Write all steps in detail.

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ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electrical Circuits + Control Systems [All Topics]

Name : ....

Roll No :

Test Centres				Student's Signature
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	46
Q.2	30
Q.3	54
Q.4	
Section-B	
Q.5	55
Q.6	
Q.7	20
Q.8	
Total Marks Obtained	205

Signature of Evaluator   Cross Checked by

Sourabh Kumar ....

Try to avoid calculation mistake

Improve presentation

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
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1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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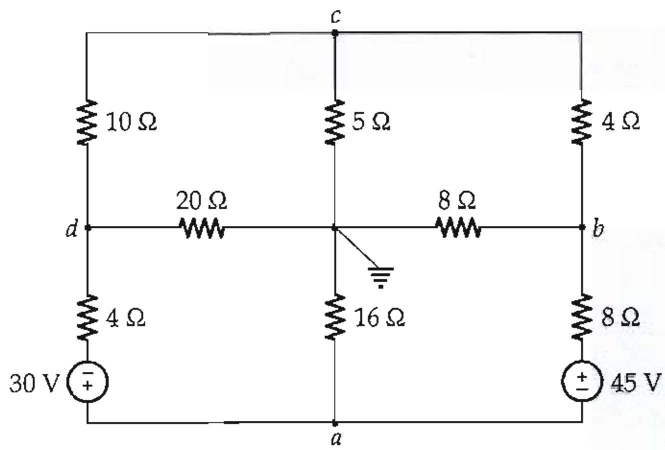
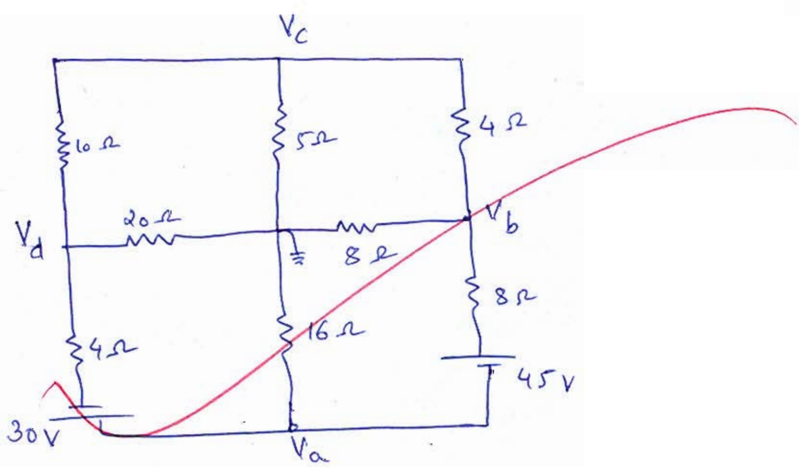
Section A : Electrical Circuits

Q.1 (a) Find the voltages at nodes *a*, *b*, *c* and *d* in the circuit shown.



[12 marks]



Applying Nodal analysis in the above circuit, we get

at node A $\quad \dfrac{V_a - 30 - V_d}{4} + \dfrac{V_a}{16} + \dfrac{V_a + 45 - V_b}{8} = 0$

or $\quad \dfrac{7}{16} V_a - \dfrac{1}{8} V_b + 0 V_c - \dfrac{1}{4} V_d = \dfrac{15}{8} \quad$ — (i)

at node B $\quad \dfrac{V_b - 45}{8} + \dfrac{V_b}{8} + \dfrac{V_b - V_c}{4} = 0$

or $\quad 0 V_a + \dfrac{1}{2} V_b - \dfrac{1}{4} V_c + 0 V_d = \dfrac{45}{8} \quad$ — (ii)

at node C $\quad \dfrac{V_c - V_b}{4} + \dfrac{V_c}{5} + \dfrac{V_c - V_d}{10} = 0$

or $\quad 0 V_a - \dfrac{1}{4} V_b + \dfrac{11}{20} V_c - \dfrac{1}{10} V_d = 0 \quad$ — (iii) at node D $\frac{V_d - V_c}{10} + \frac{V_d}{20} + \frac{V_d + 30 - V_a}{4} = 0$

$\cancel{0V_a + 0V_b}$ $-\frac{1}{4}V_a + 0V_b - \frac{V_c}{10} + \frac{2}{5}V_d = -\frac{15}{2}$ — (iv)

from eq^n (iii), we have

$$V_c = \frac{20}{11}\left[\frac{V_b}{4} + \frac{V_d}{10}\right]$$

substituting in eq (i) (ii) & (iv) we get

$\frac{7}{16}V_a - \frac{1}{8}V_b - \frac{1}{4}V_d = \frac{15}{8}$

$\frac{V_b}{2} - \frac{5}{11}\left[\frac{V_b}{4} + \frac{V_d}{10}\right] = \frac{45}{8} \Rightarrow 0V_a + \frac{17}{44}V_b - \frac{1}{22}V_d = \frac{45}{8}$

$-\frac{1}{4}V_a - \frac{2}{11}\left[\frac{V_b}{4} + \frac{V_d}{10}\right] + \frac{2}{5}V_d = -\frac{15}{2} \Rightarrow -\frac{1}{4}V_a - \frac{2}{44}V_b + \frac{21}{55}V_d = -\frac{15}{2}$

or

$$\begin{bmatrix} \frac{7}{16} & -\frac{1}{8} & -\frac{1}{4} \\ 0 & \frac{17}{44} & -\frac{1}{22} \\ -\frac{1}{4} & -\frac{2}{44} & \frac{21}{55} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_d \end{bmatrix} = \begin{bmatrix} \frac{15}{8} \\ \frac{45}{8} \\ -\frac{15}{2} \end{bmatrix}$$

2

Calculation mistake

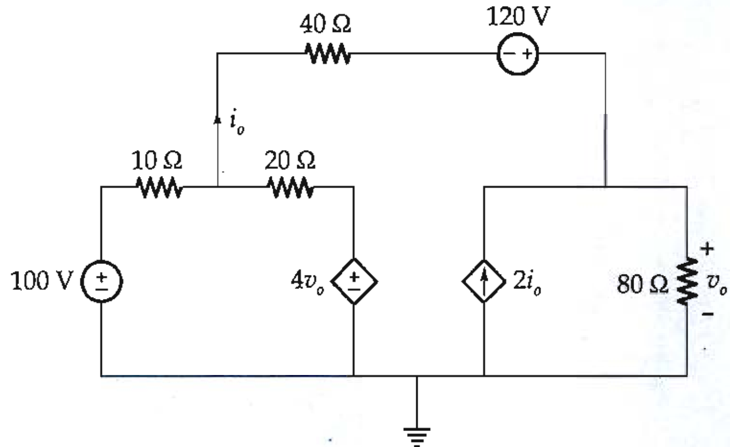
$\therefore V_a = -4.25\,V$

$V_b = 12.09\,V$

$V_d = -20.99\,V$

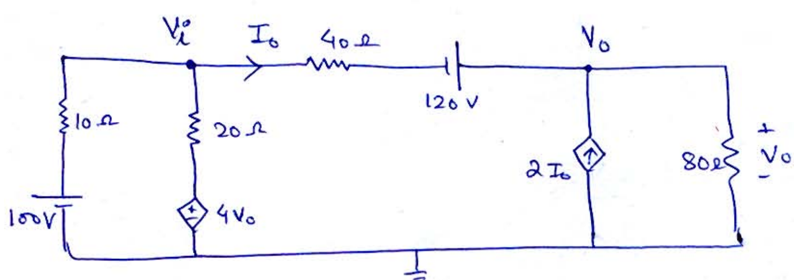
$\Rightarrow V_c = \frac{20}{11}\left[\frac{V_b}{4} + \frac{V_d}{10}\right] = 1.68\,V$

Q.1 (b) Using nodal analysis, find v_o and i_o in the circuit shown in figure.



[12 marks]

The circuit can be redrawn as



using nodal analysis

at node V_i,
$$\frac{V_i-100}{10}+\frac{V_i-4V_o}{20}+\frac{V_i+120-V_o}{40}=0$$

$$\frac{7}{40}V_i-\frac{9}{40}V_o=7 \qquad \text{— (i)}$$

at node V_o,
$$\frac{V_o-120-V_i}{40}+\frac{V_o}{80}=2I_o$$

$$-\frac{1}{40}V_i+\frac{3}{80}V_o=2I_o+3 \qquad \text{— (ii)}$$

also
$$I_o=\frac{V_i+120-V_o}{40} \qquad \text{— (iii)}$$

∴ eq (ii) becomes

$$-\frac{1}{40}V_i+\frac{3}{80}V_o=\frac{2}{40}(V_i+120-V_o)+3$$

Do not write in this margin

$$-\frac{1}{40}V_i + \frac{3}{80}V_o = \frac{1}{20}V_i + 6 - \frac{1}{20}V_o + 3$$

$$-\frac{3}{40}V_i + \frac{7}{80}V_o = 9 \quad -(\text{IV})$$

Solving eq (I) & (IV)

$$\begin{bmatrix} \frac{7}{40} & -\frac{9}{40} \\ -\frac{3}{40} & \frac{7}{80} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

$\Rightarrow V_i = -1688\ V$

$V_o = -1344\ V$

Improve presentation

$$\therefore I_o = \frac{V_i + 120 - V_o}{40}$$

$$= -5.6$$

$\therefore V_o = -1344\ V$

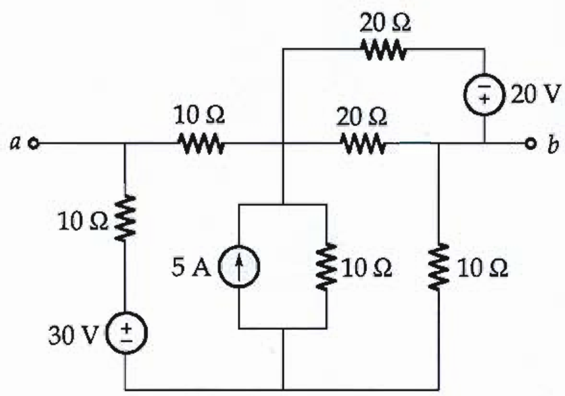
$I_o = -5.6A$

11

Good Approach

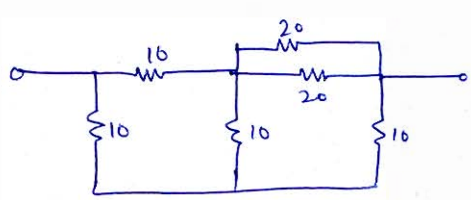
Do not write in this margin

Q.1 (c) For the circuit shown, find the Thevenin's equivalent network between terminals *a* and *b*.

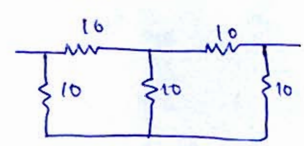


[12 marks]

To obtain Z_{th}, sources are deactivated

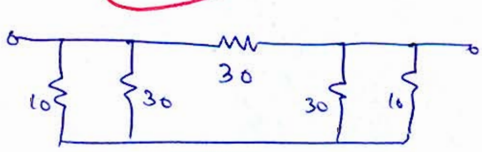


⇒

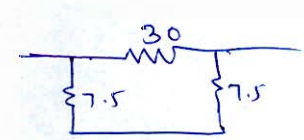


converting the 10Ω Y into delta

$R_\Delta = 3R_Y = 30\Omega$

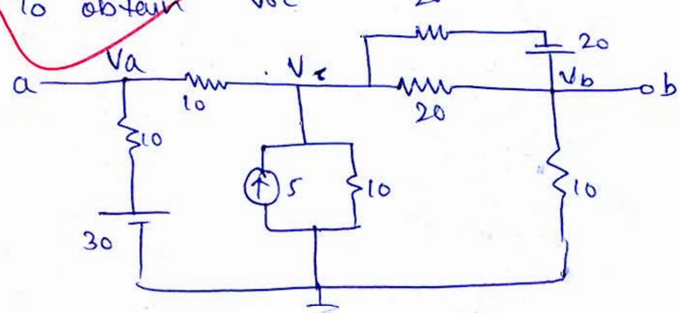


⇒



⇒ $R_{eq} = (7.5 + 7.5) || 30 = 10\Omega$

To obtain V_{oc}



node a $\quad \frac{V_a - 30}{10} + \frac{V_a - V_c}{10} = 0$

$2V_a - V_c = 30 \quad —(I)$

node c $\quad \frac{V_c - V_a}{10} + \frac{V_c - V_b}{20} + \frac{V_c + 20 - V_b}{20} + \frac{V_c}{10} = 5$

$-\frac{V_a}{10} - \frac{V_b}{10} + \frac{3V_c}{10} = 4 \quad —(II)$

node b

$\frac{V_b}{10} + \frac{V_b - V_c}{20} + \frac{V_b - 20 - V_c}{20} = 0$

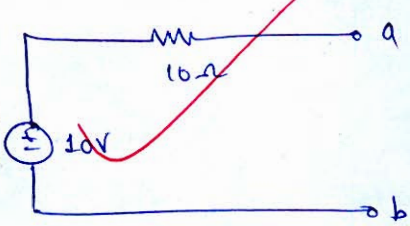
$\frac{V_b}{5} - \frac{V_c}{10} = 1 \quad —(III)$

Solving (I), (II), (III), we get

$V_a = 30V \qquad V_b = 20V \qquad V_c = 30V$

$V_{th} = V_a - V_b = 10V$

∴ Thevenin equivalent is



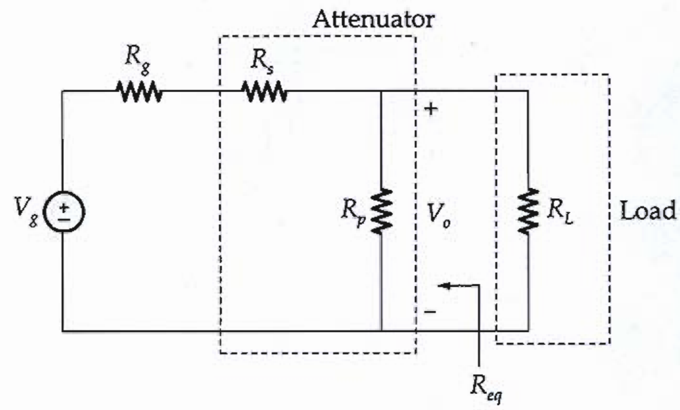
11

Good Approach

Q.1 (d) An attenuator is an interface circuit that reduces the voltage level without changing the output resistance. By specifying R_s and R_P of the interface circuit shown in figure, design an attenuator that will meet the following requirements :

$$\frac{V_o}{V_g} = 0.125, R_{eq} = R_{th} = R_g = 100\ \Omega$$

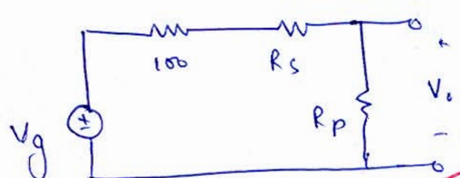
Using the interface designed, also calculate the current through a load of $R_L = 50\ \Omega$ when $V_g = 12$ V.



[12 marks]

For the attenuator circuit

Write all steps in detail



$$V_o = V_g \times \frac{R_p}{R_s + R_p + 100} \Rightarrow \frac{V_o}{V_g} = \frac{R_p}{R_s + R_p + 100} = 0.125$$

$$\therefore 8R_p = R_s + R_p + 100 \quad \text{or} \quad \boxed{7R_p = R_s + 100} \text{ — (1)}$$

now for R_{eq}, $V_g = 0$

$$\therefore R_{eq} = (R_g + R_s) \| R_p = 100$$

$$\Rightarrow \frac{(100 + R_s) R_p}{100 + R_s + R_p} = 100$$

$$\Rightarrow \frac{7R_p \cdot R_p}{8R_p} = 100$$

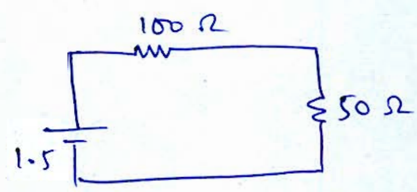
$R_P = \frac{800}{7}\ \Omega$ $\qquad R_S = 700\ \Omega$

given $V_g = 12\text{ V}$

Z_{th} to left of load $= 100\ \Omega$

$V_{th} = 0.125 \times V_g = 1.5\text{ V}$

$\therefore$



Thevenins equivalent

$\therefore I_L = \frac{1.5}{100+50}$

$= 0.01\text{ A}$

$= 10\text{ mA}$

11

Good Approach

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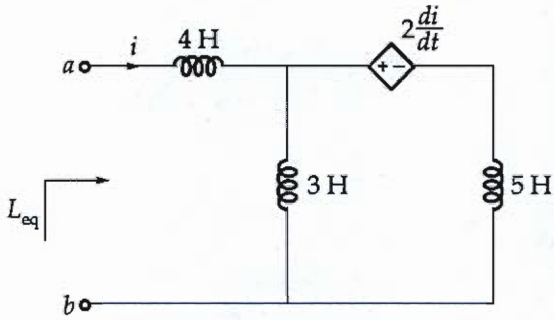
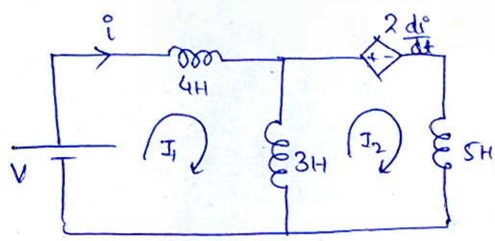
Q.1 (e) Determine the equivalent inductance, L_{eq} that can be used to represent the inductive network shown in figure.



[12 marks]

To compute Leq, let us consider the circuit shown below



then $V = L_{eq} \frac{di}{dt}$

we conclude $i = I_1$

using mesh analysis

loop 1 $\quad -V + 4\frac{dI_1}{dt} + 3\frac{d(I_1 - I_2)}{dt} = 0$

$$7\frac{dI_1}{dt} - \frac{3dI_2}{dt} = V \quad -①$$

Write all steps in detail

loop 2 $\quad 2\frac{dI_1}{dt} + 5\frac{dI_2}{dt} + 3\frac{d(I_2 - I_1)}{dt} = 0$

$$-\frac{dI_1}{dt} + 8\frac{dI_2}{dt} = 0$$

$$\text{or} \quad \frac{1}{8}\frac{dI_1}{dt} = \frac{dI_2}{dt}$$

Substituting this in eq ①

$$7\frac{dI_1}{dt} - \frac{3}{8}\frac{dI_1}{dt} = V$$

$$\text{or} \quad V = \frac{53}{8}\frac{dI_1}{dt}$$

$$\therefore \quad V = \frac{53}{8} \frac{di}{d\theta}$$

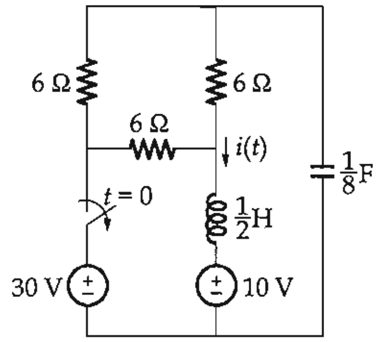
Hence $\quad L_{eq} = \frac{53}{8}$ H

or 6.625 H

11

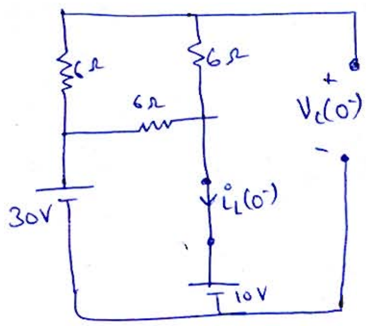
Good Approach

Q.2 (a) For the network shown in figure, find the current $i(t)$ for $t > 0$.

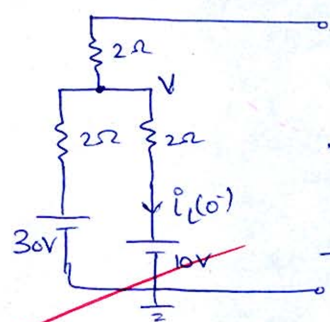


[20 marks]

At $t = 0^-$



converting delta 6Ω to star 2Ω



Write all steps in detail

using nodal analysis

$$\frac{V-30}{2} + \frac{V-10}{2} = 0 \Rightarrow V = \frac{40}{2}\,V = 20V$$

$$\therefore \; i_L(0^-) = \frac{20-10}{2} = \cancel{15A} \quad \& \quad V_c(0^-) \cancel{= V = 40V}$$

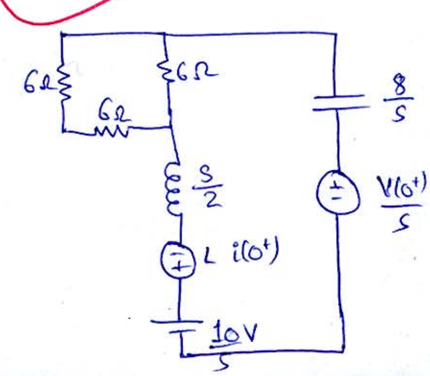
$$= V$$

$$= 20V$$

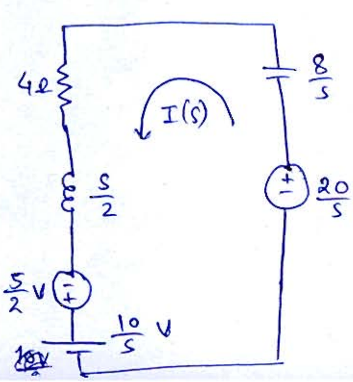
$$= 5A$$

$$\Rightarrow i_L(0^+) = i_L(0^-) = 5A, \quad V_c(0^+) = V_c(0^-) = 20V$$

Now for $t>0$, circuit becomes (in Laplace domain)



$\Longrightarrow$ on simplification



$$\therefore \quad I(s) = \frac{\frac{20}{s} + 2.5 - \frac{10}{s}}{\frac{8}{s} + \frac{s}{2} + 4} = \frac{\frac{10}{s} + 2.5}{4 + \frac{8}{s} + \frac{s}{2}}$$

$$= \frac{10 + 2.5s}{4s + 8 + \frac{s^2}{2}} = \frac{20 + 5s}{s^2 + 8s + 16}$$

$$= \frac{5(s+4)}{(s+4)^2} = \frac{5}{s+4}$$

$$i(t) = L^{-1}\{I(s)\} = 5e^{-4t}, \quad t > 0$$

11

Good Approach

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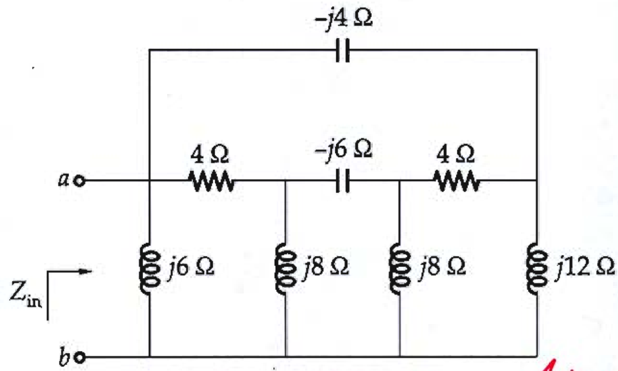
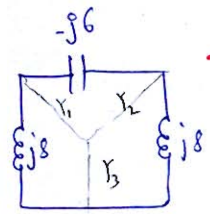
Q.2 (b) Determine the equivalent impedance of the circuit shown



you can use figure given in question.

[20 marks]



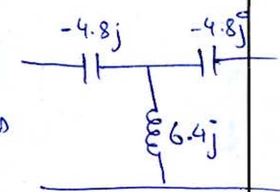
converting this Δ-network into Y, we get

$$Y_1 = \frac{j8 \times (-j6)}{j8 + j8 - j6} = -4.8j\ \Omega$$

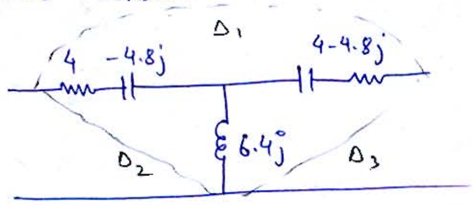
$$Y_2 = Y_1 = -4.8j\ \Omega$$

$$Y_3 = \frac{j8 \times j8}{j8 + j8 - j6} = 6.4i\ \Omega$$

$\Rightarrow$



Now consider 4Ω resistances as well

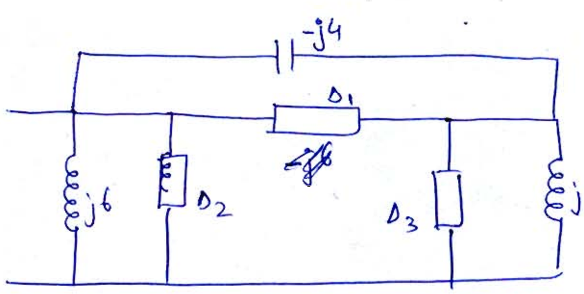


converting Y to Δ

$$\Delta_1 = 39.04 \angle -100.39^\circ\ \Omega$$

$$\Delta_2 = \Delta_3 = 39.99 \angle 39.81^\circ\ \Omega$$

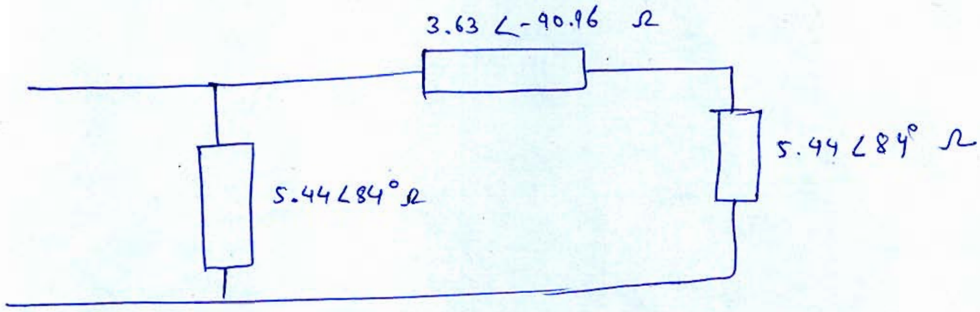
$\therefore$



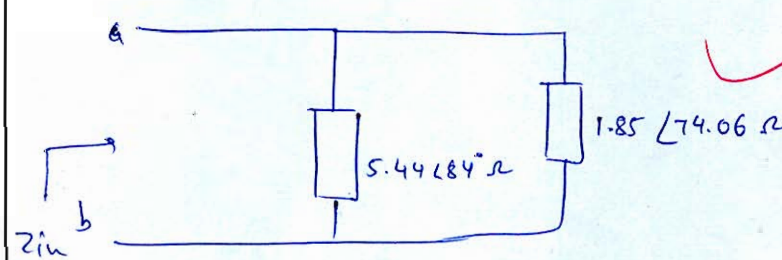
$$\Delta_2 \| j6 = \Delta_3 \| j6 = 5.44 \angle 84^\circ\ \Omega$$

$$\Delta_1 \| (-j4) = 3.63 \angle -90.96^\circ\ \Omega$$

Do not write in this margin



$3.63\angle -90.96 + 5.44\angle 84° = 1.85\angle 74.06°\ \Omega$



$\therefore\ Z_{in} = 5.44\angle 84 \parallel 1.85\angle 74.06$

$Z_{in} = 1.38\angle 76.58°\ \Omega$

5

$\therefore\ Z_{in} = 1.38\angle 76.58°\ \Omega$

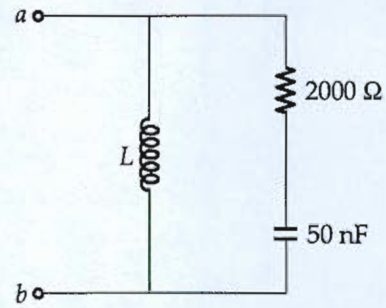
or $= (0.32 + j1.35)\Omega$

Calculation mistake

$Z_{in} = (0.66 + j1.48)\Omega$

Do not write in this margin

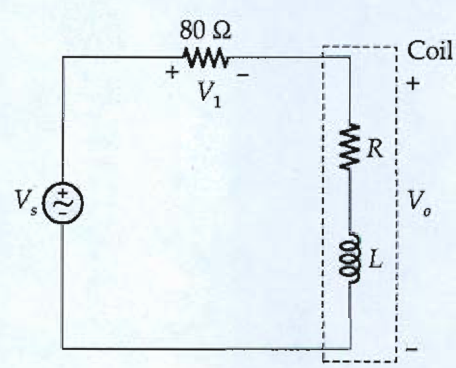
Q.2 (c) **(i)** An industrial load is modelled as a series combination of a capacitance and a resistance as shown in figure. Calculate the value of an inductance L across the series combination so that net impedance is resistive at a frequency of 50 kHz.



(ii) An industrial coil is modelled as a series combination of an inductance L and resistance R, as shown in figure. Since an AC voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz. When the circuit operates in steady state :

$$|V_s| = 145\text{ V},\ |V_1| = 50\text{ V},\ |V_o| = 110\text{ V}$$

Use these measurements to determine the values of L and R.



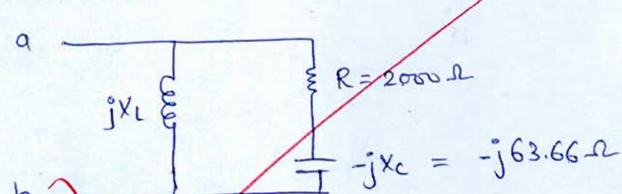
[10 + 10 marks]

(i)

~~For net impedance to be resistive at 50kHz~~

~~⇒ circuit must be at resonance at 50 Hz~~

~~redrawing we get~~



$X_L = 2\pi f L$

$X_c = \dfrac{1}{2\pi f C} = 63.66\,\Omega$

$$Y_{ab} = \frac{1}{jX_L} + \frac{1}{2000 - j63.66}$$

$$= \frac{-j}{X_L} + \frac{2000 + j63.66}{4004052.596}$$

$$Img(Y_{ab}) = \frac{-1}{X_L} + \frac{63.66}{4004052.596} = 0 \quad \text{(for resonance)}$$

$$\therefore \frac{1}{X_L} = 1.5899 \times 10^{-5}$$

$$X_L = 62897.46$$

$$2\pi f L = \cancel{6289.7} \ 62897.46$$

$$\boxed{\therefore L = 0.2 \ H}$$

9

Good Approach

(ii) ~~let $V_s = 145\angle 0°$ be the reference for phasors~~

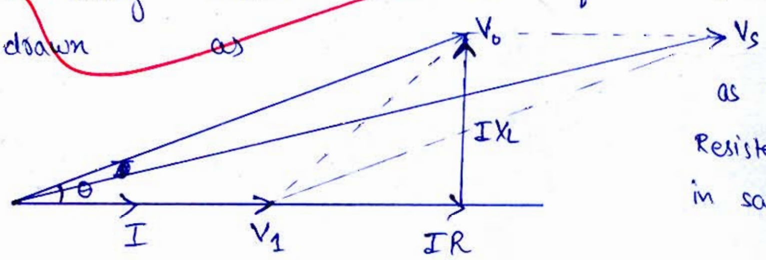
~~$\tilde{V}_s = \tilde{V}_1 + \tilde{V}_0$ & $\tilde{I}_s = \tilde{I}_{ser} = I$~~

~~as the coil is in~~

Try to avoid

(ii) Since the elements are connected in series, the current flowing through each element will be same.

$\therefore$ Taking current $\tilde{I}$ as reference, the phasors can be drawn as



as V_1 is drop across Resistance, $\therefore$ It will be in same phase with I

and coil is inductive

$\therefore I$ lags V_0 by θ

where $\theta = \tan^{-1} \frac{X_L}{R}$

as per vector addition

$$V_s^2 = V_1^2 + V_0^2 + 2V_1V_0\cos\theta$$

$$145^2 = 50^2 + 110^2 + 2\times 50\times 110 \times\cos\theta$$

$$\therefore \cos\theta = 0.584$$

$$\therefore \theta = 54.26°$$

also $$|I| = \frac{|V_1|}{80\,\Omega}$$

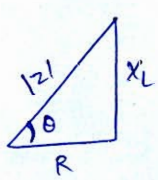
$$\therefore |I| = \frac{50}{80} = 0.625\text{ A}$$

and $$|V_s| = |I|\,|z|$$

$$145 = 0.625\,|80 + R + jX_L|$$

$$(80+R)^2 + (X_L)^2 = 60712.96$$

5



from impedance triangle

$$R = z\cos\theta \;,\; X_L = z\sin\theta$$
$$= 0.584z \;,\; = 0.812z$$

$$\therefore (80 + 0.584z)^2 + (0.812z)^2 = 60712.96$$

$$\cancel{\Rightarrow z = 190.94\,\Omega}$$

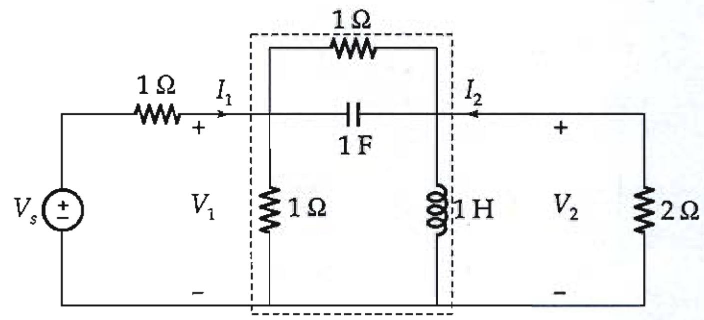
$$\therefore \cancel{\boxed{R = 111.51\,\Omega}}$$

$$X_L = 155.04\,\Omega$$

$$\therefore \boxed{L = \frac{X_L}{2\pi\times 60} = \cancel{0.41\text{ H}}}$$

Correct value
R = 102.8 Ω
L = 0.3789 H

Q.3 (a) Determine the y-parameters of two port network (1). Also determine $V_2(s)$ for $V_s = 2u(t)$ V.



Network (1)

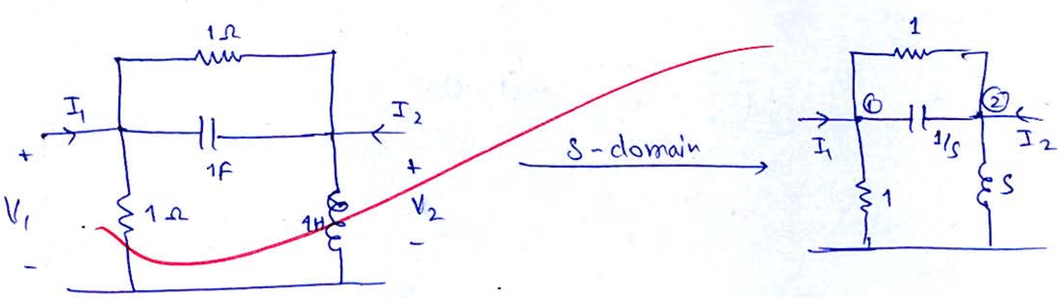
[10 + 10 marks]

we know

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Write all steps in detail

N/w (1) is drawn as



Applying nodal technique

at node (i)

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{1/s} + \frac{V_1 - V_2}{1}$$

$$I_1 = (2 + s) V_1 + (-1 - s) V_2 \quad \text{— (i)}$$

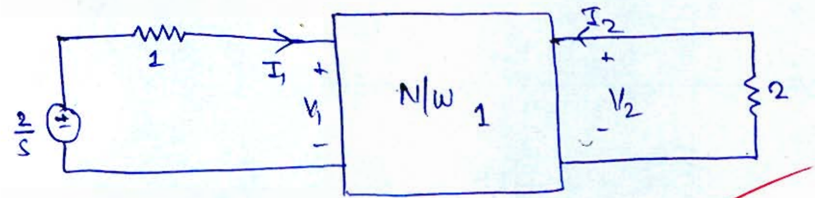
at node (ii)

$$I_2 = \frac{V_2}{s} + \frac{V_2 - V_1}{1/s} + \frac{V_2 - V_1}{1}$$

$$I_2 = (-1 - s) V_1 + \left(1 + s + \frac{1}{s}\right) V_2 \quad \text{— (ii)}$$

$$\therefore \quad Y = \begin{bmatrix} 2 + s & -(1 + s) \\ -(1 + s) & \left(1 + s + \frac{1}{s}\right) \end{bmatrix}$$

Now $V_s = 2u(t) \Rightarrow V_s(s) = \frac{2}{s}$



From the above circuit

$$\frac{2}{s} - I_1(1) = V_1 \Rightarrow V_1 = \frac{2}{s} - I_1$$

& $V_2 = -2I_2$

substituting these values in eqn (I) & (II)

$$I_1 = (2+s)\left(\frac{2}{s} - I_1\right) - (1+s)V_2 \quad \text{—(III)}$$

$$\frac{-V_2}{2} = (-1-s)\left(\frac{2}{s} - I_1\right) + \left(1 + s + \frac{1}{s}\right)V_2 \quad \text{—(IV)}$$

or $I_1 = (2+s)\frac{2}{s} - I_1(2+s) - (1+s)V_2$ {from (III)}

$$\Rightarrow I_1(3+s) = \frac{4}{s} + 2 - (1+s)V_2$$

$$\Rightarrow I_1 = \frac{\frac{4}{s} + 2 - (1+s)V_2}{3+s}$$

Substituting in eq (IV)

$$V_2\left(1 + s + \frac{1}{s} + \frac{1}{2}\right) = (1+s)\left\{\frac{2}{s} - \left[\frac{\frac{4}{s} + 2 - (1+s)V_2}{3+s}\right]\right\}$$

$$V_2\left(\frac{3}{2}+s+\frac{1}{s}\right) = (1+s)\left[\frac{6+2s-4-2s+s(1+s)V_2}{s(s+3)}\right]$$

$$V_2\left(\frac{3}{2}+s+\frac{1}{s}\right) = (1+s)\left[\frac{2+s(1+s)V_2}{s(s+3)}\right]$$

$$V_2\left[\frac{3}{2}+s+\frac{1}{s}\right] = \frac{2(s+1)}{s(s+3)} + \frac{s(s+1)^2}{s(s+3)}V_2$$

$$V_2\left[\frac{3}{2}+s+\frac{1}{s}-\frac{(s+1)^2}{(s+3)}\right] = \frac{2(s+1)}{s(s+3)}$$

$$V_2\left[\frac{3s+2s^2+2}{2s}-\frac{(s^2+2s+1)}{s+3}\right] = \frac{2(s+1)}{s(s+3)}$$

$$V_2\left(\frac{(2s^2+3s+2)(s+3)-(s^2+2s+1)(2s)}{2s(s+3)}\right) = \frac{2(s+1)}{s(s+3)}$$

$$V_2\left[\begin{array}{l}2s^3+6s^2+3s^2+9s+2s+6\\ -(2s^3+4s^2+2s)\end{array}\right] = \frac{2(s+1)}{s(s+3)}\cdot 2s(s+3)$$

$$V_2[5s^2+9s+6] = 4(s+1)$$

$$V_2(s) = \frac{4(s+1)}{5s^2+9s+6}$$

$$\therefore V_2(s) = \frac{4(s+1)}{5s^2+9s+6}$$

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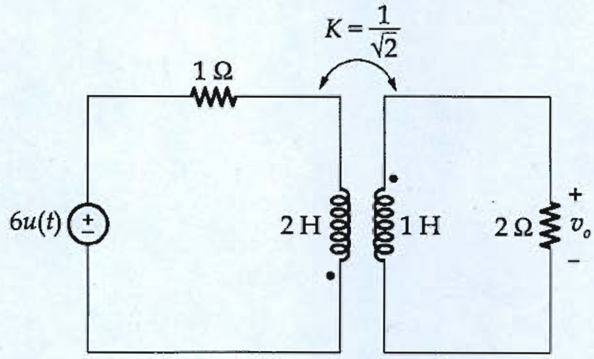
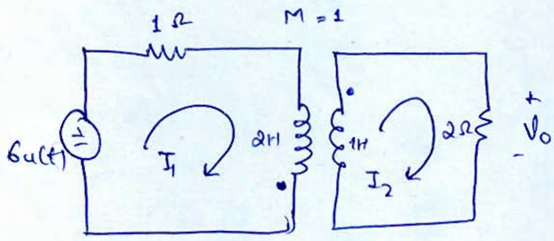
Good Approach

Q.3 (b) For the circuit shown in figure, find $V_o(t)$ for $t > 0$.



[20 marks]



given $K = \frac{1}{\sqrt{2}}$

$\therefore \frac{M}{\sqrt{L_1 L_2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow M = 1$

using mesh technique

$$6u(t) = I_1(1) + 2\frac{dI_1}{dt} + 1\frac{dI_2}{dt} \quad -\text{(I)}$$

[both currents are leaving the dot]

$$1\frac{dI_2}{dt} + 1\frac{dI_1}{dt} + 2I_2 = 0 \quad -\text{(II)}$$

Taking Laplace on both sides on eq (I) & eq (II)

$$\frac{6}{s} = I_1(s) + 2s\,I_1(s) + s\,I_2(s)$$

or $$(1+2s)\,I_1(s) + s\,I_2(s) = \frac{6}{s} \quad -\text{(III)}$$

&

$$s\,I_2(s) + s\,I_1(s) + 2I_2(s) = 0$$

or $$s\,I_1(s) + (2+s)\,I_2(s) = 0 \quad -\text{(IV)}$$

$$\Rightarrow I_1(s) = \frac{-(2+s)\,I_2(s)}{s}$$

Substituting in eqn (iii), we get

$$-(1+2s)\left(\frac{2+s}{s}\right) I_2(s) + s\,I_2(s) = \frac{6}{s}$$

$$\Rightarrow \quad I_2(s)\left[s - \frac{1}{s}\left(2+s+4s+2s^2\right)\right] = \frac{6}{s}$$

$$I_2(s)\left[s^2 - \left(2+5s+2s^2\right)\right] = 6$$

$$I_2(s)\left[s^2 - 2 - 5s - 2s^2\right] = 6$$

$$I_2(s) = \frac{6}{-s^2-5s-2}$$

$$= \frac{-6}{s^2+5s+2} \quad A$$

$$\therefore \quad V_0(s) = I_2(s) \cdot 2$$

$$= \frac{-12}{s^2+5s+2} = \frac{-12}{(s+0.438)(s+4.562)}$$

18

$$= \frac{-2.91}{(s+0.438)} + \frac{2.91}{(s+4.562)} \quad V$$

Good Approach

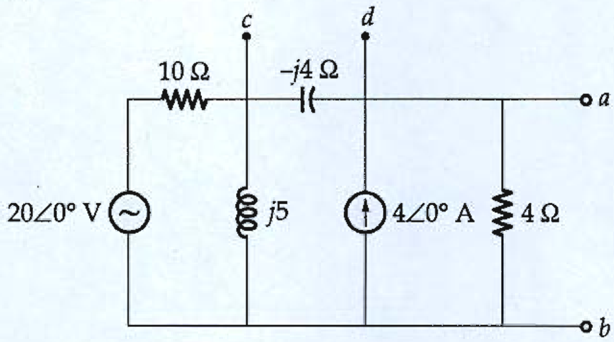
$$v_0(t) = L^{-1}\left[V_0(s)\right] = -2.91e^{-0.438t} + 2.91e^{-4.562t}$$

$$\therefore v_0(t) = 2.91\left[e^{-4.562t} - e^{-0.438t}\right] \quad t \geq 0$$

Q.3 (c) Find the Thevenin's equivalent of circuit shown in figure as seen from :

(i) terminals *a-b*

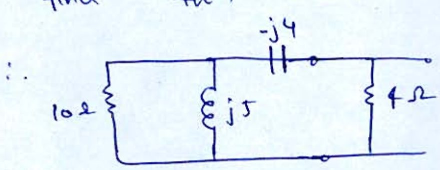
(ii) terminals *c-d*



[20 marks]

(i) To find Z_{th}, sources needs to be deactivated

∴

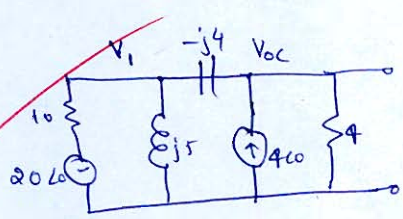


Write steps in detail

$$Z_{eq} = \left\{(10\,||\,j5) - j4\right\} \,||\, 4$$

$$= 4/3\ \Omega$$

To find V_{oc}, we have



$$\frac{V_1 - 20}{10} + \frac{V_1}{j5} + \frac{V_1 - V_{oc}}{-j4} = 0$$

$$V_1\left(\frac{1}{10} + \frac{1}{20}j\right) + V_{oc}\left(-\frac{1}{4}j\right) = 2 \quad -(i)$$

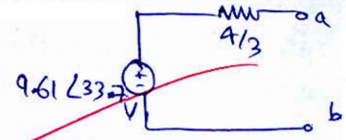
$$\frac{V_{oc}}{4} + \frac{V_{oc} - V_1}{-j4} = 4$$

$$V_{oc}\left(\frac{1}{4} + \frac{1}{4}j\right) + V_1\left(-\frac{1}{4}j\right) = 4 \quad -(ii)$$

solving (i) & (ii) we get

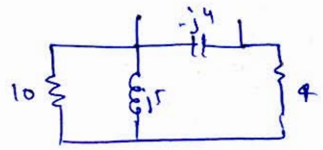
$$V_{oc} = 8 + \frac{16}{3}j = 9.61\angle 33.69^\circ\ V$$

$\therefore$ Thevenin's equivalent is



Circuit

(ii) To find Z_{th}



$$Z_{eq} = \left\{(10 \| j5) + 4\right\} \| (-j4) = \frac{8}{3} - 4j = 4.81 \angle -56.31 \ \Omega$$

To find V_{oc}

From part (i) calculations, we have

$$V_{oc(i)} = 9.61 \angle 33.69^\circ \ V$$

$$\& \quad V_1 = 18.85 \angle 45.02^\circ \ V$$

Hence from the circuit we can conclude that

$$V_{oc(ii)} = V_1 - V_{oc(i)}$$

$$\therefore \quad V_{oc(ii)} = 9.62 \angle 56.34^\circ \ V$$

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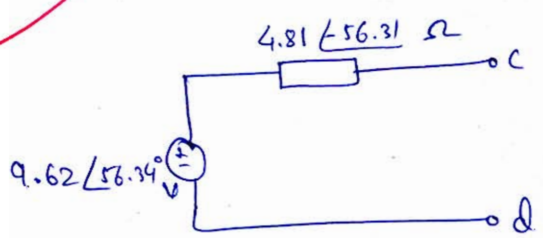
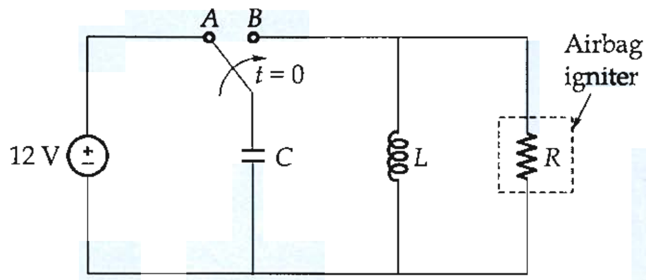
$\therefore$ Thevenins equivalent circuit is



Good Approach

Do not write in this margin

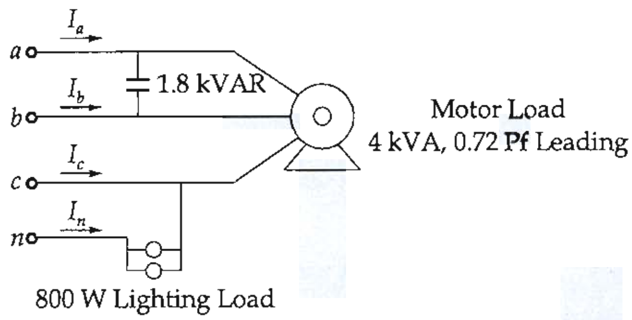
Q.4 (a) An automobile airbag igniter is modelled by the circuit shown in figure. Determine the time, it takes the voltage across the igniter to reach its first extreme (minimum or maximum) after switching from A to B. Let $R = 3\ \Omega$, $C = \frac{1}{30} F$ and $L = 60$ mH.



[20 marks]

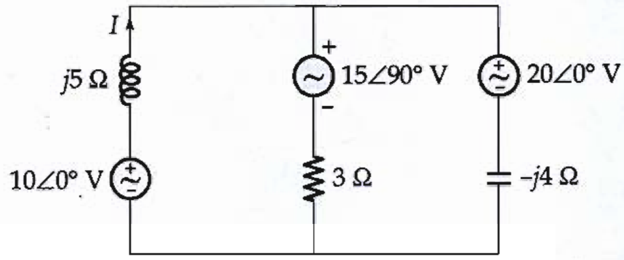
Do not write in this margin

Q.4 (b) In the figure shown, a 3-phase delta connected motor load which is connected to a line of 440 V, draws 4 kVA at a power factor of 0.72 leading. In addition, a single 1.8 kVAR capacitor is connected between line a and b, while 800 W lighting load is connected between line C and neutral. Assuming abc phase sequence and taking $V_{an} = V_p\angle 0°$, find the magnitude and phase angle of currents I_a, I_b, I_c and I_n.



[20 marks]

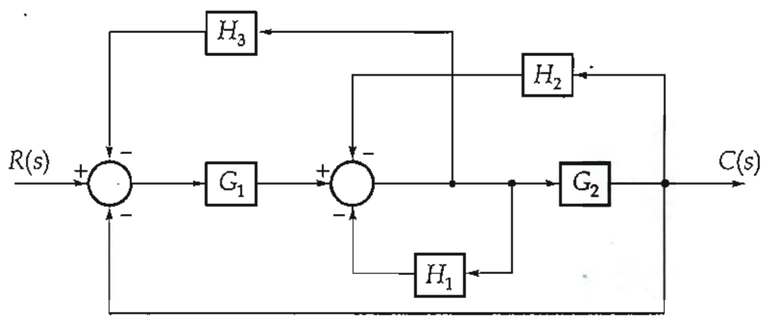
Q.4 (c) Find current I through $j5\ \Omega$ branch using superposition theorem for the network shown.



[20 marks]

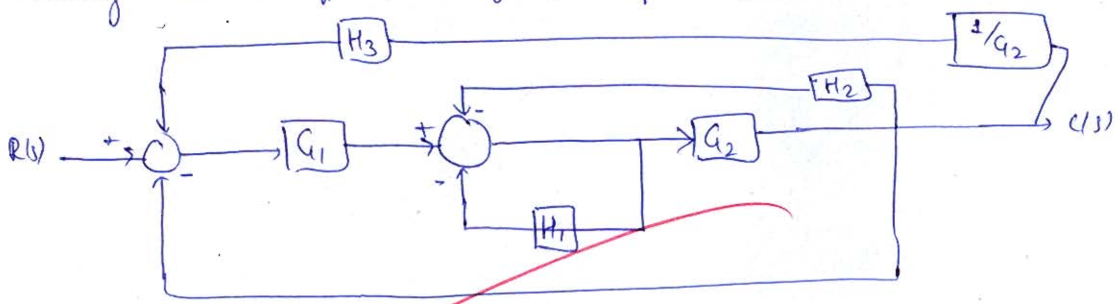
Section B : Control Systems

Q.5 (a) Using block-diagram reduction technique, find the transfer function $\frac{C(s)}{R(s)}$.

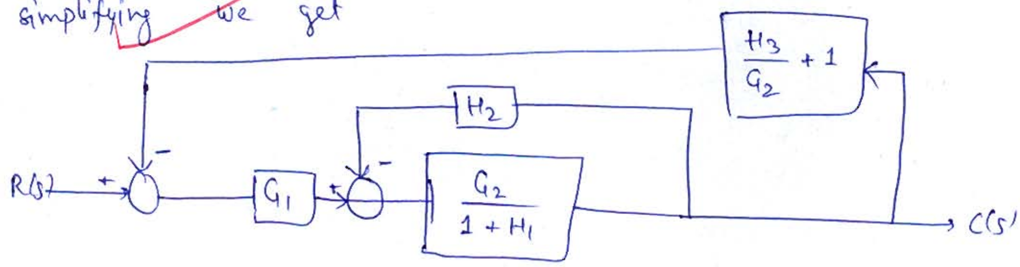


[12 marks]

Moving the take off point of H_3 after G_2

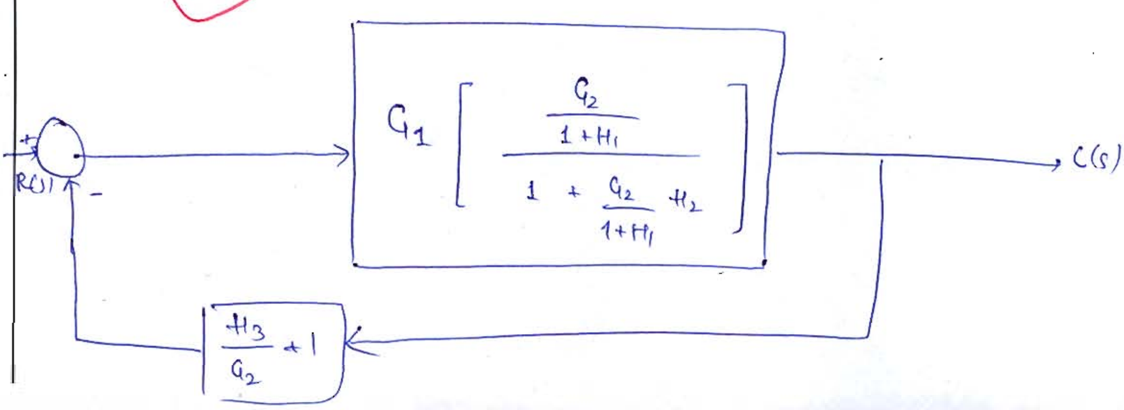


on simplifying we get



again simplifying H_2 as -ve feedback

& ~~H3~~ G_1 as forward gain after H_2 feedback

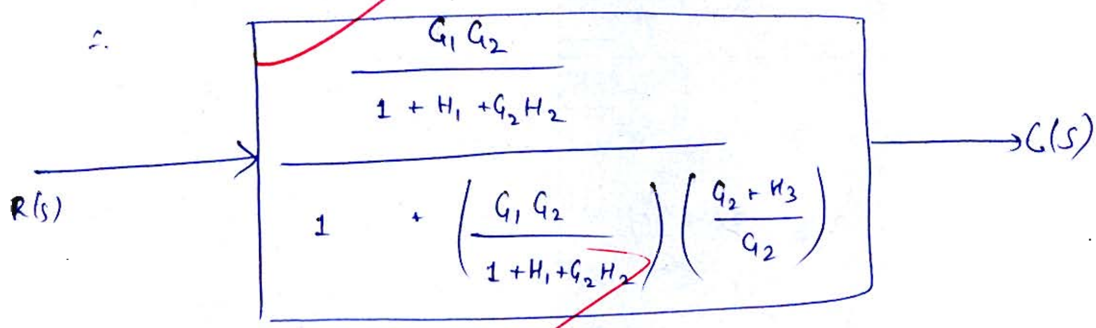


On simplification

Forward path $\Rightarrow \dfrac{G_1 G_2}{1 + H_1 + G_2 H_2}$

Feedback path $\Rightarrow \dfrac{G_2 + H_3}{G_2}$

$\therefore$

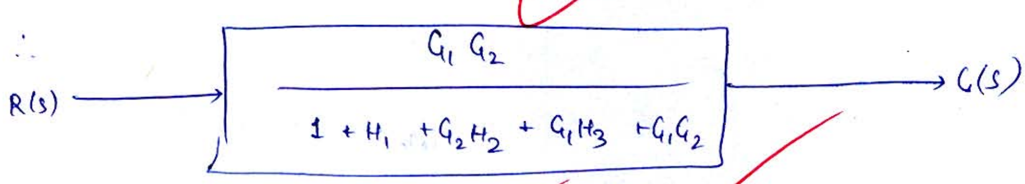


11

on simplification

$$\frac{C(s)}{R(s)} = TF(s) = \frac{G_1 G_2}{1 + H_1 + G_2 H_2 + G_1 G_2 + G_1 H_3}$$

$\therefore$



Good Approach

Do not write in this margin

Q.5 (b) The closed loop transfer function of a system is

$$\frac{C(s)}{R(s)} = \frac{100}{s^6+3s^5+8s^4+18s^3+20s^2+24s+16}$$

Determine the number of poles on the RHP, LHP and on the $j\omega$-axis and comment on the stability of the system.

[12 marks]

Using RH criteria to find location of poles

$$s^6 + 3s^5 + 8s^4 + 18s^3 + 20s^2 + 24s + 16 = 0$$

s^6	1	8	20	16
s^5	3	18	24	
s^4	2	12	16	
s^3	0	0		

as we got all terms in s^3 row as zero

$\Rightarrow$ there are 4 poles symmetric to the origin

forming auxiliary eq^n from s^4 row

$$A(s) = 2s^4 + 12s^2$$

$$\frac{d}{ds}A(s) = 8s^3 + 24s$$

roots of this polynomial are $(A(s)=0)$

$0, 0, \pm j\sqrt{6}$

$\therefore$ RH table becomes

s^3	8	24
s^2	6	16
s^1	8/3	0
s^0	16	

∴ the complete table with 1st column is

s^6	1
s^5	3
s^4	2
s^3	8
s^2	6
s^1	8/3
s^0	16

Before the zero row, there is no sign change

⇒ 2 poles in LHP

After the zero row, there is no sign change

⇒ 4 poles symmetric to origin

& none of them are in RHP

⇒ none of them in LHP

⇒ all 4 are on $j\omega$ axis

& two of them are at origin as evident from auxiliary eq^n. $A(s) = 0$.

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Good Approach

∴ Out of 6 poles

2 poles — LHP

0 poles — RHP

4 poles — $j\omega$ - axis

↳ at $j\sqrt{6}$, $-j\sqrt{6}$ & repeated at origin

∴ System is unstable due to repeated poles at the origin.

Q.5 (c) The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K(s+\alpha)}{s(s^2+12s+32)}$$

Find the value of K and α so that the velocity error constant is 6.25 and the second-order response has a natural frequency of 5 rad/s. Assume that the system is stable.

[12 marks]

given $\omega_n = 5$ rad/s

& $K_v = 6.25$

we know $K_v = \lim_{s\to 0} s\,G(s)$

$$6.25 = \lim_{s\to 0} \frac{K(s+\alpha)}{(s^2+12s+32)}$$

$$6.25 = \frac{K\alpha}{32}$$

$$\therefore K\alpha = 200 \quad \text{— (1)}$$

Now $q(s) = s^3 + 12s^2 + 32s + Ks + K\alpha$

$$= s^3 + 12s^2 + (32+K)s + 200 \quad \text{— (ii)}$$

a third order system will have the characteristic eqⁿ of the form

$$(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$$

where the 2nd order term will be dominant

$$s^3 + (2\zeta\omega_n + p)s^2 + (\omega_n^2 + 2\zeta\omega_n p)s + p\omega_n^2 = 0 \quad \text{(iii)}$$

compairing eq (ii) & eq (iii) we get

$$2\zeta\omega_n + p = 12$$

$$32 + K = \omega_n^2 + 2\zeta\omega_n p$$

$$200 = p\omega_n^2$$

given $\omega_n = 5 \Rightarrow \omega_n^2 = 25$

$\therefore \quad 10\zeta + P = 12$

$25 + 10\zeta P = 32 + K$

$25P = 200$

$\Rightarrow P = 8$

$\zeta = 0.4$

$\cancel{K \neq \angle 7}$

$K = 25$

$\therefore \quad \alpha = \frac{200}{K} = 8$

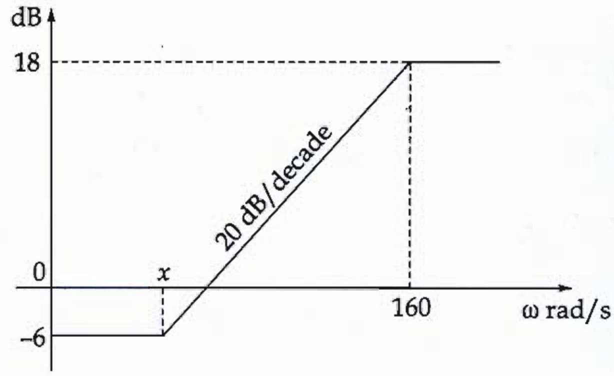
Hence $K = 25$

$\alpha = 8$

11

Good Approach

Q.5 (d) An asymptotic bode magnitude plot is shown in figure.



Find the transfer function and gain cross-over frequency.

[12 marks]

initial slope is 0

$\Rightarrow$ no pole or zero at origin

at $\omega = x$, slope = 20 dB/dec

$\Rightarrow$ zero at $\omega = 20$

at $\omega = 160$ slope = 0 again

$\Rightarrow$ pole at $\omega = 160$

$$\therefore \quad TF = \frac{K\left(\frac{s}{x}+1\right)}{\left(\frac{s}{160}+1\right)}$$

at $\omega = x$, gain = -6dB & slope = 20 dB/decade

at $\omega = 160$ gain = 18 dB

$$\therefore \quad \frac{18-(-6)}{\log 160 - \log x} = 20 \quad \Rightarrow \quad \log\frac{160}{x} = \frac{6}{5}$$

~~$\Rightarrow x = 48.19$ rad/sec~~

$\Rightarrow \quad x = 10.095$

$\therefore$

we know $-20r \log \omega + 20 \log K = \text{gain dB}$

$r = 0$

$\Rightarrow \quad 20 \log K = -6 \quad (\text{at } \omega = x)$

$\Rightarrow \quad K = 0.5$

$$\therefore \quad TF(s) = \frac{0.5\left(1 + \frac{s}{10.095}\right)}{\left(1 + \frac{s}{160}\right)}$$

at ω_{gc}, gain $= 0\,dB$ or $|gain| = 1$

from the graph

$$\frac{18' - 0}{\log 160 - \log \omega_{gc}} = 20$$

$$\log \frac{160}{\omega_{gc}} = \frac{18}{20}$$

$\omega_{gc} = 20.14$ rad/sec

11

Good Approach

Q.5 (e) A system is represented by the state model, $\dot{X} = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$ and $y = [1 \quad 2]X$. If the initial state vector is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find zero input response of the system.

[12 marks]

To obtain ZIR   Zero input response

$i/p = 0$

$\therefore r(t) = 0$

$\therefore \dot{X} = \begin{bmatrix} 0 & 0 \\ +3 & -3 \end{bmatrix} X \quad \Rightarrow \quad \dot{X} = AX$

$y = [1 \quad 2] X \quad \Rightarrow \quad y = CX$

applying Laplace, we get

$sX(s) - x(0) = A X(s) \quad , \quad Y(s) = C X(s)$

$(sI - A) X(s) = x(0)$

$X(s) = (sI - A)^{-1} x(0)$

$\Rightarrow \quad Y(s) = C \left\{ (sI - A)^{-1} x(0) \right\}$

$= [1 \quad 2] \left[\begin{bmatrix} s & 0 \\ -3 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right]$

$= \dfrac{[1 \quad 2] \begin{bmatrix} s+3 & 0 \\ 3 & s \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{s(s+3)}$

$$= \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+3 \\ 3+2s \end{bmatrix}}{s(s+3)}$$

$$= \frac{s+3 + 6 + 4s}{s(s+3)}$$

$$= \frac{5s + 9}{s(s+3)}$$

$$= \frac{3}{s} + \frac{2}{s+3}$$

$$Y(s) = \frac{3}{s} + \frac{2}{s+3}$$

$$\therefore y(t) = \cancel{3 + 2e^{-3t}}$$

$$(3 + 2e^{-3t})\, u(t)$$

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$$\therefore ZIR = (3 + 2e^{-3t})\, u(t)$$

Good Approach

Do not write in this margin

Q.6 (a) The forward path gain of a first-order unity negative feedback system is $G(s) = \dfrac{K}{s+a}$. The unit step response reveals that the time constant is 1/6 sec. When the location of the pole is moved toward the origin by half its distance, the new time constant is found to be 1/4 sec. Find the value of a and K. For the time constant to be 1/8 sec, find the location of the closed-loop pole.

[20 marks]

Q.6 (b) A unity feedback system has open-loop transfer function

$$G(s) = \frac{3(2-s)}{(s+1)(s+5)}$$

Using Nyquist stability criterion, check whether the closed-loop system is stable or not. If the system is stable, find the gain margin and phase margin.

[20 marks]

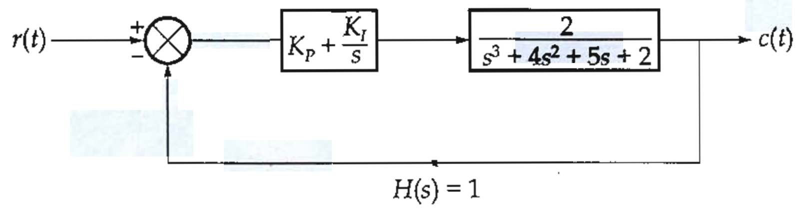
Q.6 (c) A feedback system has open-loop transfer function

$$G(s)H(s) = \frac{K(s+5)}{(s+1)^2}$$

Sketch the root locus.

[20 marks]

Q.7 (a) The stability of overall system shown in figure is controlled by tuning the PI parameters K_P and K_I. Find the maximum value of K_I that can be selected so as to keep overall system stable or in worst case, marginally stable.



[20 marks]

Write all steps

$$G(s) = \frac{2(K_I + sK_P)}{s(s^3+4s^2+5s+2)}$$

$$TF(s) = s^4 + 4s^3 + 5s^2 + 2s + 2K_Ps + 2K_I$$

$$= s^4 + 4s^3 + 5s^2 + (2+2K_P)s + 2K_I$$

applying RH criteria

$2 + 2K_P > 0 \Rightarrow K_P > -1$

$2K_I > 0 \Rightarrow K_I > 0$

forming RH table

s^4	1	5	$2K_I$
s^3	4	$2+2K_P$	
s^2	$\frac{20-2-2K_P}{4}$	$2K_I$	
s^1	$(2+2K_P) - \frac{8K_I}{\frac{(20-2-2K_P)}{4}}$		
s^0	$2K_I$		

~~at marginally~~

for stability all coffiicients of first column > 0

$20-2-2K_p > 0$

$\cancel{20}$ $18 > 2K_p$

$\boxed{K_p < 9}$

3

$2 + 2K_p > \frac{32 K_I}{20-2-2K_p}$

$(2+2K_p)\,\cancel{20-2}\,(18-2K_p) > 32 K_I$

$\boxed{(2+2K_p)(18-2K_p) > 32K_I}$

$\therefore \boxed{-1 < K_p - 9}$

$\boxed{K_I > 0}$

for stability

$K_{I\max} = \frac{25}{8}$ (for marginally stable system)

at $K_p = 4$

Q.7 (b) The open loop transfer function of a unity feedback system is $G_p(s) = \dfrac{K}{s(s+2)}$. Design a lead compensator to have a velocity-error constant of $20s^{-1}$ and phase margin of at least $50°$. Indicate each step that you are using.

[20 marks]

Do not write in this margin

Q.7 (c) A negative unity feedback control system is expected to meet the following specifications : damping ratio is 0.5, natural frequency is $\sqrt{10}$ rad/sec and the steady-state error is 10%. The open-loop transfer function is $G(s) = \dfrac{K(s+\alpha)}{(s+\beta)^2}$. Find the values of K, α and β.

[20 marks]

$$G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$$

$$TF(s) = \frac{K(s+\alpha)}{s^2 + 2s\beta + \beta^2 + Ks + K\alpha}$$

$$q(s) = s^2 + (2\beta + K)s + (\beta^2 + K\alpha)$$

given $\xi = 0.5$, $\omega_n = \sqrt{10}$

$$\Rightarrow q(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$
$$= s^2 + \sqrt{10}s + 10$$

~~also since the system is type 0~~

$\therefore$ ~~$E_{ss} = 0$~~

Try to avoid

$$\Rightarrow \boxed{2\beta + K = \sqrt{10}} \quad , \quad \boxed{\beta^2 + K\alpha = 10}$$

as the system is type 0 system

$$\Rightarrow K_p \neq 0$$

$$E_{ss} = \frac{1}{1 + K_p} = 0.1$$

$$\Rightarrow K_p = 9$$

$$\Rightarrow \frac{K\alpha}{\beta^2} = 9$$

$$\Rightarrow \boxed{K\alpha = 9\beta^2}$$

Hence $K\alpha = 9\beta^2$ — (i)

$\beta^2 + K\alpha = 10$ — (ii)

$2\beta + K = \sqrt{10}$ — (iii)

$\therefore$ from (i) & (ii)

$\beta^2 + 9\beta^2 = 10$

$\beta = \pm 1$

$\beta = -1$ is not possible

Case I

$\beta = 1$

$K\alpha = 9$

$2 + K = \sqrt{10}$

$K = 1.16$

$\alpha = 7.74$

$\beta = 1$

Case II

$\beta = -1$

$K\alpha = 9$

$-2 + K = \sqrt{10}$

$K = 5.16$

$\alpha = 1.74$

$\beta = -1$

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$\therefore$ By setting the open loop pole to either +1 or -1 we may get two set of values.

In order to follow open loop stability, we may go for $\beta = -1$

Q.8 (a) Draw the log-magnitude asymptotic plot for the transfer function.

$$G(s)H(s) = \frac{1000s}{(s+10)(s+100)}$$

Also find the gain cross-over frequency and the frequencies at 3 dB attenuation.

[20 marks]

Q.8 (b) Consider the system with state equation

$$\dot{X}(t) = AX(t) + BU(t)$$

where
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

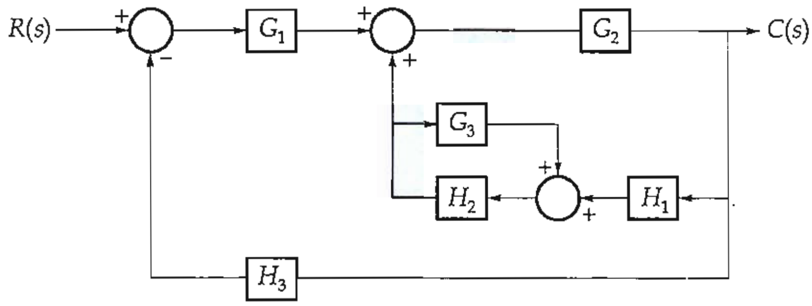
By using the state feedback control $u = -KX$, it is desired to have the closed-loop poles at

$$u_1 = -2 + j4, u_2 = -2 - j4, u_3 = -10$$

Determine the state feedback gain matrix K. Also check the validity of arbitrary pole placement.

[20 marks]

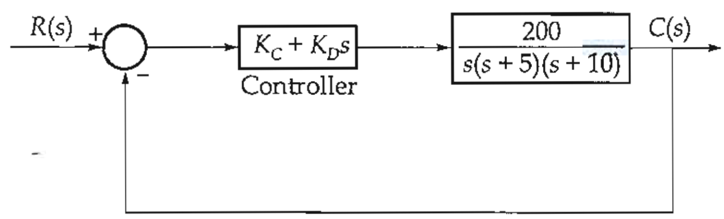
Q.8 (c) **(i)** For the block-diagram representation for a system shown below :



Draw the signal flow graph and determine the overall transfer function using Mason's gain formula.

(ii) A unity feedback system has plant transfer function $G_C(s) = \dfrac{200}{s(s+5)(s+10)}$.

The plant is controlled by a PD controller. Find the ranges of controller gains (K_C, K_D) for the system shown below to be stable. Also draw the region of stability.



[12 + 8 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

$i = C\frac{dv}{dt}$

$i = CV(0^+)$

$V = \frac{CV(0^+)}{5\Omega}$

$\frac{1}{1+\mathcal{H}}$

$245.76 - 45.056j$

$$\frac{4 - V_{oc}\left[\frac{1}{4} + \frac{1}{4}j\right]}{} \times \frac{-4}{j}$$

$$\left[16j + (1-j)V_{oc}\right]\left[\frac{1}{10} + \frac{j}{20}\right] + V_{oc}\left(-\frac{j}{4}\right) = 2$$

$$\left[-\frac{4}{5} + \frac{8}{5}j\right] + \left(\frac{3}{20} - \frac{j}{20}\right)V_{oc} + V_{oc}\left(-\frac{j}{4}\right) = 2$$