 2022]

**Solution:**

(i)

**Note:** The units given in this question are incorrect. This question is out of scope of mechatronics syllabus.

$$k_T = 0.08 \text{ Nm/A}$$

$$I_{max} = 3 \text{ A}$$

$$k_e = 0.06 \text{ V/rad/s}$$

$$(\omega)_{max} = 600 \text{ rad/sec}$$

It is an armature controlled dc motor i.e. PMDC motor

$$\therefore \phi = \text{constant}$$

$$E_b = V - 1.5i$$

Speed is max@ No lag

$$\therefore @i = 0, \omega = \text{max}$$

$$\therefore k_e(\omega)_{max} = V$$

$$\therefore V = 0.06 \times 600 = 36 \text{ V}$$

and this also equals to maximum armature voltage.

$$\therefore (V)_{max} = 36 \text{ V}$$

$$T_{o/p} = T = k_T i_a = (0.08) i_a$$

O/p torque is maximum if current is maximum.

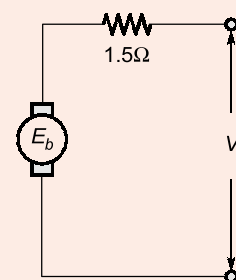
$$\therefore T_{o/p,max} = 0.08 \times 3 = 0.24 \text{ Nm}$$

$$P = (E_b) i_a = (k_e \omega) i_a$$

$$\text{Also, } E_b = V - 1.5 i_a$$

$$\therefore i_a = \frac{V - E_b}{1.5}$$

$$P_{o/p} = (k_e \omega) \left( \frac{V - k_e \omega}{1.5} \right)$$



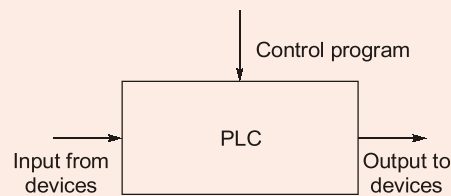
$$\frac{dP}{d\omega} = 0$$

$$\therefore k_e \omega = \frac{V}{2}$$

$$P_{o/p, \max} = \frac{V}{2} \cdot \frac{V}{3} = \frac{V^3}{6} = \frac{(36)^2}{6} = 216W$$

(ii)

A programmable logic controller (PLC) is a digital electronic device that uses a programmable memory to store instruction and to implement functions such as logic, sequencing, timing, counting and arithmetic in order to control machines and processes and has been specifically designed to make programming easy. The term logic is used because the programming is primarily concerned with implementing logic and switching operations. Input devices e.g. switches, and output devices e.g. motors, being controlled are connected to the PLC and then the controller monitors the inputs and outputs according to the program stored in the PLC by the operator and so controls the machine or process. Originally they were designed as a replacement for hard-wired relay and timer logic control systems. PLCs have the great advantages that is possible to modify a control system without having to rewire the connections to the input and output devices, the only requirement being that an operator has to key in a different set of instructions. Also they are much faster than relay-operated systems. The result is a flexible system which can be used to control systems which vary quite widely used for the implementation of logic control functions because they are easy to use and program.



**Programmable logic**

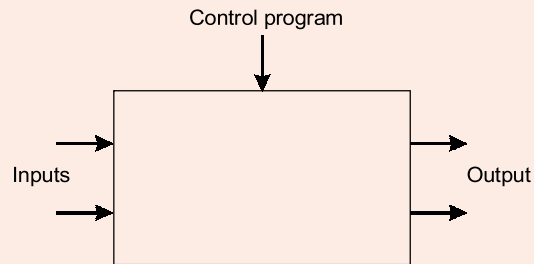
PLCs are similar to computers but have certain features which are specific to their use as controllers. These are:

1. They are rugged and designed to withstand vibrations, temperature, humidity and noise.
2. The interfacing for inputs and outputs is inside the controller.
3. They are easily programmed and are easily understood.

**Differences between PLCs and computers:**

- PLC are optimized for control task and industrial environment. While, computers are optimized for calculation and display tasks.
- Amount of memory required for PLC is less as it is application specific while computers require huge amount of memory for storing programmes.

- When power is lost, PLC require battery backup relays to start from same place of executing the logic while computer programs are stored in memory so they can be retrieved.
- Programming is simple with respect to PLC as they are concerned with simple logic and switching operations while computer programming requires knowledge of computer language skills like C, C++, Java etc.



*End of Solution*