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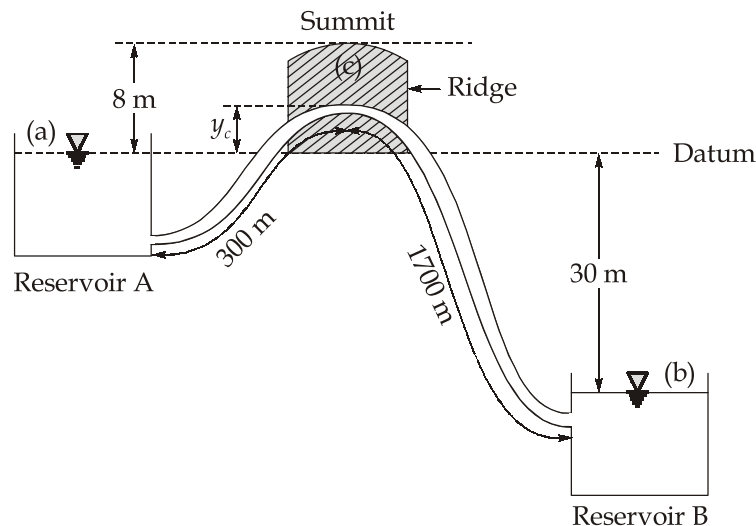
Detailed Solutions

**ESE-2022  
Mains Test Series**

**Civil Engineering  
Test No : 5**

**Section A**

**Q.1 (a) Solution:**



Applying Bernoulli's equation at (a) and (c),

$$\frac{P_a}{\rho g} + y_a + \frac{v_a^2}{2g} = \frac{P_c}{\rho g} + y_c + \frac{v_c^2}{2g} + \text{friction loss in AC} \quad \dots(i)$$

Given,

$$\begin{aligned} \frac{P_c}{\rho g} &= \text{Atmospheric pressure} - 7.2 \\ &= 10.2 - 7.2 = 3 \text{ m} \end{aligned}$$

By applying Bernaulli's equation at (a) and (b),

$$\text{Head loss,} \quad h_f = Z_B - Z_A$$

$$\Rightarrow \quad h_f = 30 \text{ m}$$

$$\Rightarrow \quad \frac{4fLv^2}{2gd} = 30$$

$$\Rightarrow \quad \frac{4 \times 0.008 \times 2000 \times v^2}{2 \times 9.81 \times 0.2} = 30$$

$$\Rightarrow \quad v = 1.356 \text{ m/s}$$

$$\begin{aligned} \text{Discharge,} \quad Q &= A \times v \\ &= \frac{\pi}{4} (0.2)^2 \times 1.356 = 0.0426 \text{ m}^3/\text{s} \end{aligned}$$

Substituting values in eq. (i)

$$\frac{P_a}{\rho_g} + y_a + \frac{v_a^2}{2g} = \frac{P_c}{\rho_g} + y_c + \frac{v_c^2}{2g} + \text{friction loss in AC}$$

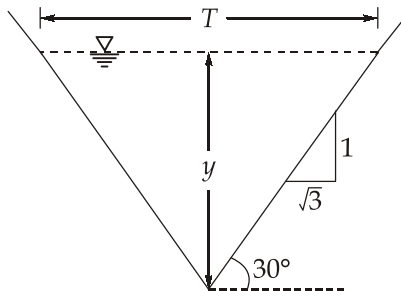
$$\Rightarrow \quad 10.2 = 3 + y_c + \frac{1.356^2}{2 \times 9.81} + \frac{4 \times 0.008 \times 300 \times 1.356^2}{2 \times 9.81 \times 0.2}$$

$$\Rightarrow \quad 10.2 = 3 + y_c + 0.0937 + 4.5$$

$$\Rightarrow \quad y_c = 2.606 \text{ m}$$

$$\begin{aligned} \therefore \text{Maximum depth of pipe below the summit of ridge} \\ = 8 - 2.606 = 5.394 \text{ m} \end{aligned}$$

**Q.1 (b) Solution:**



Assume the section with discharge ( $Q$ ), depth of flow ( $y$ ) and top width ( $T$ ).

$$T = 2\sqrt{3}y$$

$$A = \frac{1}{2} \times 2\sqrt{3}y \times y = \sqrt{3}y^2$$

Depth of C.G from surface,  $\bar{z} = \frac{y}{3}$

Using momentum equation,

$$\frac{P_1 + M_1}{r} = \frac{P_2 + M_2}{r}$$

$$\Rightarrow A_1 \bar{z}_1 + \frac{Q^2}{gA_1} = A_2 \bar{z}_2 + \frac{Q^2}{gA_2}$$

$$\Rightarrow \sqrt{3}y_1^2 \times \frac{y_1}{3} + \frac{Q^2}{g(\sqrt{3}y_1^2)} = \sqrt{3}y_2^2 \times \frac{y_2}{3} + \frac{Q^2}{g(\sqrt{3}y_2^2)}$$

$$\Rightarrow \sqrt{3} \frac{y_1^3}{3} + \frac{Q^2}{g \times \sqrt{3}y_1^2} = \sqrt{3} \frac{y_2^3}{3} + \frac{Q^2}{g(\sqrt{3}y_2^2)}$$

$$\Rightarrow \sqrt{3} \frac{Q^2}{g} + \left[ \frac{1}{y_1^2} - \frac{1}{y_2^2} \right] = \frac{\sqrt{3}}{3} (y_2^3 - y_1^3)$$

$$\Rightarrow \frac{Q^2}{g} \left[ \frac{1}{0.5^2} - \frac{1}{1.5^2} \right] = (1.5^3 - 0.5^3)$$

$$\Rightarrow Q^2 \left( \frac{3.56}{g} \right) = 3.25$$

$$\Rightarrow Q = 2.993 \text{ m}^3/\text{s}$$

$$\therefore Fr_1^2 = \frac{Q^2 T_1}{gA_1^3} = \frac{(2.993)^2 \times (2\sqrt{3} \times 0.5)}{9.81 \times (\sqrt{3} \times 0.5^2)^3}$$

$$\Rightarrow Fr_1 = 4.41 > 1, \text{ i.e. supercritical flow.}$$

Now,

$$Fr_2^2 = \frac{Q^2 T_2}{gA_2^3} = \frac{(2.993)^2 \times (2\sqrt{3} \times 1.5)}{9.81 \times (\sqrt{3} \times 1.5^2)^3}$$

$$\Rightarrow Fr_2 = 0.283 < 1, \text{ i.e. subcritical flow.}$$

### Q.1 (c) Solution:

For moving vane:

Assuming jet velocity =  $V$

Nozzle area =  $a$  and vane velocity =  $U$

Relative velocity,  $V_r = V - U$

$$\begin{aligned}\text{Force in } x\text{-direction, } F_x &= \dot{m}(V_r \cos\theta + V_r) \\ &= \rho a [((V - U) \cos\theta) + V - U] (V - U) \\ &= \rho a (V - U)^2 (1 + \cos\theta)\end{aligned}$$

$$\text{Power, } P = F_x U = \rho a (V - U)^2 (1 + \cos\theta) U$$

$$\text{Kinetic energy of water, KE} = \frac{\rho a V^2}{2}$$

$$\begin{aligned}\text{Efficiency, } \eta &= \frac{\text{Work done by vane}}{\text{Kinetic energy of water}} \\ &= \frac{2\rho a (V - U)^2 (1 + \cos\theta) U}{\rho a V^3}\end{aligned}$$

For semicircular vane,  $\theta = 0^\circ$

$$\eta = \frac{4(V - U)^2 U}{V^3}$$

For maximum efficiency,  $\frac{\partial \eta}{\partial U} = 0$

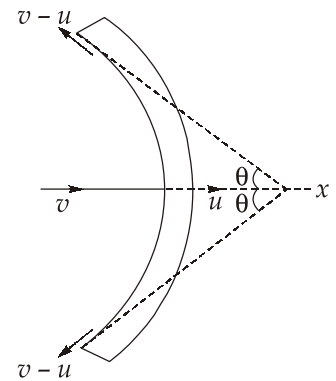
$$\Rightarrow 4(V - U)^2 + 8(V - U)(-U) = 0$$

$$\Rightarrow (V - U) = 2U \Rightarrow V = 3U$$

For  $V = U, \eta = 0$

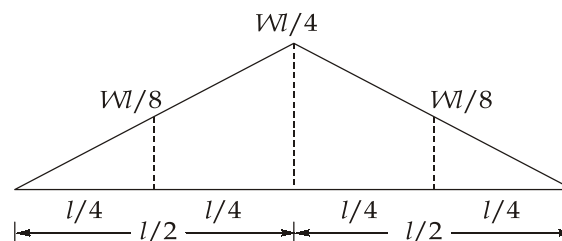
Therefore for maximum efficiency,  $V = 3U$

$$\text{So, Maximum efficiency, } \eta_{\max} = \frac{4(3U - U)^2 U}{(3U)^3} = 0.593 \simeq 59.3\%$$



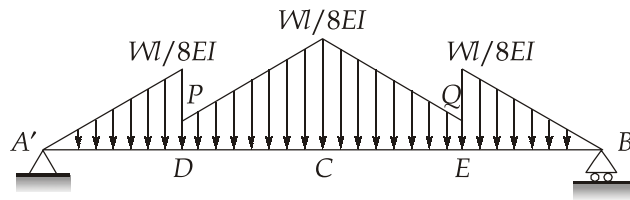
**Q.1 (d) Solution:**

**BMD for a prismatic beam**





Conjugate beam with  $\frac{M}{EI}$  loading and considering non-prismatic beam.



$$PD = QE = \frac{Wl}{16EI}$$

$$\text{Ordinate of loading at } D^- \text{ and } E^+ = \frac{Wl}{8EI}$$

$$\text{Ordinate of loading at } D^+ \text{ and } E^- = \frac{Wl}{16EI}$$

$$\text{Ordinate of loading at } C = \frac{Wl}{8EI}$$

$$\text{Consider conjugate beam, } R'_A = R'_B = \frac{\text{Total load}}{2}$$

$$\begin{aligned} \Rightarrow R'_A &= \frac{1}{2} \times \frac{Wl}{8EI} \times \frac{l}{4} + \frac{1}{2} \times \left[ \frac{Wl}{8EI} + \frac{Wl}{16EI} \right] \times \frac{l}{4} \\ &= \frac{Wl^2}{64EI} \times \frac{3Wl^2}{128EI} = \frac{10Wl^2}{256EI} = \frac{5Wl^2}{128EI} \end{aligned}$$

$$\therefore R'_A = \frac{5Wl^2}{128EI} = R'_B$$

$$\text{Slope at A in real beam} = \frac{5Wl^2}{128EI}$$

$$\text{And, slope at B in real beam} = -\frac{5Wl^2}{128EI}$$

$$\text{Central deflection, } y_c = M'_C$$

$$M'_C = \text{Moment at centre of conjugate beam}$$

$$\Rightarrow M'_C = R'_A \times \frac{l}{2} - \frac{Wl^2}{64EI} \left[ \frac{l}{4} + \frac{1}{3} \times \frac{l}{4} \right] - \frac{3}{128} \frac{Wl^2}{EI} \times \left[ \frac{l/4}{3} \left[ 2 \times \frac{Wl}{16EI} + \frac{Wl}{8EI} \right] \right]$$

$$\Rightarrow M'_C = \frac{5}{128} \frac{Wl^2}{EI} \times \frac{l}{2} - \frac{Wl^3}{192EI} - \frac{Wl^3}{384EI} = \frac{3Wl^3}{256EI}$$

## Q.1 (e) Solution:

$$\begin{aligned}
 \text{(i)} \quad A &= 300 \times 300 = 90000 \text{ mm}^2 \\
 A_{sc} &= 1257 \text{ mm}^2 \\
 A_c &= 90000 - 1257 = 88743 \text{ mm}^2 \\
 \text{Equivalent concrete area, } A_c &= A_c + mA_{sc} \\
 &= 88743 + (13.33 \times 1257) \\
 &= 105498.81 \text{ mm}^2
 \end{aligned}$$

$$\therefore \text{Stress in concrete, } f_c = \frac{W}{A_e} = \frac{440 \times 10^3}{105498.81}$$

$$\Rightarrow f_c = 4.17 \text{ MPa}$$

$$\text{Stress in steel, } f_{sc} = mf_c = 13.33 \times 4.17$$

$$\Rightarrow f_{sc} = 55.59 \text{ MPa}$$

$$\text{(ii) Maximum bending moment} = \frac{16.30 \times 6.25^2}{8} = 79.59 \text{ kNm}$$

Depth of critical neutral axis,

$$\frac{m\sigma_{cbc}}{\sigma_{st}} = \frac{x_c}{d - x_c}$$

$$\Rightarrow x_c = \left( \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} \right) d$$

$$\Rightarrow x_c = 0.2886 d$$

$$\text{Lever arm, } a = d - \frac{x_c}{3} = 0.9038 d$$

$$\therefore MR = \text{B.M} = \frac{1}{2} \times \sigma_{cbc} \times x_c \times B \times a$$

$$\Rightarrow 79.59 \times 10^6 = \frac{1}{2} \times 7 \times 0.2886 d \times 350 \times 0.9038 d$$

$$d = 499 \text{ mm}$$

Total compression = Total tension

$$\Rightarrow Bx_c \frac{\sigma_{cbc}}{2} = A_{st} \sigma_{st}$$

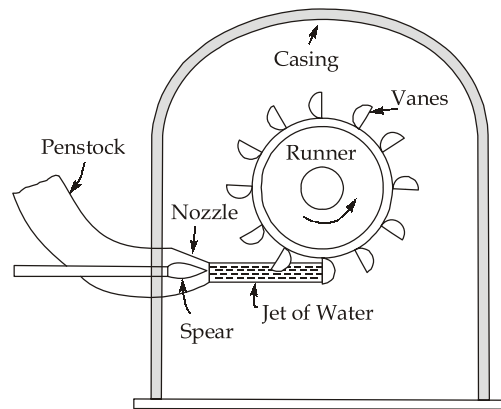
$$\Rightarrow 350 \times (0.2886 \times 499) \times \frac{7}{2} = A_{st} \times 230$$

$\Rightarrow$ 

$$A_{st} = 767 \text{ mm}^2$$

**Q.2 (a) Solution:**

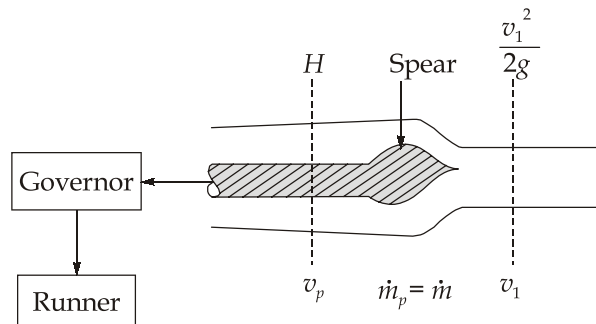
Pelton turbine is a tangential flow impulse turbines. It is high head low discharge turbine.



Pelton wheel turbine

**Components:**

1. **Casing:** No hydraulic function and is used to avoid the splashing of water.
2. **Nozzle and spear**



If  $\eta_{\text{nozzle}} = 100\%$ ,

$$\text{then } H = \frac{v_1^2}{2g},$$

If  $\eta_{\text{nozzle}} < 100\%$

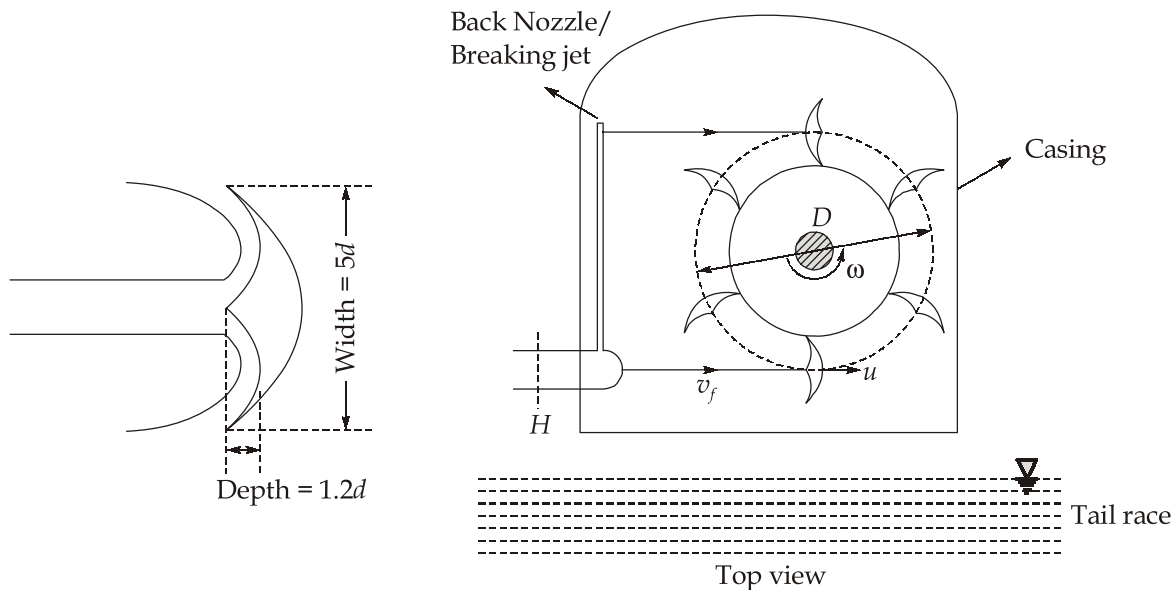
$$\text{then } \frac{v_1^2}{2g} < H$$

$$\Rightarrow v_1 = \sqrt{2gH},$$

$$\Rightarrow v_1 = C_V \sqrt{2gH} < \sqrt{2gH}$$

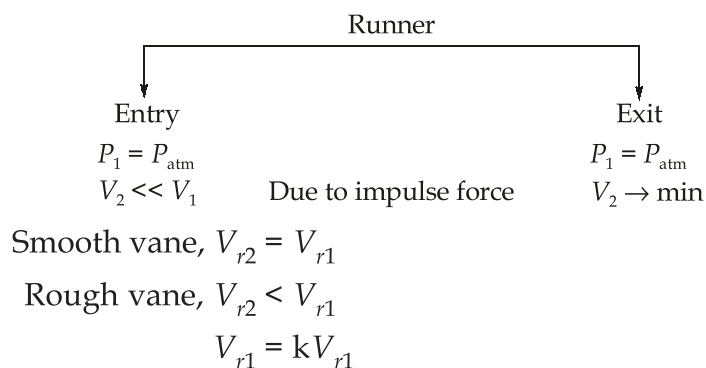
- To control the discharge through turbine a spear is provided, which can move forward and backward with the help of governor in order to increase or decrease the flow through the nozzle.
  - The process of maintaining constant speed irrespective of change in load is known as governing of turbine.
3. **Back nozzle/Breaking jet:** Jet helps to stop the runner as soon as possible.

#### 4. Runner: Double hemispherical vane/bucket.



#### Principle:

- The water is supplied by penstock from reservoir to turbine. At the exit of penstock, a nozzle is installed which is used to convert the head available with water fully into kinetic energy, therefore water exits from the nozzle in the form of jet. As the jet strikes over the runner, it will apply impulse force due to kinetic energy of water and rotates the runner. The impulse force produces the work and therefore, Pelton turbine is categorized as impulse turbine.
- In impulse turbine, only kinetic energy of water contributes into runner power.



$k$  = Coefficient of vane friction.

#### Q.2 (b) Solution:

For rectangular channel

Upstream depth,  $y_1 = 1.6 \text{ m}$

Downstream depth,  $y_2 = 0.4 \text{ m}$

By sudden closure of gate, a positive surge moving upstream will be created.

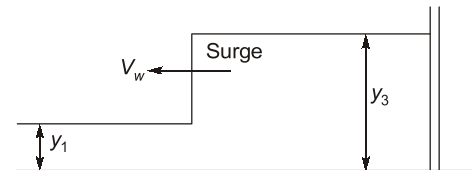
Assuming surge velocity as  $V_w$

Using equation of continuity

$$\begin{aligned} V_1 y_1 &= V_2 y_2 \\ \Rightarrow V_1 \times 1.6 &= V_2 \times 0.4 \\ \Rightarrow V_2 &= 4V_1 \end{aligned}$$

Applying Bernoulli's equation

$$\begin{aligned} y_1 + \frac{V_1^2}{2g} &= y_2 + \frac{V_2^2}{2g} \\ \Rightarrow 1.6 + \frac{V_1^2}{2g} &= 0.4 + \frac{(4V_1)^2}{2g} \\ \Rightarrow V_1 &= 1.25 \text{ m/s} \end{aligned}$$



**For surge,**

Applying equation of continuity,

$$y_1(V_1 + V_w) = y_3(V_w + V_3)$$

As flow is completely stopped,  $V_3 = 0$

$$\therefore 1.6(1.25 + V_w) = y_3 V_w \quad \dots(i)$$

Also applying momentum equation

$$\frac{1}{2}(y_3^2 - y_1^2) = \frac{1}{9.81} y_3 V_w V_1 \quad \dots(ii)$$

From eq. (i) and (ii),

$$\frac{1}{2}(y_3^2 - (1.6)^2) = \frac{1}{9.81} y_3 \left( \frac{2}{y_3 - 1.6} \right) \times 1.25$$

$$\Rightarrow y_3^3 - 1.6y_3^2 - 3.07 y_3 + 4.096 = 0$$

$$\therefore y_3 = 2.14 \text{ m}$$

$$\text{Height of surge} = y_3 - y_1 = 2.14 - 1.6 = 0.54 \text{ m}$$

**Q.2 (c) Solution:**

$$\begin{aligned} P_u &= 1.5 \times 1800 = 2700 \text{ kN} \\ M_{ux} &= 1.5 \times 160 = 240 \text{ kNm} \\ M_{uy} &= 1.5 \times 36 = 54 \text{ kNm} \\ L_{ox} &= 14000 - 600 = 13400 = 13.4 \text{ m} \\ L_{oy} &= 14000 - 900 = 13100 = 13.1 \text{ m} \end{aligned}$$

$$L_{xe} = 13.4 \times 0.65 = 8.71 \text{ m}$$

$$L_{ye} = 0.65 \times 13.1 = 8.515 \text{ m}$$

Slenderness ratio along  $x$ -axis,  $\lambda_x = \frac{L_{xe}}{D} = \frac{8710}{700} = 12.44 > 12$  i.e. long column

Slenderness ratio along  $y$ -axis,  $\lambda_y = \frac{L_{ye}}{B} = \frac{8515}{500} = 17.03 > 12$  i.e. long column.

Minimum moments,

$$(e_{\min})_{x-x} = \left( \frac{L_{ox}}{500} + \frac{D}{30} \right) \text{ or } 20 \text{ mm}$$

$$\Rightarrow (e_{\min})_{x-x} = 50.13 \text{ mm}$$

$$(e_{\min})_{y-y} = \left( \frac{L_{oy}}{500} + \frac{B}{30} \right) \text{ or } 20 \text{ mm} = 42.87 \text{ mm}$$

$$\begin{aligned} (Mu_{\min})_{x-x} &= P_u \times (e_{\min})_{x-x} \\ &= 2700 \times \frac{50.13}{100} \text{ kNm} \\ &= 135.351 \text{ kNm} < M_{ux} (= 240 \text{ kNm}) \end{aligned}$$

$$\begin{aligned} (Mu)_{\min, y-y} &= P_u \times (e_{\min})_{y-y} \\ &= 2700 \times \frac{42.87}{1000} \text{ kNm} \\ &= 115.75 \text{ kNm} > M_{uy} (= 54 \text{ kNm}) \end{aligned}$$

So,  $M_{uy} = 115.75 \text{ kNm}$

Additional moments due to long column,

$$\begin{aligned} M_{ax} &= \frac{P_u D}{2000} \left( \frac{L_{ex}}{D} \right)^2 \\ &= \frac{2700 \times 700}{2000} \times \left( \frac{8710}{700} \right)^2 \text{ Nm} = 146.31 \text{ kNm} \end{aligned}$$

$$M_{ay} = \frac{P_u B}{2000} \left( \frac{L_{ey}}{B} \right)^2$$

$$= \frac{2700 \times 500}{2000} \times \left( \frac{8515}{500} \right)^2 \text{ Nm} = 195.76 \text{ kNm}$$

$$(M_{ux} + M_{ax}) = 240 + 146.31 = 386.31 \text{ kNm}$$

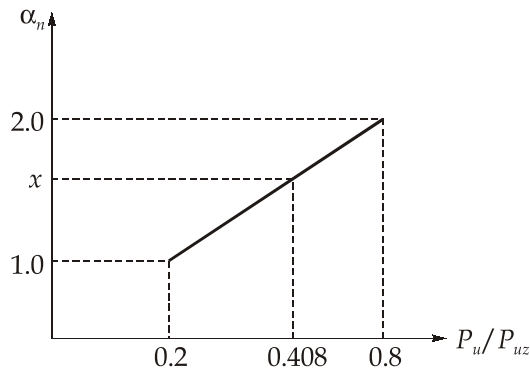
$$(M_{uy} + M_{ay}) = 115.75 + 195.76 = 311.50 \text{ kNm}$$

Let 1.5% steel is provided.

$$\therefore \text{Area of steel, } A_{sc} = \frac{1.5}{100} \times 500 \times 700 = 5250 \text{ mm}^2$$

$$\begin{aligned} P_{uz} &= 0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc} \\ &= 0.45 \times 30 \times (500 \times 700 - 5250) + 0.75 \times 500 \times 5250 \text{ N} \\ &= 6622.875 \text{ kN} \end{aligned}$$

$$\frac{P_u}{P_{uz}} = \frac{2700}{6622.875} = 0.408$$



For

$$\frac{P_u}{P_{uz}} = 0.408$$

$$\frac{2-1}{0.8-0.2} = \frac{2-x}{0.8-0.408}$$

$\Rightarrow$

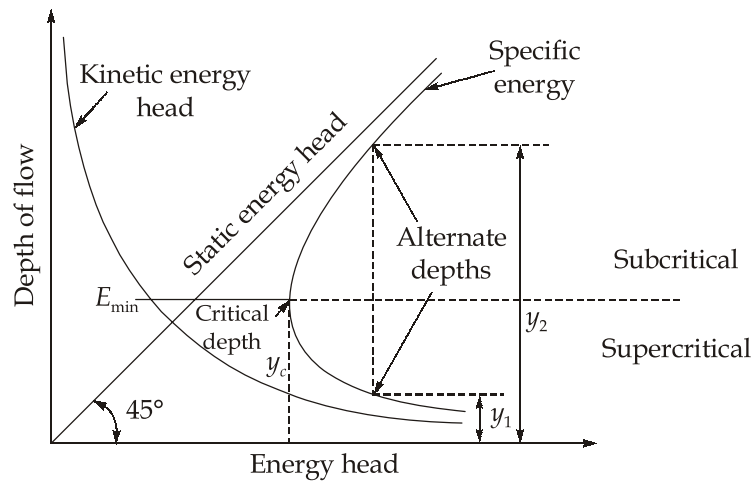
$$x = 1.347$$

$$\left( \frac{M_{ux} + M_{ax}}{M_{ux1}} \right)^{\alpha_n} + \left( \frac{M_{uy} + M_{ay}}{M_{uy1}} \right)^{\alpha_n} < 1.0$$

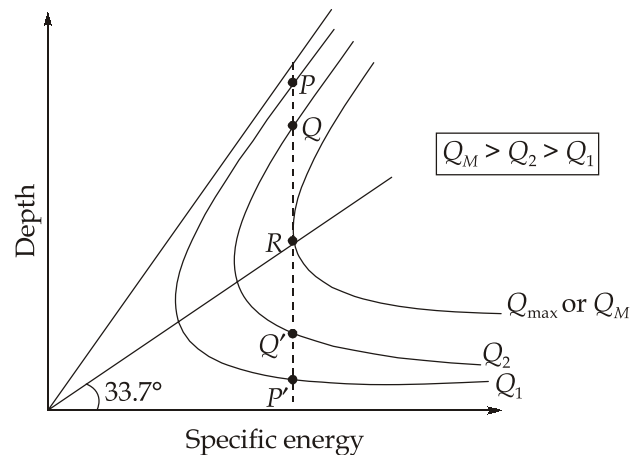
$$\left( \frac{386.31}{735} \right)^{1.345} + \left( \frac{311.5}{472.5} \right)^{1.345} = 0.991 < 1.0 \quad (\text{O.K.})$$

## Q.3 (a) Solution:

(i)



Specific energy depth relationship  
(Constant discharge)

**Observations:**

1. As discharge increases, kinetic head and critical depth increases due to which specific energy curve shifts right upward.
2. As the discharge increases, difference between alternate depths reduces.

(ii) Given:

$$y_1 = 1.5 \text{ m}$$

$$y_2 = 2.5 \text{ m}$$

$$B = 10 \text{ m}$$

$$S_0 = \frac{1}{10000}$$

$$n = 0.02$$



$$V_1 = \frac{1}{n} R_1^{2/3} S_0^{1/2}$$

$$\Rightarrow V_1 = \frac{1}{0.02} \times \left( \frac{B y_1}{B + 2 y_1} \right)^{2/3} (S_0)^{1/2}$$

$$\Rightarrow V_1 = \frac{1}{0.02} \times \left( \frac{10 \times 1.5}{10 + 2 \times 1.5} \right)^{2/3} \times \left( \frac{1}{10000} \right)^{1/2} = 0.55 \text{ m/s}$$

From continuity equation,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} = \frac{15 \times 0.55}{25} = 0.33 \text{ m/s}$$

$$\text{Length of back water curve, } \Delta x = \frac{E_2 - E_1}{S_0 - \bar{S}_f}$$

$$\text{Average flow of depth, } y = \frac{y_1 + y_2}{2}$$

$$\Rightarrow y = \frac{1.5 + 2.5}{2} = 2 \text{ m}$$

$$Q = \frac{A}{n} \times (R)^{2/3} \times (\bar{S}_f)^{1/2}$$

$$\Rightarrow A_1 V_1 = \frac{B y}{n} \left( \frac{B y}{B + 2 y} \right)^{2/3} (\bar{S}_f)^{1/2}$$

$$\Rightarrow 15 \times 0.55 = \frac{10 \times 2}{0.02} \left( \frac{10 \times 2}{10 + 2 \times 2} \right)^{2/3} \times (\bar{S}_f)^{1/2}$$

$$\Rightarrow \bar{S}_f = 4.23 \times 10^{-5}$$

$$\therefore E_1 = y_1 + \frac{V_1^2}{2g} = 1.5 + \frac{0.55^2}{2 \times 9.81}$$

$$\Rightarrow E_1 = 1.515 \text{ m}$$

$$\text{Now, } E_2 = y_2 + \frac{V_2^2}{2g} = 2.5 + \frac{0.33^2}{2 \times 9.81}$$

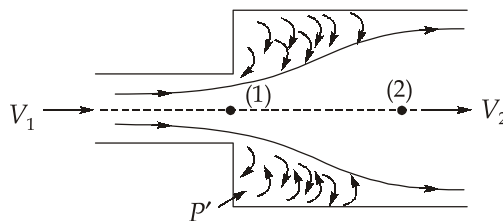
$$\Rightarrow E_2 = 2.506 \text{ m}$$

$$\begin{aligned}
 \therefore \Delta x &= \frac{E_2 - E_1}{S_0 - \bar{S}_f} \\
 &= \frac{2.506 - 1.515}{0.0001 - 0.0000423} \\
 &= 17175 \text{ m} = 17.175 \text{ km}
 \end{aligned}$$

**Q.3 (b) Solution:**

(i) Minor head losses:

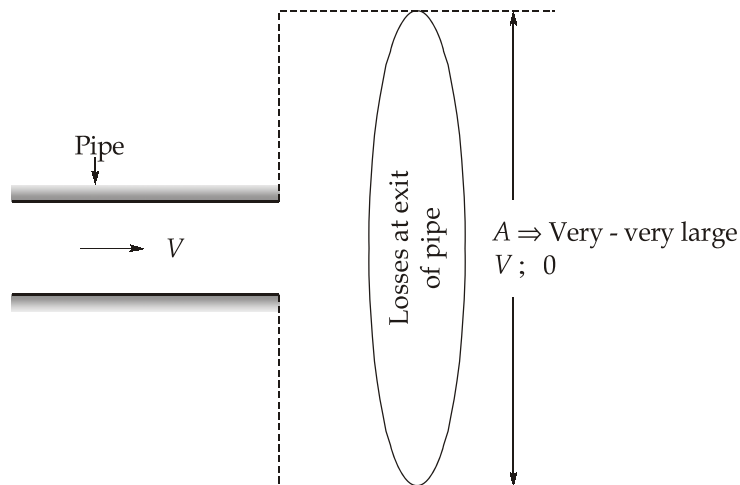
1. Due to sudden expansion:



$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$P' = P_1$$

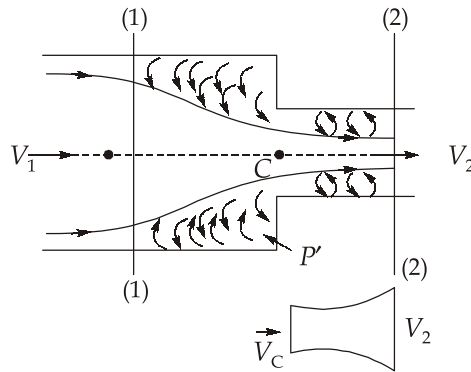
2. At the exit of pipe:



$$\frac{V^2}{2g} = \text{Minor head loss in pipe flow}$$

$$\Rightarrow h_L = \frac{(V-0)^2}{2g} = \frac{V^2}{2g}$$

### 3. Due to sudden contraction:



Here, practically section is (1) to (2) but 1 to c values are very small so we neglect these and considering only part C to 2.

$$h_L = \frac{(V_C - V_2)^2}{2g}$$

Apply continuity:

$$A_C V_C = A_2 V_2$$

$$\Rightarrow (C_C A_2) V_C = A_2 V_2$$

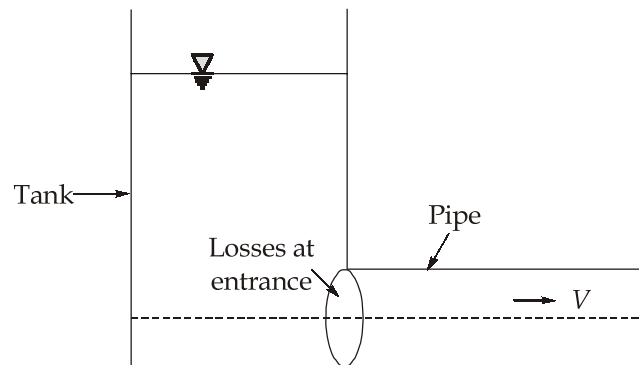
$$\Rightarrow V_C = \frac{V_2}{C_C}$$

$$\text{Now, } h_L = \frac{\left[ \frac{V_2}{C_C} - V_2 \right]^2}{2g} = \left[ \frac{1}{C_C} - 1 \right]^2 \times \frac{V_2^2}{2g}$$

If  $C_C$  is not given then

$$\left[ \frac{1}{C_C} - 1 \right]^2 = 0.5$$

$$\therefore h_L = 0.5 \frac{V_2^2}{2g}$$

**4. At the entrance of pipe:**

$$h_L = 0.5 \frac{V^2}{2g}$$

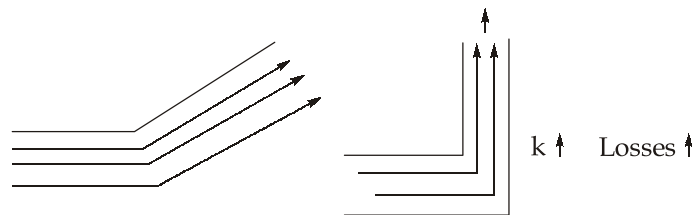
**5. Due to pipe bend:**

$$h = k \times \frac{V^2}{2g}$$

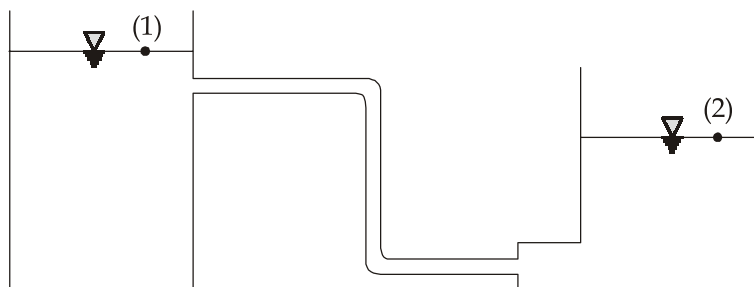
where,  $k$  = Coefficient of pipe bend

**Effects:**

Bend angle: Bend angle  $\uparrow$ , Losses  $\uparrow$ , because collision between particles  $\uparrow$  increase.



In a typical system with long pipes, these losses are minor compared to the head loss in the straight sections (the major losses i.e. due to friction) and are called minor losses. Although this is generally true that in some cases, the minor losses may be greater than the major losses. For example: In systems with several turns and valves in a short distance. Then, these minor losses cannot be neglected.

**(ii)**

Applying Bernoulli's equation between (1) and (2),

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} - Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} - Z_2 + h_L + h_f$$

Here,  $\frac{P_1}{\rho g} = \frac{P_2}{\rho g}$  and  $v_1 = v_2 = 0$

$$Z_1 = Z_2 + h_L + h_f \quad \dots(i)$$

$$\therefore h_L = k_{L1} \frac{v^2}{2g} + \left( k_{L2} \cdot \frac{v^2}{2g} \right) \times 2 + \frac{(v - v')^2}{2g} + k_{L3} \cdot \frac{v^2}{2g}$$

Velocity in pipe,  $v = \frac{Q}{A} = \frac{6 \times 10^{-3} \text{ m}^3/\text{s}}{\frac{\pi}{4} (0.05)^2 \text{ m}^2}$

$$v = 3.055 \text{ m/s}$$

Velocity in expanded pipe,

$$v' = \frac{Q}{A'} = \frac{6 \times 10^{-3}}{\frac{\pi}{4} (0.15)^2}$$

$$\Rightarrow v' = 0.34 \text{ m/s}$$

$$h_L = (0.5 + (2 \times 0.3) + 1.06) \times \frac{(3.056)^2}{2 \times 9.81} + \frac{(3.056 - 0.34)^2}{2 \times 9.81}$$

$$\Rightarrow h_L = 1.0282 + 0.376 = 1.404 \text{ m}$$

$$h_f = h_{f1} + h_{f2}$$

$$\Rightarrow h_f = \frac{fL_1 v^2}{2gD_1} + \frac{fL_2 (v)^2}{2gD_2}$$

$$\Rightarrow h_f = \frac{0.0315 \times (70 + 9) \times 3.056^2}{2 \times 9.81 \times 0.05} + \frac{0.0315 \times 10 \times 0.34^2}{2 \times 9.81 \times 0.15}$$

$$\Rightarrow h_f = 23.691 + 0.0124$$

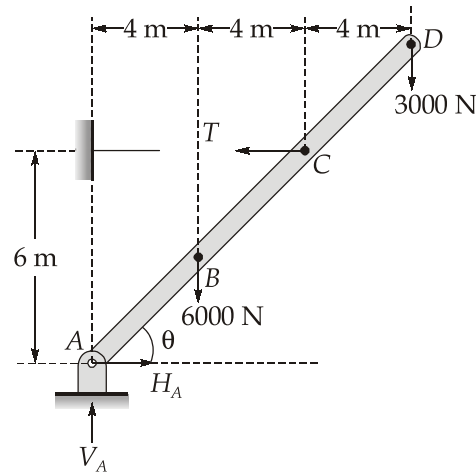
$$\Rightarrow h_f = 23.7034 \text{ m}$$

From equation (1),

$$Z_1 = Z_2 + h_L + h_f$$

$$\Rightarrow Z_1 = 4 + 1.404 + 23.7037$$

$$\Rightarrow Z_1 = 29.1074 \text{ m}$$

**Q.3 (c) Solution:**

$$\cos\theta = \frac{4}{5}$$

$$\therefore \sin\theta = \frac{3}{5}$$

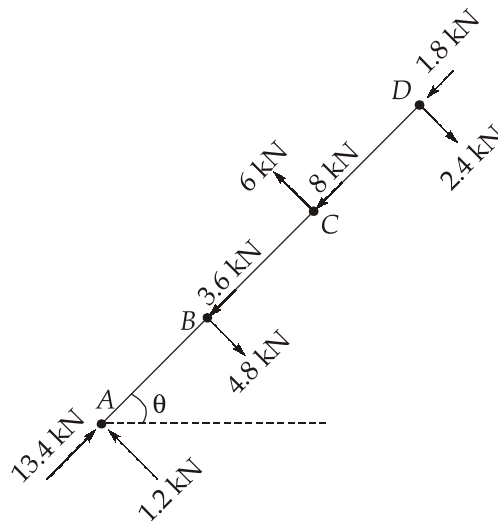
$$\tan\theta = \frac{3}{4}$$

$$\Sigma M_A = 0 \Rightarrow 6000 \times 4 + 3000 \times 12 = T \times 6$$

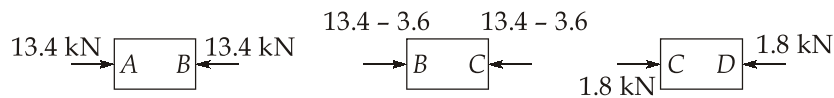
$$\Rightarrow T = 10000 \text{ N} = 10 \text{ kN}$$

$$\therefore H_A = 10 \text{ kN}, V_A = 9 \text{ kN}$$

Calculation of external forces along and tangential to sections A, B, C, D (resolving forces)



Let us divide the beam into various segments and analyze them for axial forces.

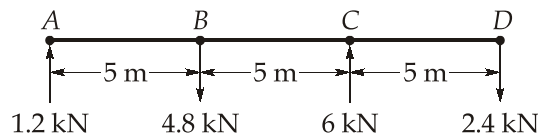


$$\begin{aligned}
 \text{(i)} \quad A(\sigma_{AB}) &= -\frac{P}{A} - \frac{M}{Z} = -\frac{13400}{180 \times 300} - \frac{6 \times 10^6}{180 \times 300^2} \\
 &= -0.248 - 2.222 \\
 &= -2.47 \text{ MPa (compression)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad A(\sigma_{BC}) &= -\frac{P}{A} - \frac{M}{Z} = -\frac{9800}{180 \times 300} - \frac{12 \times 10^6}{180 \times 300^2} \\
 &= -0.181 - 4.444 \\
 &= -4.625 \text{ MPa (compression)}
 \end{aligned}$$

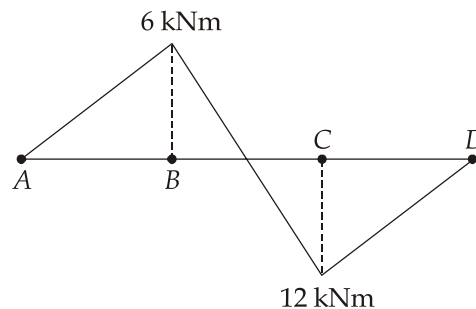
Largest stress value = max. (2.47, 4.625) = 4.625 MPa

BMD for the beam due to transverse loading



$$\begin{aligned}
 BM_A = 0 \quad \Rightarrow \quad BM_B &= 1.2 \times 5 = 6 \text{ kNm (Sagging)} \\
 BM_C &= 1.2 \times 10 - 4.8 \times 5 = 12 \text{ kNm (Hogging)}
 \end{aligned}$$

$$BM_D = 0$$



Possible cases of largest stress in the beam,

- (i) At section B,  $P = 13.4$  (Comp.)  
 $M = 6 \text{ kNm}$  (Sagging)
- (ii) At section C,  $P = 9.8$  (Comp.)  
 $M = 12 \text{ kN}$  (Sagging)

**Q.4 (a) Solution:**

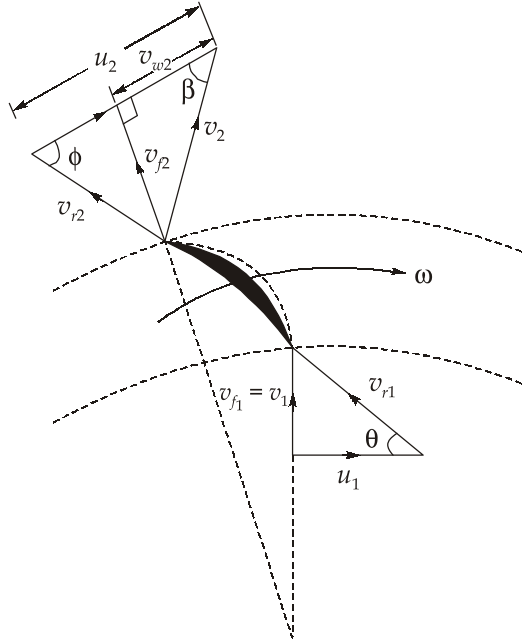
Given: For centrifugal pump,

Outside diameter,  $d_2 = 1.2$  m, Inside diameter,  $d_1 = 60$  cm = 0.6 m, Discharge,

$Q = 1800$  L/s =  $1.8$  m<sup>3</sup>/s, Manometric head,  $H = 6$  m, Speed,  $N = 200$  rpm,

Velocity of flow,  $v_{f1} = v_{f2} = 2.5$  m/s

For minimum resistance,  $v_{w1} = 0$



(i) Water velocity at outlet,

$$u_2 = \omega r_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.566 \text{ m/s}$$

From outlet velocity triangle,  $\tan \phi = \frac{v_{f2}}{u_2 - v_{w2}}$

$$\Rightarrow \tan(180^\circ - 150^\circ) = \frac{2.5}{12.566 - v_{w2}}$$

$$\Rightarrow 12.566 - v_{w2} = \frac{2.5}{\tan 30^\circ}$$

$$\Rightarrow v_{w2} = 8.236 \text{ m/s}$$

$$\therefore \text{Water velocity at outlet, } v_2 = \sqrt{v_{w2}^2 + v_{f2}^2} = \sqrt{(8.236)^2 + (2.5)^2} \\ = 8.607 \text{ m/s}$$

$$\text{Water velocity at inlet, } v_1 = v_{f1} = 2.5 \text{ m/s}$$



(ii) Manometric efficiency of pump,

$$\eta = \frac{gH_m}{v_{w2} \times u_2} \times 100 = \frac{9.81 \times 6}{8.236 \times 12.566} \times 100$$

$$\Rightarrow \eta = 56.87\%$$

(iii) For minimum starting speed, N

$$\left( \frac{\pi \times d_2 N}{60} \right)^2 - \left( \frac{\pi \times d_1 N}{60} \right)^2 = 2gH$$

$$\Rightarrow \left( \frac{\pi N}{60} \right)^2 (d_2^2 - d_1^2) = 2gH$$

$$\Rightarrow \left( \frac{\pi N}{60} \right)^2 = \frac{2 \times 9.81 \times 6}{(1.2^2 - 0.6^2)}$$

$$\Rightarrow N = \frac{60}{\pi} \sqrt{109} = 199.395 \text{ rpm} \simeq 199.4 \text{ rpm}$$

(iv) Specific speed of pump,  $N_s$

$$N_s = \frac{N \sqrt{Q}}{H^{3/4}} = \frac{200 \sqrt{1.8}}{6^{3/4}} (\text{SI units})$$

$$\Rightarrow N_s = 69.993 \simeq 70 (\text{SI units})$$

(v) Width of vane at outlet,  $Q = A_{f2} v_{f2} = (\pi d_2 b_2) v_{f2}$

$$\Rightarrow 1.8 = (\pi 1.2 b_2) 2.5$$

$$\Rightarrow b_2 = 0.191 \text{ m}$$

Width of vane at inlet,  $Q = A_{f1} v_{f1} = (\pi d_1 b_1) v_{f1}$

$$\Rightarrow 1.8 = (\pi 0.6 b_1) 2.5$$

$$\Rightarrow b_1 = 0.382 \text{ m}$$

#### Q.4 (b) Solution:

(i) The magnitude of the rise in pressure is deduced by considering the energy changes of the system.

$$\text{Kinetic energy of the moving fluid} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} (\rho A l) v^2$$

Strain energy stored in water (after closure of valve)

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times P \times \frac{P}{K} \times (Al) = \frac{P^2}{2K} (Al)$$

Where  $K$  is the bulk modulus of the liquid. When the whole column of fluid has been instantaneously, stopped (quick closure of the valve), kinetic energy will be stored uniformly throughout the length of fluid as strain energy if the pipe is assumed to be perfectly rigid.

$$\therefore \frac{1}{2}(\rho Al)v^2 = \frac{P^2}{2K}(Al)$$

$$\Rightarrow P = v\sqrt{k\rho} = v\sqrt{\frac{K}{\rho}}\rho^2$$

$$\Rightarrow \frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{K}{\rho}}$$

- (ii) For sudden closure of the valve provided in an elastic pipe, the pressure rise is given by:

$$\frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{K}{\rho\left(1+\frac{DK}{Et}\right)}}$$

$$\text{Flow velocity, } v = \frac{Q}{A} = \frac{1.75}{\frac{\pi}{4}(0.8)^2}$$

$$\Rightarrow v = 3.48 \text{ m/s}$$

$$\therefore \frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{k}{\rho\left(1+\frac{Dk}{Et}\right)}}$$

$$= \frac{3.48}{9.81}\sqrt{\frac{2 \times 10^9}{10^3 \times \left(1 + \frac{0.8 \times 2 \times 10^9}{2 \times 10^{11} \times 0.01}\right)}}$$

$$\frac{P}{\rho g} = 373.93$$

$$P = 3.668 \times 10^6 \text{ N/m}^2 = 3.668 \text{ MPa}$$

Longitudinal stress,  $f_1 = \frac{Pd}{4t} = \frac{3.668 \times 0.8}{4 \times 0.1} = 73.36 \text{ MPa}$

Circumferential stress,  $f_2 = \frac{Pd}{2t} = \frac{3.668 \times 0.8}{2 \times 0.01} = 146.72 \text{ MPa}$

#### Q.4 (c) Solution:

Effective width of isolated T-section,

$$B_F = \frac{l_0}{\frac{l_0}{b} + 4} + B_w$$

$$\Rightarrow B_F = \frac{5000}{\frac{5000}{1500} + 4} + 250 = 931.8 \simeq 932 \text{ mm}$$

Let us assume that neutral axis lies in the flange and beam section is an under-reinforced section.

$$T = C_1 + C_2$$

$$\Rightarrow 0.87f_y \cdot A_{st} = 0.36f_{ck}B_fx_u + (f_{sc} - 0.45f_{ck})A_{sc}$$

$$\Rightarrow 0.87 \times 415 \times 2700 = 0.36 \times 25 \times 932 \times x_u + (f_{sc} - 0.45 \times 25) \times 1000 \quad \text{..(i)}$$

Let  $f_{sc} = 353 \text{ MPa}$

$$\therefore 974835 = 8388x_u + (353 - 11.25) \times 1000$$

$$\Rightarrow x_u = 75.47 \simeq 76 \text{ mm}$$

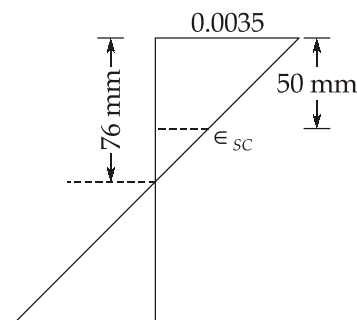
$$\therefore x_u < D_f (= 120 \text{ mm})$$

$\therefore$  Our assumption is correct and NA lies in flange.

For F415,  $(x_u)_{\lim} = 0.48d = 0.48 \times 450 = 216 \text{ mm}$

Since,  $x_u < (x_u)_{\lim}$

$\therefore$  Beam section is under-reinforced.



$$\frac{\epsilon_{sc}}{(76 - 50)} = \frac{0.0035}{76}$$

$$\Rightarrow \epsilon_{sc} = 0.00120$$

Since, 0.00120 strain is less than  $\left(\frac{0.695 f_y}{E_s} \text{ i.e. } 0.00144\right)$

Stress can be calculated directly,  $f_{sc} = \epsilon_{sc} \times E_s$   
 $= 0.00120 \times (2 \times 10^5) = 240 \text{ MPa}$

By putting the value of  $f_{sc}$  in eq. (i),

$$x_u = 88.95 \simeq 89 \text{ mm}$$

Assuming  $x_u = 82.5 \text{ mm}$

$$\epsilon_{sc} = \frac{0.0035}{82.5} \times (82.5 - 50)$$

$$\Rightarrow \epsilon_{sc} = 0.00138$$

Corresponding stress,  $f_{sc} = \epsilon_{sc} \times E_s$

$$\Rightarrow f_{sc} = 0.00138 \times (2 \times 10^5)$$

$$\Rightarrow f_{sc} = 276 \text{ MPa}$$

Moment of resistance,

$$\text{MOR} = 0.36 f_{ck} B_f x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$$

$$\Rightarrow \text{MOR} = 0.36 \times 25 \times 932 \times 82.5 \times (450 - 0.42 \times 82.5) + (276 - 0.45 \times 25) \times 1000 \times (450 - 50)$$

$$\Rightarrow \text{MOR} = 393.33 \text{ kNm}$$

### Section B

#### Q.5 (a) Solution:

Shear stress, 
$$\tau = -\frac{1}{2} \left( \frac{\partial P}{\partial x} \right) (H - 2y)$$

For shear stress at plates, 
$$\tau = -\frac{1}{2} \frac{\partial P}{\partial x} H$$

Pressure gradient, 
$$\left( -\frac{\partial P}{\partial x} \right) = \frac{12 \mu \bar{v}}{H^2}$$

Average velocity, 
$$\bar{v} = \frac{u_{\max}}{1.5}$$

$$\Rightarrow \quad \bar{v} = \frac{1.5}{1.5} = 1 \text{ m/s}$$

$$\therefore \quad \left(-\frac{\partial P}{\partial x}\right) = \frac{12 \times 2 \times 1}{(0.08)^2} = 3750 \text{ N/m}^2/\text{m}$$

$$\text{Shear stress at plates,} \quad \tau = -\frac{1}{2} \left(\frac{\partial P}{\partial x}\right) H$$

$$\Rightarrow \quad \tau = -\frac{1}{2} (-3750) \times (0.08) = 150 \text{ N/m}^2$$

$$\text{Pressure difference,} \quad P_1 - P_2 = \frac{12\mu\bar{v}L}{H^2}$$

$$\Rightarrow \quad P_1 - P_2 = \frac{12 \times 2 \times 1 \times 25}{(0.08)^2} = 93750 \text{ N/m}^2$$

$$\Rightarrow \quad P_1 - P_2 = 93.75 \text{ kN/m}^2 = 93.75 \text{ kPa}$$

Velocity at 2 cm from the plate,

$$u = \frac{1}{2\mu} \left(\frac{-\partial P}{\partial x}\right) (Hy - y^2)$$

$$\Rightarrow \quad u = \frac{1}{2 \times 2} (3750) (0.08 \times 0.02 - 0.02^2)$$

$$\Rightarrow \quad u = 1.125 \text{ m/s}$$

#### Q.5 (b) Solution:

$$\text{Discharge,} \quad Q = 20 \text{ m}^3/\text{s}$$

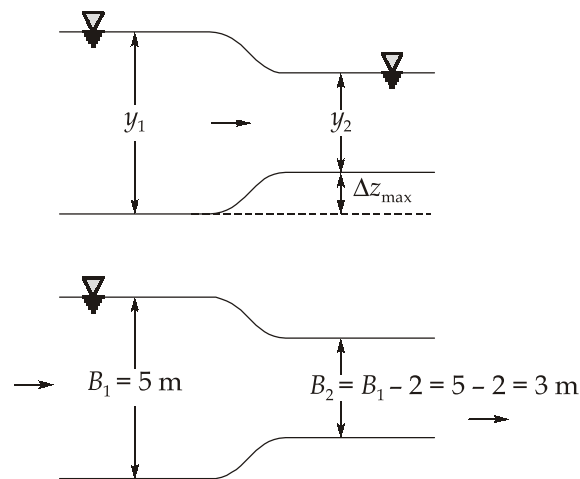
$$\text{Upstream section:} \quad y_1 = 3 \text{ m and } B_1 = 5 \text{ m}$$

$$\text{Velocity,} \quad v_1 = \frac{Q}{B_1 y_1} = \frac{20}{3 \times 5} = 1.33 \text{ m/s}$$

$$\text{Froude number,} \quad Fr_1 = \frac{v_1}{\sqrt{g y_1}} = \frac{1.33}{\sqrt{9.81 \times 3}}$$

$$Fr_1 = 0.245 < 1$$

The upstream flow is subcritical. The water surface will drop down at the contracted section.



For  $\Delta z$  to be maximum without altering the upstream depth, flow must be critical at the contracted section.

$$q_2 = \frac{Q}{B_2} = \frac{20}{3} = 6.67 \text{ m}^3/\text{s}/\text{m}$$

But 
$$y_2 = y_{c2} = \left( \frac{q_2^2}{g} \right)^{1/3} = \left( \frac{6.67^2}{9.81} \right)^{1/3} = 1.655 \text{ m}$$

$\therefore E_1 = E_2 + \Delta z_{\max}$

$\Rightarrow y_1 + \frac{V_1^2}{2g} = \frac{3}{2} y_{c2} + \Delta z_{\max}$

$\Rightarrow = 3 + \frac{1.33^2}{2 \times 9.81} = \frac{3}{2} \times 1.655 + \Delta z_{\max}$

$\Rightarrow \Delta z_{\max} = 0.608 \text{ m}$

### Q.5 (c) Solution:

The draft-tube is a pipe of gradually increasing area which connects the outlets of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race. One end of the draft-tube is connected to the outlet of the runner while the other end is sub-merged below the level of water in the tail race. The draft-tube, in addition to serve a passage for water discharge, has the following two purposes also:

1. It permits a negative head to be established at the outlet of the runner and thereby increases the net head on the turbine. The turbine may be placed above the tail race without any loss of net head and hence turbine may be inspected properly.
2. It converts a large proportion of the kinetic energy ( $V_2^2/2g$ ) rejected at the outlet of the turbine into useful pressure energy. Without the draft tube, the kinetic energy rejected at the outlet of the turbine will go waste to the tail race.

Hence by using draft-tube, the net head on the turbine increases. The turbine develops more power and also the efficiency of the turbine increases.

If a reaction turbine is not fitted with a draft-tube, the pressure at the outlet of the runner will be equal to atmospheric pressure. The water from the outlet of the runner will discharge freely into the tail race. The net head on the turbine will be less than that of a reaction turbine fitted with a draft-tube. Also without a draft-tube, the kinetic

energy  $\left( \frac{V_2^2}{2g} \right)$  rejected at the outlet of the runner will go waste to the tail race.

Consider a capital draft-tube as shown in figure.

Let  $H_s$  = Vertical height of draft-tube above the tail race,  
 $y$  = Distance of bottom of draft-tube from tail race.

Applying Bernoulli's equation to inlet (section 1-1) and outlet (section 2-2) of the draft-tube and taking section 2-2 as the datum line, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f \quad \dots(i)$$

where  $h_f$  = loss of energy between sections 1-1 and 2-2.

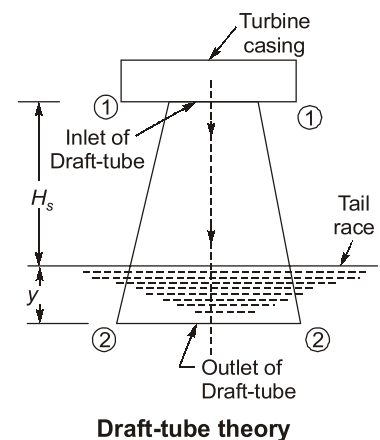
$$\Rightarrow \frac{p_2}{\rho g} = \text{Atmospheric pressure head} + y \\ = \frac{p_a}{\rho g} + y.$$

Substituting this value of  $\frac{p_2}{\rho g}$  in equation (i), we get

$$\Rightarrow \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{p_a}{\rho g} + y + \frac{V_2^2}{2g} + h_f$$

$$\Rightarrow \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{p_a}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$\therefore \frac{p_1}{\rho g} = \frac{p_a}{\rho g} + \frac{V_2^2}{2g} + h_f - \frac{V_1^2}{2g} - H_s \\ = \frac{p_a}{\rho g} - H_s - \left( \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f \right) \quad \dots(ii)$$



In equation (ii),  $\frac{p_1}{\rho g}$  is less than atmospheric pressure.

**Q.5 (d) Solution:**

Given:  $r_1 = \frac{d_1}{2} = \frac{8}{2} = 4 \text{ cm}$ ,  $r_2 = \frac{d_2}{2} = \frac{12}{2} = 6 \text{ cm}$ ,  $p_1 = 120 \text{ MPa}$ ,  $p_2 = 50 \text{ MPa}$

$$\sigma_{\theta} = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2} + \frac{r_1^2 r_2^2 (p_1 - p_2)}{r^2 (r_2^2 - r_1^2)}$$

$$\Rightarrow \sigma_{\theta} = \frac{120 \times 16 - 50 \times 36}{36 - 16} + \frac{16 \times 36 (120 - 50)}{r^2 (36 - 16)} = 6 + \frac{2016}{r^2}$$

At  $r = 4 \text{ cm}$ ,

$$\sigma_{\theta} = 6 + \frac{2016}{4^2} = 132 \text{ MPa}$$

At  $r = 6 \text{ cm}$ ,

$$\sigma_{\theta} = 6 + \frac{2016}{6^2} = 62 \text{ MPa}$$

At the mean radius,

$$r = \frac{4 + 6}{2} = 5 \text{ cm}$$

$$\sigma_{\theta} = 6 + \frac{2016}{5^2} = 86.64 \text{ MPa}$$

$$\text{Radial stress, } \sigma_r = 6 - \frac{2016}{25} = -74.64 \text{ MPa}$$

**Q.5 (e) Solution:**

Let the diameter of the column be  $D$ .

$$\text{Gross sectional area of the column} = \frac{\pi D^2}{4}$$

$$\text{Providing 2\% steel, } A_{sc} = 0.02 A_g$$

$$\text{Area of concrete, } A_c = 0.02 A_g = 0.98 A_g$$

$$\text{Ultimate load, } P_u = 1.5 \times P = 1.5 \times 1500 = 2250 \text{ kN}$$

Assuming column to be short and the minimum eccentricity does not exceed  $0.05D$ .

$$\therefore P_u = (0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \times 1.05$$



$$\Rightarrow \frac{2250 \times 10^3}{1.05} = 0.4 \times 20 \times 0.98 A_g + 0.67 \times 415 \times 0.02 A_g$$

$$\Rightarrow \frac{2250 \times 10^3}{1.05} = 13.401 A_g$$

$$\Rightarrow A_g = 159902.78 \text{ mm}^2$$

$$\Rightarrow \frac{\pi}{4} D^2 = 159902.78$$

$$\Rightarrow D = 451.2 \text{ mm}$$

Provide circular column of diameter of 450 mm.

$$\text{Slenderness ratio} = \frac{l_{eff}}{D} = \frac{2500}{450} = 5.6 < 12$$

Hence, short column (O.K)

$$\text{Minimum eccentricity, } e_{\min} = \max. \left\{ \begin{array}{l} \frac{l_0}{500} + \frac{D}{30} \\ 20 \text{ mm} \end{array} \right.$$

$$\Rightarrow e_{\min} = \max. \left\{ \begin{array}{l} \frac{2500}{500} + \frac{450}{30} = 5 + 15 = 20 \\ 20 \text{ mm} \end{array} \right.$$

$$\Rightarrow e_{\min} = 20 \text{ mm}$$

$$\text{Also, } 0.05D = 0.05 \times 450 = 22.5 \text{ mm}$$

$$\therefore e_{\min} < 0.05D \quad (\text{O.K})$$

$\therefore$  Column is axially loaded short column.

$$\text{Gross area of column, } A_g = \frac{\pi}{4} \times 450^2 = 159043.13 \text{ mm}^2$$

$$\text{Area of concrete, } A_c = A_g - A_{sc} = 159043.13 - A_{sc}$$

$$\therefore P_u = 2250 \times 10^3 = P_u = 1.05[0.4 \times 20 \times (159043.13 - A_{sc}) + 0.67 \times 415 \times A_{sc}]$$

$$\Rightarrow A_{sc} = 3223.5 \text{ mm}^2$$

So, provide 8 bars of 25 mm dia (3927.2 mm<sup>2</sup>)

Providing 8 mm $\phi$  helical bars at a pitch of 45 mm with 40 mm cover.

$$\text{Diameter of the core} = 450 - (2 \times 40) + (2 \times 8)$$

$$D_c = 386 \text{ mm}$$

$$\therefore A_c = \frac{\pi}{4} (D_c)^2 = 117021.18 \text{ mm}^2$$

Volume of helix in 1 m length,

$$V_h = \frac{1000}{p} (\pi d_h) \left( \frac{\pi}{4} \phi_h^2 \right) \quad (\because d_h = 386 - 16 = 370 \text{ mm})$$

$$\Rightarrow V_h = \frac{58428058.05}{p}$$

$$\begin{aligned} \text{Volume of core in 1 m length, } V_c &= A_c \times 1000 \\ &= 117021180 \text{ mm}^3 \end{aligned}$$

$$\therefore \frac{58428058.05}{p} \leq \frac{V_h}{V_c}$$

$$\Rightarrow 0.36 \times \frac{20}{415} \left( \frac{159043.12}{117021.18} - 1 \right) \leq \frac{58428058.05}{p \times 117021180}$$

$$\Rightarrow p \leq 80.14 \text{ mm}$$

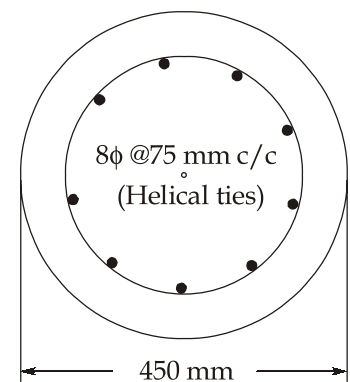
$$\text{Pitch of helical reinforcement, } p \leq 75 \text{ mm}$$

$$\leq \frac{D_c}{6} = 64.33 \text{ mm}$$

$$\geq 25 \text{ mm}$$

$$\geq 3\phi_h = 24 \text{ mm}$$

Hence, pitch of helical reinforcement = 75 mm



#### Q.6 (a) Solution:

$$(i) \text{ Given: } P = 4500 \text{ kW, } N = 200 \text{ rpm, } H = 120 \text{ m}$$

$$\text{Specific speed of turbine, } N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$\Rightarrow N_s = \frac{200\sqrt{4500}}{(120)^{5/4}}$$

$$\Rightarrow N_s = 33.78 \text{ (SI units)}$$

Since,  $N_s = 33.78$ , therefore, the turbine is multi-jet Pelton wheel turbine.

(ii) Given:  $C_v = 0.98; k_u = 0.46; \eta = 88\%$

$$\text{Efficiency, } \eta = \frac{\text{Shaft power}}{\text{Water power}} \times 100 = \frac{P}{\rho g Q H} \times 100$$

$$\Rightarrow 88 = \frac{4500 \times 1000}{1000 \times 9.81 \times Q \times 120} \times 100$$

$$\Rightarrow Q = 4.344 \text{ m}^3/\text{s}$$

$$\therefore \text{Speed ratio, } k_u = \frac{u}{\sqrt{2gH}}$$

$$\Rightarrow 0.46 = \frac{u}{\sqrt{2 \times 9.81 \times 120}}$$

$$\Rightarrow u = 22.32 \text{ m/s}$$

$$\text{Wheel velocity, } u = \frac{\pi D N}{60}$$

$$\Rightarrow 22.32 = \frac{\pi D \times 200}{60}$$

$$\Rightarrow \text{Impeller diameter, } D = 2.13 \text{ m}$$

$$\text{Now, } \frac{\text{Impeller diameter, } D}{\text{Jet diameter, } d} = 9$$

$$\Rightarrow \frac{2.13}{d} = 9$$

$$\Rightarrow \text{Jet diameter, } d = \frac{2.13}{9} \text{ m}$$

$$\Rightarrow d = 0.237 \text{ m} \simeq 0.24 \text{ m}$$

$$\text{Water velocity, } V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 120}$$

$$\Rightarrow V = 47.55 \text{ m/s}$$

$$\therefore \text{Number of jets required, } n = \frac{Q}{A_{\text{jet}} V}$$

$$\Rightarrow n = \frac{4.344}{\frac{\pi}{4} (0.24)^2 \times 47.55} = 2.019$$

$$\Rightarrow n \simeq 3$$

## Q.6 (b) Solution:

(i) Total energy, T.E.

$$TE = H = z + y + \frac{Q^2}{2gA^2}$$

Differentiating on both sides w.r.t  $x$ ,

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{Q^2}{2g} \times \frac{-2}{A^3} \times \frac{dA}{dy} \times \frac{dy}{dx}$$

$$\Rightarrow -S_f = -S_0 + \frac{dy}{dx} \left[ 1 - \frac{Q^2 T}{gA^3} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}, \text{ where } F_r^2 = \frac{Q^2 T}{gA^3} \quad \dots(i)$$

As per Manning's equation,  $Q = \frac{1}{n} AR^{2/3} S^{1/2}$ 

$$\Rightarrow Q = \frac{1}{n} (B y_0) (y_0)^{2/3} S_0^{1/2}$$

For very wide rectangular channel,  $R \simeq y$ 

$$Q = \frac{1}{n} B y_0^{5/3} S_0^{1/2}$$

$$Q = \frac{1}{n} B y^{5/3} S_f^{1/2}$$

As  $Q = \text{constant}$ ,

$$y_0^{5/3} \cdot S_0^{1/2} = y^{5/3} \cdot S_f^{1/2}$$

$$\Rightarrow \frac{S_f}{S_0} = \left( \frac{y_0}{y} \right)^{10/3} \quad \dots(ii)$$

From eq. (i)

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{S_0 \left( 1 - \frac{S_f}{S_0} \right)}{1 - F_r^2}$$

But  $F_r^2 = \frac{Q^2 T}{g A^3} = \frac{q^2 B^2 B}{g B^3 y^3} = \frac{q^2}{g y^3} = \left( \frac{y_c}{y} \right)^3$

$$\therefore \frac{dy}{dx} = \frac{S_0 \left( 1 - \left( \frac{y_0}{y} \right)^{10/3} \right)}{\left( 1 - \left( \frac{y_c}{y} \right)^3 \right)}$$

(ii) Area of flow,  $A = by = 1.2 \times 0.6 = 0.72 \text{ m}^2$

$\therefore$  Wetted perimeter,  $P = b + 2y = 1.2 + 2 \times 0.6 = 2.4 \text{ m}$

Hydraulic mean depth,  $R = \frac{A}{P} = \frac{0.72}{2.4} = 0.3 \text{ m}$

Flow velocity,  $v = \frac{Q}{A} = \frac{0.85}{0.72} = 1.18 \text{ m/s}$

$$v = C \sqrt{RS_f}$$

$$\Rightarrow 1.18 = 57 \times \sqrt{0.3 \times S_f}$$

$$\Rightarrow S_f = 0.0014285$$

1. When bed slope,  $S_0 = \frac{1}{700} = 0.0014285$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{0.0014285 - 0.0014285}{1 - \frac{1.18^2}{9.81 \times 0.6}} = 0$$

Zero value of  $\frac{dy}{dx}$  indicates that the depth of flow is constant, and hence the flow velocity remains constant at all sections. This is a case of uniform flow.

$$\begin{aligned}
 2. \text{ When } S_0 &= \frac{1}{500} = 0.002 \\
 \frac{dy}{dx} &= \frac{0.002 - 0.0014285}{1 - \frac{1.18^2}{9.81 \times 0.6}} \\
 \Rightarrow \frac{dy}{dx} &= 7.486 \times 10^{-4}
 \end{aligned}$$

Positive value of  $\frac{dy}{dx}$  indicates that the depth of flow goes on increasing on the downstream side. The water surface has a concave profile upwards and is called back water curve.

**Q.6 (c) Solution:**

$$M_u = 200 \times 1.5 = 300 \text{ kNm}$$

$$V_u = 1.5 \times 300 = 450 \text{ kN}$$

$$T_u = 1.5 \times 400 = 600 \text{ kNm}$$

$$\begin{aligned}
 \text{Equivalent shear force, } V_{ue} &= V_u + 1.6 \frac{T_u}{B} \\
 &= 450 + \frac{1.6 \times 600}{0.75} = 1730 \text{ kN}
 \end{aligned}$$

$$\therefore \tau_{ve} = \frac{V_u}{B.d} = \frac{1730 \times 10^3}{750 \times 840} = 2.746 \text{ N/mm}^2$$

Maximum permissible strength of concrete,

$$\tau_{c\max.} = 0.625 \sqrt{f_{ck}} = 0.625 \sqrt{35} = 3.7 \text{ N/mm}^2$$

$$\therefore \tau_{ve} < \tau_{c\max} \quad (\text{O.K.})$$

Equivalent bending moment,

$$\begin{aligned}
 M_{Tu} &= \frac{T_u}{1.7} \left( 1 + \frac{D}{B} \right) \\
 &= \frac{600}{1.7} \left( 1 + \frac{900}{750} \right) = 776.47 \text{ kNm} \simeq 777 \text{ kNm}
 \end{aligned}$$

Now,  $M_{Tu} > M_u (= 300 \text{ kNm})$

We have to design the beam for  $M_{ue1} = M_{Tu} + M_u$   
 $= 777 + 300 = 1077 \text{ kNm}$

and  $M_{ue2} = M_{Tu} - M_u$   
 $= 777 - 300 = 477 \text{ kNm}$

$$\begin{aligned} \text{(I) For } M_{ue1}: \quad (M_u)_{\text{lim}} &= Q \cdot B \cdot d^2 \\ &= 0.133 f_{ck} B \cdot d^2 \quad (\text{for Fe500}) \\ &= 0.133 \times 35 \times 750 \times \frac{840^2}{10^6} \text{ kNm} \\ &= 2463.43 \text{ kNm} \end{aligned}$$

$$\therefore M_{ue1} < (M_u)_{\text{lim}}$$

$\therefore$  Section is under-reinforced.

$$\begin{aligned} A_{st1} &= \frac{0.5 f_{ck}}{f_y} \left[ 1 - \sqrt{1 - \frac{4.6 M_{ue1}}{f_{ck} B d^2}} \right] B d \\ &= \frac{0.5 \times 35}{500} \left[ \sqrt{1 - \frac{4.6 \times 1077 \times 10^6}{35 \times 750 \times 840^2}} \right] 750 \times 840 \end{aligned}$$

$$\Rightarrow A_{st1} = 3178 \text{ mm}^2$$

Using 32 mm dia. bars,

$$n \times \frac{\pi}{4} (32)^2 = 3178 \Rightarrow n = 4$$

$\therefore$  Provide 4 – 32 mm $\phi$  bars

$$\begin{aligned} \text{(II) For } M_{ue2}: \quad A_{st2} &= \frac{M_{ue2}}{0.87 f_y (d - d_c)} = \frac{477 \times 10^6}{0.87 \times 500 \times (840 - 60)} \\ &= 1405.84 \simeq 1406 \text{ mm}^2 \end{aligned}$$

Using 25 mm dia. bars,

$$n \times \frac{\pi}{4} (25)^2 = 1406$$

$$\Rightarrow n = 3$$

∴ Provide 3-25 mm $\phi$  bars on compression side.

(III) Design of shear reinforcement

$$b_1 = B - 2 \times \text{eff. cover} = 750 - 2 \times 60 = 630 \text{ mm}$$

$$d_1 = D - 2 \times \text{eff. cover} = 900 - 2 \times 60 = 780 \text{ mm}$$

Adopt 2-legged 18 mm vertical stirrups, (Fe415)

$$(a) \quad S_v \leq \frac{0.87 f_y A_{sv} d_1}{\left( \frac{T_u}{b_1} + \frac{V_u}{2.5} \right)}$$

$$\Rightarrow S_v \leq \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times (8)^2 \times 780}{\left( \frac{600 \times 10^3}{630} + \frac{450}{2.5} \times 10^3 \right)}$$

$$\Rightarrow S_v \leq 156.46 \text{ mm}$$

$$(b) \quad S_v \leq \frac{0.87 f_y A_{sv} d}{(\tau_{ve} - \tau_c) B d}$$

$$\Rightarrow S_v \leq \frac{0.87 \times 415 \times \left( 2 \times \frac{\pi}{4} \times 8^2 \right)}{(2.746 - \tau_c) \times 750}$$

$$p_t \% = \frac{100 \times A_{st}}{B d} = \frac{100 \times \left( \frac{\pi}{4} \times (32)^2 \right) \times 4}{750 \times 840} = 0.51 \%$$

$$\therefore \tau_c = 0.5036$$

$$\therefore S_v \leq \frac{0.87 \times 415 \times \left( 2 \times \frac{\pi}{4} \times 8^2 \right)}{(2.746 - 0.5036) \times 750}$$

$$\Rightarrow S_v \leq 21.6 \text{ mm}$$

Using 4-legged - 12 mm $\phi$  stirrups,

$$S_v \leq 97.12 \text{ mm}$$

∴ Provide 4-legged 12 $\phi$  vertical stirrups @95 mm c/c.

**Check:**  $S_v \neq (1) \quad x_1 = b_1 + \phi_m + \phi_{st} = 630 + 32 + 12 = 674 \text{ mm}$



$$(2) \quad \frac{x_1 + y_1}{4} = \frac{674 + 818}{4} = 373 \text{ mm}$$

$$(3) \quad 300 \text{ mm}$$

$$\therefore S_v \neq 300 \text{ mm}$$

(IV) Side face reinforcement,

$$\text{As } D = 900 \text{ mm} > 450 \text{ mm}$$

$\therefore$  Side face reinforcement is required.

$$\therefore A_{ssf} = \frac{0.1}{100} \times (750 \times 900) = 675 \text{ mm}^2$$

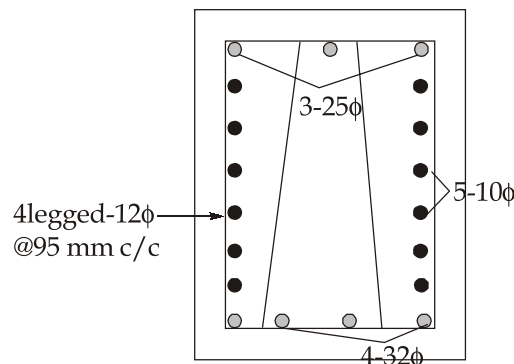
Using 10 mm dia. bars,

$$n \times \frac{\pi}{4} \times (10)^2 = 675 \text{ mm}^2$$

$$\Rightarrow n = 8.6$$

Equally distributed on both sides.

Provide 5 nos. – 10 $\phi$  bars on each face equally distributed.



**Q.7 (a) Solution:**

(i) Side length of the triangular channel,

$$S = \sqrt{3^2 + 3^2}$$

$$\Rightarrow S = 4.243 \text{ m}$$

**Subsection (1):**

$$\text{Flow area, } A_{c1} = 2 \times 6 + \frac{1}{2} \times 6 \times 3 = 21 \text{ m}^2$$

$$\text{Wetted perimeter, } P_1 = 2 + 2(4.243) = 10.486 \text{ m}$$

$$\therefore \text{Hydraulic radius, } R_1 = \frac{A_{c1}}{P_1} = \frac{21}{10.486} = 2 \text{ m}$$

**Subsection (2):**

$$\text{Flow area, } A_{c2} = 2 \times 8 = 16 \text{ m}^2$$

$$\text{Wetted perimeter, } P_2 = 8 + 2 = 10 \text{ m}$$

$$\therefore R_2 = \frac{A_{c2}}{P_2} = \frac{16}{10} = 1.6 \text{ m}$$

$$\begin{aligned} \text{Entire channel: } A_c &= 37 \text{ m}^2 \\ P &= 20.486 \text{ m} \end{aligned}$$

$$R = \frac{37}{20.486} = 1.806 \text{ m}$$

$$\text{Total flow rate, } Q = Q_1 + Q_2$$

$$\Rightarrow Q = \frac{A_{c1}}{n_1} \times R_1^{2/3} \times S_0^{1/2} + \frac{A_{c2}}{n} \times R_2^{2/3} \times S_0^{1/2}$$

$$\Rightarrow Q = \left[ \frac{21 \times 2^{2/3}}{0.03} + \frac{16 \times (1.6)^{2/3}}{0.05} \right] \times (0.003)^{1/2}$$

$$\Rightarrow Q = 84.84 \text{ m}^3/\text{s}$$

For effective Manning's coefficient,

$$Q = \frac{A_c}{n_{\text{eff}}} \times R^{2/3} \times S_0^{1/2}$$

$$\Rightarrow n_{\text{eff}} = \frac{37}{84.84} \times (1.806)^{2/3} \times (0.003)^{1/2}$$

$$\Rightarrow n_{\text{eff}} = 0.0354$$

Weighted average of Manning's coefficient,

$$n_{\text{avg}} = \frac{n_1 P_1 + n_2 P_2}{P}$$

$$\Rightarrow n_{\text{avg.}} = \frac{0.03 \times 10.486 + 0.05 \times 10}{20.486}$$

$$\Rightarrow n_{\text{avg.}} = 0.0398$$

$$\therefore \quad \% \text{ error} = \frac{n_{\text{eff}} - n_{\text{avg}}}{n_{\text{eff}}} \times 100$$

$$\Rightarrow \quad \% \text{ error} = \frac{0.0354 - 0.0398}{0.0354} \times 100$$

$$(ii) \Rightarrow \quad \% \text{ error} = (-) 12.43\%$$

$$(ii) \quad \text{Top width, } T = 2my_c$$

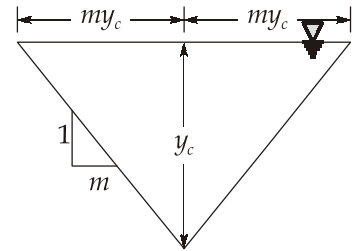
$$\text{Flow area of channel, } A_c = my_c^2$$

$$\text{and} \quad F_r = 1 \quad (\text{For critical flow})$$

$$\Rightarrow \quad \frac{V}{\sqrt{g \frac{A_c}{T}}} = 1$$

$$\Rightarrow \quad \frac{2V^2}{gy_c} = 1$$

$$\Rightarrow \quad \frac{V^2}{2g} = \frac{y_c}{4}$$



Specific energy at critical depth,

$$E = y_c + \frac{V^2}{2g}$$

$$\Rightarrow \quad E = y_c + \frac{y_c}{4}$$

$$\Rightarrow \quad E = \frac{5y_c}{4}$$

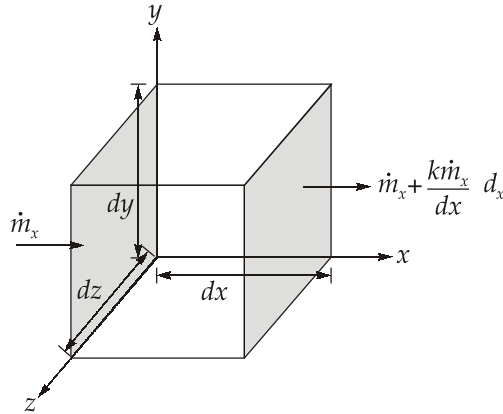
$$\Rightarrow \quad y_c = \frac{4}{5}E$$

#### Q.7 (b) Solution:

- (i) Continuity equation is based on the principle of conservation of mass i.e. when a fluid flowing through any section, the quantity of fluid flowing per second remains constant.

Velocity vector,

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$



$$\dot{m}_x = \rho(dz \cdot dy)u$$

$$d\forall = dx \cdot dy \cdot dz$$

$$\Rightarrow \quad \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \dot{m}_{\text{storage}}$$

$$(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_x + (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_y + (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_z = \dot{m}_{\text{storage}} \quad \dots(i)$$

From above figure,

$$\begin{aligned} (\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_x &= \dot{m}_x - \left( \dot{m}_{\text{in}} + \frac{\partial \dot{m}_x}{\partial x} dx \right)_x \\ &= -\frac{\partial \dot{m}_x}{\partial x} dx \\ &= -\frac{\partial \rho}{\partial x} (dz \cdot dy) u \cdot dx \end{aligned}$$

$$(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_x = \frac{-\partial}{\partial x} (\rho u) d\forall$$

Similarly,

$$(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_y = \frac{-\partial}{\partial y} (\rho v) d\forall$$

$$(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})_z = \frac{-\partial}{\partial z} (\rho w) d\forall$$

From eq. (i),

$$-\frac{\partial}{\partial x} (\rho u) d\forall - \frac{\partial}{\partial y} (\rho v) d\forall - \frac{\partial}{\partial z} (\rho w) d\forall = \frac{\partial \rho}{\partial t} d\forall$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}(\rho u) + \frac{\partial \rho}{\partial y}(\rho v) + \frac{\partial \rho}{\partial z}(\rho w) = 0$$

(ii) For 3-D incompressible steady flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Given,  
and

$$u = x^2 + z^2 + 5$$

$$v = y^2 + z^2 - 3$$

$$\frac{\partial u}{\partial x} = 2x; \quad \frac{\partial v}{\partial y} = 2y$$

$$2x + 2y + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \quad \frac{\partial w}{\partial z} = -2(x + y)$$

$$\Rightarrow \quad \partial w = -2(x + y) \partial z$$

Integrating on both sides

$$w = -2(x + y)z + c$$

Given at  $(1, 1, 0), w = 0$

$$\therefore \quad 0 = -2(1 + 1) \times 0 + c$$

$$\Rightarrow \quad c = 0$$

$$\text{So,} \quad w = -2(x + y)z$$

Now, for flow to be irrotational,  $(\nabla \times \vec{V}) = 0$

$$\Rightarrow \quad \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\Rightarrow \quad \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2 + z^2 + 5) & y^2 + z^2 - 3 & -2(x + y)z \end{vmatrix}$$

$$\begin{aligned}
 \Rightarrow \quad \nabla \times \vec{V} &= \hat{i} \left[ \frac{\partial}{\partial y} \{-2(x+y)z\} - \frac{\partial}{\partial z} (y^2 + z^2 - 3) \right] \\
 &\quad - \hat{j} \left[ \frac{\partial}{\partial x} (-2(x+y)z) - \frac{\partial}{\partial z} (x^2 + z^2 + 5) \right] \\
 &\quad + \hat{k} \left[ \frac{\partial}{\partial x} (y^2 + z^2 - 3) - \frac{\partial}{\partial y} (x^2 + z^2 + 5) \right] \\
 \nabla \times \vec{v} &= \hat{i}(-2z - 2z) - \hat{j}(-2z - 2z) + \hat{k}(0) \\
 &= -4z\hat{i} + 4z\hat{j} \neq 0
 \end{aligned}$$

Thus the curl of velocity vector is not zero and hence, the flow is not irrotational i.e. flow is rotational.

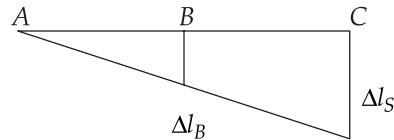
### Q.7 (c) Solution:

$$\begin{aligned}
 \text{Free expansion of steel bar, } \Delta l_s &= l \propto \Delta T \\
 &= 300 \times 12 \times 10^{-6} \times 30 \\
 &= 0.108 \text{ mm}
 \end{aligned}$$

$$\text{Free contraction of brass bar, } \Delta l_B = 400 \times 20 \times 10^{-6} \times 30 = 0.24 \text{ mm}$$

For no temperature stress to be developed,

$$\frac{\Delta l_B}{AB} = \frac{\Delta l_S}{AC}$$



$$\Rightarrow \quad \Delta l_B = \Delta l_S \times \frac{AB}{AC} = 0.108 \times \frac{0.3}{0.7} = 0.0463 \text{ mm}$$

$$\text{But,} \quad \Delta l_B = 0.24 \text{ mm}$$

Hence, in the equilibrium position attained due to rigidity of bar ABC, tensile stress will be developed in the brass bar, and consequently, tensile stress will also be developed in steel bar.

Let  $\Delta_B$  and  $\Delta_S$  be final deformations in the two bars at the equilibrium position,

$$\therefore \quad \frac{\Delta_B}{AB} = \frac{\Delta_S}{AC}$$

$$\Rightarrow \quad \Delta_B = \Delta_S \times \frac{3}{7} = 0.429 \Delta_S \quad \dots (i)$$

Let  $P_B$  and  $P_S$  be the forces induced in brass and steel bars respectively.

$$\Delta_B = \Delta l_B - \Delta_B^P = 0.24 - \frac{P_B l_B}{A_B E_B} \quad \dots \text{(ii)}$$

and

$$\Delta_S = \Delta l_S + \Delta_S^P = 0.108 + \frac{P_S l_S}{A_S E_S} \quad \dots \text{(iii)}$$

Using (i),

$$\Delta_B = \Delta_S \times \frac{3}{7} = 0.429 \Delta_S$$

$$\begin{aligned} 0.24 - \frac{P_B \times 400}{400 \times 0.9 \times 10^5} &= 0.429 \left[ 0.108 + \frac{P_S \times 300}{250 \times 2 \times 10^5} \right] \\ &= 0.046 + \frac{0.429 \times P_S \times 300}{250 \times 2 \times 10^5} \end{aligned}$$

$$\Rightarrow 0.24 - 0.046 = \frac{0.429 \times P_S \times 300}{250 \times 2 \times 10^5} + \frac{P_B \times 400}{400 \times 0.9 \times 10^5}$$

$$\Rightarrow 0.194 = \frac{0.2574 P_S}{10^5} + \frac{1.111 P_B}{10^5}$$

$$\Rightarrow 0.194 \times 10^5 = 0.2574 P_S + 1.111 P_B \quad \dots \text{(iv)}$$

Now,

$$\Sigma M_A = 0$$

$$\Rightarrow P_B \times 0.3 = P_S \times 0.7$$

$$\Rightarrow P_B = \frac{7}{3} P_S = 2.333 P_S \quad \dots \text{(v)}$$

Using (v) in (iv)

$$0.194 \times 10^5 = 0.2574 \times P_S + 1.111 \times 2.333 P_S$$

$$\Rightarrow P_S = 6808.5 \text{ N}$$

$$\therefore \sigma_S = \frac{6808.5}{250} = 27.234 \text{ MPa (Tension)}$$

Also, if

$$P_S = 6808.5$$

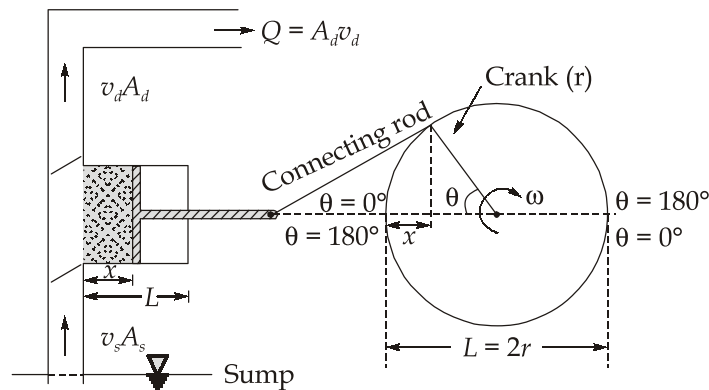
than

$$\begin{aligned} P_B &= 2.333 \times 6808.5 \\ &= 15884.23 \text{ N} \end{aligned}$$

$$\sigma_B = \frac{15884.23}{400} = 39.719 \text{ MPa (Tension)}$$

### Q.8 (a) Solution:

Reciprocating pumps are used to lift water against high head at low discharge. These provides pulsating discharge.



Let

$D$  = Diameter of cylinder (Bore)/piston diameter

$$A = \frac{\pi}{4} D^2$$

$N$  = Speed (rpm)

$r$  = Crank radius

$L$  = Stroke  $L = 2r$

$\theta = \omega t$

$$x = r - r \cos \theta = r - r \cos \omega t$$

$$V_p = \frac{dx}{dt} = 0 - r\omega(-\sin \omega t)$$

$$\Rightarrow V_p = r\omega \sin \omega t = r\omega \sin \theta$$

Volume of water discharged per second,

$$Q_{th} = \frac{ALN}{60} \text{ m}^3/\text{s}$$

where,

$A$  = Area of cylinder (in  $\text{m}^2$ )

$L$  = Length of cylinder (in m)

$N$  = Crank speed (in rpm)

If the head against which water is to be lifted is,  $H_s$ , then,

$$H_s = h_s + h_d$$

where,

$h_s$  = suction head (sump to pump)

$h_d$  = delivery head (pump to delivery tank)

$$\text{Work done per second} = \gamma Q(h_s + h_d)$$

To increase discharge and to maintain it more uniform, double acting reciprocating pumps are used.

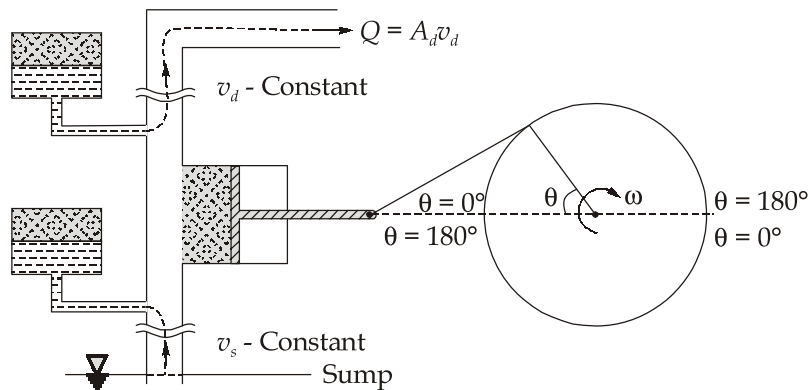


For double acting reciprocating pump,

$$Q_{th} = \frac{2ALN}{60}$$

**Air vessel:**

- It is the container of water placed near to pump in suction and delivery pipe.

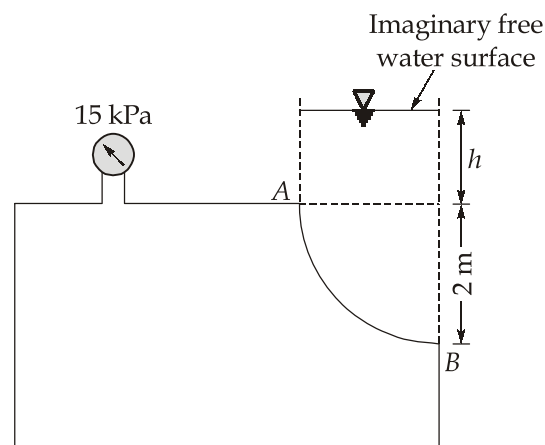


**Advantages:**

- It helps to maintain constant discharge.
- It helps to reduce power input to the pump.
- The pump can be operated at higher speed and therefore, the pump can handle more discharge.

**Q.8 (b) Solution:**

(i)



The water head equivalent to the given pressure of 15 kPa is

$$h = \frac{15 \times 10^3}{9810} = 1.53 \text{ m}$$

Hence the free water surface can be imagined to be 1.53 m above the top of the tank.

Horizontal force,  $F_h = \rho g A_v \bar{h}$

$$\Rightarrow F_h = 9810 \times (2 \times 2.5) \times \left(1.53 + \frac{2}{2}\right)$$

$$\Rightarrow F_h = 124096 \text{ N} = 124.096 \text{ kN} (\rightarrow)$$

Vertical force,  $F_v = \rho g \nabla$

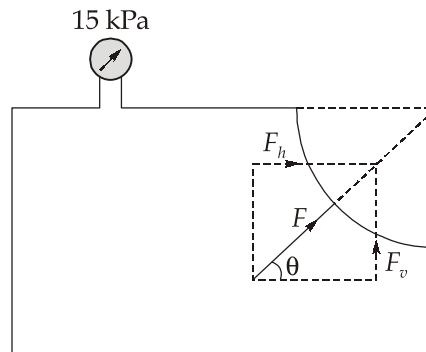
$$\Rightarrow F_v = 9810 \times \left(1.53 \times 2 \times 2.5 + \frac{\pi}{4} (2)^2 \times 2.5\right)$$

$$\Rightarrow F_v = 152094 \text{ N} = 152.094 \text{ kN} (\uparrow)$$

Resultant force,  $F = \sqrt{F_h^2 + F_v^2}$

$$\Rightarrow F = \sqrt{124.096^2 + 152.094^2}$$

$$\Rightarrow F = 196.3 \text{ kN}$$



Direction: Resultant force is inclined an angle  $\theta$  from the horizontal where,

$$\tan \theta = \frac{F_v}{F_h} = \frac{152.094}{124.096} = 1.225$$

$$\therefore \theta = 50.79^\circ$$

(ii) Kinematic viscosity,  $\nu = 0.9 \text{ stokes}$   
 $= 0.9 \times 10^{-4} \text{ m}^2/\text{s}$

Reynold's number at the end of plate,

$$\text{Re}_L = \frac{u_0 l}{\nu} = \frac{6 \times 0.45}{0.9 \times 10^{-4}} = 3 \times 10^4$$

Since, the Reynold's number is less than  $5 \times 10^5$ , the flow over the plate is entirely laminar.

$\therefore$  Average skin friction coefficient,

$$C_D = \frac{1.328}{\sqrt{\text{Re}_L}}$$

$$\Rightarrow C_D = \frac{1.328}{\sqrt{3 \times 10^4}} = 0.0077$$

$\therefore$  Drag on one side of the plate,

$$\begin{aligned} F_D &= \frac{1}{2} \rho u_0^2 C_D \times \text{area} \\ &= \frac{1}{2} \times 925 \times 6^2 \times 0.0077 \times (0.15 \times 0.45) = 8.65 \text{ N} \end{aligned}$$

Since, the plate is wetted on both sides, total drag on plate

$$= 2F_D = 2 \times 8.65 = 17.3 \text{ N}$$

#### Q.8 (c) Solution:

Consider the bottom 1 meter height of the wall. Pressure intensity corresponding to the centre of the above strip of wall.

$$= 9810 \times 2.5 = 24525 \text{ N/m}^2$$

$$\text{Hoop tension, } T = \frac{pD}{2} = \frac{24525 \times 4}{2} = 49050 \text{ N}$$

$$\text{Steel required per meter wall height} = \frac{49050}{150} = 327 \text{ mm}^2$$

Minimum steel requirement = 0.3% of gross section

(Assuming a minimum thickness of 150 mm)

$$= \frac{0.30}{100} \times 150 \times 1000 = 450 \text{ mm}^2$$

$$\text{Spacing of } 8 \text{ mm } \phi, \text{ bars} = \frac{\frac{\pi}{4} \times 8^2 \times 1000}{450} = 111.7 \text{ mm say } 110 \text{ mm c/c}$$

Thickness of wall: This shall not be less than the following:

(i) 150 mm

(ii) 30 mm per meter depth + 50 mm =  $(30 \times 3) + 60 = 140 \text{ mm}$

(iii) Thickness required to limit the tensile stress in concrete to  $1.2 \text{ N/mm}^2$  ( $m = 13.33$ )

$$1.2 = \frac{49050}{1000t + (13.33 - 1)450}$$

$$\Rightarrow t = 35.33 \text{ mm}$$

∴ Provide a thickness of 150 mm.

Vertical distribution steel = 0.30% of gross section

$$= \frac{0.30}{100} \times 150 \times 1000 = 450 \text{ mm}^2$$

Provide 8 mm  $\phi$  bars @110 mm c/c.

