MADE EASY
India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2022 Mains Test Series

Mechanical Engineering Test No : 6

Section A : Renewable Sources of Energy + Industrial and Maintenance Engineering
Section B : Production Engineering and Material Science-1 + Strength of Materials and Mechanics-2

Section : A

1. (a)

A fuel cell is an electrochemical energy conversion device that continuously converts chemical energy of a fuel directly into electrical energy. Continuous operation requires supply of fuel and oxidant and removal of water vapour, spent fuel, spent oxidant, inert residue and heat, etc. It is known as a cell because of some similarities with a primary cell. Like a conventional primary cell it also has two electrodes and an electrolyte between them and produces dc power. It is also a static power conversion device. However, active materials are generally supplied from outside unlike conventional cell where it is contained inside the cell. Fuel is supplied at the negative electrode, also known as fuel electrode or anode and oxidant is supplied at positive electrode, also known oxidant electrode or cathode.

Fuel cells can be classified in several ways:

1. **Based on the type of electrolyte**
   (a) Phosphoric Acid Fuel Cell (PAFC)
   (b) Alkaline Fuel Cell (AFC)
   (c) Polymer Electrolytic Membrane Fuel Cell (PEMFC)
   or Solid Polymer Fuel Cell (SPEC)
   or Proton Exchange Membrane Fuel Cell (PEMFC)

   (d) Molten Carbonate Fuel Cell (MCFC)
   (e) Solid Oxide Fuel Cell (SOFC)

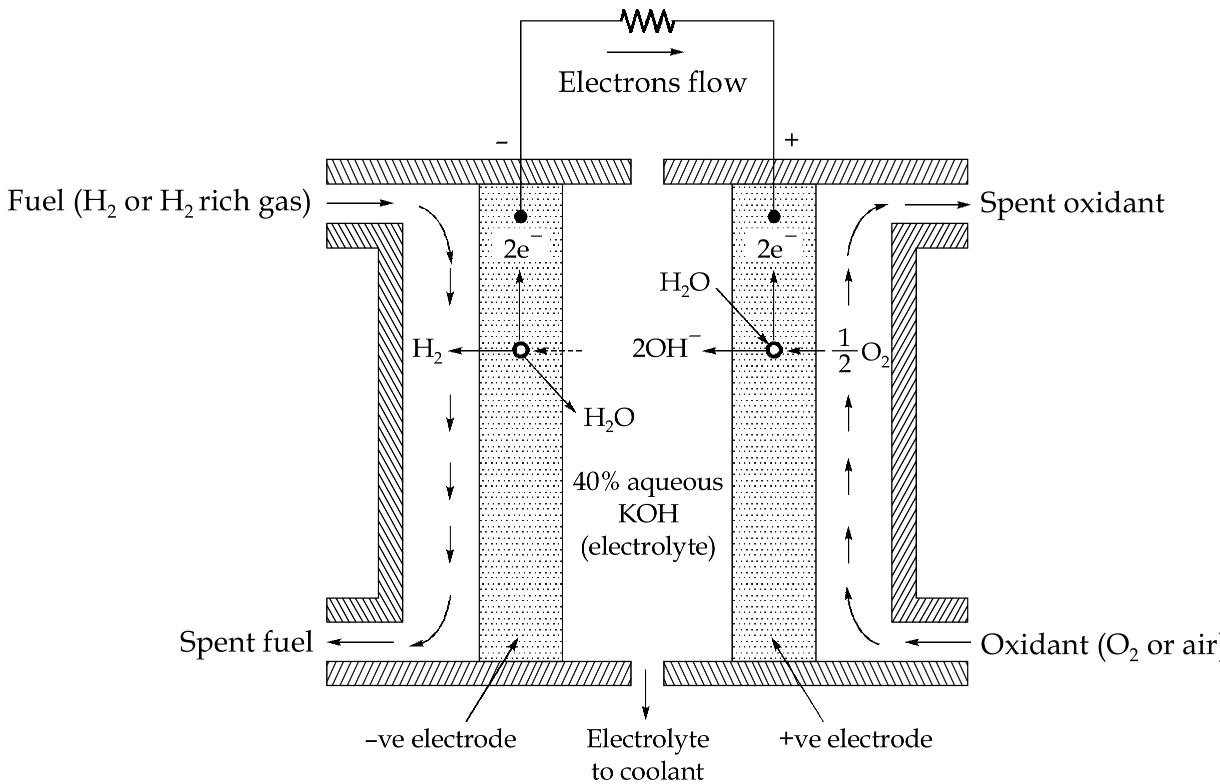
2. **Based on the Types of the Fuel and Oxidant**
   (a) Hydrogen (pure) - Oxygen (pure) fuel cell
   (b) Hydrogen rich gas - air fuel cell
   (c) Hydrazine - Oxygen/hydrogen peroxide fuel cell
   (d) Ammonia - air fuel cell
   (e) Synthesis gas - air fuel cell
   (f) Hydrocarbon (gas) - air fuel cell
   (g) Hydrocarbon (liquid) - air fuel cell

3. **Based on Operating Temperature**
   (a) Low temperature fuel cell (below 150°C)
   (b) Medium temperature fuel cell (150°C - 250°C)
   (c) High temperature fuel cell (250°C - 800°C)
   (d) Very high temperature fuel cell (800°C - 1100°C)

4. **Based on Application**
   (a) Fuel cell for space applications
   (b) Fuel cell for vehicle propulsion
   (c) Fuel cell for submarines
   (d) Fuel cell for defense applications
   (e) Fuel cell for commercial applications

5. **Based on Chemical Nature of Electrolyte**
   (a) Acidic electrolyte type
   (b) Alkaline electrolyte type
   (c) Neutral electrolyte type

Alkaline fuel cell : An alkaline fuel cell, the oldest of all fuel uses 40% aqueous KOH as electrolyte. The operating temperature is about 90°C. It consists of two electrodes of porous conducting material to collect charge; with aqueous KOH filled between them, to work as an electrolyte. Pure hydrogen or a hydrogen-rich gas is supplied at the negative electrode and oxygen or air is supplied at the positive electrode. The pores provide an opportunity for the gas, electrolyte and electrode to come into contact for electro-chemical reaction. The reaction is normally very slow and a catalyst is required in electrodes to accelerate the reaction. It works with H_2 and O_2 active materials and same level of emf is produced. The operation and movement of charge carriers is shown in figure below.



Alkaline Fuel Cell

At positive electrode oxygen, water (from electrolyte) and returning electrons from the external load combine to produce OH^- ions:

$$\frac{1}{2}O_2 + H_2O + 2e^- \rightarrow 2OH^-$$

These OH^- ions migrate from positive to negative electrode through electrolyte. On reaching negative electrode these OH^- ions combine with H_2 to produce water. An equivalent number of electrons are liberated that flow through external load towards positive electrode. Thus:

$$H_2 + 2OH^- \rightarrow 2H_2O + 2e^-$$

The overall reaction is :

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

The fuel used in AFC must be free from CO_2, because this gas can get combined with potassium hydroxide electrolyte to form potassium carbonate. This increases the electrical resistance of the cell, which in turn decreases the available output voltage of the cell. Similarly, if air is used instead of pure oxygen the CO_2 must first be removed from the air by scrubbing with lime.

1. (b)

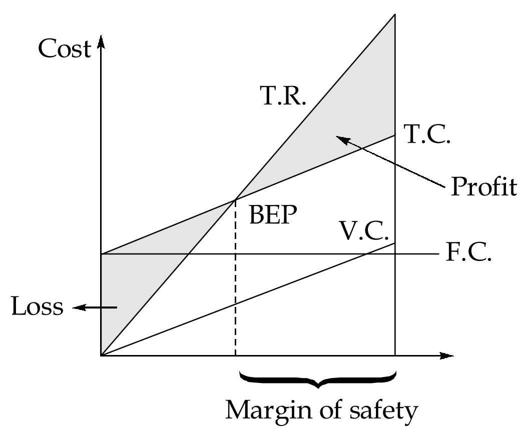
(i)

Break even analysis : Break even charts offer opportunities for several different types of analysis in addition to the break even point, other measures of worth or criterion measures may be derived from the charts. A measure, called profit ratio, is presented here for the purpose of obtaining a further comparative basis for competing projects. Profit ratio is defined as the ratio of the profit and loss areas in break even chart. The following terms are used in break even analysis.

$$\text{Profit ratio} = \frac{\text{Area of the profit region}}{\text{Area of profit and loss region}}$$

- **Fixed cost (FC) :** It does not vary with production quantity.
- **Variable cost (VC) :** It is proportional to production quantity.
- **Total cost (TC) :** FC + VC
- **Total revenue (TR) :** Q × SP

Where *Q* is quantity of the product, SP is sales price per unit.



Quantity (Output)

- **Profit :** TR – TC
- The quantity at which there is no profit – no loss, this is called break even point (BEP).
- The number of units sold beyond break even point is called margin of safety.

  Margin of safety = Actual sales – BEP sales

(ii)

F = 800000/-, Net sales = 140000/- Direct cost = 35% of sales (in rupee)

$$\text{Direct cost} = 35\% \text{ of sales (in rupee)}$$

$$\text{BEP in sales value} = \frac{F}{\left(\frac{s-v}{s}\right)} = \frac{800000}{\left(\frac{x-0.35x}{x}\right)} = ₹1230769.23/- \quad \textbf{Ans.(1)}$$

Now,

$$\text{Sales} = \text{Fixed cost} + \text{Variable cost} + \text{Profit}$$

$$x = 800000 + 0.35x + 160000$$

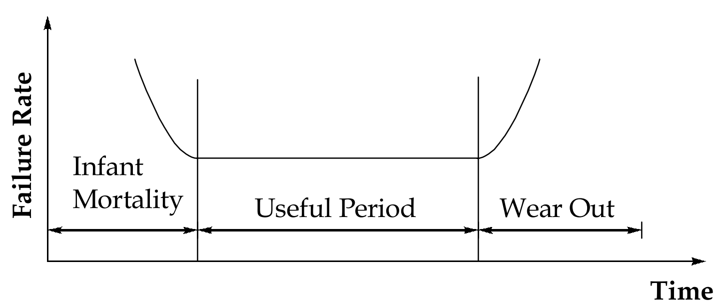
$$0.65x = 960000$$

$$\therefore \quad x = ₹1476923/- \quad \textbf{Ans. (2)}$$

1. (c)

(i)

Reliability often describes the life time of a population of products using a graphical representation called bath tub curve. The bath tub curve does not depict the failure rate of a single item, but describes the relative failure rate of an entire population of products over time.



- The typical machine failure rate versus time plot shown above has three distinct zones : the infant mortality zone, the useful period and the wear out zone.
- The infant mortality zone with high failure rate occurs in the early stage of the machines. Reasons for this high failure rate are faulty installation at the site, ignorance and unfamiliarity of the machine operator, improper electrical power supply, non-availability of a user for training module, improper specifications and the choice of machines.
- In this zone failure rate decreases with time.
- Once the reasons for infant mortality are sorted out, the machine's failure rate reduces significantly and this stage continues for a considerable time, which is known as useful life of tools or products. Reliability is calculated during this period.

- Finally, towards the end of the useful period, the failure rate of the machine again increases. It is due to excessive wear and tear on the machine and fatigue failure of the machine components.
- Though by maintenance, the failure rates can be controlled and reduced, a time comes when cost of maintenance or upkeep is so high that it is better to completely replace the machine with a new one.
- The shape of the curve is in the form of a bath tub, hence it is known as bath tub curve.

(ii)

The failure rate of each component is given by

$$\lambda = \frac{0.05}{4000} = 1.25\times10^{-5} \text{ per hour}$$

The reliability of each component for 2000 hours of operation,

$$r = e^{-\lambda t} = e^{-(1.25\times10^{-5})\times2000}$$

$$= 0.975 \qquad \textbf{Ans.}$$

The reliability (R) of the module

$$R = r^{10} = (0.975)^{10} = 0.779 \qquad \textbf{Ans.}$$

The mean time to failure of the module

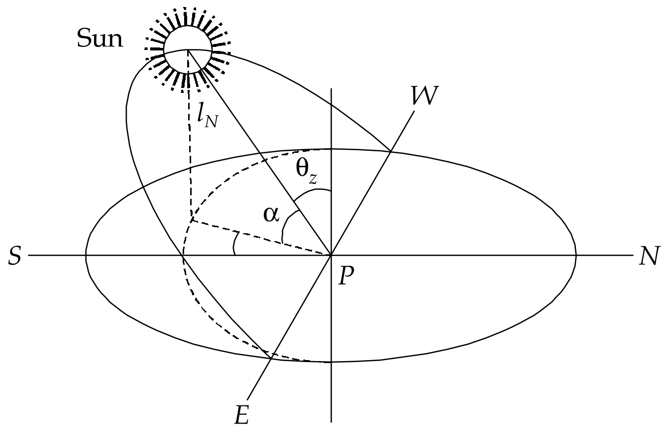
$$\text{MTTF} = \frac{1}{n\lambda} = \frac{1}{10\times1.25\times10^{-5}}$$

$$= 8000 \text{ hrs.} \qquad \textbf{Ans.}$$

1. (d)

(i) Hour angle (ω) : Hour angle ω is the angle through which the earth must rotate to bring the meridian of the point directly under the sun. It is the angular measure of time at the rate of 15° per hour. Hour angle is measured from solar noon, based on local apparent time being positive in the afternoon and negative in the forenoon.

(ii) Altitude angle (α) : It is a vertical angle between the direction of the sun's rays (passing through the point) and its projection on the horizontal plane.



Sun's zenith, altitude and azimuth angles (northern hemisphere)

(iii) Zenith angle (θ_z) : It is the vertical angle between the sun's rays and the line perpendicular to the horizontal plane through the point. It is the complimentary angle of the sun's altitude angle.

Thus, $$\theta_z + \alpha = \frac{\pi}{2}$$

(iv) Surface azimuth angle (γ) : It is an angle subtended in the horizontal plane of the normal to the surface on the horizontal plane. By convention, the angle is taken positive if the normal is west of south and negative when it is east of south in northern hemisphere, and vice versa for southern hemisphere.

1. (e)

Here, arrival rate, λ = 10 patient per hour, service rate, $\mu = \frac{60}{5} = 12$ patient per hour

$$\therefore \quad \rho = \frac{\lambda}{\mu} = \frac{10}{12} = \frac{5}{6}$$

Average number of patients in the queue,

$$L_q = \frac{\rho^2}{1-\rho} = \frac{\left(\frac{5}{6}\right)^2}{1-\frac{5}{6}} = \frac{25}{6}$$

Now, when the average queue size is decreased from $\frac{25}{6}$ patients to one patient, we have to determine the value of new service time, i.e. μ'.

$\therefore \qquad L'_q = \dfrac{\lambda^2}{\mu'(\mu'-\lambda)}$

$1 = \dfrac{10^2}{\mu'(\mu'-10)}$

$\therefore \qquad \mu'^2 - 10\mu' - 100 = 0$

On solving, we get

$\mu' = 16.18$ patients per hour

$\therefore$ Average rate of treatment required = $\dfrac{1}{\mu'} = 3.7$ minutes

$\therefore$ Decrease in the average rate = 5 – 3.7 = 1.3 minutes

$\therefore$ Budget per patient = 100 + 1.3 × 10 = ₹113/-

Hence, in order to get the required size of the queue, the budget should be increased from ₹100/- to ₹113/- per patient.

2. (a)

Given : $\phi = 13.08°$, $L = 80.27°$, $\bar{t}_d = 9.5$ h

For 16 March, $n = 75$,

Now, Declination angle, $\delta = 23.45 \sin\left[\dfrac{360}{365}(284+75)\right]$

$= -2.41°$

Hour angle at sunrise, $\omega_s = \pm\cos^{-1}\{-\tan(13.08°)\tan(-2.41)\}$

$\omega_s = 89.44°$ (or 1.56 rad)

$\therefore$ Maximum day length, $\bar{t}_{d,\max} = \dfrac{2\times\omega_s}{15} = \dfrac{2\times 89.44°}{15} = 11.92$ hr

Average daily extraterrestrial radiation,

$$\bar{H}_0 = 3600 \times \frac{24}{\pi} \times I_{sc}\left[1 + 0.033\cos\frac{360n}{365}\right] \times \left(\cos\phi\cos\delta\sin\omega_s + \omega_s\sin\delta\sin\phi\right)\ \text{kJ/m}^2$$

Substituting the values, we get

$$\overline{H}_0 = 3600 \times \frac{24}{\pi} \times 1.367\left[1 + 0.033\cos\left(\frac{360 \times 75}{365}\right)\right] \times$$

$$\{\cos(13.08°)\cos(-2.41°)\sin(89.44°) + 1.56\sin(-2.41°)\sin(13.08°)\}\ \text{kJ/m}^2$$

$$\therefore \qquad \overline{H}_0 = 36355.78\ \text{kJ/m}^2\text{/day}$$

Therefore, the average daily global radiation, on horizontal surface,

$$\overline{H}_g = \overline{H}_0 \times \left\{a + b\left(\frac{\overline{t}_d}{t_{d\max}}\right)\right\}$$

$$\overline{H}_g = 36355.78 \times \left\{0.42 + \frac{0.27 \times 9.5}{11.92}\right\}$$

$$= 23092.63\ \text{kJ/m}^2\text{/day}$$ **Ans.**

2. (b)

Given : D = 12000 units, C = ₹15/-, C_0 = ₹40/-, C_h = 0.3 × 15 = ₹4.5/ unit/year

Now,
$$\text{EOQ, } Q^* = \sqrt{\frac{2 \times C_0 \times D}{C_h}} = \sqrt{\frac{2 \times 40 \times 12000}{4.5}}$$

$$Q^* = 461.88 \text{ units}$$

or
$$Q^* \simeq 462 \text{ units}$$

Total cost at EOQ with Q^* is

$$T.C_1 = D.C + \frac{D}{Q^*} \times C_0 + \frac{Q^*}{2} \times C_h$$

or
$$T.C_1 = 12000 \times 15 + \frac{12000}{462} \times 40 + \frac{462}{2} \times 4.5$$

$$\therefore \qquad T.C_1 = ₹182078.461/-$$

When 1% discount is offered, total cost is

$$T.C_2 = 12000 \times 0.99 \times 15 + \frac{12000}{3000} \times 40 + \frac{3000}{2} \times 4.5 \times 0.99$$

$$\therefore \qquad T.C_2 = ₹185042.5/-$$

Since, the total cost comes out to be higher when one percent discount is offered, the company should not accept the offer.

Now, let x be the percentage minimum discount acceptable to the company. Then it can be determined by setting the total cost equal to the total cost associated with the policy of ordering EOQ.

Accordingly,

$$15\times12000\times\left(\frac{100-x}{100}\right)+\frac{12000}{3000}\times40+\frac{1}{2}\times3000\times0.3\times15\times\left(\frac{100-x}{100}\right) = 182078.461$$

or $$\left(\frac{100-x}{100}\times186750\right) = 182078.461 - 160$$

or $$100 - x = 100\times\left(\frac{182078.461-160}{186750}\right)$$

or $$x = 2.58\%$$

Hence, minimum discount acceptable to company is 2.58%.

2. (c)

Given : U_H = 20 m/s, H = 12 m, z = 100 m, ρ = 1.3 kg/m^3, α = 0.14, D = 80 m

Let U_0 and U_1 are the upstream wind velocity and wind velocity at turbine rotor.

Now, Area of rotor, $A = \frac{\pi}{4}\times80^2 = 5026.54\text{ m}^2$

From power law,

$$U_z = U_H\left(\frac{z}{H}\right)^{\alpha} = 20\left(\frac{100}{12}\right)^{0.14}$$

$$= 26.91\text{ m/s} = U_0$$

and $$U_1 = 0.75U_0 = 0.75\times26.91$$

$\therefore$ $$U_1 = 20.18\text{ m/s}$$

Now, power available, $$P_0 = \frac{1}{2}\rho AU_0^3 = \frac{1}{2}\times1.3\times5026.54\times26.91^3$$

$\therefore$ $$P_0 = 63.66\text{ MW}$$ **...Ans. (i)**

Power extracted, $$P_T = c_p\cdot P_0$$

Interference factor, $$a = \frac{U_0-U_1}{U_0} = \frac{26.91-20.18}{26.91} = 0.25$$

Power coefficient, $$c_p = 4a(1-a)^2$$

$\therefore$ $$c_p = 4\times0.25(1-0.25)^2$$

$$= 0.562$$

$\therefore$ Power, $p_T = 0.562 \times 63.66$

$= 35.77$ MW **...Ans. (ii)**

Electrical power, $p_E = 0.8 \times p_T$

$= 0.8 \times 35.77$

$= 28.61$ MW **...Ans. (iii)**

$$\text{Axial thrust, } F_a = 4a(1-a)\rho A\frac{U_0^2}{2}$$

$$= 4 \times 0.25(1-0.25) \times 1.3 \times 5026.54 \times \frac{26.91^2}{2}$$

$= 1774.48$ kN **...Ans. (iv)**

Maximum axial thrust occurs when $a = 0.5$,

$$\therefore \quad F_{a,\max} = \rho A\frac{U_0^2}{2}$$

$$F_{a,\max} = 1.3 \times 5026.54 \times \frac{26.91^2}{2}$$

$F_{a,\max} = 2365.97$ kN **...Ans. (v)**

3. (a)

Heat energy required for cooking,

$E_1 = 50 \times 1750$

$= 87500$ kJ/day

Since burner efficiency is 70%

$$\therefore \quad \text{Actual energy required, } E = \frac{E_1}{\eta_{\text{Burner}}} = \frac{87500}{0.7} = 125000 \text{ kJ/day}$$

$$\text{Biogas required} = \frac{125000}{17500} = 7.14 \text{ m}^3\text{/day}$$

Required dry matter to produce 7.14 m^3/day of gas

$$= \frac{7.14}{0.34} = 21 \text{ kg of dry matter}$$

$$\text{Required wet dung} = \frac{21}{0.18} = 116.66 \text{ kg/day}$$

$$\text{Net collectable dung obtained} = 10 \times 0.7 = 7 \text{ kg/cattle}$$

$$\therefore \text{Number of cattles required } (n) = \frac{116.66}{7} = 16.66$$

or $n \simeq 17$ cattles **Ans.**

$$\text{Daily feed of slurry} = \text{Wet dung} + \text{Water}$$
$$= 116.66 + 116.66$$
$$= 233.32 \text{ kg/day}$$

$$\text{Volume of slurry} = \frac{233.32}{1090} = 0.214 \text{ m}^3\text{/day}$$

$\therefore$ Volume of digestor required considering 50 days retention period

$$= \frac{0.214 \times 50}{0.9} = 11.88 \text{ m}^3$$ **Ans.**

3. (b)

The data of the given problem can be summarized as follows:

Nutrient constituents	Nutrient content in the product		Minimum amount of nutrient
	A	B	
x	36	6	108
y	3	12	36
z	20	10	100
Cost	₹20	₹40	

Mathematical formulation of the linear programming is

Minimize $z = 20x_1 + 40x_2$

Subjected to:
$$36x_1 + 6x_2 \geq 108$$
$$3x_1 + 12x_2 \geq 36$$
$$20x_1 + 10x_2 \geq 100$$

and $x_1, x_2 \geq 0$

where, x_1 = number of units of product A, x_2 = number of units of product B.

The constraints of the given problem are plotted by treating them as equation:

$$36x_1 + 6x_2 = 108$$

$$\Rightarrow \qquad \frac{x_1}{3} + \frac{x_2}{18} = 1$$

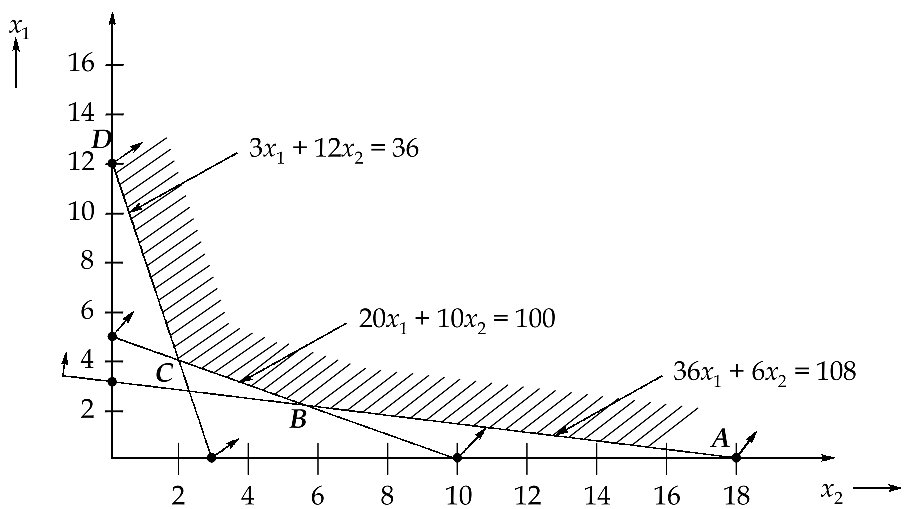
$$3x_1 + 12x_2 = 36$$

$$\Rightarrow \qquad \frac{x_1}{12} + \frac{x_2}{3} = 1$$

$$20x_1 + 10x_2 = 100$$

$$\Rightarrow \qquad \frac{x_1}{5} + \frac{x_2}{10} = 1$$

The feasible region of the problem is shown below:



The coordinate of the extreme points of the feasible region are:

$$A = (0, 18), B = (2, 6), C = (4, 2) \text{ and } D = (12, 0)$$

The value of objective function at each of the extreme point can be evaluated as follows:

Extreme point	(x_1, x_2)	$z = 20x_1 + 40x_2$
A	(0, 18)	720
B	(2, 6)	280
C	(4, 2)	160 ← Minimum
D	(12, 0)	240

Hence, the optimum solution is to purchase 4 units of product *A* and 2 units of product *B* in order to maintain a minimum cost of ₹160/-.

3. (c)

1. **Vibration monitoring :** The noise and vibration are the most important parameters to monitor a machine, particularly in the moving parts such as shafts, rotors, cutting tools, gears etc. The vibration level is recorded by attaching a transducer like velocity probe, accelerometer, or proximity probe to the machine. Special equipment is also available for using the output from the sensor to indicate the nature of vibration problem and even its precise cause. In some cases, it may become necessary to use the principles of sonics and acoustics.
2. **Wear debris monitoring :** This works on the principle that the working surfaces of a machine are washed by the lubricating oil, and any damage to them should be detectable from particles of wear debris in the oil. If the debris consists of relatively large ferrous lumps such as those generated by the fatigue of rolling element bearings and gears or the pitting of cams and taproots, these can be picked up by removable magnetic plugs inserted in the oil return lines. For small debris particle, spectrographic analysis or microscopic examination of oil samples after magnetic separation are commonly used techniques. Another popular technique is SOAP analysis for debris analysis.
3. **Thermography** is a technique that can be used to monitor the condition of plant machinery, structures and system. It uses instrumentation designed to monitor the emission of infrared energy, that is temperature to determine their operating condition. By detecting thermal anomalies or areas which are hotter or colder than they should be, an experienced surveyor can locate and define incipient problems within the plant. The intensity of infrared radiation from an object is function of its surface temperature. Inclusion of thermography into a condition monitoring programme will enable the user to monitor the thermal efficiency of critical processes system relying on heat transfer or retention that will improve reliability of the plant.
4. **Visual inspection :** Regular visual inspection of the machinery and systems in a plant is a necessary part of any condition monitoring programme. In many cases, visual inspection will detect potential problems that will be missed using other condition monitoring techniques. Routine visual inspection of all critical part system will augment the other gytechniques and ensure potential that problems are detected before serious damage can occur, since the incremental cost of visual observations are small, this technique should be incorporated in all condition monitoring programme.

4. (a)

(i)

Tidal power plants can be broadly classified into the following four categories:

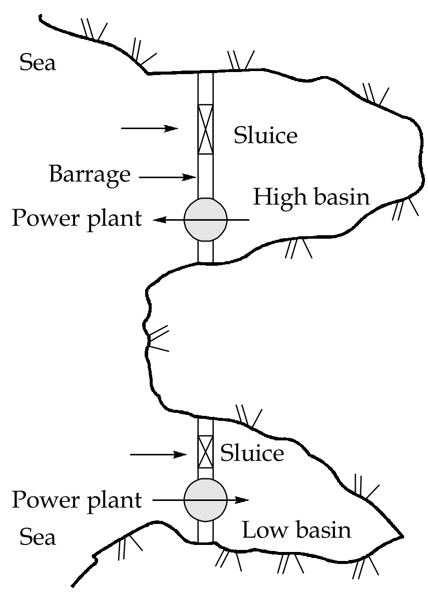
(1) Single-basin single-effect plant

(2) Single-basin double-effect plant

(3) Double-basin with linked-basin operation

(4) Double-basin with paired-basin operation

Double-basin with paired-basin operation : The paired basin scheme consists of two single-basin single-effect separate schemes located at a distance from each other. The locations are so selected that there is a difference in tidal phase between them. Both the schemes never exchange water, but are interconnected electrically. Both the basins operate in single-basin single effect mode. One basin generates electrical energy during the 'filling' process while the other during the 'emptying' process. The scheme is shown in figure below.



Double-basin with paired-basin operation

This arrangement affords a little more flexibility in operation of the plants to meet power demands. More benefit can be derived if there is a difference in tidal phase of the sea near the two basins. In case where there is no difference in tidal phase, variations in power output can be evened out by resorting to ebb tide operation in one plant and flood tide operation in the other.

The paired-basin operation leads to a continuous output, still its power supply remains irregular and there is no solution for equalizing the great difference in output between the spring and the neap tide operation. Further, it is difficult to find two tidal sites within reasonable distance of each other having the requisite difference in time of high water.

(ii)

Given : $A = 2\text{ km}^2$, $H = 10$ m, $H' = 5$ m, $t = 4$ hrs. $\eta = 0.8$, $\rho = 1025\text{ kg/m}^3$

$$\text{Volume of basin, } V = AH = 2 \times 10^6 \times 10 = 2 \times 10^7\text{ m}^3$$

$$\text{Average discharge, } Q = \frac{V}{t} = \frac{2\times 10^7}{4\times 3600}$$

$$\therefore \quad Q = 1388.88\text{ m}^3/\text{s}$$

Theoretical power obtained by Q quantity of water falling through H' metres is

$$P_T = \rho Q H'\text{ kg-m/s}$$

$$\text{Power generated, } P = \frac{\rho Q H'}{75}\times \eta \times 0.736\text{ kW}$$

$$\because \quad 1\text{ hp} = 75\text{ kg m/s} = 0.736\text{ kW}$$

$$\therefore \quad P = \frac{1025\times 1388.88\times 5\times 0.8\times 0.736}{75}$$

or $\quad P = 55.88$ MW ...**Ans.**

Energy generated per tidal cycle $= 55.88 \times 10^3 \times 4$ kWh

$= 223520$ kWh

$\because$ Tidal number of tidal cycle in a year = 705

$\therefore$ Yearly power generation $= 223520 \times 705$ kWh/year

$= 157.58 \times 10^6$ kWh/year

4. (b)

Step 1 : Converting maximization problem into minimization problem by subtracting from the highest element i.e. 154.

The resulting opportunity loss matrix is

	A	B	C	D
P	14	42	56	0
Q	64	82	91	55
R	44	66	77	33
S	74	90	98	66

Step 2 : For optimum assignment, subtract the minimum element of each row from all the elements of that row. Then subtract the minimum element of each column from all the elements of the column. In the reduced matrix make assignments in rows and columns that have single zeros as usual. After that draw the minimum number of lines to cover all the zeros of the reduced matrix. Thus, we get table

	A	B	C	D	
P	6	18	24	[0]	✓
Q	1	3	4	~~0~~	✓
R	3	9	12	~~0~~	✓
S	[0]	~~0~~	0	0	
				✓	

Step 3 : Modify the reduced opportunity loss matrix by subtracting element '1' from all the elements not converted by the lines and adding the same at the intersection of two lines. Thus, we get table

	A	B	C	D	
P	5	17	23	[0]	✓
Q	[0]	2	3	~~0~~	
R	2	8	11	~~0~~	✓
S	~~0~~	[0]	~~0~~	1	
				✓	

Step 4 : Modify further the reduced table by subtracting element '2' from all the elements not covered by the lines and adding the same at the intersection of two lines. Thus, we get table

	A	B	C	D	
P	3	15	21	[0]	✓
Q	[0]	2	3	~~2~~	✓
R	~~0~~	6	9	~~0~~	✓
S	~~0~~	[0]	~~0~~	3	
	✓			✓	

Step 5 : Modify further the reduced table by subtracting element '2' from all the elements not covered by the lines and adding the same at the intersection of two lines.

	A	B	C	D
P	3	13	19	[0]
Q	~~0~~	[0]	1	2
R	[0]	4	7	~~0~~
S	2	~~0~~	[0]	5

Thus the optimum solution is obtained and the assignment is

$P \rightarrow D,\ Q \rightarrow B,\ R \rightarrow A$ and $S \rightarrow C$

Maximum sales (in thousands) = ₹(154 + 72 + 110 + 56)

= ₹392 (in thousands)

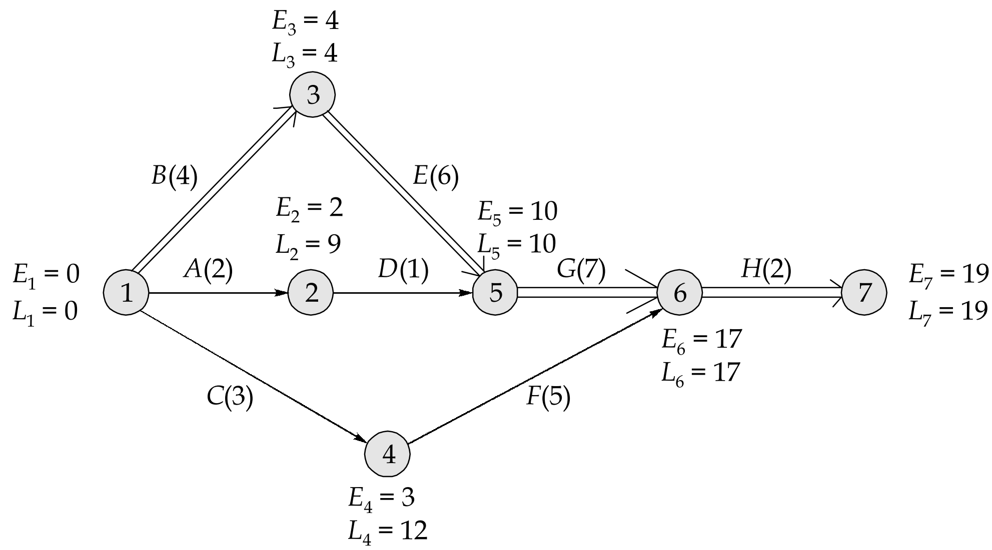
= ₹392000 **Ans.**

4. (c)

The expected time and variance of each activity is computed in table below:

Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\left(\frac{t_p - t_0}{6}\right)^2$
A	1	1	7	2	1
B	1	4	7	4	1
C	2	2	8	3	1
D	1	1	1	1	0
E	2	5	14	6	4
F	2	5	8	5	1
G	3	6	15	7	4
H	1	2	3	2	$(2/6)^2$

Using the precedence relationship among the activities, the resulting network is show in figure below.



From the above network diagram, we observe:

CPM : $1 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 7$ i.e. $B \rightarrow E \rightarrow G \rightarrow H$

Expected duration of the project is 19 days **Ans. (i)**

$$\text{Standard deviation, } \sigma = \sqrt{1+4+4+0.108} = 3.02$$

Since, $$P(z \le 1.645) = 0.5 + 0.45 = 0.95$$

$\therefore$ We get $$\frac{T_s - T_e}{\sigma_e} = 1.645$$

$$T_s = 19 + 3.02 \times 1.645$$

$$T_s = 24 \text{ days}$$

Hence, 24 days of project completion time will have 95% confidence of completion in the scheduled time.

Section : B

5. (a)

At a distance x m from the smaller end (left end)

$$\text{Depth of section} = 60 + (180 - 60) \cdot \frac{x}{2.5}$$

$$= 60 + 48x = 12(5 + 4x) \text{ mm}$$

Depth of centroid from top edge

$$= 6(5 + 4x) = (30 + 24x) \text{ mm}$$

$$\text{Eccentricity of load} = 6(5 + 4x) - 30 = 24x \text{ mm}$$

$$\text{Area of section} = 30 \times 12(5 + 4x) = 360(5 + 4x) \text{ mm}^2$$

$$I = \frac{30 \times 12^3 (5+4x)^3}{12} = 4320(5 + 4x)^3 \text{ mm}^4$$

Tensile stress,

$$\sigma = \frac{54 \times 10^3}{360(5+4x)} + \frac{(54 \times 10^3 \times 24x)}{4320(5+4x)^3} \times (30+24x)$$

$\because$ σ (tensile stress) is the sum of direct stress and bending stress.

$$\Rightarrow \quad \sigma = \frac{150}{(5+4x)} + \frac{1800x}{(5+4x)^2} \quad \text{...(i)}$$

For maximum value,

$$\frac{d\sigma}{dx} = 0$$

$$\frac{-150 \times 4}{(5+4x)^2} + \frac{1800[(5+4x)^2 - x.2(5+4x).4]}{(5+4x)^4} = 0$$

$$\frac{-150 \times 4}{(5+4x)^2} + \frac{1800[5+4x-8x]}{(5+4x)^3} = 0$$

On multiplying throughout by $(5 + 4x)^3$

$$-150 \times 4(5 + 4x) + 1800(5 - 4x) = 0$$

$$-3000 - 2400x + 9000 - 7200x = 0$$

$$6000 - 9600x = 0$$

$$\Rightarrow \quad x = \frac{6000}{9600} = 0.625 \text{ m}$$

So, the location of maximum tensile stress is 0.625 m from left end and its value is

$$\sigma = \frac{150}{(5+4 \times 0.625)} + \frac{1800 \times 0.625}{(5+4 \times 0.625)^2} \quad \text{(By eqn. (i))}$$

$$\Rightarrow \quad \sigma = 20 + 20 = 40 \text{ MPa}$$

So, value of maximum tensile stress is 40 MPa and its position is 0.625 m from smaller end.

5. (b)

The angle θ which the surface makes with the horizontal is to be calculated first.

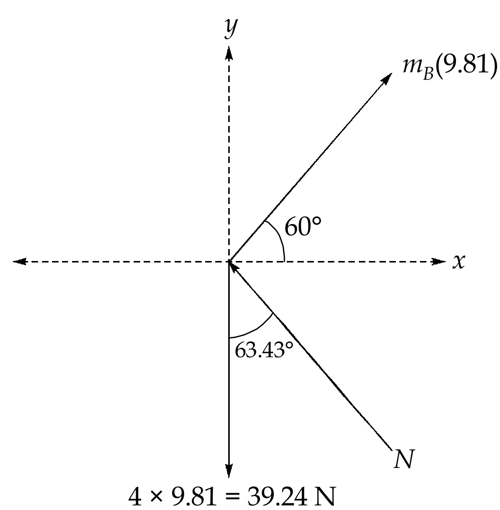
$$\tan\theta = \left(\frac{dy}{dx}\right)_{x=0.4\text{ m}} = (2.5\times 2x)_{x=0.4\,\text{m}}$$

$$\tan\theta = 5 \times 0.4 = 2$$

$$\theta = \tan^{-1}(2) = 63.43°$$

Now, the tension in the cord is the same throughout the cord and is equal to weight of block B, $w_B = m_B \times 9.81$

Free Body Diagram of A



Equations of equilibrium,

$\Sigma F_x = 0;$

$$m_B(9.81)\cos 60 - N\sin 63.43 = 0$$

$$N = 5.48\, m_B \quad \text{...(i)}$$

$\Sigma F_y = 0;$

$$m_B(9.81)\sin 60 + N\cos 63.43 - 39.24 = 0$$

$$8.496\, m_B + 0.447\, N = 39.24 \quad \text{...(ii)}$$

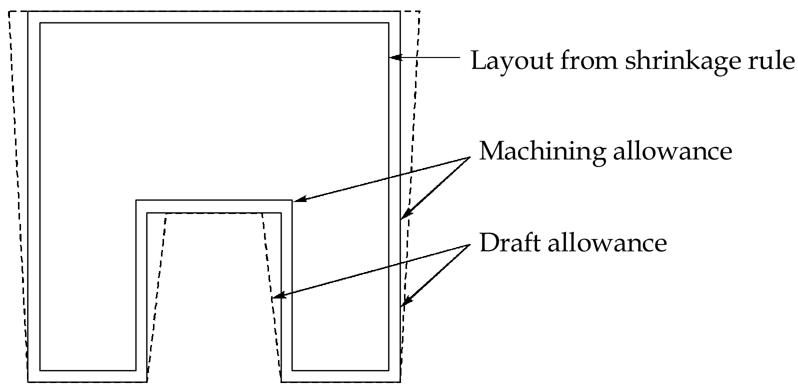
Solving equations (i) and (ii),

$$m_B = 3.58 \text{ kg and } N = 19.7 \text{ N}$$

5. (c)

Various pattern allowances: Patterns are not prepared exactly of the same dimensions as the casting. Some allowances are provided over its dimensions to allow for metal contraction (shrinkage), easy withdrawal from mould, machining after casting, distortion and pattern shake (or rapping) before withdrawal. Pattern allowances include (a) Shrinkage (b) Draft (c) Machining (d) Distortion, and (e) Shake.

A simple single piece pattern is shown in figure below.



Allowances on a single piece pattern

Pattern drawing is first prepared by using a shrink rule (during drawing and fabrication). Over this size machining allowance is added and then draft is provided on external and internal vertical surfaces for easy removal of pattern from mould. The draft is 1-2% on external and 6.25% on interior holes.

For steel, $\alpha_s = 21 \text{ mm/°C m}$

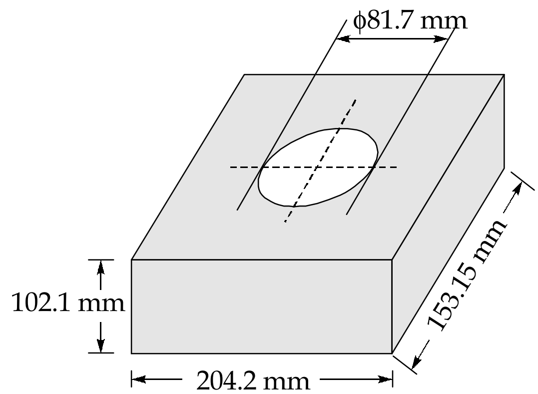
For 200 mm dimension, allowance = $200 \times \frac{21}{1000} = 4.20 \text{ mm}$

For 150 mm dimension, allowance = $150 \times \frac{21}{1000} = 3.15 \text{ mm}$

For 100 mm dimension, allowance = $100 \times \frac{21}{1000} = 2.10 \text{ mm}$

For 80 mm dimension, allowance = $80 \times \frac{21}{1000} = 1.68 \simeq 1.70 \text{ mm}$

Dimensions of Aluminium pattern:



Now,

For aluminium, $\alpha_{Al} = 13 \text{ mm/°C m}$

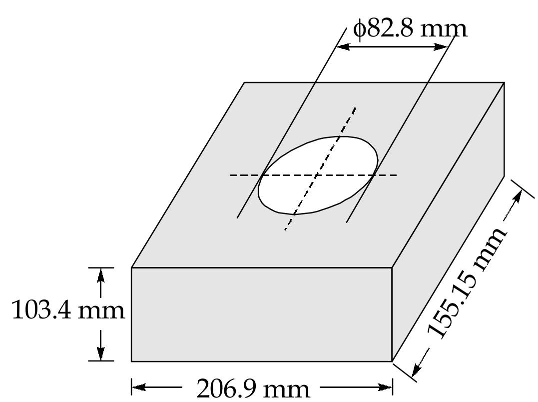
For 204.2 mm dimension, allowance = $204.2 \times \frac{13}{1000} = 2.6546 \simeq 2.70 \text{ mm}$

For 153.15 mm dimension, allowance = $153.15 \times \frac{13}{1000} = 1.9905 \simeq 2.00 \text{ mm}$

For 102.1 mm dimension, allowance = $102.1 \times \frac{13}{1000} = 1.3273 \simeq 1.30 \text{ mm}$

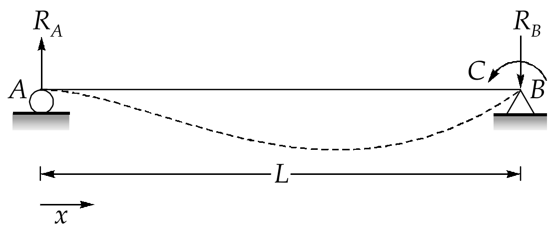
For 81.7 mm dimension, allowance = $81.7 \times \frac{13}{1000} = 1.0621 \simeq 1.10 \text{ mm}$

Dimensions of wooden pattern:



5. (d)

(i)



Since there is no external vertical load, the reactions R at A and B will be equal and opposite so as to form a clockwise couple to resist the ACW couple.

Hence, $$R_A = \frac{C}{L}(\uparrow) \text{ and } R_B = \frac{C}{L}(\downarrow)$$

Measuring x from A

$$EI\frac{d^2y}{dx^2} = -M = -\frac{C}{L}x$$

On integrating,

$$EI\frac{dy}{dx} = \frac{-C}{L}\cdot\frac{x^2}{2} + P_1 \quad \text{...(i)}$$

and $$EIy = \frac{-C}{L}\cdot\frac{x^3}{6} + P_1x + P_2 \quad \text{...(ii)}$$

where P_1, P_2 is constant of integration.

At $x = 0$, $y = 0$

Hence, $P_2 = 0$

At $x = L$, $y = 0$

$$\Rightarrow \quad \frac{-C}{L}\cdot\frac{L^3}{6} + P_1L = 0$$

$$\Rightarrow \quad P_1 = \frac{CL}{6}$$

So, slope and deflection equation for elastic curve is

$$EI\frac{dy}{dx} = \frac{-C}{2L}x^2 + \frac{CL}{6} \quad \text{...(1)}$$

$$EIy = \frac{-C}{6L}x^3 + \frac{CLx}{6} \quad \text{...(2)}$$

So, equation of elastic curve

$$y = \frac{C}{6EI}\left(Lx - \frac{x^3}{L}\right) \quad \text{...Ans.}$$

For maximum deflection $\frac{dy}{dx} = 0$. Hence, from equation (1)

$$0 = \frac{-C}{2L}x^2 + \frac{CL}{6} \quad \Rightarrow \quad x = \frac{L}{\sqrt{3}}$$

Now, from equation (1) at $x = \frac{L}{\sqrt{3}}$

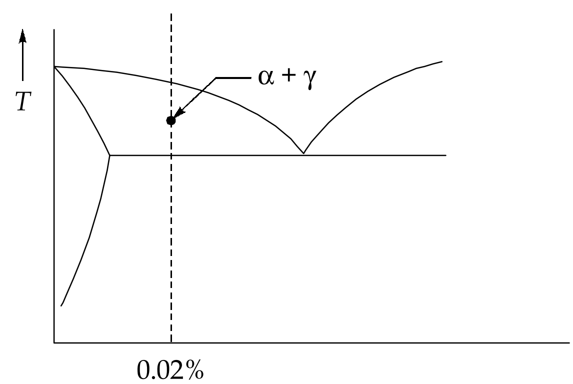
$$EIy_{max} = \frac{-C}{6L}\left(\frac{L}{\sqrt{3}}\right)^3 + \frac{CL}{6} \times \frac{L}{\sqrt{3}}$$

$$\Rightarrow \quad y_{max} = \frac{CL^2}{9\sqrt{3}EI} \quad \text{...Ans.}$$

5. (e)

Dual phase steels:

- DP-steels are high strength steels that has a ferritic martensite microstructure.



- They are formed when very low carbon steels when they are heated in the range where two phase mixture α and γ are present and from this temperature material is cooled rapidly at a rate equal to or greater than critical cooling rate.
- Since, TTT-diagram is applicable only for austenite, so there will not be any change in α but austenite present will convert into micropockets of martensite.
- α-ferrite is already a very strong material and the micropockets of martensite creates further obstacles in the movement of dislocation, so strength of material increases exponentially.

Important characteristics are :

(a) Low yield strength

(b) Good uniform elongation

(c) High ultimate tensile strength

(d) A high strain rate sensitivity

(e) High initial strain hardening rates

(f) A good fatigue resistance

Applications:

- Automobile body panels
- Wheels
- Bumpers
- TMT-bars etc

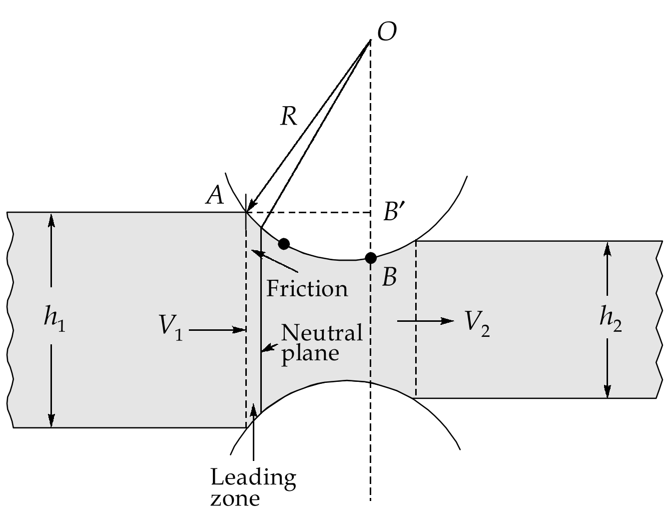
Maraging steel:

- Since, Ti is very expensive material so the replacement is maraging steel.
- Maraging steel is having extensive application in defense and aerospace industries.

It contains:

- Ni – 17-19% : Increases ductility
- Co – 8-19% : Increases strength
- Mo – 3-35% : Increases strength
- Ti – 0.15% : Increases strength
- Al – 0.05-0.15% : as an impurity

6. (a)



At entry the velocity of material will be lower than the velocity of rolls, so friction will be in the forward direction. There exist a plane in the deformation zone where the velocity of the material is equal to the velocity of roll which is called neutral plane. Beyond this plane the velocity of rolls and the friction will be in the backward direction.

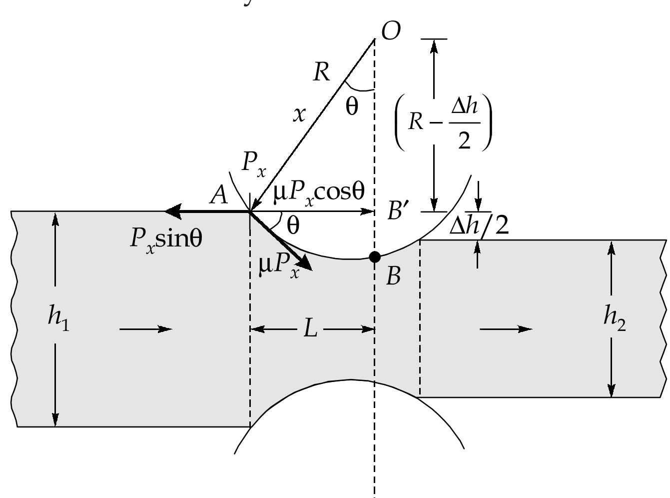
The outgoing plate thickness will be slightly higher than the minimum gap between the rolls due to elastic recovery.

Let, V_r = Velocity of roll

then $$\text{Backward slip} = \frac{V_r - V_1}{V_1} \times 100$$

and $$\text{forward slip} = \frac{V_2 - V_r}{V_r} \times 100$$

Condition for unaided or self entry:



P_x = Roller separating force

For self entry,

$$\mu P_x \cos\theta \geq P_x \sin\theta$$

$$\mu \geq \frac{\sin\theta}{\cos\theta}$$

$$\mu \geq \tan\theta$$

and $$\tan\theta = \frac{L}{R - \frac{\Delta h}{2}} = \frac{\sqrt{R\Delta h}}{R - \frac{\Delta h}{2}}$$

We get, $$\mu \geq \frac{\sqrt{R\Delta h}}{R - \frac{\Delta h}{2}}$$

Since, $$R >> \frac{\Delta h}{2}$$

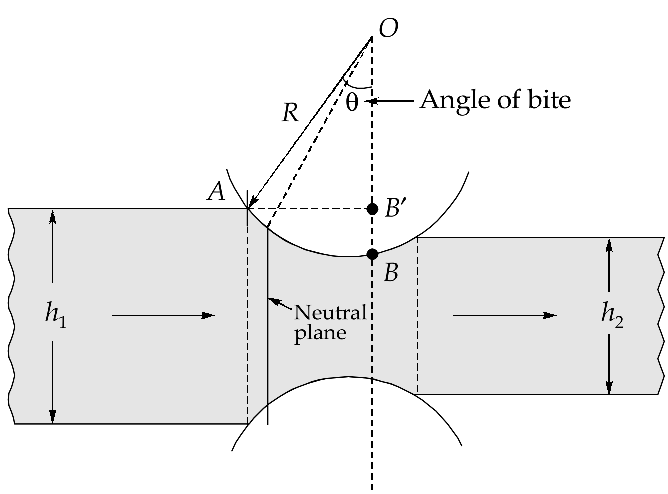
$$\mu \geq \frac{\sqrt{R\Delta h}}{R}$$

For the condition for self entry,

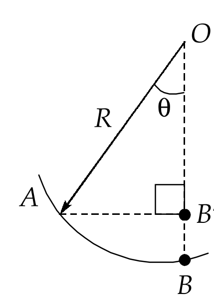
$$\mu = \sqrt{\frac{(\Delta h)_{max}}{R}}$$

$$(\Delta h)_{max} = \mu^2 R$$

During the rolling operation by increasing the co-efficient of friction the neutral plane shifts towards left. By maintaining the same parameters if the thickness of the incoming plate is increased, external pressure has to be applied at entry and this shifts the neutral plane towards right. There will be a situation at which neutral plane coincides with the vertical axis so there will not be any deformation in the material and roll will slip over the material. This is called rolling limit and the bite angle at the rolling limit is called angle of nip.



Now, consider $\Delta AOB'$



$$\cos\theta = \frac{OB'}{OA} = \frac{OB - BB'}{OA}$$

$$\cos\theta = \frac{R - \left(\frac{h_1 - h_2}{2}\right)}{R}$$

$$\cos\theta = 1 - \frac{h_1 - h_2}{2R} \qquad (\text{say } h_1 - h_2 = \Delta h)$$

$$\Rightarrow \qquad \cos\theta = 1 - \frac{\Delta h}{2R} = 1 - \frac{\Delta h}{D}$$

$$\text{So, angle of bite } (\theta) = \cos^{-1}\left(1 - \frac{\Delta h}{D}\right)$$

$$\text{And, angle of nip } (\alpha_n) = 2\mu \qquad \text{(radians)}$$

6. (b)

Given : $\rho_w = 1000 \text{ kg/m}^3$; $\rho_{CI} = 7000 \text{ kg/m}^3$; $\sigma_t = 20$ MPa; $\sigma_c = 50$ MPa; $L = 12$ m

Let's take bottom part of channel as reference for neutral axis calculation.

$$\bar{y} = \frac{500 \times 20 \times 10 + 2 \times (320 - 20) \times 20 \times 170}{500 \times 20 + 2 \times (320 - 20) \times 20}$$

$$= 97.27 \text{ mm}$$

$$I_{NA} = \frac{500 \times 20^3}{12} + 500 \times 20 \times (97.27 - 10)^2 + 2\left[\frac{20 \times 300^3}{12} + 20 \times 300 \times (170 - 97.27)^2\right] \times 10^{-12}$$

$$\approx 23 \times 10^{-5} \text{ m}^4$$

Moment of resistance in compression

$$M_c = \frac{\sigma_c I_{NA}}{y_c} = \frac{50 \times 23 \times 10^{-5} \times 10^6}{(320 - 97.27) \times 10^{-3}}$$

$$= 51.632 \text{ kN.m}$$

Moment of resistance in tension

$$M_t = \frac{\sigma_t I_{NA}}{y_t} = \frac{20 \times 10^6 \times 23 \times 10^{-5}}{97.27 \times 10^{-3}}$$

$$= 47.291 \text{ kN.m}$$

So, safe bending moment is 47.291 kN.m

$$\sigma_t = 20 \text{ MPa}$$

$$\Rightarrow \qquad \sigma_c = \frac{20 \times (320 - 97.27)}{97.27} = 45.796 \text{ MPa}$$

Let w = Total weight of channel and water per meter length (kN/m)

$$M = \frac{wl^2}{8} = \frac{w \times 12^2}{8} = 18w \text{ kN.m}$$

Now, $18w = 47.291$

$\Rightarrow$ $w = 2.62728$ kN/m

Weight of channel per meter

$$w_1 = (500 \times 20 + 2 \times 300 \times 20) \times 10^{-6} \times 7000 \times 9.81$$
$$= 1510.74 \text{ N/m}$$

Weight of water, $w_2 = w - w_1 = 2627.28 - 1510.74$
$$= 1116.54 \text{ N/m}$$

Let x = Depth of water in channel (mm)

$$w_2 = 460 \times x \times 10^{-6} \times 1000 \times 9.81$$
$$1116.54 = 460 \times x \times 10^{-6} \times 1000 \times 9.81$$

$\Rightarrow$ $x = 247.42$ mm

6. (c)

Computer Aided Manufacturing (CAM) : "Computer aided manufacturing is the use of computer to help in the manufacturing or production after the design process.

- CAM basically produces instructions for the controller, a cutting or processing machine to automatically perform a set of operations to produce the final product.
- The design outputs of a CAD applications can directly become the input to a CAM system and form the design directly such a machine control program called otherwise as "Numerical Control Code" can be generated.

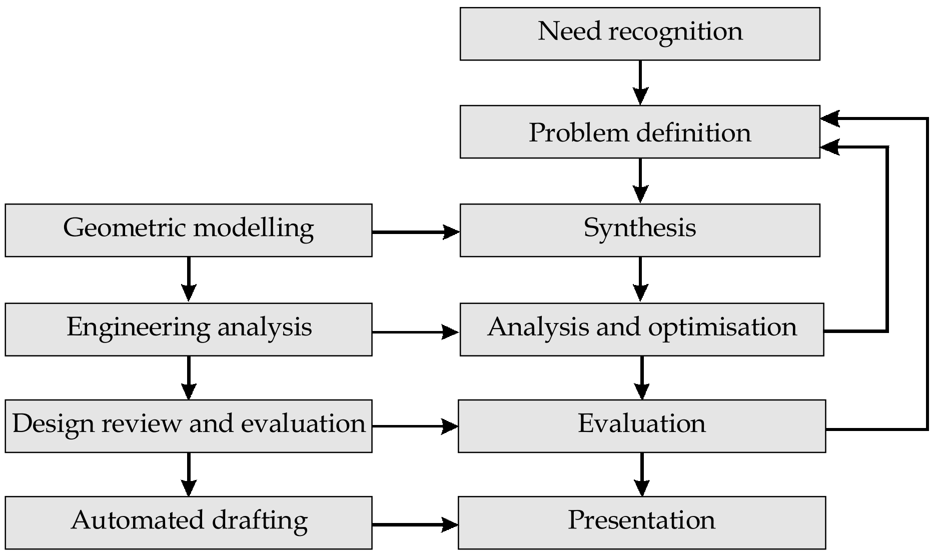
Different machine controllers are available with every type of NC numerical control machine. Each of these controllers support one or more of these languages; ISO, APT, UNIAPT. The output of the CAM system can directly be input into these machines to perform the machining or processing required.

Applications of CAM : Following are the applications of CAM:

1. **Computer monitoring and control :** These are the direct applications in which the computer is connected directly to the manufacturing process for the purpose of monitoring or controlling the process.
2. **Manufacturing support applications :** These are the indirect applications in which computer is used in support of the product operations in the plant, but there is no direct interface between the computer and the manufacturing process.

- Computer monitoring and control can be separated into monitoring applications and control applications. "Computer process monitoring" involves a direct computer interface with the manufacturing process and associated equipment and collecting data from process. The computer is not used to control the operation directly. The control of the process remains in the hands of human operators, who may be guided by the information compiled by the computer.

The various steps involved in computer aided engineering design problems are shown in figure below.



Steps involved in computer aided engineering problems

7. (a)

As the torque is applied to the rigid plate, it rotates through an angle θ. This torque is resisted by opposite torques at two ends which are known as fixing torques. Let fixing torque at D be T_d and at C be T_c.

Fixing torque at C, $T_c = (2500 - T_d)$ N.m

Polar MOI for brass, $J_b = \frac{\pi}{32}\times 16^4 = 2048\pi \text{ mm}^4$

Polar MOI for steel, $J_s = \frac{\pi}{32}\times(32^4 - 24^4) = 22400\pi \text{ mm}^4$

Polar MOI for aluminium, $J_a = \frac{\pi}{32}\times(48^4 - 32^4) = 133120\pi \text{ mm}^4$

Thus, Angle of twist, $\theta = \frac{(2500 - T_d)l}{G_aJ_a + G_bJ_b} = \frac{T_dl}{G_sJ_s}$

or $$\frac{(2500 - T_d)}{T_d} = \frac{G_aJ_a + G_bJ_b}{G_sJ_s}$$

$$\frac{2500}{T_d} - 1 = \frac{27000 \times 133120\pi + 40000 \times 2048\pi}{82000 \times 22400\pi}$$

$\Rightarrow$ T_d = 832.95 N.m

and T_c = 2500 – 832.95 = 1667.05 N.m

The fixing torque is shared by two shafts. Let T_a be torque taken by aluminium shaft and T_b by brass shaft.

Then, $T_a = 1667.05 - T_b$ ...(a)

and $$\frac{T_a}{T_b} = \frac{G_a J_a \theta / l}{G_b J_b \theta / l} = \frac{G_a J_a}{G_b J_b} = \frac{27000 \times 133120\pi}{40000 \times 2048\pi}$$

$\Rightarrow$ $\frac{T_a}{T_b} = 43.875$ ...(b)

On solving equation (a) and (b)

T_b = 37.15 N.m and T_a = 1629.90 N.m

So, σ_b (stress in brass) $= \frac{16T_b}{\pi d_b^3} = \frac{16 \times 37.15 \times 10^3}{\pi \times 16^3}$ = 46.19 MPa

σ_a (stress in aluminium) $= \frac{16T_a \times D_a}{\pi\left(D_a^4 - d_a^4\right)}$

$$\sigma_a = \frac{16 \times 1629.90 \times 10^3 \times 48}{\pi(48^4 - 32^4)} = 93.53 \text{ MPa}$$

σ_s (stress in steel shaft) $= \frac{16 \times T_d \times D_s}{\pi(D_s^4 - d_s^4)}$

$$= \frac{16 \times 832.95 \times 10^3 \times 32}{\pi(32^4 - 24^4)}$$

$$= 189.38 \text{ MPa}$$

7. (b)

The structure of all crystals can be described in terms of a lattice with a group of atoms attached to every lattice point. The group of atoms is called basis. When they are repeated in a space it forms the crystal structure. The basis consist of primitive cell, containing one single lattice point.

Different unit cells and space lattice under bravais crystal system:

Unit cell	Space lattice	Materials
(1) Cubic $a = b = c$ $\alpha = \beta = \gamma = 90°$	Simple cubic	Mn, NaCl
	BCC	Na, W
	FCC	Ni, Au, Ag
(2) Tetragonal $a = b \neq c$ $\alpha = \beta = \gamma = 90°$	Simple Tetragonal	Pb, In
	BCT	Martensite
(3) Orthorhombic $a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$	Simple orthorhombic	As, Bi
	End centered orthorhombic	$MgSO_4$, KNO_3
	Body centered orthorhombic	Cementite
	Face centered orthorhombic	Ga
(4) Rhombohedral $a = b = c$ $\alpha = \beta = \gamma \neq 90°$	Simple Rhombohedral	$CaCO_3$, SiO_2
(5) Hexagonal $a = b \neq c$ $\alpha = \beta = 90°$ $\gamma = 120°$	Hexagonal closed packed	Cobalt, Zinc, Magnesium
(6) Monoclinic $a \neq b \neq c$ $\alpha = \gamma = 90° \neq \beta$	Simple monoclinic	
	End centered monoclinic	
(7) Triclinic $a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$	Simple triclinic	$CuSO_4$

Given : Zn has HCP structure

For HCP, volume of unit cell,

$$V_C = 4.82185\, a^3$$

where, a = side of hexagon and $\frac{c}{a} = 1.856$

Also, $a = 2r$ (where, r = atomic radius of element)

and, $r = 0.133$ nm $= 1.33 \times 10^{-8}$ cm

$$\Rightarrow \quad V_C = 4.82185\,(2r)^3 = 38.575\, r^3$$

$$V_C = 38.575\ (1.33 \times 10^{-8})^3 = 90.753 \times 10^{-24}\ \text{cm}^3$$

$$N_A = \text{Avogadro's number} = 6.023 \times 10^{23}\ \text{atoms/mol}$$

Now,

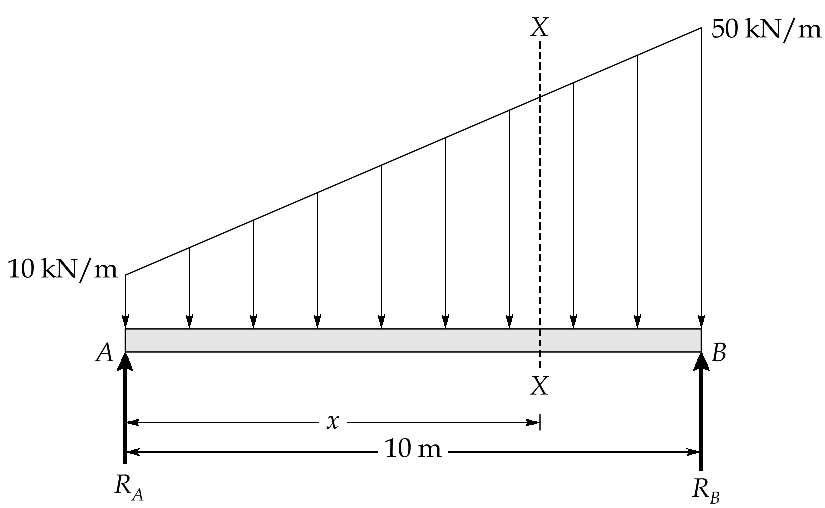
$$\text{Density } (\rho) = \frac{nA_{zinc}}{V_C N_A}$$

$$= \frac{(6)(65.39)}{90.753 \times 10^{-24} \times 6.023 \times 10^{23}} \qquad [\because n = 6 \text{ for HCP}]$$

$$\rho_{\text{zinc}} = \frac{6 \times 65.39}{54.66} = 7.18\ \text{g/cc}$$

So, theoretical density of zinc is 7.18 g/cc.

7. (c)



Given : $L = 10$ m, $h = 400$ mm, $\sigma_{b,\max} = 90$ MPa, $E = 205$ GPa

At a distance x from left end A, intensity of loading is

$$\omega_x = 10 + \left(\frac{50-10}{10}\right)x = 10 + 4x$$

$\Sigma M_B = 0$:

$$10R_A = 10 \times 10 \times 5 + \frac{1}{2} \times 10 \times 40 \times \frac{10}{3}$$

$$\Rightarrow \qquad R_A = 116.67\ \text{kN}$$

$$M_x = R_A \times x - 10 \times x \times \frac{x}{2} - \frac{1}{2} \times x \times 4x \times \frac{x}{3}$$

$$= 116.67x - 5x^2 - \frac{2x^3}{3}$$

Now, $$EI\frac{d^2y}{dx^2} = -Mx = -116.67x + 5x^2 + \frac{2}{3}x^3$$

$$EI\frac{dy}{dx} = -58.333x^2 + \frac{5}{3}x^3 + \frac{x^4}{6} + C_1$$

$$EIy = -19.444x^3 + \frac{5x^4}{12} + \frac{x^5}{30} + C_1x + C_2$$

At $x = 0, y = 0$

$\Rightarrow$ $$C_2 = 0$$

At $x = 10$ (m), $$y = 0$$

$$0 = -19.444 \times 10^3 + \frac{5 \times 10^4}{12} + \frac{10^5}{30} + C_1 \times 10$$

$\Rightarrow$ $$C_1 = 1194.44$$

So, at $x = 5$ m

$$EIy = -19.444 \times 5^3 + \frac{5 \times 5^4}{12} + \frac{5^5}{30} + 1194.44 \times 5$$

$$EIy = 3906.26$$

$$y = \frac{3906.26}{EI} \quad \text{...(i)}$$

For M_x to be maximum $\frac{dM_x}{dx} = 0$

$$116.667 - 10x - 2x^2 = 0$$

$$x = -10.536 \text{ or } 5.536$$

So, at $x = 5.536$ m there is maximum bending moment.

Now, $$M_{max} = 116.67 \times 5.536 - 5 \times 5.536^2 - \frac{2}{3} \times 5.536^3$$

$$= 379.54 \text{ kN.m}$$

We also know that

$$M_{max} = \sigma_{b,max} \times Z$$

$\Rightarrow$ $$Z = \frac{M_{max}}{\sigma_{b,max}} = \frac{379.54 \times 10^6}{90}$$

Section modulus, $$Z = 4.217 \times 10^6 \text{ mm}^3$$

and also $$Z = \frac{bh^2}{6}$$

So, $$b = \frac{4.217\times10^6\times6}{400^2}$$

$$b = 158.14 \text{ mm}$$

$$I = \frac{bh^3}{12} = \frac{158.14\times400^3}{12}$$

$$I = 843.422 \times 10^6 \text{ mm}^4$$

So, by equation (i)

$$y_{(x=5m)} = \frac{3906.26\times10^3\times10^9}{205\times10^3\times843.422\times10^6}$$

$\Rightarrow$ $$y_{(x=5m)} = 22.59 \text{ mm}$$ **...Ans.**

8. (a)

(i)

When austenite is cooled down by quenching at a temperature below 723°C, the following microstructures are formed depending upon cooling rate:

(a) Coarse pearlite

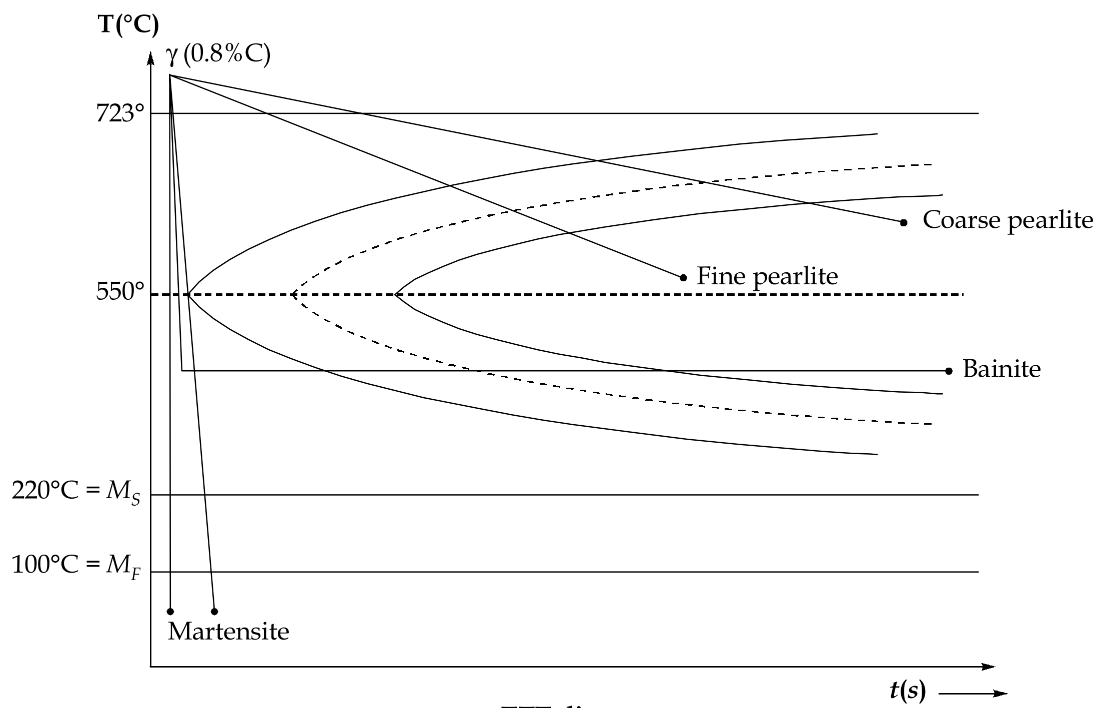
(b) Fine pearlite

(c) Bainite

(d) Martensite

- If the cooling rate is such that it just touches the nose of TTT diagram then it is called critical cooling rate.
- Any cooling rate equal to or greater than critical cooling curve will produce martensite.
- If cooling rate is less than the critical cooling rate then resulting microstructure will be pearlite. If the cooling rate is too low then coarse pearlite is produced otherwise at a high cooling rate fine pearlite is produced.
- Bainite cannot be produce by continuous cooling.
- Austenite is quenched to a temperature below the nose but above martensite start line, sample is kept at this temperature for a substantial period of time so that the cooling curve enters the TTT diagram. This is called austempering and produces bainite.

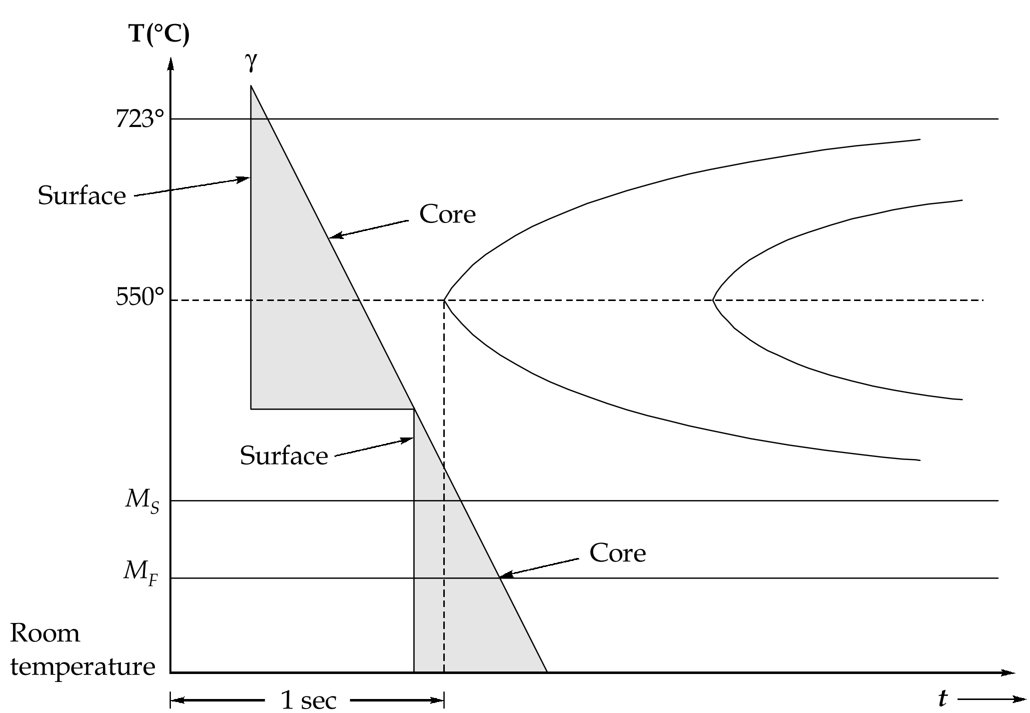
(ii)



TTT diagram

(iii)

Martempering:



- When a sample is quenched in some medium, surface and core experiences different cooling rates. Since, surface comes in contact with the quenching medium, immediately it will convert into martensite and becomes rigid but core is still austenite (high density).
- After some time when core converts into martensite its volume will expand but surface is rigid, this produces cracks on the surface so cracks during quenching are due to density difference. To avoid these cracks martempering is done.
- In martempering, sample is initially quenched to a temperature below the nose but above the martensite start line. Sample is kept in this hot bath for some time period, so that the temperature and core becomes uniform. After that sample is kept in cold bath maintained at room temperature.
- This procedure decreases the temperature difference and hence density gradient decreases between the surface and the core.
- Martempering decreases the possibility of crack but it does not completely eliminate it.

(iv)

Hardening

- Sample is heated to a temperature at which austenite is stable and then it is quenched at a rate equal to or greater the critical cooling rate to produce martensite.
- Main objective of hardening is to get martensite structure and the moment we deviate from eutectoid composition there will be proeutectoid phase and proeutectoid phase does not contribute in formation of martensite because only eutectoid composition produces martensite.

Importance of tempering after hardening:

- After hardening process resulting microstructure is extremely hard and brittle.
- Tempering is mainly performed for reducing hardness and brittleness and to increase ductility or toughness.

Type of tempering	Temperature range	Applications
High temperature tempering	500 - 650°C (Sorbite)	Engineering applications
Medium temperature tempering	350 - 500°C (Troosite)	In springs
Low temperature	~250°C (No change in microstructure)	Metrology and Agricultural tools

8. (b)

(i) The following assumptions have been made in developing the equations for stresses and deformations in a bar subjected to pure torsion :

1. Shaft is loaded with twisting couples in planes that are perpendicular to the axis of the shaft.
2. Torsion is uniform along the length, i.e., all normal cross-sections which are at the same axial distance suffer equal relative rotations.
3. Circular sections remain circular. Thus, radii remain straight after torsion.
4. Plane normal sections of shaft remain plane after twisting, i.e., no warping or distortion of parallel planes normal to the axis of the shaft takes place.
5. Stress is proportional to strain, i.e., stresses do not exceed proportional limit.
6. Material is homogeneous and isotropic.

(ii) The shaft is subjected to variable torque along its lengths.

For portion AB : This portion is subjected to uniformly distributed torque $t = 1$ kN-m/m.

∴ $T_x = t.x = 1.x = x$ kNm (↺) (linear)

at A : $x = 0$. Hence $T_A = 0$

at B : $x = 2$. Hence $T_B = 2$ kN.m (↺)

For portion BC : This portion is subjected to a constant torsion

$= 1 \times 2 = 2$ kN.m

For portion CD : At C, the shaft is subjected to a concentrated torque of 4 kN.m in the ACW direction.

Now T_c(left) = 2 kN.m (↺)

T_c(right) = 2 – 4 = –2 kN.m, i.e., 2 kN.m (↻)

Thus, the torque changes sign at C and torque is constant from C to D.

For portion DE : This portion is subjected to a uniformly variable torque, having intensity $t = 0$ at D to $t = 2$ kN.m/m at E. Consider a section X at a distance x meters from D.

The intensity of torque at that section is

$$t_x = \frac{2}{4}x = \frac{x}{2} \text{ kN.m/m}$$

Hence, total torque at X is

$$T_x = (1 \times 2) - 4 + \frac{1}{2}x\left(\frac{x}{2}\right) = -2 + \frac{x^2}{4} \; (\circlearrowleft) \text{ (parabolic)}$$

At D, $x = 0$

Hence, $T_D = -2$ kN.m (as earlier)

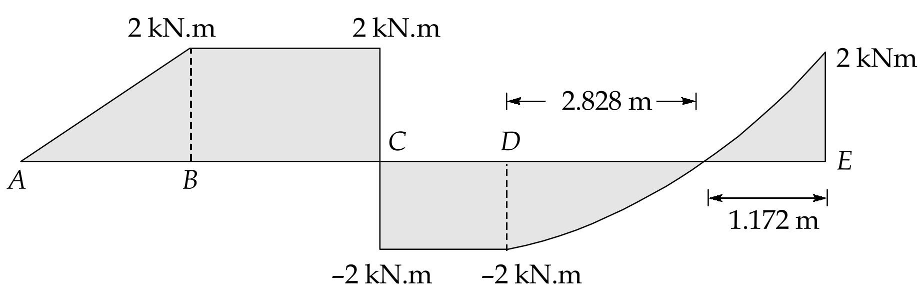
At D, $x = 4$

Hence, $T_E = -2 + \frac{1}{4}(4^2) = 2$ kN.m $(\circlearrowleft)$

Thus, the torque changes sign in the portion DE, the torque is zero at

$$0 = -2 + \frac{x^2}{4}$$

$\Rightarrow$ $x = \sqrt{8} = 2.828$ m from D



Variation of torque

8. **(c)**

(i)

Ceramics are essentially inorganic and non-metallic materials. Most of the ceramics are typically crystalline in nature and are compounds formed between metallic and non-metallic elements.

The general characteristics of ceramic material are high hardness, brittleness and high temperature resistance.

The common ceramics that are used as tool materials:

(i) Alumina,

(ii) Carbides (Tungsten Carbide, Titanium Carbide, SiC),

(iii) Nitrides (CBN, Si-Al-O-N, Cermets),

(iv) Diamond

Ceramics are typically produced by the application of heat upon processed clays and other natural raw materials to form rigid products. Chemically prepared powders are also used as starting materials for some products. These synthetic materials can be controlled to produce powders with precise chemical compositions and particle size. After addition of additives like binders, lubricants; these powder mixture are subjected to shape forming process. Some common forming methods for ceramics are extrusion, slip casting and injection molding. After compaction, these "Green" ceramics undergo sintering process to promote bond formation between particles resulting in strong, rigid and finished ceramic product.

(ii)

Volume fraction of carbon fibre in composite,

$$\frac{V_{cf}}{V_c} = 0.35$$

Volume fraction of epoxy in composite,

$$\frac{V_e}{V_c} = 1 - 0.35 = 0.65$$

Tensile strength of carbon fibre, σ_{cf} = 1700 MPa, tensile strength of epoxy, σ_e = 300 MPa. Assuming given fibre is a parallel fibre and hence, area fraction will be same as that of volume fraction.

Let the composite is subjected to a load of P_c which is shared between fibres and epoxy.

So, $$P_c = P_{cf} + P_e$$

$\Rightarrow$ $$\sigma_c A_c = \sigma_{cf} A_{cf} + \sigma_e A_e$$

$$\sigma_c = \sigma_{cf} \frac{A_{cf}}{A_c} + \sigma_e \frac{A_e}{A_c}$$

$$\sigma_c = 1700\,(0.35) + 300\,(0.65)$$

$$\sigma_c = 790 \text{ MPa}$$

❍❍❍❍