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ESE 2022

Main Exam Detailed Solutions

Electronics & Telecom. Engineering

PAPER-I

EXAM DATE : 26-06-2022 | 9:00 AM to 12:00 PM

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ANALYSIS

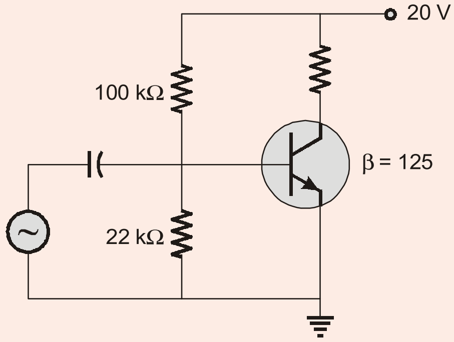
Electronics and Telecom. Engineering Paper-I
ESE 2022 Main Examination

Sl.	Subjects	Marks
1.	Basic Electronics Engineering (EDC)	33
2.	Materials Science	65
3.	Electronic Measurements and Instrumentation	80
4.	Network Theory	150
5.	Analog Circuits	67
6.	Digital Circuits	55
7.	Basic Electrical Engineering	30
	Total	**480**

Scroll down for detailed solutions

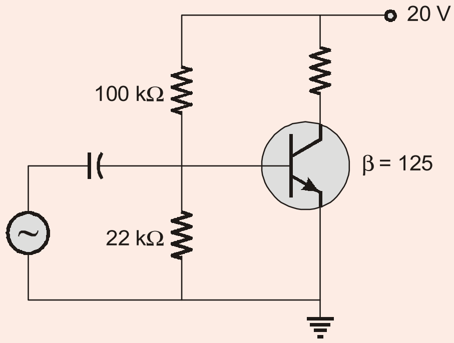
Section-A

Q.1 (a) The transistor in the figure below has maximum ratings of $P_{D(max)}$ = 500 mW, $V_{CE(max)}$ = 15 V and $I_{C(max)}$ = 100 mA. Determine the maximum value of V_{CC} which would not exceed the device ratings. Take V_{BE} = 0.7 V.

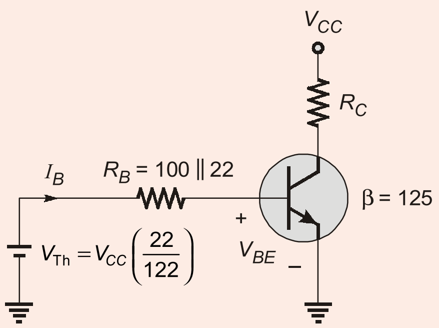


[10 marks : 2022]

Solution:



Drawing the Thevenin equivalent DC circuit,



$$V_{Th} = V_{CC}\left(\frac{22\text{ k}\Omega}{100\text{ k}\Omega + 22\text{ k}\Omega}\right) = \left(\frac{11}{61}\right)V_{CC}$$

$$R_B = \frac{100 \times 22}{100 + 22} = 18.03\ \text{k}\Omega$$

Hence,
$$I_B = \frac{V_{Th} - V_{BE}}{R_B} = \frac{\left(\frac{11}{61}\right)V_{CC} - 0.7}{18.03 \times 10^3}$$

We have,
$$I_C = \beta I_B = 6.93\left[\left(\frac{11}{61}\right)V_{CC} - 0.7\right]\text{mA}$$

Given :
$$I_{C(\max)} = 100\ \text{mA}$$

Therefore,
$$6.93\left[\left(\frac{11}{61}\right)V_{CC} - 0.7\right] < 100$$

$$\left(\frac{11}{61}\right)V_{CC} < 14.43 + 0.7$$

$\Rightarrow$
$$V_{CC} < 83.9\ \text{V} \quad \text{...(i)}$$

Since, the value of R_C is not given. Hence, V_{CE} corresponding to I_C cannot be determined. Assume R_C = 100 Ω.

$$V_{CE} = V_{CC} - I_C R_C$$

$$V_{CE} = V_{CC} - 6.93\left[\left(\frac{11}{61}\right)V_{CC} - 0.7\right] \times 100 \times 10^{-3}$$

$$V_{CE} = V_{CC} - 0.125\ V_{CC} + 0.4851$$

$$V_{CE} = 0.875\ V_{CC} = 0.4851$$

Given: $V_{CE(\max)}$ = 15 V. Therefore,

$$0.875 V_{CC} + 0.4851 < 15$$

$$V_{CC} < 16.59\ \text{V} \quad \text{...(ii)}$$

From equations (i) and (ii), $V_{CC(\max)}$ = 16.59 V

For $V_{CC(\max)}$ = 16.59 V,
$$P_D = V_{CE} \times I_C$$

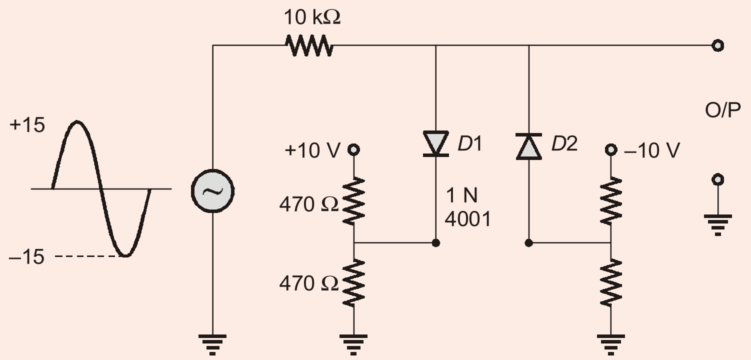
$$= 15 \times 6.93\left[\left(\frac{11}{61}\right) \times 16.59 - 0.7\right]\text{mW}$$

$$= 238.22\ \text{mW which is less than } P_{D(\max)}$$

Hence, maximum value of V_{CC} which would not exceed the device ratings = 16.59 V.

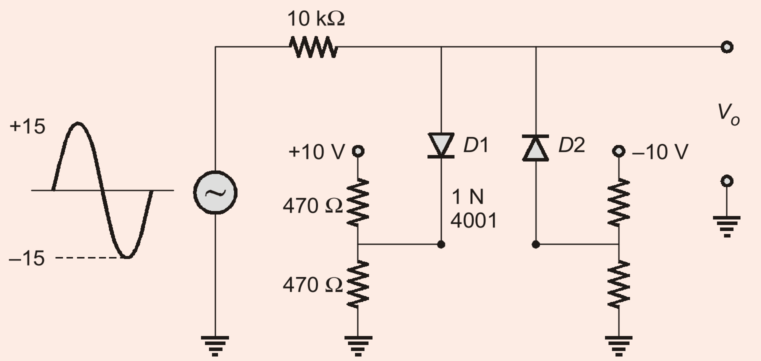
End of Solution

Q.1 (b) Determine the output waveforms for the circuit given below by showing the necessary analysis. Differentiate between the operations of shunt clippers and series clippers

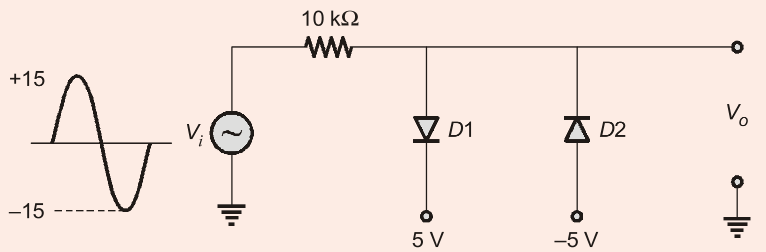


[10 marks : 2022]

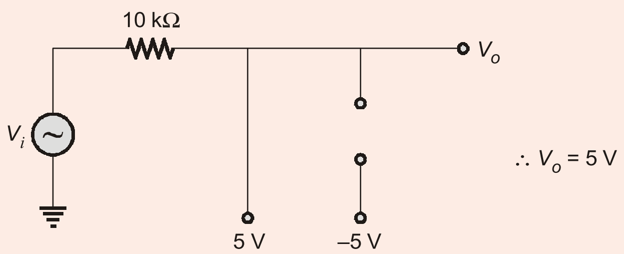
Solution:



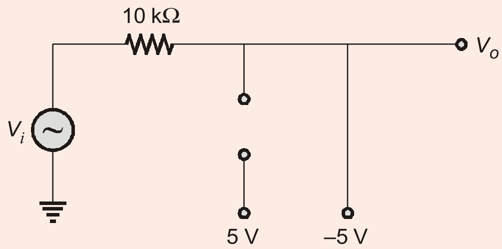
Simplifying the resistive voltage dividers, the circuit can be drawn as below:



Case-I: When $V_i > 5$ V, diode D_1 is forward biased and diode D_2 is reverse biased.



Case-II : When $V_i < -5$ V, diode D_1 is reverse biased and diode D_2 is forward-biased

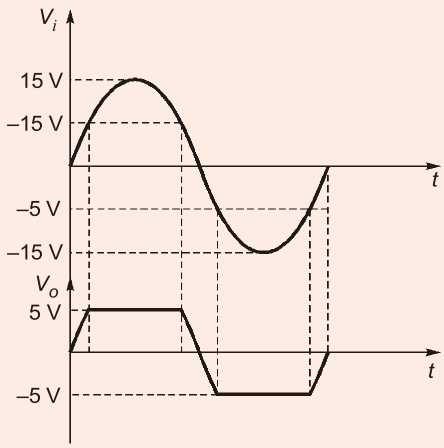


$\therefore V_o = -5$ V

Case-II: When -5 V $< V_i < 5$ V, both the diodes are reverse biased. Hence,

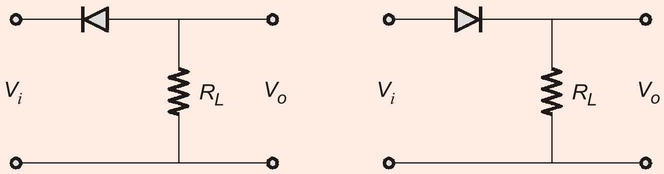
$$V_o = V_i$$

The output waveform can be sketched as below:



Difference between the operations of shunt clippers and series clippers.

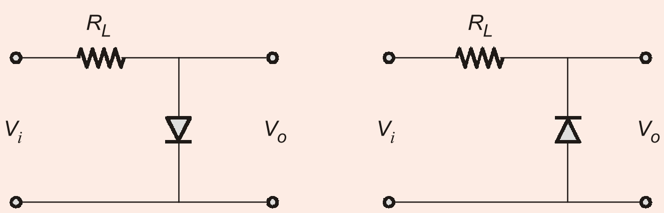
In series clippers, the diode is connected in series with the output load resistance as shown below:



Positive series clipper **Negative series clipper**

In the series clippers, when the diode is in off position, there is no transmission of the input signal to the output, thereby clipping the input signal.

In shunt clippers, the diode is connected in parallel with the output load resistance as shown below:

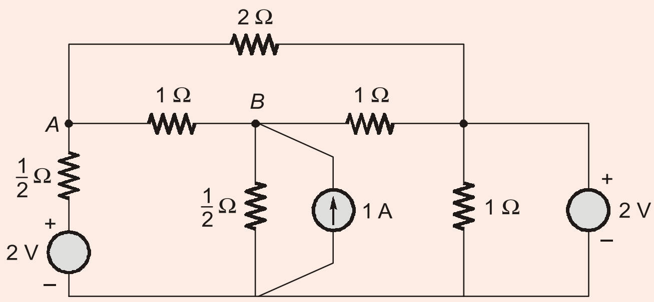


Shunt positive clipper **Shunt negative clipper**

In the shunt clippers, when the diode is 'ON', the input signal is clipped.

End of Solution

Q.1 (c) For the network shown, determine the current through the branch *AB*, using nodal analysis.



[20 marks : 2022]

Solution:

Using nodal analysis, $\frac{2-V_A}{1/2} = \frac{V_A-V_B}{1}+\frac{V_A-2}{2}$

$$4-2V_A = V_A-V_B+\frac{V_A}{2}-1$$

$$3V_A+\frac{V_A}{2}-V_B = 5$$

$$\frac{7V_A}{2}-V_B = 5$$

$$7V_A-2V_B = 10 \quad \text{...(i)}$$

Applying KCL at node *B*, $1 = \frac{V_B}{1/2}+\frac{V_B-V_A}{1}+\frac{V_B-2}{1}$

$$1 = 2V_B+V_B-V_A+V_B-2$$

$$-V_A+4V_B = 3 \quad \text{...(ii)}$$

Multiply equation (ii) by 7

$$-7V_A+28V_B = 21 \quad \text{...(iii)}$$

Adding equations (i) and (iii)

$$26V_B = 31$$

$$V_B = \frac{31}{26}\text{ Volt} \quad \text{put in equation (i)}$$

$$7V_A = 10+2V_B = 10+\frac{31}{13}$$

$$V_A = \frac{10+\frac{31}{13}}{7} = 1.769 \text{ Volts}$$

$$V_B = 1.192 \text{ Volt}$$

Current through branch *AB*,

$$I = \frac{V_A-V_B}{1} = 0.577 \text{ A}$$

End of Solution

Q.1 (d) (i) The concentration of silicon in an iron-silicon alloy is 0.25 wt%. What is the concentration in kilograms of silicon per cubic metre of alloy?
[Assume Si and iron as 2.33 and 7.87 gm/cm^3 respectively].

(ii) Molybdenum has the BCC crystal structure, has a density of 10.22 g cm^{-3} and an atomic mass of 95.94 g mol^{-1}. What is the atomic concentration, lattice parameter *a*, and atomic radius of molybdenum.

[5 + 5 marks : 2022]

Solution:

(i) Given: concentration of Si in Fe-Si alloy = 0.25 wt%

$$\text{Density of Si} = 2.33 \text{ gm/cm}^3$$

$$\text{Density of Fe} = 7.87 \text{ g/cm}^3$$

Assume a mass of 1 kg of Fe-Si alloy. Hence,

$$\text{Mass of Si} = \frac{0.25}{100} \times 1\text{kg} = 0.0025 \text{ kg} = 2.5 \text{ gm}$$

Calculating volumes of Fe and Si using the densities,

$$\text{Volume of Si} = \frac{\text{Mass}}{\text{Density}} = \frac{2.5 \text{ gm}}{2.33 \text{ gm/cm}^3} = 1.073 \text{ cm}^3$$

$$\text{Volume of Fe} = \frac{\text{Mass}}{\text{Density}} = \frac{1000 - 2.5}{7.87 \text{ gm/cm}^3} = 126.747 \text{ cm}^3$$

Hence, total volume of 1 kg of Fe-Si alloy = 1.073 + 126.747

$$= 127.82 \text{ cm}^3 = 127.82 \times 10^{-6} \text{ m}^3$$

$$\text{Concentration of Si} = \frac{\text{Mass of silicon (kg)}}{\text{Volume of alloy (m}^3)} = \frac{0.0025 \text{ kg}}{127.82 \times 10^{-6} \text{ m}^3}$$

$$= 19.56 \text{ kg/m}^3$$

(ii) For the BCC crystal structure, number of atoms per unit cell (N) =2

The expression for density is given as

$$d = \frac{NM_{at}}{N_A \cdot a^3}$$

where,

d = Density = 10.22g cm^{-3}

M_{at} = Atomic mass = 95.94 $gmol^{-1}$

a = Lattice parameter

N_A = Avogadro's number = 6.023×10^{23}

The expression for lattice parameter is obtained as

$$a = \left(\frac{NM_{at}}{dN_A}\right)^{1/3}$$

Substituting the values, we get,

$$a = \left(\frac{2 \times 95.94}{10.22 \times 6.023 \times 10^{23}}\right)^{1/3} = 3.147 \times 10^{-8} \text{ cm}$$

For BCC crystal, $4R = \sqrt{3}a$, where R = Atomic radius of Molybdenum

$$\therefore \qquad R = \frac{\sqrt{3}\times 3.147\times 10^{-8}}{4} = 1.36\times 10^{-8}\text{ cm}$$

$$\text{Atomic concentration of Molybdenum} = \frac{N_A\times d}{M_{at}} = \frac{6.023\times 10^{23}\times 10.22}{95.94}$$

$$= 6.42\times 10^{22}\text{ atoms/cm}^3$$

End of Solution

Q.1 (e) Describe the events that occur during sintering. Should green compacts be brought up to the sintering temperature slowly or rapidly? Comment.

[10 marks : 2022]

Solution:

Sintering: Sintering is defined as the thermal treatment of a powder or compact at a temperature below the melting point of the main constituent, for the purpose of increasing its strength by bonding together of the particles.

What happens during sintering is described below:

1. Atomic diffusion takes place and the welded areas formed during compaction grow until eventually they may be lost completely.
2. Recrystallisation and grain growth may follow, and the pores tend to become rounded and the total porosity, as a percentage of the whole volume tends to decrease.
3. In the pressing operation the powder particles are brought together and deformed at the points of contact.
4. At elevated temperature – the sintering temperature the atoms can move more easily and quickly migrate along the particle surfaces (the technical term is diffusion).
5. Metals consist of crystallites. At the sintering temperature new crystallites foam at the points of contact so that the original inter-particle boundaries disappear, or become recognizable merely as grain boundaries (this process is called recrystallisation).
6. The total internal surface area of the pressed body is reduced by sintering.
7. Neck-like junctions are formed between adjacent particles as can seen on the adjoining scanning electron micrograph.

\# Rapid heating can cause excessive thermal stresses in the part being sintered and can lead to distortion or cracking; on the other hand, it reduces cycle times. Slow heating has the advantage of allowing heating and diffusion to occur more uniformly.

So, green compacts be brought upto the sintering temperature slowly.

End of Solution

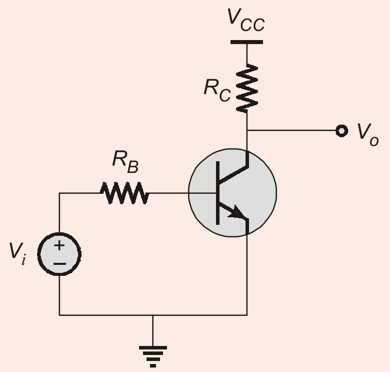
Q.2 (a) (i) Write the circuit Schematics, transfer characteristics and contrast the performances of logic inverters based on BJT, CMOS and NMOS devices.

(ii) Show how resistors and capacitors are fabricated in Integrated circuits, with the help of necessary illustrations.

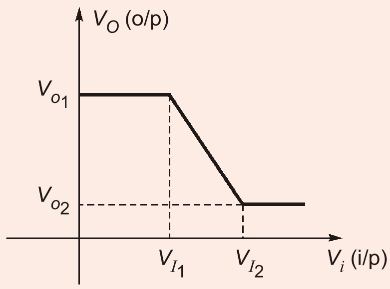
[12 + 8 marks : 2022]

Solution:

(i) BJT inverters:

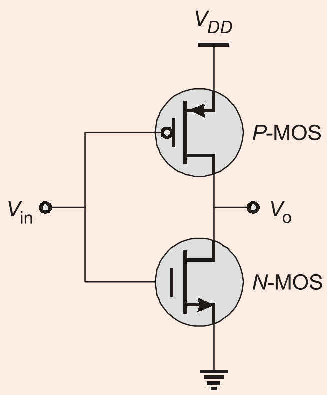


Circuit

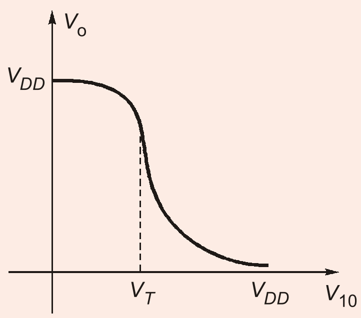


Transfer characteristic

CMOS inverters:

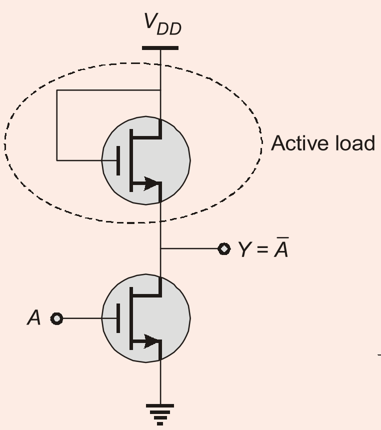


Circuit

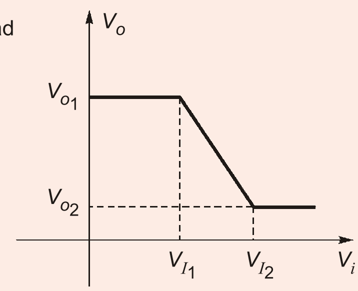


Transfer characteristic

NMOS inverters:



Circuit



Transfer characteristic

Performance of BJT inverter :

- High speed of operation.
- Low fan-out then MOS.
- Lower noise immunity.

Performance of CMOS inverter :

- Less speed than BJT.
- Highest fan-out.
- Lower power-dissipation.

Performance of NMOS inverter :

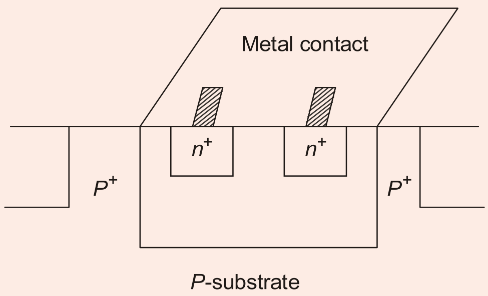
- Higher speed than CMOS but lower than BJT.
- Smaller power dissipation than BJT but higher than CMOS.
- High noise immunity.

(ii) Show how resistor and capacitor are fabricated in integrated circuits, with the help of necessary illustrations.

Integrated Resistors : A resistor in a monolithic integrated circuit is obtained by utilising the bulk resistivity of the diffused volume of semiconductor region. The commonly used methods for fabricating integrated resistors are

1. Diffused 2. Epitaxial 3. Pinched 4. Thin film techniques

1. **Diffused Resistor :** The diffused resistor is formed in any one of the isolated regions of epitaxial layer during base or emitter diffusion process.
2. **Epitaxial Resistor :**

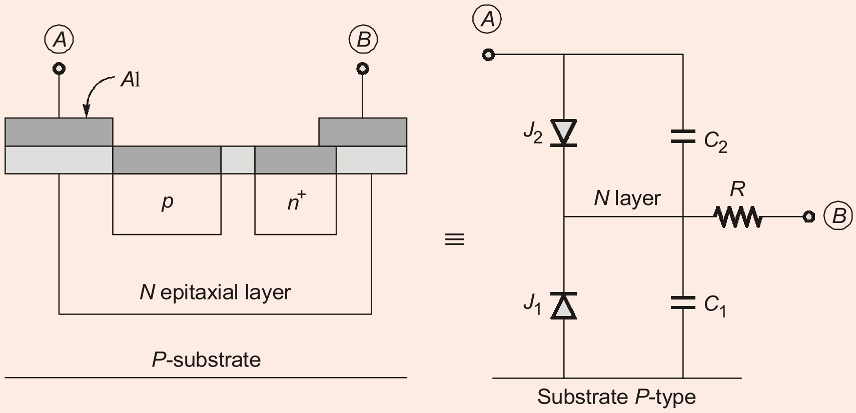


Integrated capacitors :

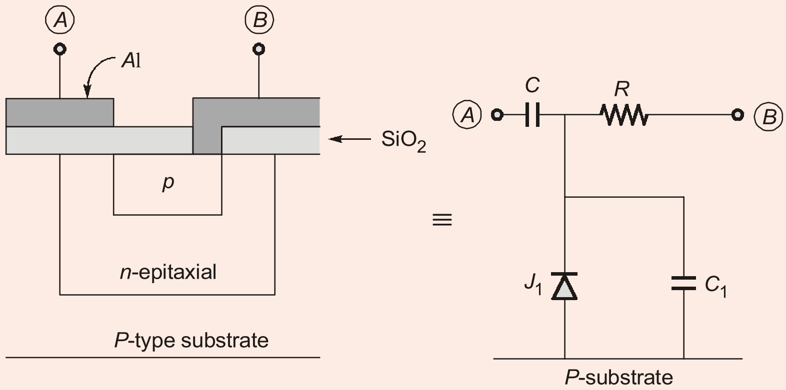
⇒ Diffused junction capacitor

⇒ Thin film MOS capacitor

1. Diffused junction capacitor:

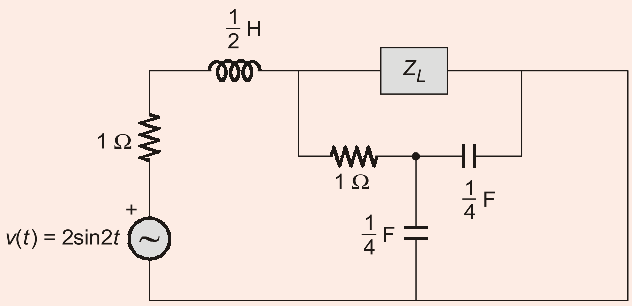


2. Thin film MOS capacitor:



End of Solution

Q.2 (b) Determine the value of Z_L for maximum power transfer in Z_L, in the given network and hence, find the value of this power.



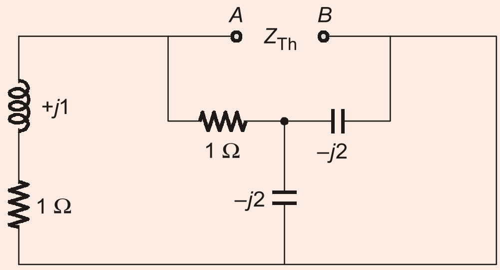
[20 marks : 2022]

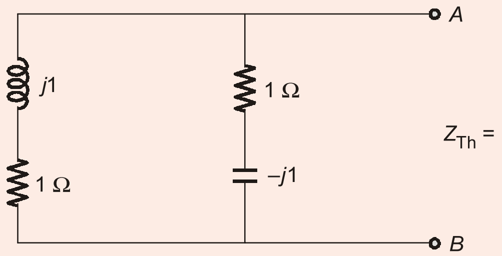
Solution:

Case-I : $[Z_{Th}]$

$$jX_L = j\omega L = j1$$

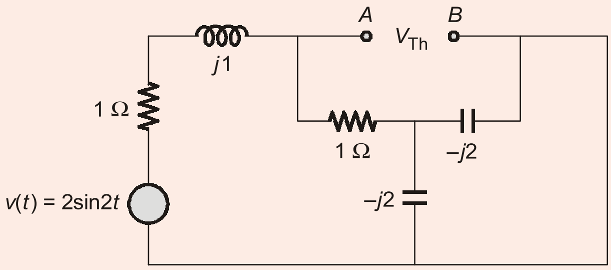
$$-jX_c = \frac{-j}{\omega C} = -j2$$

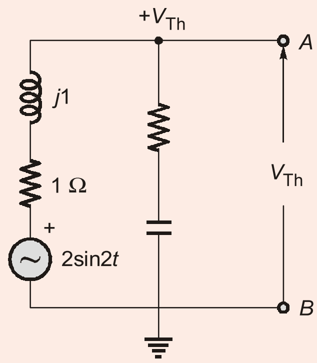




$$Z_{Th} = \frac{(1+j1)(1-j1)}{1+j1+1-j1} = 1\Omega$$

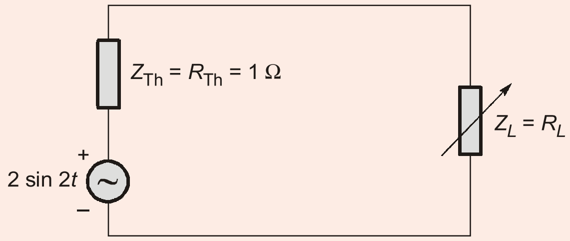
Case-II: [V_{Th}]





By KCL,
$$\frac{V_{Th} - \frac{2}{\sqrt{2}}}{1+j1} + \frac{V_{Th}}{1-j1} = 0$$

Case-III: [Equivalent circuit]



$$R_L = \text{RTh} = 1\text{ W}$$
$$Z_L = 1\ \Omega$$
$$P = \frac{V_{Th}^2}{4R_L} = \frac{1^2}{4\times 1} = 0.25\text{ W}$$

End of Solution

Q.2 (c) (i) Prove that superconductors are perfectly diamagnetic.

(ii) Describe a method of producing superconducting wires or strips from powdered materials (such as Ceramics). What are the possible difficulties involved in this process?

[8 + 12 marks : 2022]

Solution:

(i) A superconducting material kept in a magnetic field expels the magnetic flux out of its body when cooled below the critical temperature, T_C.

For the normal state ($T > T_C$), the magnetic flux density inside the specimen is given by

$$B = \mu_o(H + M)$$

where H is the applied magnetic field

M is the magnetization is in superconducting state,

$$B = 0$$

$$\Rightarrow \quad \mu_o(H + M) = 0$$

$$\Rightarrow \quad H = -M$$

$$\Rightarrow \quad \chi_m = \frac{M}{H} = -1$$

Since, the value of susceptibility is negative and equal to –1, the superconductor exhibits perfect diamagnetism and produces strong repulsion to the external magnets.

(ii) Superconducting wires are electrical wires made of superconductive material. When cooled below their transition temperatures, they have zero electrical resistance.

Most commonly, conventional superconductors such as niobium titanium are used.

- Normal wire drawing process can be used for malleable alloys such as niobium-titanium.
- The wire drawing process is quite simple in concept. The wire is prepared by shrinking the beginning of it, by hammering, filing, rolling or swagging, so that it will fit through the die; the wire is then pulled through the die. As the wire is pulled through the die, its volume remains the same, so as the diameter decreases, the length increases.

Usually the wire will require more than one draw, through successively smaller dies, to reach the desired size.

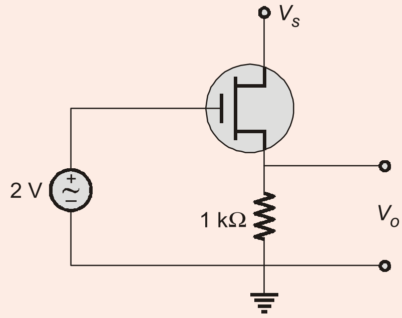
The difficulties involved in this process is

1. Problems caused by improper selection of working cone angle.
2. Center fracture and 'I' shape crack.
3. Residual stress and work hardening phenomena.

End of Solution

Q3 (a) (i) What are dependent sources? Give the model of a voltage controlled current source (VCCS) and determine the expressions for current gain, voltage gain and power gain.

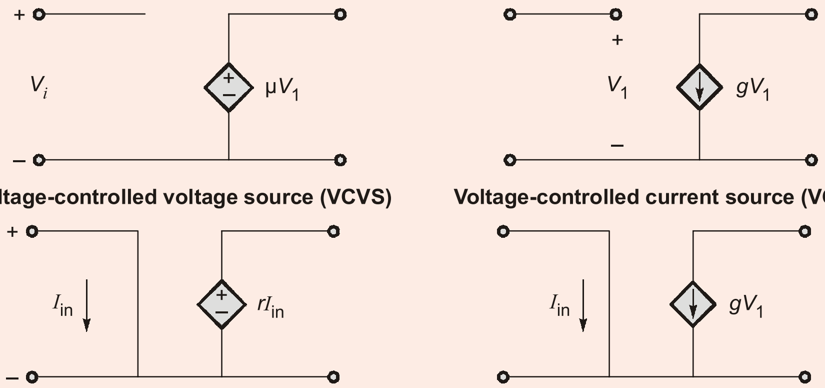
(ii) What is the purpose of a source follower? Figure below shows a source follower circuit using a MOSFET. Assuming it operates in the saturation region, determine the output voltage and the output current, given V_{IN} = 2 V, K = 2 mA/V^2, V_T = 1 V.



[10 + 10 marks : 2022]

Solution:

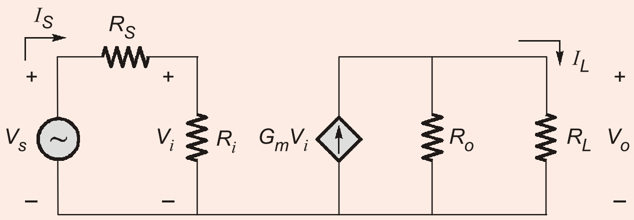
(i) Dependent source:
"A dependent source is voltage source or a current source whose value depends on a voltage or current elsewhere in the network".



Voltage-controlled voltage source (VCVS) **Voltage-controlled current source (VCCS)**

Current-controlled voltage source (CCVS) **Current-controlled current source (CCCC)**

Voltage controlled current source is a transconductance amplifier.



G_M is short-circuit transconductance gain.

$$I_L = \frac{G_mV_i \times R_o}{R_o + R_L} = G_MV_i$$

$$G_M = \frac{G_mR_o}{R_o + R_L}$$

G_M is transconductance gain when $R_L \neq 0$.

$$I_L = G_MV_i = G_M \times \frac{V_SR_i}{R_S + R_i} \quad ...(1)$$

Voltage Gain:

$$A_V = \frac{V_o}{V_s} = \frac{I_LR_L}{V_s} = \frac{\frac{G_MV_sR_i}{R_s + R_i} \times R_L}{V_s}$$

$$A_V = \frac{G_MR_iR_L}{R_s + R_i}$$

Current Gain:

$$A_I = \frac{I_L}{I_S} = \frac{G_MV_i}{\frac{V_i}{R_i}} \Rightarrow A_I = G_MR_i$$

Power Gain:

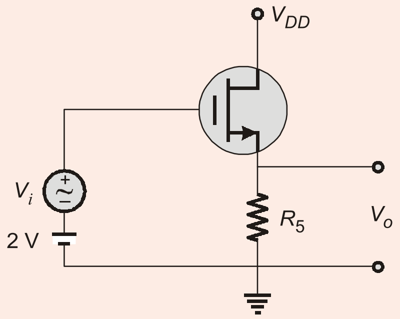
$$A_P = \frac{\text{Output power}}{\text{Input power}} = \frac{V_o \times I_L}{V_s \times I_s}$$

$$A_P = A_V \times A_I = \frac{G_MR_iR_L}{R_s + R_i} \times G_MR_i$$

$$A_P = \frac{G_M^2R_i^2R_L}{R_S + R_i}$$

(ii) The source follower, also known as common-drain amplifier is typically used as a voltage buffer as it has unity gain, very high input impedance and low output impedance. The source follower can also be used as an impedance matching transformers.

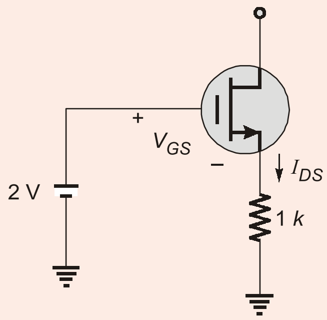
Considering the given source follower circuit,
Assume DC gate voltage as 2 V



DC analysis: Disable V_i

$$V_{GS} = 2 - 1 \times I_{DS}$$

$$I_{DS} = 2 - V_{GS} \quad ...(i)$$



MOSFET is considered in saturation

$$I_{DS} = k(V_{GS} - V_T)^2$$

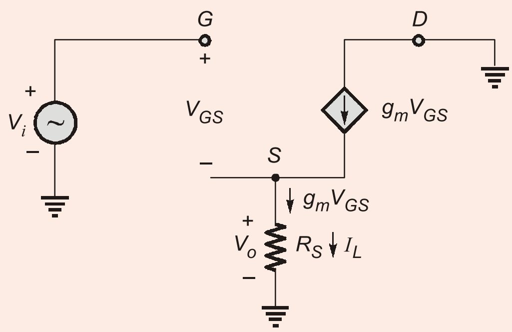
$$2 - V_{GS} = 2(V_{GS} - 1)^2 = 2(V_{GS}^2 + 1 - 2V_{GS})$$

$$2 - V_{GS} = 2V_{GS}^2 + 2 - 4V_{GS}$$

$$2V_{GS}^2 - 3V_{GS} = 0 \Rightarrow V_{GS} = 1.5 \text{ V}, 0$$

But $V_{GS} > V_T \Rightarrow V_{GS} = 1.5$ V

Small signal AC analysis:



$$I_{DS} = k(V_{GS} - V_T)^2$$

$$\frac{\partial I_{DS}}{\partial V_{GS}} = g_m = 2k(V_{GS} - V_T) = 2 \times 2\,(1.5 - 1)$$

$$g_m = 2 \text{ m℧}$$

$$\frac{V_o}{V_i} = \frac{g_m V_{gs} R_s}{V_{gs} + g_m V_{gs} R_s}$$

$$A_V = \frac{g_m R_s}{1 + g_m R_s}$$

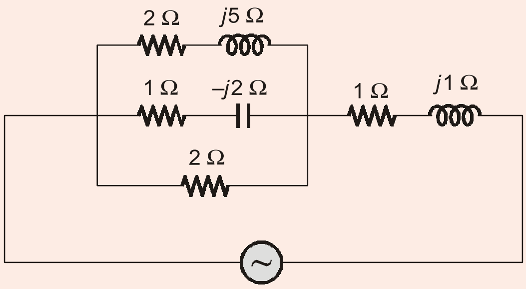
$$A_V = \frac{2\times 1}{1+2\times 1} = \frac{2}{3} = 0.667$$

$$V_o = A_V V_i = 0.667 V_i$$

$$I_L = \frac{V_o}{R_s} = \frac{0.667 V_i}{1k} = 0.667 V_i \text{ mA}$$

End of Solution

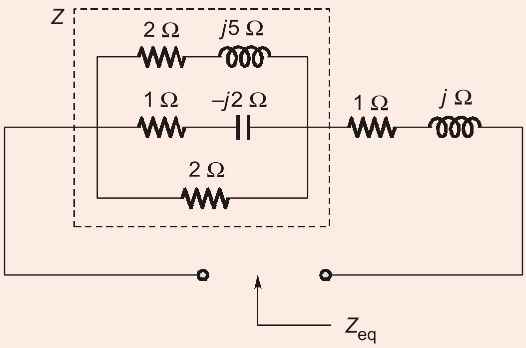
Q.3 (b) For the circuit shown in figure, calculate (i) the total impedance, (ii) the total current, (iii) the power factor, (iv) the total apparent power, (v) the active power, and (vi) reactive power.



[20 marks : 2022]

Solution:

(i) Finding equivalent impedance,



$$(2+j5)\|(1-j2) = \frac{(2+j5)(1-j2)}{(2+j5+1-j2)} = \frac{(2-j4+j5+10)(3-j3)}{(3+j3)(3-j3)}$$

$$= \frac{1}{18}[(12+j)(3-j3)]$$

$$= \frac{1}{18}[36 - j36 + j3 + 3] = \frac{1}{18}[39 - j33]$$

$$= \frac{1}{18}[39 - j33]$$

$$Z = \frac{(39 - j33)}{18} \Big\| 2 = \frac{(2.167 - j1.84)2}{2.167 - j1.84 + 2}$$

$$= \frac{2(2.84\angle -40.33^\circ)}{(4.55\angle -23.82^\circ)}$$

$$Z = (1.248\ \angle{-16.51^\circ})\ \Omega$$

$$Z = (1.196 - j0.354)\ \Omega$$

$$Z_{eq} = Z + 1 + j = (2.196 + j0.646)\ \Omega$$

(ii) Total current,

$$I = \frac{V}{Z_{eq}} = \frac{40}{2.196 + j0.646} = \frac{40}{2.29\angle 16.4^\circ}\ \text{A}$$

$$I = (17.46\ \angle{-16.4^\circ})\ \text{A}$$

(iii) Power factor $= \cos\theta$

$$= \cos(16.4^\circ) = 0.959 \text{ lagging}$$

(iv) Total apparent power,

$$S = VI^*$$

$$= 40 \times 17.46\ \angle 16.4^\circ \text{ VA} = 698\ \angle{+16.4^\circ} \text{ VA}$$

(v) Active power,

$$P = VI\cos\theta$$

$$= 698.4 \times 0.959 = 669.76 \text{ Watt}$$

(vi) Reactive power,

$$Q = VI\sin\theta$$

$$= 698.4 \times 0.282 = 197.18 \text{ VAR}$$

End of Solution

Q.3 (c) (i) Find out the velocity of light in a material which has a dielectric constant ϵ_r of 5.5 and a magnetic susceptibility of -2.17×10^{-5}.

(ii) The data for weight gain w.r.t. time for oxidation of a metal at an elevated temperature are as follows:

W (mg/cm^3) →	1.54	23.24	95.37
Time (min) →	10	150	620

1. Determine whether the oxidation kinetics obey a logarithmic, linear or parabolic rate expression.
2. Also calculate W after a time of 1200 min.

(iii) Consider a current of 10 A flowing through a coil of wire. The coil of wire 0.20 m long has 200 turns.

1. What is the magnitude of magnetic field strength H?
2. Calculate the flux density B if the coil is in a vacuum.
3. Calculate the flux density inside a bar of titanium (susceptibility $= 1.81 \times 10^{-4}$) that is positioned within the coil
4. Calculate the magnitude of magnetisation M.

[5 + 5 + 10 marks : 2022]

Solution:

(i) $\epsilon_r = 5.5$

Magnetic susceptibility $= \chi_m = -2.17 \times 10^{-5}$

$\Rightarrow \quad \mu_r - 1 = -2.17 \times 10^{-5}$

$\Rightarrow \quad \mu_r = 1 - 2.17 \times 10^{-5}$

$\Rightarrow \quad \mu_r = 0.9999783$

$$\text{Velocity of light} = v = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_r \mu_o \epsilon_r \epsilon_o}} = \frac{1}{\sqrt{\mu_r \epsilon_r} \times \sqrt{\mu_o \epsilon_o}}$$

$$\Rightarrow \quad v = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{5.5 \times 0.9999783}}$$

$\Rightarrow \quad v = 1.28 \times 10^8$ m/s

(ii) The data for weight gain w.r.t. time for oxidation of a metal at an elevated temp:

W (mg/cm^3) $\rightarrow$	1.54	23.24	95.37
Time (min) $\rightarrow$	10	150	620

1. The oxidation kinetics equations are :

$W^2 = K_1 t + K_2$ ...(i) for parabolic rate expression

$W = K_3$ ...(ii) for linear rate expression

$W = K_3 \log(K_5 t + K_6)$ ...(iii) for linear rate expression

To solve this problem we need trial and error trial and error method.

$\Rightarrow$ First let us assume that the rate expression is parabolic.

$W^2 = K_1 t + K_2$

$(1.54)^2 = K_1 \times 10 + K_2$ ...(iv)

$(23.24)^2 = 150K_1 + K_2$ ...(v)

$(95.37)^2 = 620K_1 + K_2$ ...(vi)

Solving equations (iv) and (v)

$K_1 = 3.8409$; $K_2 = -36.0374$

Solving equations (v) and (vi)

$K_1 = 18.202$; $K_2 = 0.1978$

Hence, we conclude that parabolic rate expression is not obeyed.

Let us now investigate linear kinetics

$W = K_3 t$

$1.53 = 10K_3 \Rightarrow K_3 = 0.154$

$23.24 = 150K_3 \Rightarrow K_3 = 0.154$

$95.37 = 620K_3 \Rightarrow K_3 = 0.154$

Hence we can conclude that oxidation kinetics obeys linear rate expression.

2. Now, $W = 0.154t$

$\therefore$ After a time $t = 1200$ min

$W = 0.154 \times 1200 = 184.8$ mg/cm^3

(iii)
$$I = 10 \text{ A}$$
$$N = 200 \text{ turns}$$
$$l = 0.20 \text{ m}$$

1.
$$H = \frac{NI}{l} = \frac{200 \times 10}{0.2} \text{ A/m}$$
$$H = 10000 \text{ A/m}$$

2. Since coil is in vacuum, so flux density
$$B = \mu_o H = 4\pi \times 10^{-7} \times 1000 \text{ Wb/m}^2$$
$$B = 12.56 \text{ mWb/m}^2$$

3. Susceptibility, $\chi_m = 1.81 \times 10^{-4} = \mu_r - 1$

$\Rightarrow$ $\mu_r = 1.00181$

$\therefore$ $B = \mu_r \mu_o H = 1.00181 \times 4\pi \times 10^{-7} \times 10^4$
$$B = 12.56227 \text{ mWb/m}^2$$

4. Magnetisation, $M = \chi_m H = 1.81 \times 10^{-4} \times 10^4$
$$M = 1.81 \text{ A/m}$$

End of Solution

Q.4 (a) (i) Both LED and junction diode are made of *p-n* junctions. Contrast their construction and working. How does a Laser diode differ from LED?

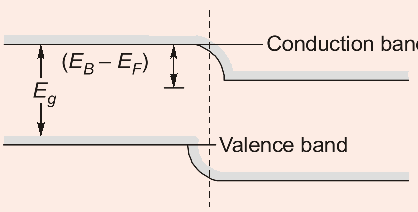
(ii) By writing an equivalent circuit for a *p-i-n* photo-detector, obtain the relation for the SNR of an optical receiver, considering the various noise sources.

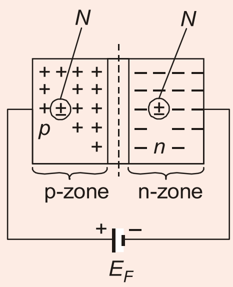
[8 + 12 marks : 2022]

Solution:

(i) Construction and Working of LED

- A light emitting diode works on the phenomenon of spontaneous emission when it is forward biased and conducting current. Radiation from an LED is caused due to recombination of holes and electrons that are injected into the junction by forward bias voltage.
- One side of junction is a n type material consisting of mostly free electrons while another side is p-type material consisting of holes.
- When no voltage is applied, a depletion region is formed at the junction consisting of immobile charge carriers.





- With application of forward bias electrons of n-side moves towards the ions near the boundary and holes on p-side moves towards the ions, thus reduces the width of depletion region and also the potential barrier get reduced.

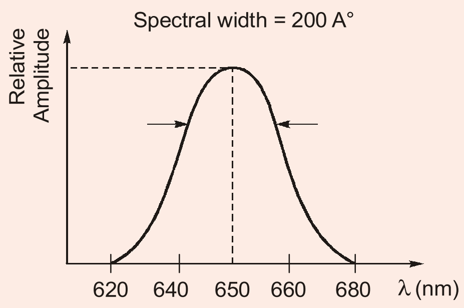
- The width of depletion region reduces until the holes and electrons are free to cross the barrier to conduct current. In this phenomenon, electrons injected into p-region encounter holes and recombines, so as holes.
- When each electron-hole pair recombines, a simple photon of energy is released whose wavelength is given by

$$\lambda = \frac{hc}{E_g} = \frac{1.24}{E_g(ev)}\,\mu\text{m}$$

- Material for the LED must be a direct band gap and generally used material are GaAs, GaAsP, GaP.
- For fabrication of LED, we use Group III–V compound material (binary/ternary/quaternary).
- The quaternary/ternary materials are used so as we can vary the bandgap by varying the concentration.
- Quaternary alloy $In_{1-x}\,Ga_x\,As_y\,P_{1-y}$ is used for higher wavelength. It is used for wavelength range of 1.0 μm to 100 μm.
- As the energy gap varies, wavelength emitted will vary hence the colour of LED will vary. The following table summarize the colour of LED with energy gap.

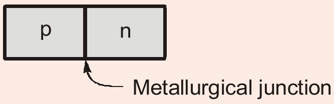
S.No.	Energy gap (in eV)	Colour of LED
1.	$1.7 < E_g < 1.9$	Red
2.	$1.9 < E_g < 2.15$	Yellow
3.	$2.15 < E_g < 2.15$	Green
4.	$2.5 < E_g < 2.7$	Blue
5.	$2.7 < E_g < 3.2$	Violet

- The value of energy gap less than 1.7 eV corresponds to infra red region. Hence GaAs emits light in infra red region.
- GaAsP have energy gap of 1.8 eV hence gives red colour of light with a spectrum having peak wavelength around 660 nm. The spectrum of GaAsP is shown in the figure.



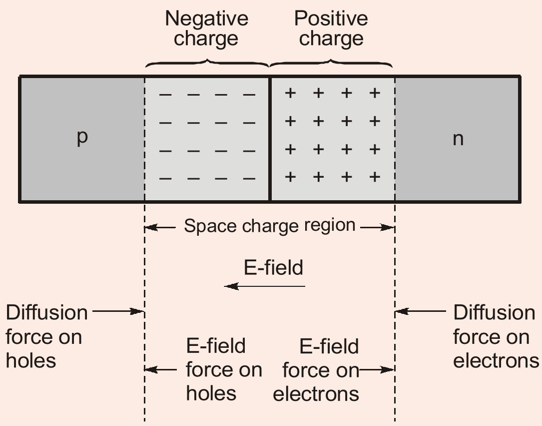
- Spectral width for LED is around 200 A° which is very broad and hence non-monochromatic. Thus LED can not be used for excitation of modes in single mode fibre.

Basic Structure of the p-n Junction:

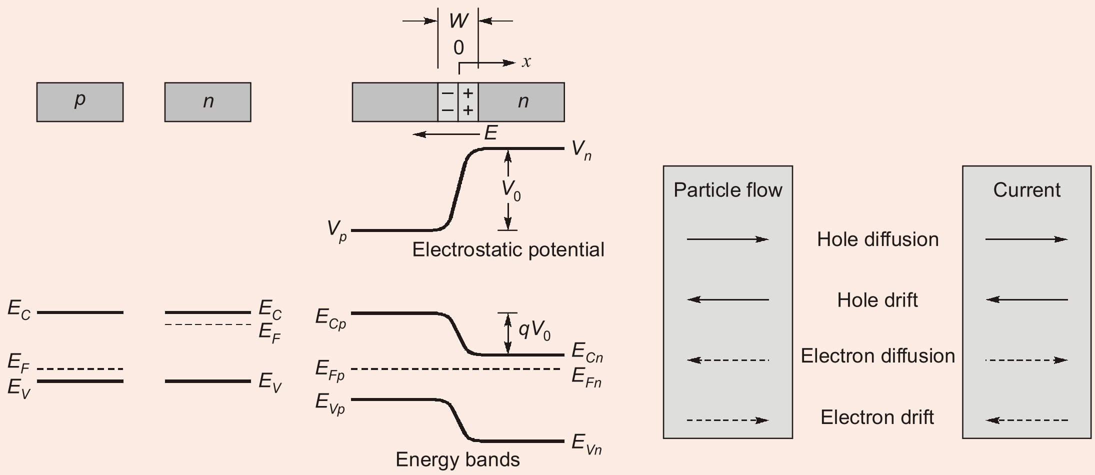


If donor impurities are introduced into one side and acceptors into the other side of a single crystal of a semiconductor as shown in the figure above. The interface separating the '*n*' and '*p*' regions is referred to as the metallurgical junction.

Majority carrier electrons in the n-region will begin diffusing into the p-region and majority carrier holes in the p-region will be diffusing into the n-region. If we assume there are no external excitation to the semiconductor, then this diffusion process cannot continue indefinitely. As electrons diffuse from the n-region, positively charged donor atoms are left behind. Similarly, as holes diffuse from the p-region, they uncover negatively charged acceptor atoms. The unneutralized ions in the neighbourhood of the junction are referred to as uncovered charges. The general shape of the charge density 'ρ' depends upon how the diode is doped. Since the region of the junction is depleted of mobile charges, it is called the depletion region, the space-charge region, or the transition region.

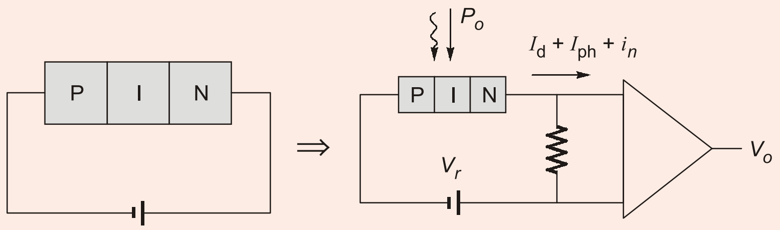


The net positive and negative charges in the n and '*p*' regions induce an electric fields in the region near the metallurgical junction, in the direction from the positive to the negative charge, or from the n to the p region.
Density gradients still exist in the majority carrier concentrations at each edge of the space charge region and producing a "diffusion force" that acts on the majority carriers as shown in figure above. The electric field in the space charge region produces another force on the electrons and holes which is in the opposite direction to the diffusion force for each type of particle. In thermal equilibrium, the diffusion force and the electric field force exactly balance each other.



(a) (b) (c)

(ii) PIN photodetector:



The lowest signal that a photo detector can detect is determined by the extent of random fluctuations in the current through the detector and the voltage across it as a result of various statistical processes in the device.

When PIN photodiode is reverse biased there is still a dark current I_d present which mainly due to thermal generation of electron-hole pairs in the depletion layer and within diffusion lengths to the depletion layer.

The root mean square (rms) value of the fluctuations in the dark current represents the shot noise current $i_{n\text{-dark}}$

$$(i_{n\text{-dark}})^2 = 2eI_dB$$

The photodetection process involves the interaction of discrete photons with valence electrons.

The quantum nature of photon therefore gives rise to the statistical randomness in the EHP photogeneration process. This type of fluctuation is called quantum noise.

$$(I_{n\text{-quantum}})^2 = 2eI_{ph}B$$

Total rms shot noise current is

$$i_n = \left[2e(I_d + I_{ph})B\right]^{1/2}$$

The mean thermal noise power in the resistor R is 4 KTB.

In receiver design we are often interested in the signal to noise ratio, which is defined as

$$\text{SNR} = \frac{\text{Signal power}}{\text{Noise power}}$$

For photodetector alone, SNR is simply the ratio of I_{ph}^2 to i_n^2. SNR for the receiver must include the noise power generated in the sampling resistor R (thermal noise)

$$\text{SNR} = \frac{I_{ph}^2 R}{i_n^2 R + 4KTB}$$

$$\text{SNR} = \frac{I_{ph}^2}{i_n^2 + \dfrac{4KTB}{R}}$$

$$\text{SNR} = \frac{I_{ph}^2}{2e(I_d + I_{ph}) + \dfrac{4KTB}{R}}$$

End of Solution

Q.4 (b) A 75 KVA, 230 V/115 V, 60 Hz transformer was tested with the following results:
Short circuit test : 9.5 V, 326 A, 1200 W
Open circuit test : 115 V, 16.3 A, 750 W
Determine the (i) equivalent impedance in high-voltage terms, (ii) equivalent impedance in per unit, (iii), voltage regulation at rated load and 0.8 power factor lagging (iv) efficiency at rated load and 0.8 power factor lagging and also at 1/2 load and unity power factor.

[20 marks : 2022]

Solution:

Short circuit test: 9.5 V, 326 A, 1200 W

The series impedance,

$$Z_{SC} = \frac{V_{SC}}{I_{SC}} \angle\phi$$

$$\phi = \cos^{-1}\left(\frac{1200}{9.5 \times 326}\right) = 67.20°$$

$$Z_{SC} = \frac{9.5}{326}\angle 67.20° = 0.02914\angle 67.20°\ \Omega$$

$$Z_{SC} = 0.02914\angle 67.20°\ \Omega = (0.0113 + j0.0269)\ \Omega$$

Open circuit test: 115 V, 16.3 A, 750 W

$$\bar{I}_{OC} = 16.3\angle -\cos\left(\frac{750}{115 \times 16.3}\right) = 16.3\angle -66.415°$$

$$\bar{I}_{OC} = (6.25 - j14.938)\ \Omega$$

$$R_C = \frac{115}{6.52} = 17.63\Omega$$

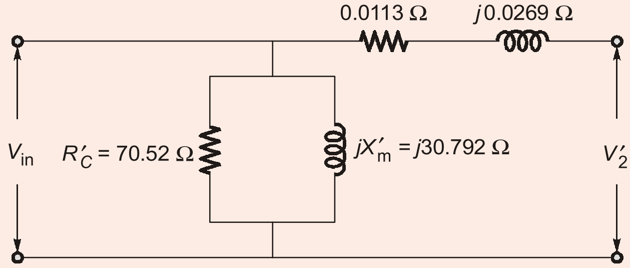
$$X_m = \frac{115}{14.938} = 7.698\ \Omega$$

R_C and X_m referred to high voltage side

$$R'_c = 17.63\left(\frac{230}{115}\right)^2 = 70.52\ \Omega$$

$$X'_m = 7.698\left(\frac{230}{115}\right)^2 = 30.792\ \Omega$$

(i) The equivalent circuit referred to high voltage side,



(ii) Base impedance, $$Z_{base} = \frac{230^2}{75\times10^3} = 0.705\ \Omega$$

The per unit value, $$X_{pu} = \frac{X_{value}}{X_{base}}$$

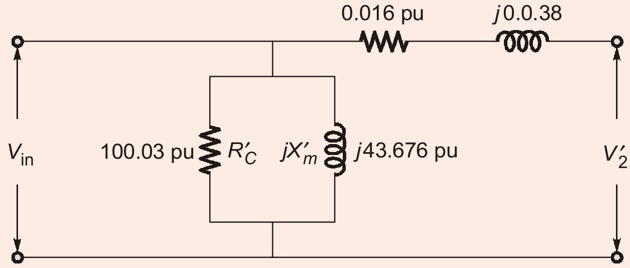
$$Z_{pu} = \frac{0.02914\angle 67.2}{0.705} = 0.0413\angle 67.2$$

$$Z_{pu} = (0.016 + j0.038)\ \text{pu}$$

$$(R'_c)_{pu} = \frac{70.52}{0.705} = 100.03\ \text{pu}$$

$$(X'_m)_{pu} = \frac{30.792}{0.705} = 43.676\ \text{pu}$$

The equivalent circuit in per unit value



(iii)
$$Z = (0.016 + j0.038)\ \text{pu}$$
$$\text{pf} = 0.8\ \text{lagging}$$
$$V.R = (R_{pu}\cos\phi + X_{pu}\sin\phi)\times 100\%$$
$$= (0.016\times0.8 + 0.038\times0.6)\times100\%$$
$$= 0.0356\times100\%$$
$$V.R = 3.56\%$$

(iv) Per unit iron loss,

$$P_i = \frac{1}{100.03} = 9.997 \times 10^{-3}$$

Per unit copper loss, $P_{cu} = 0.016$ pu

For rated load and 0.8 pf lagging load i.e. $x = 1$

$$\%\eta = \frac{1 \times 0.8 \times 1}{1 \times 0.8 + 9.997 \times 10^{-3} + 0.016} \times 100\%$$

$$\%\eta = 96.852\%$$

For $\frac{1}{2}$ load and unity power factor,

$$\%\eta = \frac{\frac{1}{2} \times 1 \times 1}{\frac{1}{2} \times 1 \times 1 + 9.997 \times 10^{-3} + (0.016) \times \left(\frac{1}{2}\right)^2} \times 100$$

$$\%\eta = 97.28\%$$

End of Solution

Q.4 (c) (i) 1. "The ferroelectric behaviour of $BaTiO_3$ ceases above its ferroelectric Curie temperature." Explain whether this statement is correct, giving reasons.

2. Differentiate between Curie temperature and Neel temperature, and also draw a graph between susceptibility (χ) and Temperature (K) to demonstrate this difference for paramagnetic, Ferromagnetic and Antiferromagnetic materials.

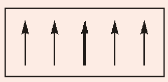
(ii) A laminate composed of 0.1 mm thick aluminium sandwiched around a 20 mm thick layer of polysterene foam (styrofoam) is produced as an insulation material. Calculate the thermal conductivity of the laminate parallel and perpendicular to the layers. The thermal conductivity of aluminium is 238.6 W m^{-1} K^{-1} and that of foam is 0.032 W m^{-1} K^{-1}.

[5 + 5 + 10 marks : 2022]

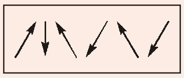
Solution:

(i) 1. Ferroelectric material :

- In ferroelectric material, dipoles are parallel to each other i.e. it will have permanent dipole at temperature 'T'.



- As the temperature of material increases, dipole start moving from its original position.
- When temperature of material reaches its Curie temperature, all the dipole become random in nature.

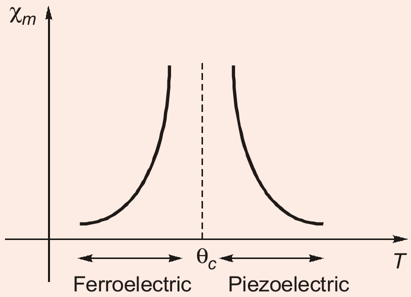


- At temperature $T = \theta_c$ (Curie temperature) we get piezoelectric material from ferroelectric material.

i.e magnetic susceptibility $(\chi_m) = \dfrac{C}{T - \theta_c}$ ... Curie Weiss law

where, $C = \dfrac{Np_B^2 \cdot \mu_o}{K}$ ... Curie constant

$\theta_c = \dfrac{Np_B^2 \cdot \mu_o \gamma}{K}$ ... Curie temperature



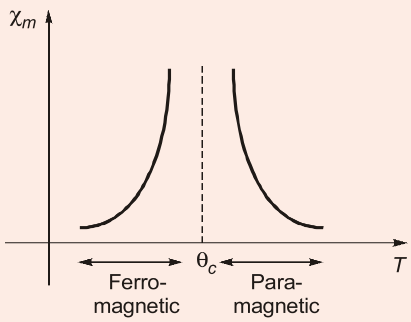
- From above explanation we can say ferroelectric behaviour of $BaTiO_3$ ceases above its ferroelectric Curie temperature.

2. Difference between Curie temperature and Neel temperature
   - Curie temperature is the temperature at or above, certain materials lose their permanent magnetic properties i.e. ferromagnetic material will become paramagnetic material.
   - Neel temperature is the temperature above which certain antiferromagnetic materials become paramagnetic.

→ For ferromagnetic material :

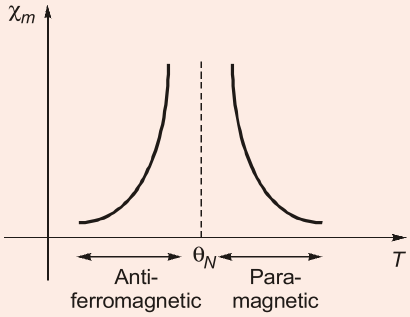
$$\chi_m = \frac{C}{T - \theta_c}$$

θ_c = Curie temperature



→ For antiferromagnetic :

$$\chi_m = \frac{C}{T + \theta_N}$$



(ii) The volume fraction of aluminium in the laminate,

$$f_{Al} = \frac{V_{Al}}{V_{Al} + V_{foam}} = \frac{2t_{Al} \cdot A}{2t_{Al} \cdot A + t_{foam} \cdot A}$$

$$f_{Al} = \frac{2t_{Al}}{2t_{Al} + t_{foam}}$$

Given: t_{Al} = 0.1 mm and t_{foam} = 20 mm

$$f_{Al} = \frac{2 \times 0.1}{2 \times 0.1 + 20} = 0.01$$

∴ $f_{foam} = 1 - 0.01 = 0.99$

Thermal conductivity of aluminium, $K_{Al} = 238.6\ Wm^{-1}\ K^{-1}$

Thermal conductivity of foam, $K_{foam} = 0.032\ Wm^{-1}\ K^{-1}$

Thermal conductivity of laminate parallel to layers,

$$K_1 = \Sigma K_i f_i$$

$$K_1 = K_{Al}\, f_{Al} + K_{foam} \cdot f_{foam}$$

$$K_1 = 238.6 \times 0.01 + 0.032 \times 0.99$$

$$K_1 = 2.41768\ Wm^{-1}\ K^{-1}$$

Thermal conductivity of laminate perpendicular to layers,

$$\frac{1}{K_2} = \Sigma \frac{f_i}{K_i} = \frac{f_{Al}}{K_{Al}} + \frac{f_{foam}}{K_{foam}}$$

$$\frac{1}{K_2} = \frac{0.01}{238.6} + \frac{0.99}{0.032} = \frac{236.21432}{7.6352}$$

$$K_2 = 0.03231\ Wm^{-1}\ K^{-1}$$

End of Solution

Section-B

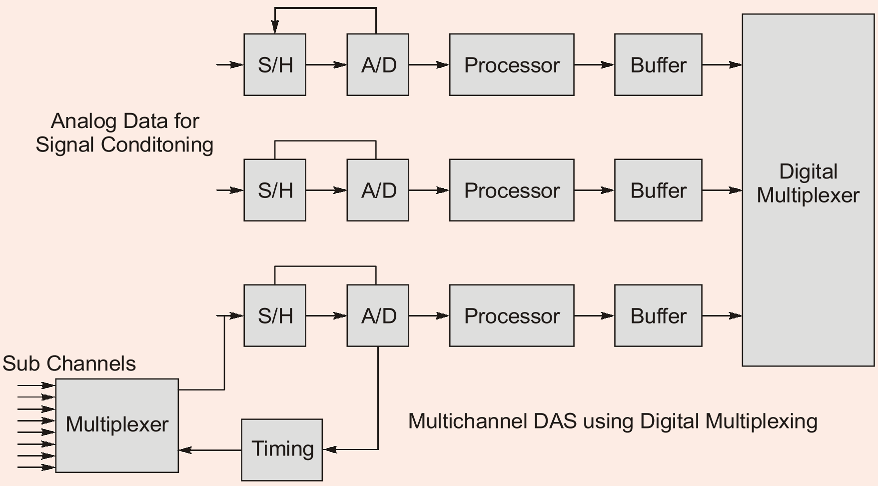
Q.5 (a) What are the objectives of a Data Acquisition System (DAS)? Write the block schematic of a multichannel DAS, handling analog signals as input.

[10 marks : 2022]

Solution:

Objective of Data Acquisition System :

- It must acquire the necessary data, at correct speed.
- use of all data efficiently to inform the operator about the state of the.
- It must monitor the complete plant operation to maintain on-line optimum and safe operations.
- It must provide an effective human communication system and be able to identify problem areas, thereby minimising unit availability and maximising unit through point at minimum cost.
- It must be able to collect, summarise and store data for diagnosis of operation and record purpose.
- It must be able to compute unit performance indices using on-line, real-time data.
- It must be flexible and cabable of being expanded for future require.
- It must be reliable and not have a down time greater than 0.1%



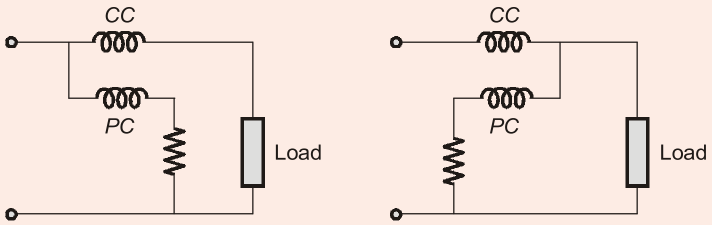
Multichannel DAS using Digital Multiplexing

End of Solution

Q.5 (b) A wattmeter has a current coil of 0.1 Ω resistance and a pressure coil of 6.5 kΩ resistance. Calculate the percentage errors, due to resistance only with each of the two methods of connection shown in figure, when reading the input to an apparatus which takes:

(i) 12 A at 250 V with unity power factor and

(ii) 12 A at 250 V and 0.4 power factor.



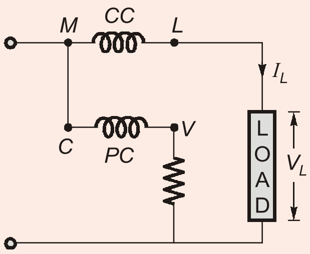
First connection   Second connection

[10 marks : 2022]

Solution:

Given that $R_{cc} = 0.1\ \Omega$, $R_{pc} = 6.5\ \text{k}\Omega = 6500\ \Omega$

(i) $I = 12$ amp, $V = 250$ volts, P.F = 1



M-C-Short:

$$P_T = V_L \cdot I_L \cos(\phi)$$

$$P_T = 250 \times 12 \times 1 = 3000 \text{ Watt}$$

$$P_m = P_T + I_L^2 R_{CC}$$

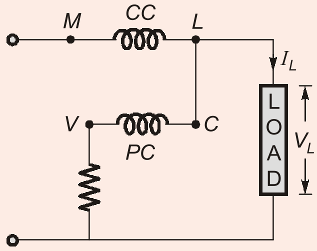
$$P_m = 3000 + (12)^2 \times 0.1$$

$$P_m = 3014.4 \text{ Watt}$$

$$\%\text{ Error} = \frac{P_m - P_T}{P_T} \times 100$$

$$\%\text{ Error} = \frac{14.14}{3000} \times 100$$

$$\%\text{ Error} = 0.40\%$$



L-C-Short:

$$P_T = V_L \cdot I_L \cos(\phi)$$

$$P_T = 250 \times 12 \times 1 = 3000 \text{ Watt}$$

$$P_m = P_T + \frac{V_L^2}{R_{pc}}$$

$$P_m = 3000 + \frac{(250)^2}{6500} = 3009.61$$

$$\% \text{ Error} = \frac{P_m - P_T}{P_T} \times 100 = \frac{9.61}{3000} \times 100$$

$$\% \text{ Error} = 0.32\%$$

(ii) $I = 12$ A, $V = 250$ Volt; P.F = 0.4

M-C-Short

$$P_T = 250 \times 12 \times 0.4 = 1200 \text{ Watt}$$

$$P_m = P_T + I_L^2 R_{CC}$$

$$P_m = 1200 + (12)^2 \times 0.1 = 1214.4 \text{ Watt}$$

$$\% \text{ Error} = \frac{P_m - P_T}{P_T} \times 100 = \frac{14.14}{1200} \times 100$$

$$\% \text{Error} = 1.2\%$$

L-C-Short

$$P_T = 250 \times 12 \times 0.4 = 1200 \text{ Watt}$$

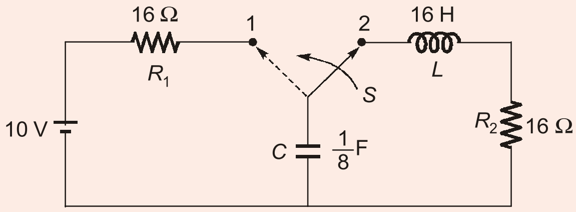
$$P_m = P_T + \frac{V_1^2}{R_{PC}} = 1200 + \frac{(250)^2}{6500} = 1209.6 \text{ Watt}$$

$$\% \text{ Error} = \frac{P_m - P_T}{P_T} \times 100 = \frac{9.61}{1200} \times 100$$

$$\% \text{Error} = 0.8\%$$

End of Solution

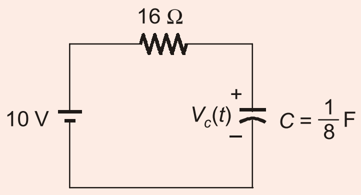
Q.5 (c) The switch S shown in figure is initially at position '2' and is moved to position '1' at $t = 0$ second. Then, at $t = 2$ seconds the switch S is again moved to position '2'. Find the voltage across the resistor R_2 for $t > 2$ seconds.



[20 marks : 2022]

Solution:

At $t = 0^+$



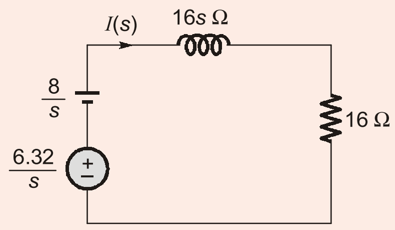
$$V_c(0^+) = 0 \text{ V}$$
$$V_c(\infty) = 10 \text{ V}$$
$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-t/\tau}$$
$$\tau = R_c = 16 \times \frac{1}{8} = 2 \text{ sec}$$
$$V_c(t) = 10 - 10e^{-t/2}$$
$$= 10(1 - e^{-t/2}) \text{ volt}$$

At $t = 2$ sec, voltage across capacitor.

$$V_c(2) = 10(1 - e^{-1})$$
$$= 6.32 \text{ Volt}$$

Now At $t = 2$ sec.

Switch is at position 2;



$$I(s) = \frac{6.32/s}{\frac{8}{s} + 16s + 16} = \frac{6.32}{16s^2 + 16s + 8} = \frac{6.32/16}{s^2 + s + \frac{1}{2}}$$

$$= \frac{6.32/16}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{6.32}{16 \times \frac{1}{2}}\left[\frac{1/2}{\left(\frac{s+1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}\right]$$

Current through resistor R_2,

$$i(t) = 0.79e^{\frac{-(t-2)}{2}} \sin\left(\frac{t-2}{2}\right)$$

Voltage across resistor R_2.

$$V(t) = i(t)R_2$$

$$= 0.79 \times 16e^{-(t-2)/2} \sin\left(\frac{t-2}{2}\right) \text{ for } t > 2$$

$$V(t) = 12.64e^{-(t-2)/2}\sin\left(\frac{t-2}{2}\right) \quad t > 2$$

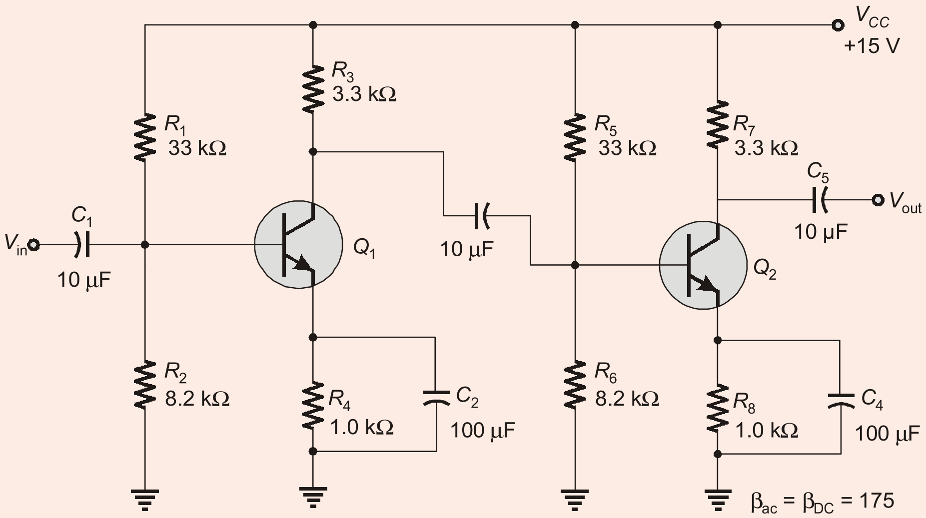
End of Solution

Q5 (d) For a two-stage, capacitively coupled amplifier as shown below, find the following:

(i) Voltage gain of each stage

(ii) Overall voltage gain

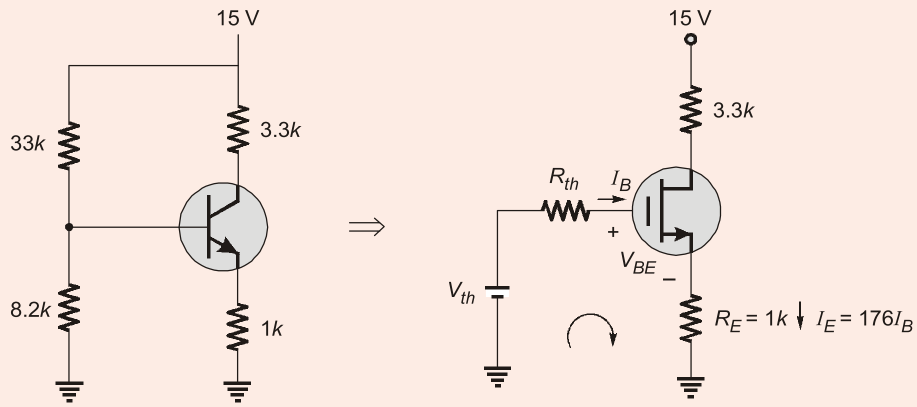
(iii) Express the gains found in (a) and (b) in dB.



[10 marks : 2022]

Solution:

DC analysis of each stage:



$$V_{th} = \frac{15 \times 8.2}{33 + 8.2} = 2.985 \text{ V}$$

$$R_{th} = 33 || 8.2 = 6.57 \text{ k}\Omega$$

KVL: $V_{th} = I_B R_{th} + V_{BE} + 176\, I_B \times R_E$

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + 176 R_E} = \frac{2.985 - 0.7}{6.57 + 176 \times 1} = 12.52 \times 10^{-3} \text{ mA}$$

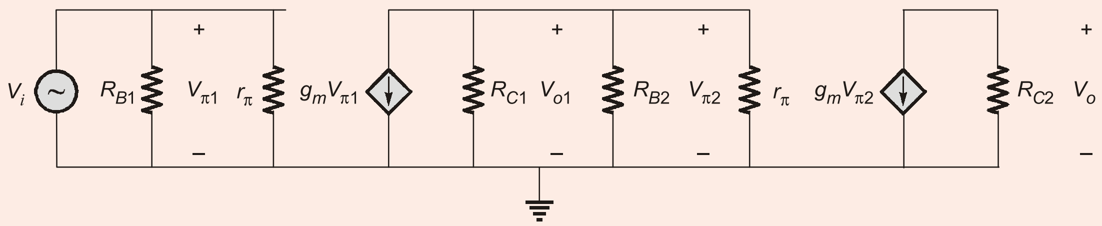
$$I_C = \beta I_B = 2.19 \text{ mA}$$

Small signal AC analysis:

Assume $V_T = 26$ mV

$$g_m = \frac{I_C}{V_T} = \frac{2.19 \text{ mA}}{26 \text{ mA}} = 84.23 \text{ m℧}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{175}{84.23} = 2.08 \text{ k}\Omega$$



(i) 2[nd] stage:

$$V_o = -g_m V_{\pi_2} R_{C_2}$$

$$\Rightarrow \quad \frac{V_o}{V_{\pi_2}} = -g_m R_{C_2} = -84.23 \times 3.3 = -278$$

1[st] stage:

$$V_{o1} = -g_m V_{\pi_1} (R_{C_1} \,||\, R_{B_2} \,||\, r_\pi)$$

$$\frac{V_{o1}}{V_{\pi_1}} = \frac{V_{o1}}{V_i} = -g_m (R_{C_1} \,||\, R_{B_2} \,||\, r_\pi)$$

$$= -84.23\,(3.3 \,||\, 6.57 \,||\, 2.02) = -90$$

(ii) Overall gain, $A_V = A_{V1} \times A_{V2} = -90 \times -278$

$A_V = 25020$

(iii) Voltage gain in dB

$$A_{V1} \text{ in dB} = 20 \times \log_{10} |A_{V1}| = 39.08 \text{ dB}$$

$$A_{V2} \text{ in dB} = 20 \times \log_{10} |A_{V2}| = 48.88 \text{ dB}$$

$$A_V \text{ in dB} = 20 \times \log_{10} |A_V| = 87.96 \text{ dB}$$

End of Solution

Q.5 (e) The operation of a relay switch for a particular application is controlled by the output Y of a logic circuit which is having four inputs A, B, C, D. The relay switch must be switched ON when Y is '1' for the following inputs ($ABCD$):]

0000, 0001, 0100, 1000, 1001, 1011.

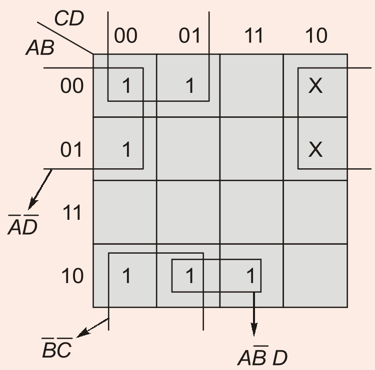
The inputs states 0010, and 0110 do not occur and for the remaining input states, relay must be switched OFF. Design the logic circuit using only 2-input NOR gates.

[10 marks : 2022]

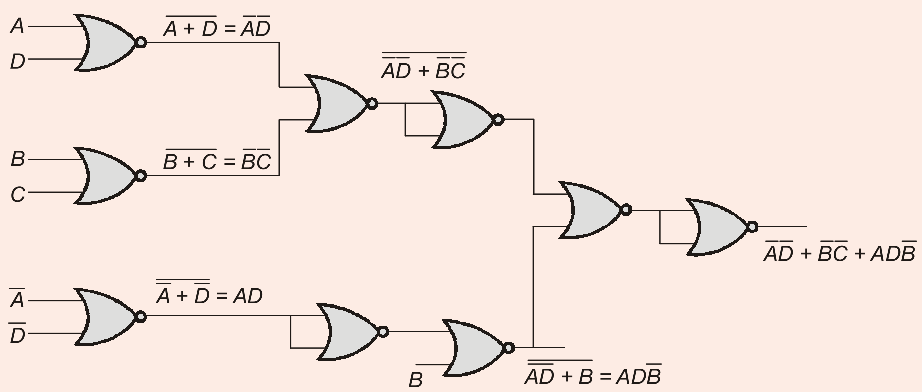
Solution:

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	1
0	0	1	0	x
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	x
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

By using K-map



$$Y = \bar{A}\bar{D} + \bar{B}\bar{C} + A\bar{B}D$$



End of Solution

Q.6 (a) (i) What is the reason for using Amplitude modulation principle in certain measurement systems? Develop a mathematical model for its frequency spectrum. Apply this analysis to the following cases :

a. Vibration and noise of shafts with gears (20 teeth gear running at 4000 Hz).

b. Recording very small voltages of a strain gauge and amplifying its output, assuming strain gauge bridge is excited by an AC signal of amplitude 5 V and a frequency of 3000 Hz.

(ii) Which technique would you employ for transmitting slow changing low bandwidth data using telemetry? Write the block schematic and illustrate the principle of working.

[12 + 8 marks : 2022]

Solution:

(i) Amplitude modulation principle is used in certain measurement systems mainly for two considerations.

1. Physical data that are to be measured and interpreted sometimes are amplitude modulated.
2. Certain type of measurement systems intentionally introduce amplitude modulation for one or more benefits.

Consider an AM signal,

$$s_{AM}(t) = A_c A_m \cos\omega_m t \cdot \cos\omega_c t$$

where, $m(t) = A_m \cos\omega_m t$ is the message signal

and $c(t) = A_c \cos\omega_c t$ is the carrier signal

$$s_{AM}(t) = \frac{A_c A_m}{2}\left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right]$$

The frequency spectrum of this AM signal is a discrete spectrum existing only at frequencies $(\omega_c - \omega_m)$, called side frequencies.

If the modulating signal is a periodic function $m(t)$, it may be expanded in Fourier series to get the output of modulation as

$$s(t) = \left[A_o + \sum_{n=1}^{\infty} A_n \cos(n\omega_o t) + \sum_{n==1}^{\infty} B_n \sin(n\omega_o t)\right] \cdot A_c \cos(\omega_c t)$$

$$s(t) = A_o A_c \cos(\omega_c t) + [A_1 A_c \cos(\omega_o t)\cos(\omega_c t) + A_2 A_c \cos(2\omega_o t)\cos(\omega_c t) + ...]$$
$$+[B_1 A_c \sin(\omega_o t)\cos(\omega_c t) + B_2 A_c \sin(2\omega_o t)\cos(\omega_c t) + ...]$$

$$s(t) = A_o A_c \cos(\omega_c t)\ c_1 \cos(\omega_o t - \alpha_1)\cos(\omega_c t) + c_2 \cos(2\omega_o t - \alpha_2)\cos(\omega_c t) + ...$$

where $c_n = \dfrac{A_c}{\sqrt{A_n^2 + B_n^2}}$ and $\alpha_n = \tan^{-1}\left(\dfrac{B_n}{A_n}\right)$

$$s(t) = A_o A_c \cos(\omega_c t) + \frac{c_1}{2}[\cos[\omega_c + \omega_o] + -\alpha_1] + \cos[(\omega_c - \omega_o)t + \alpha_2]$$
$$+\frac{c_2}{2}[\cos[(\omega_c + 2\omega_o)t - \alpha_2]] + \cos[(\omega_c - 2\omega_o)t + \alpha_2 + ...]$$

The spectrum of the signal is a discrete spectrum containing the frequencies, ω_c, $\omega_c \pm \omega_o$, $\omega_c \pm 2\omega_o$, etc i.e. each frequency component of the modulating signal produces one pair of side frequencies.

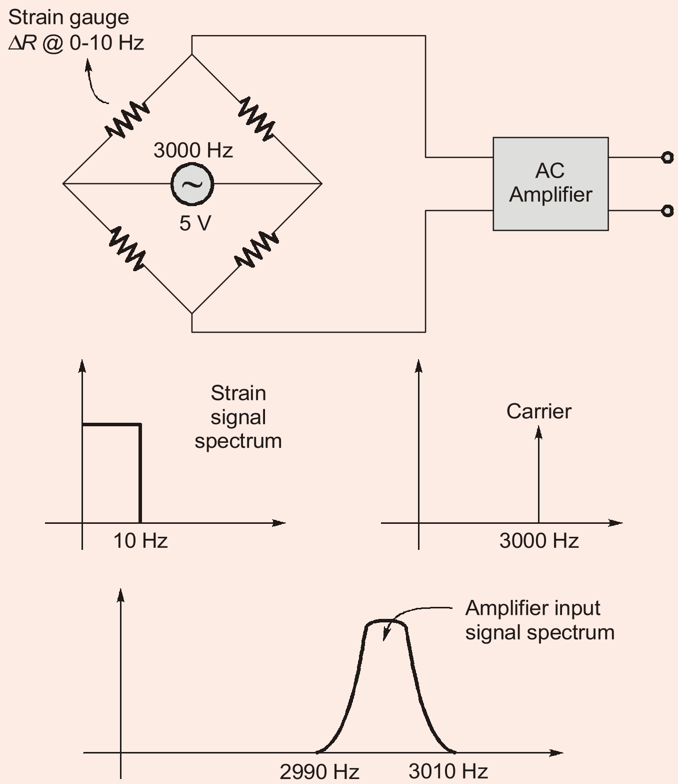
(a) Vibration and noise of shafts with gears (20 teeth gear running at 4000 Hz).

The 20 teeth gear running at 4000 Hz will have rotational speed = $\dfrac{4000}{20}$ = 200 rev/sec.

The tooth forces (which cause vibration and thus noise) would have a fundamental frequency equal to the tooth meshing frequency, which is 4000 Hz. These forces would not be pure sine waves, but would be periodic. In an actual i.e. the gears are closer together at some points in their rotation than at the others. This runout leads to a force amplitude that varies as the gear rotates i.e. the tooth force amplitude is modulated as a function of rotational position.

Hence, the frequencies of generated noise (corresponding to tooth force frequencies) would be expected to be the side frequencies generated by amplitude modulating the 4000 Hz tooth meshing frequency with the 200 Hz (once per rotation) runout frequencies i.e. 3800 Hz and 4200 Hz.

(b) Recording very small voltages of a strain gauge and amplifying its output its output, assuming strain gauge bridge is excited by an AC signal of amplitude 5 V and a frequency of 3000 Hz.

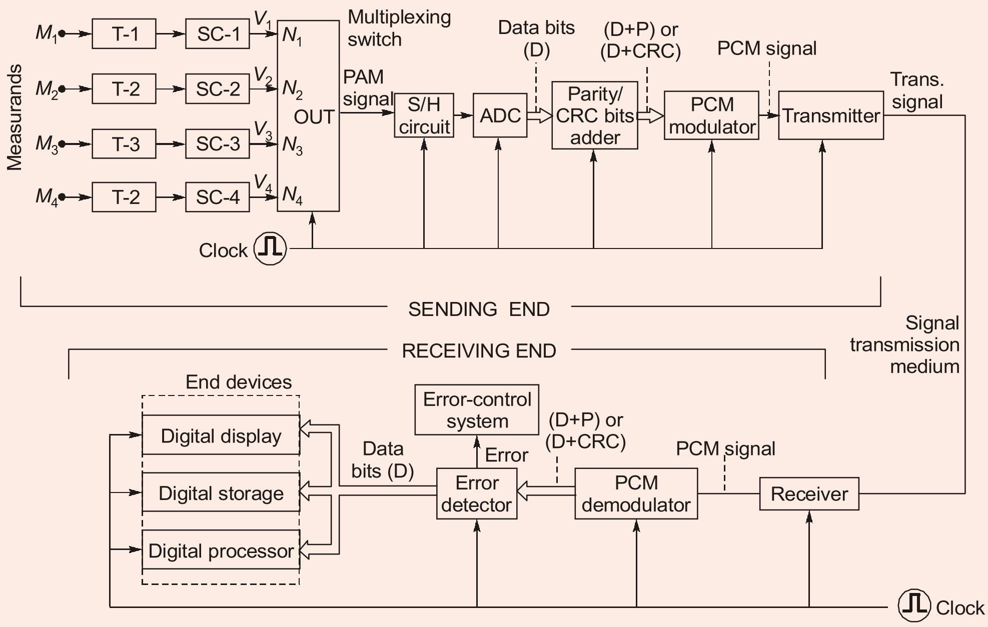


If we are measuring strains which vary from, say 0 Hz (static) to 10 Hz, the amplifier will have an input frequency spectrum bounded by 2990 Hz and 3010 Hz. This shifting of information frequencies from one part of frequency range to another corresponds to amplitude modulation.

(ii) PCM Telemetry system is employed for transmitting slow changing low bandwidth data.

The transmission signal in PCM telemetry system takes the form of a coded sequence of binary pulses. The multi-channel operation is achieved through time-division multiplexing (TDM).

A 4-channel PCM telemetry system is shown in the figure below:



Block schematic of a 4-channel PCM (digital) telemetry system

The physical variable or measurand M is applied to an appropriate transducer, the output of which is processed suitably by the signal conditioner to produce a dc voltage, Vdc , proportional (analogous) to M. This analog voltage is sampled at regular intervals (or sampling frequency) and converted to its equivalent digital value by an analog to digital converter (ADC). As the analog voltage follows the time-variations in the value of the measurand, its value changes continuously. Any change in the analog input to the ADC of successive-approximation type, while A-to-D conversion is going on, can cause serious error in the output of the ADC. The problem is avoided by using a sample-and-hold circuit (S/H circuit), rather than a simple sampling circuit, before the ADC as shown in the block schematic. He ADC is followed by the circuits described below:

Parity/CRC Bits Adder: The data transmission can be either asynchronous or synchronous. In case of asynchronous transmission, this circuit adds a parity bit to each character, that is, n-bit data output of the ADC. In case of synchronous transmission, the circuit adds a certain number of cyclic-redundancy-check bits (CRC bits) at the end of the data bits.

PCM Modulator: It encodes the augmented data (comprising data bits and parity/ CRC bits) into a series of coded pulses using a certain PCM code.

Transmitter: The transmitter has two functions to perform:

(a) **Addition of Synchronization Information:** In case of asynchronous transmission, it adds start and stop pulses before and after each data character. In case of synchronous transmission, it adds a synchronization (sync) bit pattern in front of the entire data block.

(b) **Signal Conversion:** The transmitter converts the coded voltage pulses (PCM signal) into a transmission signal to suit the given transmission medium and the desired range of transmission.

Since the S/H circuit, ADC and all other circuits mentioned above operate in tandem, their operations need to be synchronized using a common reference clock, as shown in the diagram.

Receiving-End Scheme: The receiver has two functions to perform here:

(a) It synchronizes its own operation with the transmitter by using the start and stop pulses (for asynchronous transmission) or the sync bit pattern (for synchronous transmission).

(b) It carries out reverse signal conversion, that is, it converts the transmission signal received by it into coded voltage pulses (PCM signal).

Thereafter, a PCM demodulator decodes the coded voltage pulses back into 'augmented data' and converts them into parallel bits. This data is passed on to an error detector, which verifies the correctness of the data from the parity or CRC bits. In case an error is detected, the information is passed on to the error control system for suitable action. If no error is found, then the data bits alone are passed on to the end devices. All the circuits, including the end devices, operating in tandem, are synchronized by a clock in the receiving end of the telemetry system, as shown in the diagram. This clock has the same frequency as the one in the sending end.

In the block schematic, three commonly used end devices are shown:

(a) A multiplexed digital display unit for displaying the values of the measurands one by one.

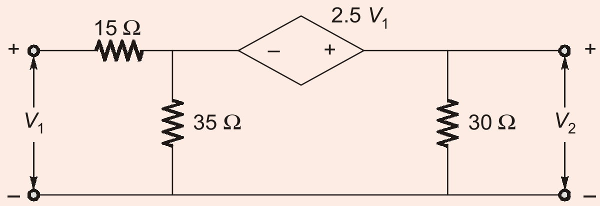
(b) A digital storage unit for storing values of all the measurands for a later analysis.

(c) A data processor for analyzing the data immediately.

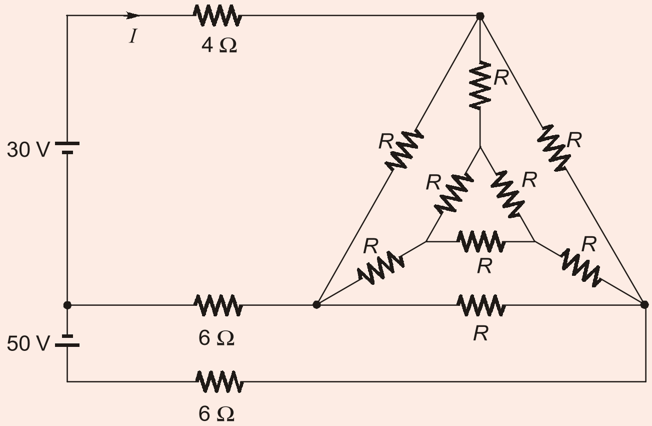
The receiver sends channel information along with data (for identifying the channel number for each data) to these end devices so that the data reaches the correct end device.

End of Solution

Q.6 (b) (i) Determine the transmission parameters for the following two port network shown.



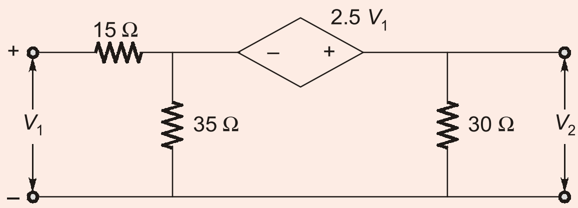
(ii) Using delta-to-star transformation, determine the current I in the circuit shown in figure when the value of the resistor R = 15 Ω.



[12 + 8 marks : 2022]

Solution:

(i)

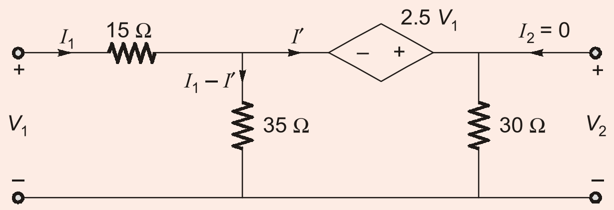


Transmission parameters are given as

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

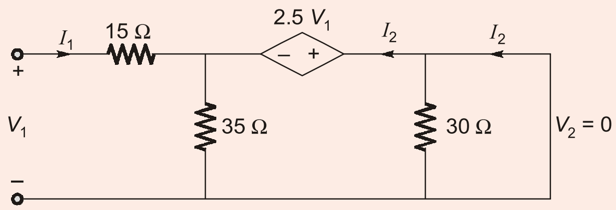
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$



$$V_1 = 15I_1 + 35(I_1 - I')$$
$$(I_1 - I')35 = 30I' - 2.5\,V_1$$
$$V_2 = 30I'$$
$$35(I_1 - I') = 30I' - 2.5[15I_1 + 35(I_1 - I')]$$
$$35(I_1 - I')35 = 30I' - 37.5I_1 - 87.5I_1 + 87.5I'$$
$$35I_1 + 37.5\,I_1 + 87.5I_1 = 30I' + 87.5I' + 35I'$$
$$160I_1 = 152.5I'$$
$$I' = 1.049I_1$$
$$V_2 = 30 \times 1.049I_1$$
$$C = \frac{I_1}{V_2} = 0.0317\,℧$$
$$V_1 = 15I_1 + 35(I_1 - 1.049I_1)$$
$$V_1 = 13.285I_2$$
$$V_2 = 31.47\,I_1$$
$$A = \frac{V_1}{V_2} = 0.422$$

Now, put $V_2 = 0$.

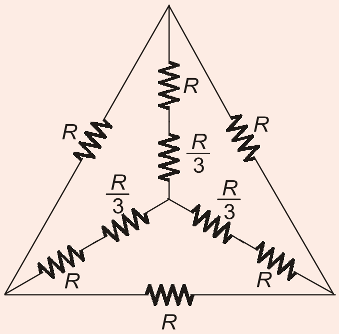


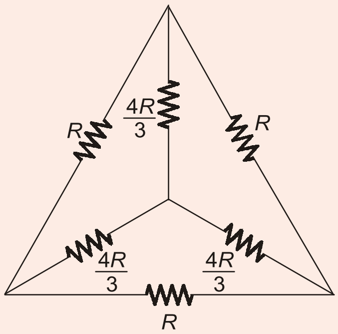
$$V_1 = 15I_1 + (I_1 + I_2)35$$
$$V_1 = 50I_1 + 35I_2 \quad \text{...(1)}$$
$$(I_1 + I_2)35 = -2.5\,V_1 \quad \text{...(2)}$$
$$50I_1 + 35I_2 = -14I_1 - 14I_2$$
$$64I_1 = -49I_2$$
$$I_1 = -0.765I_2$$
$$V_1 = 50(-0.765)I_2 + 35I_2$$
$$V_1 = -3.25I_2$$
$$B = \frac{-V_1}{I_2} = 3.25\ \Omega$$
$$D = \frac{-I_1}{I_2} = 0.765$$

∴ Transmission parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.422 & 3.25 \\ 0.0317 & 0.765 \end{bmatrix}$$

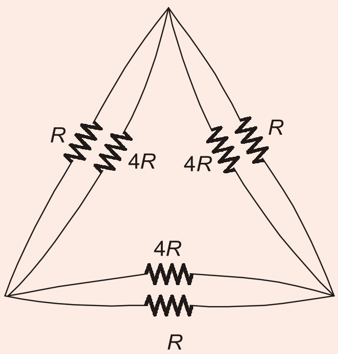
(ii) Using delta to star conversion





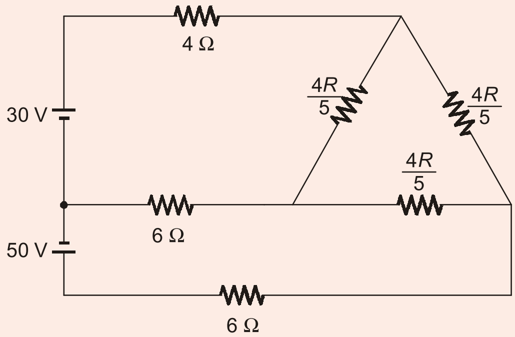
Again converting star to delta.

Using star to delta conversion,

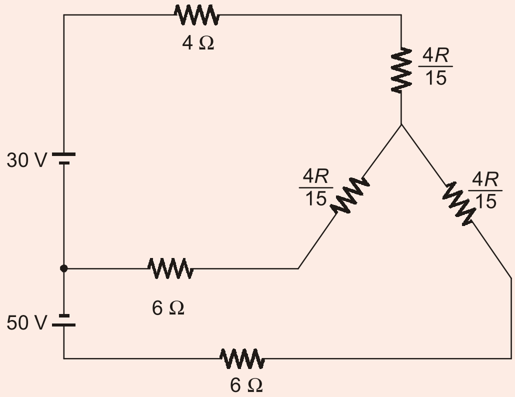


$$R \,||\, 4R = \frac{4R^2}{5R} = \frac{4R}{5}$$

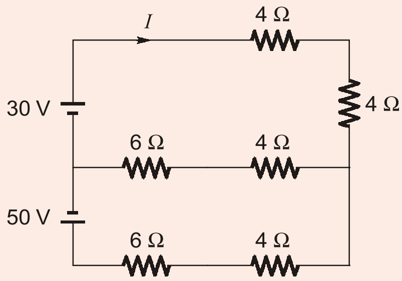
Circuit can be simplified as,

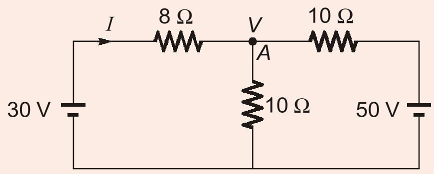


Again converting delta to star.



Put $R = 15\ \Omega$





Now, using nodal analysis at node A,

$$\frac{30-V}{8} = \frac{V}{10} + \frac{V-50}{10}$$

$$\frac{30}{8} - \frac{V}{8} = \frac{V}{10} + \frac{v}{10} - 5$$

$$\frac{2V}{10} + \frac{V}{8} = \frac{15}{4} + 5$$

$$\frac{8V+5V}{40} = \frac{35}{4}$$

$$13\text{ V} = 350$$

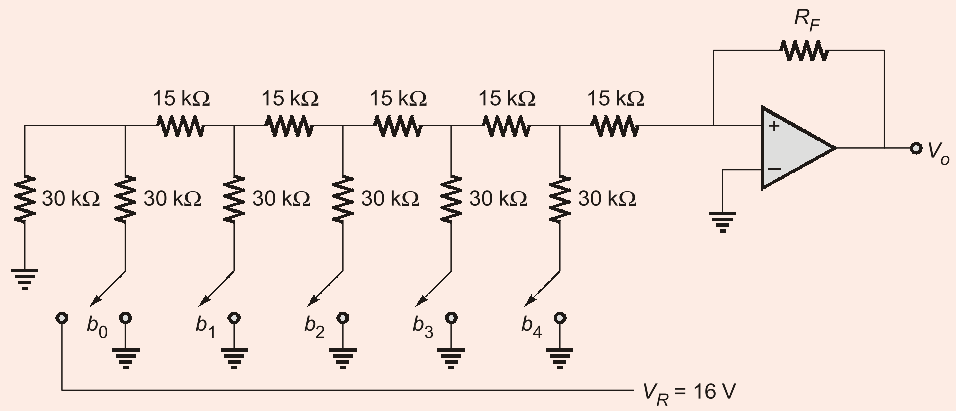
$$V = \frac{350}{13}\text{ Volt}$$

$$\text{Current I} = \frac{30-V}{8} = \frac{30-\frac{350}{13}}{8}$$

$$= \frac{40}{8\times 13} = \frac{5}{13}\text{A}$$

$$I = 0.384$$

(iii) R-2R ladder DAC



Given V_R = 16 V, input binary = $(11010)_2$
D = Decimal equivalent = $(26)_{10}$
Output voltage, $V_o = RDG$
Here, let G = 1

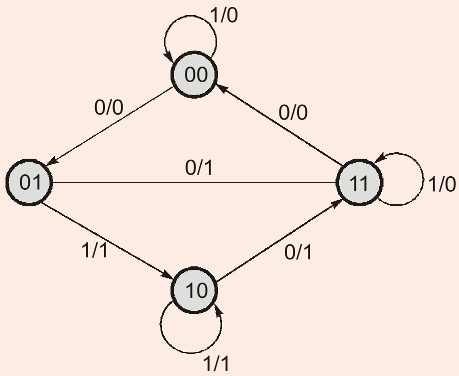
$$V_o = RD$$

$$V_o = \frac{V_R}{2^n}\times 26 = \frac{16}{2^5}\times 26 = \frac{16}{32}\times 26$$

$$V_o = 13\text{ Volt}$$

End of Solution

Q.6 (c) A sequential circuit has one input and one output. The state diagram of the sequential circuit is given.



Design the sequential circuit using *M-N* Flip-Flop. The truth table for the *M-N* flip-flop is given as

M	N	Q_{n+1}
0	0	1
0	1	$\overline{Q}_n$
1	0	Q_n
1	1	0

[20 marks : 2022]

Solution:

Excitation table for M-N Flip flop

M	N	Q_n	Q_{n+1}
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Excitation table:

Q_n	Q_{n+1}	M	N
0	0	1	X
0	1	0	X
1	0	X	1
1	1	X	0

Let the input be '*X*' and output be '*Z*'

Here, there are 4 states. So, 2 M-N flip flops are required.

State Table:

Present state Q_1	Present state Q_0	Input X	Next State Q_1^+	Next State Q_0^+	Output Z	M-N Flip flop M_1	N_1	M_0	N_0
0	0	0	0	1	0	1	X	0	X
0	0	1	0	0	0	1	X	1	X
0	1	0	1	1	1	0	X	X	0
0	1	1	1	0	1	0	X	X	1
1	0	0	1	1	1	X	0	0	X
1	0	1	1	0	1	X	0	1	X
1	1	0	0	0	0	X	1	X	1
1	1	1	1	1	0	X	0	X	0

K-map for M_1:

Q_1 \ Q_0X	00	01	11	10
0	1	1	0	0
1	X	X	X	X

$$M_1 = \bar{Q}_0$$

K-map for N_1:

Q_1 \ Q_0X	00	01	11	10
0	X	X	X	X
1	0	0	0	1

$$N_1 = Q_0\bar{X}$$

K-map for M_0:

Q_1 \ Q_0X	00	01	11	10
0	0	1	X	X
1	0	1	X	X

$$M_0 = X$$

K-map for N_0:

Q_1 \ Q_0X	00	01	11	10
0	X	X	1	0
1	X	X	0	1

$$N_0 = \bar{Q}_1X + Q_1\bar{X}$$

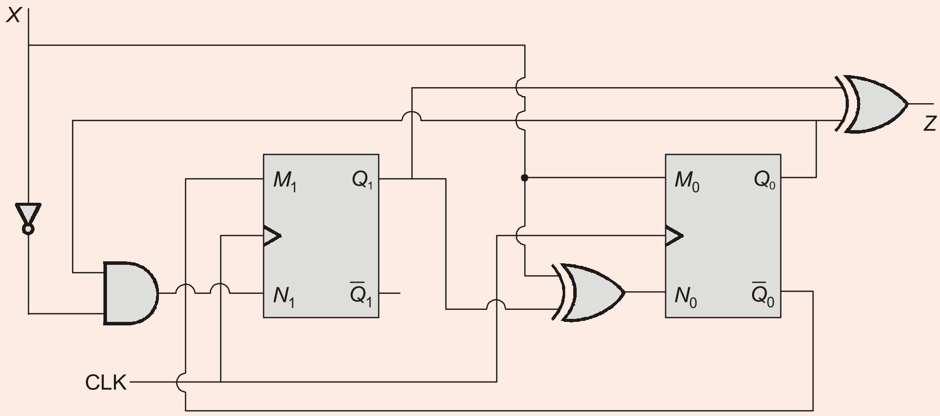
$$N_0 = Q_1 \oplus X$$

K-map for Z:

$Q_1 \backslash Q_0X$	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$$Z = \bar{Q}_1Q_0 + Q_1\bar{Q}_0$$

$$Z = Q_1 \oplus Q_0$$



End of Solution

Q.7 (a) In a low-voltage Schering bridge designed for the measurement of permittivity, the branch AB consists of two electrodes between which the specimen under test may be inserted; arm BC is a non-reactive resistor R_3 in parallel with a standard capacitor C_3; arm CD is a non-reactive resistor R_4 in parallel with a standard capacitor C_4; arm DA is a standard air capacitor of capacitance C_2. Without the specimen between the electrodes, balance is obtained with the following values:

$C_3 = C_4 = 120$ pF, $C_2 = 150$ pF, $R_3 = R_4 = 5$ kΩ.

With the specimen inserted these values become :

$C_3 = 200$ pF, $C_4 = 1000$ pF, $C_2 = 900$ pF and $R_3 = R_4 = 5$ kΩ.

Find the relative permittivity of the specimen if $\omega = 5$ k rad/s.

[20 marks : 2022]

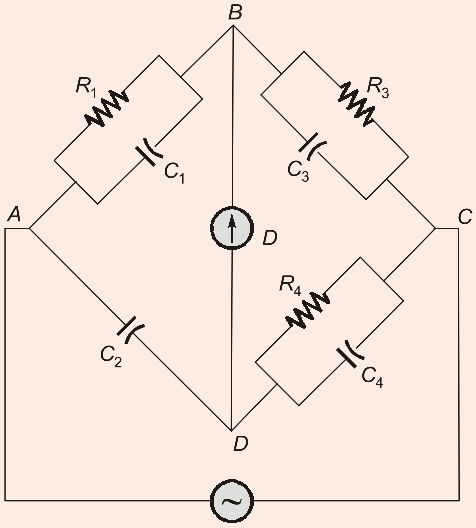
Solution:

Schering Bridge:

Given that Arm $BC = R_3 \parallel C_3$

Arm CD $R_4 \parallel C_4$

Arm $DA \Rightarrow C_2$



Without specimen inserted:

$$C_3 = C_4 = 120 \text{ pf}, \; C_2 = 150 \text{ pf},$$
$$R_3 = R_4 = 5 \text{ k}\Omega = 5000 \;\Omega$$

With specimen inserted:

$$C_3 = 200 \text{ pf}, \; C_4 = 1000 \text{ pf}, \; C_2 = 900 \text{ pf}$$
$$R_3 = R_4 = 5000 \;\Omega, = 5 \text{ k}\Omega,$$
$$W = 5000 \text{ rad/sec}, \; W = 5 \text{ K rad/sec}$$

For balance condition $\Rightarrow$ $Y_1 \cdot Y_4 = Y_2 \cdot Y_3$

$$\left(\frac{1}{R_1} + j\omega C_1\right)\left(\frac{1}{R_4} + j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$\left(\frac{1}{R_1 R_4} - \omega^2 C_1 C_4\right) + j\omega\left[\frac{C_1}{R_4} + \frac{C_4}{R_1}\right] = j\omega\frac{C_2}{R_3} - \omega^2 C_2 C_3$$

Real Part:

$$\frac{1}{R_1 R_4} - \omega^2 C_1 C_4 = -\omega^2 C_2 C_3 \qquad ...(1)$$

Img. part:

$$\frac{C_1}{R_4} + \frac{C_4}{R_1} = \frac{C_2}{R_3} \qquad ...(2)$$

by solving equation (1) and (2),

$$C_1 = \frac{C_2 R_4}{R_3} + \frac{\omega^2 C_2 C_3 C_4 R_4^2}{1 + \omega^2 R_4^2 C_4^2}$$

$$C_1 = \frac{C_2 \cdot R_4}{R_3} = 150\text{ pf} \times \frac{5000}{5000}$$

$$C_1 = 150\text{ pf} \quad \Rightarrow \quad \text{Without specimen inserted.}$$

$$C_1' = C_2 \cdot \frac{R_4}{R_3} = 900 \times \frac{5000}{5000} = 900\text{ pf}$$

$\Rightarrow$ $$C_1' = 900\text{ pf} \quad \Rightarrow \quad \text{With specimen inserted}$$

$$C_1 = \frac{A\,\epsilon_0}{t} \quad \text{and} \quad C_1' = \frac{A\,\epsilon_0 \epsilon_r}{t}$$

Let $t \Rightarrow$ Theveness of specimen.

Let $\epsilon_r \Rightarrow$ Relative permittivity of specimen.

$$\epsilon_r = \frac{C_1'}{C_1} \quad \Rightarrow \quad \epsilon_r = \frac{900\text{ pf}}{150\text{ pf}} = 6 \Rightarrow \epsilon_r = 6$$

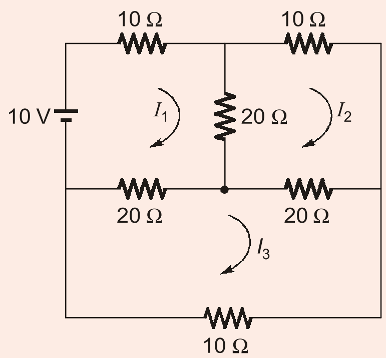
End of Solution

Q.7 (b) For the circuit diagram shown below, draw the graph and

(i) Obtain incidence matrix, tie-set and cut-set matrix.

(ii) How many trees are possible for this circuit.

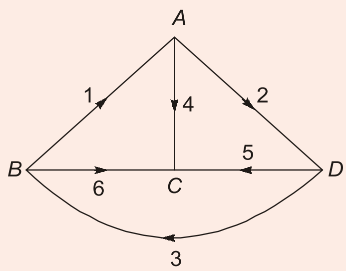
(iii) Obtain network equilibrium equations in matrix form using KVL and find the loop currents I_1, I_2 and I_3.



[20 marks : 2022]

Solution:

Step1: Develop Graph



Step 2:

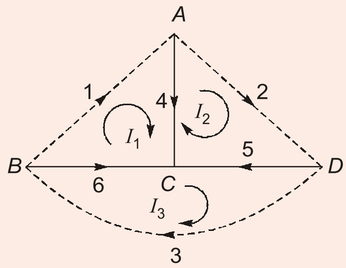
(i) Develop Incidence Matrix

$[A]_a =$

	1	2	3	4	5	6
A	–1	+1		+1		
B	+1		–1			+1
C				–1	–1	–1
D		–1	+1		+1	

Step 3:

Develop tree for Tie-set matrix

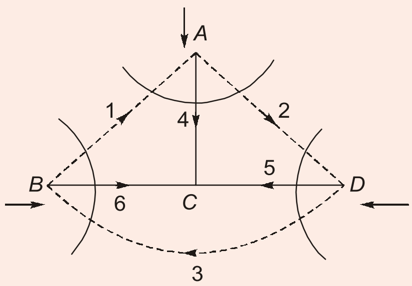


$[B] =$

	1	2	3	4	5	6
I_1	+1			+1		–1
I_2		+1		–1	+1	
I_3			+1		–1	+1

Step 4:

Develop tree for cut-set matrix



$[C] =$

	1	2	3	4	5	6
A	–1	+1		+1		
B	+1		–1			+1
D		–1	+1		+1	

(ii) Total number of possible trees = $N^{N-2} = 4^{4-2} = 16$

From equilibrium equations

$$[Z_L][I_L] = [B][V_s] - [B][Z_b][I_s]$$

where, $[Z_b]$ = Branch impedance matrix

$$[Z_b] = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix}$$

$[B]$ = Tie-set matrix

$$[B] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

$$[B^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$[Z_L]$ = Loop impedance matrix

$$[Z_L] = [B][Z_b][B^T]$$

$$[Z_L] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[Z_L] = \begin{bmatrix} 50 & -20 & -20 \\ -20 & 50 & -20 \\ -20 & -20 & 50 \end{bmatrix}$$

$[I_L]$ = Loop current matrix

$$[I_L] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$[I_s]$ = Source current matrix

$$[I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$[V_s]$ = Source voltage matrix

$$[V_s] = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(iii) $$[Z_L][I_L] = [B][V_s] - [B][Z_b][I_s]$$

By substituting all respective matrices in above equation.

$$\begin{bmatrix} 50 & -20 & -20 \\ -20 & 50 & -20 \\ -20 & -20 & 50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

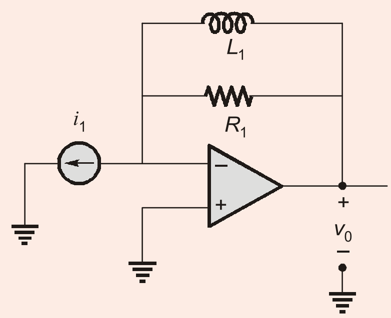
$$I_1 = \frac{3}{7}\text{ A}$$

$$I_2 = \frac{2}{7}\text{ A}$$

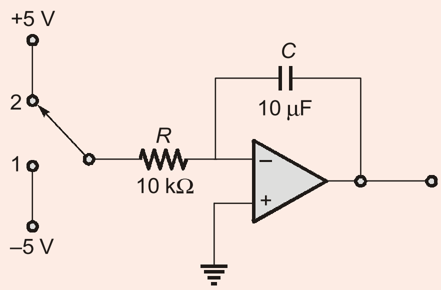
$$I_3 = \frac{2}{7}\text{ A}$$

End of Solution

Q.7 (c) (i) The circuit below represents a filter between the input current i_1 and output voltage v_o. Assume the op-amp is ideal. Identify the type of filter (low pass/ band pass/high pass/band stop). Provide justification for your answer.



(ii) In the wave-shaping circuit shown, the switch is initially in position 1. The switch is thrown into position 2, and held there fore 10 ms, then back to position 1 for 10 ms, and so on. Sketch the resulting output waveform if its initial value is 0 V. The saturated output levels of the op-amp are ±12 V.

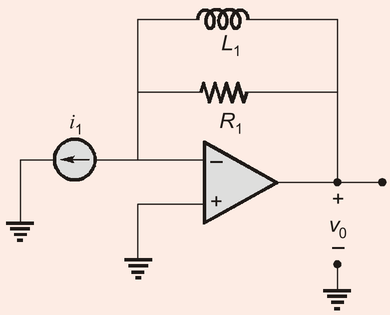


(iii) Sketch a five-stage ladder network (DAC) using 15 kΩ and 30 kΩ resistors. For a reference voltage of 16 V, calculate the output voltage for an input of 11010 to the circuit.

[5 + 10 + 5 marks : 2022]

Solution:

(i) Drawing the given circuit in frequency domain,



Using the concept of virtual ground

$$V_- = V_+ = 0\text{ V}$$

Applying KCL at non-inverting terminal, we get

$$i_1 + \frac{0 - V_0}{R_1} + \frac{0 - V_0}{j\omega L_1} = 0$$

$$\Rightarrow \quad V_0\left[\frac{1}{R_1} + \frac{1}{j\omega L_1}\right] = i_1$$

$$\Rightarrow \quad V_0 = i_1\left(\frac{j\omega L_1 R_1}{R_1 + j\omega L_1}\right)$$

$$\Rightarrow \quad V_0 = i_1\left(\frac{R_1}{1 + \frac{R_1}{j\omega L_1}}\right)$$

$$\Rightarrow \quad V_0 = i_1\left(\frac{R_1}{1 + \frac{\omega_C}{j\omega}}\right), \text{ where } \omega_c = \frac{R_1}{L_1}$$

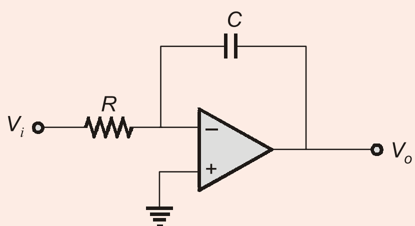
At $\omega \to 0$, $V_0 \to 0$

For $\omega \to \infty$, $V_0 = i_1 R_1$

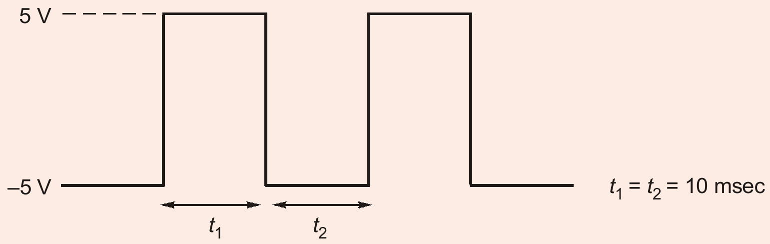
Hence, the given circuit represents a high-pass filter.

(ii) $R = 10\text{ k}\Omega$

$C = 10\ \mu\text{F}$



V_i is a square wave signal.

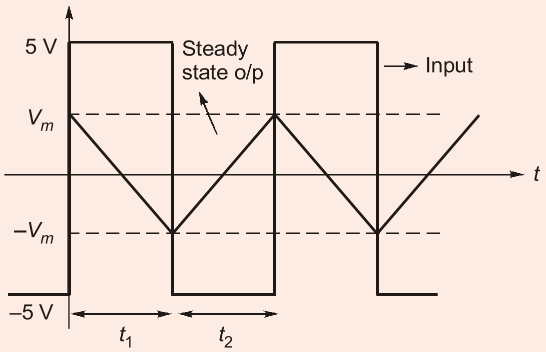


KCL at inverting node,

$$\frac{V_i - 0}{R} + C \times \frac{dV_o}{dt} = 0$$

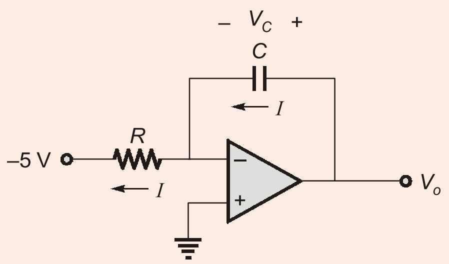
$$\frac{dV_o}{dt} = -\frac{V_i}{RC} \Rightarrow V_o = -\frac{1}{RC}\int_t V_i dt$$

∴ Given circuit is integrator. It converts square wave input into triangular wave output.



Here capacitor charges and discharges through constant current because current through R remains constant.

Consider t_2 interval.



$$V_i = -5 \text{ V}$$

$$I = \frac{0 - V_i}{R} = \frac{0+5}{10K}$$

$$I = 0.5 \text{ mA}$$

As current through capacitor is constant, we can use

$$I \times \Delta t = C \times \Delta V_C$$

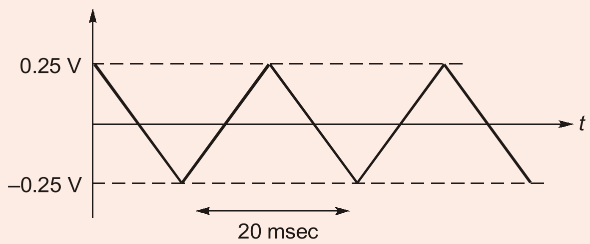
Here, $I = 0.5$ mA

$$\Delta t = t_2 = 10 \text{ msec}$$
$$C = 10\,\mu F$$
$$\Delta V_C = V_m - (-V_m) = 2V_m$$

ΔV_C is change in capacitor voltage. V_C or V_o changes from $-V_m$ to $+V_m$ in t_2 interval.

$$\Delta V_C = \frac{I \times \Delta t}{C}$$
$$2V_m = \frac{0.5 \times 10^{-3} \times 10 \times 10^{-3}}{10 \times 10^{-6}}$$
$$V_m = 0.25 \text{ V}$$

Hence, steady state V_o is as shown:



End of Solution

Q.8 (a) (i) List 6 parameters which can be sensed using optical fibres. How can this sensor be advantageous compared to electric transducers?

(ii) A CR tube has an anode-screen distance of 30 cm. The accelerating potential is 1 KV. The tube is placed with its axis vertical. Find the maximum deflection of the spot due to earth's magnetic field having $B = 0.018 \times 10^{-3}$ Wb/m^2.

[10 + 10 marks : 2022]

Solution:

(i) The parameters can be sensed using optical fibers.

(1) temperature
(2) relative humidity
(3) refractive index
(4) strain
(5) bending etc.

	Optical Sensor	Electrical Transducer
Response time	Fast, seconds	Slow, minutes
Accuracy and resolution	High	Low
Contactless measurement	Available	Unavailable
Sample consumption	No	Yes
Stirring sensitivity	No	Yes
Lifetime	Upto several years	Upto several months

(ii) Given that CR tube

$L = 30$ cm, $V_a = 1$ kV, $B = 0.018 \times 10^{-3}$ Wb/m^2

Find deflection $= D$

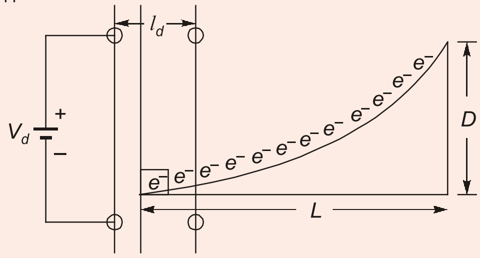
Let l_d = length of deflecting plate

Let V_d = Applied potential difference

Let D = Deflection

Let V_a = Accelerating potential

Let B = Magnetic flux density



We know that $$D = L \cdot l_d \cdot B\sqrt{\frac{q}{2mV_a}}$$

$$\frac{D}{l_d} = 30\times10^{-2}\times0.018\times10^{-3}\times\sqrt{\frac{1.6\times10^{-19}}{2\times9.11\times10^{-31}\times1\times10^{3}}}$$

Let q = charge of electron = 1.6×10^{-19} Coulomb

Let m = mass of electron = 9.11×10^{-31} kg

$$\frac{D}{l_d} = 0.54\times10^{-5}\times0.29\times\sqrt{10^{-19}\times10^{28}}$$

$$= 0.1566\times10^{-5}\times\sqrt{10^9} = 0.1566\times10^{-5}\times10^4\times\sqrt{10}$$

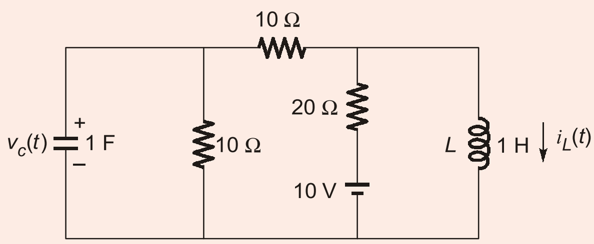
$$\frac{D}{l_d} = 0.4952\times10^{-1} \Rightarrow \frac{D}{l_d} = 0.04952$$

For $l_d = 1$ cm $\Rightarrow$ Max. deflection $= D_{max} = 0.04952$ cm

End of Solution

Q.8 (b) (i) The zeros of the admittance function of a parallel *RLC* circuit are located at $-6 + j8$ and $-6 - j8$. If the value of the resistance R is 25 Ω, find the quality factor of the circuit.

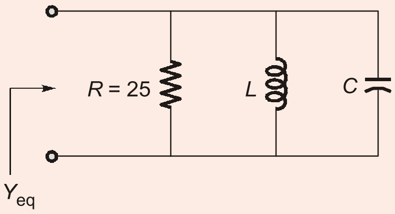
(ii) Obtain the state equations for the circuit shown below by considering $v_c(t)$ and $i_L(t)$ as state variables.



[12 + 8 marks : 2022]

Solution:

(i)



$$Y_{eq} = \frac{1}{R} + \frac{1}{sL} + sC = \frac{s^2RLC + R + sL}{sRL}$$

$$Y_{eq} = \frac{C\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right)}{sRL}$$

Comparing zeroes of admittance function

$$(s+6)^2 + 8^2 = s^2 + \frac{s}{RC} + \frac{1}{LC}$$

$$s^2 + 12s + 100 = s^2 + \frac{s}{25C} + \frac{1}{LC}$$

$$\frac{1}{25C} = 12$$

$$C = \frac{1}{25 \times 12} = \frac{1000}{100 \times 3} = \frac{10}{3} \text{ mF}$$

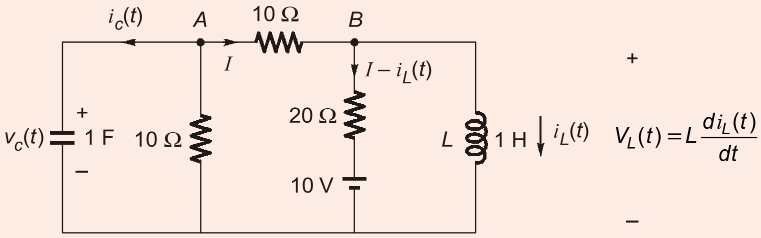
$$\frac{1}{LC} = 100$$

$$L = 3H$$

$$Q \text{ [Quality factor]} = R\sqrt{\frac{C}{L}}$$

$$Q = 0.833$$

(ii)



KCL at node B, $V_A = v_c(t)\,;\quad V_B = V_L(t)$

$$\frac{V_L(t) - V_L(t)}{10} = \frac{V_L(t) - 10}{20} + i_L(t)$$

$$\frac{V_L(t)}{10} + \frac{1}{10}\frac{di_L(t)}{dt} = \frac{1}{20}\frac{di_L(t)}{dt} - \frac{1}{2} + i_L(t)$$

$$\frac{1}{20}\hat{i}_L(t) = i_L(t) - \frac{v_c(t)}{10} - \frac{1}{2}$$

$$\hat{i}_L(t) = 20i_L(t) - 2V_c(t) - \frac{1}{2}$$

KCL at node A,

$$i_C(t) + \frac{v_c(t)}{10} + \frac{v_c(t) - v_L(t)}{10} = 0$$

$$C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{10} + \frac{v_c(t)}{10} - \frac{L}{10}\frac{di(t)}{dt} = 0$$

$$\frac{dv_c(t)}{dt} = \frac{\hat{i}_L(t)}{10} - \frac{v_c(t)}{5}$$

$$\hat{v}_c(t) = \frac{20i_L(t) - 2v_c(t) - 1/2}{10} - \frac{v_c(t)}{5}$$

$$\hat{v}_c(t) = 2i_L(t) - \frac{2}{5}v_c(t) - \frac{1}{20}$$

$\therefore$ State equations are

$$\hat{i}_L(t) = 20i_L(t) - 2v_c(t) - \frac{1}{2}$$

$$\hat{v}_c(t) = 2i_L(t) - \frac{2}{5}v_c(t) - \frac{1}{20}$$

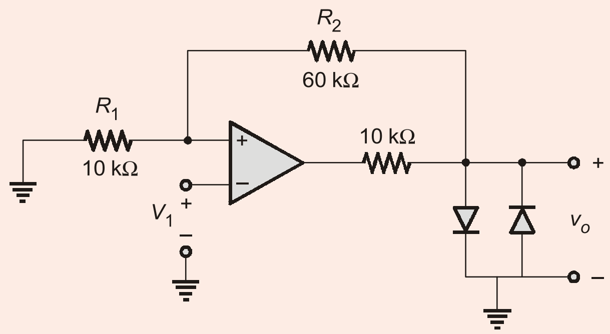
$$\begin{bmatrix} \hat{i}_L(t) \\ \hat{v}_c(t) \end{bmatrix} = \begin{bmatrix} 20 & -2 \\ 2 & -2/5 \end{bmatrix}\begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} -1/2 \\ -1/20 \end{bmatrix} u$$

End of Solution

Q.8 (c) (i) Realize the following Boolean functions using 4 : 1 multiplexer :

$$Y(A, B, C, D) = \bar{A}\bar{D} + B\bar{D} + \bar{B}\bar{C}D$$

(ii) 1. For the circuit shown, sketch and label the transfer characteristic $v_o - v_i$. The diodes are assumed to have a constant 0.7 V drop when conducting, and the op-amp saturates at ±12 V. What is the maximum diode current?



2. In the above circuit, if R_1 is open circuit and R_2 is short-circuit, redraw the characteristics.

[8 + 7 + 5 marks : 2022]

Solution:

(i)

$$Y(A, B, C, D) = \bar{A}\bar{D} + B\bar{D} + \bar{B}\bar{C}D$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$	
$\bar{A}\bar{B}$	1 (0)	1 (1)	(3)	1 (2)	→ I_0
$\bar{A}B$	1 (4)	(5)	(7)	1 (6)	→ I_1
AB	1 (12)	(13)	(15)	1 (14)	→ I_3
$A\bar{B}$	(8)	1 (9)	(11)	(10)	→ I_2

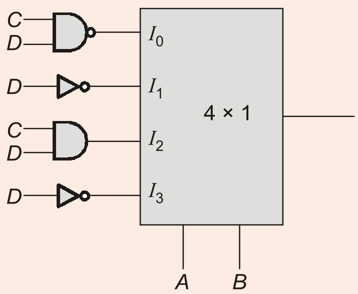
$$Y(A, B, C, D) = \Sigma m(0, 1, 2, 4, 6, 9, 12, 14)$$

$$I_0 = \bar{C} + \bar{D} = \overline{CD}$$

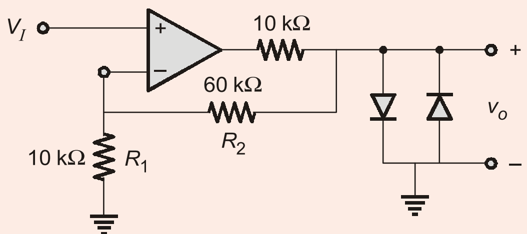
$$I_1 = \bar{C}\bar{D} + C\bar{D} = \bar{D}[\bar{C} + C] = \bar{D}$$

$$I_2 = \bar{C}D$$

$$I_3 = \bar{C}\bar{D} + C\bar{D} = \bar{D}[\bar{C} + C] = \bar{D}$$



(ii) 1. The circuit can be redrawn as shown below :



$$V_+ = V_o\left(\frac{R_1}{R_2 + R_2}\right)$$

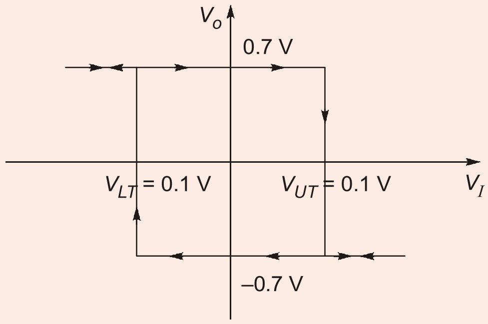
When $V_o' = +V_{sat}$, $\quad V_o = +0.7\text{ V}$

and $\quad V_{UT} = 0.7\left(\frac{10}{10+60}\right) = 0.1\text{ V}$

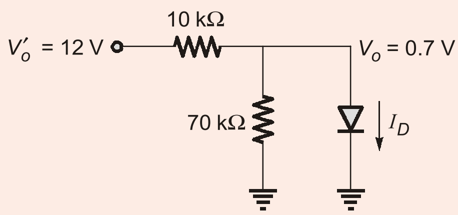
When $V_o' = -V_{sat}$, $\quad V_o = -0.7\text{ V}$

and $\quad V_{LT} = -0.7\left(\frac{10}{10+60}\right) = -0.1\text{ V}$

The transfer characteritics of the circuit can be drawn as below:



For maximum diode current, consider $V_o' = +12$ V



Applying KCL, we get,

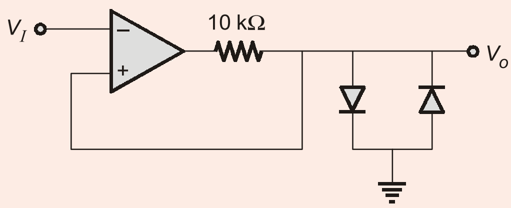
$$\frac{V_o' - V_o}{10\,\text{k}\Omega} = I_D + \frac{V_o}{70\,\text{k}\Omega}$$

$$\Rightarrow \quad \frac{12 - 0.7}{10} = I_D(\text{mA}) + \frac{0.7}{70}$$

$$\Rightarrow \quad 1.13 = I_D(\text{mA}) + 0.01$$

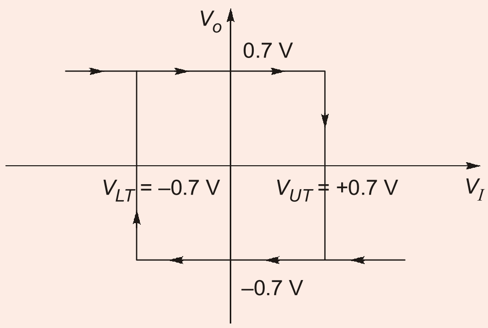
$$\Rightarrow \quad I_D = 1.12 \text{ mA}$$

2. If R_1 is open-circuited and R_2 is short-circuited, the circuit can be redrawn as below:



In the above circuit, V_{UT} = 0.7 V and V_{LT} = –0.7 V

The transfer characteristics can therefore be drawn as below:



End of Solution

■■■■