



**MADE EASY**  
India's Best Institute for IES, GATE & PSUs

Detailed Solutions

**ESE-2022  
Mains Test Series**

**Mechanical Engineering  
Test No : 1**

**Section A : Thermodynamics [All Topics]**

**Section B : Refrigeration and Air Conditioning [All Topics]**

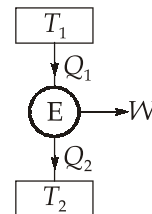
**Section : A**

1. (a)

Consider a Carnot heat engine, employing an ideal gas as the working medium, operating between two reservoirs at temperature  $T_1$  and  $T_2$  as measured by an ideal gas thermometer, then efficiency of Carnot engine is given by

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \dots (i)$$



The absolute thermodynamic temperature scale is established through the relation,

$$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2} \quad \dots (ii)$$

where  $\theta_1$  and  $\theta_2$  are the temperature on the thermodynamic temperature scale.

From equation (i) and (ii), we get

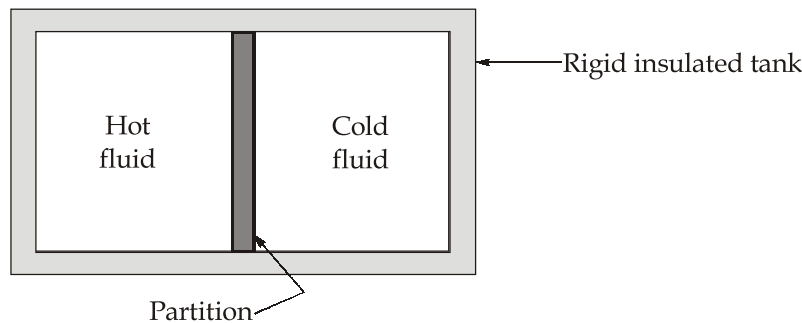
$$\frac{\theta_1}{\theta_2} = \frac{T_1}{T_2}$$

Since, the fixed point for the establishment of the ideal gas temperature scale is the same as the fixed point for the establishment of the thermodynamic temperature scale, these two temperature scales are equivalent.

## 1. (b)

The principle of maximum entropy states that in a state of stable equilibrium, the entropy of an isolated system is maximum for a given value of internal energy.

Consider a rigid and insulated tank which is divided into two compartments by a partition as shown in figure. The first compartment contains a hot fluid while the second compartment contains a cold fluid. If the partition is removed the fluids mix and attain a state of equilibrium.



Consider the fluid contained in both the compartments as the system. Since the tank is rigid and adiabatic, we have

$$W = 0 \text{ and } Q = 0$$

From first law of thermodynamics,

$$\Rightarrow Q = \Delta U + W$$

$$\Rightarrow \Delta U = 0$$

$$\Rightarrow U_1 = U_2 \quad \dots(i)$$

Equation (i) shows that the system occupies only that state for which the internal energy is equal to the initial value.

The second law of thermodynamics (the principle of entropy increase) tells that the entropy of an isolated system either increases or remains constant.

If there is any irreversibility in the process, the entropy of the system must increase. Since mixing of fluids at different states is an irreversible process, the entropy of the system increases.

The system changes from one state to another of higher entropy and finally settles in that state for which the entropy is maximum for the given value of the internal energy. This state of maximum entropy is the equilibrium state of the system.

1. (c)

**Assumptions:**

1. Steady flow of energy
2. No external flow except heat transfer.

From energy equation,

$$\dot{W} + \dot{Q}_F + \dot{Q}_C = \dot{Q}_A \quad \dots (i)$$

For minimum power, refrigerator needs to be reversible,

$$\Rightarrow -\frac{\dot{Q}_A}{T_A} + \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_F}{T_F} = 0$$

$$\dot{Q}_A = T_A \left( \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_F}{T_F} \right) \quad \dots (ii)$$

Putting value of  $\dot{Q}_A$  from equation (ii) in equation (i) we get,

$$\dot{W} + \dot{Q}_F + \dot{Q}_C = T_A \left( \frac{\dot{Q}_C}{T_C} + \frac{\dot{Q}_F}{T_F} \right)$$

$$\dot{W} = \left( \frac{T_A}{T_C} \dot{Q}_C - \dot{Q}_C \right) + \left( \frac{T_A}{T_F} \dot{Q}_F - \dot{Q}_F \right)$$

So, the expression for minimum work required by refrigerator is

$$\dot{W} = \dot{Q}_C \left( \frac{T_A}{T_C} - 1 \right) + \dot{Q}_F \left( \frac{T_A}{T_F} - 1 \right)$$

Now,

$$T_A = 20^\circ\text{C} = 293 \text{ K}$$

$$T_C = 5^\circ\text{C} = 278 \text{ K}$$

$$T_F = -10^\circ\text{C} = 263 \text{ K}$$

and

$$\dot{Q}_F = \dot{Q}_C = 3 \text{ kW}$$

So,

$$\dot{W} = 3 \left( \frac{293}{278} - 1 \right) + 3 \left( \frac{293}{263} - 1 \right)$$

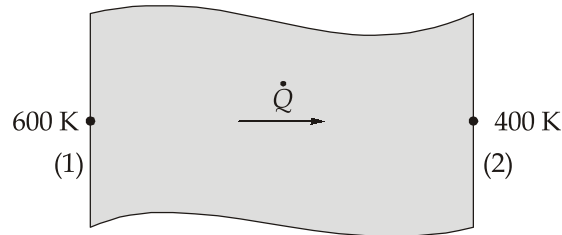
$$\dot{W} = 3(0.1680)$$

$$\dot{W} = 0.5040 \text{ kW or } 504 \text{ Watts}$$

1. (d)

Let ambient temperature ( $T_{amb}$ ) =  $27^{\circ}\text{C} = 300\text{ K}$

$$\dot{Q} = 250\text{ W}$$



$$(dS)_u = dS_1 + dS_2$$

$$(dS)_u = -\frac{\dot{Q}}{600} + \frac{\dot{Q}}{400}$$

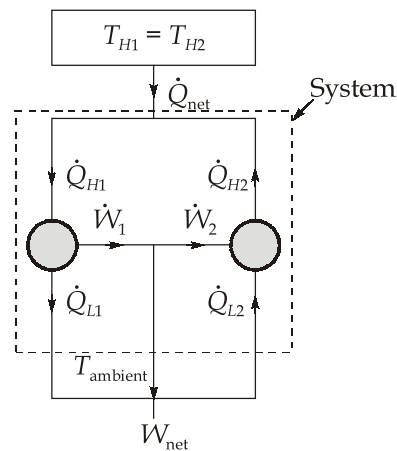
$$(dS)_u = 250 \left( \frac{1}{400} - \frac{1}{600} \right) = \frac{5}{24}\text{ W/K}$$

So, exergy destruction in the wall,

$$X_{des} = \text{Irreversibility} = T_o (ds)_u$$

$$X_{des} = 300 \left( \frac{5}{24} \right) = 62.5\text{ W}$$

1. (e)



For case (a)

$$\dot{Q}_{net} = \dot{Q}_{H1} - \dot{Q}_{H2} = 3 \quad [\text{Second law possible}]$$

$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 = 1 \quad [\text{Second law possible}]$$

For case (b)

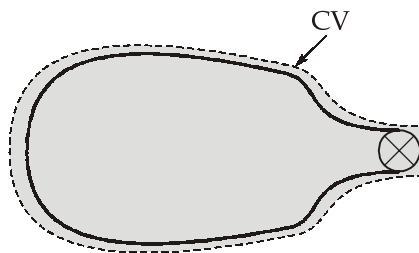
$$\left. \begin{aligned} \dot{Q}_{net} &= \dot{Q}_{H_1} - \dot{Q}_{H_2} = 1 \\ \dot{W}_{net} &= \dot{W}_1 - \dot{W}_2 = 1 \end{aligned} \right\} \begin{aligned} &[100\% \text{ efficiency, second law}] \\ &[\text{violates (Kelvin's statement)}] \end{aligned}$$

For case (c)

$$\left. \begin{aligned} \dot{Q}_{net} &= \dot{Q}_{H_1} - \dot{Q}_{H_2} = -1 \\ \dot{W}_{net} &= \dot{W}_1 - \dot{W}_2 = 0 \end{aligned} \right\} \begin{aligned} &[2\text{nd law violates}] \\ &[(\text{Clausius statement})] \end{aligned}$$

	1st law	2nd law
a	YES	YES (Possible), process is irreversible
b	YES	No, Kelvin Plank's statement violated
c	YES	No, Clausis statement violated

2. (a)



**Assumption:**

1. Rigid insulated tank is initially filled with an ideal gas at  $P_o$  and  $T_o$ .
2.  $P_f$  and  $T_f$  denotes the final pressure and temperature of the gas left in the cylinder.

First law of thermodynamics for a control volume is

$$\dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) - \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) = \dot{Q} - \dot{W}_s - \frac{dE}{dt} \quad \dots (i)$$

Insulated cylinder:  $\dot{Q} = 0$

No shaft work:  $\dot{W}_s = 0$

No gas enters the control volume:  $\dot{m}_i = 0$

Neglecting KE and PE changes, then equation (i) becomes,

$$\dot{m}_e (h_e) = -\frac{dE}{dt} = -\frac{d(mu)}{dt} \quad \dots (ii)$$

and for ideal gas,

$$h = h(T) \text{ only} \Rightarrow h = C_p T$$

$$u = u(T) \text{ only} \Rightarrow u = C_v T$$

At any instant of time, gas left in the cylinder is at the same conditions as the gas leaving the cylinder.

$$\Rightarrow T_e = T,$$

where  $T$  is the temperature of the gas left in the cylinder and  $m$  is the mass of gas in control volume at time  $t$ .

Now, equation (ii) becomes,

$$\dot{m}_e C_P T = -\frac{d}{dt}(m C_V T) \quad \dots \text{(iii)}$$

and 
$$\frac{dm}{dt} = -\dot{m}_e \quad \dots \text{(iv) (Mass conservation)}$$

From equation (iii) and (iv), we get

$$C_P T \frac{dm}{dt} = \frac{d}{dt}(m C_V T)$$

$$\Rightarrow \gamma T \frac{dm}{dt} = \frac{d}{dt}(m T) = \frac{m dT}{dt} + T \frac{dm}{dt}$$

$$\Rightarrow (\gamma - 1) T \frac{dm}{dt} = m \frac{dT}{dt}$$

$$\begin{aligned} \Rightarrow \frac{dm}{m} &= \left( \frac{1}{\gamma - 1} \right) \frac{dT}{T} \\ &= \ln \frac{m_f}{m_o} = \frac{1}{\gamma - 1} \ln \frac{T_f}{T_o} \quad \dots \text{(v)} \end{aligned}$$

We have,  $V = \text{Volume of the cylinder}$

From ideal gas equation,  $P_o V = m_o R T_o$

$$P_f V = m_f R T_f$$

Putting values of  $m_f$  and  $m_o$  in equation (v) we get,

$$\Rightarrow \ln \left( \frac{\frac{P_f V}{R T_f}}{\frac{P_o V}{R T_o}} \right) = \frac{1}{\gamma - 1} \ln \frac{T_f}{T_o}$$

$$\Rightarrow \ln \left( \frac{P_f T_o}{P_o T_f} \right) = \frac{1}{\gamma - 1} \ln \frac{T_f}{T_o}$$

$$\Rightarrow \frac{P_f T_o}{P_o T_f} = \left( \frac{T_f}{T_o} \right)^{\frac{1}{\gamma-1}}$$

$$\Rightarrow \frac{P_f}{P_o} = \left( \frac{T_f}{T_o} \right)^{\frac{1}{\gamma-1} + 1}$$

$$\Rightarrow \frac{P_f}{P_o} = \left( \frac{T_f}{T_o} \right)^{\frac{\gamma}{\gamma-1}}$$

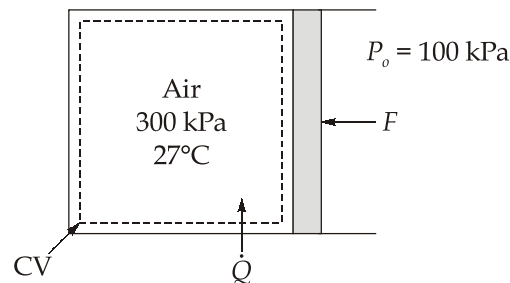
$$\Rightarrow \frac{T_f}{T_o} = \left( \frac{P_f}{P_o} \right)^{\frac{\gamma-1}{\gamma}} \quad \dots \text{(vi)}$$

Equation (vi) is identical with relation for a reversible adiabatic expansion of an ideal gas. Thus when an ideal gas escapes from a tank the temperature of the gas remaining in the tank can be estimated by using the relation which is true for a reversible adiabatic expansion.

2. (b)

**Assumption:**

Air behaves as an ideal gas.



$$P_1 = P_o + CV_1^{1.5}$$

$$300 = 100 + C(0.2)^{1.5}$$

$$\Rightarrow C = 2236.06$$

and,

$$P_2 = P_o + CV_2^{1.5}$$

$$425 = 100 + 2236.06(V_2)^{1.5}$$

$$\Rightarrow V_2 = 0.2764 \text{ m}^3$$

From ideal gas equation,  $m = \frac{P_1 V_1}{RT_1} \quad \dots \text{(i)}$

So,  $P_2 V_2 = mRT_2 \quad \dots \text{(ii)}$

From equation (i) and (ii), we get

$$P_2 V_2 = \frac{P_1 V_1}{T_1} T_2$$

$$(425) \times (0.2764) = \frac{300 \times 0.2}{300} T_2$$

$$\Rightarrow T_2(\text{Final temperature}) = 587.35 \text{ K or } 314.35^\circ\text{C}$$

$$\text{Work done} = \int_1^2 P dV = \int_1^2 (P_o + CV^{1.5}) dV$$

$$W = P_o(V_2 - V_1) + \frac{C(V_2^{2.5} - V_1^{2.5})}{2.5}$$

$$= 100[(0.2764) - 0.2] + \frac{2236.06}{2.5} [(0.2764)^{2.5} - (0.2)^{2.5}]$$

$$W = 27.564 \text{ kJ}$$

2. (c)

### Free Expansion

Initially, vessel (1) contains air and the vessel (2) is completely evacuated, as shown in figure. Finally air occupies both the vessels (1) and (2)

$$\text{Initial volume} = V_1$$

$$\text{Final volume} = V_1 + V_1 = 2V_1$$

Let the vessel (1) is filled with air at a certain pressure and vessel (2) is completely evacuated. When valve is opened air in vessel (1) will expand and will fill both the vessels (1) and (2) rapidly. The pressure finally will be lower than the initial pressure in vessel (1). This is known as unrestricted expansion or free expansion.

The process is not reversible, since external work would have to be done to restore the fluid to its initial condition. The non-flow energy equation can be applied between the initial and final states.

$$\text{i.e.} \quad Q = (U_2 - U_1) + W = 0 \text{ (insulated)}$$

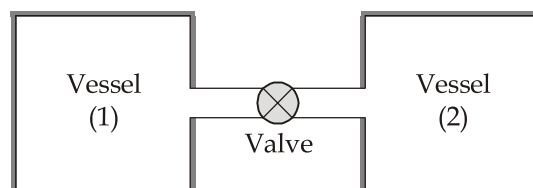
Since the boundary of system does not move, there is no work transfer.

$$\Rightarrow W = 0$$

$$\therefore U_2 - U_1 = 0$$

$$\Rightarrow U_1 = U_2$$

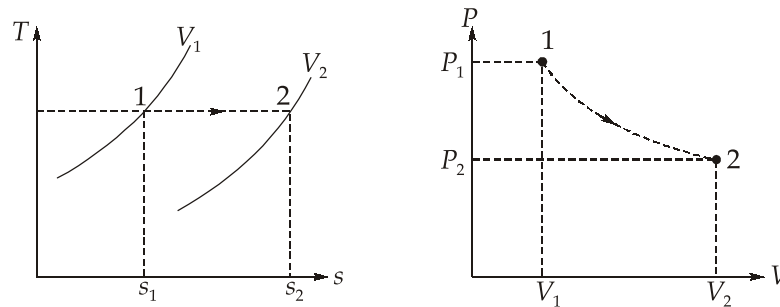
$$\text{For perfect gas,} \quad T_1 = T_2$$





End states are marked on T-s diagram as shown below.

Process 1-2 is irreversible process(drawn dotted).



Change of entropy regardless of path of the process,

$$S_2 - S_1 = m \left( C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \right) \quad \dots (i)$$

and, for perfect gas (air) undergoing a free expansion, the final and initial temperature are equal (same).

So,

$$S_2 - S_1 = 1 \left( R \ln \frac{V_2}{V_1} \right) = 1 \times 0.287 \ln \left( \frac{2V_1}{V_1} \right) = 0.199 \text{ kJ/K}$$

So, Increase in entropy = 0.199 kg/K (Irreversible process)

**Throttling process:**

A flow of fluid is said to be throttled:

- (i) when there is some restriction to the flow.
- (ii) when the velocities before and after the restriction are either equal or negligibly small.
- (iii) when there is negligible heat loss to the surroundings.

The restriction to flow can be partly open valve, an orifice, or any other sudden reduction in cross-section of the flow.

Flow equation can be applied.

$$\dot{m} \left( h_1 + \frac{C_1^2}{2} \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} \right) + \dot{W}$$

Since,

$$\dot{Q} = 0, \dot{W} = 0$$

$$C_1 \approx C_2 \text{ or } C_1, C_2 \text{ very small}$$

$$\Rightarrow h_1 = h_2$$

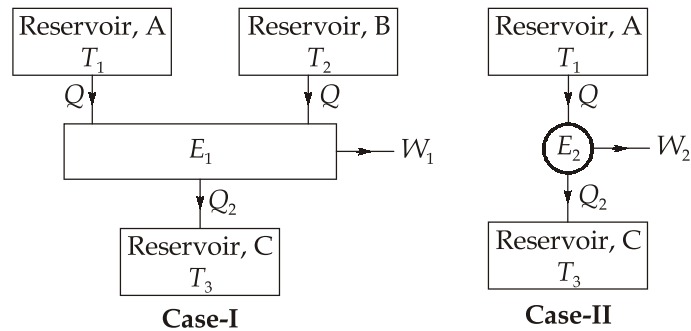
For perfect gas,

$$h = C_p T$$

$$\Rightarrow T_1 = T_2$$

The process of throttling can be used to find dryness fraction of steam.

3. (a)



Now,  $\eta_I = k\eta_{II} \quad \dots (i)$

and for case-I  $\eta_I = 1 - \frac{Q_2}{2Q} \quad \dots (ii)$

and, for reversible engine,

$$\frac{Q}{T_1} - \frac{Q_2}{T_3} + \frac{Q}{T_2} = 0$$

$$\Rightarrow Q \left( \frac{1}{T_1} + \frac{1}{T_2} \right) = \frac{Q_2}{T_3}$$

$$\Rightarrow \frac{Q_2}{Q} = T_3 \left( \frac{1}{T_1} + \frac{1}{T_2} \right)$$

Putting in equation (ii), we get

$$\eta_I = 1 - \frac{1}{2} \left[ T_3 \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \right] \quad \dots (iii)$$

For case-II,  $\eta_{II} = 1 - \frac{Q_2}{Q} = 1 - \frac{T_3}{T_1} \quad \dots (iv)$

Putting values of efficiencies from question (iii) and (iv) in equation (i) we get,

$$1 - \frac{1}{2} \left[ T_3 \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \right] = k \left( 1 - \frac{T_3}{T_1} \right)$$

$$\Rightarrow 1 - \frac{1}{2} \left( \frac{T_3 T_1 + T_2 T_3}{T_1 T_2} \right) = k \left( \frac{T_1 - T_3}{T_1} \right)$$

$$\Rightarrow k \left( \frac{T_1 - T_3}{T_1} \right) = \frac{2T_1 T_2 - T_3 T_1 - T_2 T_3}{2T_1 T_2}$$

$$\Rightarrow k = \frac{1}{2} \left( \frac{T_1}{T_1 - T_3} \right) \left[ \frac{T_1 T_2 - T_1 T_3 + T_1 T_2 - T_2 T_3}{T_1 T_2} \right]$$

$$\Rightarrow k = \frac{1}{2} \left( \frac{1}{T_1 - T_3} \right) [T_1(T_2 - T_3) - T_2(T_3 - T_1)] \times \frac{1}{T_2} \times \frac{T_1}{T_1}$$

$$\Rightarrow k = \frac{1}{2} \left( \frac{T_1}{T_2} \right) \left[ \frac{T_1(T_2 - T_3) - T_2(T_3 - T_1)}{T_1(T_1 - T_3)} \right]$$

$$\Rightarrow k = \frac{1}{2} \left( \frac{T_1}{T_2} \right) \left[ \frac{(T_2 - T_3)}{(T_1 - T_3)} + \frac{T_2}{T_1} \right]$$

### 3. (b)

#### Assumption

(i) Piston and springs are massless

(ii) Compression in spring is linear.

Given:  $P_{\text{atm}} = 100 \text{ kPa}$ ,

At bottom state,  $V_o = 0 \text{ m}^3$ ;  $P_o = P_{\text{atm}} = 100 \text{ kPa}$

At state (i)  $x = 0.13$

From table, at  $-2^\circ\text{C}$

$$P_1 = 399.2 \text{ kPa}$$

$$\begin{aligned} v_1 &= v_f + x(v_g - v_f) \\ &= 0.00156 + 0.13(0.310 - 0.00156) \end{aligned}$$

$$v_1 = 0.0417 \text{ m}^3/\text{kg}$$

and

$$V_1 = 1 \text{ m}^3$$

At state (ii),

$$V_2 = 2 \text{ m}^3$$

We know,

Spring constant ( $k$ ) = Slope of PV curve

$$k = \frac{P_1 - P_o}{V_1 - V_o} = \frac{399.2 - 100}{(1 - 0)} = 299.2 \text{ kPa/m}^3$$

For process 1 - 2 only one spring is in contact then,

$$\Rightarrow k = \frac{P_2 - P_1}{V_2 - V_1} = \frac{P_2 - 399.2}{(2 - 1)}$$

$$P_2 = k(1) + 399.2 = 299.2 + 399.2$$

$$P_2 = 698.4 \text{ kPa}$$

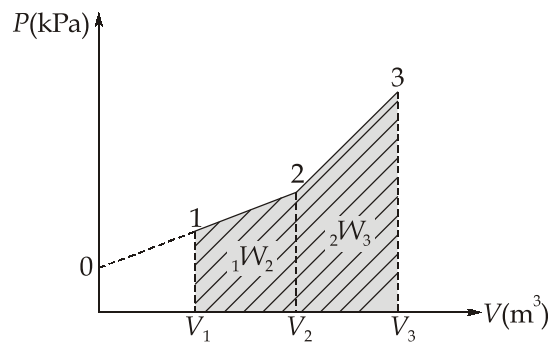
So, pressure when piston just touches the second spring is 698.4 kPa.

For process, 2-3 two springs are in contact, so slope of PV diagram will be equal to  $2k$ .

$$\Rightarrow 2k = \frac{P_3 - P_2}{V_3 - V_2}$$

$$\Rightarrow 2(299.2) = \frac{1200 - 698.4}{V_3 - 2}$$

$$\Rightarrow V_3 = 2 + 0.838 = 2.838 \text{ m}^3$$



Now, total work done,  ${}_1W_3 = {}_1W_2 + {}_2W_3$

$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + \frac{1}{2}(P_3 + P_2)(V_3 - V_2)$$

$$= \frac{1}{2}(399.2 + 698.4)(2 - 1) + \frac{1}{2}(1200 + 698.4)(2.838 - 2)$$

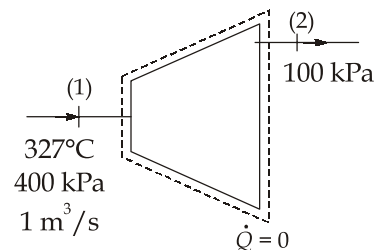
$$= 548.8 + 795.65$$

$${}_1W_3 = 1344.45 \text{ kJ}$$

3. (c)

**Assumption:**

1. Steady flow
2. Air is an ideal gas ( $\gamma = 1.4$ )
3. Single inlet and exit flow
4.  $C_{p,\text{air}} = 1.005 \text{ kJ/kgK}$



Given:  $T_1 = 327^\circ\text{C} = 600 \text{ K}$ ,  $P_1 = 400 \text{ kPa}$ ,  $\dot{V}_1 = 1 \text{ m}^3/\text{s}$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287(600)}{400} = 0.4305 \text{ m}^3/\text{kg}$$

So,

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{1}{0.4305} = 2.323 \text{ kg/s}$$

**Case-1:**

For lowest exit temperature, work output from the turbine should be maximum i.e. reversible case.

Process 1-2 will be reversible adiabatic.

So,  $\Delta S = 0$

and  $\frac{T_{exit}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$T_{exit} = 600 \left(\frac{100}{400}\right)^{\frac{1.4-1}{1.4}} = 403.8 \text{ K}$$

From energy equation,

$$\begin{aligned} \left(\frac{dU}{dt}\right)_{CV}^{(1)} &= \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{CV} \\ \dot{W}_{CV} &= \dot{m}_i h_i - \dot{m}_e h_e \quad (\because \dot{m}_i - \dot{m}_e = \dot{m}) \\ \dot{W}_{CV} &= \dot{m}(h_1 - h_2) = \dot{m}(C_p T_1 - C_p T_2) \\ \dot{W}_{CV} &= 2.323(1.005)(600 - 403.8) \\ \dot{W}_{CV} &= 458.05 \text{ kW} \end{aligned}$$

and,  $\dot{S}_{gen} = 0$

### Case-2:

For highest exit temperature, it occurs when there is no work output i.e. throttling process,

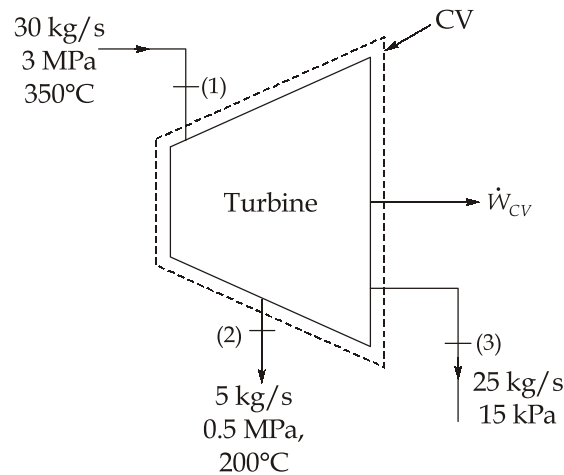
So,  $Q = 0, W = 0$  and  $h_1 = h_2$

So,  $T_{exit} = T_1 = 600 \text{ K}$

Entropy change associated with throttling process,

$$\begin{aligned} \left(\frac{dU}{dt}\right)_{CV}^{(1)} &= \dot{S}_i + \dot{S}_{gen} - \dot{S}_e \\ 0 &= \dot{S}_i - \dot{S}_e + \dot{S}_{gen} \\ \dot{S}_{gen} &= \dot{m}(s_e - s_i) \\ \dot{S}_{gen} &= \dot{m} \left( C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right) \\ \dot{S}_{gen} &= 2.323 \left[ -0.287 \ln \left( \frac{100}{400} \right) \right] \\ \dot{S}_{gen} &= 0.9242 \text{ kW/K} \end{aligned}$$

4. (a)

**Assumptions:**

1. Insulated turbine ( $Q = 0$ )
2. Steady flow
3. Neglecting KE and PE changes

Now, availability at any point for the steam entering or leaving the turbine is given by

$$\psi = (h - h_o) - T_o(s - s_o) + \frac{V^2}{2} + g(z - z_o)$$

After applying assumptions, we get,

$$\psi = (h - h_o) - T_o(s - s_o)$$

From table,

$$T_o = 25 + 273 = 298 \text{ K}, \quad s_o = 0.3674 \text{ kJ/kg}$$

$$h_o = 104.9 \text{ kJ/kg}, \quad s_1 = 6.587 \text{ kJ/kg}$$

$$h_1 = 3095.1 \text{ kJ/kg}, \quad \dot{m}_1 = 30 \text{ kg/s}$$

$$(h_g)_{0.5 \text{ MPa}} = 2747.5 \text{ kJ/kg}$$

$$(s_g)_{0.5 \text{ MPa}} = 6.819 \text{ kJ/kgK}$$

$$\dot{m}_2 = 5 \text{ kg/s}; \quad \dot{m}_3 = 25 \text{ kg/s}$$

$$h_3 = [h_f + x(h_g - h_f)]_{0.0015 \text{ MPa}} = [226 + 0.90(2599.2 - 226)] \\ = 2361.88 \text{ kJ/kg}$$

$$s_3 = [s_f + x(s_g - s_f)]_{0.0015 \text{ MPa}} \\ = [0.755 + 0.90(8.009 - 0.755)]$$

$$s_3 = 7.2836 \text{ kJ/kg},$$

Now, availability per kg of steam,

$$\begin{aligned}\text{At (1),} \quad \psi_1 &= (h_1 - h_o) - T_o(s_1 - s_o) \\ &= (3095.1 - 104.9) - 298(6.587 - 0.3674) \\ &= 1136.7592 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{At (2),} \quad \psi_2 &= (h_2 - h_o) - T_o(s_2 - s_o) \\ &= (2747.5 - 104.9) - 298(6.819 - 0.3674) \\ &= 720.02 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{At (3),} \quad \psi_3 &= (h_3 - h_o) - T_o(s_3 - s_o) \\ &= (2361.88 - 104.9) - 298(7.2836 - 0.3674) \\ &= 195.95 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{For isentropic process,} \quad s_1 &= s_2 \\ 6.587 &= 1.860 + x(6.819 - 1.860) \\ x_s &= 0.9532\end{aligned}$$

$$\begin{aligned}\text{So,} \quad h_{2s} &= 640.1 + x_s(2747.5 - 640.1) \\ h_{2s} &= 2648.90 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{and} \quad s_1 &= s_3 \\ 6.587 &= 0.755 + x_s(8.009 - 0.755) \\ x_s &= 0.8039\end{aligned}$$

$$\begin{aligned}\text{So,} \quad h_{3s} &= 226 + 0.8039(2599.2 - 226) \\ &= 2133.98 \text{ kJ/kg}\end{aligned}$$

For ideal isentropic turbine,

$$\begin{aligned}\dot{W}_s &= \dot{m}_1 h_1 - \dot{m}_2 h_{2s} - \dot{m}_3 h_{3s} \\ &= 30(3095.1) - 5(2648.90) - 25(2133.98) \\ \dot{W}_s &= 26259 \text{ kW}\end{aligned}$$

$$\begin{aligned}\text{For actual turbine,} \quad \dot{W}_a &= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \\ &= 30(3095.1) - 5(2747.5) - 25(2361.88) \\ &= 20068.5 \text{ kW}\end{aligned}$$

$$\text{So, Isentropic efficiency, } \eta_s = \frac{\dot{W}_a}{\dot{W}_s} = \frac{20068.5}{26259} = 0.7642$$

$$\text{and second law efficiency, } \eta_{II} = \frac{\dot{W}_a}{\text{Change in availability}} = \frac{20068.5}{\dot{m}_1 \psi_1 - \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3}$$

$$= \frac{20068.5}{30(1136.7592) - 5(720.02) - 25(195.95)}$$

$$= \frac{20068.5}{25603.026} = 0.7838$$

4. (b)

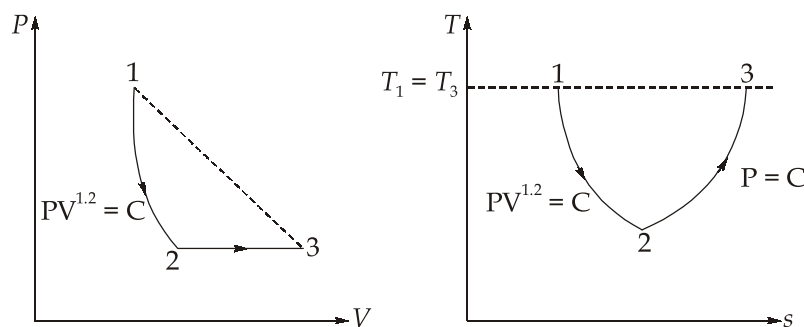
Given data: At initial condition:

$$V_1 = 0.025 \text{ m}^3, P_1 = 700 \text{ kPa}, T_1 = 97^\circ\text{C} = 370 \text{ K}$$

At state 2:

$$P_2 = P_3 = 70 \text{ kPa}$$

$$T_3 = T_1 = 370 \text{ K}$$



For polytropic process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{T_2}{370} = \left(\frac{70}{700}\right)^{\frac{1.2-1}{1.2}}$$

$$\Rightarrow T_2 = 252.08 \text{ K}$$

$$P_1 V_1^n = P_2 V_2^n$$

$$\left(\frac{V_2}{V_1}\right)^n = \frac{P_1}{P_2}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n} = \left(\frac{700}{70}\right)^{1/1.2}$$

$$\Rightarrow V_2 = 0.025 \times (10)^{1/1.2} = 0.1703 \text{ m}^3$$

$$R = C_p - C_v = 1.047 - 0.775 = 0.272 \text{ kJ/kgK}$$

For constant pressure process 2-3:

$$\frac{V_3}{V_2} = \frac{T_3}{T_2}$$



$$V_3 = \frac{370 \times 0.1703}{252.08} = 0.2499 \approx 0.25 \text{ m}^3$$

Applying equation of state at state-1

$$P_1 V_1 = mRT_1$$

$$\Rightarrow m = \frac{P_1 V_1}{RT_1} = \frac{700 \times 0.025}{0.272 \times 370} = 0.1739 \text{ kg}$$

Work done for process 1-2:

$$\begin{aligned} W_{1-2} &= \frac{P_1 V_1 - P_2 V_2}{n-1} \text{ or } \frac{mR(T_1 - T_2)}{n-1} \\ &= \frac{0.1739 \times 0.272 \times (370 - 252.08)}{1.2 - 1} = 27.88 \text{ kJ} \end{aligned}$$

Applying first law of thermodynamics for process 1-2,

$$\begin{aligned} Q_{1-2} &= dU + W_{1-2} \\ &= mC_V(T_2 - T_1) + W_{1-2} \\ &= 0.1739 \times 0.775 \times (252.08 - 370) + 27.88 \\ &= 11.99 \text{ kJ} \end{aligned}$$

$$\begin{aligned} S_2 - S_1 &= mC_P \ln \frac{T_2}{T_1} - mR \ln \frac{P_2}{P_1} \\ &= 0.1739 \left[ 1.047 \ln \frac{252.08}{370} - 0.272 \ln \frac{70}{700} \right] \\ &= 0.03904 \text{ kJ/K} \end{aligned}$$

Work done for process 2-3:

$$\begin{aligned} W_{2-3} &= P_2(V_3 - V_2) \\ &= 70 \times (0.25 - 0.1703) \\ &= 5.579 \text{ kJ} \\ Q_{2-3} &= mC_P(T_3 - T_2) = 0.1739 \times 1.047(370 - 252.08) \\ &= 21.47 \text{ kJ} \end{aligned}$$

$$\begin{aligned} S_3 - S_2 &= mC_P \ln \frac{T_3}{T_2} = 0.1739 \times 1.047 \times \ln \frac{370}{252.08} \quad [\because P_2 = P_3] \\ &= 0.06987 \text{ kJ/K} \end{aligned}$$

Work done for whole process,

$$\begin{aligned} W_{1-2-3} &= W_{1-2} + W_{2-3} \\ &= 27.88 + 5.579 \approx 33.46 \text{ kJ} \end{aligned}$$

Heat transfer for whole process,

$$\begin{aligned} Q_{1-2-3} &= Q_{1-2} + Q_{2-3} \\ &= 11.99 + 21.47 = 33.46 \text{ kJ} \end{aligned}$$

Change in entropy for whole process,

$$\begin{aligned} (S_3 - S_1) \text{ or } \Delta S &= (S_2 - S_1) + (S_3 - S_2) \\ &= 0.03904 + 0.06987 \\ &= 0.10891 \text{ kJ/K} = 108.91 \text{ J/K} \end{aligned}$$

4. (c)

Given:  $\dot{W}_{in} = 250 \text{ W}$ ,  $T_{i,iron} = 80^\circ\text{C}$ ,  $V = 90 \text{ litre}$ ,  $T_{i,water} = 20^\circ\text{C}$ ,  $T_f = 25^\circ\text{C}$ ,  $T_o = 20^\circ\text{C}$ ,

$C_{P,iron} = 0.45 \text{ kJ/kgK}$ ,  $\rho = 1000 \text{ kg/m}^3$ ,  $C_{P,w} = 4.18 \text{ kJ/kgK}$

According to exergy balance,

$$E_{in} - E_{out} = \Delta E_{system}$$

$$m_{iron} C_{P,iron} (T_f - T_{i,iron}) + m_w C_{P,w} (T_f - T_{i,water}) - \dot{W}_{in} \times t = 0$$

$$\therefore m_{iron} \times 0.45 \times (25 - 80) + 90 \times 10^{-3} \times 10^3 \times 4.18(25 - 20) - \frac{250 \times 30 \times 60}{1000} = 0$$

$$\therefore m_{iron} = 57.81 \text{ kg}$$

Now, Entropy change of iron,

$$\Delta S_{iron} = m_{iron} \times C_{P,iron} \ln\left(\frac{T_f}{T_{i,iron}}\right)$$

$$\Delta S_{iron} = 57.81 \times 0.45 \times \ln\left(\frac{25 + 273}{80 + 273}\right) = -4.406 \text{ kJ/kgK}$$

For water,

$$\Delta S_w = m_w \times C_{P,w} \times \ln\left(\frac{T_f}{T_{i,water}}\right)$$

$$= 90 \times 4.18 \times \ln\left(\frac{25 + 273}{20 + 273}\right)$$

$$= 6.365 \text{ kJ/kgK}$$

Entropy balance gives,

$$S_{gen} = \Delta S_{system} = \Delta S_{iron} + \Delta S_{water}$$

$$\therefore S_{gen} = -4.406 + 6.365 = 1.959 \text{ kJ/kgK}$$

Now, Exergy balance,  $(\Delta X)_{system} = X_{in} - X_{destroyed} = \dot{W}_{in} - T_o \dot{S}_{gen}$

$$\begin{aligned} &= \frac{250 \times 30 \times 60}{1000} - 293 \times 1.959 \\ &= -123.987 \text{ kJ} \end{aligned}$$

Negative sign indicates that exergy of system is decreased.

### Section : B

#### 5. (a)

**Secondary Refrigerants :** In large refrigeration plants, secondary refrigerants or coolants such as water, brines, glycols and sometimes even halocarbons are used for carrying refrigeration from the plant room to the space where it is usefully applied, instead of directly obtaining it by the evaporating refrigerant at the place of application. This is done in order to reduce the quantity of the refrigerant charge in the system and to reduce pressure losses in lines. The desirable properties of secondary coolants are low freezing point, low viscosity, non-flammability, good stability and low vapour pressure. Chilled water is used as a secondary refrigerant in air-conditioning applications. For low-temperature applications, brines, glycols and hydrocarbons are used.

**Brines :** Brines are secondary refrigerants and are generally used where temperatures are required to be maintained below the freezing point of water, i.e.,  $0^{\circ}\text{C}$ . In case the temperature involved is above the freezing point of water ( $0^{\circ}\text{C}$ ), then water is commonly used as a secondary refrigerant.

Brine is a solution of salt in water. It may be noted that when salt is mixed in water, then the freezing temperature of the solution becomes lower than that of the water. This is due to the fact that the salt while dissolving in water takes off its latent heat from the solution and cools it below the freezing point of water. The mass of the salt in the solution expressed as the percentage of the mass of the solution is known as **concentration** of the solution. As the concentration of the solution increases, its freezing point decreases. But if the concentration of the salt is increased beyond a certain point, the freezing point increases instead of decreasing. The point at which the freezing temperature is minimum, is known as **eutectic temperature** and the concentration at this point is known as **eutectic concentration**. The brine used in a particular application should have a concentration for which the freezing point of the brine is at least  $5^{\circ}\text{C}$  to  $8^{\circ}\text{C}$  lower than the brine temperature required.

The brines commonly used are calcium chloride ( $\text{CaCl}_2$ ), sodium chloride, i.e., common salt ( $\text{NaCl}$ ) and glycols such as ethylene glycol, propylene glycol etc.

The calcium chloride brine has the eutectic temperature of  $-55^{\circ}\text{C}$  at salt concentration of 30% by mass. This brine is primarily used where temperatures below  $-18^{\circ}\text{C}$  are required.

It is generally used in industrial process cooling and product freezing. The chief disadvantages of calcium chloride brine are its dehydrating effect and its tendency to impart a bitter taste to food products.

The sodium chloride brine has the eutectic temperature of  $-21.1^{\circ}\text{C}$  at salt concentration of 23% by mass. This brine is used in chilling and freezing of meat and fish.

Both of the above two brines are corrosive in nature for metallic containers which put limitation on their use. Also the thermal properties of the above two brines are less satisfactory.

Other water soluble compounds known as **antifreeze** are also used for decreasing the freezing point of water for certain refrigeration uses. Ethylene and propylene glycol have a number of good properties. Since they are non-corrosive and non-electrolytic even in the presence of water, therefore, these brines are most extensively used as antifreeze elements. The following table shows typical applications of various brines.

Application	Brine Used
1. Breweries	Propylene glycol
2. Chemical plants	Sodium chloride, Calcium chloride, Ethylene glycol
3. Dairies	Sodium chloride, Calcium chloride, Propylene glycol
4. Food process	Sodium chloride, Calcium chloride, Propylene glycol
5. Ice-creams	Calcium chloride, Propylene glycol
6. Ice-plant	Sodium chloride
7. Meat packing	Sodium chloride, Calcium chloride
8. Skating ring	Calcium chloride, Ethylene glycol
9. Special low temperature	Calcium chloride, Ethylene glycol

**Inhibited Glycols :** Ethylene glycol and propylene glycol are also used as freezing point depressants and heat transfer media in solution with water. Freezing points of ethylene glycol solutions are lower than those of propylene glycol. It is more commonly used except when, because of its toxicity, it is found to attack a food product by direct contact, in which case propylene glycol is used.

Glycol solutions have lower corrosivity when properly inhibited. The specific gravities of ethylene glycol and propylene glycol are 1.1155 and 1.0381 respectively. Their specific heats at  $15^{\circ}\text{C}$  are 9.82 and 10.38 kJ/(kg.K) respectively.

Some commonly used refrigerants are :

**Ammonia :** Used with reciprocating and screw compressors in cold storages.

**Water :** Used in Lithium-Bromide absorption system.

**Carbon dioxide :** Used in frozen food transport refrigeration.

**R-11 :** Used in central air conditioning.

**R-12 :** Used in domestic refrigerator.

**R-22 :** Used in room air conditioner.

**R-502 :** For large supermarket frozen food cabinets involving high pressure ratio applications.

5. (b)

The enthalpy of moist air can also be written in the form

$$\begin{aligned}
 h &= (C_{pa} + \omega C_{pv})t + \omega(h_{fg})_{0^\circ\text{C}} \\
 &= C_p t + \omega(h_{fg})_{0^\circ\text{C}} \quad \dots (1)
 \end{aligned}$$

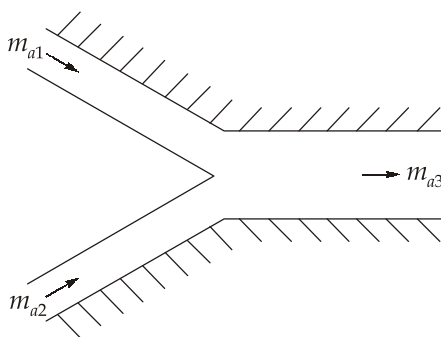
Let subscripts 1 and 2 refer to the two streams of air, and let  $m_a$  refer to the mass of dry air in the stream. Then by moisture balance, we have for the specific humidity of the mixture,

$$\begin{aligned}
 m_{a3} \omega_3 &= m_{a1} \omega_1 + m_{a2} \omega_2 \\
 \Rightarrow \omega_3 &= \frac{m_{a1} \omega_1 + m_{a2} \omega_2}{m_{a3}}
 \end{aligned}$$

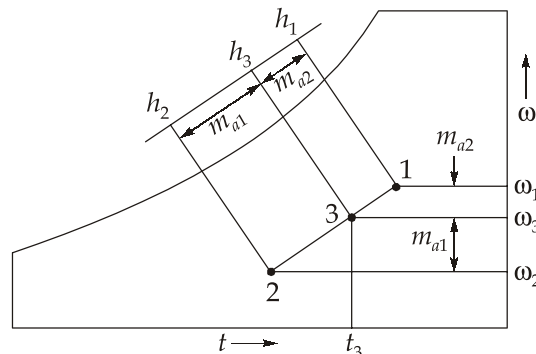
where by dry air mass balance,

$$m_{a3} = m_{a1} + m_{a2}$$

is the mass of dry air in the mixture.



(a) Adiabatic mixing of air streams



(b) Mixing process on psychrometric chart

Also, by energy balance, we similarly get the expression for the enthalpy of the mixture

$$h_3 = \frac{m_{a1} h_1 + m_{a2} h_2}{m_{a3}}$$

Substituting expressions from eqn. (1) for the enthalpies in the above equation, we have

$$(C_p t_3 + h_{fg0} \omega_3) = \frac{m_{a1}}{m_{a3}} (C_p t_1 + h_{fg0} \omega_1) + \frac{m_{a2}}{m_{a3}} (C_p t_2 + h_{fg0} \omega_2)$$

Simplifying, we get an expression for the temperature of the mixture

$$t_3 = \frac{m_{a1}t_1 + m_{a2}t_2}{m_{a3}} + \frac{h_{fg_o}}{C_p} \left[ \frac{m_{a1}}{m_{a3}}\omega_1 + \frac{m_{a2}}{m_{a3}}\omega_2 - \omega_3 \right]$$

The second term in the above expression being negligible, we can write

$$t_3 \approx \frac{m_{a1}t_1 + m_{a2}t_2}{m_{a3}}$$

The sign of approximation has been used since an assumption has been made that the humid specific heat  $C_p$  is the same for all three streams.

Thus, if the psychrometric chart had been plotted on a  $\omega$ - $h$  coordinate system, the state point for the mixture would lie on the straight line joining the two states. On the  $\omega$ - $t$  coordinate system, it is only approximately so. The position of the mixture state is such that it divides the straight line joining states 1 and 2 in the inverse ratio of the masses  $m_{a1}$  and  $m_{a2}$  of the two dry air streams.

(ii) For the first stream :

$$\omega_1 = 0.0035 \text{ kg w.v./kg of d.a.}$$

$$h_1 = 10.78 \text{ kJ/kg d.a.}$$

Mass of dry air per unit mass of moist air

$$m_{a1} = \frac{1}{1 + \omega_1} = \frac{1}{1 + 0.0035} = 0.9965 \text{ kg}$$

For the second stream :

$$\omega_2 = 0.00765 \text{ kg w.v./kg of d.a.}$$

$$h_2 = 49.67 \text{ kJ/kg d.a.}$$

Mass of dry air per two unit mass of moist air

$$m_{a2} = \frac{2}{1 + \omega_2} = \frac{2}{1.00765} = 1.9848 \text{ kg}$$

For the mixture :

$$h = \frac{0.9965 \times 10.78 + 1.9848 \times 49.67}{0.9965 + 1.9848} = 36.68 \text{ kJ/kg d.a.}$$

$$\omega = \frac{0.9965 \times 0.0035 + 1.9848 \times 0.00765}{0.9965 + 1.9848}$$

$$= 0.00627 \text{ kg w.v./kg of d.a.}$$

5. (c)

The volumetric efficiency of a reciprocating compressor is given by,

$$\eta_v = 1 + C - C \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \quad \dots(i)$$

where,

$C$  = Clearance ratio,

$P_1$  = Suction pressure,

$P_2$  = Discharge pressure, and

$n$  = Polytropic index of expansion.

The various factors which affect the volumetric efficiency are discussed below :

1. **Effect of clearance ratio:** The equation (i) may be rewritten as

$$\eta_v = 1 + C \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

We know that the discharge pressure ( $P_2$ ) is always more than the suction pressure

( $P_1$ ). Therefore, the ratio  $\left( \frac{P_2}{P_1} \right)^{\frac{1}{n}}$  will be more than unity and the expression

$\left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$  will be negative. Thus, the increase in clearance ratio ( $C$ ) will increase

the quantity to be subtracted from (i) to give volumetric efficiency. Hence, the volumetric efficiency decreases as the clearance ratio increases.

2. **Effect of valve pressure drops :** The  $P$ - $v$  diagram of a reciprocating compressor considering the pressure drop at the suction and discharge valves is shown in figure. In actual practice, the compression will start from pressure  $P_s$  at point 1' and expansion from pressure  $P_d$  at point 3'. The pressure  $P_1 = P_4$  is the pressure at the flange on suction side and  $P_2 = p_3$  is the pressure at the flange on discharge side.

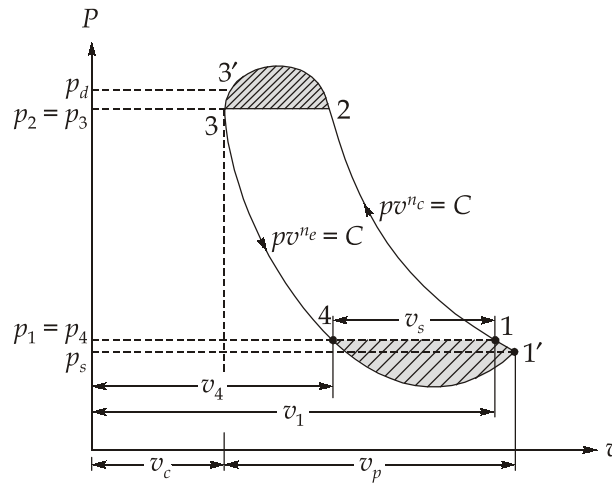
Applying  $Pv^{n_c} = C$  to points 1 and 1',

$$P_1(v_1)^{n_c} = P_s(v_c + v_p)^{n_c}$$

$$\therefore v_1 = (v_c + v_p) \left( \frac{P_s}{P_1} \right)^{\frac{1}{n_c}}$$

Similarly, applying  $Pv^{n_e} = C$  to points 3' and 4,

$$P_d(v_c)^{n_e} = P_1(v_4)^{n_e}$$



**P-v Diagram with valve pressure drops**

$$\therefore v_4 = v_c \left( \frac{P_d}{P_1} \right)^{\frac{1}{n_e}}$$

We know that volumetric efficiency,

$$\eta_v = \frac{v_1 - v_4}{v_p}$$

Substituting the values of  $v_1$  and  $v_4$

$$\eta_v = \frac{(v_c + v_p) \left( \frac{P_s}{P_1} \right)^{\frac{1}{n_c}} - v_c \left( \frac{P_d}{P_1} \right)^{\frac{1}{n_e}}}{v_p}$$

$$= (C + 1) \left( \frac{P_s}{P_1} \right)^{\frac{1}{n_c}} - C \left( \frac{P_d}{P_1} \right)^{\frac{1}{n_e}} \quad \left( \because \frac{v_c}{v_p} = C \right)$$

$$= (1 + C) \left( \frac{P_s}{P_1} \right)^{\frac{1}{n_c}} - C \left( \frac{P_d}{P_1} \right)^{\frac{1}{n_e}}$$



**Note :**

- (a) When the pressure drop at suction valve is only considered, then  $P_d = P_2 = P_3$ . In this case,

$$\eta_v = (1+C) \left( \frac{P_s}{P_1} \right)^{\frac{1}{n_c}} - C \left( \frac{P_2}{P_1} \right)^{\frac{1}{n_e}}$$

- (b) When the pressure drop at discharge valve is only considered, then  $P_s = P_1 = P_3$ . In this case,

$$\eta_v = 1 + C - C \left( \frac{P_d}{P_1} \right)^{\frac{1}{n_e}}$$

3. **Effect of heat gain from the cylinder walls and re-expansion index :** The effect of heat gain from the cylinder walls is to increase the expanded clearance volume ( $v_4$ ) and to decrease the suction volume ( $v_s$ ) which decreases the volumetric efficiency. In order to reduce the work of compression, the normal practice is to cool the cylinder either by water jackets provided around the cylinder as in the case of ammonia compressors or by surrounding air by natural convection through fins provided on the external surface of the cylinder as in the case of R-12 compressors. The re-expansion index ( $n_e$ ) is generally less than the isentropic index ( $\gamma$ ) which gives a reduction in volumetric efficiency. Since the isentropic index for ammonia is high, therefore it will result in high discharge and cylinder wall temperatures. The cooling by water jacketing of ammonia compressors will result in lower discharge and cylinder wall temperatures and thus higher volumetric efficiency. The isentropic index for the freon group of refrigerants is quite low. This gives large expanded clearance volume ( $v_4$ ) and hence low volumetric efficiency. It is, therefore, necessary to provide minimum possible clearance in these compressors. It may be noted that the compressors used for freon group of refrigerants need to be air-cooled only, because of low isentropic index.
4. **Effect of valve and piston leakage :** The effect of gas leakage through the suction or discharge valves or past the piston rings is to decrease the volumetric efficiency of compressor. This factor is accounted for by allowing one percent of the compression ratio ( $r$ ), i.e.,

$$\text{Leakage loss} = 0.01r = 0.01 \left( \frac{P_d}{P_s} \right)$$

5. (d)

Given :

$$P = 2 \text{ bar}, T_E = -5^\circ\text{C} = 268 \text{ K}, RC = 20TR$$

$$\text{Actual COP} = 70\% \text{ of } \text{COP}_{\max}, T_C = 30^\circ\text{C} = 303 \text{ K}$$

$$T_G = T_{\text{sat@2 bar}} = 120.2^\circ\text{C} = 393.2 \text{ K}, h_{fg} = 2201.6 \text{ kJ/kg}$$

$$\text{So, Maximum COP} = \left[ \frac{T_E}{T_C - T_E} \right] \left[ \frac{T_G - T_C}{T_G} \right]$$

$$\text{COP}_{\max} = \left( \frac{268}{303 - 268} \right) \left( \frac{393.2 - 303}{393.2} \right) = 1.757$$

**Mass of steam required per hour :**

$$\begin{aligned} \text{Actual COP} &= 0.7 \times \text{COP}_{\max} = 0.7 \times 1.757 \\ &= 1.23 \end{aligned}$$

$$\begin{aligned} \text{Actual heat supplied} &= \frac{RC}{\text{Actual COP}} = \frac{20 \times 210}{1.23} \\ &= 3414.63 \text{ kJ/min} \end{aligned}$$

As only latent heat is used for heating purposes. So, mass of steam required per hour

$$\begin{aligned} &= \frac{\text{Actual heat supplied}}{h_{fg}} \\ &= \frac{3414.63}{2201.6} \times 60 \text{ kg/h} = 93.06 \text{ kg/h} \end{aligned}$$

5. (e)

(i) Outside air sensible heat

$$\begin{aligned} \text{OASH} &= 0.0204 \text{ cmm } \Delta T = 0.0204 \times 70 \times (40 - 25) \\ &= 21.42 \text{ kW} \end{aligned}$$

Outside air latent heat

$$\begin{aligned} \text{OALH} &= 50 \text{ cmm } \Delta \omega = 50 \times 70 \times (0.0175 - 0.01) \\ &= 26.25 \text{ kW} \end{aligned}$$

Outside air total heat or ventilation load

$$\text{OATH} = \text{OASH} + \text{OALH} = 21.42 + 26.25 = 47.67 \text{ kW}$$

(ii) Total sensible heat

$$\text{TSH} = \text{RSH} + \text{OASH} = 58.15 + 21.42 = 79.57 \text{ kW}$$

Total latent heat

$$\text{TLH} = \text{RLH} + \text{OALH} = 14.54 + 26.25 = 40.79 \text{ kW}$$

Grand total heat

$$\text{GTH} = \text{TSH} + \text{TLH} = 79.57 + 40.79 = 120.36 \text{ kW}$$

(iii) Bypass factor,  $x = 0.15$

Effective room sensible heat

$$\begin{aligned} \text{ERSH} &= \text{RSH} + x(\text{OASH}) \\ &= 58.15 + 0.15(21.42) = 61.36 \text{ kW} \end{aligned}$$

Effective room latent heat

$$\begin{aligned} \text{ERLH} &= \text{RLH} + x(\text{OALH}) \\ &= 14.54 + 0.15(26.25) = 18.48 \text{ kW} \end{aligned}$$

Effective sensible heat factor

$$\text{ESHF} = \frac{\text{ERSH}}{\text{ERSH} + \text{ERLH}} = \frac{61.36}{61.36 + 18.48} = 0.77$$

6. (a)

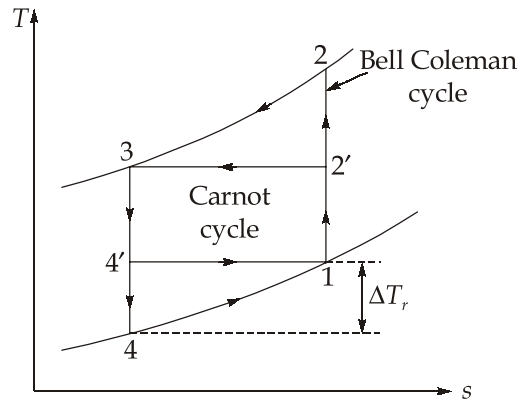
Both cycles are shown on T-s diagram considering compression and expansion being isentropic. The COP of Bell-Coleman and reversed Carnot cycles for the temperatures are given by

$$(\text{COP})_{\text{Bell-Coleman}} = \left( \frac{T_4}{T_3 - T_4} \right)$$

$$(\text{COP})_{\text{Carnot}} = \left( \frac{T_1}{T_3 - T_1} \right)$$

$$\text{Ratio of COP} = \frac{\frac{T_4}{(T_3 - T_4)}}{\frac{T_1}{(T_3 - T_1)}} = \frac{T_4}{T_1} \left( \frac{T_3 - T_1}{T_3 - T_4} \right) \quad \dots (i)$$

In this case,  $T_3 = T_a$  and  $T_4 = T_r - \Delta T_r$  where  $T_1 = T_r$  and  $\Delta T_r = T_1 - T_4$ .



$$\begin{aligned}
 \therefore \text{Ratio of COP} &= \left( \frac{T_r - \Delta T_r}{T_r} \right) \left( \frac{T_a - T_r}{T_a - (T_r - \Delta T_r)} \right) = \frac{(T_r - \Delta T_r)}{(T_a - T_r + \Delta T_r)} \times \frac{(T_a - T_r)}{T_r} \\
 &= \frac{T_a T_r - \Delta T_r \cdot T_a - T_r^2 + \Delta T_r \cdot T_r}{T_a T_r - T_r^2 + \Delta T_r \cdot T_r} \\
 &= 1 - \frac{T_a \cdot \Delta T_r}{T_a T_r - T_r^2 + \Delta T_r \cdot T_r} \\
 &= 1 - (T_a \cdot \Delta T_r) [T_a T_r - T_r^2 + \Delta T_r \cdot T_r]^{-1}
 \end{aligned}$$

When  $T_a$  and  $\Delta T_r$  are constant, the above ratio becomes maximum when

$$\frac{d}{dT_r} [\text{Ratio of COP}] = 0$$

$$\frac{d}{dT_r} [1 - (T_a \cdot \Delta T_r) (T_a T_r - T_r^2 + \Delta T_r \cdot T_r)^{-1}] = 0$$

$$\therefore (T_a \cdot T_r - T_r^2 + \Delta T_r \cdot T_r)^{-2} (T_a - 2T_r + \Delta T_r) = 0$$

$$\therefore T_a - 2T_r + \Delta T_r = 0$$

$$\Delta T_r = 2T_r - T_a$$

Substituting this value in equation (i), we get

$$\text{Ratio of COP} = \frac{(T_r - 2T_r + T_a)}{(T_a - T_r + 2T_r - T_a)} \times \frac{T_a - T_r}{T_r}$$

$$\text{Ratio of COP} = \frac{(T_a - T_r)^2}{T_r^2} = \left( \frac{T_a - T_r}{T_r} \right)^2$$

6. (b)

**Effective Temperature :** The degree of warmth or cold felt by a human body depends mainly on the following three factors : (i) Dry bulb temperature; (ii) Relative humidity; and (iii) Air velocity.

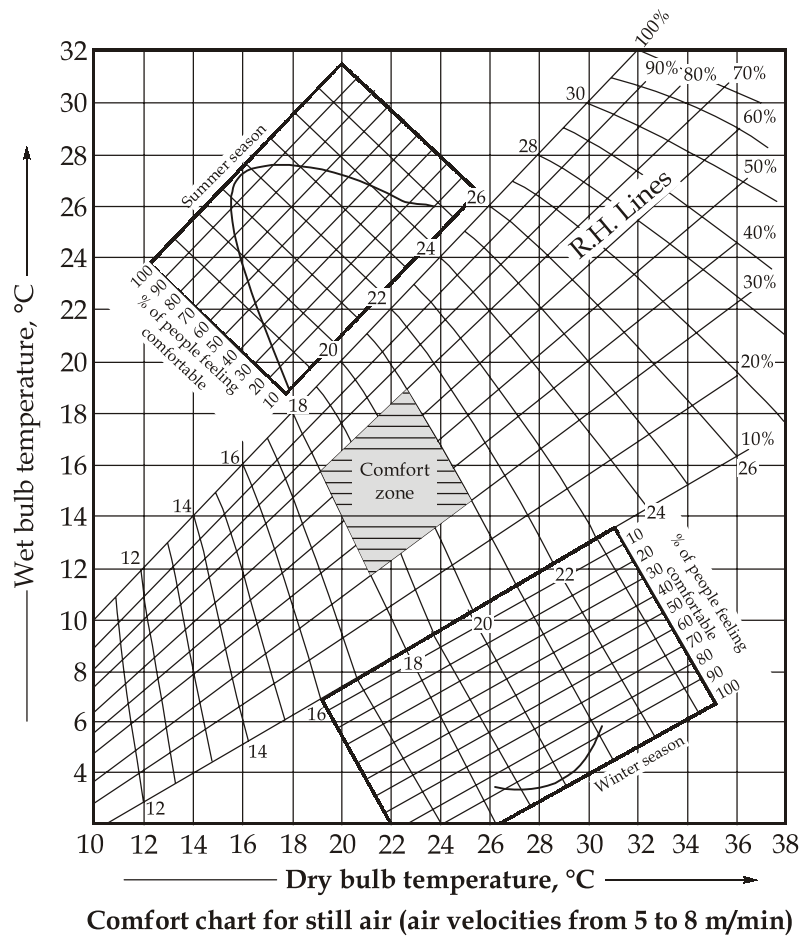
In order to evaluate the combined effect of these factors, the term **effective temperature** is employed. It is defined as that index which correlates the combined effects of air temperature, relative humidity and air velocity on the human body. The numerical value of effective temperature is made equal to the temperature of still (i.e., 5 to 8 m/min air velocity) saturated air, which produces the same sensation of warmth or coolness as produced under the given conditions.

The practical application of the concept of effective temperature is presented by the **comfort chart**, as shown in figure. This chart is the result of research made on different kinds of people subjected to wide range of environmental temperature, relative humidity and air movement by the American Society of Heating, Refrigeration and Air Conditioning Engineers (ASHRAE). It is applicable to reasonably still air (5 to 8 m/min air velocity) to situations where the occupants are seated at rest or doing light work and to spaces whose enclosing surfaces are at a mean temperature equal to the air dry bulb temperature.

In the comfort chart, as shown in figure, the dry bulb temperature is taken as abscissa and the wet bulb temperature as ordinates. The relative humidity lines are replotted from the psychrometric chart. The statistically prepared graphs corresponding to summer and winter season are also superimposed. These graphs have effective temperature scale as abscissa and % of people feeling comfortable as ordinate.

A close study of the chart reveals that the several combinations of wet and dry bulb temperatures with different relative humidities will produce the same effective temperature. However, all points located on a given effective temperature line do not indicate conditions of equal comfort or discomfort. The extremely high or low relative humidities may produce conditions of discomfort regardless of the existent effective temperature. The moist desirable relative humidity range lies between 30 and 70 per cent. When the relative humidity is much below 30 per cent, the mucous membranes and the skin surface become too dry for comfort and health. On the other hand, if the relative humidity is above 70 per cent, there is a tendency for a clammy or sticky sensation to develop. The curves at the top and bottom, as shown in figure indicate the percentages of person participating in tests, who found various effective temperatures satisfactory for comfort.

The comfort chart shows the range for both summer and winter condition within which a condition of comfort exists for most people. For summer conditions, the chart indicates that a maximum of 98 percent people felt comfortable for an effective temperature of 21.6°C. For winter conditions, chart indicates that an effective temperature of 20°C was desired by 97.7 percent people. It has been found that for comfort, women require 0.5°C higher effective temperature than men. All men and women above 40 years of age prefer 0.5°C higher effective temperature than the persons below 40 years of age.



It may be noted that the comfort chart, as shown in figure, does not take into account the variations in comfort conditions when there are wide variations in the mean radiant temperature (MRT). In the range of  $26.5^{\circ}\text{C}$ , a rise of  $0.5^{\circ}\text{C}$  in mean radiant temperature above the room dry bulb temperature raises the effective temperature by  $0.5^{\circ}\text{C}$ . The effect of mean radiant temperature on comfort is less pronounced at high temperatures than at low temperatures.

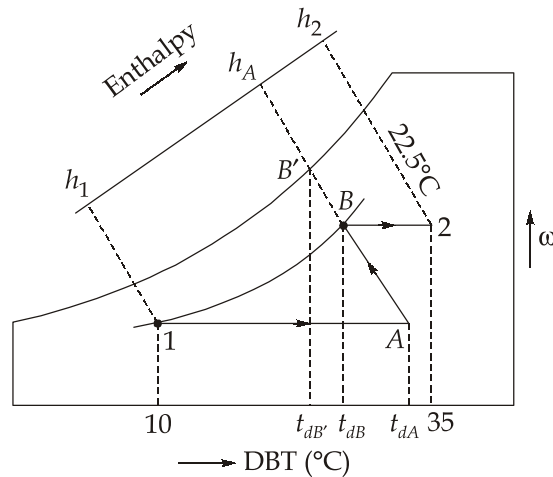
The comfort conditions for persons at work vary with the rate of work and the amount of clothing worn. In general, the greater the degree of activity, the lower the effective temperature necessary for comfort.

6. (c)

Given:  $t_{dbt_1} = 10^{\circ}\text{C}$ ,  $\phi_1 = 90\%$ ,  $t_{dbt_2} = 35^{\circ}\text{C}$ ,  $t_{wbt_2} = 22.5^{\circ}\text{C}$

First of all, mark the initial condition of air, i.e., at  $10^{\circ}\text{C}$  DBT and 90% RH, on the psychrometric chart at point-1 as shown in figure. Now mark final condition of air, i.e., at  $35^{\circ}\text{C}$  DBT and  $22.5^{\circ}\text{C}$  WBT at point-2.

From point-1, draw a horizontal line to represent sensible heating and from point-2 draw horizontal line to intersect 90% RH curve at point B. Now from point B, draw a constant WBT line which intersects the horizontal line drawn through point-1 at point A. The line 1-A represents preheating of air, line AB represents cooling and humidification and line B-2 represents sensible heating to final condition.



- (i) **Temperature to which the air should be preheated :** From the psychrometric chart, the temperature to which the air should be preheated (corresponding to point A) is

$$t_{dA} = 30^{\circ}\text{C}$$

- (ii) **Total heating required :** From the psychrometric chart, we find

Enthalpy of air at point 1,

$$h_1 = 29 \text{ kJ/kg of d.a}$$

Enthalpy of air at point A,

$$h_A = 49.5 \text{ kJ/kg of d.a}$$

Enthalpy of air at point 2,

$$h_2 = 66 \text{ kJ/kg of d.a}$$

We know that heat required for preheating of air

$$= h_A - h_1 = 49.5 - 29 = 20.5 \text{ kJ/kg of d.a}$$

and heat required for reheating of air

$$= h_2 - h_B = 66 - 49.5 = 16.5 \text{ kJ/kg of d.a}$$

So, Total heat required = 20.5 + 16.5 = 37 kJ/kg of d.a

- (iii) **Make up water required in the air washer :**

From the psychrometric chart, we find that specific humidity of entering air

$$\omega_1 = 0.0070 \text{ kg/kg of d.a}$$

Specific humidity of leaving air,

$$\omega_2 = 0.0120 \text{ kg/kg of d.a}$$

Make up water required in the air washer

$$= \omega_B - \omega_A = \omega_2 - \omega_1$$

$$= 0.0120 - 0.0070$$

$$= 0.005 \text{ kg/kg of d.a} = 5.0 \text{ g/kg of d.a}$$

(iv) **Humidifying efficiency of air washer :**

From psychrometric chart, we find that

$$t_{dB} = 18.5^\circ\text{C}, t_{dB'} = 17.5^\circ\text{C}, t_{dA} = 30^\circ\text{C}$$

We know that humidifying efficiency of air washer,

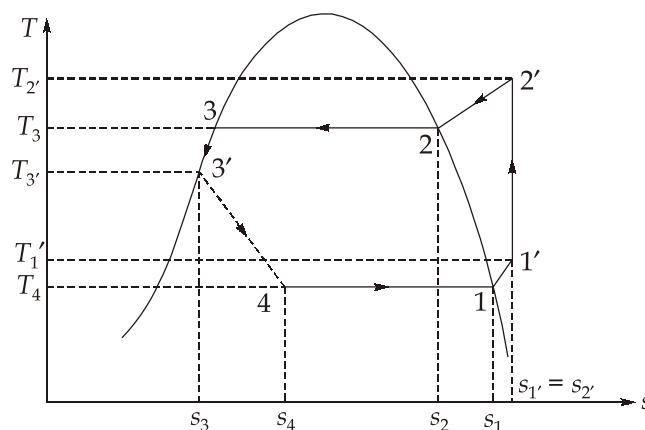
$$\eta_H = \frac{\text{Actual drop in DBT}}{\text{Ideal drop in DBT}}$$

$$= \frac{t_{dA} - t_{dB}}{t_{dA} - t_{dB'}} = \frac{30 - 18.5}{30 - 17.5} = \frac{11.5}{12.5} = 0.92 = 92.00\%$$

7. (a)

Given:  $T_2 = T_3 = 40^\circ\text{C} = 313 \text{ K}$ ,  $T_1 = T_4 = -20^\circ\text{C} = 253 \text{ K}$ ,  $T_3' = 26^\circ\text{C} = 299 \text{ K}$ ,  $Q = 10 \text{ kW}$ ,  
 $C_{pV} = 1.03 \text{ kJ/kgK}$ ,  $v_1 = 0.093 \text{ m}^3/\text{kg}$ ,  $h_1 = 243.25 \text{ kJ/kg}$ ,  $s_1 = 0.9638 \text{ kJ/kgK}$ ,

$h_{f3'} = h_4 = 79.74 \text{ kJ/kg}$ ,  $h_{f3} = 97.94 \text{ kJ/kg}$ ,  $h_2 = 263.21 \text{ kJ/kg}$ ,  $s_{f3} = 0.3563 \text{ kJ/kgK}$ ,  
 $s_2 = 0.8822 \text{ kJ/kgK}$



Considering thermal equilibrium of heat exchanger.

Heat lost by liquid refrigerant = Heat gained by vapour refrigerant

$$h_{f3} - h_{f3'} = h_{1'} - h_1$$



$$97.94 - 79.74 = h_{1'} - 243.25$$

$$\Rightarrow h_{1'} = 261.45 \text{ kJ/kg}$$

$$\text{and } C_{PV}(T_{1'} - T_1) = h_{1'} - h_1$$

$$\Rightarrow T_{1'} = \frac{261.45 - 243.25}{1.03} + 253$$

$$T_{1'} = 270.67 \text{ K}$$

$$\begin{aligned} \text{Entropy at point 1', } s_{1'} &= s_1 + C_{PV} \ln\left(\frac{T_{1'}}{T_1}\right) = 0.9638 + 1.03 \ln\left(\frac{270.67}{253}\right) \\ &= 1.0333 \text{ kJ/kgK} \end{aligned}$$

$$\text{Entropy at point 2', } s_{2'} = s_2 + C_{PV} \ln\left(\frac{T_{2'}}{T_2}\right)$$

$$\text{and we know that } s_{2'} = s_{1'}$$

$$\Rightarrow 1.0333 = 0.8822 + 1.03 \ln\left(\frac{T_{2'}}{313}\right)$$

$$\Rightarrow T_{2'} = 362.45 \text{ K}$$

$$\text{Enthalpy at point 2', } h_{2'} = h_2 + C_{PV}(T_{2'} - T_2) = 263.21 + 1.03(362.45 - 313)$$

$$h_{2'} = 314.14 \text{ kJ/kg}$$

### Power requirement :

Refrigerant effect (R.E.) per kg of refrigerant

$$\begin{aligned} RE &= h_1 - h_{f3'} && (\because h_{f3'} = h_4) \\ &= 243.25 - 79.74 \\ &= 163.51 \text{ kJ/kg} \end{aligned}$$

$$\text{Mass flow of refrigerant, } \dot{m} = \frac{Q}{RE} = \frac{10}{163.51} = 0.0611 \text{ kg/s}$$

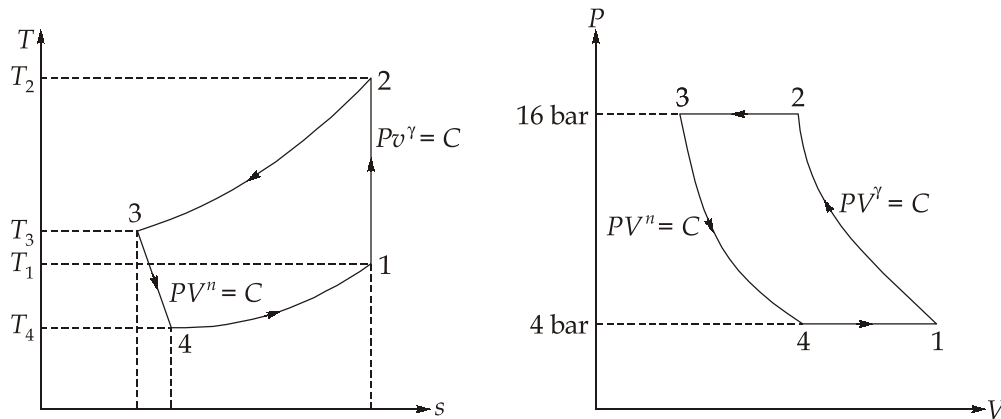
and power required during compression of refrigerant

$$\begin{aligned} P_{in} &= \dot{m}(h_{2'} - h_{1'}) \\ &= 0.0611(314.14 - 261.45) \\ &= 3.22 \text{ kJ/s} \\ &= 3.22 \text{ kW} \end{aligned}$$

### Coefficient of performance :

$$\text{C.O.P.} = \frac{RE}{P_{in}} = \frac{163.51}{\left(\frac{3.22}{0.0611}\right)} = 3.1$$

7. (b)



Given: R.C. =  $Q = 10$  TR,  $P_1 = P_4 = 4$  bar,  $P_2 = P_3 = 16$  bar,  $T_1 = 0^\circ\text{C} = 273$  K,  $T_3 = 25^\circ\text{C} = 298$  K,  $\eta_e = 80\% = 0.80$ ,  $\eta_c = 85\% = 0.85$ ,  $N = 250$  rpm,  $L = 250$  mm = 0.25 m,  $n = 1.25$

For isentropic compression process 1-2

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$$T_2 = T_1 \times 1.486 = 273 \times 1.486 = 405.7 \text{ K}$$

For polytropic expansion process 3-4

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{n-1}{n}} = \left(\frac{16}{4}\right)^{\frac{1.25-1}{1.25}} = 1.32$$

$$T_4 = \frac{T_3}{1.32} = 225.7 \text{ K}$$

(i) During process 1-2, workdone by the compressor

$$\begin{aligned} W_c = W_{1-2} &= \frac{\gamma}{\gamma-1} R(T_2 - T_1) \times \frac{1}{\eta_c} \\ &= \frac{1.4}{0.4} \times 0.287(405.7 - 273) \times \frac{1}{0.85} = 156.8 \text{ kJ/kg} \end{aligned}$$

Work done by expander during process 3-4

$$\begin{aligned} W_e = W_{3-4} &= \frac{n}{n-1} R(T_3 - T_4) \eta_e \\ &= \frac{1.25}{1.25-1} \times 0.287 \times (298 - 225.7) \times 0.8 = 83 \text{ kJ/kg} \end{aligned}$$

So, net work done  $W_{\text{net}} = W_c - W_e = 156.8 - 83 = 73.8 \text{ kJ/kg}$

Refrigeration effect,

$$RE = C_p(T_1 - T_4) = 1.005(273 - 225.7) = 47.5 \text{ kJ/kg}$$

$$\text{COP} = \frac{RE}{W_{\text{net}}} = \frac{47.5}{73.8} = 0.644$$

(ii) Power required :

Let

$$\dot{m}_a = \text{Mass of air circulated in kg per sec}$$

$$RC = 10TR = 10 \times 3.5 = 35 \text{ kW}$$

So,

$$\dot{m}_a = \frac{RC}{RE} = \frac{35}{47.5} = 0.74 \text{ kg/s}$$

So,

$$\text{Power required} = \dot{m}_a \times W_{\text{net}} = 0.74 \times 73.8$$

$$\text{Power required} = 54.612 \text{ kW}$$

(iii) Bore of compression and expansion cylinders :

Let

$$D = \text{Bore of compression cylinder}$$

$$d = \text{Bore of expansion cylinder}$$

$$v_1 = \text{Piston displacement of compressor}$$

$$v_4 = \text{Piston displacement of expander}$$

We know that,

$$P_1 v_1 = m_a R_a T_1$$

$$v_1 = \frac{m_a R_a T_1}{P_1} = \frac{0.74 \times 287 \times 273}{4 \times 10^5}$$

⇒

$$v_1 = 0.145 \text{ m}^3/\text{sec}$$

We also know that for double acting cylinder

$$v_1 = \left( \frac{\pi}{4} D^2 \times L \times 2 \right) \frac{N}{60}$$

$$0.145 = \frac{\pi}{4} \times D^2 \times 0.25 \times 2 \times \frac{250}{60}$$

⇒

$$D = 0.2977 \text{ m} = 297.7 \text{ mm}$$

Now for constant pressure process 4-1

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

⇒

$$v_4 = v_1 \times \frac{T_4}{T_1} = 0.145 \times \frac{225.7}{273}$$

⇒

$$v_4 = 0.12 \text{ m}^3/\text{sec}$$

Now,

$$v_4 = \left( \frac{\pi}{4} \times d^2 \times L \times 2 \right) \frac{N}{60}$$

$$0.12 = \frac{\pi}{4} d^2 \times 0.25 \times 2 \times \frac{250}{60}$$

$$d = 0.2707 \text{ m} = 270.7 \text{ mm}$$

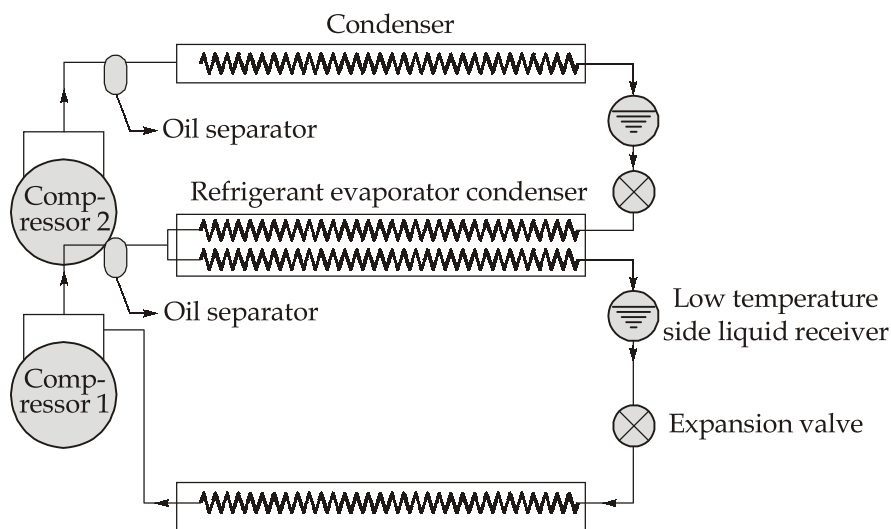
So, Bore of compression cylinder = 297.7 mm

Bore of expansion cylinder = 270.7 mm

7. (c)(i)

S.No.	Aspect	VARs	VCRs
1.	Quality of Energy	Low grade energy sources are more than capable of running VAR system. These sources can be waste heat from furnaces, exhaust steam etc. Solar power can also be used to run it.	VCR system needs high grade energy. It needs electrical or mechanical energy to run compressor which is essential part of VCR system.
2.	Moving Parts in the system	The only moving part of VAR system is pump.	In this moving part is compressor which is operated by electric motor.
3.	Effect of evaporator pressure	Very little effect is seen in refrigeration capacity with lowering evaporator pressure.	Performance reduces with varying load.
4.	Evaporator exit	In VAR system, if the liquid refrigerant leaves the evaporator, the refrigeration effect is reduced but the system functions well.	Liquid refrigerant at exit of evaporator is not desirable in this because it may damage the compressor. So, in VCR system refrigerant at exit of evaporator is in superheated state.
5.	COP	The COP of VAR system is poor.	The COP of VCR system is very good.
6.	Workability at varying load	Load variation does not have any effect on VAR system.	At partial load VCR system does not work well and its performance is very poor.
7.	Lowest Temperature	When water is used as refrigerant then the temperature attained is above 0°C.	-150° or even lower temperature can be achieved by the cascading system.
8.	Capacity	Capacity above 1000 TR is easily achievable.	It is difficult to achieve capacity above 1000 TR with single compression system.
9.	Noise and Vibration	Noise and vibration are minimised.	High noise and vibration.
10.	Maintenance	Extremely low maintenance is required.	Usual maintenance is required.

- (ii) **Cascade Refrigeration System :** The cascade refrigeration system is a freezing system that uses two kinds of refrigerants having different boiling points, which run through their own independent freezing cycle and are joined by a heat exchanger. Multistage compression is employed when low evaporator temperatures are required and when the pressure ratio is high. If vapour compression systems are to be used for the production of low temperatures, the common alternative to stage compression is the cascade systems in which series of refrigerants, with progressively lower boiling points are used in a series of single-stage units. The use of single refrigerant in a simple vapour compression cycle for production of low temperatures is limited by the following reasons :

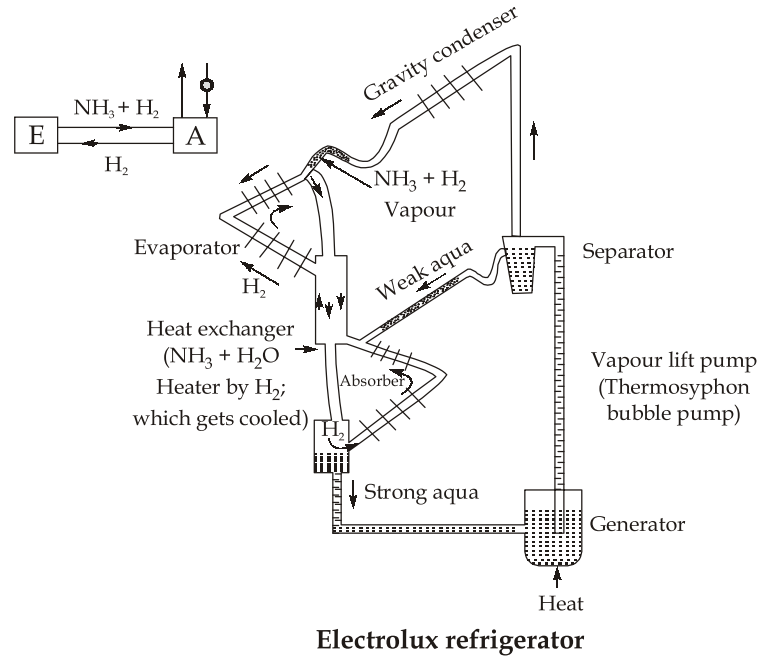


- (i) Solidification temperature of the refrigerant.
- (ii) Extremely low pressures in the evaporator and large suction volumes if a high-boiling refrigerant is selected.
- (iii) Extremely high pressures in the condenser if a low-boiling refrigerant is selected.
- (iv) Very high pressure ratio and therefore low coefficient of performance.

If lower and upper cycles have COPs  $C_1$  and  $C_2$  respectively, then combined COP of the system will be :

$$C = \frac{C_1 \times C_2}{1 + C_1 + C_2}$$

8. (a)



Electrolux principle works on 3-fluid system. There is no solution circulation pump. Total pressure is the same throughout the system. The third fluid remains mainly in the evaporator thus reducing partial pressure of refrigerant to enable it to evaporate at low pressure and hence low temperature.

The schematic diagram of the electrolux refrigerator working on  $\text{NH}_3\text{-H}_2\text{O}$  system with  $\text{H}_2$  as the third fluid is shown in figure. Liquid  $\text{NH}_3$  evaporates in the evaporator in the presence of  $\text{H}_2$ . Hydrogen is chosen as it is non-corrosive and insoluble in water.

A **thermosyphon bubble pump** is used to lift the weak aqua from the generator to the separator. The discharge tube from the generator is extended down below the liquid level in the generator. The bubbles rise and carry slugs of weak  $\text{NH}_3\text{-H}_2\text{O}$  solution into the separator.

Two U-bends are provided as vapour-locks to prevent  $\text{H}_2$  from getting into the high side or solution circuit.

Partial pressure of  $\text{H}_2$  provides the pressure difference of  $\text{NH}_3$  between the condenser and the evaporator. Accordingly, we have :

In condenser, Pure  $\text{NH}_3$  vapour pressure = Total pressure

In evaporator,  $\text{NH}_3$  vapour pressure = Total pressure - Partial pressure of  $\text{H}_2$

For example, consider the condenser temperature as  $50^{\circ}\text{C}$ , and evaporator temperature as  $-15^{\circ}\text{C}$ . The corresponding vapour pressures of  $\text{NH}_3$  are :

$$\text{Condenser, } p_k = 20.33 \text{ bar}$$

$$\text{Evaporator outlet, } p_{0_2} = 2.36 \text{ bar}$$

The approximate pressures in various parts of the system, then will be as given in table.

**Table : Partial pressures in electrolux refrigerator in bar**

Section	$\text{NH}_3$	$\text{H}_2\text{O}$	$\text{H}_2$	Total
Condenser	20.33	0	0	20.33
Evaporator inlet	1.516	0	18.814	20.33
Evaporator exit	2.36	0	17.97	20.33
Generator top	15.54	4.79	0	20.33

It has been assumed that vapours leaving generator top are in equilibrium with entering rich solution at  $40^{\circ}\text{C}$ , at which temperature saturation pressure of  $\text{NH}_3$  is 15.54 bar. It has also been assumed that the temperature at evaporator inlet is  $-25^{\circ}\text{C}$  at which temperature saturation pressure of  $\text{NH}_3$  is 1.516 bar.

- (ii) In an ideal vapour absorption refrigeration system
- the heat ( $Q_C$ ) is given to the refrigerant in the generator,
  - the heat ( $Q_C$ ) is discharged to the atmosphere or cooling water from the condenser and absorber,
  - the heat ( $Q_E$ ) is absorbed by the refrigerant in the evaporator, and
  - the heat ( $Q_p$ ) is added to the refrigerant due to pump work.

Neglecting the heat due to pump work ( $Q_p$ ), we have according to First Law of Thermodynamics,

$$Q_C = Q_G + Q_E \quad \dots(i)$$

Let  $T_G$  = Temperature at which heat ( $Q_G$ ) is given to the generator,

$T_C$  = Temperature at which heat ( $Q_C$ ) is discharged to atmosphere or cooling water from the condenser and absorber, and

$T_E$  = Temperature at which heat ( $Q_E$ ) is absorbed in the evaporator.

Since the vapour absorption system can be considered as a perfectly reversible system, therefore the initial entropy of the system must be equal to the entropy of the system after the change in its condition.

$$\therefore \frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_C}{T_C} \quad \dots(\text{ii})$$

$$= \frac{Q_G + Q_E}{T_C} \quad \dots[\text{From equation (i)}]$$

$$\frac{Q_G}{T_G} - \frac{Q_G}{T_C} = \frac{Q_E}{T_C} - \frac{Q_E}{T_E}$$

$$Q_G \left( \frac{T_C - T_G}{T_G \times T_C} \right) = Q_E \left( \frac{T_E - T_C}{T_C \times T_E} \right)$$

$$\begin{aligned} \therefore Q_G &= Q_E \left[ \frac{T_E - T_C}{T_C \times T_E} \right] \left[ \frac{T_G \times T_C}{T_C - T_G} \right] \\ &= Q_E \left[ \frac{T_C - T_E}{T_C \times T_E} \right] \left[ \frac{T_G \times T_C}{T_G - T_C} \right] \\ &= Q_E \left( \frac{T_C - T_E}{T_E} \right) \left( \frac{T_G}{T_G - T_C} \right) \quad \dots(\text{iii}) \end{aligned}$$

Maximum coefficient of performance of the system is given by

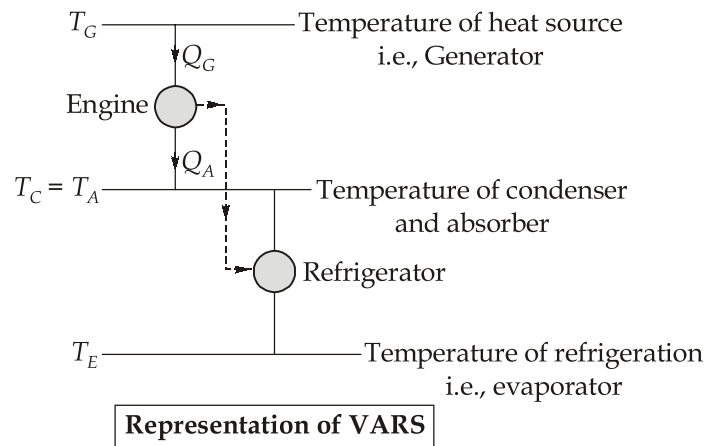
$$\begin{aligned} (\text{C.O.P.})_{\max} &= \frac{Q_E}{Q_G} = \frac{Q_E}{Q_E \left( \frac{T_C - T_E}{T_E} \right) \left( \frac{T_G}{T_G - T_C} \right)} \\ &= \left( \frac{T_E}{T_C - T_E} \right) \left( \frac{T_G - T_C}{T_G} \right) \quad \dots(\text{iv}) \end{aligned}$$

It may be noted that :

1. The expression  $\frac{T_E}{T_C - T_E}$  is the C.O.P. of a Carnot refrigerator working between the temperature limits of  $T_E$  and  $T_C$ .
2. The expression  $\frac{T_G - T_C}{T_G}$  is the efficiency of a Carnot engine working between the temperature limits of  $T_G$  and  $T_C$ .

Thus, an ideal vapour absorption refrigeration system may be regarded as a combination of a Carnot engine and a Carnot refrigerator to produce the desired refrigeration effect as shown in figure.





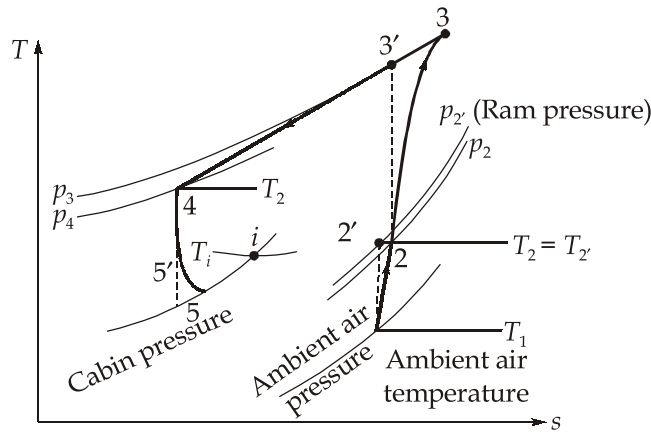
The maximum C.O.P. may be written as

$$(C.O.P.)_{\max} = (C.O.P.)_{\text{Carnot}} \times \eta_{\text{Carnot}}$$

8. (b)

Given: Ambient air pressure,  $P_1 = 0.35$  bar, Ambient air temperature,  $T_1 = -15^\circ\text{C} = 158$  K, Cockpit temperature =  $25^\circ\text{C} = 298$  K, Speed of jet plane,  $C = 1000$  km/hr, Pressure of air leaving the cooling turbine,  $P_5 = 1.06$  bar, Cockpit cooling load, R.E. = 58.05 kW

Assuming turbine inlet temperature to be  $T_4 = T_2 = 296.5$  K



(i) Speed of aircraft,  $C = \frac{1000 \times 1000}{3600} = 277.8$  m/s

Stagnation temp.,  $T_2 = T_1 + \frac{C^2}{2C_p}$

$$= 258 + \frac{277.8^2}{2 \times 1.005 \times 10^3}$$

$$= 296.5 \text{ K}$$

$$\begin{aligned} \text{Stagnation Pressure, } P_2 &= 0.35 \times \left( \frac{296.5}{258} \right)^{\frac{1.4}{0.4}} \\ &= 0.57 \text{ bar} \end{aligned}$$

(ii) Discharge pressure from jet compressor

$$P_3 = 3 \times 0.57 = 1.71 \text{ bar}$$

Discharge temperature from jet compressor

$$T_3 = T_2 \left( \frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = 296.5(3)^{0.286}$$

$$T_3 = 406 \text{ K} = 133^\circ\text{C}$$

Assuming turbine inlet temperatures to be

$$T_4 = T_2 = 296.5 \text{ K}$$

$$P_4 = 1.71 - 0.1 = 1.61 \text{ bar}$$

$$P_5 = 1.06 \text{ bar}$$

$$T_5 = T_4 \left( \frac{P_5}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = 296.5 \times \left( \frac{1.06}{1.61} \right)^{0.286}$$

$$T_5 = 263 \text{ K} = -10^\circ\text{C}$$

Refrigerating effect and mass flow rate of air

$$\text{RE or } q_o = 1.005(298 - 263) = 35.18 \text{ kJ/kg}$$

$$\dot{m} = \frac{58.05 \times 3600}{35.18} = 5940.31 \text{ kg/h}$$

(iii) Volume handled by compressor

$$\dot{V}_C = \frac{\dot{m}RT_2}{P_2} = \frac{5940.31 \times 287 \times 296.5}{0.57 \times 10^5} = 8868.31 \text{ m}^3/\text{h}$$

Volume handled by expander

$$\dot{V}_E = \frac{\dot{m}RT_5}{P_5} = \frac{5940.31 \times 287 \times 263}{1.06 \times 10^5} = 4230.01 \text{ m}^3/\text{h}$$

(iv) Ram work (done by engine in overcoming the drag of ram diffusion).

$$|W_R| = C_p(T_2 - T_1) = 1.005 \times 38.5 = 38.7 \text{ kJ/kg}$$

$$\text{Compressor work, } |W_C| = C_p(T_3 - T_2) = 1.005(406 - 296.5) = 111.0 \text{ kJ/kg}$$

$$\text{Expander work, } |W_E| = C_p(T_4 - T_5) = 1.005(296.5 - 263) = 33.7 \text{ kJ/kg}$$

Net work, 
$$W_{\text{net}} = |W_R + W_C| - W_E$$

$$= 38.7 + 111.0 - 33.7 = 116.0 \text{ kJ/kg}$$

Net power delivered by engine

$$= \dot{m} \times W_{\text{net}} = \frac{5940.31 \times 116.0}{3600} = 191.04 \text{ kW}$$

(v) 
$$\text{COP} = \frac{q_0 \text{ or RE}}{W_{\text{net}}} = \frac{35.18}{116.0} = 0.303$$

8. (c)

If total pressure is  $P$  then

$$P = P_v + P_a \text{ or } P_a = P - P_v$$

where

$P_v$  = Partial pressure of water vapour

$P_a$  = Partial pressure of dry air

So, for air 
$$P_a V = m_a R_a T \quad \dots(i)$$

for vapour, 
$$P_v V = m_v R_v T \quad \dots(ii)$$

Specific humidity, 
$$\omega = \frac{m_v}{m_a} \Rightarrow \omega = \frac{\frac{P_v V}{R_v T}}{\frac{P_a V}{R_a T}}$$

Putting values of  $P_a$  &  $\frac{R_a}{R_v}$

$$\omega = \frac{P_v}{P - P_v} \times 0.622 \quad \dots(iii) \quad \left[ \because \frac{R_a}{R_v} = \frac{M_v}{M_a} = 0.622 \right]$$

$$\omega = \frac{0.622 P_v}{P - P_v}$$

Again as relative humidity, 
$$\phi = \frac{P_v}{P_{vs}} \quad \dots(iv)$$

So from equation (iii),

$$\frac{0.622}{\omega} = \frac{P}{P_v} - 1 \text{ or } P_v = \frac{\omega P}{0.622 + \omega}$$

Similarly, 
$$P_{vs} = \frac{\omega_s P}{0.622 + \omega_s}$$

So, from equation (iv)

$$\phi = \left( \frac{\omega}{0.622 + \omega} \right) \left( \frac{0.622 + \omega_s}{\omega_s} \right)$$

Given:  $P_{vs@22^\circ\text{C}} = 0.02642 \text{ bar}$

$$t_d = 22^\circ\text{C}, \phi = 0.3,$$

$$P = 760 \text{ mm Hg}$$

$$= 760 \times 133.3 = 101308 \text{ N/m}^2$$

**Vapour Pressure : ( $P_v$ )**

As we know that

$$\phi = \frac{P_v}{P_{vs}}$$

$\Rightarrow$

$$P_v = \phi \times P_{vs}$$

$$= 0.3 \times 0.02642 = 0.007926 \text{ bar}$$

**Humidity ratio ( $\omega$ ) :**

$$\omega = \frac{0.622 P_v}{P - P_v} = \frac{0.622 \times 0.007926}{1.01308 - 0.007926}$$

$$= 0.0049 \text{ kg w.v./kg of d.a.}$$

**Vapour density ( $\rho_v$ ) :**

$$\rho_v = \frac{\omega(P - P_v)}{R_a T_d} = \frac{0.0049 \times (1.01308 - 0.007926) \times 10^5}{287 \times (273 + 22)}$$

$$= 0.00582 \text{ kg/m}^3 \text{ of d.a}$$

**Degree of saturation ( $\mu$ ) :**

We know that,

$$\mu = \frac{\omega}{\omega_s} = \phi \left( \frac{P - P_{vs}}{P - P_v} \right)$$

$$= 0.3 \times \left( \frac{1.01308 - 0.02642}{1.01308 - 0.007926} \right)$$

$$= 0.2945$$

○○○○