1. Very good presentation and try to avoid strike outs....
2.better increase your font size so that there is clear in visibility of answer.
3.Little calculation errors so try to improve it....

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ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-3 : Analog and Digital Communication Systems [All topics]
Signals and Systems-1 + Microprocessors and Microcontroller [Part Syllabus]
Network Theory-2 + Control Systems-2 [Part Syllabus]

Name : ....

Roll No :

Test Centres				Student's Signature
Delhi ☐	Bhopal ☑	Jaipur ☐		
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	207

Signature of Evaluator ....

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DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

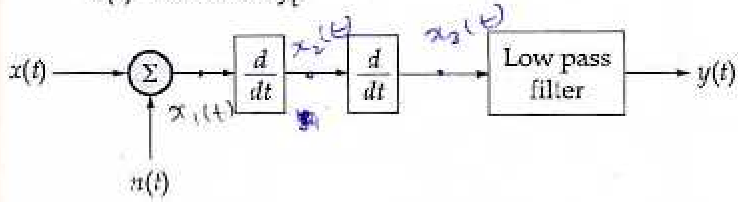
1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

Section A : Analog and Digital Communication Systems

Q.1 (a) Consider the system shown in figure. The signal $x(t)$ is defined by:

$$x(t) = A\cos 2\pi f_c t$$



The low pass filter has unity gain in the passband and bandwidth W, where $f_c < W$. The noise $n(t)$ is white with two sided power spectral density $\frac{1}{2}N_0$. Determine the signal to noise ratio at the output $y(t)$.

[12 marks]

$x_1(t) = x(t) + n(t)$

$= A\cos 2\pi f_c t + n(t)$

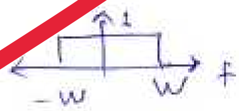
$x_2(t) = \dfrac{dx_1(t)}{dt}$

$= -A\, 2\pi f_c \sin 2\pi f_c t + n'(t)$

$x_3(t) = \dfrac{dx_2(t)}{dt}$

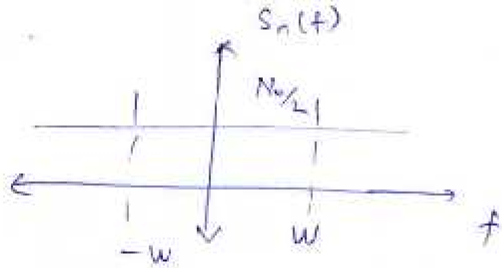
$= -A(2\pi f_c)^2 \cos 2\pi f_c t + n''(t)$

$x_3(t)$ After passing through LPF
given $f_c < W$ therefore



$y(t) = \underbrace{-A(2\pi f_c)^2 \cos 2\pi f_c t}_{S_o} + n_o(t)$

Power of signal at o/p (S_o) $= \dfrac{A^2(2\pi f_c)^4}{2}$



$N_o' = \dfrac{N_o}{2} \times 2W$

$N_o' = N_o W$

$$(SNR)_{o/p} = \frac{S_o}{N_o}$$

$$= \frac{\frac{A^2 (2\pi f_c)^4}{2}}{N_o \omega}$$

$$= \frac{8 A^2 \pi^4 f_c^4}{N_o \omega}$$

Q.1 (b) Consider a continuous input signal whose amplitude V lies in the range $[-V_{max}, +V_{max}]$. This is applied to a uniform quantizer of mid-rise type where the step size is given by Δ and L denotes the number of representation levels. Let σ_Q^2 represent the variance of the quantization error and 'n' represent the number of bits per sample. Show that $\sigma_Q^2 = \frac{1}{3} V_{max}^2 \cdot 2^{-2n}$ and that the output signal to noise ratio of a uniform quantizer is

$$(SNR_0) = \frac{3P}{V_{max}^2} \cdot 2^{2n} \quad \text{where } P \text{ is signal power}$$

[12 marks]

Sol^n Given

i/p range $\in [-V_{max}, +V_{max}]$

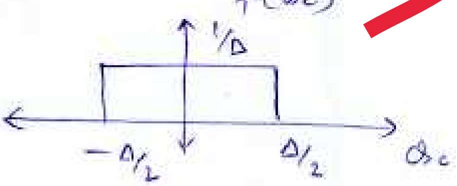
step size $= \Delta$

levels $= L$

bits per sample $= n$

$\sigma_Q^2 =$ variance of quantization error

PDF of quantization error is ~~~~ given by



$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} Q_e^2 \, f(Q_e) \, dQ_e$$ { since mean of quantization error is 0 }

$$= \int_{-\Delta/2}^{\Delta/2} Q_e^2 \frac{1}{\Delta} \, dQ_e$$

$$= \frac{1}{\Delta} \left| \frac{Q_e^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta} \left[\frac{\frac{\Delta^3}{8} + \frac{\Delta^3}{8}}{3} \right]$$

$$\sigma_Q^2 = \frac{\Delta^2}{12} \quad \text{--- (A)}$$

$$\Delta = \frac{V_{p-p}}{2^n} \text{ or } \frac{V_{p-p}}{L}$$

$$= \frac{2V_{max}}{2^n}$$

Putting value of Δ in eq^n (A)

$$\sigma_Q^2 = \frac{\frac{\cancel{4} V_{max}^2}{2^{2n}}}{\cancel{12}\,3}$$

$$\boxed{\sigma_Q^2 = \frac{1}{3} V_{max}^2 \, 2^{-2n}}$$

O/P SNR = 9

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Q.1 (c) The random process $X(t)$ is defined by

$X(t) = X\cos 2\pi f_0 t + Y\sin 2\pi f_0 t$

where X and Y are two zero mean independent Gaussian random variable each with variance σ^2.

(i) Find $m_X(t)$.

(ii) Find $R_X(t+\tau, t)$. Is $X(t)$ stationary? Is it cyclostationary?

[12 marks]

Sol^n

(i) $X(t) = X\cos 2\pi f_0 t + Y\sin 2\pi f_0 t$

$E[X(t)] = E[X\cos 2\pi f_0 t + Y\sin 2\pi f_0 t]$

$= \cos 2\pi f_0 t\, E(X) + \sin 2\pi f_0 t\, E(Y)$

$E(X) = E(Y) = 0$ ∵ given

$E[X(t)] = 0$

$\boxed{M_X(t) = 0}$

(ii) $R_X(t+\tau, t) = E[X(t+\tau)X(t)]$

$= E[(X\cos\omega_0(t+\tau) + Y\sin\omega_0(t+\tau))(X\cos\omega_0 t + Y\sin\omega_0 t)]$

$R_X(t+\tau, t) = E[X^2\cos\omega_0(t+\tau)\cos\omega_0 t + XY\cos\omega_0(t+\tau)\sin\omega_0 t + XY\sin[\omega_0(t+\tau)]\cos\omega_0 t + Y^2\sin\omega_0(t+\tau)\sin\omega_0 t]$

$= \frac{1}{2}E[\cos\omega_0(2t+\tau) + \cos\omega_0\tau]\,E(X^2)$

$+ \frac{1}{2}[\sin\omega_0(2t+\tau) \pm \sin(t\omega\tau)]\,\cancel{E(XY)}^{0}$

$+ \frac{1}{2}[\sin\omega_0(2t+\tau) + \sin\omega_0\tau]\,\cancel{E(XY)}^{0} + \frac{1}{2}E[\cos\omega_0(2t+\tau) - \cos\omega_0\tau]$

{Given $E(X) = E(Y) = 0$; X, Y independent

$= \frac{1}{2}E[\cancel{\cos\omega_0(2t+\tau)} + \cos\omega_0\tau]\,\sigma^2 - \frac{1}{2}E[\cancel{\cos\omega_0(2t+\tau)} - \cos\omega_0\tau]\,\sigma^2$

$= \sigma^2\left[\frac{\cos\omega_0\tau}{2} + \frac{\cos\omega_0\tau}{2}\right]$

$\boxed{R_X(t+\tau, t) = \sigma^2\cos\omega_0\tau}$

Since $R_X(t+\tau, t)$ is independent of 't' therefore $X(t)$ is stationary

Q.1 (d) A PCM system uses a uniform quantizer followed by a 8-bit binary encoder. The bit rate of the system is equal to 60 Mbps.

(i) What is the maximum message bandwidth for which the system operates satisfactory?

(ii) Determine signal to quantization noise ratio for uniform distributed sample of message signal having uniform quantization level.

[12 marks]

Sol^n Given $n = 8$ bits

$R_b = 60 \times 10^6$ bps

(i) considering $(BW)_{Nch} = \frac{R_b}{2}$

$R_b = n f_s$ { f_s is sampling frequency }

$60 \times 10^6 = 8 \times 2 f_m$ { $f_s = 2 f_m$ Nyquist rate }

$60 \times 10^6 = 16 f_m$

$\frac{60}{16} \times 10^6 = f_m \rightarrow$ message BW.

$\boxed{f_m = 3.75 \text{ MHz}}$

6

(ii) $SQNR = \dfrac{S_o}{N_Q}$ considering msg signal sinusoidal

$$= \frac{\frac{A_m^2}{2}}{\frac{\Delta^2}{12}} = \frac{6A_m^2}{\Delta^2} = \frac{6A_m^2}{\frac{(2A_m)^2}{2^{2n}}} = \frac{3}{2}\, 2^{2n}$$

for $n = 8$ bit

$$SQNR = \frac{3}{2}\, 2^{16}$$

$$= 98304$$

$$\boxed{SQNR = 49.92\ dB}$$

Q.1 (e) What are the capture effect and threshold effect in an FM system? List two different methods used for FM threshold improvement.

[12 marks]

Q.2 (a) A communication channel has a bandwidth of 100 kHz. This channel is to be used for transmission of an analog source $m(t)$, where $|m(t)| < 1$, whose bandwidth is 4 kHz. The power content of the message signal is 0.1 W.

(i) Find the ratio of the output SNR of an FM system that utilizes the whole bandwidth, to the output SNR of a conventional AM system with a modulation index of $\mu = 0.85$. What is this ratio in dB?

(ii) Show that if an FM system and a PM system are employed and these systems have same output signal to noise ratio, we have

$$\frac{BW_{\text{PM}}}{BW_{\text{FM}}} = \frac{\sqrt{3}\beta_f + 1}{\beta_f + 1} \qquad (\beta_f = \text{Modulation index of FM})$$

[10 + 10 marks]

Q.2 (b) An analog signal having 5 kHz bandwidth is sampled at twice the Nyquist rate and each sample is quantized into one of 256 equally likely levels. Assume the samples to be statistically independent.

(i) Calculate the information rate of the source.

(ii) Can the output of the source be transmitted without error over an AWGN channel with a bandwidth of 10 kHz and $\left(\frac{S}{N}\right)$ ratio of 40 dB?

(iii) Find the $\left(\frac{S}{N}\right)$ ratio so that the output of this source is transmitted without error over an AWGN channel with a bandwidth of 10 kHz.

(iv) Find the bandwidth requirement for an AWGN channel for an error free transmission of the output of this source if $\left(\frac{S}{N}\right)$ ratio is 40 dB.

[20 marks]

Q.2 (c) **(i)** The two sided power spectral density of the channel noise is 1×10^{-11} W/Hz and the carrier used in the transmitter is 15 cos $(2\pi f_c t)$ mV. Binary data (equiprobable bits) with a rate of 0.5 Mbps is transmitted through an AWGN channel using different modulation schemes. In each case of different modulation schemes, the signal are received by their respective correlator receiver with exact phase synchronisation and with optimum threshold detection. Find the average symbol error probability for modulation schemes BASK, BFSK and BPSK.

(ii) For a minimum hamming distance of "5",

1. How many errors can be detected?
2. How many errors can be detected and corrected?

[14 + 6 marks]

Q.3 (a) A Gaussian signal pulse given by,

$$x(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t^2/2\sigma^2)}$$

is applied to the input of matched filter and the noise on the channel is a white noise with power density spectrum of $\frac{N_0}{2} = 10^{-20}$ Watt/Hz, then calculate the maximum signal to noise ratio $\left(\frac{S}{N}\right)_{max}$ in dB achieved by this filter with $\sigma = 1$.

[20 marks]

Sol^n

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

For matched filter

$$h(t) = x^*(T - t)$$

$$\frac{N_0}{2} = 10^{-20} \text{ watt/Hz}$$

Max (SNR) in matched filter is given by

$$(SNR)_{max} = \frac{2E_b}{N_0}$$

$$N_0 = 2 \times 10^{-20} \text{ watt/Hz}$$

~~E_b = Power~~

$$E_b = var[x(t)]$$

$$E_b = \sigma^2$$

$$= 1$$

Do not write in this margin

~~$(SNR)_{max} = \frac{2\times 1}{2\times 10^{-20}}$~~

~~$= 10^{20}$~~

~~$\boxed{(SNR)_{max} = 200\,dB}$~~

Q.3 (b) For each of the following processes, find the power spectral density.

(i) $X(t) = A\cos(2\pi f_0 t + \theta)$, where A is a constant and θ is a random variable uniformly distributed on $\left[0, \frac{\pi}{4}\right]$.

(ii) $X(t) = x + y$, where x and y are independent, x is uniformly distributed on [–1, 1] and y is uniformly distributed on [0, 1].

[10 + 10 marks]

(i) $R_X(\tau) = E[X(t)\,X(t+\tau)]$

$= E[A\cos(2\pi f_0 t + \theta)\,A\cos(2\pi f_0(t+\tau) + \theta)]$

$= A^2 E[\cos(\omega_0 t + \theta)\cos(\omega_0(t+\tau) + \theta)]$

$= \frac{A^2}{2} E[\cos(\omega_0(2t+\tau) + 2\theta) + \cos\omega_0\tau]$

$= \frac{A^2}{2}\cos\omega_0\tau + \frac{A^2}{2} E[\cos(\omega_0(2t+\tau) + 2\theta)]$

$E[\cos\{\omega_0(2t+\tau) + 2\theta\}] = \frac{4}{\pi}\int_0^{\pi/4} \cos(\omega_0(2t+\tau) + 2\theta)\,d\theta$

$= \frac{4}{\pi}\left.\frac{\sin(\omega_0(2t+\tau) + 2\theta)}{2}\right|_0^{\pi/4}$

$= \frac{4}{2\pi}\left[\sin(\omega_0(2t+\tau) + \pi/2) - \sin\omega_0(2t+\tau)\right]$

$$= \frac{2}{\pi}\left[\cos[\omega_0(2t+\tau)] - \sin(\omega_0(2t+\tau))\right]$$

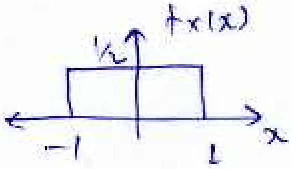
$$R_X(\tau) = \frac{A^2}{2}\cos\omega_0\tau + \frac{A^2}{\pi}\left[\cos[\omega_0(2t+\tau)] - \sin(\omega_0(2t+\tau))\right]$$

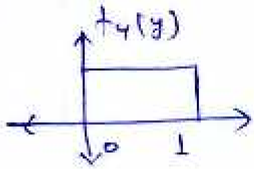
PSD $\leftrightarrow$ FT[ACF]

taking FT of $R_X(\tau)$

$$\boxed{\begin{aligned} S_X(\omega) &= \frac{A^2}{2}\cos\omega_0\tau \times 2\pi\delta(\omega) \\ &+ \frac{A^2}{\pi}\Big[\pi\{\delta(\omega+2\omega_0) + \delta(\omega-2\omega_0)\} \\ &\quad - \pi j\{\delta(\omega+2\omega_0) - \delta(\omega-2\omega_0)\}\Big] \end{aligned}}$$

(ii) $X(t) = x+y$





$$R_X(\tau) = E[X(t)X(t+\tau)]$$
$$= E[(x+y)(x+y)]$$
$$= E[x^2 + 2xy + y^2]$$
$$= E(x^2) + 2E(xy) + E(y^2)$$

$$\mathrm{Var}(x) = \frac{[1-(-1)]^2}{12} = \frac{1}{3} \qquad \mathrm{Var}(y) = \frac{1^2}{12} = \frac{1}{12}$$

$$\mathrm{Var}(x) = E(x^2) - |E(x)|^2 \qquad \mathrm{Var}(Y) = E(Y^2) - |E(Y)|^2$$

$$\frac{1}{3} = E(x^2) - 0 \qquad \frac{1}{12} = E(Y^2) - 0$$

$$\boxed{\frac{1}{3} = E(x^2)} \qquad \boxed{\frac{1}{12} = E(Y^2)}$$

$$R_X(\tau) = E(x^2) + 2\cancel{E(x)}^{0}\,\cancel{E(Y)}^{0} + E(Y^2)$$

$\because$ $E(XY) = E(X)E(Y)$ because X, Y are independent

$$= \frac{1}{3} + \frac{1}{12}$$

$$= \frac{5}{12}$$

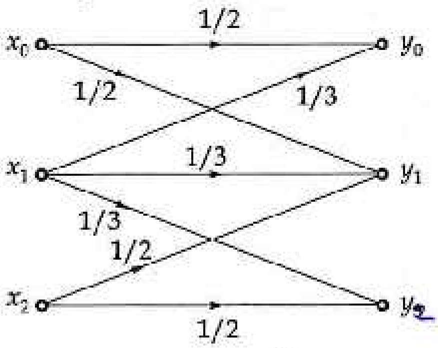
$PSD[X(t)] = FT[R_X(\tau)]$

$= FT\left[\frac{5}{12}\right]$

$S_X(\omega) = \frac{5}{12} \times 2\pi\, \delta(\omega)$

$$\boxed{S_X(\omega) = \frac{5\pi}{6}\delta(\omega)}$$

Q.3 (c) Consider the discrete memoryless channel shown below:



If the input probabilities are $P(x_0) = P(x_2) = \frac{1}{4}$ and $P(x_1) = \frac{1}{2}$, then determine the mutual information $I(X;Y)$.

[20 marks]

Soln $I(X;Y) = H(Y) - H(Y/X)$

$$P(Y/X) = \begin{array}{c} x_0 \\ x_1 \\ x_2 \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix} \\ \begin{matrix} y_0 & y_1 & y_2 \end{matrix} \end{array}$$

$P(Y) = P(X)\, P(Y/X)$

$$= \left[\tfrac{1}{4} \;\; \tfrac{1}{2} \;\; \tfrac{1}{4}\right] \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$= \left[\tfrac{1}{8}+\tfrac{1}{6} \qquad \tfrac{1}{8}+\tfrac{1}{6}+\tfrac{1}{8} \qquad \tfrac{1}{6}+\tfrac{1}{8}\right]$$

$$P(Y) = \left[\tfrac{7}{24} \quad \tfrac{10}{24} \quad \tfrac{7}{24}\right]$$

$$H(Y) = -\tfrac{7}{24}\times 2 \log_2 \tfrac{7}{24} + \tfrac{10}{24}\log_2 \tfrac{10}{24}$$

$$= \cancel{o}(2)\ \tfrac{7}{24}\times 1.77 + \tfrac{10}{24}\times 1.26$$

$$= 1.0325 + 0.525$$

$\boxed{H(Y) = 1.5575}$ bits / symbol

$P(X,Y) = P(X)_d\ P(Y/X)$

$$= \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$P(X,Y) = \begin{bmatrix} 1/8 & 1/8 & 0 \\ 1/6 & 1/6 & 1/6 \\ 0 & 1/8 & 1/8 \end{bmatrix}$$

$$H(Y/X) = -\sum_{i,j} P(x_i, y_j) \log_2 P(y_j/x_i)$$

$$= -\tfrac{1}{8}\log_2 \tfrac{1}{2} - \tfrac{1}{8}\log_2 \tfrac{1}{2} - \tfrac{1}{6}\log_2 \tfrac{1}{3}\times 3$$

$$- 2\times \tfrac{1}{8}\log_2 \tfrac{1}{2}$$

$= 0.125 \times 2 + 0.7924 + 0.5$

$H(Y/X) = 1.2924$ bits / symbol

$I(X;Y) = H(Y) - H(Y/X)$

$= 1.5575 - 1.2924$

$\boxed{I(X;Y) = 0.2651}$ bits / symbol

Q.4 (a) An AM signal has the form

$u(t) = [20 + 2\cos 3000\pi t + 10\cos 6000\,\pi t]\cos 2\pi f_c t$

where $f_c = 10^5$ Hz.

(i) Sketch the (voltage) spectrum of $u(t)$.

(ii) Determine the power in each of the frequency components.

(iii) Determine the modulation index.

(iv) Determine the power in the sidebands, the total power, and the ratio of the sidebands power to the total power.

[5 × 4 marks]

(i)

$u(t) = [20 + 2\cos 3000\pi t + 10\cos 6000\pi t]\cos 2\pi f_c t$

$= 20\cos 2\pi f_c t + 2\cos 3000\pi t \cos 2\pi f_c t + 10\cos 6000\pi t \cos 2\pi f_c t$

$u(t) = 20\cos 2\pi f_c t + \cos 2\pi[f_c + 1500]t + \cos 2\pi[f_c - 1500]t$

$+ 5\{\cos 2\pi[f_c + 3000]t + \cos 2\pi[f_c - 3000]t\}$

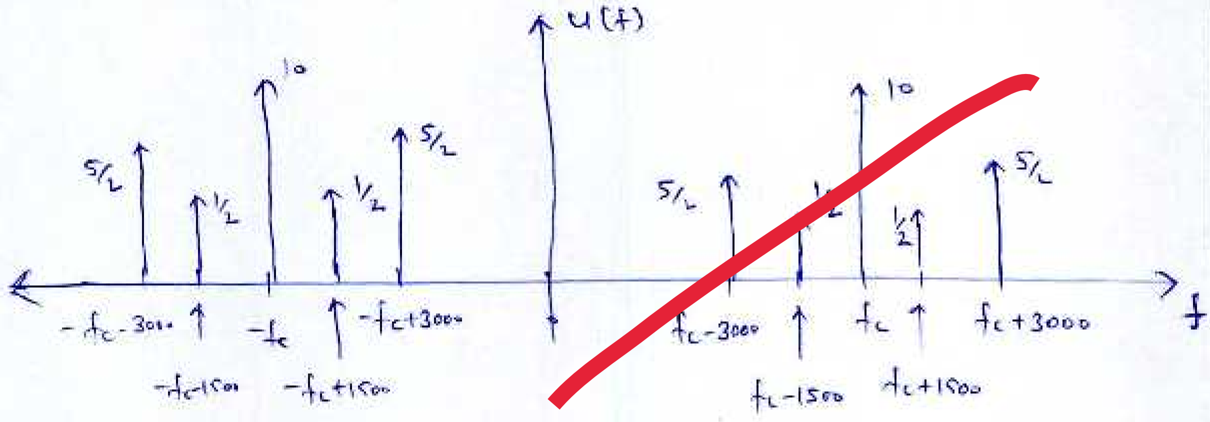
taking Fourier transform

$U(f) = 10[\delta(f + f_c) + \delta(f - f_c)] + \frac{1}{2}[\delta(f + f_c + 1500)$

$+ \delta[f - f_c - 1500)] + \frac{1}{2}[\delta(f + f_c - 1500) + \delta(f - f_c + 1500)]$

$+ \frac{5}{2}[\delta(f + f_c + 3000) + \delta(f - f_c - 3000)] + \frac{5}{2}[\delta(f + f_c - 3000)$

$+ \delta(f - f_c + 3000)]$



(ii) Power in f_c frequency $= \frac{10^2}{2} + 10^2$ ($= 10^5$ Hz)

$= 200$ watt

Power in $|(f_c+1500)|$ Hz $= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$ watt

Power in $|f_c - 1500|$ Hz $= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$ watt

Power in $|f_c + 3000|$ Hz $= \frac{25}{4} + \frac{25}{4}$

$= \frac{25}{2}$ watt

Power in $|f_c - 3000|$ Hz $= \frac{25}{4} + \frac{25}{4}$

$= \frac{25}{2}$ watt

(iii) $u(t) = 20\left[1 + \frac{1}{10}\cos 3000\pi t + \frac{1}{2}\cos 6000\pi t\right]\cos 2\pi f_c t$

$\beta_1 = K_{a_1} A_1 = \frac{1}{10}$ $\qquad \beta_2 = K_{a_2} A_2 = \frac{1}{2}$

$\beta = \sqrt{\beta_1^2 + \beta_2^2}$

Modulation index $\boxed{\beta = 0.51}$

(iv) SB are $= |f_c + 1500|, |f_c - 1500|, |f_c + 300|, |f_c - 300|$

Power in SB $= \frac{1}{2} + \frac{1}{2} + \frac{25}{2} + \frac{25}{2}$

$= 1 + 25$

$= 26$ watt

total power $= 200 + \frac{1}{2} + \frac{1}{2} + \frac{25}{2} + \frac{25}{2}$

$= 226$ watt

Ratio of sideband to total power $= \frac{26}{226}$

$= 0.115$

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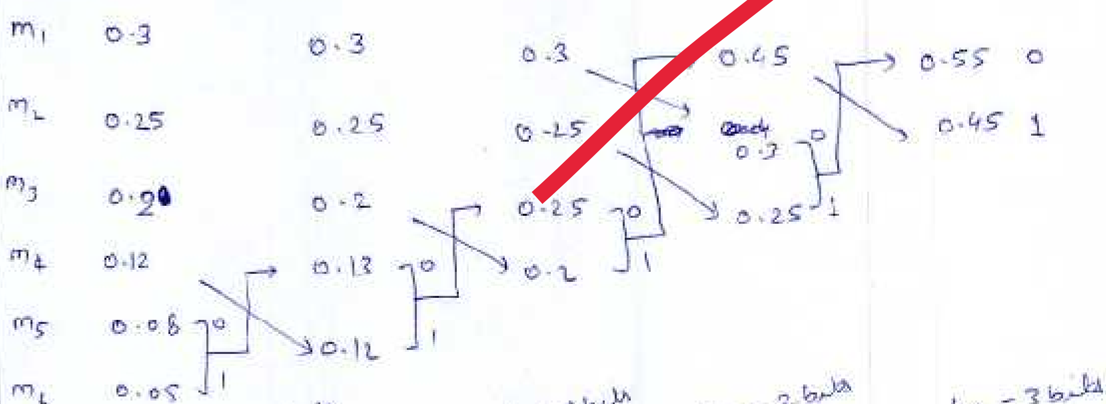
Q.4 (b) (i) A message source generates six message symbols $m_1, m_2, \ldots m_6$ with probabilities 0.3, 0.2, 0.08, 0.25, 0.12, 0.05 respectively. Give Huffman code for these symbols. Determine the efficiency and redundancy of the code.

(ii) For an AM modulator with carrier frequency f_c = 200 kHz and a maximum modulating signal frequency $f_{m(\max)}$ = 6 kHz, determine,

1. Frequency limits for the upper and lower sidebands.
2. Bandwidth
3. Upper and lower side frequencies produced when the modulating signal is a single frequency 2 kHz tone.

[10 + 10 marks]

Solⁿ



$L_1 = 2$ bits, $L_2 = 2$ bits, $L_3 = 2$ bits, $L_4 = 3$ bits

$m_1 = 00$; $m_2 = 01$; $m_3 = 11$; $m_4 = 101$

$L_5 = 4$ bits, $L_6 = 4$ bits

$m_5 = 1000$; $m_6 = 1001$

$$L = \underset{\rightarrow \text{bits}}{L_1} P(m_1) + L_2 P(m_2) + L_3 P(m_3) + L_4 P(m_4) + L_5 P(m_5) + L_6 P(m_6)$$

$$= 2 \times 0.3 + 2 \times 0.25 + 2 \times 0.2 + 3 \times 0.12 + 4 \times 0.08 + 4 \times 0.05$$

$$= 0.6 + 0.5 + 0.4 + 0.36 + 0.32 + 0.20$$

$$= 2.38 \text{ bits/symbol}$$

$$H = \sum_k -P_k \log_2 P_k$$

$$= -0.3 \log_2 0.3 - 0.2 \log_2 0.2 - 0.08 \log_2 0.08 - 0.25 \log_2 0.25 - 0.12 \log_2 0.12 - 0.05 \log_2 0.05$$

$= 0.5210 + 0.464 + 0.2914 + 0.5 + 0.3670 + 0.2160$

$= 2.3595$

Efficiency $= \frac{\cancel{K}}{\cancel{H}} \; \frac{H}{L}$

$= \frac{2.3595}{2.38}$

$\eta = 99.13\%$

or

$\eta = 0.9913$

Redundancy $= 1 - \eta$

$= 1 - 0.9913$

$= 8.7 \times 10^{-3}$

(ii) Given $f_c = 200\,kHz$

$f_m|_{max} = 6\,kHz$

① Frequency limit for upper sideband $= f_c + f_m$

$= (200 + 6)\,kHz$

$= 206\,kHz$

upper SB $\in [200 - 206]\,kHz$

Frequency limit for lower sideband $= f_c - f_m$

$= (200 - 6)\,kHz$

$= 194\,kHz$

lower SB $\in [194 - 200]\,kHz$

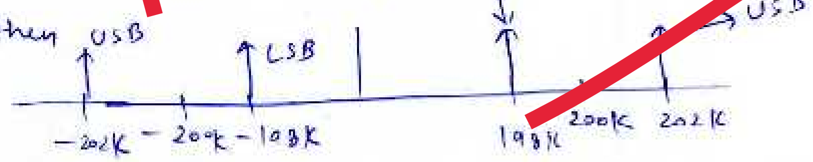
② BW $= 2f_m$

$= 2 \times 6K$

$= 12\,kHz$

③ when $f_m = 2\,kHz$

then



Q.4 (c) A single-tone modulating signal $m(t) = A_m \cos(2\pi f_m t)$ is used to generate the VSB signal

$$S(t) = \frac{1}{2} a A_m A_c \cos\left[2\pi(f_c + f_m)t\right] + \frac{1}{2} A_m A_c (1-a) \cos\left[2\pi(f_c - f_m)t\right]$$

where 'a' is a constant, less than unity, representing the attenuation of the upper side frequency.

(i) Find the quadrature component of the VSB signal $S(t)$.

(ii) The VSB signal, plus the carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced in recovering the message signal.

(iii) What is the value of constant 'a' for which this distortion reaches its worst possible condition?

[20 marks]

Sol^n $s(t) = \frac{1}{2} a A_m A_c \cos\left[2\pi(f_c+f_m)t\right] + \frac{1}{2} A_m A_c \cos\left[2\pi(f_c-f_m)t\right]$

$- \frac{A_m A_c a}{2} \cos\left[2\pi(f_c-f_m)t\right]$

$$s(t) = \frac{a A_m A_c}{2} \cos\left[2\pi(f_c+f_m)t\right] + \frac{A_m A_c}{2}(1-a)\cos\left[2\pi(f_c-f_m)t\right]$$

$$= \frac{a A_m A_c}{2}\left[\cos \omega_c t \cos \omega_m t - \sin \omega_c t \sin \omega_m t\right]$$

$$+ \frac{A_m A_c}{2}(1-a)\left[\cos \omega_c t \cos \omega_m t + \sin \omega_c t \sin \omega_m t\right]$$

Do not write in this margin

From the above $s(t)$ expression quadrature component of $s(t)$ is

$$S_q(t) = -\frac{a A_m A_c}{2} \sin\omega_m t + \frac{A_m A_c}{2}(1-a)\sin\omega_m t$$

(ii) when VSB signal is added with $A_c \cos\omega_c t$

$$s'(t) = \frac{a A_m A_c}{2}\cos\omega_m t \cos\omega_c t - \frac{a A_m A_c}{2}\sin\omega_m t \sin\omega_c t + \frac{A_m A_c}{2}(1-a)\cos\omega_m t \cos\omega_c t + \frac{A_m A_c}{2}(1-a)\sin\omega_m t \sin\omega_c t + A_c \cos\omega_c t$$

ED o/p would be

$$(o/p)_{ED} = \sqrt{\left[A_c + \frac{a A_m A_c}{2}\cos\omega_m t + \frac{A_m A_c}{2}(1-a)\cos\omega_m t\right]^2 + \left[\frac{A_m A_c}{2}(1-a)\sin\omega_m t - \frac{a A_m A_c}{2}\sin\omega_m t\right]^2}$$

$$(o/p)_{ED} = \sqrt{\left(A_c + \frac{A_m A_c}{2}\cos\omega_m t\right)^2 + \left(\frac{A_c A_m}{2} - a A_m A_c \sin\omega_m t\right)^2}$$

$$= \sqrt{A_c^2 + \frac{A_m^2 A_c^2}{4}\cos^2\omega_m t + A_m A_c^2 \cos\omega_m t + \frac{A_c^2 A_m^2}{4} + a^2 A_m^2 A_c^2 \sin^2\omega_m t - a A_c^2 A_m^2 \sin\omega_m t}$$

$$= \sqrt{A_c^2 + A_m^2 A_c^2\left\{\frac{\cos^2\omega_m t}{4} + a^2 \sin^2\omega_m t\right\} + \frac{A_c^2 A_m^2}{4} + A_c^2 A_m\left\{\cos\omega_m t - a A_m \sin\omega_m t\right\}}$$

DC will be blocked $\cos^2\omega_m t$ and $\sin^2\omega_m t$ term will produce distortion in msg signal

Do not write in this margin

(ii) for $\boxed{a=0.1}$ $\sin^2 \omega_m t$ term magnitude will be maximum hence it will produce maximum distortion

Section B : Signals and Systems-1 + Microprocessors and Microcontroller-1 + Network Theory-2 + Control Systems-2

Q.5 (a) Consider a system described by the differential equation $\ddot{y}(t)+2y(t)+3\dot{y}(t)=x(t)$ with $x(t)=3e^{-4t}$, $y(0)=3$ and $\dot{y}(0)=4$. Find its Z.I.R and Z.S.R.

[12 marks]

For ZIR $x(t)=0$

$\ddot{y}(t)+2y(t)+3\dot{y}(t)=0$

taking LT

$s^2Y(s)-sy(0)-\dot{y}(0)+2Y(s)+3[sY(s)-y(0)]=0$

$s^2Y(s)-3s-4+2Y(s)+3sY(s)-3\times 3=0$

$Y(s)[s^2+2+3s]=3s+4+9$

$Y(s)=\frac{3s+13}{s^2+3s+2}=\frac{3s+13}{(s+2)(s+1)}$

$\frac{3s+13}{(s+2)(s+1)}=\frac{A}{s+2}+\frac{B}{s+1}$

$A+B=3$ $\quad$ $A+2B=13$

$A+B=3$
$A+2B=13$
$-B=-10$
$B=10$

$A=13-20$
$=-7$

$\Rightarrow \frac{-7}{s+2}+\frac{10}{s+1}$

$\boxed{y(t)=(-7e^{-2t}+10e^{-t})u(t)}$ ZIR

<u>ZSR</u> $\ddot{y}(t) + 2y(t) + 3\dot{y}(t) = x(t)$

$$s^2 Y(s) + 2Y(s) + 3sY(s) = \frac{3}{s+4}$$

$$Y(s) = \frac{3}{(s+4)(s^2+3s+2)} = \frac{3}{(s+4)(s+1)(s+2)} = \frac{A}{s+4} + \frac{B}{s+1} + \frac{C}{s+2}$$

After solving $A = \frac{1}{2}$ $B = 1$ $C = -\frac{3}{2}$

$$y(s) = \frac{1/2}{s+4} + \frac{1}{s+1} + \frac{-3/2}{s+2}$$

$$\boxed{y(t) = \left(\frac{1}{2}e^{-4t} + e^{-t} - \frac{3}{2}e^{-2t}\right)u(t)}$$

Q.5 (b) Describe the following instructions of 8086:

(i) LDS R_d, M

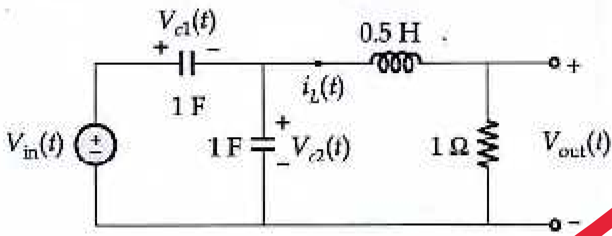
(ii) AAM

(iii) DAS

(iv) CLI

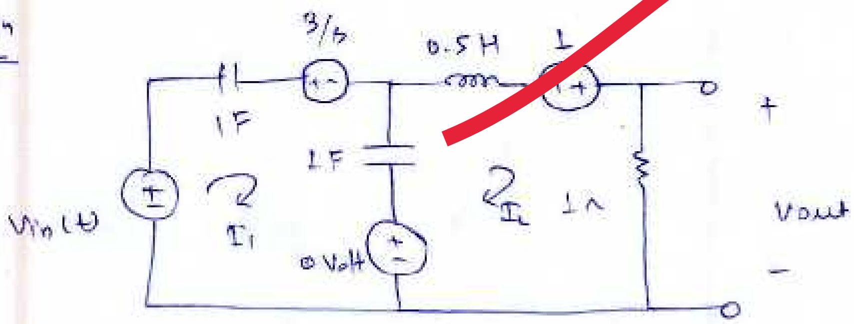
[12 marks]

Q.5 (c) Consider the circuit below in which $V_{in}(t) = 5u(t)$ V, $V_{c1}(0^-) = 3$ V, $V_{c2}(0^-) = 0$ V and $i_L(0^-) = 2$ A. Find $V_{out}(t)$ and also obtain V_{out} at $t = 1$ sec.



[12 marks]

Sol^n



KVL in loop 1

$$V_{in}(s) = \frac{I_1(s)}{sC_1} + \frac{3}{s} + \frac{I_1(s) - I_2(s)}{sC_2}$$

$$\frac{5}{s} = I_1(s)\left[\frac{1}{s} + \frac{1}{s}\right] - \frac{I_2(s)}{s} + \frac{3}{s}$$

$$\frac{5}{s} = \frac{2I_1(s)}{s} - \frac{I_2(s)}{s} + \frac{3}{s}$$

$$\frac{2}{\cancel{s}} = \frac{2}{\cancel{s}} I_1(s) - \frac{I_2(s)}{\cancel{s}}$$

$$\boxed{2 = 2I_1(s) - I_2(s)} \quad — (1)$$

$$I_2(s)\frac{s}{2} + 1 + I_2(s) + \frac{I_2(s) - I_1(s)}{s} = 0$$

$$I_2(s)\left[\frac{s}{2} + 1 + \frac{1}{s}\right] - \frac{I_1(s)}{s} + 1 = 0$$

$$\frac{I_1(s)}{s} - I_2(s)\left[1 + \frac{s}{2} + \frac{1}{s}\right] = -1$$

$$\frac{I_1(s)}{s} - I_2(s)\left[\frac{2s + s^2 + 2}{2s}\right] = -1 \quad — (B)$$

from eq^n (1) $\quad I_1(s) = \dfrac{2 + I_2(s)}{2}$

Putting $I_1(s)$ value in eq^n (B)

$$\frac{2+I_2(s)}{2} - I_2(s)\left[\frac{s^2+2s+2}{2s}\right] = -1$$

$$I_2(s)\left[\frac{1}{2} - \frac{s^2+2s+2}{2s}\right] = -2$$

$$I_2(s)\left[\frac{s-s^2-2s-2}{2s}\right] = -2$$

$$I_2(s)\left[\frac{-s^2-s-2}{2s}\right] = -2$$

$$I_2(s) = \frac{4s}{s^2+s+2}$$

$$V_{out} = I_2(s) \times 1$$

$$= \frac{4s}{s^2+s+2}$$

$$V_{out}(s) = \frac{4s}{s^2+2\times\frac{1}{2}s+\frac{1}{4}+\frac{7}{4}}$$

$$= \frac{4s}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}$$

$$= \frac{4\left(s+\frac{1}{2}-\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}$$

$$= \frac{4\left(s+\frac{1}{2}\right)-2}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}$$

$$V_{out}(t) = 4e^{-t/2}\cos\frac{\sqrt{7}}{2}t - 2\times\frac{2}{\sqrt{7}}e^{-t/2}\sin\frac{\sqrt{7}}{2}t$$

$$= 4e^{-t/2}\left[\cos\frac{\sqrt{7}}{2}t - \frac{1}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right]$$

at $t = 1$

$$V_{out} = 2.4261\,[0.2453 - 0.366]$$

$$\boxed{V_{out} = -0.2928} \text{ volt}$$

Q.5 (d) The transfer function of a controller is given by,

$$G_c(s) = \frac{10s+4}{s}$$

If this controller is realised using an operational amplifier, then find the other parameters of the controller assuming the capacitor value of 25 µF.

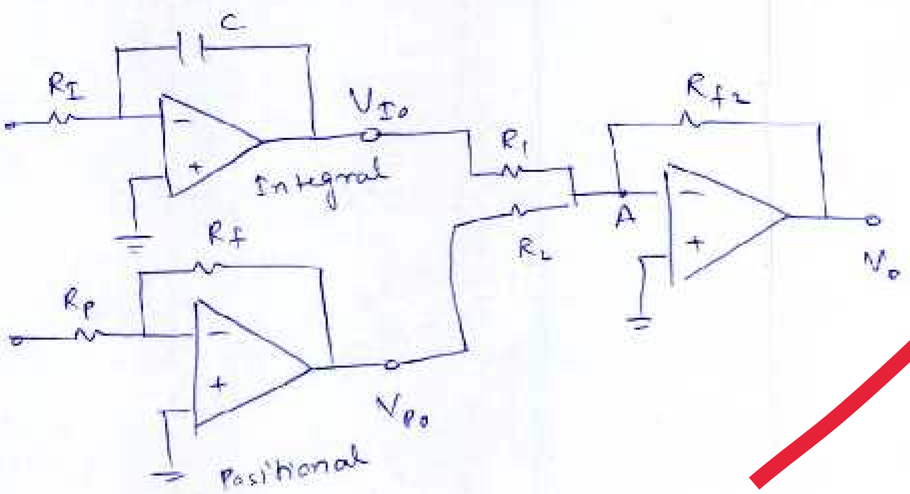
[12 marks]

Sol^n

$$G_c(s) = \frac{10s+4}{s}$$

$$= 10 + 4/s$$

It's an PI controller therefore its circuit will look like



TF of integral ckt = $\frac{1}{sR_I C}$

TF of proportional ckt = $-\frac{R_F}{R_p}$

KCL at Node

$$\frac{V_{Io}}{R_1} + \frac{V_{Po}}{R_2} = -\frac{V_o}{R_{f2}}$$

$$V_o = -V_{Io}\frac{R_{f2}}{R_1} - V_{Po}\frac{R_{f2}}{R_2}$$

$$\frac{V_o}{V_{in}} = -\frac{R_{f2}}{R_1}\cdot\frac{1}{sR_I C} - \frac{R_{f2}}{R_2}\cdot\frac{R_F}{R_p}$$

considering only magnitude of $\frac{V_o}{V_{in}}$ as -ve sign can be taken care by using another op-amp of inverting configuration

$$\frac{R_{f2}}{sR_1R_I C} + \frac{R_{f2}}{R_2}\frac{R_F}{R_p} = 10 + 4/s$$

After comparing

$\rightarrow \frac{R_{f2}}{R_1R_I C} = 4$ $\rightarrow \frac{R_{f2}}{R_2}\cdot\frac{R_F}{R_p} = 10$

$\frac{R_{f2}}{R_1R_I} = 4 \times C$ $\qquad \frac{\cancel{R}}{\cancel{R}} \times \frac{R_F}{R_p} = 10$

$\frac{R_{f2}}{R_1R_I} = 4\times 25\times 10^{-6}$ $\qquad R_F = 10R_p$

for $\boxed{R_p = 1\,k\Omega}$

$\boxed{R_F = 10\,k\Omega}$

Let $R_1 = R_2 = R_{f2} = R$

$$\frac{\cancel{R}}{R\cdot R_I} = 4\times 25\times 10^{-6}$$

$R_I = 10^4\,\Omega$

$\boxed{R_I = 10\,k\Omega}$

Rest R_1, R_2 and R_{f2} can be choosen according to our desired gain

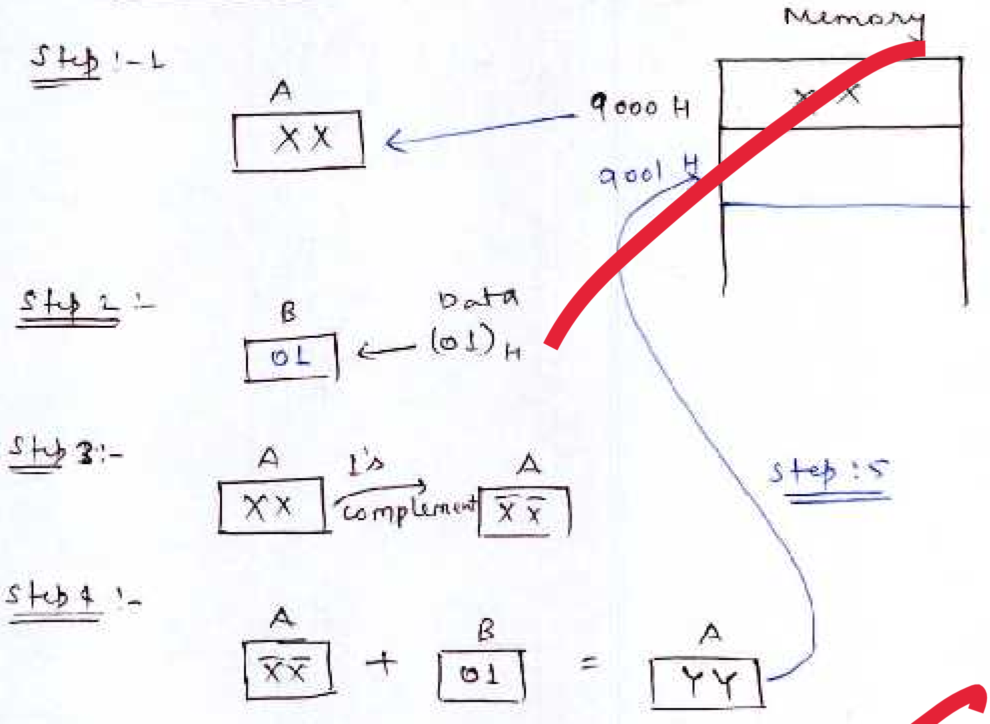
Q.5 (e) Write a 8085 program to find 2's complement of the number stored in memory location 9000 H, and store the result in memory location 9001 H. Also give the flow chart of the program and calculate execution time of program if operating frequency is 5 MHz.

[12 marks]

Solⁿ

```
LDA   9000H     // Loading data @ 9000H into ACCUM
MVI   B, 01H    // setting B = 01H
CMA             // taking 1's complement of Accumulator
ADD B           // [A] = (A) + [B]
                              ↳ (01)H
STA   9001H     // storing content of Accumu-
                   -lator from accumulator to
                   memory location 9001 H
```

Flow chart



→ Step 1-1

4T + 3T×2 + 3T×1 = 13T
↓ opcode fetch, ↓ 2 MEMR, ↓ 1 MEMR

→ Step 1-2

4T + 3T = 7T
↓ Opcode fetch, ↓ 1 MEMR

Step 3 :- 4T

Step 4 :- 4T

Step 5 :- 4T + 3T × 2 + 3T = 13T

OF 2MEMR 1MEMW

Total T-state = 13T + 7T + 4T + 4T + 13T
= 41T

total time = $\frac{41 \times 10^{-6}}{5}$

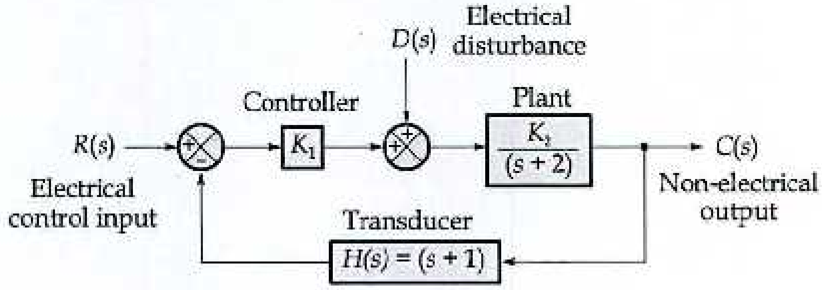
= 8.2 μsec

Q.6 (a) **(i)** Explain all the basic machine cycles of 8085 microprocessor and differentiate between instruction cycle (IC) and machine cycle (MC).

(ii) Draw the timing diagram of OUT instruction for 8085 microprocessor.

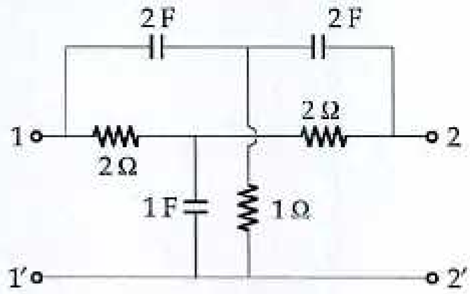
[10 + 10 marks]

Q.6 (b) For the system shown in the figure below, both the electrical control input and the disturbance are unit step signals. Find the sensitivity of the steady-state error for changes in K_1 and in K_2 individually, when $K_1 = 100$ and $K_2 = 0.10$.

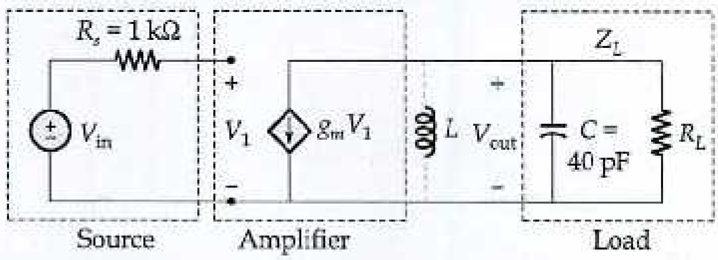


[20 marks]

Q.6 (c) **(i)** Determine the Y parameters of given network.



(ii) Below given figure displays an amplifier model containing a VCCS with $g_m = 2$ mS (milli-Siemens) and R_L = 20 kΩ. The applied sinusoidal voltage $V_{in}(j\omega)$ has a magnitude of 0.1 V at 10 MHz. The load is modeled by the parallel combination of R_L and the 40-pF capacitor. The capacitance accounts for such real-world phenomena as wiring capacitance, the device input capacitance, and other embedded capacitances. This capacitance cannot be removed from the circuit and often has deleterious effects on the amplifier performance.



1. With the load connected directly as shown (without L), find the magnitude of the output voltage.
2. If an inductance L is connected across the load to tune out the effect of the capacitance, find the value of L and the resulting $|V_{out}|$. What is the impact on the amplifier gain?

[8 + 12 marks]

Q.7 (a) (i) Explain the addressing modes of 8086 with one example each.

(ii) Obtain the physical address and effective address for different addressing modes of 8086 with the contents of register as given below:

Offset = 1000 H; [AX] = 5000 H; [BX] = 2000 H; [SI] = 3000 H; [BP] = 5000 H; [DI] = 4000 H; [SP] = 6000 H, [DS] = 7000 H

1. Register indirect addressing mode (assuming DI).
2. Based addressing mode (assuming BX)
3. Based index addressing mode (assuming DX).
4. Based index with displacement addressing mode (assuming BX).

[14 + 6 marks]

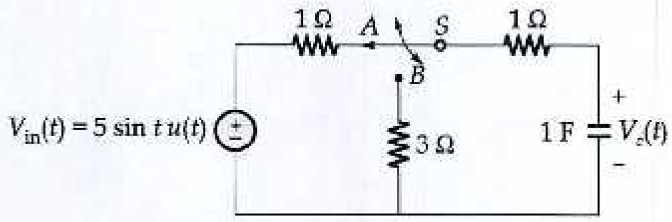
Q.7 (b) A system is described by the following state and output equations:

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)\,; \qquad \frac{dx_2(t)}{dt} = -2x_2(t) + u(t)\,; \qquad y(t) = x_1(t)$$

If $u(t)$ is the input and $y(t)$ is the output, then find the system transfer function and state transition matrix of the above system.

[20 marks]

Q.7 (c) In the circuit given below $V_{in}(t) = 5 \sin t\, u(t)$ V and $V_c(0^-) = 0$. The switch is initially in position A. The switch 'S' moves from position 'A' to position 'B' at t = 1s and from position 'B' to position 'A' at t = 2s, where it remains for all subsequent time. Find $V_c(t)$ for $t \geq 0$.



[20 marks]

Q.8 (a) Determine the unilateral Laplace transform of the signals given below. Specify the property used, if any, in each step.

(i) $x(t) = [u(t-1) + u(-t-4)] * e^{-2t}u(t-1)$

(ii) $x(t) = t \cdot \frac{d}{dt}\left[e^{-t} \cdot \cos t\, u(t) + e^{-(t+1)}u(-(t+1))\right]$

[10 + 10 marks]

Soln (i) $x(t) = [u(t-1) + u(-t-4)] * e^{-2t}u(t-1)$

$= \underbrace{u(t-1) * e^{-2t}u(t-1)}_{x_1(t)} + \underbrace{u(-t-4) * e^{-2t}u(t-1)}_{x_2(t)}$

$x_1(t) = u(t-1) * e^{-2t}u(t-1)$

using property $x_1(t) * x_2(t) \Leftrightarrow X_1(s)X_2(s)$
and $x(t-T) \Leftrightarrow e^{-sT}X(s)$

$X_1(s) = \frac{e^{-s}}{s} \times e^{-2} \cdot \frac{e^{-s}}{s+2}$

$\boxed{X_1(s) = \frac{e^{-(s+2)}}{s(s+2)}}$ — ©

$x_2(t) = u(-t-4) * e^{-2t}u(t-1)$

$u(t) \Leftrightarrow 1/s$

$u(t-4) \Leftrightarrow \frac{e^{-4s}}{s}$ {using time shifting property}

$u(-t-4) \Leftrightarrow \frac{e^{4s}}{-s}$ — (A) { Using property $x(-t) \Leftrightarrow X(-s)$ }

$e^{-2t}u(t-1) = e^{-2(t-1+1)}u(t-1)$

$= e^{-2(t-1)}\, e^{-2}\, u(t-1)$

$= e^{-2}\cdot e^{-2(t-1)}u(t-1)$

$e^{-2t}u(t) \Leftrightarrow \frac{1}{s+2}$

Using time shifting property $x(t-T) \Leftrightarrow e^{-sT}X(s)$

$e^{-2(t-1)}u(t-1) \Leftrightarrow \frac{e^{-s}}{s+2}$

$e^{-2}\, e^{-2(t-1)}u(t-1) \Leftrightarrow \frac{e^{-(s+2)}}{s+2}$ — (B)

$x_2(t) = u(-t-4) * e^{-2t}u(t-1)$

→ From eqn (A) and (B) and using property

$x_1(t) * x_2(t) \Leftrightarrow X_1(s)X_2(s)$

$X_2(s) = \frac{-e^{4s}}{s} \times \frac{e^{-(s+2)}}{s+2}$ — (D)

$x(t) = x_1(t) + x_2(t)$

$X(s) = X_1(s) + X_2(s)$ { From eqn (C) and (D)

$$\boxed{X(s) = \frac{e^{-(s+2)}}{s(s+2)} - \frac{e^{4s}e^{-(s+2)}}{s(s+2)}}$$

(ii) $x(t) = t\cdot \frac{d}{dt}\left[e^{-t}\cos t\, u(t) + e^{-(t+1)}u(-t-1)\right]$

→ $\cos t\, u(t) = \frac{s}{s^2+1} \Rightarrow e^{-t}\cos t\, u(t) = \frac{s+1}{(s+1)^2+1}$ { freq shifting property }

→ $e^{-t}u(t) = \frac{1}{s+1} \Rightarrow e^{-(t)}u(-t) = \frac{-1}{s+1}$ { time reversal prop } $x(-t) \Leftrightarrow X(-s)$

→ $e^{-(t+1)}u[-(t+1)] = \frac{-e^{s}}{s+1}$ { time shifting property } $x(t-T) \Leftrightarrow e^{-sT}X(s)$

therefore

$$\underbrace{e^{-t}\cos t\, u(t) + e^{-(t+1)}u[-(t+1)]}_{y(t)} = \frac{s+1}{(s+1)^2+1} - \frac{e^{s}}{s+1}$$

Using time differentiation prop $\frac{d}{dt} x(t) = s X(s)$

$$\frac{d}{dt} y(t) = \frac{s(s+1)}{(s+1)^2+1} - \frac{s e^{s}}{s+1}$$

Using frequency integration property $t^n x(t) = (-1)^n \int_{s}^{\infty} X(s)\, ds$

$$t \cdot \frac{d}{dt} y(t) = (-1) \int \left(\frac{s^2+s}{s^2+1+4+1} - \frac{s e^{s}}{s+1} \right) ds$$

$$(-1) \int \frac{s^2+s+2-2}{s^2+2s}$$

Q.8 (b) A series circuit consists of a 300 Ω non-inductive resistor, a 7.95 μF capacitor and a 2.06 H inductor of negligible resistance.

If the supply voltage is

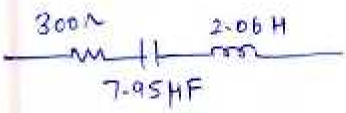
$v(t) = 250\sqrt{2}\sin(314t + 30°)$ V, calculate

(i) the circuit current,

(ii) the voltage drop across each component in the circuit,

(iii) the power consumed in the circuit.

[5 + 10 + 5 marks]



$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 7.95 \times 10^{-6}} = 400.6\,\Omega$$

$$X_L = \omega L = 314 \times 2.06 = 646.84\,\Omega$$

$$Z = R + jX_L - jX_C$$

$$= R + j[X_L - X_C]$$

$$= 300 + j[646.84 - 400.6]$$

$$Z = 300 + j\,246.24$$

$$Z = 388.116 \angle 39.38°$$

$$\boxed{I = \frac{V}{Z} = \frac{250\sqrt{2} \angle 30°}{388.116 \angle 39.38°} = 0.9109 \angle -9.38°\ A}$$

(ii) $V_R = IR$

$= 0.9109 \angle -9.38° \times 300$

$V_R = 273.28 \angle -9.38°$ Volt

$V_L = jIX_L$

$= (0.9109 \angle -9.38°)(646.84) \angle 90°$

$\boxed{V_L = 589.206 \angle 80.62°}$ Volt

$V_C = -jIX_C$

$= (0.9109 \angle -9.38°)(400.6 \angle -90°)$

$\boxed{V_C = 364.90 \angle -99.38°}$ Volt

(iii) Power consumed

$S = VI^*$

$= (250\sqrt{2} \angle 30°)(0.9109 \angle 9.38°)$

$\boxed{S = 322 \angle 39.38°}$ VA

$\boxed{\begin{array}{l} P = 248.43 \text{ watt} \\ Q = 204.29 \text{ VAR} \end{array}}$

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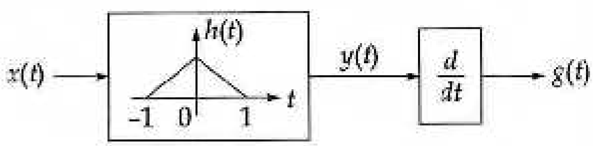
Q.8 (c) (i) Consider an LTI system has the impulse response $h(t)$ shown in figure below:



LTI

If the input $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$, then sketch output $g(t)$.

(ii) A voltage waveform $V(t)$ has a period $T = 2$ second, its Fourier series coefficient values are:

$C_0 = 1, C_1 = 2j, C_2 = 2$

Obtain the value of $V(t)$ at $t = 0$.

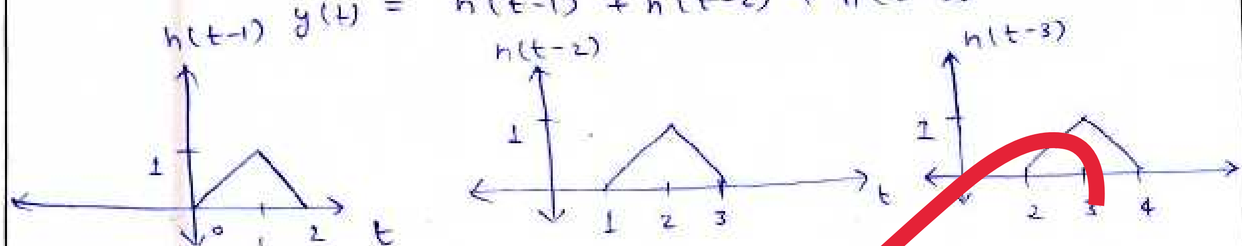
[10 + 10 marks]

Sol^n

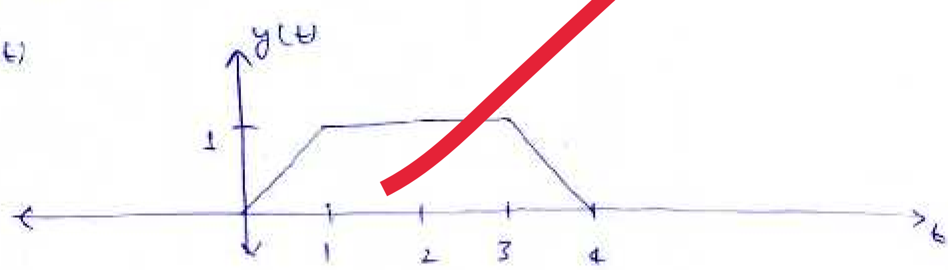
$x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

$y(t) = h(t) * x(t)$ using property $\{ x(t) * \delta(t-t_0) = x(t-t_0) \}$

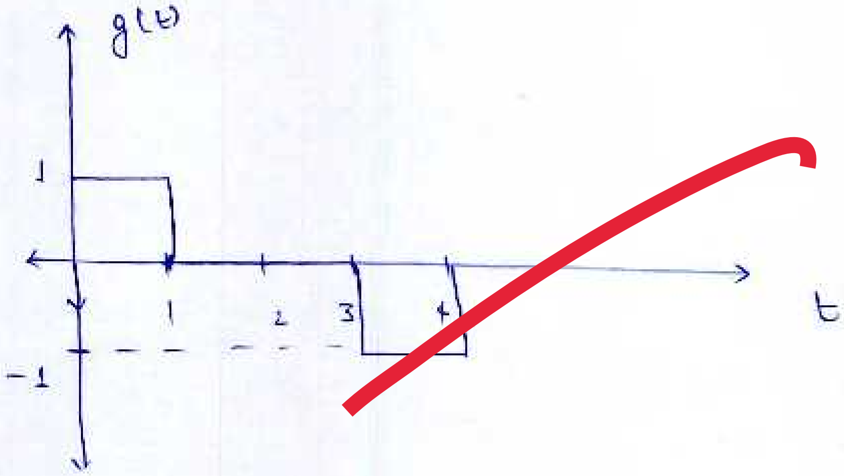
$y(t) = h(t-1) + h(t-2) + h(t-3)$



y(t)



$g(t) = \frac{d}{dt} y(t)$



(ii) $v(t)$ periodic with $T_0 = 2$

$c_0 = 1$ $\quad c_1 = 2j$ $\quad c_2 = 2$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

exponential fourier series is given by

$v(t) = \sum_k c_k e^{jk\omega_0 t}$

~~$v(t) = c_0 + c_1 e^{j\omega_0 t} + c_2 e^{j2\omega_0 t}$~~

~~$v(t) = 1 + 2j e^{j\pi t} + 2e^{j2\pi t}$~~

~~Put $t=0$ in above eq^n~~

~~$v(0) = 1 + 2j + 2$~~

~~$v(0) =$~~

Since $v(t)$ voltage source will be real

$v(t) = v^*(t)$

therefore $c_k = c^*_{-k}$

$C_{-1} = C_1^*$

$\boxed{C_{-1} = -2j}$

$\boxed{C_{-2} = 2}$

$$V(t) = C_{-2} e^{-j2\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t}$$

$$= 2 e^{-j2\pi t} - 2j e^{-j\pi t} + 1 + 2j e^{j\pi t} + 2 e^{j2\pi t}$$

Put $t=0$ in above eqn

$V(0) = 2 - \cancel{2j} + 1 + \cancel{4} + 2$

$\boxed{V(0) = 5 \text{ Volt}}$

Space for Rough Work

Space for Rough Work

Space for Rough Work

$\frac{d}{dt}x(t)\ \frac{d}{dt}x(t+\tau)$

$\boxed{\frac{d}{dt}\ R_x(\tau)}$

$-7s-7+10s+20$

$3s+13$

$\wedge$

$e^{-2t}\ u(t-1)$

$e^{-2(t-1+1)}\ u(t-1)$

$e^{-2(t-1)}\ e^{-2}\ u(t-1)$

$e^{-2}\ \frac{e^{-s}}{s+2}$

$v(t) = v^{+}(t)$

$c_k = c_{-k}^{*}$