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Detailed Solutions

ESE-2022 Mains Test Series

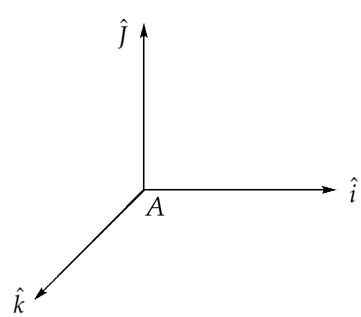
Mechanical Engineering Test No : 13

Full Syllabus Test (Paper-II)

Section : A

1. (a)

We will consider equilibrium of forces acting at 'A'.



$$\vec{P} = P\hat{i}$$

$$\vec{W} = (200 \times 9.81)(-\hat{j}) = -1962\hat{j}$$

$$\vec{T}_{AB} = \frac{T_{AB}\left(-1.2\hat{i} + 10\hat{j} + 8\hat{k}\right)}{12.86} \left[\because T_{AB} = \sqrt{(-1.2)^2 + 10^2 + 8^2} = 12.86\right]$$

Similarly,

$$\vec{T}_{AC} = T_{AC}\frac{\left(-1.2\hat{i} + 10\hat{j} - 10\hat{k}\right)}{14.19}$$

Now, equating forces in three perpendicular direction,

x-direction

$$P - 1.2\left(\frac{T_{AB}}{12.86} + \frac{T_{AC}}{14.19}\right) = 0 \qquad \text{... (i)}$$

y-direction

$$-1962 + 10\left(\frac{T_{AB}}{12.86} + \frac{T_{AC}}{14.19}\right) = 0 \qquad \text{... (ii)}$$

z-direction,

$$\frac{8T_{AB}}{12.86} - \frac{10T_{AC}}{14.19} = 0 \qquad \text{... (iii)}$$

From equation (i), (ii) and (iii)

$$P = 235.44 \text{ N}$$

$$T_{AB} = 1401.74 \text{ N}$$

$$T_{AC} = 1237.37 \text{ N}$$

1. (b)

Given : Maximum concentrated load, $W = 50$ kN

Permissible bending stress, $\sigma_{b\,max} = 120$ N/mm^2

(i) Let l_{max} be maximum possible length of beam.

$$\therefore \qquad M_{max} = \frac{Wl_{max}}{4} = \frac{50 l_{max}}{4}$$

$$= 12.5\, l_{max} \text{ kN.m} = 12.5 l_{max} \times 10^6 \text{ Nmm}$$

We know,

$$\frac{M_{max}}{I} = \frac{\sigma_{b\,max}}{y_{max}}$$

Where,

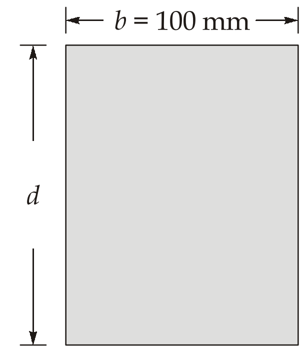
$$I = \frac{20 \times 260^3}{12} + 150 \times 20(130+10)^2 + 150 \times 20 \times (140)^2$$

$$= 146.893 \times 10^6 \text{ mm}^4$$

$$\Rightarrow \qquad \frac{12.5 l_{max} \times 10^6}{146.893 \times 10^6} = \frac{120}{150}$$

$$l_{max} = 9.4 \text{ mm}$$

(ii) Let 'd' be the depth of equivalent rectangular section,



$$\therefore \quad \frac{M}{I} = \frac{\sigma_{b\max}}{y}$$

$$\frac{12.5 \times 9.4 \times 10^6 \times 12}{100 \times d^3} = \frac{120 \times 2}{d}$$

$$\therefore \quad d = 242.4 \text{ mm}$$

We know, Weight of beam, $W = \rho_g \times$ Area of cross-section (A) × Length

$$\Rightarrow \quad W \propto A$$

Let weight of equivalent rectangular beam be W_R. Then,

$$W_R \propto A_R$$

Where, $A_R = bd = 242.384 \times 100 = 24238.4 \text{ mm}^2$

and, weight of I section beam, $W_I \propto A_I$

where, $A_I = 150 \times 20 \times 2 + 260 \times 20 = 11200 \text{ mm}^2$

Hence, Percentage increase $= \dfrac{24238.4 - 11200}{11200} \times 100 = 116.41\%$

1. (c)

Given: Mass of ship, m = 10 tonnes = 10000 kg, Radius of gyration, k = 0.8 m,

$$N = 2000 \text{ rpm or } \omega = \frac{2\pi N}{60} = 209.44 \text{ rad/s}$$

$$V = 200 \text{ kmph} = 55.55 \text{ m/s}; \; R = 80 \text{ m}$$

Mass moment of inertia of rotor, $I = mk^2$

$$I = 10000 \times (0.8)^2 = 6400 \text{ kg-m}^2$$

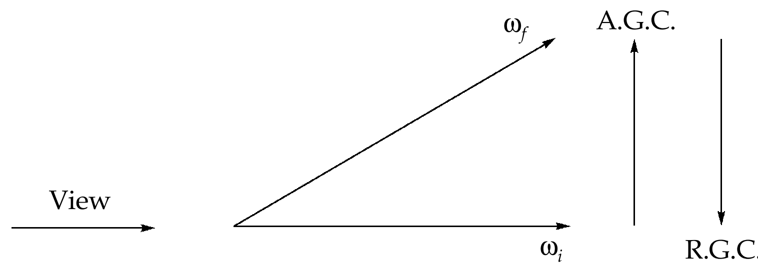
Angular velocity of precession,

$$\omega_p = \frac{V}{R} = \frac{55.55}{80} = 0.694 \text{ rad/s}$$

Gyroscopic couple is given as, $C = I\omega\omega_p$

$$C = 6400 \times 209.44 \times 0.694$$

$$= 930.248 \text{ kN}$$



Effect : Rotor of ship is rotating in clockwise direction when looking from stern and ship turns left, then bow will go up and stern will go down.

1. (d)

Given: N = 100 rpm, P = 350 kW

$$P = T\omega$$

$$\Rightarrow \quad 350 \times 10^3 = T \times 2 \times \pi \times \frac{100}{60}$$

$$T = 33.41 \times 10^3 \text{ Nm}$$

Now,
$$\tau = \frac{16T}{\pi D_s^3}$$

$$90 = \frac{16 \times 33.41 \times 10^3 \times 10^3}{\pi D_s^3}$$

$$\Rightarrow \quad D_s = 123.63 \text{ mm}$$

For hollow shaft, internal diameter,

$$D_i = 0.65D$$

$$\therefore \quad J = \frac{\pi\left(D^4 - (0.65D)^4\right)}{32} = \frac{0.8215\pi D^4}{32}$$

Now,
$$\frac{\tau}{r} = \frac{T}{J}$$

$$\frac{90}{D/2} = \frac{33.41 \times 10^6}{\frac{0.8215\pi D^4}{32}}$$

$$\frac{90}{D} = \frac{33.41 \times 10^6}{\frac{0.8215\pi D^4}{16}}$$

$$D^3 = \frac{33.41 \times 10^6 \times 16 \times 7}{90 \times 0.8215 \times 22}$$

$$D = 132.01 \text{ mm}$$

$$D_i = 0.65D = 85.81 \text{ mm}$$

$$\text{Area of hollow shaft, } A_2 = \frac{\pi}{4}\left(D^2 - D_i^2\right) = \frac{\pi}{4} \times 10063.284$$

$$\text{Area of solid shaft, } A_1 = \frac{\pi}{4} D_s^2 = \frac{\pi}{4} \times 15284.37$$

$$\text{Percentage saving in weight} = \frac{|A_2 - A_1|}{A_1} \times 100 = 34.16\%$$

1. (e)

Let E_A, K_A, G_A and E_B, K_B, G_B be the modulus of elasticity, bulk modulus and modulus of rigidity of materials A and B respectively.

Given: $K_A = K_B$ and $E_B = 1.01E_A$

We know,
$$E = \frac{9KG}{3K + G}$$

$$3KE + EG = 9KG$$

$$3K(3G - E) = EG$$

$$\Rightarrow \quad K = \frac{EG}{3(3G - E)}$$

Hence,
$$\frac{E_A G_A}{3(3G_A - E_A)} = \frac{E_B G_B}{3(3G_B - E_B)}$$

$$E_A G_A (3G_B - E_B) = E_B G_B (3G_A - E_A)$$

$$3E_A G_A G_B - E_A G_A E_B = 3G_A G_B E_B - E_A E_B G_B$$

$$G_B(3E_A G_A - 3G_A E_B + E_A E_B) = E_A G_A E_B$$

$$\therefore \quad G_B = \frac{E_A G_A E_B}{3E_A G_A - 3G_A E_B + E_A E_B}$$

$$G_B = \frac{1.01 E_A G_A E_A}{3E_A G_A - 3 \times 1.01 E_A G_A + 1.01 E_A E_A}$$

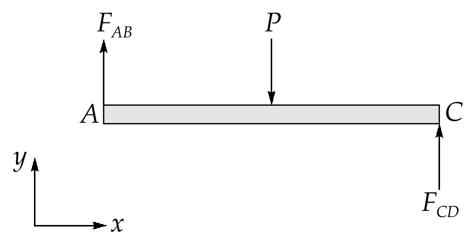
$$= \frac{1.01 E_A G_A}{3G_A - 3.03 G_A + 1.01 E_A}$$

$$= \frac{101E_A G_A}{101E_A - 3G_A}$$

2. (a)

Given: Load, P = 60 kN, Length of strut, L_{AB} = 2 m, Diameter of strut, d_{AB} = 50 mm, Length of column, L_{CD} = 0.5 m, Diameter of column, d_{CD} = 80 mm.

Consider FBD of beam AC.



F_{AB} : Force in strut

F_{CD} : Force in column

- $\Sigma F_Y = 0 \Rightarrow \quad F_{AB} + F_{CD} - P = 0$

$$F_{AB} + F_{CD} = 60 \quad \text{... (i)}$$

- $\Sigma F_A = 0 \Rightarrow P \times 0.75 - F_{CD} \times 2 \times 0.75 = 0$

$$F_{CD} = \frac{P}{2} = 30 \text{ kN}$$

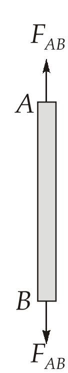
Thus, from (i), $\quad F_{AB} = 30$ kN

Now from stress-strain diagram,

Modulus of elasticity = Slope in elastic region

$$= \frac{32.2}{0.01} = 32.2 \times 10^2 \text{ MPa}$$

Hence, Elongation in strut AB,



$$\delta_{AB} = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{30\times10^3\times2000}{\frac{\pi}{4}(50)^2\times32.2\times10^2} = 9.5\text{ mm}$$

and, elongation in column CD,

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{30\times10^3\times0.5\times10^3}{\frac{\pi}{4}(80)^2\times32.2\times10^2} = 0.927\text{ mm}$$

Thus, angle of tilt of beam, $\alpha = \tan^{-1}\left(\frac{\delta_{AB}-\delta_{CD}}{L_{AC}}\right)$

$$\alpha = \tan^{-1}\left(\frac{9.5-0.927}{1500}\right) = 0.327^\circ$$

2. (b)

(i)

Given : Total mass of machine, m = 100 kg,

Unbalanced mass, m_o = 2 kg, Stroke length = $2r$ = 6 cm $\Rightarrow r$ = 0.03 m

$$\text{Crank shaft rotation, } \omega = \frac{2\pi N}{60} = \frac{2\times\pi\times1500}{60} = 50\pi\text{ rad/s}$$

Maximum unbalanced periodic force, $F_o = m_o r\omega^2$

$$F_o = 2\times0.03\times(50\pi)^2 = 1480.44\text{ N}$$

Equivalent stiffness, $6k = k_{eq}$

Transmissibility, $\varepsilon = \frac{1}{10}$, when $\xi = 0$

$$\varepsilon = \pm\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

Taking positive sign first,

$$\frac{1}{10} = +\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$10 = 1-\left(\frac{\omega}{\omega_n}\right)^2$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = -9 \qquad \text{(Not possible)}$$

Now taking negative sign,

$$\frac{1}{10} = -\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$-10 = 1-\left(\frac{\omega}{\omega_n}\right)^2$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 11$$

$$\frac{\omega}{\omega_n} = \sqrt{11} = 3.316$$

$\therefore$ $$\omega_n = \frac{\omega}{3.316} = \frac{50\pi}{3.316} = 47.37 \text{ rad/s}$$

Now, $$\omega_n = \sqrt{\frac{k_{eq}}{m}} = 47.37$$

$\therefore$ $$k_{eq} = (47.37)^2 \times 100 = 224391.69 \text{ N/m}$$

$\therefore$ $$k = \frac{k_{eq}}{6} = \frac{224391.69}{6} = 37398.61 \text{ N/m}$$

Now, after damping, $$x_1 = (1 - 0.1)\, x_o$$

$\therefore$ $$\frac{x_o}{x_1} = \frac{x_2}{x_1} = \frac{1}{0.9} = 1.111 = e^{\delta}$$

$\Rightarrow$ $$\delta = \ln\left(\frac{1}{0.9}\right) = \ln(1.111)$$

$\Rightarrow$ $$\delta = 0.1053$$

Also, $$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$\Rightarrow$ $$0.1053 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$\Rightarrow$ $$\xi = 0.0167$$

Now transmissibility,
$$\varepsilon = \frac{\sqrt{1+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

$$\varepsilon = \frac{\sqrt{1+(2\times0.0167\times3.316)^2}}{\sqrt{\left[1-(3.316)^2\right]^2+(2\times0.0167\times3.316)^2}}$$

$$\varepsilon = 0.10065 \qquad \text{Ans.}$$

(ii) Amplitudes of vibration at resonance ($\omega = \omega_n$)

$$A = \frac{F_o/k_{eq}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} = \frac{F_o/k_{eq}}{2\xi}$$

where,
$$F_0 = m_0 r\omega_n^2$$
$$= 2 \times 0.03 \times (47.37)^2$$
$$= 134.63 \text{ N}$$

$$A = \frac{134.63}{224391.69\times2\times0.0167} = 0.01796 \text{ m} = 17.96 \text{ mm} \qquad \text{Ans.}$$

$$\varepsilon = \frac{\sqrt{1+(2\xi)^2}}{2\xi} = \frac{\sqrt{1+(2\times0.0167)^2}}{(2\times0.0167)} = 29.9568$$

∴ The force transmitted in foundation,
$$F_T = \varepsilon \times F_o$$
$$F_T = 29.9568 \times 134.63 = 4033.08 \text{ N} \qquad \text{Ans.}$$

2. (c)

Given: $n = 1200$ rpm; $\mu = 0.2$; $r_m = 2b$, $P_a = 0.1$ N/mm^2, $\alpha = 12.5°$

For Machine, $m = 180$ kg, $k = 300$ mm, $t = 30$ sec, $\omega_1 = 0$, $\omega_2 = \frac{2\pi n}{60} = 125.6$ rad/s

$$\alpha = \text{Angular acceleration} = \frac{\omega_2-\omega_1}{t} = \frac{125.6-0}{30}$$
$$= 4.186 \text{ rad/s}^2$$
$$M_t = I\alpha = mk^2\alpha = 180 \times (0.3)^2 \times 4.186$$

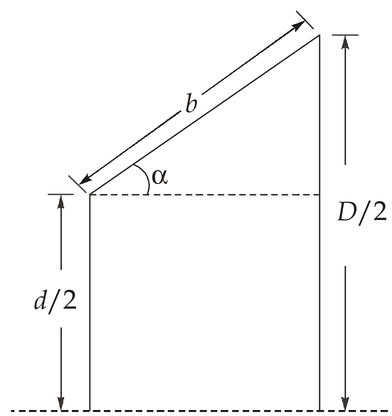
$$= 67813.2 \text{ N-mm}$$

Also, $$M_t = \frac{\pi \mu P_a d}{8 \sin \alpha}\left(D^2 - d^2\right)$$

$\therefore$ $$67813.2 = \frac{\pi (0.2)(0.1)(d)\left(D^2 - d^2\right)}{8 \sin 12.5°}$$

$\therefore$ $$d(D^2 - d^2) = 1868792.61 \quad \text{... (i)}$$

From figure, $$D - d = 2b\sin\alpha \quad \text{... (ii)}$$



Since the mean radius of the clutch is twice the face width,

$$\frac{D + d}{4} = 2b$$

or, $$D + d = 8b \quad \text{... (iii)}$$

Dividing equation (iii) by (ii)

$$\frac{D + d}{D - d} = \frac{8b}{2b \sin \alpha} = \frac{4}{\sin \alpha}$$

Therefore, $$\frac{D}{d} = \frac{4 + \sin \alpha}{4 - \sin \alpha} = \frac{4 + \sin 12.5}{4 - \sin 12.5}$$

$$\frac{D}{d} = 1.1143$$

$$D^2 = (1.1143)^2 d^2 = 1.242 d^2$$

Substituting this relation in equation (i)

$$d(1.242d^2 - d^2) = 1868792.61$$

$$d = 197.66 \text{ mm}$$

$$D = 1.1143d = 220.25 \text{ mm}$$

Face width of friction lining, $b = \frac{D+d}{8} = \frac{220.25+197.66}{8} = 52.23 \text{ mm}$

Force required to engage the clutch

$$F = \frac{4M_t \sin\alpha}{\mu(D+d)} = \frac{4\times 67813.2\times \sin 12.5}{0.2(220.25+197.66)}$$

$$= 702.42 \text{ N}$$

Heat generated during each engagement,

$$\omega_{avg} = \frac{\omega_1+\omega_2}{2} = \frac{125.6+0}{2} = 62.8 \text{ rad/s}$$

$$\theta = \omega_{avg} \times \text{time} = 62.8 \times 30 = 1884 \text{ radians}$$

Heat generated during = Work done by frictional torque;

$$H_g = M_t \times \theta = 67.813 \times 1884 = 127.76 \text{ kJ}$$

3. (a)

Divide the mass of rod into two parts.

Mass at crank pin, $M_a = \frac{300\times(2-0.7)}{2} = 195 \text{ kg}$

Mass at gudgeon pin, $m = 300 - 195 = 105$ kg

Inertia force, $F_I = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$= 105\times 0.5\times\left(\frac{2\pi\times 220}{60}\right)^2\left(\cos 20 + \frac{\cos 40}{2/0.5}\right)$$

$$F_I = 31521.2 \text{ N}$$

For vertical engine, $F_{net} = 31521.2 - 105 \times 9.81$

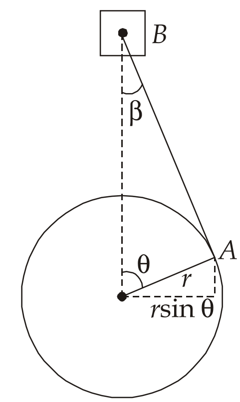
$$= 30491.15 \text{ N (Upwards)}$$

Torque, $T_b = F\times r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$

or $T_b = 30491.15\times 0.5\left[\sin 20° + \frac{\sin 40°}{2\sqrt{4^2-\sin^2 20°}}\right]$

$$T_b = 6443.75 \text{ Nm (ACW)}$$

$$\omega = \frac{2\pi \times 220}{60} = 23.038 \text{ rad/s}$$



We have, $L = b + \frac{k^2}{b}$

Where, $b = 2 - 0.7 = 1.3$ m

Now, $T = 2\pi\sqrt{\frac{L}{g}}$

or $\frac{25}{10} = 2\pi\sqrt{\frac{L}{9.81}}$

∴ $L = 1.553$ m

∴ $1.3 + \frac{k^2}{1.3} = 1.553$

⇒ $k = 0.5735$ m **Answer**

Angular acceleration, $\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{\left(n^2 - \sin^2\theta\right)^{3/2}}\right]$

$$\alpha_c = -(23.038)^2 \sin 20\left[\frac{4^2 - 1}{\left(4^2 - \sin^2 20\right)^{3/2}}\right]$$

$$\alpha_c = -43.016 \text{ rad/s}^2$$

Correction couple, $\Delta T = m\alpha_c b(l - L)$

$= 300 \times (-43.016) \times 1.3\,(2 - 1.553)$

$= -7498.98$ Nm

The direction of correction couple will be same as that of the angular acceleration, i.e. in the direction of decreasing angle β,

$$T_c = \Delta T \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -7498.98 \times \frac{\cos 20°}{\sqrt{4^2 - \sin^2 20°}}$$

$$= -1768.16 \text{ Nm (ACW)}$$

The correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards left on the upper side of the crankshaft, the correction torque is anticlockwise.

Now, Torque due to weight of mass at 'A',

$$T_a = M_a g r \sin\theta$$
$$= 195 \times 9.81 \times 0.5 \sin 20°$$
$$= 327.13 \text{ Nm (CW)}$$

∴ Total inertia torque on crankshaft,

$$T = T_b - T_c + T_a$$
$$= 6443.75 - (-1768.16) - 327.13$$
$$= 7884.78 \text{ Nm}$$

Answer

3. (b)

Given: $N = 300$ rpm or $\omega = \dfrac{2\pi N}{60} = 31.42 \text{ rad/s}$, $m = 600$ kg, $k = 0.5$ m

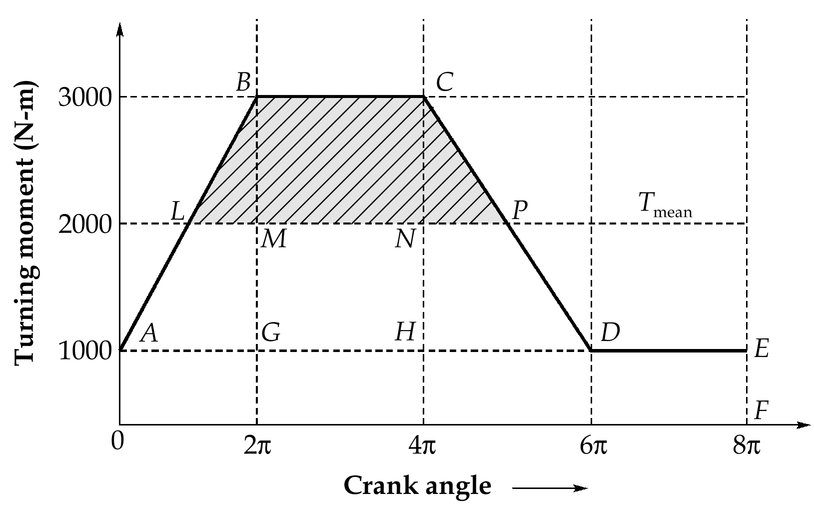
The turning moment diagram for the complete cycle is shown in figure.

We know that work done for one complete cycle

= Area of figure OABCDEF

= Area of OAEF + Area of ABG + Area of BCHG + Area of CDH

$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$



$$= 1000 \times 8\pi + \frac{1}{2} \times 2\pi \times 2000 + 2\pi \times 2000 + \frac{1}{2} \times 2\pi \times 2000$$

$$= 8000\pi + 2000\pi + 4000\pi + 2000\pi = 16000\pi J \quad \text{... (i)}$$

If T_{mean} is the mean torque in N-m, then work required for one complete cycle

$$= T_{mean} \times 8\pi \quad \text{... (ii)}$$

From equation (i) and (ii),

$$16000\pi = 8\pi \times T_{mean}$$

$$T_{mean} = 2000 \text{ N-m}$$

We know that power required to drive the machine,

$$P = T_{mean} \times \omega = 2000 \times 31.42 = 62.84 \text{ kW}$$

For calculation of coefficient of speed, we need to calculate the energy fluctuation (i.e. the area above the T_{mean} line)

From similar triangles ABG and BLM

$$\frac{LM}{AG} = \frac{BM}{BG}$$

$$\therefore \quad \frac{LM}{2\pi} = \frac{3000-2000}{3000-1000} = \frac{1}{2}$$

$$\therefore \quad LM = \pi$$

Similarly, $NP = \pi$

Also, $\text{BM} = \text{CN}$

$$3000 - 2000 = 1000 \text{ N-m}$$

Since, the area above the mean torque line represents the maximum fluctuation of energy, therefore the maximum fluctuation of energy,

$$\Delta E = \text{Area LBCP} = \text{Area LBM} + \text{Area MBCN} + \text{Area PNC}$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times \pi \times 1000 + 2\pi \times 1000 + \frac{1}{2} \times \pi \times 1000$$

$$= 500\pi + 2000\pi + 500\pi = 3000\pi J$$

$$= 9424.77 \text{ Joules}$$

Also, $\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$

$$C_s = \frac{\Delta E}{mk^2\omega^2} = \frac{9424.77}{600(0.5)^2 \times (31.42)^2}$$

$$= 0.0636$$ **Answer**

3. (c)

Given: The lower ball has dropped 5 m from *A*.

Let the time taken for the lower ball to reach 5 m from *A* be *t*.

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}10 \times t^2$$

$$t = 1 \text{ sec}$$

As balls fall at a steady rate of two per second, means every 0.5 sec the consecutive ball is dropped.

Therefore, in 1 sec the lower ball has reached 5 m from *A* and after 0.5 seconds, the consecutive ball is dropped. This consecutive ball has travelled for a distance of *x* metres in (1 – 0.5 = 0.5 seconds).

$$S = ut + \frac{1}{2}at^2$$

$$x = \frac{1}{2} \times 10 \times (0.5)^2 = 1.25 \text{ m}$$

Therefore, vertical separation (*h*) of two consecutive balls when lower one has dropped 5 m from *A* is:

$$h = (5 - 1.25) \text{ m} = 3.75 \text{ m}$$

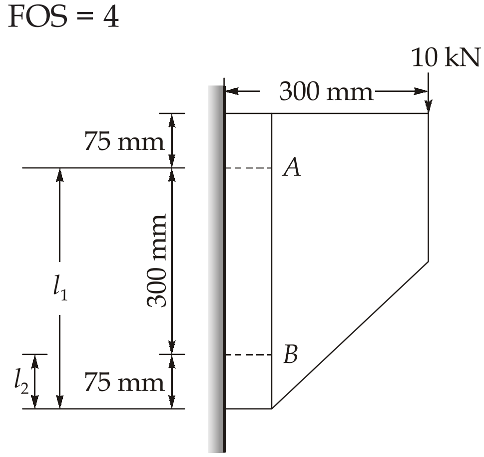
4. (a)

Given: Two bolts at point *A* and two bolts at point *B*.

Maximum load, $P = 10$ kN

Yield strength in tension, $\sigma_{yt} = S_{yt} = 380 \text{ N/mm}^2$

Factor of safety, FOS = 4



Assume two bolts at *A* denoted by 1 and two bolts at *B* denoted by 2.

$$P_{1'} = P_{2'} = \frac{P}{\text{Number of bolts}} = \frac{10000}{4}$$

$$P_{1'} = P_{2'} = 2500 \text{ N}$$

Direct shear stress, $$\tau_{direct} = \frac{P_{each}}{\text{Area}} = \frac{2500}{A} \text{N/mm}^2$$

Since, the tendency of the bracket is to tilt about edge 'C' the bolts at A denoted by '1' are at the farthest distance from 'C'. Therefore, bolts at A area subjected to maximum tensile force.

$$P_{1''} = \frac{P \times el_1}{2\left(l_1^2 + l_2^2\right)} = \frac{10000 \times 300 \times 375}{2\left(375^2 + 75^2\right)} = 3846.15 \text{ N}$$

Tensile stress in bolt at A is given by:

$$\sigma_t = \frac{3846.15}{A}$$

Principal stress in bolt, $$\sigma_{1,2} = \left(\frac{\sigma_t}{2}\right) \pm \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,2} = \left(\frac{3846.15}{2A}\right) \pm \sqrt{\left(\frac{3846.15}{2A}\right)^2 + \left(\frac{2500}{A}\right)^2}$$

$$\sigma_{1,2} = \left(\frac{1923.075}{A}\right) \pm \sqrt{\left(\frac{1923.075}{A}\right)^2 + \left(\frac{2500}{A}\right)^2}$$

$$\sigma_1 = \frac{5077.15}{A}, \qquad \sigma_2 = \frac{-1231.005}{A}$$

For safe design by maximum principal stress theory,

$$\sigma_{induced} \leq \sigma_{permissible}$$

$$\frac{5077.15}{A} \leq \frac{380}{4}$$

$$\therefore \quad A \geq 53.44 \text{ mm}^2$$

$$\therefore \quad \frac{\pi}{4} d_c^2 \geq 53.44 \text{ mm}^2$$

$$d_c \geq 8.248 \text{ mm}$$

$$d \geq \frac{8.248}{0.8} \geq 10.31 \text{ mm}$$

4. (b)

Given: σ_x = 150 MPa, σ_y = 250 MPa, $\tau_{xy} = \tau$ = 25 MPa.

$$\text{Maximum shear stress} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2}$$

Maximum/minimum principal stresses,

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \left(\frac{150 + 250}{2}\right) \pm \sqrt{\left(\frac{150 - 250}{2}\right)^2 + 25^2}$$

$$\sigma_{1,2} = 200 \pm \sqrt{50^2 + 25^2} = 200 \pm 55.90$$

$\Rightarrow$ σ_1 = 255.90 MPa

σ_2 = 144.10 MPa

Major axis strain, $\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$

$$\varepsilon_1 = \frac{255.90 - 0.3 \times 144.10}{2 \times 10^5} \quad [\mu = 0.3 \text{ for mild steel}]$$

$\varepsilon_1 = 1.06335 \times 10^{-3}$

$$\varepsilon_2 = \frac{144.1 - 0.3 \times 255.90}{2 \times 10^5} = 3.3665 \times 10^{-4}$$

Diameter of circle, D_o = 200 mm

Length of major axis, $L_1 = D_o (1 + \varepsilon_1)$

$L_1 = 200 (1 + 1.06335 \times 10^{-3}) = 200.21267$ mm

Length of minor axis, $L_2 = D_o (1 + \varepsilon_2)$

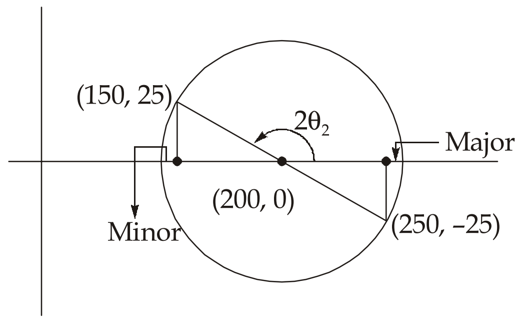
$L_2 = 200 (1 + 3.3665 \times 10^{-4}) = 200.06733$ mm

Direction:

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2 \times 25}{150 - 250}$$

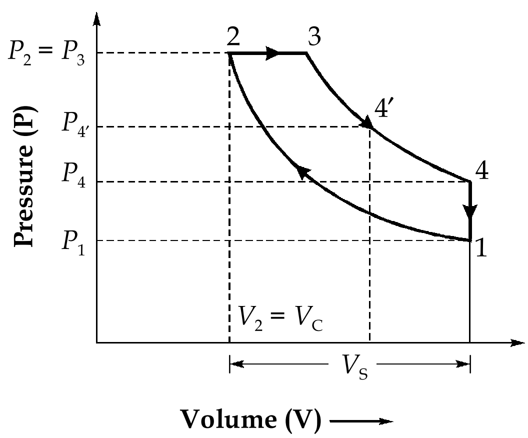
$\theta_1 = 13.280°$ (ACW) (Minor axis)

$\theta_2 = 90 - \theta_1 = 76.22°$ (CW) (Major axis)



4. (c)

Given: $d = 0.4$ m, $L = 0.6$ m, $r = \frac{L}{2} = 0.3 \text{ m}$, $l = 6 \times r = 6 \times 0.3 = 1.8$ m, $N = 240$ rpm,



$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

$$m_R = 280 \text{ kg}$$

$$\frac{V_1}{V_2} = 12$$

The P-V diagram for a diesel engine is shown above. The compression and expansion processes are isentropic i.e. PV^γ = Constant. The injection of fuel takes place at constant pressure and the exhaust is as constant volume.

$$PV^{1.35} = \text{Constant}$$

Therefore, $$P_1V_1^{1.35} = P_2V_2^{1.35}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.35} = 0.1(12)^{1.35} = 2.863 \text{ N/mm}^2$$

$$\text{Swept volume, } V_s = \frac{\pi}{4}D^2L = \frac{\pi}{4}\times(0.4)^2\times(0.6)$$

$$= 0.075 \text{ m}^3$$

and
$$\text{Compression ratio} = \frac{V_1}{V_2} = \frac{V_C+V_S}{V_C} = 1+\frac{V_S}{V_C}$$

$$12 = 1+\frac{V_S}{V_C}$$

$\therefore$
$$\frac{V_S}{V_C} = 11$$

$\therefore$
$$V_C = \frac{V_S}{11} = \frac{0.075}{11} = 0.00681 \text{ m}^3$$

Since, the injection of fuel takes place at constant pressure (i.e. $P_2 = P_3$) and continuous upto $\left(\frac{1}{10}\right)^{\text{th}}$ of the stroke, therefore volume at the end of the injection of the fuel.

$$V_3 = V_C + \frac{1}{10}V_S = 0.00681 + \frac{0.075}{10} = 0.01431 \text{ m}^3$$

When the crank displacement is 45° (θ = 45°) from the inner dead centre during expansion stroke, the corresponding displacement of the piston (marked by point 4′ on the P-V diagram) is given by,

$$x = r\left[(1-\cos\theta)+\left(n-\sqrt{n^2-\sin^2\theta}\right)\right]$$

$$= 0.3\left[(1-\cos 45°)+\left(6-\sqrt{6^2-\sin^2 45°}\right)\right]$$

$$= 0.1004 \text{ m}$$

$$V_4' = V_C + \frac{\pi}{4}\times D^2\times x = 0.00681 + \frac{\pi}{4}\times(0.4)^2\times 0.1004$$

$$= 0.0194 \text{ m}^3$$

Calculation of V_4',
$$V_4' = P_4'(V_{4'})^{1.35} = P_3(V_3)^{1.25}$$

$$P_4' = P_3\times\left(\frac{V_3}{V_{4'}}\right)^{1.35} = 2.863\times\left(\frac{0.01431}{0.0194}\right)^{1.35} = 1.898 \text{ N/mm}^2$$

∴ Difference of pressure on two sides of the piston,

$$P = P_{4'} - P_3 = 1.898 - 0.1 = 1.798 \text{ N/mm}^2$$

$$P = 1.798 \times 10^6 \text{ N/m}^2$$

∴ Net load on the piston, $F_L = P \times \frac{\pi}{4} \times D^2 = 1.798 \times 10^6 \times \frac{\pi}{4} \times (0.4)^2$

$$= 225.94 \text{ kN}$$

∴ Inertia force on the reciprocating parts,

$$F_I = m_R \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$= 280 \times (25.13)^2 \times 0.3 \left(\cos 45° + \frac{\cos(2 \times 45°)}{6} \right)$$

$$= 37.510 \text{ kN}$$

∴ Piston effort, $F_P = F_L - F_I + m_R g$

$$F_P = 225.94 - 37.510 + \frac{280 \times 9.81}{1000}$$

$$= 191.17 \text{ kN}$$

(i) Crank pin effort, $\phi = \sin^{-1}\left(\frac{\sin\theta}{n}\right) = 6.768°$

$$F_T = \frac{F_P \sin(\theta + \phi)}{\cos\phi} = \frac{191.17 \times 0.785}{0.993} = 151.13 \text{ kN}$$

(ii) Thrust on the bearings, $F_B = \frac{F_P \cos(\theta + \phi)}{\cos\phi} = \frac{191.17 \times 0.619}{0.993} = 119.13 \text{ kN}$

(iii) Turning moment on the crank shaft,

$$T = F_T \times r = 151.13 \times 0.3 = 45.339 \text{ kN-m}$$

Section : B

5. (a)

Monochromatic X-radiation wavelength,

$$\lambda = 0.1542 \text{ nm}$$

For each peak, in order to compute the interplanar spacing.

The first peak, which results from diffraction by the (110) set of planes occur at $2\theta = 45°$

Now, we have, $d_{nkl} = \dfrac{n\lambda}{2\sin\theta}$

$$d_{110} = \frac{(1)(0.1542)}{2\sin\left(\frac{45}{2}\right)} = 0.2015 \text{ nm}$$

$\therefore$ Lattice parameter, $a = d_{nkl}\sqrt{h^2 + k^2 + l^2}$

$$= 0.2015\sqrt{1^2 + 1^2 + 0^2}$$

$$= 0.2849 \approx 0.285 \text{ nm}$$

Similarly for other peaks, $d_{200} = \dfrac{n\lambda}{2\sin\theta} = \dfrac{(1)(0.1542)}{2\left(\sin\frac{65.1}{2}\right)} = 0.1433 \text{ nm}$

and Lattice parameter, $a = d_{nkl}\sqrt{h^2 + k^2 + l^2}$

$$= 0.1433\sqrt{2^2 + 0^2 + 0^2} = 0.2866 \text{ nm}$$

for Third peak, $d_{211} = \dfrac{1 \times 0.1542}{2\sin\left(\frac{82.8}{2}\right)} = 0.1166 \text{ nm}$

and Lattice parameter, $a = d_{nkl}\sqrt{h^2 + k^2 + l^2}$

$$= 0.1166\sqrt{2^2 + 1^2 + 1^2} = 0.2856 \text{ nm}$$

5. (b)

Let number of product $A = x$

Number of product $B = y$

Profit, $z = 150x + 200y$

Constraints : $2x + 3y \leq 1200$

$8x + 3y \leq 2400$

$5x + 4y \leq 2000$

$$\therefore \quad \frac{x}{600} + \frac{y}{400} \leq 1 \quad \text{... (i)}$$

$$\frac{x}{300} + \frac{y}{800} \leq 1 \quad \text{... (ii)}$$

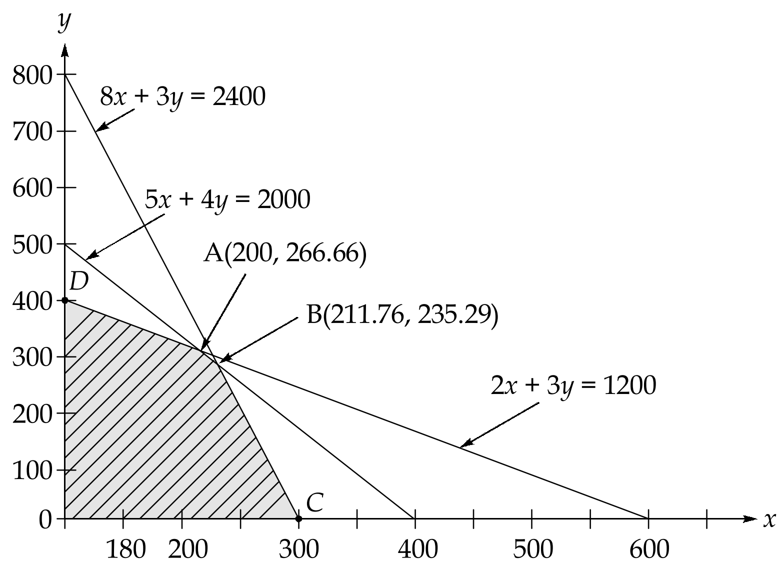
$$\frac{x}{400} + \frac{y}{500} \leq 1 \qquad \text{... (iii)}$$

From equation (i) and (ii)

$$x = 200; y = 266.66 \text{ (Point A)}$$

From equation (ii) and (iii)

$$x = 211.76; y = 235.29 \text{ (Point B)}$$



For profit, checking the points *A*, *B*, *C* and *D*.

At point *A*(200, 266.66)

$$z_A = 150 \times 200 + 200 \times 266.66 = ₹83332$$

At point *B*(211.76, 235.29)

$$z_B = 150 \times 211.76 + 200 \times 235.29 = ₹78822$$

At point *C*(300, 0)

$$Z_C = 150 \times 300 + 200 \times 0 = ₹45000$$

At point D(0, 400)

$$Z_D = 0 \times 150 + 200 \times 400 = ₹80000$$

Optimum profit will be at point A (200, 266.66) and the value of this optimum profit is ₹83332.

5. (c)

$$① \xrightarrow[4-5-12]{A} ② \xrightarrow[3-7-17]{B} ③ \xrightarrow[7-9-11]{C} ④ \xrightarrow[7-10-13]{D} ⑤$$

$t_e = 6 \quad t_e = 8 \quad t_e = 9 \quad t_e = 10$

Using equation,

$$\text{Expected mean time } (t_e) = \frac{t_p + 4t_m + t_o}{6}$$

$$\text{Standard deviation, } \sigma_o = \left(\frac{t_p - t_o}{6}\right)$$

$$\text{Variance, } (\sigma_o)^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Activity	Optimistic time (t_o)	Most likely time (t_m)	Pessimistic time (t_p)	Standard deviation	Variance
A	4	5	12	1.33	1.77
B	3	7	17	2.33	5.43
C	7	9	11	0.66	0.443
D	7	10	13	1	1

From the data, we can see that the activity C is most reliable.

Expected time of the project = T_E

$$T_E = 6 + 8 + 9 + 10 = 33 \text{ days}$$

Variance of the project, $(\sigma_o)^2 = 1.77 + 5.43 + 0.443 + 1$

$$= 8.643$$

Standard deviation of the project (σ_o)

$$\sigma_o = \sqrt{8.643} = 2.94$$

Z(Normal deviate) or probability factor for 95% probability.

$$Z = +1.647$$

$\Rightarrow$
$$1.647 = \frac{x - T_E}{\sigma_o}$$

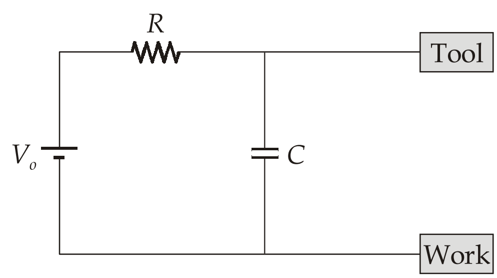
$\therefore$
$$x = 2\sigma_o + T_E = 1.647 \times 2.94 + 33$$
$$= 37.58 \text{ days}$$

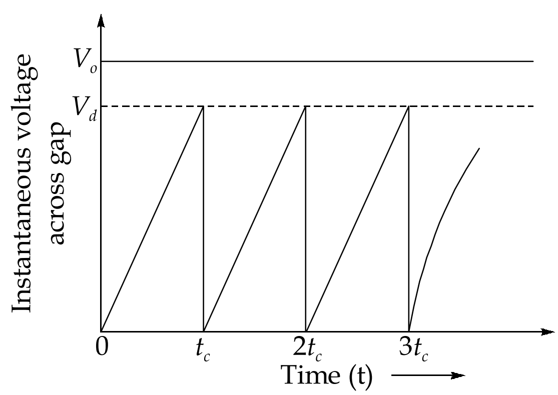
Z for 5% probability = –1.647

$$x = T_E - 2\sigma$$
$$= 33 - 1.647 \times 2.94$$
$$= 28.16 \text{ days}$$

5. (d)

Relaxation circuit used for generating pulses in EDM process is given as:





Variation of gap voltage

The voltage across the gap is given by

$$V_d = V_o\left(1 - e^{-t_c/RC}\right)$$

Energy released per spark, $E = \frac{1}{2}CV_d^2$

The average power delivered,

$$P_{av} = \frac{E}{t_c + t_d}$$

Since, the discharge time is much smaller than the charging time,

$$P_{av} = \frac{E}{t_c} = \frac{\frac{1}{2}CV_o^2\left(1 - e^{-t_c/RC}\right)^2}{t_c}$$

$$= \frac{V_o^2}{2R}\left(\frac{RC}{t_c}\right) \times \left(1 - e^{-t_c/RC}\right)^2$$

Let $$\frac{t_c}{RC} = x$$

$\therefore$ $$P_{av} = \frac{V_o^2\left(1 - e^{-x}\right)^2}{2Rx}$$

For maximum power supply,

$$\frac{dP_{av}}{dx} = 0$$

$$\frac{V_o^2}{2R} \times \frac{\left[a\left(\frac{\left(1 - e^{-x}\right)^2}{x}\right)\right]}{dx} = 0$$

$$\frac{x \times 2\left(1 - e^{-x}\right)e^{-x} - \left(1 - e^{-x}\right)^2 \times 1}{x^2} = 0$$

$$\left(\frac{1 - e^{-x}}{x^2}\right)\left(2xe^{-x} - 1 + e^{-x}\right) = 0$$

$$\left(\frac{1 - e^{-x}}{x^2}\right)\left[(2x + 1)e^{-x} - 1\right] = 0$$

$$(2x + 1)\, e^{-x} - 1 = 0 \qquad [\because x \neq 0]$$

$$(2x + 1)\, e^{-x} = 1$$

$$x = 1.2564$$

For maximum power supply,

$$\frac{V_d}{V_o} = 1 - e^{-x}$$

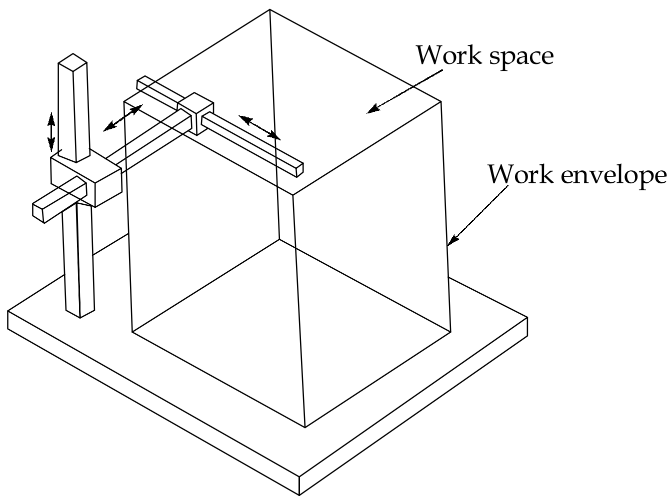
$$= 1 - e^{-1.2564} = 0.7153$$

$$\approx 0.72$$

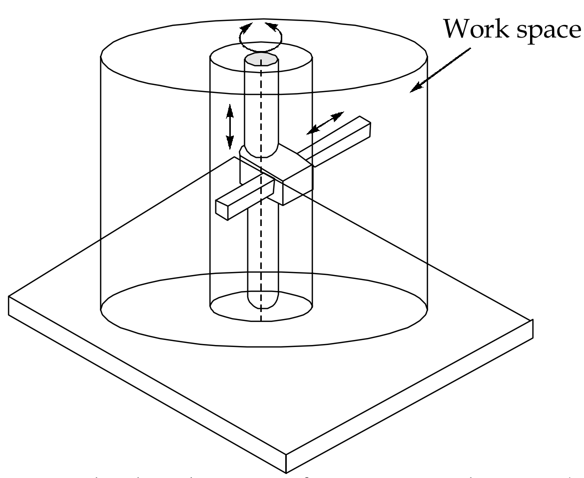
$\therefore$ $$V_d = 0.72\, V_o$$

Hence, optimum discharge voltage is 72% of supply voltage.

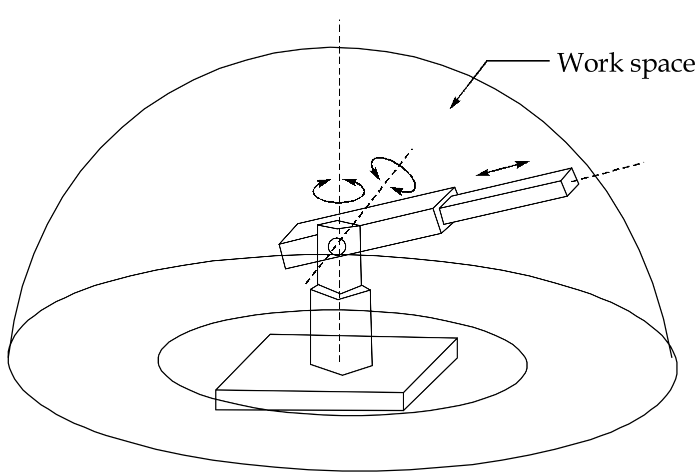
5. (e)



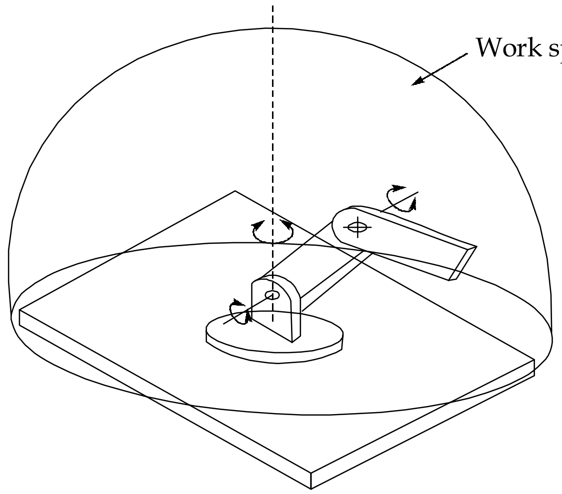
A3-DOF Cartesian arm configuration and its work space



As 3-DOF cylindrical arm configuration and its work space



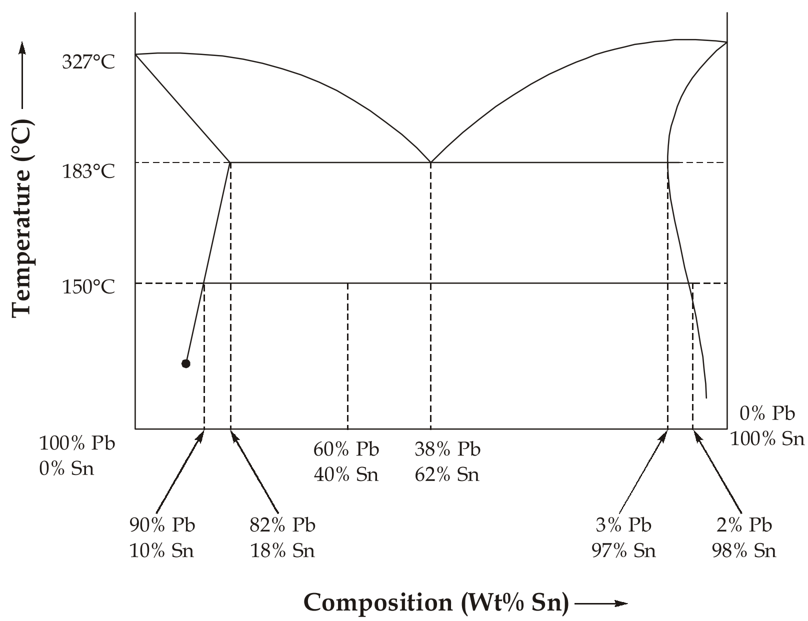
A 3-DOF polar arm configuration and its work space



A 3-DOF articulated arm configuration and its work space

6. (a)

The lead - tin phase diagram is shown in the figure.



(i) For relative amount of α and β phase present in terms of mass fraction:

Now, mass fraction can be computed in terms of weight fraction,

$$W_\alpha = \frac{C_\beta - C_o}{C_\beta - C_\alpha} = \frac{98-40}{98-10} = 0.6591 \approx 0.66$$

and

$$W_\beta = \frac{C_o - C_\alpha}{C_\beta - C_\alpha} = \frac{40-10}{98-10} = 0.3409 \approx 0.34$$

(ii) For relative amount of α and β phase present in terms of volume fraction.

Now, we need to calculate the density of each phase,

Density of α-phase,

$$\rho_\alpha = \frac{100}{\dfrac{C_{Pb(\alpha)}}{\rho_{Pb}} + \dfrac{C_{Sn(\alpha)}}{\rho_{Sn}}} = \frac{100}{\dfrac{90}{11.23} + \dfrac{10}{7.24}} = 10.643 \text{ g/cm}^3$$

and Density of β-phase, $$\rho_\beta = \frac{100}{\dfrac{C_{Pb(\beta)}}{\rho_{Pb}} + \dfrac{C_{Sn(\beta)}}{\rho_{Sn}}} = \frac{100}{\dfrac{2}{11.23} + \dfrac{98}{7.24}} = 7.292 \text{ g/cm}^3$$

∴ Volume fraction of α-phase,

$$V_\alpha = \frac{\dfrac{W_\alpha}{\rho_\alpha}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}} = \frac{\dfrac{0.66}{10.643}}{\dfrac{0.66}{10.643} + \dfrac{0.34}{7.292}} = 0.5708$$

and Volume fraction of β-phase,

$$V_\beta = \frac{\dfrac{W_\beta}{\rho_\beta}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}} = \frac{\dfrac{0.34}{7.292}}{\dfrac{0.66}{10.643} + \dfrac{0.34}{7.292}} = 0.4292$$

6. (b)

(i) First come, first served schedule

FCFS schedule is A - B - C - D - E - F and the processing time is as follows:

Job		A	B	C	D	E	F
Machine	In	0	5	9	24	32	38
	Out	5	9	24	32	38	41

(ii) Shortest processing time schedule

SPP is F - B - A - E - D - C and the processing time is as follows:

Job		F	B	A	E	D	C
Machine	In	0	3	7	12	18	26
	Out	3	7	12	18	26	41

(iii) Slack time remaining schedule

Job	Processing time, t_i (Days)	Due date, d_i (Days)	Slack time (days) $(d_i - t_i)$
A	5	15	10
B	4	18	14
C	15	23	8
D	8	25	17
E	6	11	5
F	3	19	16

Therefore, STR schedule is E - C - A - B - F - D and the processing times will be

Job		E	C	A	B	F	D
Machine	In	0	6	21	26	30	33
	Out	6	21	26	30	33	41

(iv) Earliest due date schedule.

The EDD schedule is E - A - B - F - C - D and the processing times will be

Job		E	A	B	F	C	D
Machine	In	0	6	11	15	18	33
	Out	6	11	15	18	33	41

(v) Mean flow times for various schedules are:

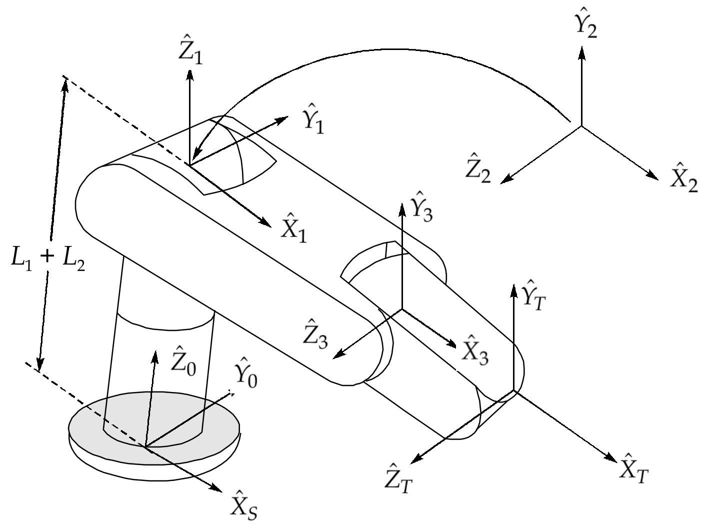
$$\text{FCFS schedule} = \frac{5+9+24+32+38+41}{6} = 24.83 \text{ days}$$

$$\text{SPT schedule} = \frac{3+7+12+18+26+41}{6} = 17.83 \text{ days}$$

$$\text{STR schedule} = \frac{6+21+26+30+33+41}{6} = 26.16 \text{ days}$$

$$\text{EDD schedule} = \frac{6+11+15+18+33+41}{6} = 20.66 \text{ days}$$

6. (c)



α_{i-1}	a_{i-1}	d_i	θ_i
0	0	$L_1 + L_2$	θ_1
90°	0	0	θ_2
0	L_3	0	θ_3
0	L_4	0	0

$$^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

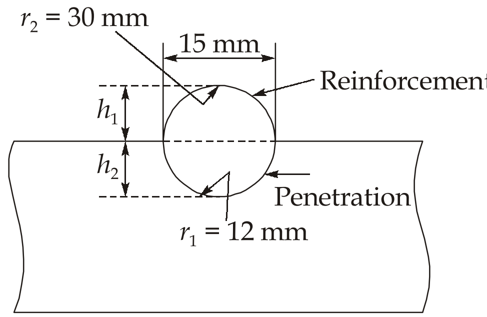
$$^2_2T = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & L_3 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

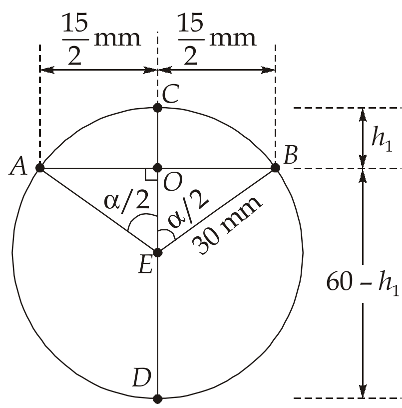
7. (a)

Reinforcement is a part of circle of radius 30 mm.

We know, perpendicular drawn from the centre of circle on the chord bisect the chord.



We have,



and $$OA \times OB = OC \times OD$$

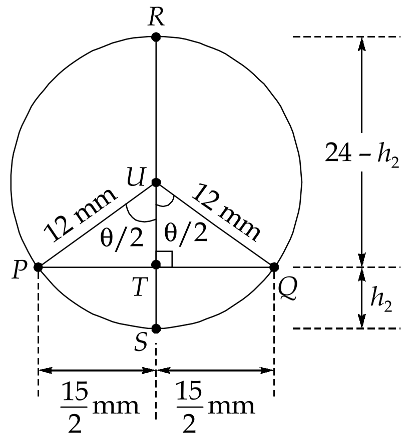
$$\frac{15}{2} \times \frac{15}{2} = h_1 \times (60 - h_1)$$

$$h_1^2 - 60h + \frac{225}{4} = 0$$

$$h_1 = 59.047 \text{ mm (Not possible)}$$

$$h_1 = 0.953 \text{ mm}$$

Now, Penetration is also a part of circle of radius 12 mm.



We know,

and $$PT \times TQ = RT \times TS$$

$$\frac{15}{2} \times \frac{15}{2} = h_2 \times (24 - h_2)$$

$$h_2^2 - 24h_2 + \frac{225}{4} = 0$$

$$h_2 = 21.367 \text{ mm (Not possible)}$$

$$h_2 = 2.633 \text{ mm}$$

Now, $$\% \text{ Dilution} = \frac{A_P}{A_P + A_R} \times 100$$

Where, A_P = Penetration area, A_R = Reinforcement area

For penetration area:

$$\sin\frac{\theta}{2} = \frac{15/2}{12} = \frac{5}{8}$$

$$\frac{\theta}{2} = 38.682°$$

$$\theta = 77.364°$$

Now, Area of sector (UPQ), $A_1 = \frac{\theta}{360} \times \pi r_1^2 = \frac{77.364}{360} \times \pi \times 12^2$

$$= 97.218 \text{ mm}^2$$

and Area of ΔPQU, $A_2 = \frac{1}{2} \times PQ \times TU = \frac{1}{2} \times 15 \times (12 - 2.633)$

$= 70.2525 \text{ mm}^2$

∴ Area of penetration $(A_P) = A_1 - A_2$

$= 97.218 - 70.2525$

$= 26.9655 \text{ mm}^2$

For area of reinforcement, $\sin\frac{\alpha}{2} = \frac{OA}{AE} = \frac{15/2}{30} = \frac{1}{4}$

$$\frac{\alpha}{2} = 14.4775°$$

$$\alpha = 28.955°$$

Area of sector (ACB), $A_3 = \frac{\alpha}{360°}\pi r_2^2 = \frac{28.955}{360} \times \pi \times 30^2$

$= 227.412 \text{ mm}^2$

Area of ΔABE, $A_4 = \frac{1}{2} \times AB \times OE = \frac{1}{2} \times 15 \times (30 - 0.953)$

$= 217.8525 \text{ mm}^2$

So, Area of reinforcement, $A_R = A_3 - A_4$

$= 227.412 - 217.8525$

$= 9.5595 \text{ mm}^2$

So, % Dilution $= \frac{A_P}{A_P + A_R} \times 100 = \frac{26.9655}{26.9655 + 9.5595} \times 100$

$= 73.8275\%$ **Answer**

7. (b)

Subtracting the lowest element of the row for each row.

	A	B	C	D	E
1	5	0	4	13	6
2	7	3	0	6	8
3	1	0	2	2	3
4	9	0	3	8	6
5	5	0	8	13	4

Subtracting the lowest element of the column from each column.

	A	B	C	D	E
1	4	[0]	4	11	3
2	6	3	[0]	4	5
3	[0]	~~0~~	2	~~0~~	~~0~~
4	8	~~0~~	3	6	3
5	4	~~0~~	8	11	1

Since, the minimum number of lines crossing all zeros is 3. Hence, this is not the optimal solution.

Iteration towards optimality: The minimum element that does not have a line through it is 1. Subtract 1 from all the elements which do not have a line through them. Add it to all the elements that lie at the intersection of two lines and leave the remaining elements of the matrix unchanged. We get the second basic feasible solution.

	A	B	C	D	E
1	3	0	3	10	2
2	6	4	0	4	5
3	0	1	2	0	0
4	7	0	2	5	2
5	3	0	7	10	0

Since, the number of lines creasing zeros are again 4. The solution is not optimal. Repeating the iteration again.

	A	B	C	D	E
1	1	0	1	8	0
2	6	6	0	4	5
3	0	3	2	0	0
4	5	0	0	3	0
5	3	2	7	10	0

Repeating the iteration again.

	A	B	C	D	E
1	0	0	1	7	0
2	5	6	0	3	5
3	0	4	3	0	1
4	4	0	0	2	0
5	2	2	7	9	0

Now, the given solution is optimal

$$Z_{min} = 10 + 6 + 4 + 9 + 10 = 39$$

7. (c)

(i)

For relatively soft material, scale B is used with ball penetrator and load of 100 kg. For relatively hard material, scale C is used in conjuction with diamond cone known as Brale indenter with 150 kg load. Scale T is used for very thin sections (blades), or parts with a thin, hard outer surface with load of 15 to 45 kg.

(ii)

Pitting is a local corrosion damage characterized by cavities. It is a particularly insidious form of corrosion, because even if one pit perforates the side of a tank, serviceability is lost until the tank is repaired. Chemical nature of the environment causes pitting which are as follows:

1. Halogen-containing solutions
2. Brackish water
3. Salt water
4. Chloride bleaches
5. Reducing inorganic acids are solutions that tend to produce pitting.

Stainless steels are particularly prone to pitting. Pitting of brass conductive tubes sometimes occurs due to dezincification. This consists in the solution of the brass followed by precipitation of copper by zinc in the brass. The net result is selective removal of zinc. A localized attack frequently occurs near the inlet ends of the condenser tubes, due to impingement of air bubbles, which carry away the corrosion products. Pumps and ship propellers are liable to an attack known as cavitation, an impact caused by the collapse of vapour bubbles.

Major preventive measure are as follows:

1. Austenitic steels pit in salt water, so most designers tend to use copper alloys, bronzes, monels and other materials having lower pitting tendencies.
2. Some are carbon steels in salt water, in which corrosion rate is much higher than with stainless steels, but attack is more uniform and no pitting takes place.

(iii)

Super alloys are nickel, cobalt or iron based alloys with excellent elevated temperature strength, creep properties and oxidation resistance.

Iron based super alloys contain	32 to 37% Iron
	15 to 22% chromium
	9 to 38% nickel

Cobalt based super alloys contain	35 to 65% cobalt
	19 to 30% chromium
	upto 35% nickel
Nickel based super alloys contain	38 to 76% nickel
	27% chromium
	20% cobalt

Properties :

- Iron based super alloys are characterized by high temperature as well as room temperature strength and resistance to creep, oxidation, corrosion and wear. Wear resistance increases with carbon content.
- Nickel based super alloys based on the formula $Ni_3(Al, Ti)$ are particularly resistant to temperature.
- Cobalt-base super alloys have excellent high-temperature creep and fatigue strengths and resistance to hot corrosion attack.

Application:

Iron based super alloys: High temperature aircraft bearings, and machinery parts subjected to sliding contact.

Nickel based super alloys: Aero engine turbine blades, turbine discs, turbo chargers.

Cobalt based super alloys: Gas turbine engines.

8. (a)

We have, $\phi = 30°, \theta = 40°, \psi = 50°$

$$\text{Euler }(\phi, \theta, \psi) = \text{Rot}(a, \phi)\ \text{Rot}(0, \theta)\ \text{Rot}(a, \psi)$$

$$\text{Euler's }(30, 40, 50) = \begin{bmatrix} C30 & -S30 & 0 & 0 \\ S30 & C30 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C40 & 0 & S40 & 0 \\ 0 & 1 & 0 & 0 \\ -S40 & 0 & C40 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C50 & -S50 & 0 & 0 \\ S50 & C50 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Euler's }(30, 40, 50) = \begin{bmatrix} 0.0434 & -0.8296 & 0.5566 & 0 \\ 0.9096 & 0.2632 & 0.3213 & 0 \\ -0.4134 & 0.4924 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now for RPY,

$$\text{RPY }(\phi_a, \phi_0, \phi_n) = \text{Rot}(a, \phi_a)\ \text{Rot}(o, \phi_0)\ \text{Rot}(n, \phi_n)$$

$$= \begin{bmatrix} n_x & O_x & a_x & 0 \\ n_y & O_y & a_y & 0 \\ n_z & O_2 & a_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For RPY system, $\text{Rot}^{-1}(a, \phi_a) \begin{bmatrix} n_x & O_x & a_x & 0 \\ n_y & O_y & a_y & 0 \\ n_z & O_2 & a_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{Rot}(O, \phi_o)\ \text{Rot}(n, \phi_n)$

$$\begin{bmatrix} n_x C\phi_a + n_y S\phi_a & O_x C\phi_a + O_y S\phi_a & a_x S\phi_a + a_y \sin\phi_a & 0 \\ n_y C\phi_a + n_x S\phi_a & O_y C\phi_a - O_x S\phi_a & a_y C\phi_a - a_x S\phi_a & 0 \\ n_z & O_z & a_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\phi_a & S\phi_o S\phi_n & S\phi_o C\phi_n & 0 \\ 0 & C\phi_n & -S\phi_n & 0 \\ -S\phi_o & C\phi_o S\phi_n & C\phi_o C\phi_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

On comparing, $\phi_a = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}\left(\frac{0.9096}{0.0434}\right) = 87.26°$

$$\phi_a = \tan^{-1}\left(\frac{-n_y}{-n_x}\right) = \tan^{-1}\left(\frac{-0.9096}{-0.0434}\right) = 267.26°$$

$$\phi_o = \tan^{-1}\left(\frac{-n_z}{n_x C\phi_a + n_y S\phi_a}\right) = \tan^{-1}\left(\frac{-(-0.4134)}{(0.0434\cos 87.26 + 0.9096\sin 87.26)}\right)$$

$$\phi_o = 24.416$$

Also, $\phi_o = \tan^{-1}\left(\frac{-(-0.4134)}{(0.0434\cos(267.26) + 0.9096\sin(267.26))}\right)$

$$= 155.523°$$

and $\phi_n = \tan^{-1}\left(\frac{-a_y \cos\phi_a + a_x S\phi_a}{O_y C\phi_a - O_x S\phi_a}\right)$

$$= \tan^{-1}\left(\frac{-0.3213\cos(87.27) + 0.5566\sin(87.6)}{0.2632\cos(87.26) - (-0.8296\sin(87.26))}\right)$$

$$= 32.726$$

Also, $\phi_n = \tan^{1}\left[\frac{-0.3213\cos(267.26) + 0.5566\sin(267.26)}{0.2632\cos(267.26) - (-0.8296\sin(267.26))}\right]$

$$= \tan^{1}\left(\frac{-0.5406}{-0.8421}\right) = 212.726°$$

8. (b)

(i)

Total cost = Tooling cost + Setting cost + Machining cost + Area head cost

Let breakeven quantity be 'x' units

Total cost = (Tooling cost) + (Setting time × Setting labour cost/min)

$$+ \frac{\text{Machining labour cost per hour} \times \text{Number of piece}}{\text{Number of pieces produced per hour}} + \text{Overhead charges}$$

For machine I

$$\text{Total cost, } TC_1 = 60 + \left(\frac{80}{60} \times 10\right) + \frac{6x}{40} + 4\left(\frac{80}{60} \times 10 + \frac{6x}{40}\right)$$

$$\therefore \quad TC_1 = 60 + \frac{40}{3} + \frac{6x}{40} + \frac{160}{3} + \frac{3x}{5} = \frac{380}{3} + \frac{3x}{4}$$

For machine II

$$\text{Total cost, } TC_2 = 120 + \left(\frac{160}{60} \times 15\right) + \frac{6x}{120} + 10\left(\frac{160}{60} \times 15 + \frac{6x}{120}\right)$$

$$TC_2 = 560 + \frac{11x}{20}$$

For break-even quantity, $TC_1 = TC_2$

$$\frac{380}{3} + \frac{3x}{4} = 560 + \frac{11x}{20}$$

$$\therefore \quad 0.20x = 433.33$$

$$\therefore \quad x = 2166.66 \text{ units} \approx 2167 \text{ units}$$

(ii) For $x = 2100$

$$TC_1 = \frac{380}{3} + \frac{3}{4} \times 2100 = ₹1701.67$$

$$TC_2 = 560 + \frac{11}{20} \times 2100 = ₹1715$$

For $x = 2200$

$$TC_1 = \frac{380}{3} + \frac{3}{4} \times 2200 = ₹1776.66$$

$$TC_2 = 560 + \frac{11}{20} \times 2200 = ₹1770$$

$\therefore$ If $x < 2167$ units, we should go with the machine I.

If $x > 2167$, we should go with machine II.

8. (c)

Major imperfections in the crystal structure of metals

1. **Point defects**

   (i) Vacancy (ii) Interstitial (iii) Impurities

2. **Line defects (Dislocations)**

   (i) Edge dislocation (ii) Screw dislocation

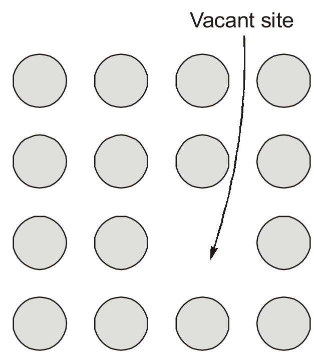
3. **Surface or Grain boundaries defects**

   (i) Grain boundaries (ii) Tilt boundaries (iii) Twin boundaries

4. **Volume defects :**

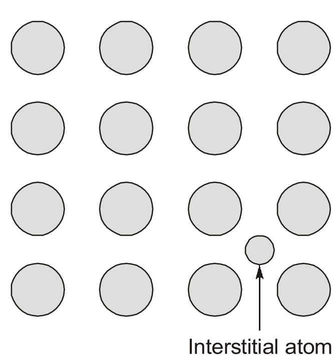
   1. **Point defects:** A point imperfection is a very localized interruption in the regular arrangement of a lattice, e.g. vacant site.

   (i) **Vacancy defect:** Vacancies are empty atomic sites in crystal lattice. Vacancies may arise in a crystal lattice due to imperfect packing during solidification or crystallization and they may also arise due to thermal vibrations of atoms.



Vacancy defect

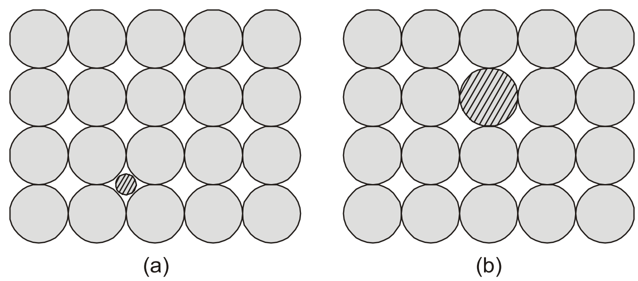
   (ii) **Interstitial defect:** Interstitial defect arises when an atom occupies an interstitial position i.e. between the atoms in the lattice of the ideal crystal.



Interstitial defect

Figure shows the interstitial defect. The interstitial atom may be either a normal atom of the crystal or a foreign atom.

(iii) Impurities: Small particles may be embedded in the structure such as slag inclusions in metals or foreign atoms in the lattice structure. Impurity atoms are of two types either substitutional or interstitial atoms.



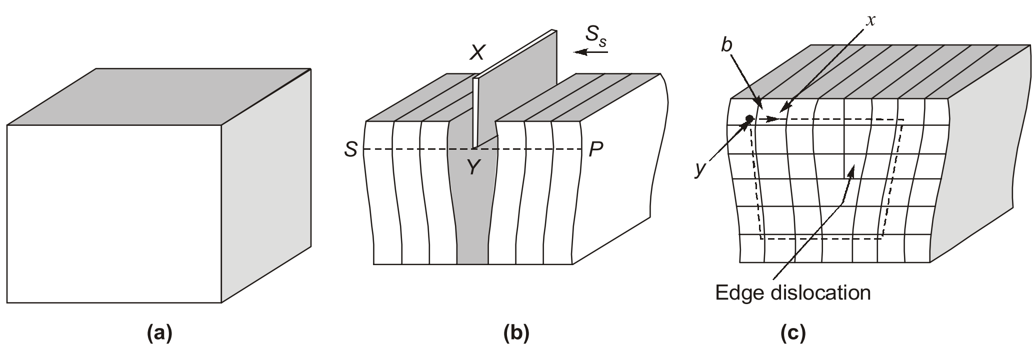
Impurities : (a) Interstitial impurity, (b) Substitutional impurity

Substitutional impurity atom refers to that atom which substitutes for or replaces a parent atom in the crystal.

Considerable distortion of lattice structure occurs due to the presence of impurities in the crystal or lattice structure.

2. Line Defects-Dislocations:

A dislocation may be defined as a disturbed region between two substantially perfect parts of a crystal.



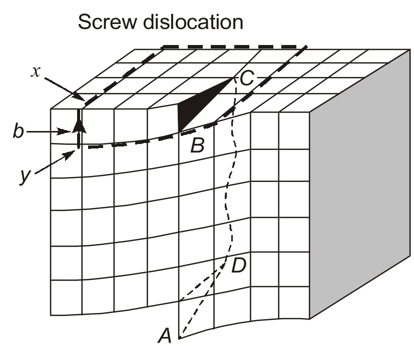
(a) The perfect crystal (b) An extra plane of atoms is inserted
(c)The bottom edge of the extra plane is an edge dislocation

(i) Edge dislocation: The line marked as SP represents the slip plane of the crystal and the dashed lines represent the crystal planes perpendicular to the slip plane.

The plane marked '*X*' at the top of the figure ends at point '*Y*' on the slip plane, whereas the planes on either side of '*X*' run continuously from the top to the bottom of the figure. In such a case where a lattice plane ends inside a crystal, an edge dislocation results.

The edge dislocation shown in figure has an incomplete plane which lies above the slip plane. Such as edge dislocation is called positive edge dislocation and is represented by the symbol ⊥ where the vertical line represents the incomplete plane and the horizontal line represents the slip plane. It is also possible to have the incomplete plane below the slip plane which can be represented by the symbol T.

(ii) Screw Dislocation : The right side portion of the crystal has been sheared by one atomic distance on the downward direction relative to the left side portion. No slip has taken place to the rear portion of the line *DC* and therefore *DC* is a dislocation line. The plane *ABCD* is the slip plane The designation screw for this lattice defect is derived from the fact that the lattice planes of the crystal, spiral the dislocation line *DC*.



Representation of a screw dislocation

3. **Surface Defects:** Surface defects may include grain boundary, tilt boundary, twist boundary, twin boundary etc.

   (i) Grain Boundaries: Grain boundary is formed when two growing grain surfaces meet each other and these grain boundaries separate crystals or grains of different orientation in polycrystalline materials.

   (ii) Tilt Boundaries: Tilt boundaries are formed by edge dislocations and are regarded as an array of edge dislocations located one above the other.

   (iii) Twist Boundaries: Other low angle boundaries formed by screw dislocations are called as twist boundaries.

   Twin Boundaries: A twin boundary separates two parts of a crystal having the same orientation.

4. **Volume Defects:** Volume defects are stacking faults which are created by a fault in the staking sequence of close packed atomic planes in crystals such as *FCC* and *HCP*.

Stacking faults are more frequently found in deformed metals. The presence of these imperfections or defects in metallic crystals impairs the physical and mechanical properties of metals and alloys.

The theoretical strength of a metal (the force required to separate the bond between adjoining atoms) turns out to be several million newtons per square metres, but the ordinary strength of metals is 100 to 1000 times less. This is because of occurrence of defects in the crystal structure.

The lower yield point of crystals than the computed yield point is because of the imperfections in the crystal and is explained by the type of defect called 'dislocation'. With grain growth, strength and hardness of a metal decrease, but ductility increases.

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