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ESE 2023 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-1 : Strength of Materials + Environmental Engineering [All Topics]

Name : .....

Roll No :

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE	
Question No.	Marks Obtained
Section-A	
Q.1	54
Q.2	58
Q.3	47
Q.4	X
Section-B	
Q.5	23
Q.6	X
Q.7	11
Q.8	X
Total Marks Obtained	193

Signature of Evaluator Cross Checked by

Akshay ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

1. SOM is excellent. Keep it up.
2. Work hard on Environmental Engineering.
3. Numerical Solving is good but theory is very sub-standard. Keep reading theory.
4. Presentation is neat & clean.

Section-A : Strength of Materials

Q.1 (a) A copper rod of cross-sectional area 305 sq. mm connects two parallel walls distant 5 m apart. When the rod is heated to 80° C, the nuts at the end gets heated up. Calculate the pull exerted by the bar on cooling at atmospheric temperature at 20° C if

(i) the ends do not yield.

(ii) if the ends yield by 2.5 mm.

Take: α_{copper} = 0.0000175 °C; μ_{copper} = 0.35 and modulus of rigidity of copper = 40000 N/mm^2.

[12 marks]

Sol.) Modulus of elasticity $(E) = 2G(1+\mu)$

$= 2 \times 40000 (1.35)$

$= 108000 \text{ N/mm}^2$ 12

i.) Pull exerted if ends do not yield.

$P \Rightarrow \alpha \Delta T E A$

$= 1.75 \times 10^{-5} \times (80 - 20) \times 108000 \times 305 \times 10^{-3}$

$P = 34.587 \text{ kN}$. ⇒ Pull exerted.

ii.) If ends yield by 2.5 mm.

Strain restrained $\Rightarrow \left(\frac{\alpha \Delta T L - 2.5}{L}\right) \text{mm}$

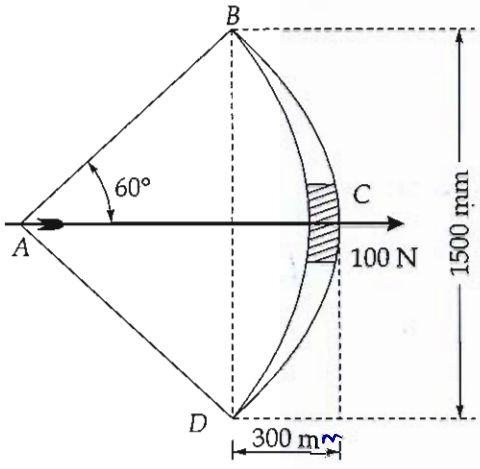
$= \frac{1.75 \times 10^{-5} \times 60 \times 5000 - 2.5}{5000}$

$\Rightarrow 5.5 \times 10^{-4}$

Pull exerted $\Rightarrow (\alpha \Delta T @ L - 2.5) \times E \times A$

$P \Rightarrow 18.117 \text{ kN}$.

Q.1 (b) At full draw, an archer applies a pull of 100 N to the bow string of the bow as shown in the figure. Determine the bending moment at the midpoint of the bow.

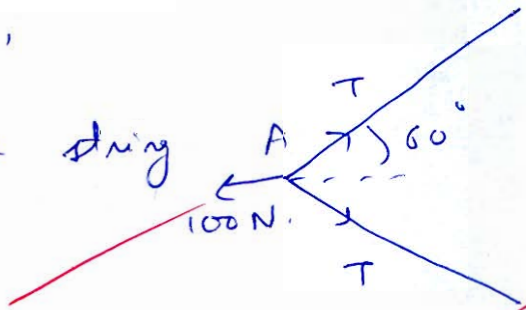


12

[12 marks]

Sol:) At A,

Tension in the string will be T.



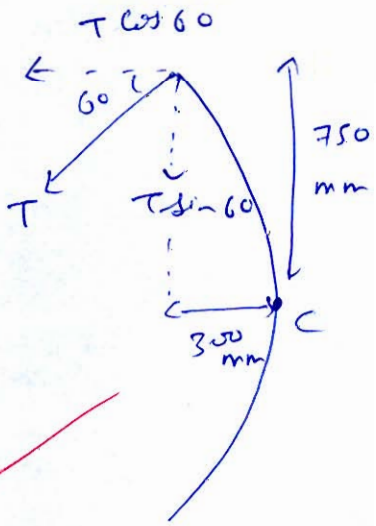
FBD = ?

So, $\Sigma F_x = 0$ at A.

$\Rightarrow 2T \cos 60^\circ = 100$

$\Rightarrow T = 100$ N.

Now, Bending moment at C,



$\Rightarrow T \cos 60 \times 750 + T \sin 60 \times 300$

$\Rightarrow 50 \times 750 + 50\sqrt{3} \times 300$

$= 63480.76$ N-mm

↳ Bending moment at C.

Q.1 (c) A brass plate is stretched by tensile forces in x and y directions at right angle to each other. The strains given by strain gauges in the two directions are as follows:

(*a*) x-direction = 0.00088 (*b*) y-direction = 0.00022

Take modulus of elasticity as 0.8×10^5 MPa and Poisson's ratio as 0.3.

Find:

(i) Stresses in the x and y-direction.

(ii) Direct strain on a plane at 30° to the x-direction.

(iii) Stresses on the plane inclined at 30° to the x-axis.

[12 marks]

Sol:)

Since there is no shear strain thus planes are principal plan.

So, $\varepsilon_{p_1} = 0.00088$ $\varepsilon_{p_2} = 0.00022$

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Now

i) $\sigma_x (\sigma_{p_1}) = \frac{E}{1-\mu^2} (\varepsilon_{p_1} + \mu \varepsilon_{p_2})$

$\sigma_x = 83.1648$ MPa

& $\sigma_y (\sigma_{p_2}) = \frac{E}{1-\mu^2} (\varepsilon_{p_2} + \mu \varepsilon_{p_1})$

$\sigma_y = 42.5494$ MPa

ii) $\varepsilon_{\theta=30^\circ} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \phi_{xy} \sin\theta \cos\theta$ ($\phi_{xy} = 0$)

$= 0.00088 \times \cos^2 30 + 0.00022 \sin^2 30$

$\varepsilon_{\theta=30^\circ} = 7.15 \times 10^{-4}$ or 0.000715

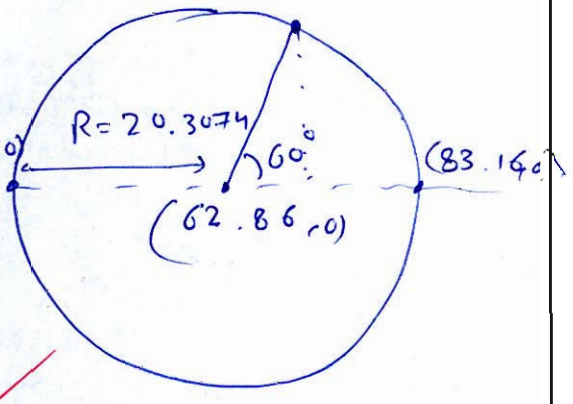
iii) ~~σ at $\theta = 30^\circ$ $\Rightarrow$ $\sigma_x \cos^2\theta + \sigma_y \sin^2\theta$~~

from

$\Rightarrow$

Stress at $\theta = 30^\circ$

from x-axis



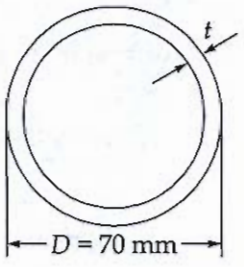
$\Rightarrow$ $\tau_{\text{at } \theta=30^\circ} = R \sin 60^\circ$

$= 17.586$ MPa.

& $\sigma = 62.86 + R \cos 60$

$\sigma_{\text{at } \theta=30^\circ} = 73.014$ MPa

Q.1 (d) A pinned end strut of steel (E = 210 GPa) with length L = 3 m is constructed of circular tubing with outside diameter D = 70 mm as shown in figure. The strut must resist an axial load P = 35 kN with a factor of safety 2.0 with respect to critical load. Determine the minimum required thickness of the tube.



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[12 marks]

Critical load = Axial load × F.O.S

= 35 × 2 = 70 kN.

As for euler equation, $P_{critical}$ for pinned end $\Rightarrow$ $P_{cr} = \frac{\pi^2 E I}{L^2}$

So, $70000 = \frac{\pi^2 \times 210 \times 1000 \times I}{3000^2}$

$I = 303963.55 \text{ mm}^4$

So, $\frac{\pi}{64}(D_o^4 - D_i^4) = 303963.55$

So, $D_{inner} = 64.97 \text{ mm}.$

So, $t = \frac{D_o - D_i}{2} = 2.515 \text{ mm}.$

↓

thickness required.

Q.1 (e) (i) A motor drives a shaft at 15 Hz and delivers 25 kW of power.

(*a*) If the shaft has a diameter of 30 mm, what is the maximum shear stress developed in the shaft?

(*b*) If the maximum allowable shear stress is 40 MPa, then what is the minimum permissible diameter of the shaft?

(ii) Describe the following theories of failure (elastic) giving necessary formulae along with associated limitations:

(*a*) St. Venant's theory. (*b*) Guest's theory.

[6 + 6 marks]

i) a) $\omega = 2\pi f \Rightarrow 30\pi = 94.247$ rad/s.

& $P = T\omega$

$\Rightarrow 25000 = T \times 94.247$

$T = 265.258$ N-m. $= 265258.23$ N-mm

Now, $\tau_{max} = \frac{16T}{\pi D^3} = \frac{16 \times 265258.25}{\pi \times 30^3}$

$\Rightarrow$ $\tau_{max} = 50.035$ MPa. (6)

b) $\tau_{max} = \frac{16T}{\pi D^3}$

$\Rightarrow 40 = \frac{16 \times 265258.23}{\pi D^3}$

$D = 32.324$ mm

↳ Minimum diameter permissible.

ii) a) St. Venant's theory → It is the theory that governs on the maximum principle strain.

Formula associated with this theory is,

$$\Rightarrow \frac{\sigma_1 - \mu(\sigma_2 + \sigma_3)}{E} \leq \frac{\sigma_y}{(FOS)E}$$

or

$$\varepsilon_1 - \mu(\varepsilon_2 + \varepsilon_3) \leq \frac{\varepsilon_y}{FOS}$$

or

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \leq \frac{\sigma_y}{FOS}$$

(1)

b) Guest's theory → It is the theory that governs on the principle of shear stress.

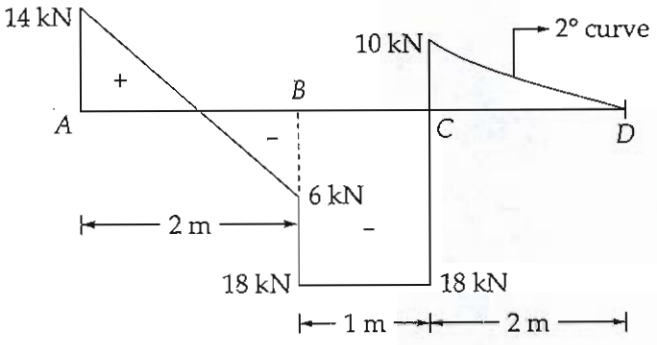
Formula associated with this theory is,

$$\text{Max}\left\{\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_1 - \sigma_3}{2}\right|\right\} \leq \frac{\sigma_y}{2\,FOS}$$

Q.2 (a) The shear force diagram for a simply supported beam AC with overhang CD is as shown in the figure.

Determine the loading on the beam and draw the bending moment diagram, assuming that a clockwise moment is acting at the point *B*. Also, the equation of shear force for the overhang portion CD (x from C) is given by $V = \frac{3x^2}{2} - 8x + 10$.

Write all the equations involved along with maximum bending moment and location of point of contraflexture.



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[20 marks]

(1) At A, a point load of 14 kN is acting upward.

(2) for AB, shear force change from +14 kN to −6 kN uniformly so, udl of intensity $\left[\frac{14-(-6)}{2}\right]$ ⇒ 10 kN is acting downwards on span AB.

(3) At B, a downward point load of 12 kN is applied.

(4) No force on span BC.

(5) At C, a point load of 28 kN is acting upward.

Now, intensity of varying load on CD at C

$\Rightarrow \frac{dV}{dx} \Rightarrow 3x - 8$, so at $x = 0$ from C,

$\Rightarrow$ intensity $= -8$ KN/m

& at $x = 2$ m, , intensity $= -2$ KN/m.

~~Loading~~

& for calculation of bending moment at B, under shear FD

Total area will give unbalanced Moment.

So, Area $\Rightarrow \frac{1}{2} \times 2 \times 20 - 6 \times 2 - 18 \times 1$

$+ \int \left(\frac{3x^2}{2} - 8x + 10\right) dx$.

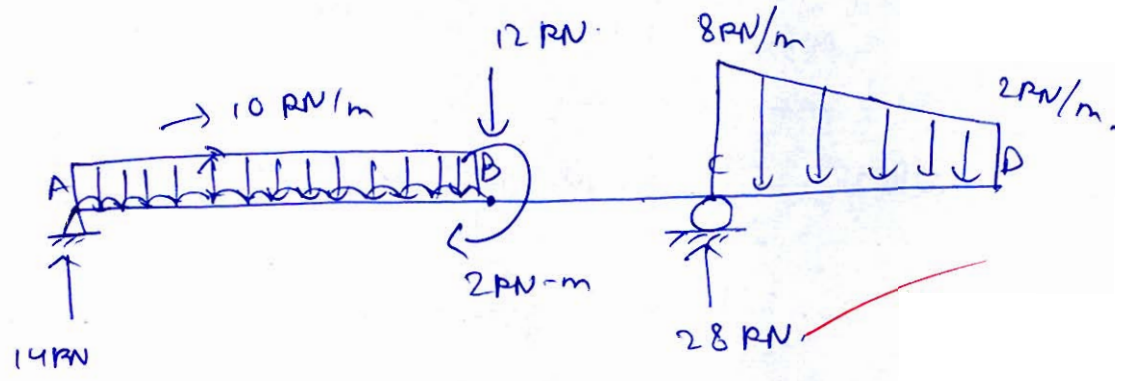
Equation = ?

$\Rightarrow 20 - 12 - 18 + 8$

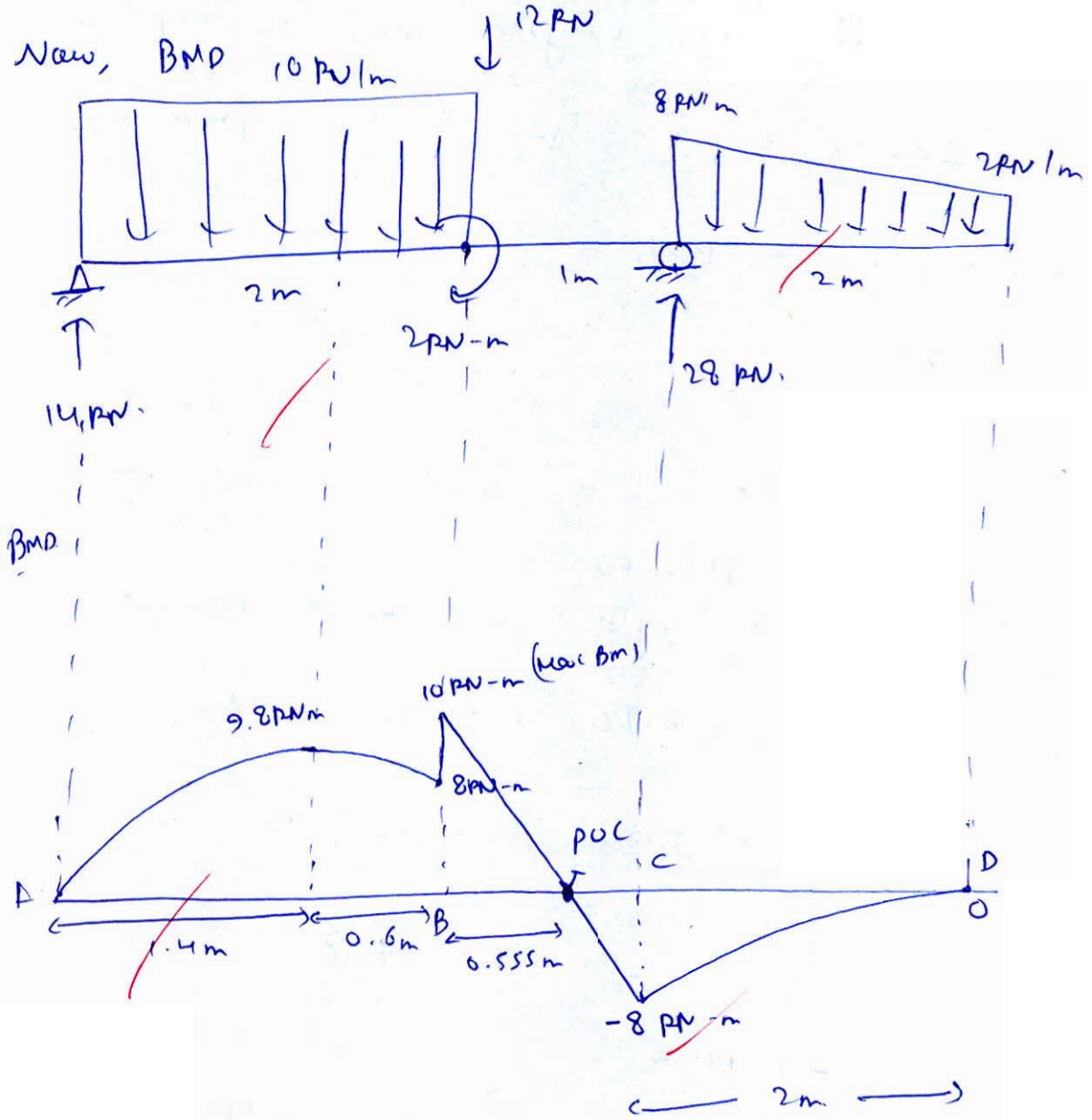
$= -2$ KN-m.

So, Bending moment at B $= 2$ KN-m.

So, Loading diagram.



Now, BMD



Location of max bending moment at B.

& Max BM = 10 kN-m.

& Location of point of contraflexure

⇒ 2.555 m from A

Q.2 (b) A uniformly tapered bar AB of solid cross section is twisted by a torque T. The diameter of the bar varies linearly from d_A at the left end to d_B at the right end.

(i) Show that the angle of twist of the tapered bar is $\theta = \dfrac{T_L}{G(I_P)_A}\left(\dfrac{\beta^2+\beta+1}{3\beta^3}\right)$,

where, $(I_P)_A$ = Polar moment of inertia of end A and, $\beta = \dfrac{d_B}{d_A}$.

(ii) For what ratio of d_B/d_A will the angle of twist of tapered bar be one-fourth the angle of twist of a prismatic bar of diameter d_A?

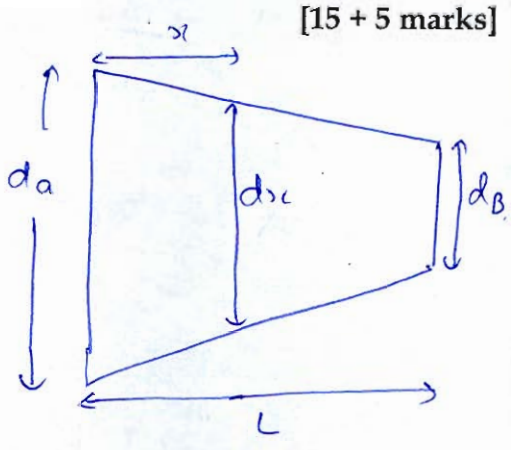
(Assume, the prismatic bar is made of the same material, has the same length and is subjected to same torque as the tapered bar).

[15 + 5 marks]

i) $d_x = d_a + \dfrac{(d_B - d_a)}{L} x$

$\Rightarrow d_a + kx$

where $k = \dfrac{d_B - d_A}{L}$



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Now, at any x,

$$d\theta = \frac{T\,dx}{G\,I_{Px}}$$

$$d\theta \Rightarrow \frac{T\,dx}{G\,\frac{\pi}{32}(d_a + kx)^4}$$

So, Integrate it. L

$$\Rightarrow \theta = \int_0^L \frac{32\,T\,dx}{G\,\pi\,(d_a+kx)^4}$$

$$\Rightarrow \frac{32\,T}{G\,\pi}\int_0^L \frac{dx}{(d_a+kx)^4}$$

$$\theta = \frac{32T}{G\pi}\left[\frac{-1}{3(d_A + kx)^3 k}\right]_0^L$$

$$\theta \Rightarrow \frac{32T}{G\pi}\left(-\frac{1}{3k}\right)\left(\frac{1}{d_B^3} - \frac{1}{d_A^3}\right)$$

$$\theta \Rightarrow \frac{32T}{G\pi 3\left(\frac{d_B - d_A}{L}\right)}\left(\frac{1}{d_A^3} - \frac{1}{d_B^3}\right)$$

$$\Rightarrow \frac{32TL}{G\pi 3(d_B - d_A)} \frac{d_B^3 - d_A^3}{d_B^3 d_A^3}$$

$$\Rightarrow \frac{32TL}{G\pi 3(d_B - d_A)} \frac{(d_B - d_B)(d_B^2 + d_A^2 + d_A d_B)}{d_B^3 d_A^3}$$

$$\Rightarrow \frac{TL}{G\frac{\pi}{32}} \frac{d_A^2(\beta^2 + \beta + 1)}{3 d_B^3 d_A^3}$$

$$\Rightarrow \frac{TL}{G\frac{\pi}{32} d_A^4} \frac{(\beta^2 + \beta + 1)}{3\frac{d_B^3}{d_A^3}}$$

$$\theta \Rightarrow \frac{TL}{G(I_p)_A} \frac{(\beta^2 + \beta + 1)}{3\beta^3}$$

ii) Now, for prismatic bar, of dia d_A.

$$\theta = \frac{TL}{G\frac{\pi}{32}d_A^4} \quad \& \quad \theta_{required} = \frac{\theta}{4}$$

So, $$\frac{TL}{G(I_P)_A}\frac{(\beta^2+\beta+1)}{3\beta^3} = \frac{TL}{GI_{PA}} \times \frac{1}{4}$$

$$\Rightarrow \quad 4(\beta^2+\beta+1) = 3\beta^3.$$

$$\Rightarrow \quad 3\beta^3 - 4\beta^2 - 4\beta - 4 = 0.$$

So, $$\beta = 2.209$$

Thus $$\boxed{\frac{d_B}{d_A} = 2.209}$$

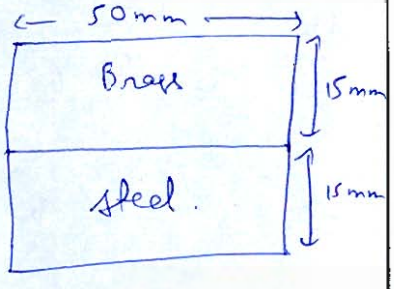
Q.2 (c) Two rectangular plates, one of steel and other of brass each 50 mm wide and 15 mm deep are placed together to form a beam of 50 mm width and 30 mm depth, on two supports 3.0 m apart, the brass plate being on the top of the steel plate. Determine the maximum load which can be applied at the centre of the beam, if the plates are:

(i) Separate and can bend independently.

(ii) Firmly secured throughout their length. Maximum allowable stress in steel = 115 N/mm^2 and that in brass = 80 N/mm^2.
Take $E_s = 2 \times 10^5$ N/mm^2 and $E_b = 8 \times 10^4$ N/mm^2.

[20 marks]

i) Let the moment taken by Brass be M_B & moment taken by steel by M_s



So, Total Moment = $M_B + M_S$.

Now, Radius of curvature will be same for independent bending.

So, $\frac{1}{R} = \frac{M_B}{E_B I_B} = \frac{M_S}{E_S I_S}$.

& $I_B = I_S$

So, $\frac{M_B}{M_S} = \frac{E_B}{E_S} = 0.4$ So $M_B = 0.4 M_S$

Now, $M_S = \frac{\sigma_S I_S}{y} \Rightarrow \sigma_S \times Z$ at max allowable stress

$= 115 \times \frac{50 \times 15^2}{6}$.

$= 215625$ N-mm.

So, $M_B = 86250$ N-mm

& $\sigma_B = \frac{M_B}{Z} \Rightarrow 46$ MPa

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Thus steel will fail first.

So, $M_{total} = M_C + M_B$

$= 215625 + 86250$

$= 301875$ N-mm

So, Max BM at centre due to load P

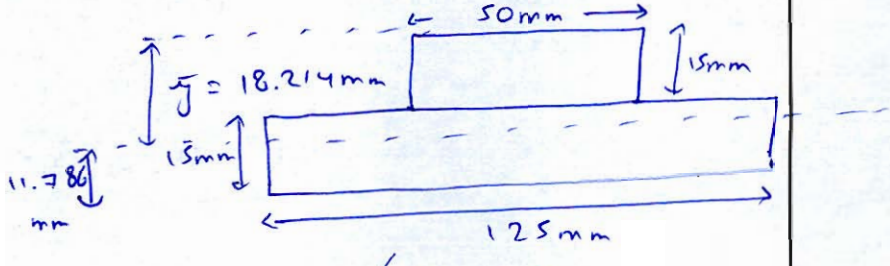
$\Rightarrow M_{max} = \frac{PL}{4}$

$\Rightarrow 301875 = \frac{P \times 3000}{4}$

$\boxed{P = 402.5 \text{ N}}$

ii) Convert steel section into brass using

$m = \frac{E_S}{E_B} = 2.5$



$$\bar{y} = \frac{50 \times 15 \times 15/2 + 125 \times 15 \times \left(15 + \frac{15}{2}\right)}{50 \times 15 + 125 \times 15}$$

$= 18.214$ mm

$$I \Rightarrow \frac{50 \times 15^3}{12} + 50 \times 15 \times \left(\bar{y} - \frac{15}{2}\right)^2 + \frac{125 \times 15^3}{12} + 125 \times 15 \times \left(\bar{y} - 22.5\right)^2.$$

$I = 169754.46\ mm^4$

Assume steel fails first, so $\sigma_{steel} = 115$ MPa at bottom

Δ $\frac{\sigma_{steel\ top}}{15 - 11.786} = \frac{115}{11.786}$ =

$\sigma_{steel\ top} = 31.36$

So, $\sigma_{brass\ bottom} = \frac{31.36}{2.5} = 12.544$

Δ $\frac{\sigma_{b\ bottom}}{3.214} = \frac{\sigma_{top\ brass}}{18.214}$

$\Rightarrow \sigma_{top} = 71.088 < 80$ MPa

So, steel fails first.

So, Max BM $\Rightarrow$ $\frac{\sigma_{brass\ top} I}{y_{max}}$

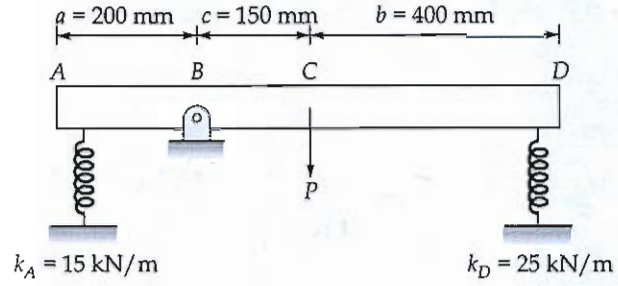
$\Rightarrow \frac{71.088 \times 169754.46}{18.214}$

$\Rightarrow 662540$ N-mm

So, Max BM = $\frac{P\ 3000}{4}$

$\Rightarrow P_{max} = 883.386$ N

Q.3 (a) A rigid bar ABCD is pinned at point B and is supported by springs at A and D having stiffness $K_A = 15$ kN/m and $K_D = 25$ kN/m respectively. The dimensions are shown in figure below. A load P acts at point C. The angle of rotation of the bar due to the action of load P is limited to 2°. What is the maximum permissible load P_{max}?



Also, determine the maximum forces resisted by the springs at A and D and the reaction force at B.

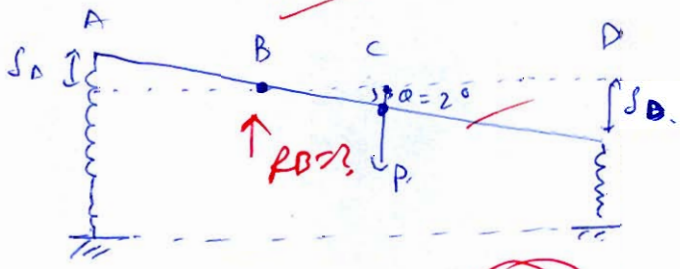
[20 marks]

Sol) $\delta_A = \theta \times a$

$= \frac{2\pi}{180} \times 200$

$= 6.9813$ mm

& $\theta = \frac{2}{180} \times \pi$



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& $\delta_D = \theta \times (b+c) = \theta \times 550$

$\Rightarrow 19.1986$ mm.

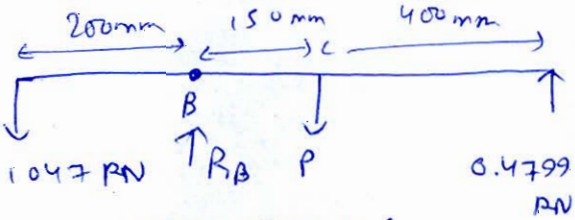
So, Reaction in spring at A = $K_A \delta_A$

$= 0.1047$ KN.

& Reaction in spring at D = $K_D \delta_D$

$= 0.4799$ KN.

So, $\Sigma M_B = 0$



$\Rightarrow$

$P \times 150 - 0.4799 \times 550 - 0.1047 \times 200 = 0$

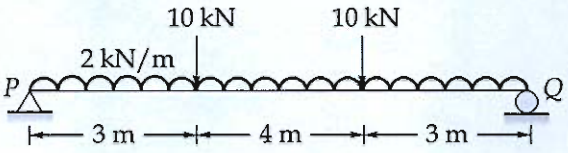
$\Rightarrow$ $P_{max} = 1.899$ KN.

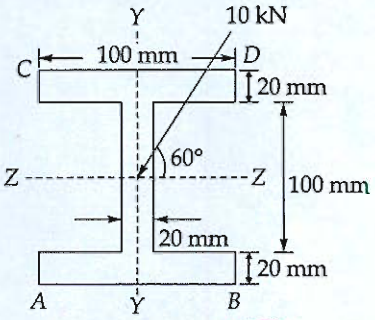
Δ $\Sigma F_y = 0$

$\Rightarrow$ $0.1047 + 1.899 - 0.4799 = R_B$

$$R_B = 1.5238 \text{ kN}$$ Reaction at B.

Q.3 (b) A simply supported beam of length 10 m is loaded as shown in figure. Determine the maximum bending stresses at points *A*, *B*, *C* and *D* on the cross-section shown below. Also determine the location of neutral axis on the cross-section.

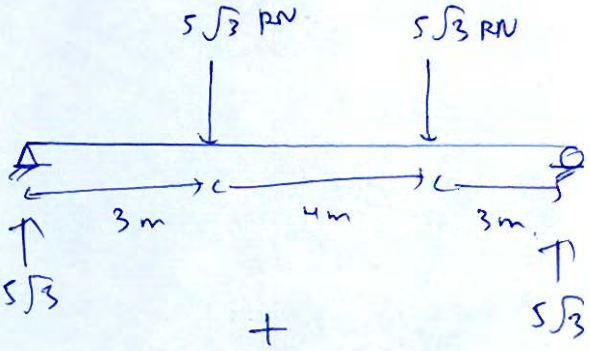




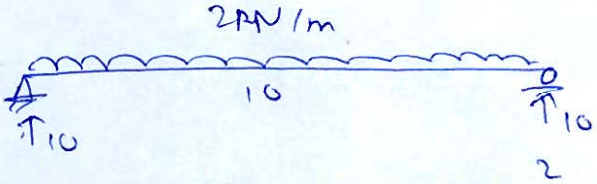
(Neglect self weight of beam)

[20 marks]

Sol:) Consider transverse loading in vertical plane.



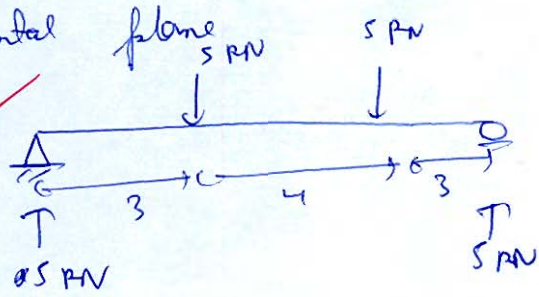
So, Max BM will be at centre due to both loading +



So, BM max in vertical plane $\Rightarrow 5\sqrt{3} \times 3 + \frac{2 \times 10^2}{8}$

$M_{ZZ} \Rightarrow 50.98$ kN-m

& BM max in horizontal plane



So, $BM_{max} = 15$ kN-m

(M_{yy})

Now, due to M_{ZZ}, C & D will be in compression & A & B will be in tension.

& due to M_{YY}, B & D will be in compression & A & C will be in tension.

Now, $I_{ZZ} = \frac{100 \times 140^3}{12} - \frac{80 \times 100^3}{12}$

$= 16.2 \times 10^6 \text{ mm}^4$

& $I_{yy} = \frac{140 \times 100^3}{12} - \frac{100 \times 80^2}{12}$

$= 7.4 \times 10^6 \text{ mm}^4$

Now, $\sigma_{at\ any\ point} = \pm \frac{M_{ZZ}\, y_{max}}{I_{ZZ}} \pm \frac{M_{yy}\, z_{max}}{I_{yy}}$

$\Rightarrow \pm 220.283 \pm 101.351$

So, at A $\Rightarrow +220.283 + 101.351$

$\sigma_A \Rightarrow 321.634$ MPa

at B $\Rightarrow +220.283 - 101.351$

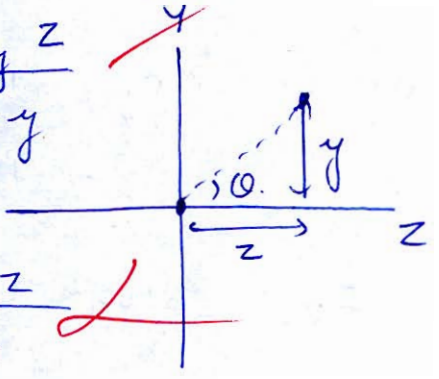
$\sigma_B \Rightarrow 118.932$ MPa

& $\sigma_C = -118.932$ MPa

& $\sigma_D = -321.634$ MPa

For neutral axis,

$$\sigma = 0 \Rightarrow -\frac{M_{zz}\, y}{I_{zz}} \; \& \; -\frac{M_{yy}\, z}{I_{yy}}$$



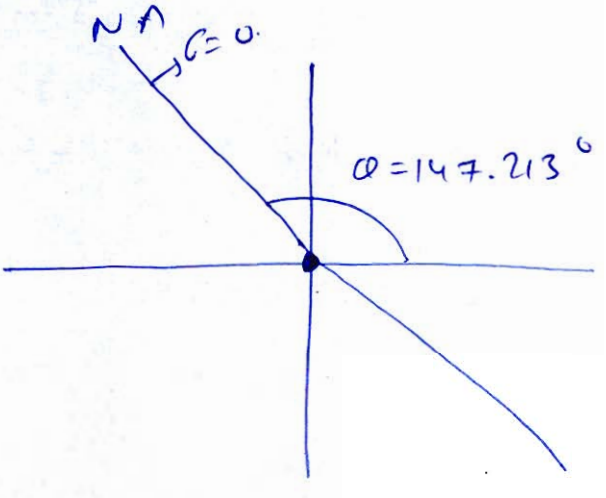
$$\Rightarrow \frac{50.98\, y}{16.2} = -\frac{15 \times z}{7.4}$$

$$\Rightarrow \frac{y}{z} = -0.644$$

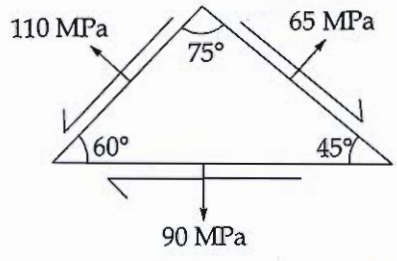
$$\Rightarrow \tan\theta = -0.644$$

So, $\theta = -32.786^\circ$ or 147.213°

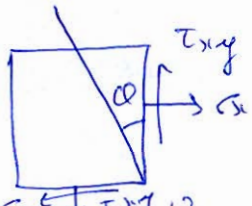
So, Neutral axis



Q.3 (c) In a strained body, the normal stresses on three planes inclined as shown in figure are 65 MPa (Tensile), 90 MPa (Tensile) and 110 MPa (Tensile). Determine the shear stresses acting on these planes. Also find the principle stresses and draw the final stress element.

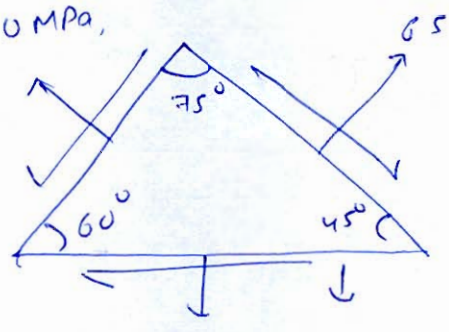


[20 marks]



$\sigma_\theta = \sigma_x \cos\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$,

20



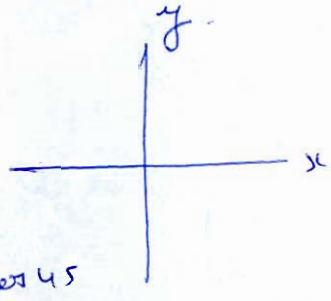
So,

$\sigma_{90^\circ} = \sigma_y$

$\Rightarrow \sigma_y = 90\ MPa.$

&

$\sigma_{45^\circ} = \sigma_x \cos^2 45 + \sigma_y \sin^2 45 + 2\tau_{xy} \sin 45 \cos 45$



$\Rightarrow 65 = \frac{\sigma_x}{2} + \frac{90}{2} + \tau_{xy}$ — (1)

& $\sigma_{150^\circ} = \sigma_x \cos^2 150^\circ + \sigma_y \sin^2 150 + 2\tau_{xy} \sin 150 \cos 150$

$\Rightarrow 110 = \sigma_x \frac{3}{4} + \frac{90}{4} + 2\tau_{xy} \times \frac{1}{2} \times \frac{-\sqrt{3}}{2}$

$110 = \frac{3\sigma_x}{4} + \frac{90}{4} - \frac{\sqrt{3}}{2}\tau_{xy}$ — (2)

Using (1) & (2).

$\sigma_x = 88.604$ & $\tau_{xy} = -29.302$

$\sigma_x = 88.605$ MPa & $\tau_{xy} = -24.302$ MPa

& $\sigma_y = 90$ MPa

So,

$$\tau_\theta = -\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

So, $\boxed{\tau_{45^\circ} = 0.6975 \text{ MPa}}$ ✓

& $\boxed{\tau_{90^\circ} = 24.302 \text{ MPa}}$ ✓

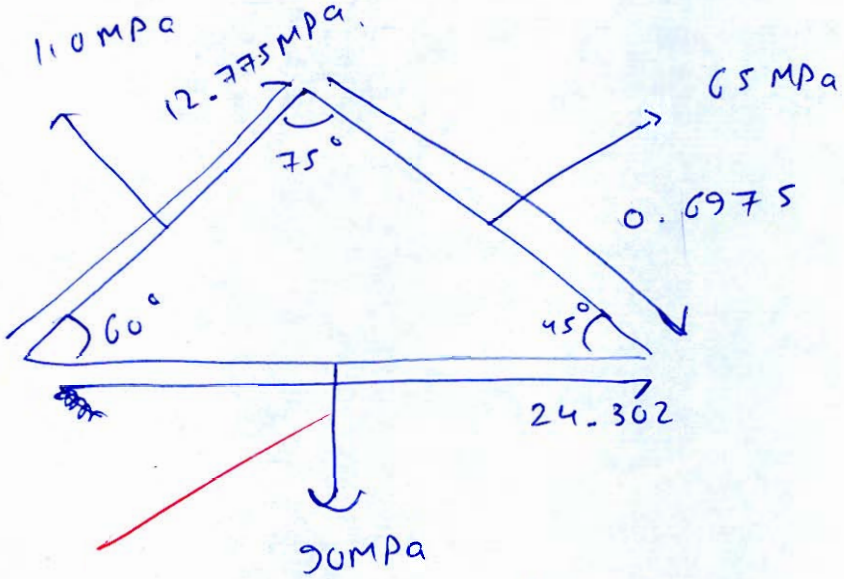
& $\boxed{\tau_{150^\circ} = -12.775 \text{ MPa}}$ ✓

& principal stress

$$\Rightarrow \sigma_{p_1/p_2} \Rightarrow \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

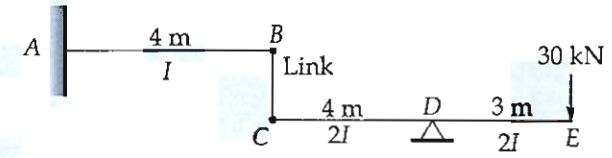
$$\Rightarrow 89.3025 \pm 24.312$$

So, $\boxed{\sigma_{p_1} = 113.614 \text{ MPa}}$ & $\boxed{\sigma_{p_2} = 64.99 \text{ MPa}}$

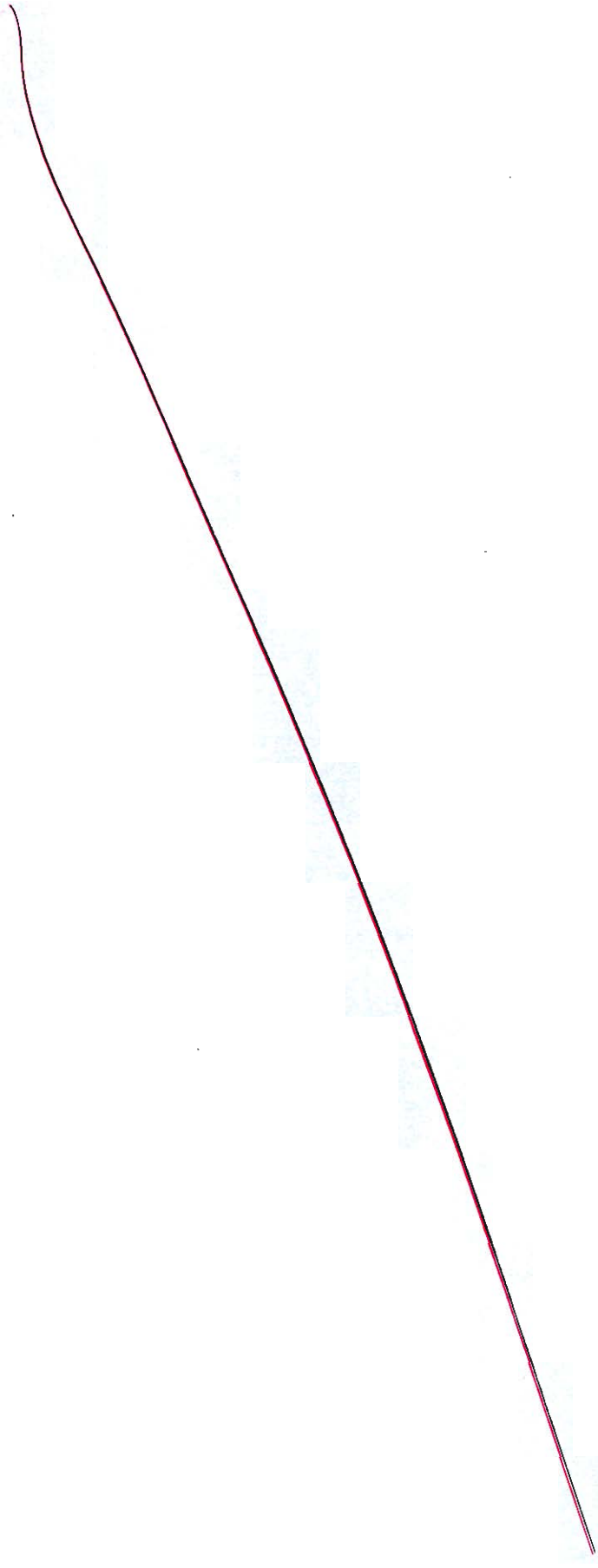


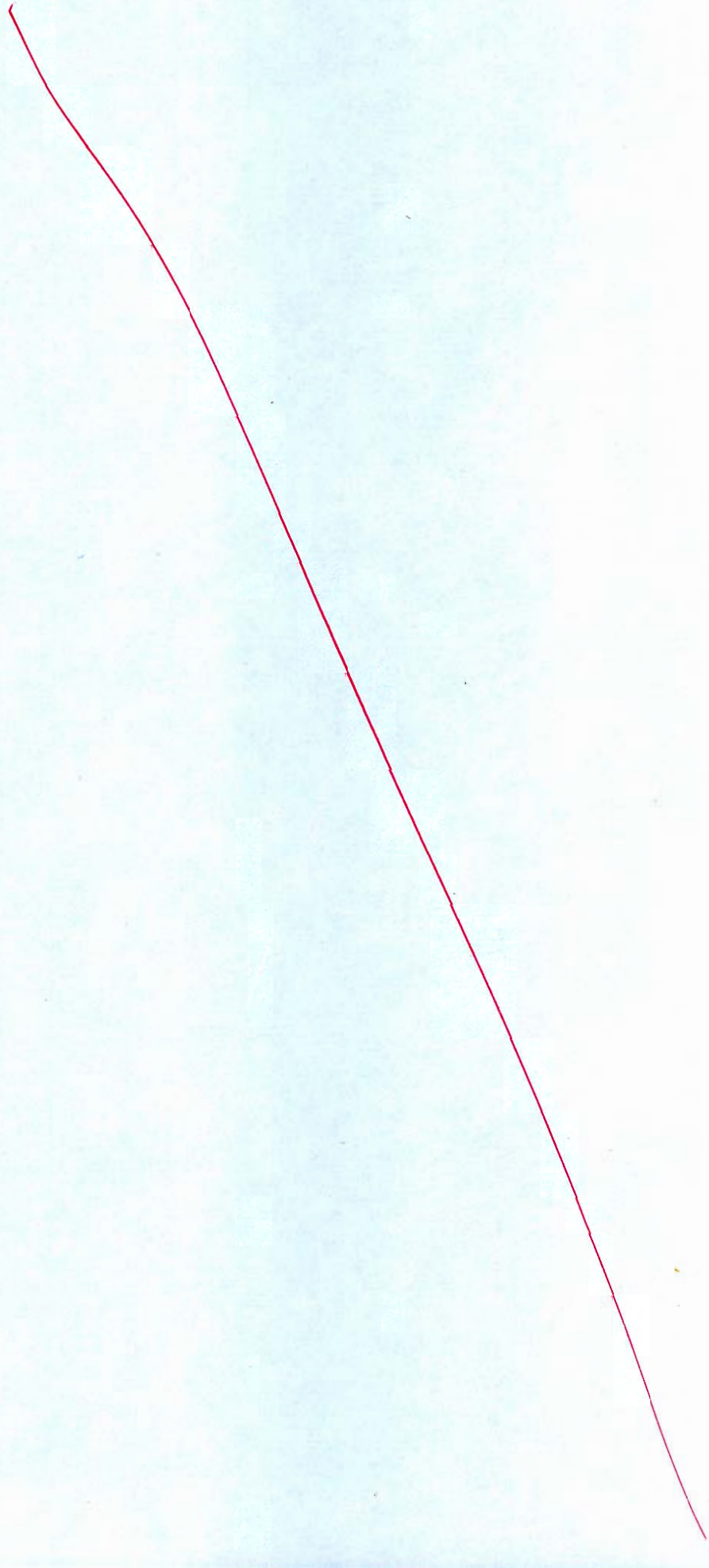
Final stress element.

Q.4 (a) For the beam shown in figure; determine **(i)** the deflection at E; **(ii)** the deflection at B; **(iii)** the slope at B and **(iv)** the slope at C, using conjugate beam method.
[Take : $I = 8.30 \times 10^7$ mm^4 and $E = 200$ kN/mm^2]



[25 marks]

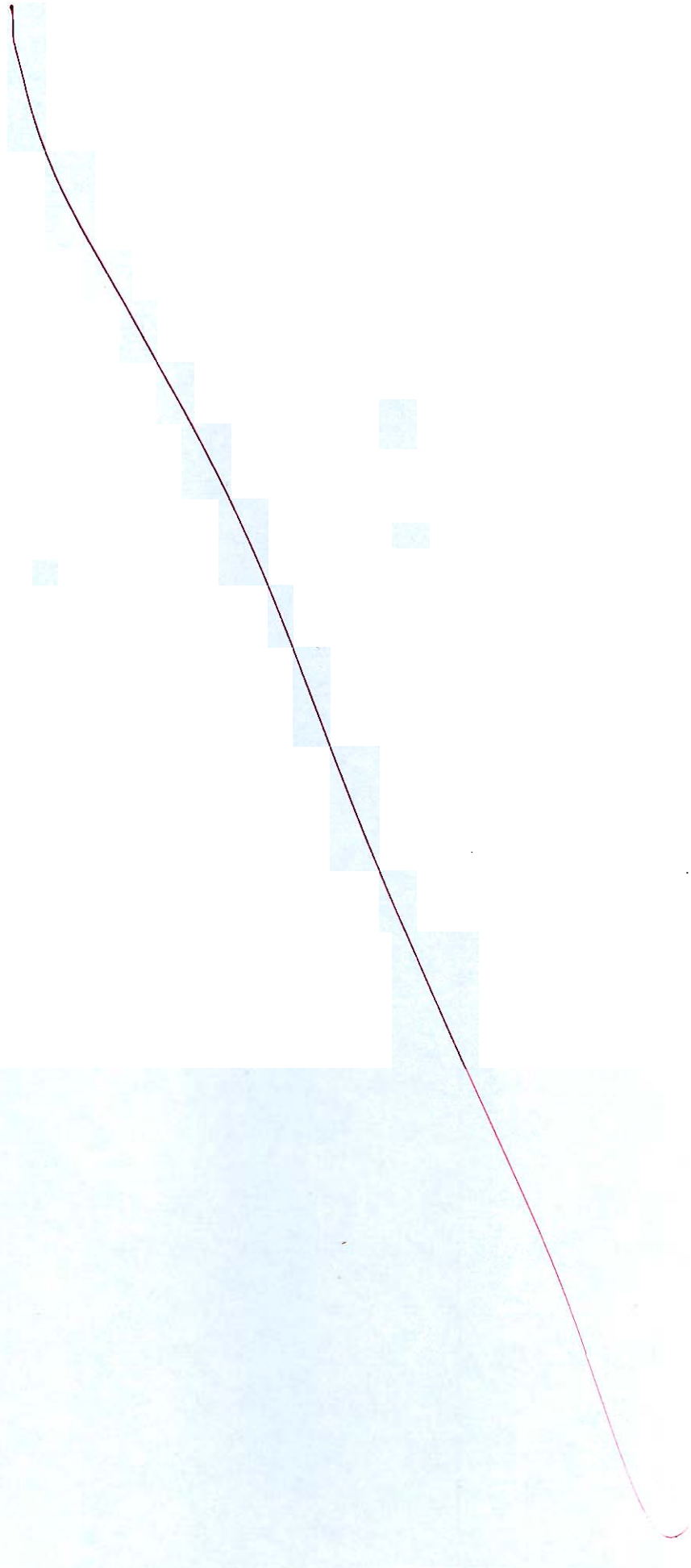


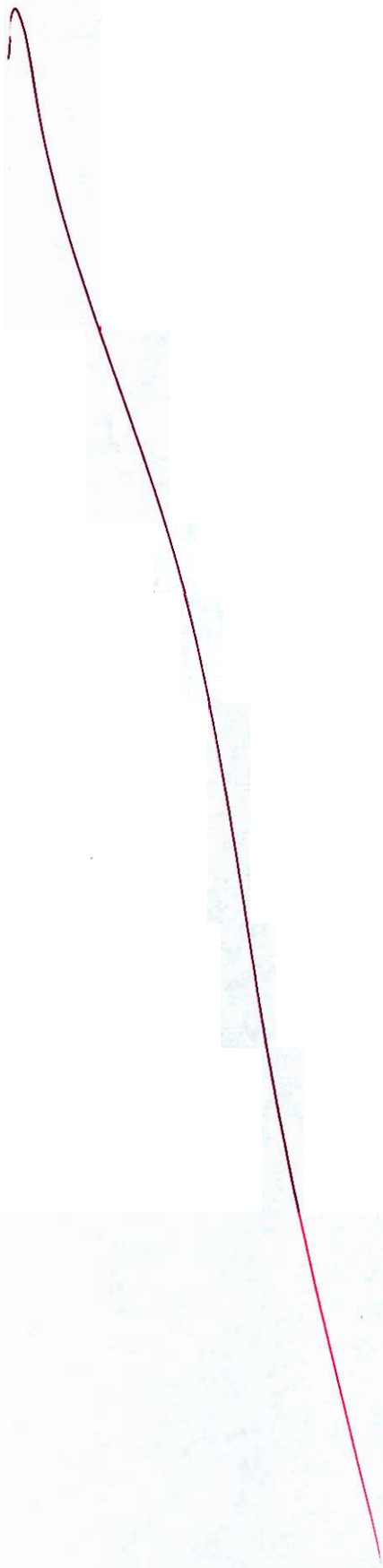


Q.4 (b) A 15 mm diameter steel rod passes centrally through a copper tube of 50 mm external diameter and 38 mm internal diameter and is 3 m long. The tube is closed at each end by 25 mm thick steel plates which are secured by nuts. The nuts are tightened until the copper tube gets reduced by 0.635 mm in length. The whole assembly is then raised in temperature by 65° C. Calculate the stresses in copper and steel before and after the rise of temperature, assuming that the thickness of plates remains unchanged.

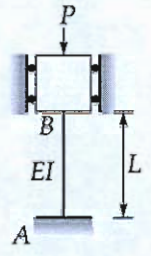
(Take $E_s = 2.1 \times 10^5$ N/mm^2 , $E_C = 1.05 \times 10^5$ N/mm^2 , $\alpha_S = 1.2 \times 10^{-5}$ per °C, $\alpha_C = 1.75 \times 10^{-5}$ per °C)

[20 marks]

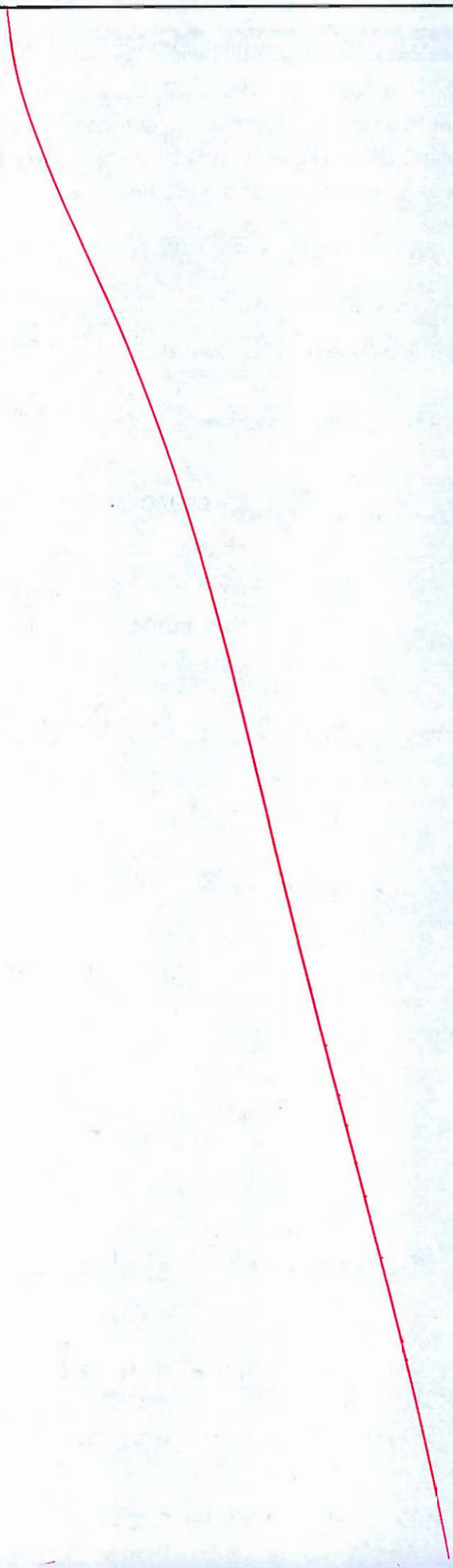




Q.4 (c) Determine the critical load P_{cr} and the equation of the buckled shape for an ideal column with both ends fixed against rotation (as shown in figure) by solving the differential equation of the deflection curve, such that the deflection at midpoint is 'δ'. (Assume single bow condition)



[15 marks]



Section-B : Environmental Engineering

Q.5 (a) What do you mean by the term 'per capita demand'? For a town, what will be the total yearly water requirement in year 2025 if per capita demand is estimated as 250 lpcd? The population in the year 2025 can be estimated by using logistic curve method. The following data can be used for estimation of population:

Year	:	1965	1985	2005
Population	:	60,000	1,20,000	1,70,000

[12 marks]

Sol) 'Per capita demand' denotes the demand of any particular item by one person in any given time interval.

Now,

$P_0 = 60000$, $P_1 = 120000$, $P_2 = 170000$

So, $$P_S \Rightarrow \frac{2P_0P_1P_2 - P_1^2(P_0+P_2)}{P_0P_2 - P_1^2}$$

$$P_S \Rightarrow 205714.28$$

& $$m = \frac{P_S - P_0}{P_0}$$ & $$n = \frac{1}{t_1} \ln\left\{\frac{P_0(P_S - P_1)}{P_1(P_S - P_0)}\right\}$$

$m = 2.428$ & $n = -0.061188$.

So, P at any $t = \frac{P_S}{1 + me^{nt}}$

So, P $\Rightarrow \frac{205714.28}{1 + 2.428\, e^{-0.061188 \times 60}}$

$t = 60$ yrs

$P_{in\ 2025} \Rightarrow 193745.66$

So, Water requirement yearly

$\Rightarrow 193745.66 \times 250 \times 365 \times 10^{-6}$

$= 17679.29$ Million litres

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Q.5 (b) Design an oxidation pond for treating sewage from a hot climate residential colony with 10000 persons, contributing sewage at the rate of 135 litre per capita per day. The 5-day BOD of sewage is 250 mg/l. Assume suitable data if necessary.

[12 marks]

Q.5 (c) The sound power from a source is 0.0018 W and is travelling in air of density 1.16 kg/m^3.

Determine:

(i) Sound intensity and sound intensity level at a distance of 10 m from source.

(ii) Sound pressure at a distance of 15 m from source.

(iii) Speed of sound in medium at distance 15 m from source.

(iv) Sound pressure level at a distance of 15 m (say N) if there is another sound source of 50 dB at N.

Assume that sound radiates from source in all directions.

[12 marks]

i) Sound Intensity $(I_w) = \dfrac{P_w}{4\pi r^2}$

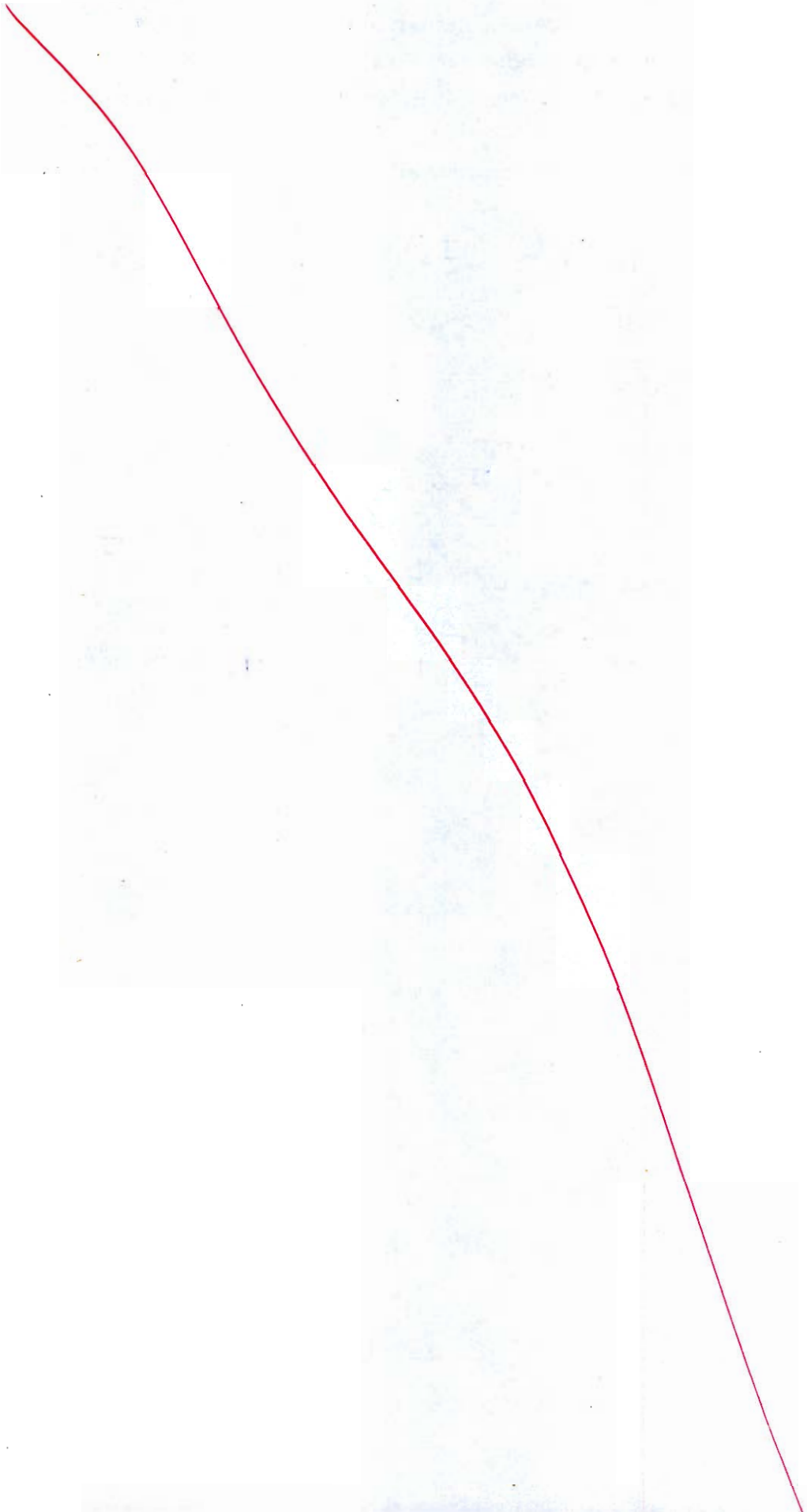
3

$= \dfrac{0.0018}{4\pi \times 10^2}$

$I_w = 1.4324 \times 10^{-6}\ W/m^2$

ii) Sound intensity level $(L_I) = 10 \log_{10} \dfrac{I_w}{10^{-12}}$ dB

$L_I = 61.56$ dB



Q.5 (d) (i) Explain the significance of COD and BOD in case of waste waters.

(ii) 5 ml of sample is diluted in a 300 ml incubation bottle. The initial and final D.O. of diluted sample are 8 mg/l and 4.5 mg/l respectively after (5 days of incubation). The temperature of incubation is kept as 20° C. When only seeded water is incubated in bottle for 5 days at the same temperature, then initial and final DO come out to be 8 mg/l and 7 mg/l respectively. Determine BOD_5 of sample.

[4 + 8 marks]

i) COD represents chemical oxygen demand i.e. oxygen required to decompose both biodegradable & non biodegradable compounds in the polluted ~~the~~ water.

9

→ BOD represents bio chemical oxygen demand i.e. oxygen demand ~~req~~ required to decompose bio degradable ~~organic~~ matter in the polluted waters.

ii) Test Sample + Seeded Water → Diluted sample in incubation bottle.

5 ml 295 ml 300 ml.

$DO_{i_{TS}}$ $DO_{i_{SW}} = 8$ mg/l $DO_{i_{IB}} = 8$ mg/l.

$DO_{f_{TS}}$ $DO_{f_{SW}} = 7$ mg/l. $DO_{f_{IB}} = 4.5$ mg/l

So, $5 \times DO_{i_{TS}} + 295 \times 8 = 300 \times 8$. — (1)

& $5 \times DO_{f_{TS}} + 295 \times 7 = 300 \times 4.5$ — (2)

Using (1) - (2)

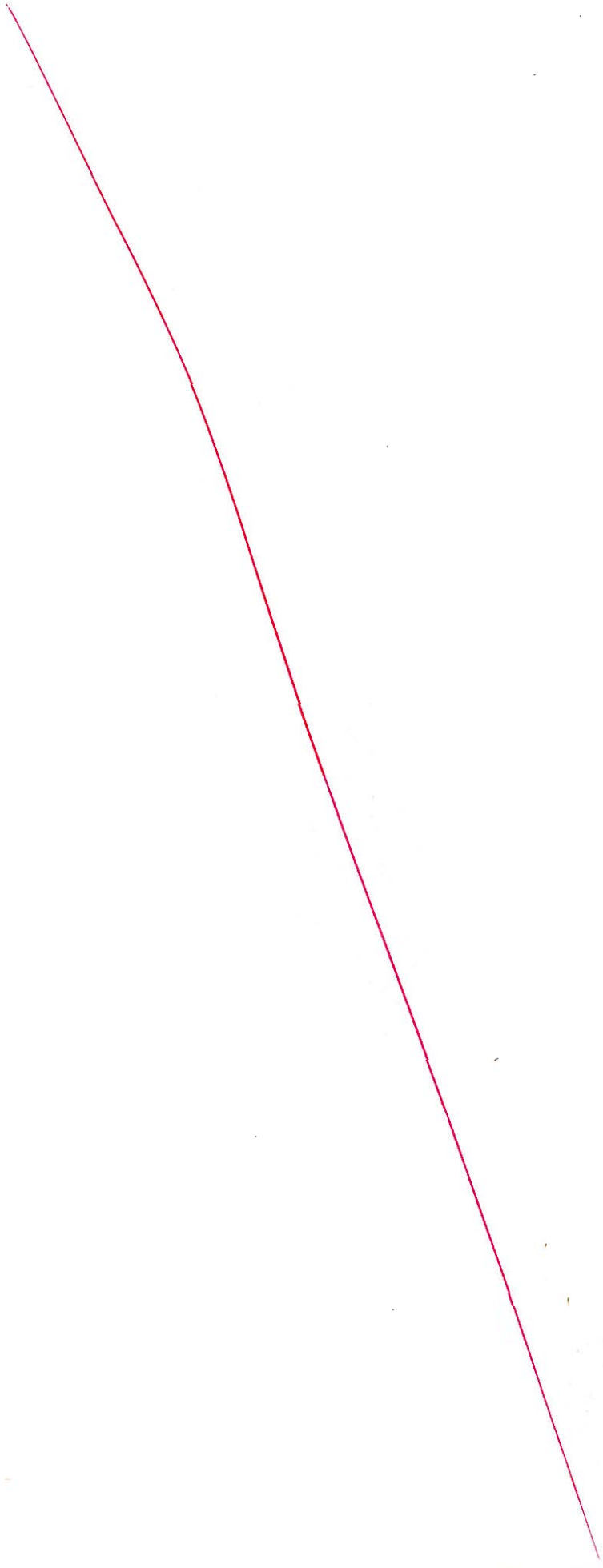
So, $5(DO_{i_{TS}} - DO_{f_{TS}}) + 295 = 300 \times 3.5$

So, $DO_{iTS} - DO_{fTS} = 151\ mg/\ell$

So, BOD's of sample $\Rightarrow$ 151 mg/ℓ.

Q.5 (e) Explain Indore and Bangalore process of composting in details.

[12 marks]



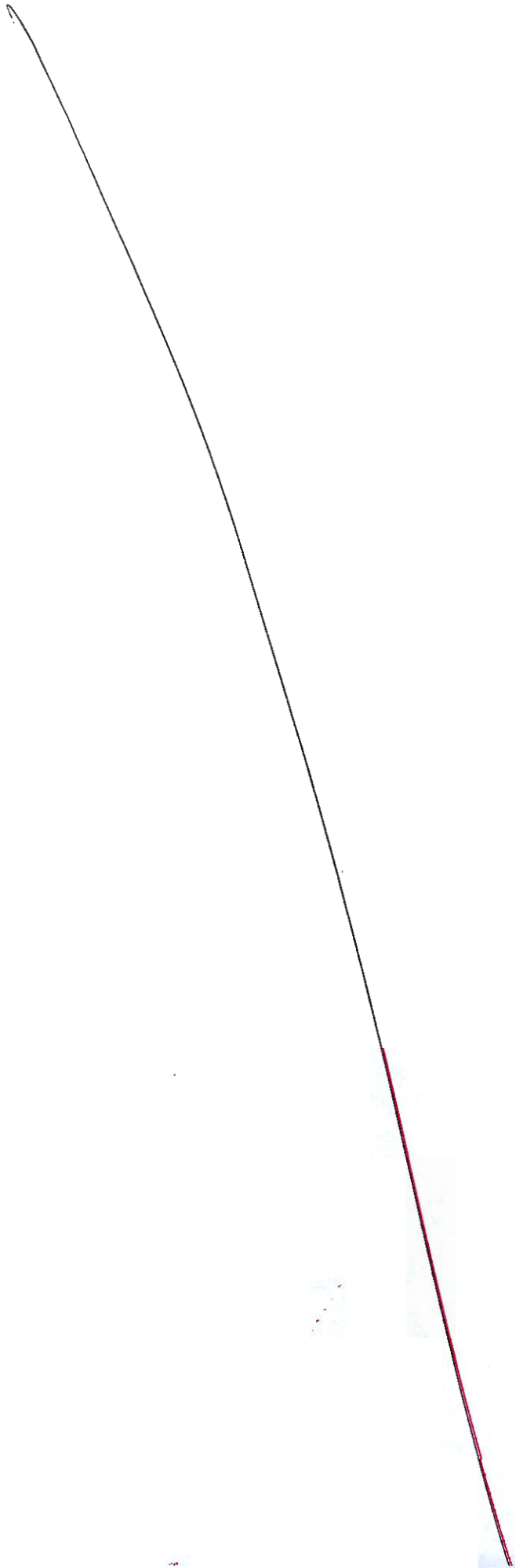
Q.6 (a) (i) Explain the following terms in context of rapid gravity filters:

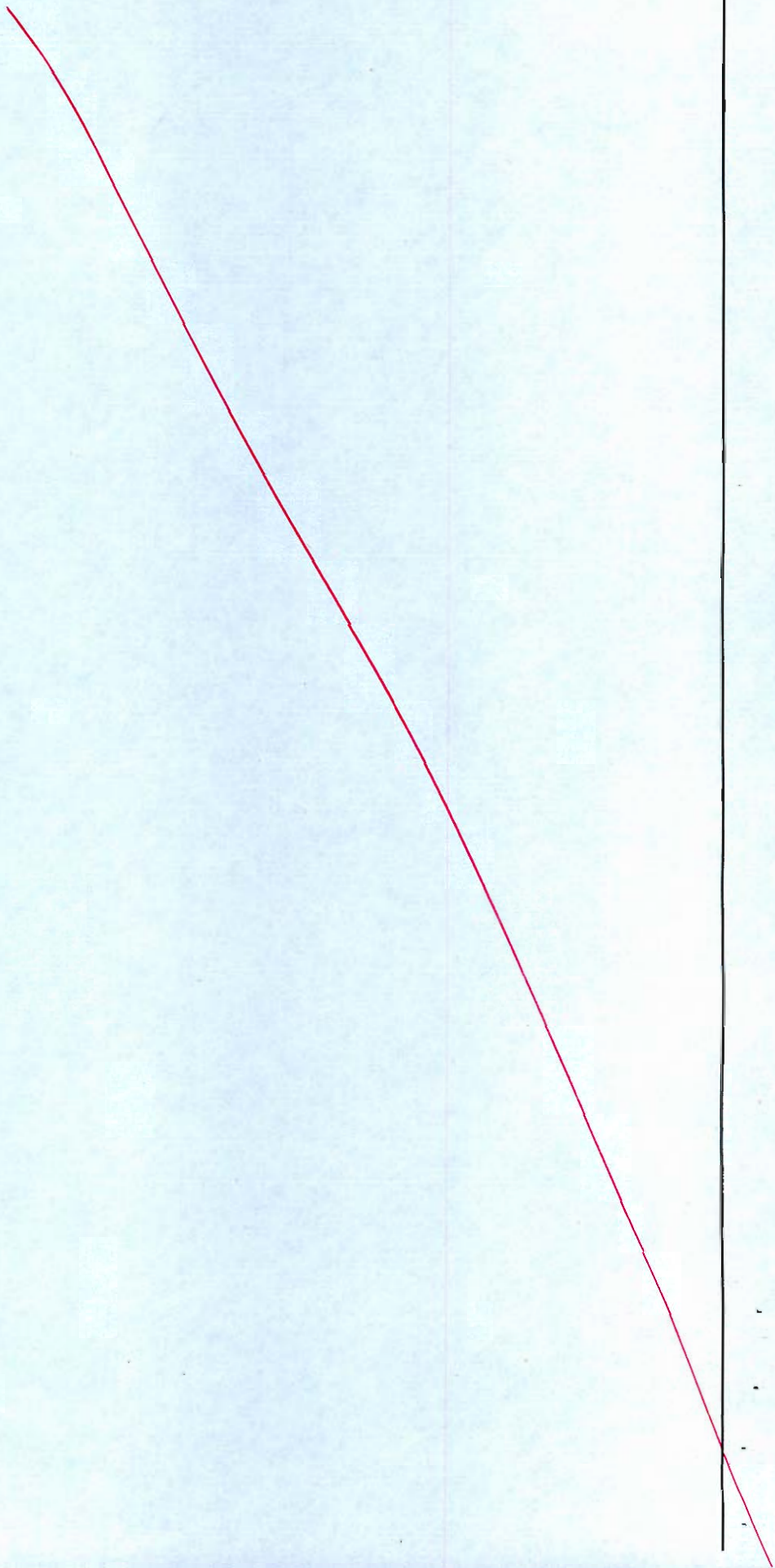
1. Formation of mud balls
2. Cracking of filters

(ii) A rapid sand filter is to be provided in a water treatment plant, to process the water for a town with a population of 3 lakh and average water demand for this town is estimated as 175 lpcd. The rate of filtration to be kept in filter is 24 $m^3/m^2/hr$. 5% of filtered water is to be provided for back washing which takes about 30 minutes daily. It is observed that during back washing, filter bed expands from 0.6 m to 0.66 m. Porosity of original bed is 0.5 m. Specific gravity of particles can be taken as 2.5. Drag coefficient is to be kept as 5.0.

1. Determine the number of beds (including stand by) if area of one filter is limited to 50 m^2.
2. What is effective diameter of particle if flow is assumed to be transitional during back washing and if the back washing upward velocity is kept as 18 cm/min?

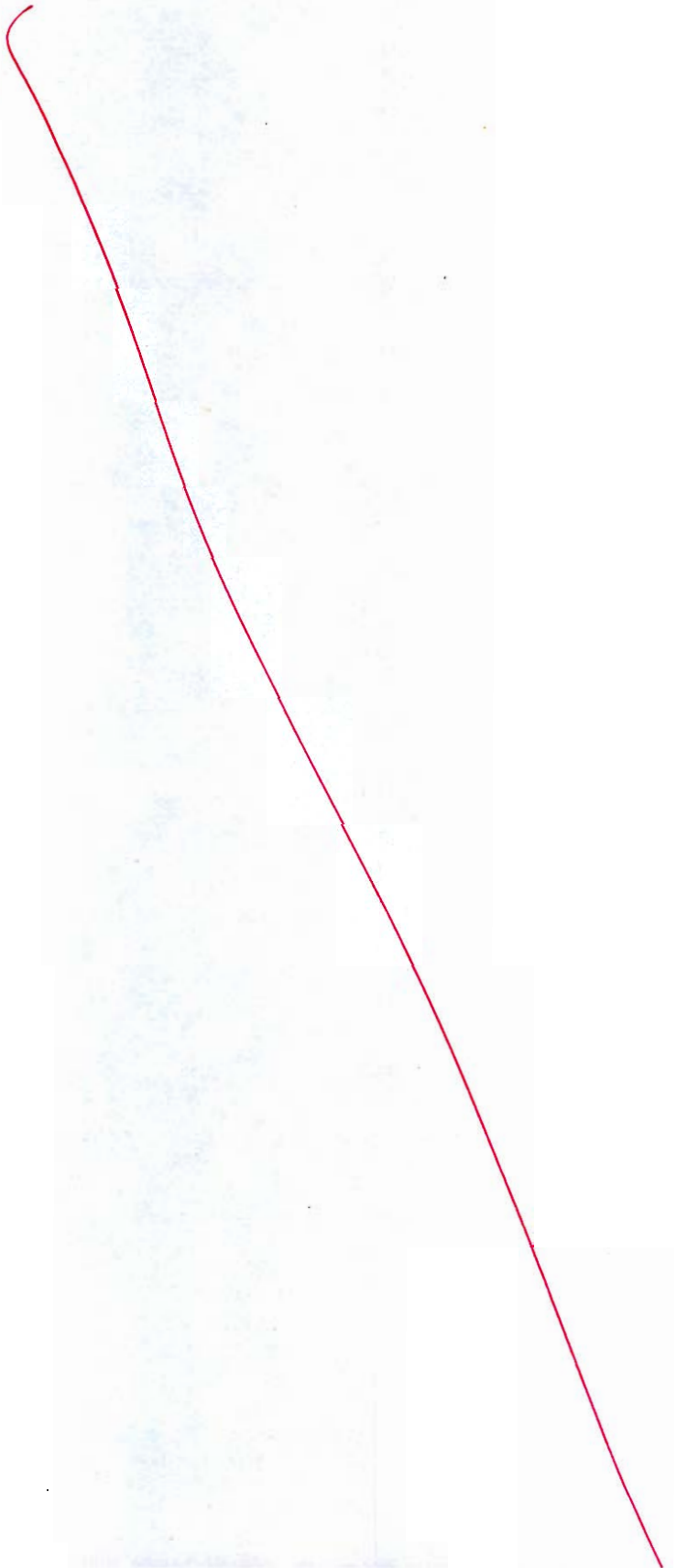
[6 + 14 marks]





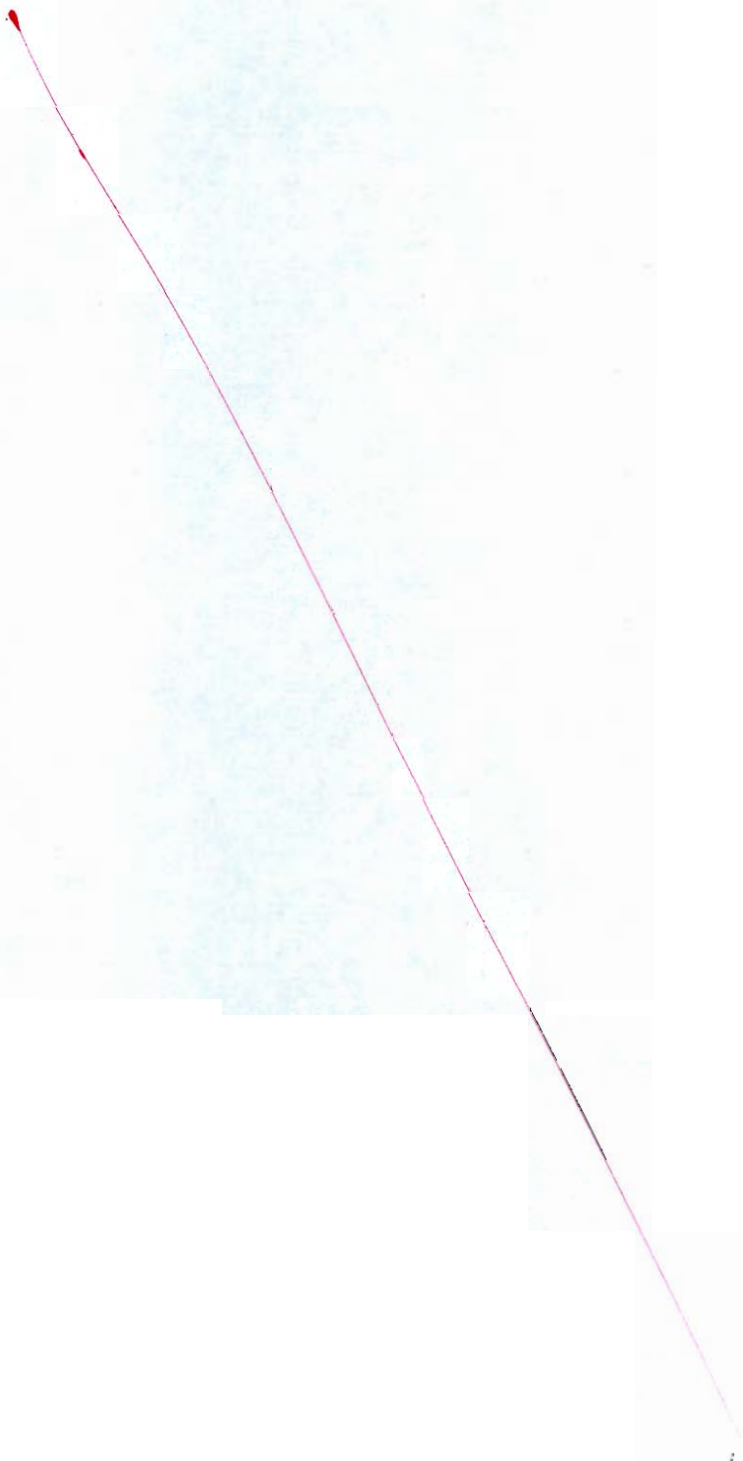
Q.6 (b) (i) What do you understand by the term 'breakpoint' chlorination? If 0.8 mg/litre of total chlorine is required for satisfactory disinfection of water at pH = 7, then what chlorine dosage will be necessary at pH = 8? (Take $K_i = 2.7 \times 10^{-8}$ mol/lit)

[12 marks]



Q.6 (b) (ii) Explain the effect of variations in demand on design component of a water supply scheme.

[8 marks]



Q.6 (c) The chemical composition of a municipal solid waste (MSW) sample of 100 kg is given in table below:

Component	Wet mass, kg	Dry mass, kg	C	H	O	N	S	Ash
Food waste	25	7.5	3.6	0.48	2.82	0.19	0.03	0.38
Paper	60	56.4	24.54	3.38	24.82	0.17	0.11	3.38
Plastics	10	9.8	5.88	0.70	2.24	–	–	0.98
Wood	5	4	1.98	0.24	1.71	0.008	0.004	0.058

The chemical formula for this compound can be written as $C_aH_bO_cN_dS$.

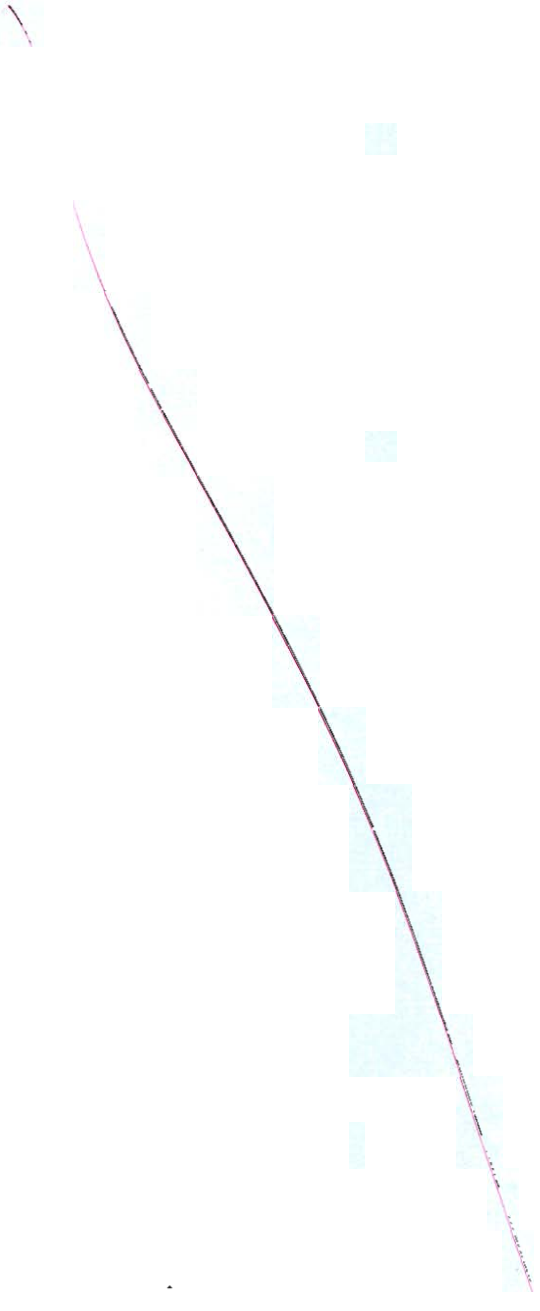
Determine :

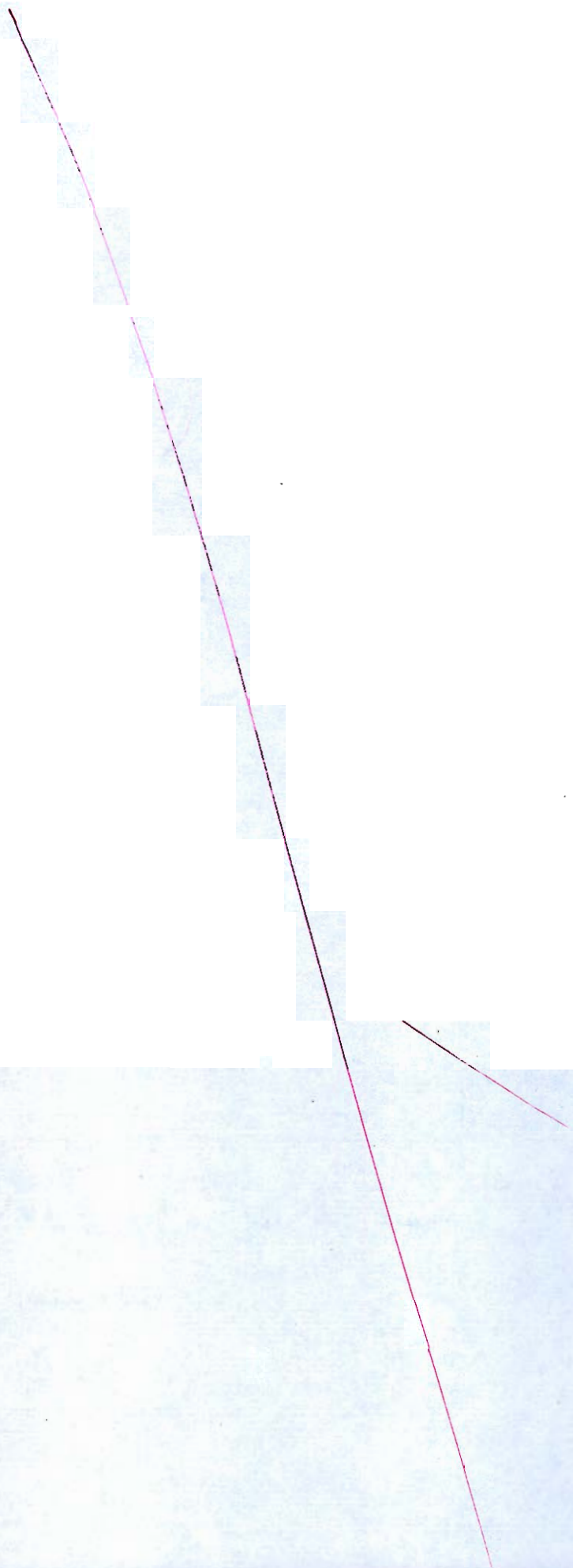
(i) The values of a, b, c, and d.

(ii) The energy content on wet basis and on dry basis of solid wastes if energy content on wet basis can be determined by the equation as:

Energy content (in kJ/kg) = $337C + 1428\left(H - \frac{O}{8}\right) + 9S$ where C, H, O and S are Carbon, Hydrogen, Oxygen and Sulphur in percent by mass.

[20 marks]





Q.7 (a) (i) Calculate the maximum horizontal velocity that can be allowed in a gravitational settling chamber which is designed to remove particles of diameter 48 μm with 100% efficiency. Use the following data:

Length of chamber = 7.5 m
Height of chamber = 1.5 m
Mass density of particle = 2000 kg/m^3
Mass density of air = 1.2 kg/m^3
Correction factor = 1
Viscosity of air = 2.1×10^{-5} kg/m-s

[12 marks]

i) $V_s = \dfrac{(G-1)\rho g d^2}{18\mu}$ (0)

$\Rightarrow$ ~~7.1753×10^{-5}~~ 7.1753×10^{-5} m/s

$\Rightarrow$ 0.07177 mm/s

So, Max horizontal U

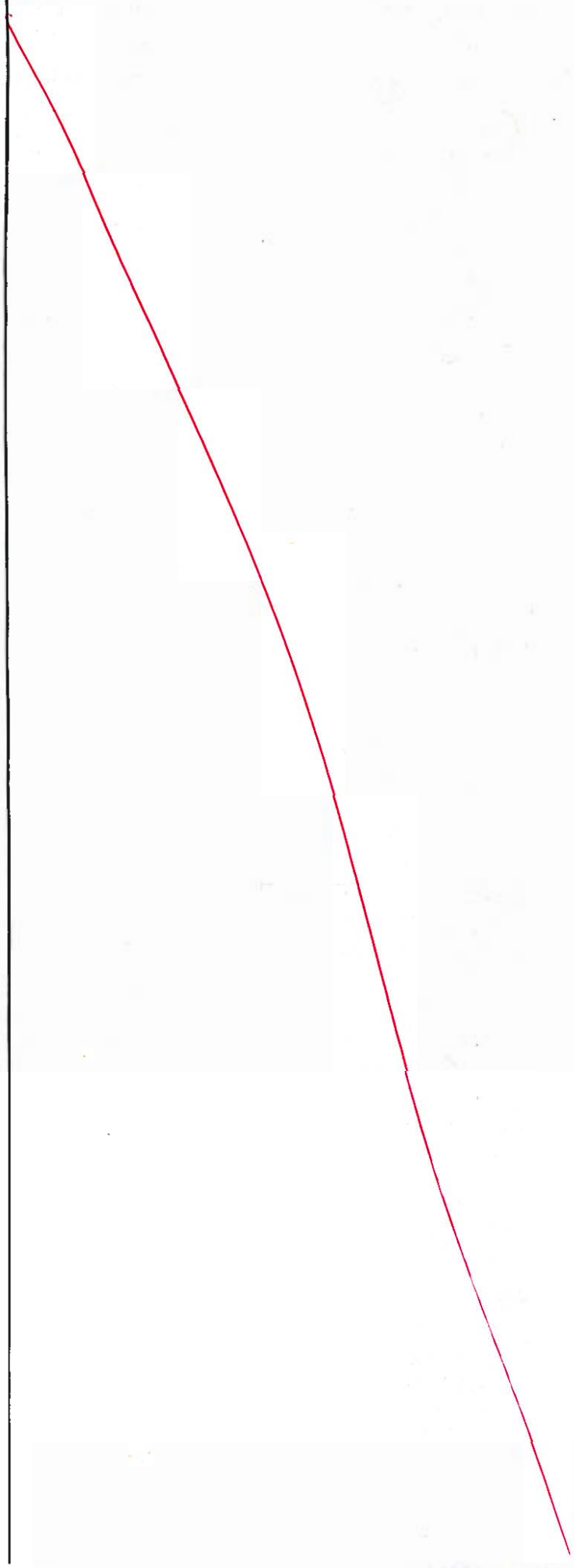
Use Pair also

$\Rightarrow \dfrac{L}{V_f} = \dfrac{H}{V_s}$

$\Rightarrow$ ~~3.588~~

$\Rightarrow$ $V_f = 0.3585$ mm/s

Max horizontal flow velocity.



Q.7 (a) (ii) An electrostatic precipitator is to be constructed to remove fly-ash particles from stack gases flowing at 15 m^3/s. Analysis of a similar system shows that drift velocity can be taken as 0.2 m/s. Determine the plate area required to collect 0.5 μm particles with 90 percent efficiency.

(iii) A fabric filter is to be constructed using bags that are 0.35 m in diameter and 5.8 m long. The bag house is to receive 12 m^3/s of air and filtering velocity has been determined to be 2.0 m/min. Determine the number of bags required for a continuously cleaned operation.

[4 + 4 marks]

ii) $\eta = 1 - e^{-\frac{AV}{Q}}$

$\Rightarrow 0.9 = 1 - e^{-\frac{A \times 0.2}{15}}$

8

$\Rightarrow$ $A = 172.693\ m^2$

$\Rightarrow$ Area of plate req.

iii). Area required $\Rightarrow$ $\frac{\text{Flow}}{\text{flow speed}}$

$\Rightarrow \frac{12}{2/60}$

$\Rightarrow 360\ m^2$

So, No. of bag req $\Rightarrow \frac{A}{\pi D L}$

$\Rightarrow 56.449$

So, 57 No. of filter bags

Q.7 (b) (i) Explain the following terms in an activated sludge process along with their methods of measurement:

1. Sludge age
2. Sludge-volume index
3. Food to biomass ratio

[15 marks]

i.) It is defined as the age time for which biomass is remained in the tank. It is given by.

$$\theta = \frac{VX}{Q_w X_u + (Q_o - Q_w) X_e}$$

(3)

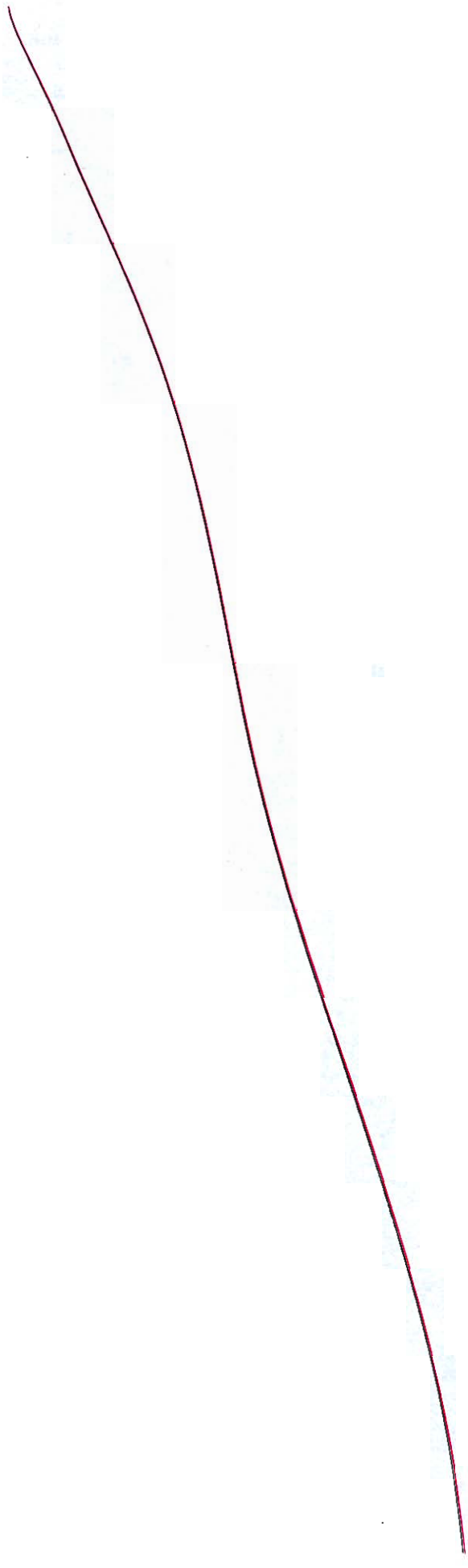
$X, X_u, X_e = ?$

ii.) SVI is defined as the volume occupied by 1 g of MLVSS suspended for 30 min in 1 litre flask.

iii.) F/M ratio is defined as total food provided to biomass available.

It is given by

$$\frac{F}{M} \Rightarrow \frac{Q_o S_o}{VX}$$

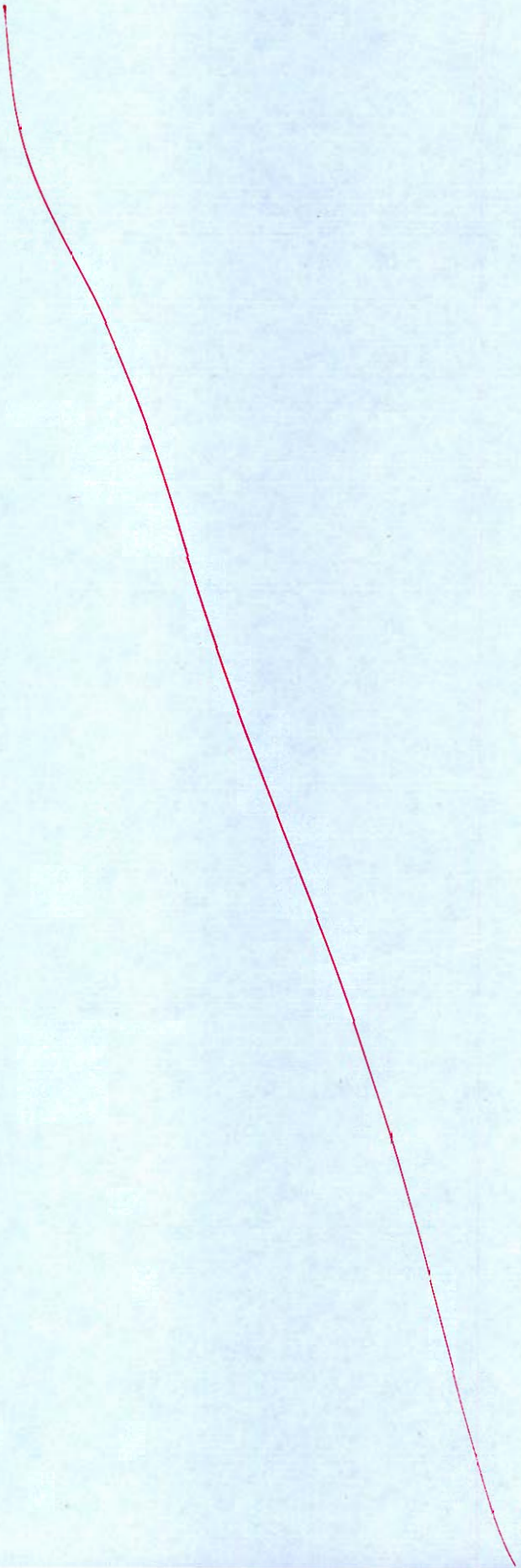


Q.7 (b) (ii) What are the advantages and disadvantages of an Activated Sludge Plants?

[5 marks]

Q.7 (c) (i) Explain the mechanism of coagulation in detail.

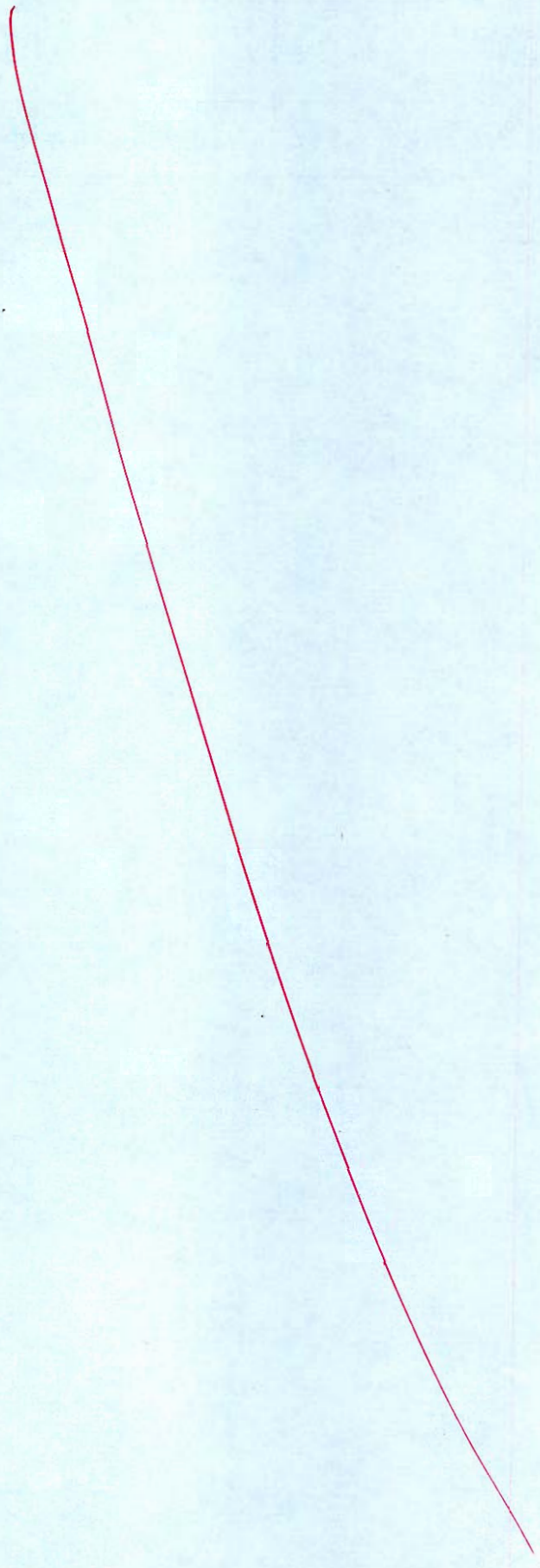
[12 marks]



Q.7 (c) (ii) A settling basin is designed to have a surface overflow rate of 32 m/day. Determine the overall efficiency of solids removal of basin with the help of table given below:

Particle size mm	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
Weight fraction greater than stated size in percent (%)	10	15	35	65	75	80	85	95	90	100
Settling velocity, v_s in mm/s	1.06	0.88	0.72	0.65	0.37	0.25	0.17	0.10	0.04	0.01

[8 marks]



Q.8 (a) (i) A milk products industry discharges its wastewater into a stream. Characteristics of the wastewater and stream are given below:

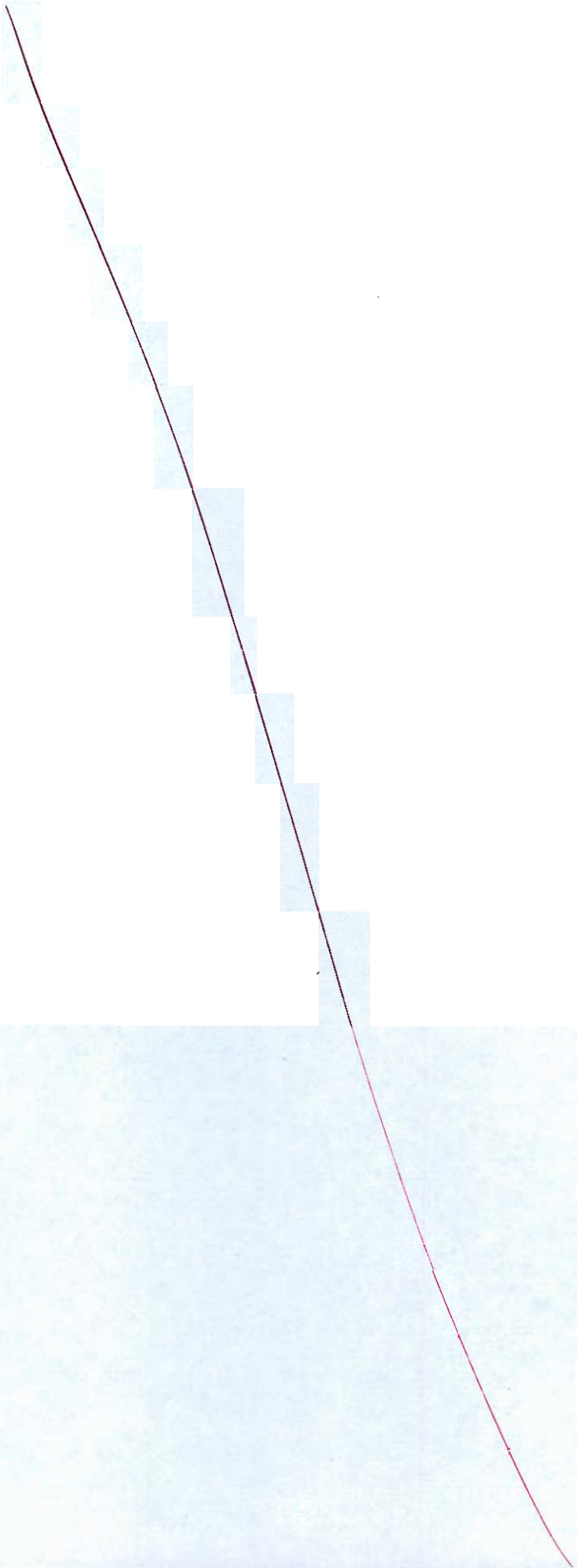
Parameter	Wastewater	Stream
Flow	1250 m^3/d	20,000 m^3/d
BOD_5 at 20° C	200 mg/l	2 mg/l
DO	0 mg/l	8 mg/l
Temperature °C	26	22
K_1 at 20° C(base e)	0.35 d^{-1}	-
K_2 at 20° C(base e)	-	0.55 d^{-1}

Equilibrium concentration of dissolved oxygen C_s for water is as follows:

Temperature °C	18	20	22	23	24	25	26
C_s (mg/l)	9.54	9.17	8.99	8.83	8.53	8.38	8.22

Temperature coefficients for K_1 and K_2 are taken as 1.04 and 1.02 respectively. If no treatment at all is given to wastewater, then what will be the lowest oxygen level in the stream as a result of discharge and its time of occurrence since mixing?

[15 marks]



Q.8 (a) (ii) Briefly describe population equivalent and relative stability.

[5 marks]

Q.8 (b) (i) Explain the concept of L_{eq} in the context of noise pollution. Traffic noise data for a city is given in table below. Compute L_{eq} for the given data:

Time (in s)	10	20	30	40	50	60	70	80	90	100
Noise (dBA)	72	74	73	77	75	80	82	84	81	78

[8 marks]

Q.8 (b) (ii) A wastewater treatment plant consists of primary treatment followed by a completely mix activated sludge secondary system. The primary and secondary sludge are mixed, thickened and treated by anaerobic digestion. The wastewater treatment plant and sludge characteristics are given below:

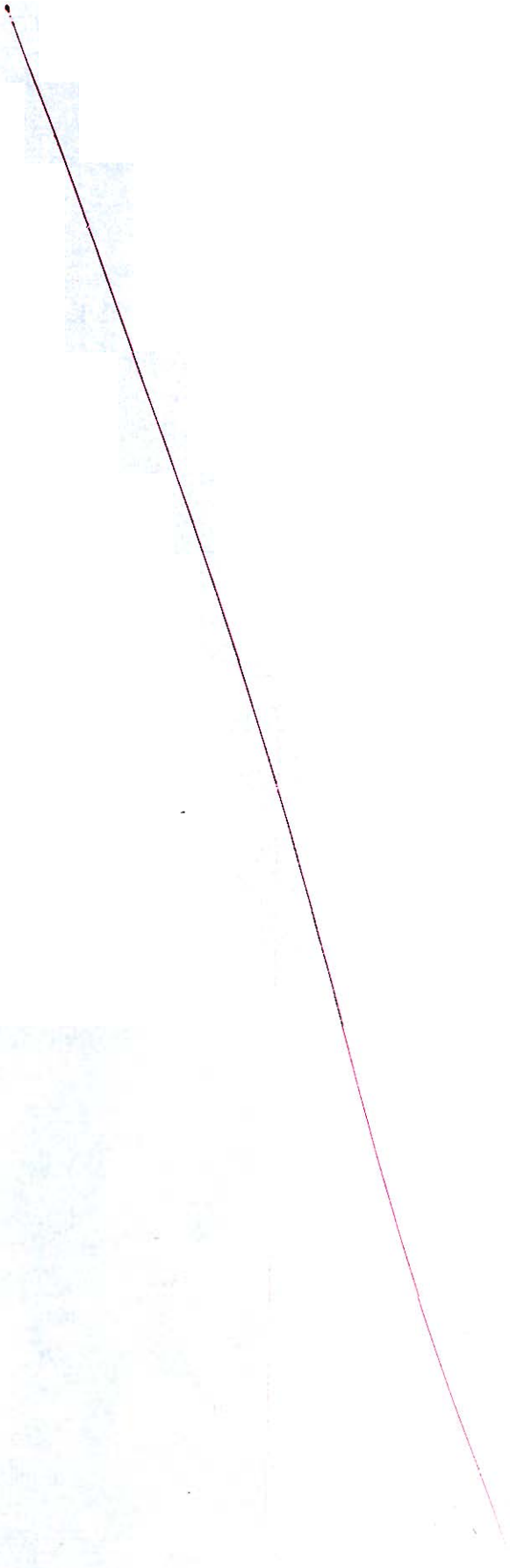
Wastewater	Treatment plant	Sludge
Influent suspended solids = 240 mg/l	Diameter of primary clarifier = 25 m	Primary sludge = 4% solids
Influent BOD = 210 mg/l	Aeration basin volume = 3600 m^3	Secondary sludge = 1% solids
Effluent BOD = 10 mg/l	MLSS in aeration tank = 2800 mg/l	Thickened sludge = 3% solids
Flow = 14350 m^3/d		Specific gravity of primary sludge = 1.02
		Specific gravity of secondary sludge = 1.03
		Specific gravity of thickened sludge = 1.3

Determine the following:

1. Solid loading to digesters.
2. Percent volume reduction in thickener.

Assume efficiency of primary clarifier in removal of suspended solids and BOD as 60% and 30% respectively. Ratio of solids formed in aerator to BOD consumed. i.e. biomass conversion factor is 0.34.

[12 marks]



1.8 (c) (i) What are the different types of water distribution network? Explain the advantages and disadvantages of each system.

[12 marks]