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- Write all steps in detail
- Do not left question incomplete

ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Systems and Signal Processing + Microprocessors + Electrical Circuits-1 + Control Systems-1

Name :

Roll No :

Test Centres	Student's Signature
Delhi <input checked="" type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Kolkata <input type="checkbox"/> Bhubaneswar <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	44
Q.2	20
Q.3	
Q.4	
Section-B	
Q.5	54
Q.6	48
Q.7	30
Q.8	
Total Marks Obtained	200 196

Signature of Evaluator

Cross Checked by

- Draw diagram properly.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Systems and Signal Processing + Microprocessors

Q.1 (a) The input $x(n]$ and the impulse response $h(n]$ of a discrete-time LTI system are given by

$$x(n) = \alpha^n u(n), h(n) = \alpha^{-n} u(-n); 0 < \alpha < 1$$

Using z-transform, find the response $y(n]$.

[12 marks]

$$x(n) = \alpha^n u(n)$$

$$0 < \alpha < 1$$

\Downarrow Z-T

$$X(z) = \frac{1}{1 - \alpha z^{-1}}$$

$$|z| > |\alpha|$$

*write
all steps*

also $h(n) = \alpha^{-n} u(n)$

$$= \left(\frac{1}{\alpha}\right)^n u(-n)$$

$$= \left(\frac{1}{\alpha}\right)^{n-1+1} u(-(n-1)+1)$$

$$= \left(\frac{1}{\alpha}\right)^{(n-1)} \cdot \left(\frac{1}{\alpha}\right) u(-(n-1)-1)$$

$$= \cancel{\left(\frac{1}{\alpha}\right)^{n-1+1}}$$

$$= \left(\frac{1}{\alpha}\right) \left(\frac{1}{\alpha}\right)^{n-1} u(-(n-1)-1)$$

applying ZT

$$h(n) \Rightarrow H(z) = \frac{1}{\alpha} \left[\frac{-1}{1 - \left(\frac{1}{\alpha}\right) z^{-1}} \right] \cdot z^{-1} \quad |z| < \left|\frac{1}{\alpha}\right|$$

using time shifting property

$$h(n) \Rightarrow H(z)$$

$$h(n-n_0) \Rightarrow H(z) \cdot z^{-n_0}$$

now $y(n) = x(n) * h(n)$

$$\mathcal{ZT}$$

$$Y(z) = X(z) \cdot H(z) \quad (ROC_1 \cap ROC_2)$$

$$\therefore Y(z) = \left(\frac{1}{1-\alpha z^{-1}} \right) \cdot \left(\frac{-1}{\alpha} \right) \frac{z^{-1}}{1-\frac{1}{\alpha} z^{-1}} \quad ; |\alpha| < |z| < \frac{1}{|\alpha|}$$

$$\frac{Y(z)}{z^{-1}} = \left(\frac{-1}{\alpha} \right) \frac{1}{(1-\alpha z^{-1})(1-\frac{1}{\alpha} z^{-1})}$$

$$= \left(\frac{-1}{\alpha} \right) \left[\frac{\left(\frac{1}{1-\frac{1}{\alpha} z^{-1}} \right)}{1-\alpha z^{-1}} + \frac{\left(\frac{1}{1-\alpha^2} \right)}{1-\frac{1}{\alpha} z^{-1}} \right]$$

$$= \left(\frac{-1}{\alpha} \right) \left[\frac{\left(\frac{\alpha^2}{\alpha^2-1} \right)}{1-\alpha z^{-1}} + \left(\frac{\left(\frac{1}{1-\alpha^2} \right)}{1-\frac{1}{\alpha} z^{-1}} \right) \right]$$

$$= \frac{-1}{\alpha(1-\alpha^2)} \left[\frac{-\alpha^2}{1-\alpha z^{-1}} + \frac{1}{1-\frac{1}{\alpha} z^{-1}} \right]$$

$$Y(z) = \frac{1}{\alpha(1-\alpha^2)} \left[\frac{\alpha^2 z^{-1}}{1-\alpha z^{-1}} + \frac{z^{-1}}{1-\frac{1}{\alpha} z^{-1}} \right] \quad ; |\alpha| < |z| < \frac{1}{|\alpha|}$$

$$\mathcal{ZT} \text{ Inverse } z^{-1}$$

$$y(n) = \frac{1}{\alpha(1-\alpha^2)} \left[\alpha^2 \alpha^{n-1} u(n-1) - \left(\frac{1}{\alpha} \right)^{n-1} u(-(n-1)-1) \right]$$

$$= \frac{1}{\alpha(1-\alpha^2)} \left[\alpha^{n+1} u(n-1) - \alpha^{1-n} u(-n) \right]$$

$$y(n) = \frac{1}{1-\alpha^2} \left[\alpha^n u(n) + \alpha^{-n} u(-n-1) \right]$$

$$\therefore y(n) = \frac{1}{\alpha(1-\alpha^2)} \left[\alpha^{n+1} u(n-1) - \alpha^{1-n} u(-n) \right]$$

Q.1 (b) For an 8085 microprocessor, explain the followings :

- (i) Logical operations
- (ii) Branching operations

[8 + 4 = 12 marks]

(i) Logical operations — These are operations which are based on some digital logic like AND, OR, XOR. 8085 supports only 3 logical operations AND, OR, XOR.

These can be used wrt various operands like AND R, AND M, ANI 8 bit data. Similarly for OR & XOR.

AND — AND operation performs bit wise AND of the specified operand & accumulator [A]

$AND\ B \Rightarrow [A] AND [B]$

Let's take $A = 11H$, $B = 23H$

then $AND\ B$ will result in

$[A] \leftarrow [A] AND [B]$

00000001

00010001 AND 00100011

$\therefore [A] \rightarrow 01H$

These are based on Boolean logic.

(ii) Branching operations — these are operations which when executed takes the execution to a different memory location

some branching operations of 8085 are

CALL 16 bit address

JMP 16 bit address

These can be conditional, which got executed when certain flag is set or reset

as it can be unconditional which executed irrespective of flag value.

CALL 8000H

when this is executed, the sequence of execution is shifted to mem address location 8000H

while the previous value of ~~stack pointer~~ program counter [PC] is stored in stack

such that after RET (return) it can go to the previous location.

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Q.1 (c) Find the inverse laplace transform of the following :

(i) $X(s) = \frac{s^2 + 6s + 7}{s^2 + 3s + 2}; \text{Re}\{s\} > -1$

(ii) $X(s) = \frac{5s + 13}{s(s^2 + 4s + 13)}; \text{Re}\{s\} > 0$

[6 + 6 marks]

i) $\frac{s^2 + 6s + 7}{s^2 + 3s + 2} \quad \sigma > -1$

Here N^o degree = D^o degree

\therefore we first divide the polynomial

$$\begin{array}{r} s^2 + 3s + 2 \overline{) s^2 + 6s + 7} \quad 1 \\ \underline{s^2 + 3s + 2} \\ 3s + 5 \end{array}$$

$$\Rightarrow \frac{s^2 + 6s + 7}{s^2 + 3s + 2} = 1 + \frac{3s + 5}{(s+1)(s+2)}$$

$$\text{as } \sigma > -1 \Rightarrow \sigma > -2$$

\Rightarrow right sided signal

$$\therefore X(s) = 1 + \frac{2}{s+1} + \frac{1}{s+2}$$

applying inverse LT

$$x(t) = \delta(t) + 2e^{-t}u(t) + e^{-2t}u(t)$$

(ii)

$$\frac{5s + 13}{s(s^2 + 4s + 13)} \quad \rightarrow > 0$$

using partial fraction we can write as

$$\frac{5s + 13}{s(s^2 + 4s + 13)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 13}$$

residue at $s=0$ gives $A = 1$

$$\therefore s^2 + 4s + 13 + Bs^2 + Cs = 5s + 13$$

$$1 + B = 0 \quad \Rightarrow B = -1$$

$$4 + C = 5 \quad \Rightarrow C = 1$$

$$\therefore X(s) = \frac{1}{s} + \frac{-s + 1}{s^2 + 4s + 13}$$

$$= \frac{1}{s} - \frac{s}{(s+2)^2 + 3^2} + \frac{1}{(s+2)^2 + 3^2}$$

$$= \frac{1}{s} - \frac{(s+2)-2}{(s+2)^2 + 3^2} + \frac{1}{(s+2)^2 + 3^2}$$

$$= \frac{1}{s} - \frac{(s+2)}{(s+2)^2 + 3^2} + \frac{3}{(s+2)^2 + 3^2}$$

Taking inverse Laplace Transform

$$x(t) = u(t) - e^{-2t} \cos 3t u(t) + e^{-2t} \sin 3t u(t)$$

$$= (1 - e^{-2t} \cos 3t + e^{-2t} \sin 3t) u(t)$$

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Good
Approach

Q.1.(d) Explain the flag register of 8086 microprocessor.

[12 marks]

In 8086 μp , there are 9 flags

they are

S — sign flag = $\begin{cases} 0 & \text{+ve data} \\ 1 & \text{-ve data} \end{cases}$

Z — zero flag = $\begin{cases} 0 & \text{non-zero data} \\ 1 & \text{zero data} \end{cases}$

P — parity flag = $\begin{cases} 0 & \text{odd no of 1's} \\ 1 & \text{even no of 1's} \end{cases}$

CF — carry flag = $\begin{cases} 0 & \text{if no carry} \\ 1 & \text{if carry generated} \end{cases}$

AC — auxiliary carry = $\begin{cases} 0 & \text{if no auxiliary carry} \\ 1 & \text{if auxiliary carry present} \end{cases}$

OF — overflow = $\begin{cases} 0 & \text{if no overflow} \\ 1 & \text{if overflow occurs} \end{cases}$

and there are 3 more flags which are

Interrupt ,

Incomplete
solution

Q.1 (e) Consider the following discrete time system :

(i) $y(n) = |x(n)|$

(ii) $y(n) = \text{sgn}[x(n)]$

Check whether these systems are static or dynamic, linear or non-linear, time varying or time-invariant, causal or non-causal and stable or unstable.

[6 + 6 marks]

$$(i) \quad y(n) = |x(n)| = \begin{cases} x(n) & x(n) \geq 0 \\ -x(n) & x(n) < 0 \end{cases}$$

as present value of o/p depends only on present value of i/p \Rightarrow static system

o/p does not depend on future i/p \Rightarrow causal system

for every bounded i/p $x(n)$; $|x(n)|$ & hence $y(n)$ is bounded \Rightarrow stable system

$$x(n) \longrightarrow y(n) \xrightarrow{\text{delay}} y(n-n_0) = |x(n-n_0)|$$

$$x(n) \xrightarrow{\text{delay}} x(n-n_0) \longrightarrow y(n) = |x(n-n_0)|$$

as the delay in i/p is reflected in o/p

\Rightarrow time invariant system

$$x_1(n) \longrightarrow y_1(n) = |x_1(n)|$$

$$x_2(n) \longrightarrow y_2(n) = |x_2(n)|$$

$$x_1(n) + x_2(n) \longrightarrow y(n) = |x_1(n) + x_2(n)| \neq |x_1(n)| + |x_2(n)|$$

\Rightarrow non-linear system

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(ii)

$$y(n) = \text{sgn}(x(n)) = \begin{cases} 1 & x(n) > 0 \\ 0 & x(n) = 0 \\ -1 & x(n) < 0 \end{cases}$$

as present value of o/p depends only on present value of i/p

\Rightarrow static system

o/p is independent of future i/p

\Rightarrow causal system

~~for $x(n) = 10$ (bounded)~~

for $x(n) > 0$ (any bounded $x(n)$)

$y(n) = 1$ is ~~unbounded~~

\Rightarrow unstable system

system is stable

4

$$x_1(n) \longrightarrow y_1(n) = \text{sgn}(x_1(n))$$

$$x_2(n) \longrightarrow y_2(n) = \text{sgn}(x_2(n))$$

$$x_1 + x_2 \longrightarrow y(n) = \text{sgn}[x_1(n) + x_2(n)] \neq \text{sgn}(x_1(n)) + \text{sgn}(x_2(n))$$

\Rightarrow non-linear system

as $y(n)$ can take only 3 discrete values
 \Rightarrow delay in input will not reflect in o/p

\Rightarrow time variant system

Q.2 (a) Derive the DFT of the sample data sequence $x(n) = \{1, 1, 2, 2, 3, 3, 0, 0\}$.

[20 marks]

- Q.2(b) Write an assembly language program in 8085 to find 1's and 2's complement of 16-bit number. Assume that the number is stored at 2040 H and store the result at 2050 H and 2052 H respectively. Also give the algorithm of the program.

[20 marks]

To find 1's complement of a 16-bit number,
we can subtract the number from FFFF H

To find 2's complement of a 16-bit number,
we can add 0001 H to its 1's complement

2 3

LXI H, 2040 H

MOV C, M

INX H

MOV B, M

MVI A, FFH

Incomplete
solution

Q.2 (c) (i) A causal and stable LTI system S has the frequency response :

$$H(\omega) = \frac{4 + j\omega}{6 - \omega^2 + 5j\omega}$$

(a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S .

(b) Determine the impulse response $h(t)$ of S .

(c) What is the output of S when the input is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.

(ii) Compute the linear convolution of the following sequence to obtain $y(n)$

$$x(n) = \{1, 3, 0, 4, -2\}$$

↑

$$h(n) = \{2, 4, -1, -3\}$$

↑

[15 + 5 marks]

(i)

(a)

$$H(\omega) = \frac{4 + j\omega}{6 - \omega^2 + 5j\omega} = \frac{4 + j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{4 + j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$\Rightarrow (j\omega)^2 Y(\omega) + 5j\omega Y(\omega) + 6 Y(\omega) = j\omega X(\omega) + 4 X(\omega)$$

applying inverse Fourier Transform

$$\frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 6y = \frac{dx}{dt} + 4x$$

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

(b)

applying LT

$$\frac{Y(s)}{X(s)} = \frac{s + 4}{s^2 + 5s + 6} = \frac{s + 4}{(s + 2)(s + 3)}$$

$$= \frac{2}{s+2} - \frac{1}{s+3}$$

$$\therefore h(t) = (2e^{-2t} - e^{-3t})u(t) \quad (\text{By inverse Laplace})$$

~~(c) $e^{-4t}u(t)$~~

$$(c) \quad x(t) = e^{-4t}u(t) - te^{-4t}u(t)$$

$$\Rightarrow X(s) = \frac{1}{s+4} - \frac{1}{(s+4)^2} \quad (\text{using Laplace})$$

$$= \frac{s+3}{(s+4)^2}$$

$$\therefore Y(s) = \frac{(s+3)}{(s+4)^2} \cdot \frac{(s+4)}{(s+2)(s+3)}$$

$$= \frac{1}{(s+2)(s+4)}$$

$$= \frac{3/2}{s+2} - \frac{1/2}{s+4}$$

using inverse Laplace

$$y(t) = \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t} \right) u(t)$$

Write all steps

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(ii)

$$x(n) = (1 \quad 3 \quad 0 \quad 4 \quad -2)$$

↑

$$h(n) = (2 \quad 4 \quad -1 \quad -3)$$

↑

to obtain $y(n)$, using table method

	1	3	0	4	-2
2	2	6	0	8	-4
4	4	12	0	16	-8
-1	-1	-3	0	-4	2
-3	-3	-9	0	-12	6

$$\therefore y(n) = \{2 \quad 10 \quad 11 \quad 2 \quad 3 \quad -12 \quad -10 \quad 6\}$$

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$x(n)$ starts at $L_1 = 1$

$h(n)$ starts at $L_2 = 2$

$\therefore y(n)$ starts at $L = L_1 + L_2 = 1 + 2 = 3$

$$\therefore y(n) = \{2 \quad 10 \quad 11 \quad 2 \quad 3 \quad -12 \quad -10 \quad 6\}$$

↑

- Q.3 (a) (i) Explain the status pins (\bar{S}_2, \bar{S}_1 and \bar{S}_0) and queue status pins (Q_{S1} and Q_{S0}) of 8086 with their function.
- (ii) Discuss the pointers and index group of registers of 8086.

[10 + 10 marks]



Q.3 (b) Design an ideal band reject filter with a desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1, & |\omega| < \frac{\pi}{3} \text{ and } |\omega| \geq \frac{2\pi}{3} \\ 0, & \text{otherwise} \end{cases}$$

Find the impulse response $h(n)$ and transfer function $H(z)$ of the filter for length $M = 11$.

[20 marks]

- Q.3 (c) Ten 8-bit numbers are stored starting from memory location 3000H. Write an 8085 assembly language program, by giving suitable flow chart to find the greatest of the ten numbers and store it at memory location 4000H.

[20 marks]

- Q.4 (a) (i) The input to a linear shift-invariant system is $x(n) = 2\cos\left(\frac{n\pi}{4}\right) + 3\sin\left(\frac{3n\pi}{4} + \frac{\pi}{8}\right)$.

Find the output if the unit sample response of the system is $h(n) = \frac{2\sin(n-1)\frac{\pi}{2}}{(n-1)\pi}$.

- (ii) Consider a system described by the difference equation

$$y(n) = y(n-1) - y(n-2) + 0.5x(n) + 0.5x(n-1).$$

Find the response of this system to the input $x(n) = (0.5)^n u(n)$, with initial conditions $y(-1) = 0.75$ and $y(-2) = 0.25$

[10 + 10 marks]



Q.4 (b) For $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$. Compute the DFT, $X(k)$ using DIF FFT algorithm.

[20 marks]





- Q.4 (c) (i) Draw the lattice filter implementation of FIR filter $H(z) = 8 + 4z^{-1} + 2z^{-2} + z^{-3}$.
- (ii) It is required to move a 16-byte long data string from offset 4000H to offset 5000H. Write an assembly language program to accomplish the above task for 8085 microprocessor.

[12 + 8 marks]

Section B : Electrical Circuits - 1 + Control Systems - 1

Q.5 (a) The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+20)}$$

Find the range of K for which the system is stable. Also show that the system response can oscillate at two different frequencies.

[12 marks]

$$G_H(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+20)}$$

$$\begin{aligned} \Rightarrow q(s) &= s(s-1)(s^2+4s+20) + K(s+1) \\ &= s(s^3 + 4s^2 + 20s - s^2 - 4s - 20) + K(s+1) \\ &= s(s^3 + 3s^2 + 16s - 20) + K(s+1) \\ &= s^4 + 3s^3 + 16s^2 + (-20+K)s + K \end{aligned}$$

Constructing R-H table

$$s^4 \quad 1 \quad 16 \quad K$$

$$s^3 \quad 3 \quad K-20$$

$$s^2 \quad \frac{48-K+20}{3} \quad K$$

$$= \left(\frac{68-K}{3} \right)$$

$$s^1 \quad \frac{\left(\frac{68-K}{3} \right)(K-20) - 3K}{\left(\frac{68-K}{3} \right)}$$

$$s^0 \quad K$$

from necessary conditions

$$-20 + K > 0$$

$$\Delta \quad K > 0$$

$$\Rightarrow K > 20$$

also for system to be stable, coefficient of 1st column of RH table should be +ve (no sign change)

$$\therefore \frac{68-K}{3} > 0 \Rightarrow K < 68$$

$$\left(\frac{68-K}{3}\right)(K-20) - 3K > 0$$

$$\text{or } (68-K)(K-20) - 9K > 0$$

$$68K - 1360 - K^2 + 20K - 9K > 0$$

$$-K^2 + 79K - 1360 > 0$$

$$K^2 - 79K + 1360 < 0$$

$$K^2 - 79K + 1360 < 0$$

$$(K - 53.65)(K - 25.35) < 0$$

$$\Rightarrow 25.35 < K < 53.65$$

Hence

range of K is

$$25.35 < K < 53.65$$

for oscillatory response

$$s^1 - \text{row coefficient} = 0$$

$$\Rightarrow K = 25.35$$

or

$$K = 53.65$$

putting these values in s^2 - row

$$\text{for } K = 25.35$$

$$14.2s^2 + 25.35 = 0 \Rightarrow$$

$$\omega = 1.3 \text{ rad/s}$$

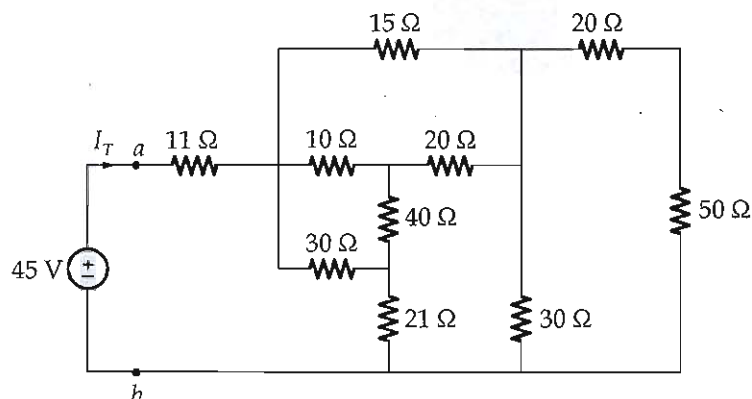
$$\text{for } K = 53.65$$

$$4.78s^2 + 53.65 = 0 \Rightarrow$$

$$\omega = 3.3 \text{ rad/s}$$

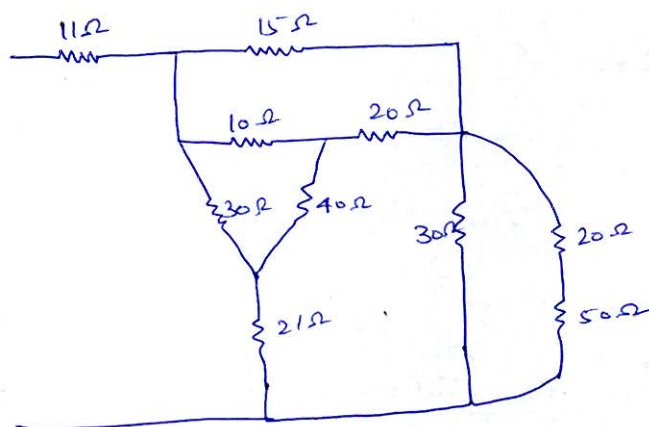
11
Good
Approach

- Q.5(b) For the circuit shown in figure below, obtain the equivalent resistance at terminals $a - b$. Also find total current I_T as indicated below.

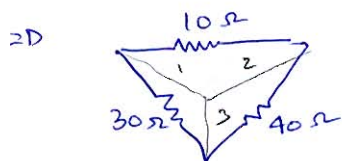


[12 marks]

for calculating R_{ab} , circuit can be redrawn as



$$\begin{aligned} & \downarrow \\ & 36\Omega \parallel (20 + 50)\Omega \\ & = 21\Omega \end{aligned}$$

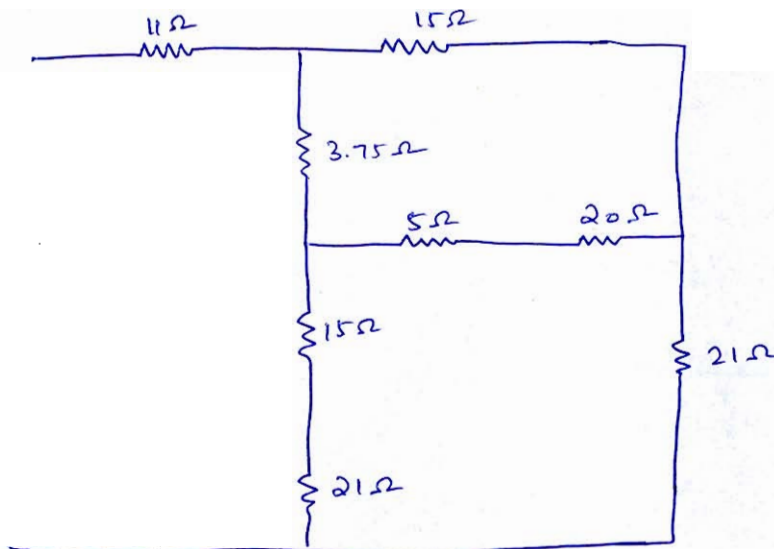


converting Δ connection to Y

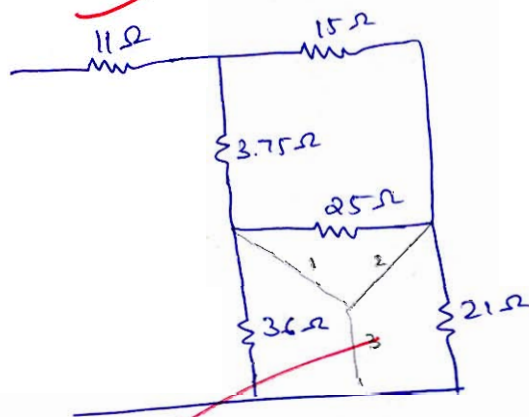
$$Y_1 = 3.75\Omega$$

$$Y_2 = 5\Omega$$

$$Y_3 = 15\Omega$$



⇓
on simplification

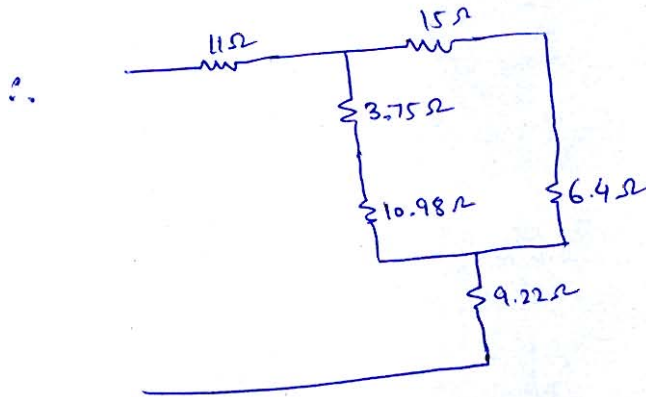


✓ converting Δ to Y , $Y_3 = \frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2 + \Delta_3}$

$$\therefore Y_1 = 10.98 \Omega$$

$$Y_2 = 6.4 \Omega$$

$$Y_3 = 9.22 \Omega$$

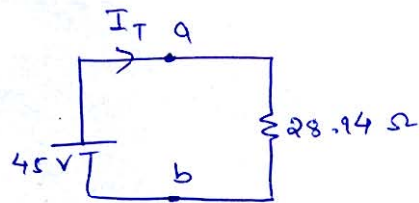


$$\begin{aligned}
 & (3.75 + 10.98) \parallel (15 + 6.4) \\
 & = 14.73 \parallel 21.4 \\
 & = 8.72 \Omega
 \end{aligned}$$

∴ $Z_{eq} = (11 + 8.72 + 9.22) \Omega = 28.94 \Omega$

∴ $R_{ab} = 28.94 \Omega$

∴ circuit can be redrawn as



Hence using Ohm's law

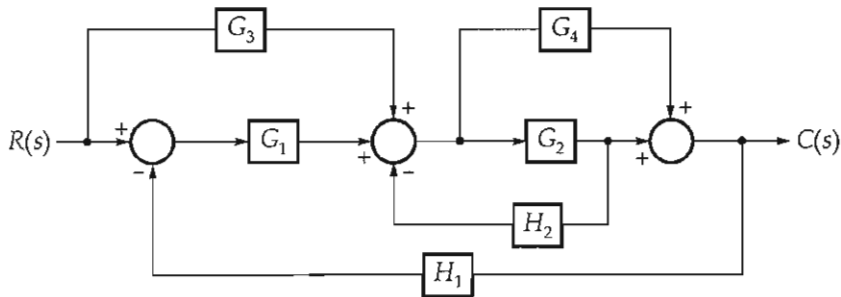
$$V = I R$$

$$I_T = \frac{45}{28.94}$$

$$I_T = 1.55 \text{ A}$$

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- Q.5 (c) Determine the transfer function $\frac{C(s)}{R(s)}$ for a system represented by the block diagram shown below :



[12 marks]

Forward paths

$$P_1 = G_1 G_2$$

$$\Delta_1 = 1$$

$$P_2 = G_1 G_4$$

$$\Delta_2 = 1$$

$$P_3 = G_3 G_2$$

$$\Delta_3 = 1$$

$$P_4 = G_3 G_4$$

$$\Delta_4 = 1$$

Loops

$$L_1 = -G_2 H_2$$

$$L_2 = -G_1 G_2 H_1$$

$$L_3 = -G_3 G_2 H_1$$

$$L_4 = -G_1 G_4 H_1$$

$$L_5 = -G_3 G_4 H_1$$

no possibility of two or two or more non-touching loops

$$TF = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4}{\Delta}$$

using mason's formula

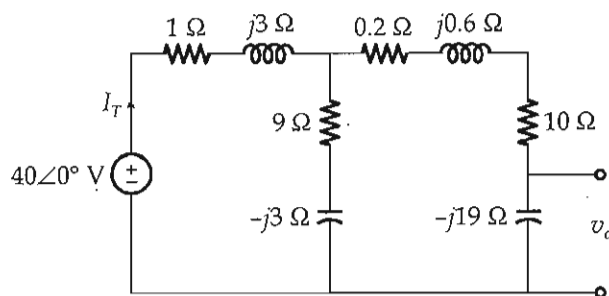
$$\therefore TF = \frac{C(s)}{R(s)}$$

$$= \frac{G_1 G_2 + G_1 G_4 + G_3 G_2 + G_3 G_4}{1 + G_2 H_2 + G_1 G_2 H_1 + G_1 G_4 H_1}$$

Good
Approach

11

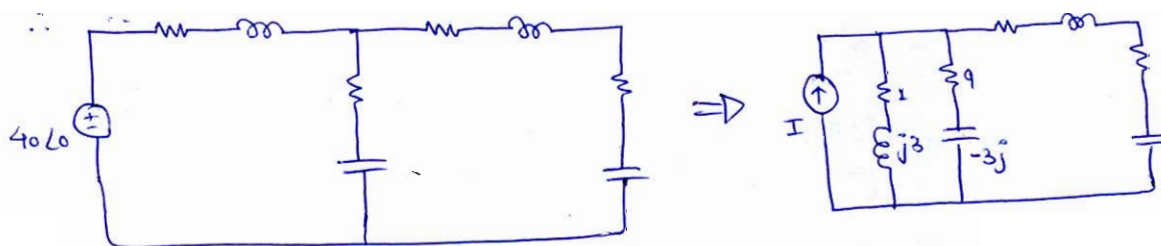
- Q.5(d) Use the concept of source transformation to find the phasor voltage v_o in the circuit shown below. Also, calculate the total current I_T of the circuit.



[12 marks]

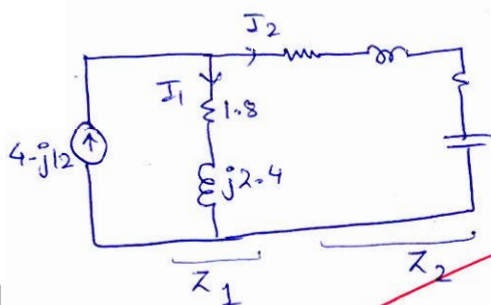
using source transformation

$$\tilde{V}, z \longleftrightarrow \tilde{I}, z \quad \text{where } \tilde{I} = \frac{\tilde{V}}{z}$$



$$I = \frac{40 \angle 0^\circ}{1 + j3} = 4 - j12$$

$$\text{Now } (1 + j3) \parallel (9 - j3) = 1.8 + j2.4$$



$$Z_1 = (1.8 + j2.4) \Omega$$

$$Z_2 = (10.2 - j18.4) \Omega$$

Now using current division rule

$$I_2 = I \cdot \frac{Z_1}{Z_1 + Z_2} = \frac{(4 - j12)(1.8 + j2.4)}{1.8 + j2.4 + 10.2 - j18.4} = (1.56 + j1.08) \text{ A}$$

$$\begin{aligned}
 \therefore V_o &= I_2 (-j19) \\
 &= (1.56 + j1.08) (-j19) \\
 &= 20.52 - j29.64 \\
 &= (36.05 \angle -55.3^\circ) \text{ V}
 \end{aligned}$$

Now to obtain I_T

$$\begin{aligned}
 Z_{eq} &= (10.2 - j18.4) \parallel (9 - 3j) + (1 + j3) \\
 &= 6.96 \angle -5.03^\circ \Omega
 \end{aligned}$$

$$\begin{aligned}
 I_T &= \frac{V}{Z_{eq}} = \frac{40 \angle 0^\circ}{6.96 \angle -5.03^\circ} \\
 &= 5.75 \angle 5.03^\circ \text{ A}
 \end{aligned}$$



$$\begin{aligned}
 \therefore V_o &= 36.05 \angle -55.3^\circ \text{ V} \\
 I_T &= 5.75 \angle 5.03^\circ \text{ A}
 \end{aligned}$$

Good
Approach

- Q.5 (e) A second-order servo-mechanism with unity feedback, has the open-loop transfer function $G(s) = \frac{K}{s(s+4)}$. Find the gain k so that the steady-state error shall not exceed 0.4 degree when the input shaft is rotated at 3 rpm. (Assume input $r(t) = \omega t$)

[12 marks]

$$G = \frac{K}{s(s+4)}$$

The system is Type 1 system \Rightarrow only velocity error is finite

we know $e_{ss} = \frac{1}{K_v} \cdot (i/p)$, $K_v = \lim_{s \rightarrow 0} sGH(s)$

$$\therefore K_v = \lim_{s \rightarrow 0} s \cdot \frac{K}{s(s+4)} = \frac{K}{4}$$

Now i/p is 3 rpm $\Rightarrow \left(3 \times \frac{2\pi}{60}\right) \text{ rad/sec}$
 $= \frac{\pi}{10} \text{ rad/sec}$

error = $0.4^\circ = \frac{\pi}{450} \text{ rad}$

$\therefore e_{ss} < 0.4^\circ$

$$\frac{\pi}{10} \times \frac{1}{K/4} < \frac{\pi}{450}$$

$$\frac{4\pi}{10K} < \frac{\pi}{450}$$

$$K > 180$$

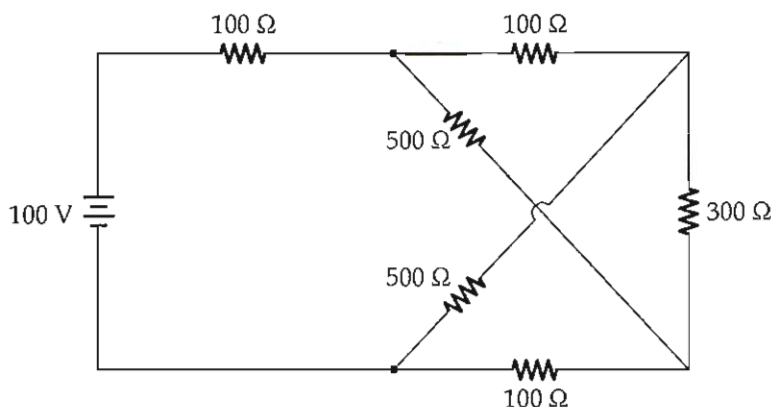
$$\therefore \boxed{K > 180}$$

$$\boxed{K_{min} = 180}$$

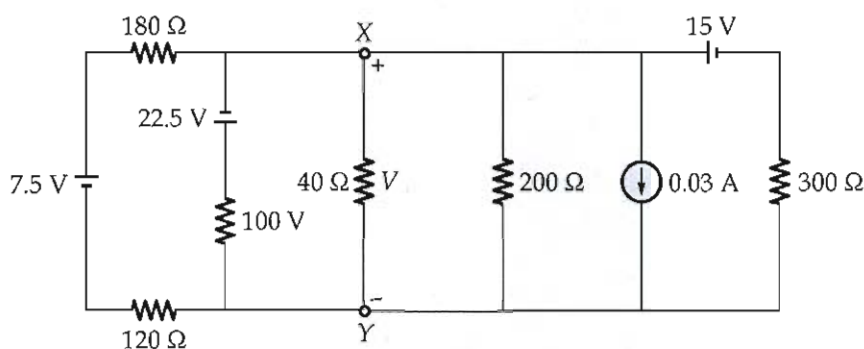
11

Good
Approach

- Q.6 (a) (i) Determine the current supplied by the battery in the circuit shown below by using Mesh Analysis only.

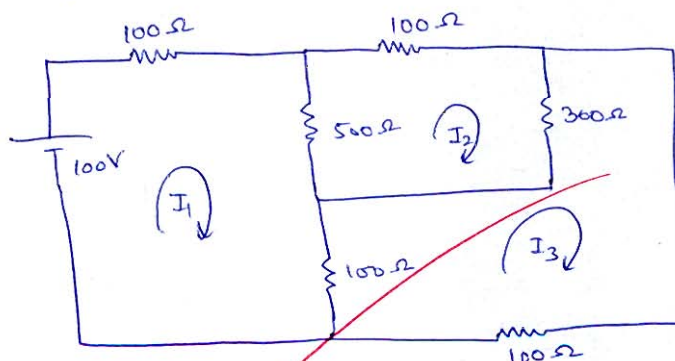


- (ii) By constructing Millman equivalent voltage source with respect to terminals x-y, find the voltage across 40 Ω resistor.



[10 + 10 marks]

(i) Redrawing the circuit we get



using mesh analysis

$$\textcircled{1} \quad -100 + 100 I_1 + 500 (I_1 - I_2) + 100 (I_1 - I_3) = 0$$

$$700 I_1 - 500 I_2 - 100 I_3 = 100$$

$$7 I_1 - 5 I_2 - I_3 = 1$$

$$\text{---} \textcircled{1}$$

$$\textcircled{2} \quad 500(I_2 - I_1) + 100 I_2 + 300(I_2 - I_3) = 0$$

$$-500 I_1 + 900 I_2 - 300 I_3 = 0$$

$$-5 I_1 + 9 I_2 - 3 I_3 = 0 \quad \text{--- (ii)}$$

$$\textcircled{3} \quad 100 I_3 + 100(I_3 - I_1) + 300(I_3 - I_2) = 0$$

$$-100 I_1 - 300 I_2 + 500 I_3 = 0$$

$$-I_1 - 3 I_2 + 5 I_3 = 0 \quad \text{--- (iii)}$$

writing in matrix form

$$\begin{bmatrix} 7 & -5 & -1 \\ -5 & 9 & -3 \\ -1 & -3 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{I_1 = 0.34 \text{ A}}$$

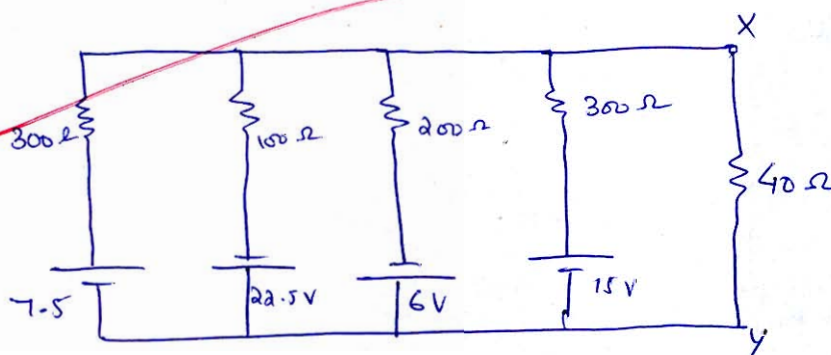
$$0.3 \text{ A}$$

(ii)

converting current source to voltage source

$$0.03 \text{ A}, 200 \Omega \Rightarrow V = IR = 6 \text{ V}, 200 \Omega$$

redrawing the circuit



As per Millman's theorem

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{13}{600}$$

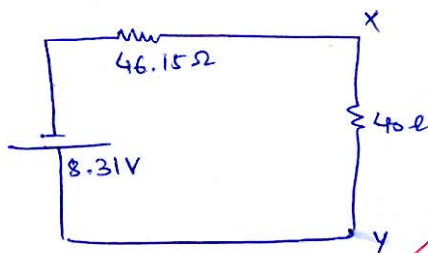
$$R_{eq} = 46.15 \Omega$$

$$\text{and } E = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3} + \frac{E_4}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

$$= \frac{\frac{7.5}{300} - \frac{22.5}{100} - \frac{6}{200} + \frac{15}{300}}{\frac{13}{600}}$$

$$= -8.31 \text{ V}$$

\therefore equivalent circuit



$$\therefore V_{40\Omega} = 8.31 \times \frac{40}{40 + 46.15}$$

using voltage division

$$\therefore V_{40\Omega} = 3.85 \text{ V}$$

$$\therefore V_{40\Omega} = 3.85 \text{ V}$$

$$V_{40\Omega} = -3.85 \text{ V}$$

9

Good
Approach

- Q.6(b) The open-loop transfer function of a system is $G(s) = \frac{K(s+4)}{(s+10)^2}$. What must be the value of K that the gain cross-over frequency is $\omega_{gc} = 30$ rad/s. Also find gain margin and phase margin for that value of K . Also comment on stability of system.

[20 marks]

given $\omega_{gc} = 30$ rad/s

$$\Rightarrow |G|_{\omega_{gc}} = 1$$

$$\Rightarrow \left| \frac{K(30j+4)}{(30j+10)^2} \right| = 1$$

$$\left| \frac{K \sqrt{30^2+4^2}}{30^2+10^2} \right| = 1$$

$$\Rightarrow \boxed{K = 33}$$

$$\phi_G = \angle G(s) = \tan^{-1} \frac{\omega}{4} - 2 \tan^{-1} \frac{\omega}{10}$$

for GM, $\angle G_{\omega_{pc}} = -180$

$$\therefore \tan^{-1} \frac{\omega}{4} - 2 \tan^{-1} \frac{\omega}{10} = -180$$

$$\tan^{-1} \frac{\omega}{4} - \frac{\tan^{-1} \frac{2\omega}{10}}{1 - \frac{\omega^2}{100}} = -180$$

$$\frac{\omega}{4} - \frac{\omega/5}{1 - \omega^2/100} = 0$$

$$\omega \left(1 - \frac{\omega^2}{100}\right) - \frac{4\omega}{5} = 0$$

$$1 - \frac{\omega^2}{100} - \frac{4}{5} = 0$$

$$\omega = 2.5$$

$$\boxed{\omega_{pc} = 2.5 \text{ rad/s}}$$

$$|G|_{\omega_{pc}} = \left| \frac{K (25s + 4)}{(25s + 10)^2} \right| = 1.65$$

$$\therefore \text{gm} = -20 \log 1.65 = -4.35 \text{ dB}$$

for PM

$$\angle G|_{\omega_{gc}} = \angle \frac{K (30s + 4)}{(30s + 10)^2}$$

$$= -60.72^\circ$$

$$\therefore \text{PM} = 180 + \phi$$

$$= 180 - 60.72^\circ$$

$$= 119.27^\circ$$

14

$$\therefore K = 33$$

$$\omega_{gc} = 30 \text{ rad/sec}$$

$$\omega_{pc} = 25 \text{ rad/sec}$$

$$\text{gm} = -4.35 \text{ dB}$$

$$\text{PM} = 119.27^\circ$$

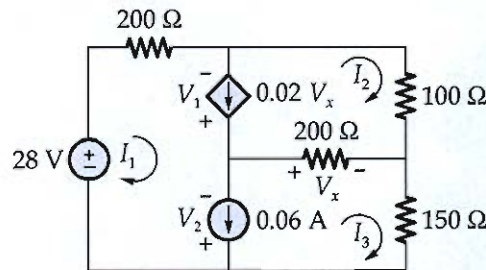
ω_{pc} is Undefined, hence Gm is also undefined.

The system is said to be stable based on positive phase margin

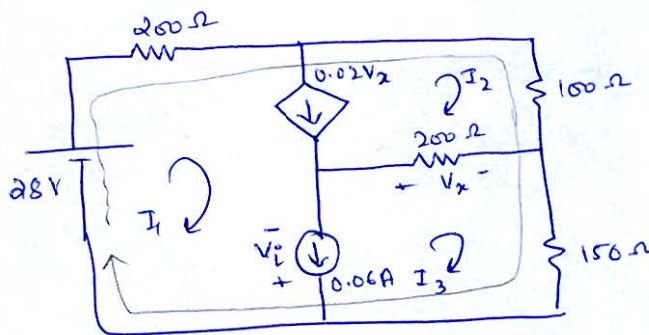
$$\text{as } \text{gm} < 0 \quad \& \quad \text{PM} > 0$$

\Rightarrow system is unstable

- Q.6 (c) Find the values for the loop currents I_1 , I_2 , I_3 and the power delivered by each independent source.



[20 marks]



$V_x = (I_3 - I_2) 200$, we'll use this in other eqⁿ wherever required

loop ①

$$-28 + 200I_1 + 100I_2 + 150I_3 = 0 \quad \text{--- ①}$$

as the loops form super mesh, we considered only the bigger outer loop

Now $I_1 - I_2 = 0.02V_x = 0.02 \times 200(I_3 - I_2)$

$$I_1 - I_2 = 4I_3 - 4I_2$$

$$I_1 + 3I_2 - 4I_3 = 0 \quad \text{--- ②}$$

&

$$I_1 - I_3 = 0.06 \quad \text{--- ③}$$

solving eqⁿ (i), (ii) & (iv), we get

$$\begin{aligned} I_1 &= 0.1 \text{ A} \\ I_2 &= 0.02 \text{ A} \\ I_3 &= 0.04 \text{ A} \end{aligned}$$

Now for power delivered by sources

in loop (3) $(I_3 - I_2)200 + I_3 \cdot 150 + V_i = 0$

$$\therefore V_i = -10$$

for current source

$$P_{\text{delivered}} = (-10)(0.06) = -0.6 \text{ W}$$

for voltage source

$$P_{\text{delivered}} = 28 \times 0.1 = 2.8 \text{ W}$$

$$P_{\text{delivered by } 28\text{V}} = 2.8 \text{ W}$$

$$P_{\text{delivered by } 0.06\text{A}} = -0.6 \text{ W} \quad (0.6 \text{ W absorbed})$$

18

Good
Approach

Q.7 (a) (i) The response of a feedback system to a unit step input is

$$C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}.$$

(a) Obtain the expression for the closed loop transfer function.

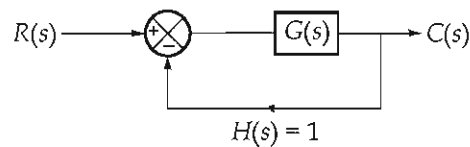
(b) Determine the undamped natural frequency and damping ratio of the system.

(ii) Consider the unity feedback system whose open loop transfer function

$$G(s)H(s) = \frac{4}{s(s+5)}.$$

When this system is excited by a unit step input then calculate

the output response and comment on the peak overshoot of the system.



[10 + 10 marks]

(i)

$$c(t) = 1 + 0.2 e^{-60t} - 1.2 e^{-10t}$$

$$r(t) = u(t)$$

$$\Rightarrow C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$R(s) = \frac{1}{s}$$

$$(a) \quad \frac{C(s)}{R(s)} = \frac{\left(\frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10} \right)}{\frac{1}{s}}$$

$$= \frac{(s+60)(s+10) + 0.2s(s+10) - 1.2s(s+60)}{(s+10)(s+60)}$$

$$= \frac{s^2 + 70s + 600 + 0.2s^2 + 2s - 1.2s^2 - 72s}{s^2 + 70s + 600}$$

$$= \frac{600}{s^2 + 70s + 600}$$

(b) comparing the denominator with standard 2nd order TF

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 70s + 600$$

$$\therefore \omega_n^2 = 600$$

$$2\zeta\omega_n = 70$$

$$\omega_n = 10\sqrt{6} = 24.5 \text{ rad/s}$$

$$\zeta = \frac{70}{2\omega_n} = 1.43$$

\therefore it is overdamped system with $\zeta = 1.43$, $\omega_n = 24.5 \text{ rad/s}$

(9)

Good
Approach

(ii)

$$\text{closed loop TF} = \frac{G(s)}{1 + GH(s)}$$

$$= \frac{4}{s(s+5)}$$

$$1 + \frac{4}{s(s+5)}$$

$$= \frac{4}{s(s+5) + 4}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 5s + 4}$$

$$\text{given } R(s) = \frac{1}{s} \quad [\text{unit step input}]$$

$$\therefore C(s) = \frac{4}{(s^2 + 5s + 4)} \cdot \frac{1}{s}$$

$$C(s) = \frac{4}{s(s+4)(s+1)}$$

$$= \frac{1}{s} - \frac{4/3}{s+1} + \frac{1/3}{s+4}$$

taking inverse LT

$$c(t) = \left(1 - \frac{4}{3} e^{-t} + \frac{1}{3} e^{-4t} \right) u(t)$$

$$q(s) \text{ of system} = s^2 + 5s + 4$$

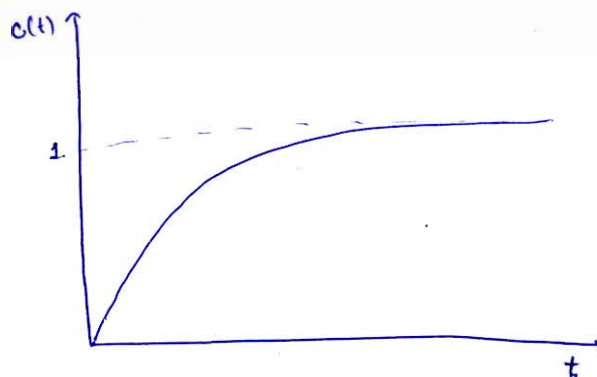
$$\Rightarrow \omega_n = 2$$

$$\gamma = 1.25$$

\Rightarrow It is an overdamped system

response will be

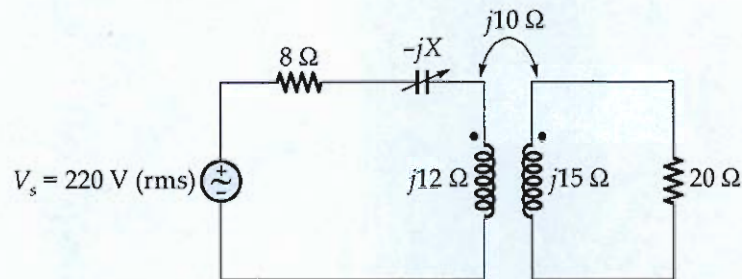
9



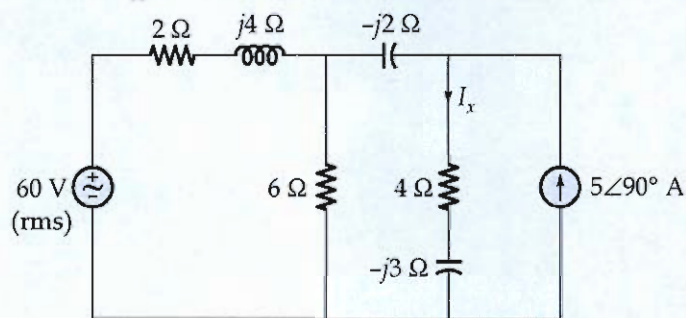
Good
Approach

\therefore there is no peak overshoot as the system is overdamped

- Q.7 (b) (i) For the circuit shown in figure, calculate the value of X that will give maximum power transfer to the $20\ \Omega$ load. Also calculate the maximum power delivered to load.

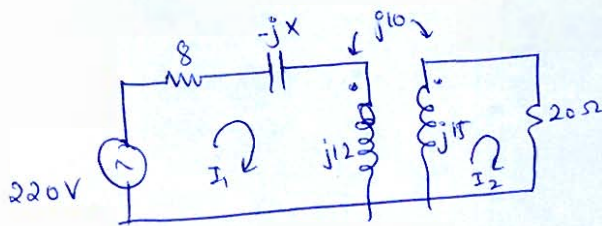


- (ii) Calculate current I_X for the circuit shown below :



[12 + 8 marks]

(i)



as I_1 enters dot, I_2 leaves dot
 \Rightarrow -ve coupling

$$-220 + (8 - jX)I_1 + j12I_1 - j10I_2 = 0$$

$$I_1[8 + (12 - X)j] - I_2(10j) = 220$$

$$I_1 = \frac{220 + 10jI_2}{8 + (12 - X)j} \quad \text{--- (1)}$$

4

$$j15 I_2 - j10 I_1 + 20 I_2 = 0$$

$$j15 I_2 + 20 I_2 = j10 I_1 \quad \text{--- (10)}$$

substituting (1) in (8)

$$j15 I_2 + 20 I_2 = j10 \left[\frac{220 + 10j I_2}{8 + (12-x)j} \right]$$

$$\frac{(j15 + 20)}{j10} I_2 [8 + (12-x)j] = 220 + 10j I_2$$

$$\left[(1.5 - j2)(8 + (12-x)j) - 10j \right] I_2 = 220$$

$$I_2 = \frac{220}{(1.5 - j2)(8 + (12-x)j) - 10j}$$

Power will be max, when I_2 will be max

$\Rightarrow I_2$ will be max, when denominator will be min

$$\therefore \frac{d}{dx} [(1.5 - j2)(8 + (12-x)j) - 10j] = 0$$

$$(1.5 - j2)(-j)$$

$$P = I_2^2 R = \frac{220^2}{[(1.5 - j2)(8 + (12-x)j) - 10j]^2} \times 20$$

$$\frac{dP}{dx} = 0 \Rightarrow 2 [(1.5 - j2)(8 + (12-x)j) - 10j] [(1.5 - j2)(-j)] = 0$$

$$\Rightarrow (1.5 - j2)(8 + 12j - jx) = 10j$$

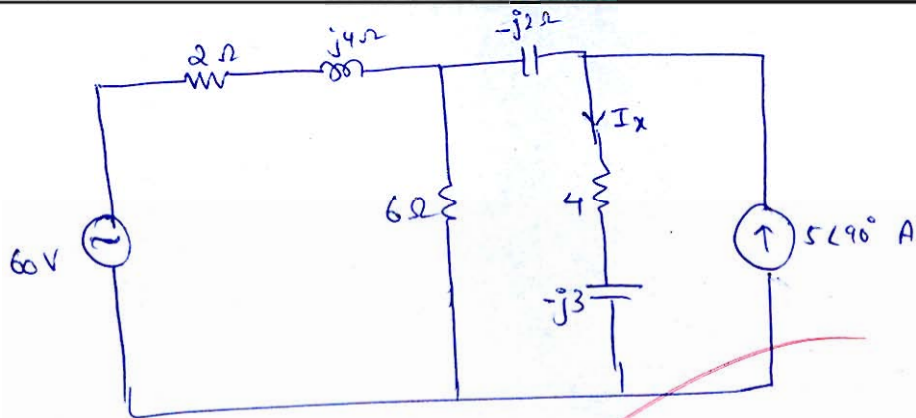
$$8 + 12j - jx = -3.2 + j2.4$$

$$8 + 12j + 3.2 - j2.4 = jx$$

$$\boxed{|x| = 14.75 \Omega}$$

Try to avoid

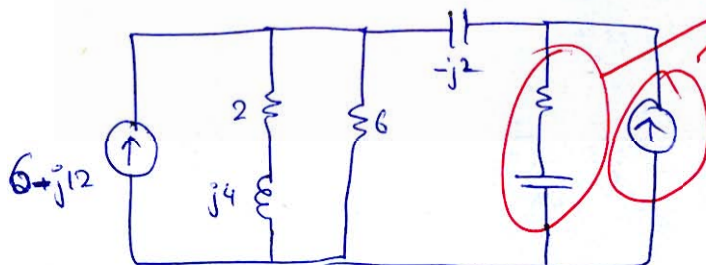
(u)



using source transformation 60V, $(2+j4)$ converted to

$$i = \frac{60}{2+j4} = \frac{60}{2+j4} (6-j12) \text{ A}, (2+j4) \Omega$$

write value also



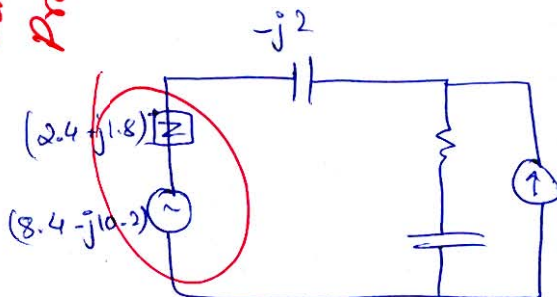
3

$$(2+j4) \parallel 6 = (2.4 + j1.8) \Omega$$

again converting $(6-j12) \text{ A}$ & $(2.4+j1.8) \Omega$ to voltage source

$$v = (6-j12)(2.4+j1.8) = (8.4 - j10.2) \text{ V}$$

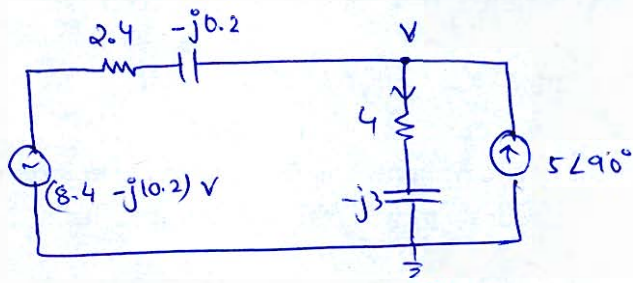
Draw diagram properly



Try to avoid

$$(2.4 + j1.8) \parallel j2 = (1.655 - j1.88) \Omega$$

$$(2.4 + j1.8) + (-j2) = (2.4 - j0.2) \Omega$$



~~again~~ using nodal analysis

$$\frac{V - (8.4 - j10.2)}{2.4 - j0.2} + \frac{V}{4 - j3} = 5 \angle 90^\circ$$

$$\therefore V \left[\frac{1}{2.4 - j0.2} + \frac{1}{4 - j3} \right] = 5 \angle 90^\circ + \frac{(8.4 - j10.2)}{(2.4 - j0.2)}$$

$$V = 6.69 \angle 0.53^\circ$$

$$\therefore I_x = \frac{V}{4 - j3}$$

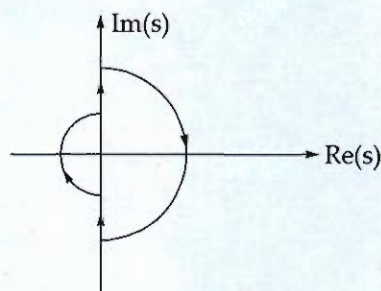
$$= \underline{1.34} = (1.06 + j0.81) \text{ A}$$

$$= 1.34 \angle 37.4^\circ \text{ A}$$

Q.7 (c) The open loop transfer function of a unity negative feedback system is given as

$$G(s) = \frac{1 + 4s}{s^2(1 + s)(1 + 2s)}$$

The Nyquist contour in s-plane encloses the entire right half plane and a small neighbour around origin in left half plane, as shown in figure. Draw the Nyquist plot of the system and examine its closed loop stability.

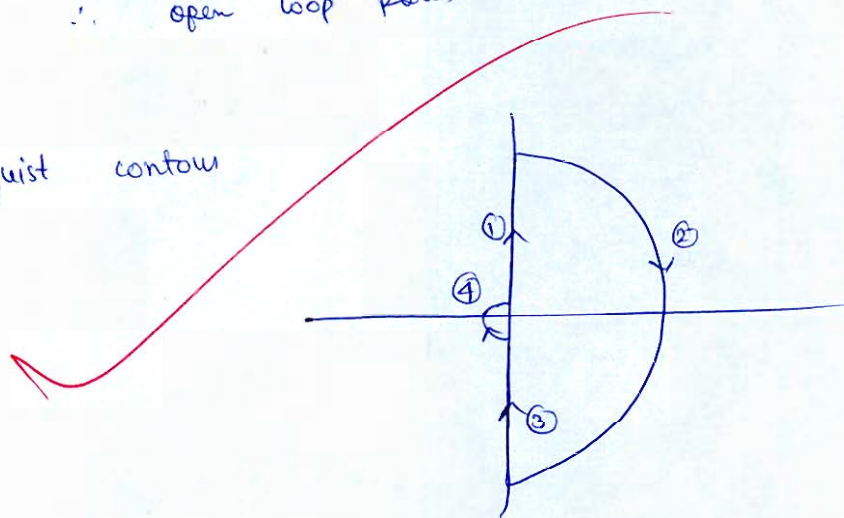


[20 marks]

There are two poles at origin, rest of the open loop poles are situated in LHP

∴ open loop poles $P = 2$

Nyquist contour



path 1

$$s = j\omega$$

$\omega: 0 \text{ to } \omega: \infty$

$$G(j\omega) = \frac{1 + 4j\omega}{(j\omega)^2 (1 + j\omega)(1 + 2j\omega)}$$

$$|G(j\omega)| = \frac{\sqrt{1+16\omega^2}}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}} \quad \angle G(j\omega) = \tan^{-1} 4\omega - 180 - \tan^{-1} \omega - \tan^{-1} 2\omega$$

$$\begin{array}{lll} \omega \rightarrow 0 & |G| = \infty & \angle G = -180^\circ \\ \omega = \infty & |G| = 0 & \angle G = -270^\circ \end{array}$$

path 2

$$S = \lim_{R \rightarrow \infty} R e^{j\theta} \quad \theta: 90^\circ \text{ to } -90^\circ \quad \text{via } 0^\circ$$

$$G(j\omega) \approx \frac{4 R e^{j\theta}}{R^2 e^{2j\theta} \cdot R e^{j\theta} \cdot 2 R e^{j\theta}} \approx \frac{1}{R^3} e^{-j3\theta}$$

$$\theta = 90^\circ \quad |G| = 0 \quad \angle G = -270^\circ$$

$$\theta = -90^\circ \quad |G| = 0 \quad \angle G = 270^\circ$$

path 3

$$S = -j\omega \quad \omega: \infty \text{ to } \omega: 0$$

$$|G(j\omega)| = \frac{\sqrt{1+16\omega^2}}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}} \quad \angle G(j\omega) = -\tan^{-1} 4\omega + 180 + \tan^{-1} \omega + \tan^{-1} 2\omega$$

$$\omega = \infty \quad |G| = 0 \quad \angle G = 270^\circ$$

$$\omega = 0 \quad |G| = \infty \quad \angle G = 180^\circ$$

path 4

$$s = \lim_{r \rightarrow 0} r e^{j\theta}$$

$$\theta: -90 \text{ to } -270 \text{ via } -180^\circ$$

$$G(j\omega) \approx \frac{1}{s^2 e^{j2\theta}}$$

$$\theta = -90$$

$$|G| = \infty$$

$$\angle G = 180^\circ$$

$$\theta = -270$$

$$|G| = \infty$$

$$\angle G = -180^\circ$$

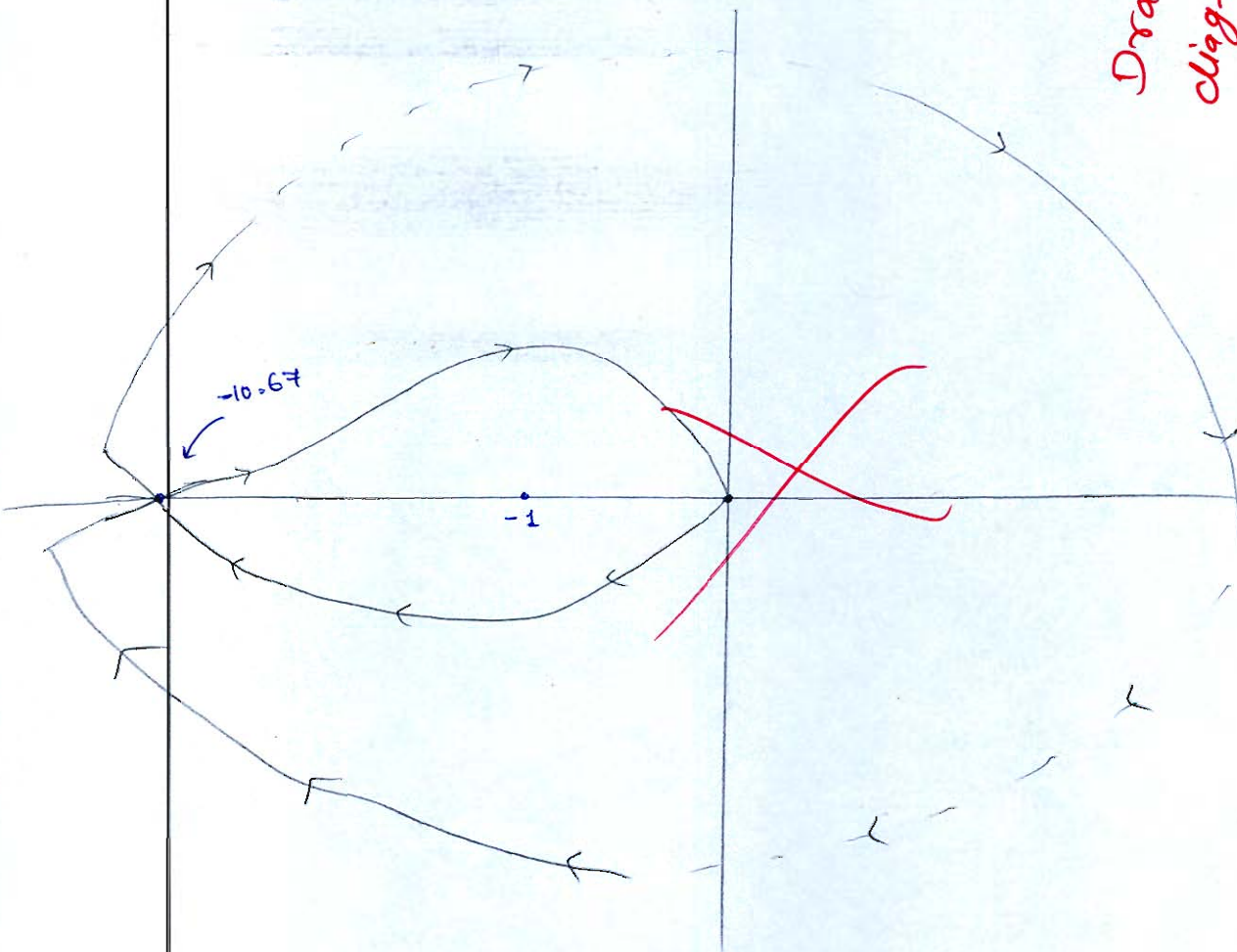
$$\text{when } \theta = -180^\circ$$

$$|G| = \infty$$

$$\angle G = 0^\circ$$

Therefore the Nyquist plot will be drawn as

Draw
diagram
properly



To find intersection with $-w$ real axis

$$\phi = -180^\circ$$

$$\tan^{-1} 4w - 180^\circ - \tan^{-1} w - \tan^{-1} 2w = -180^\circ$$

$$\tan^{-1} 4w = \tan^{-1} w + \tan^{-1} 2w$$

$$4w = \frac{w + 2w}{1 - 2w^2}$$

$$4w = \frac{3w}{1 - 2w^2}$$

$$1 - 2w^2 = \frac{3}{4}$$

$$\frac{1}{4} = 2w^2$$

$$w = \frac{1}{2\sqrt{2}}$$

at this w , gain of system is

$$|G| = +10.67$$

\Rightarrow point of intersection is -10.67

$\therefore -1$ will lie left to it

\therefore Encirclement of $-1 = 2$ in clockwise direction
 $\Rightarrow N = -2$

we know $N = P - Z$

$$-2 = 2 - Z$$

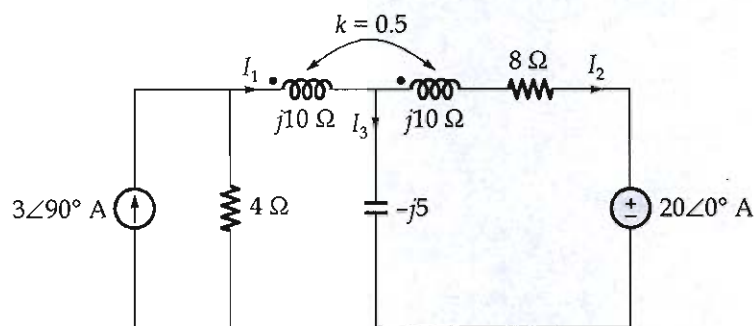
$$Z = 4$$

$\therefore 4$ poles in RHS

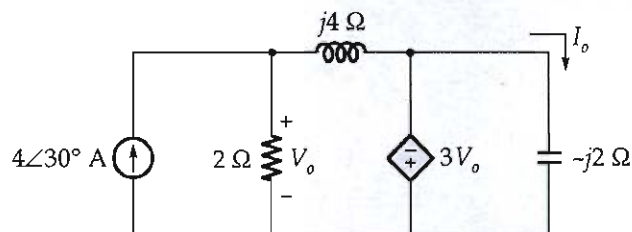
\Rightarrow system is unstable

5

- Q.8 (a) (i) Determine the current I_1 , I_2 and I_3 in the circuit shown. Take $\omega = 1000$ rad/sec.



- (ii) Calculate voltage V_o for the circuit shown below.



[15 + 5 marks]

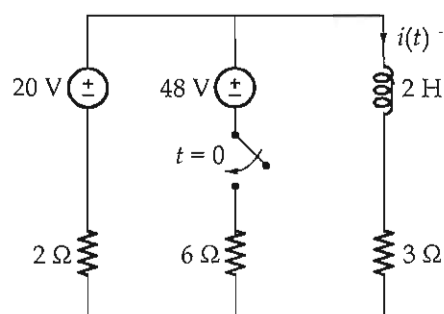


Q.8 (b) Let $G(s) = \frac{K(s-1)}{(s+2)(s+3)}$ with unity negative feedback.

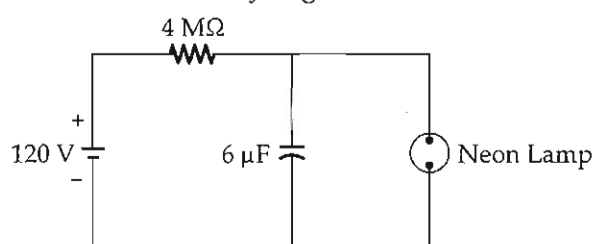
- (i) Find the range of K for closed loop stability.
- (ii) Plot the root locus for $K < 0$.
- (iii) Assuming a step input, what value of K will result in the smallest attainable settling time?

[20 marks]

- Q.8 (c) (i) Obtain the current $i(t)$ for both $t < 0$ and $t > 0$.



- (ii) A simple relaxation oscillator circuit is shown in figure. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drop to 30 V. Its resistance is 120Ω , when 'ON' and infinitely high when 'OFF'.



For how long is the lamp on each time the capacitor discharges? What is the time interval between two flashes?

[10 + 10 marks]



Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

$$\frac{1}{1 - \frac{1}{\alpha} \frac{1}{\alpha}}$$

$$\frac{1}{1 - \frac{1}{\alpha^2}}$$

$$z^T = \alpha$$

$$\frac{1}{1 - \alpha \alpha}$$