2019

RANK Improvement WORKBOOK

Electrical Engineering

Basic Electronics Engineering & Analog Electronics

Answer Key of Objective & Conventional Questions

MADE EASY Publications

1 Semiconductor Physics

LEVEL 1 Objective Solutions

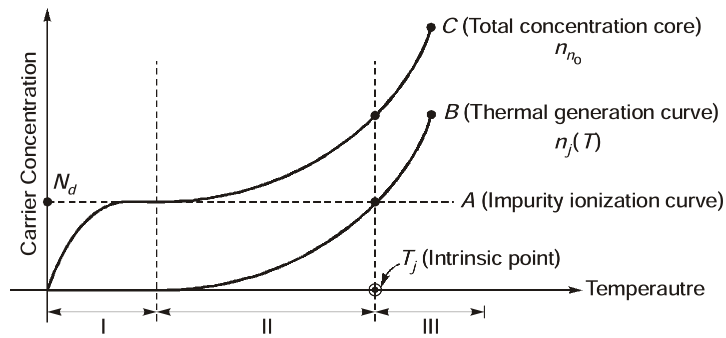
1.	(b)
2.	(6.25)
3.	(a)
4.	(c)
5.	(a)
6.	(c)
7.	(a)
8.	(a)
9.	(d)
10.	(b)
11.	(b)
12.	(c)
13.	(c)
14.	(a)
15.	(c)
16.	(d)

LEVEL 2 Objective Solutions

17.	(b)
18.	(c)
19.	(a)
20.	(c)
21.	(a)
22.	(0.075)
23.	(b)
24.	(5)
25.	(b)
26.	(b)
27.	(c)
28.	(9.36)
29.	(c)
30.	(b)

LEVEL 3 Conventional Solutions

Solution: 1



Total carrier concentration (shown by curve-*C* in the above diagram) has 2 types of carrier.

(a) Carrier due to ionized dopant atom (impurity atom).

(b) $n_j(T)$, thermally generated carrier (intrinsic carriers).

The difference between energy of dopant level and that of conduction band in *N*-type, valance band in *p*-type semiconductor is typically very low as compared to band gap energy $E_g(T)$, *T* stands for temperature variation of E_g.

$\Rightarrow$ $(E_d - E_c)$, $(E_a - E_v) << E_g(T)$ [in extrinsic range of temperature]

Where, E_d = Donor level, E_a, acceptor level E_g is also the amount of energy to generate an electron hole pair intrinsic carrier. Due to small energy requirement of impurity ionization, $(E_c - E_d)$ or $(E_a - E_v)$, the temperature required to ionize dopant atoms is very small and typically between 100-150 Kelvin.

The above diagram shows variation of carrier concentration due to increase in temperature rate for an n-type semiconductor.

(i) Impurity ionization carrier (Curve-A).

(ii) Thermally generated carrier (Curve-B) and curve-C is just the sum of two carrier concentration. Show in the above diagram are 3 region, I, II, III. Depending on temperature range.

Region-I (temperature range of partial impurity ionization): It typically stretches upto 150 K. During such small temperature $n_j(T) \approx 0$ and $n_{n_o} \approx N_d^+$ is carrier concentration due to ionized dopant atoms. If N_d be concentration of dopant atom then

$$N_d^+ = N_d\left[1+\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)\right]^{-1} \quad \text{...(i)}$$

$\left[\frac{1}{2}\text{ term accounts for degenerating of spin.}\right]$

E_F = Fermi level of the n-type specimen.

Region-II (Extrinsic range of temperature): In this range temperature is sufficiently high to ionize impurity (dopant) atoms. So

$$N_d^+ \approx N_d \text{(all impurity atom ionized)}$$

$n_j(T)$ starts to rise but still several orders of magnitude smaller than N_d.

The Region-II ends at that temperature when $n_j(T) = N_d$ at $T = T_j$(intrinsic temperature)

This particular temperature is called intrinsic point. At this temperature extrinsic behaviour starts to vanish intrinsic carrier concentration takes over the impurity carrier concentration.

$$n_{n_0} = n_i(T) + N_d$$

Region-III (Intrinsic range of temperatures): It start at intrinsic temperature point (T_j) $n_i(T)$ at $T_j = N_d$(by definition given above).

$$\Rightarrow \quad n_i(T_j) = N_d$$

In this region $n_{n_0} \approx n_i(T)$ for temperature slightly above T_j and hence called intrinsic range.

Part-2: T_j by definition is the temperature at which $n_i(T_j) = N_d$ ...(ii)

$$\Rightarrow \because \quad n_i(T) = \sqrt{N_c N_v}\exp\left[-\left(\frac{E_g(T)}{2kT}\right)\right] \quad \text{...(iii)}$$

Where N_c, N_v, $E_g(T)$, K have usual meanings. Now we put $n_i(T_j)$ in equation (ii) and we get,

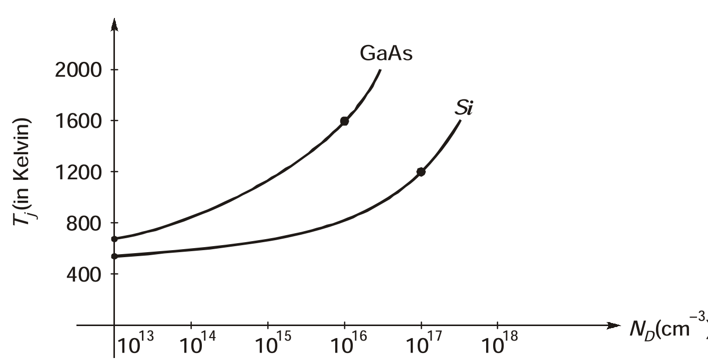
$$\sqrt{N_c N_v}\exp\left[-\frac{E_g(T_j)}{2kT_j}\right] = N_d$$

$$\Rightarrow \quad \exp\left[-\left(\frac{E_g(T_j)}{2kT_j}\right)\right] = \frac{N_d}{\sqrt{N_c N_v}}$$

$$\Rightarrow \quad \frac{-E_g(T_j)}{2kT_j} = ln\frac{N_d}{\sqrt{N_c N_v}}$$

$$\Rightarrow \quad T_j = \frac{E_g(T_j)}{2k\,ln\left[\frac{\sqrt{N_c N_v}}{N_d}\right]}$$

From here, we observe that for a given semiconductor T_j depends on N_d(doping concentration) and $E_g(T_j)$ or band-gap energy at intrinsic temperature point.



Intrinsic temperature Vs Doping

Solution: 2

$$p = \frac{n_i^2}{n} = 2.25 \times 10^5/\text{cm}^3$$

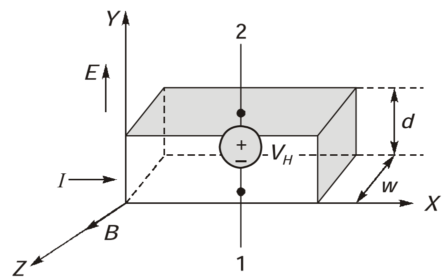
$$(E_C - E_F) = 0.307 \text{ eV}$$

Solution: 3

The given phenomenon is Hall Effect.

According to this effect the statements are:

- If a specimen (metal or semiconductor) carrying a current I is placed in a transverse magnetic field B, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon, known as the Hall effect, is used to determine whether a semiconductor is n-type or p-type and to find the carrier concentration.



- Consider a semiconductor specimen bar having volume charge density ρ_v (in C/m^3), width 'w', thickness 'd', cross-sectional area 'A' and developed Hall voltage is "V_H".

Since, $$E = \frac{F}{e}$$

$\therefore$ $$F = eE \quad \text{...(i)}$$

also, $$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{...(ii)}$$

In the equilibrium state,

Force on specimen due to electric field (E) = Force on specimen due to magnetic field (H)

$\Rightarrow$ $$eE = ev_dB \quad \text{[where } v_d = \text{drift velocity]}$$

$\Rightarrow$ $$E = v_dB \quad \text{...(iii)}$$

Also, $$E = \frac{V_H}{d} \quad \text{...(iv)}$$

$\therefore$ $$V_H = Bv_dd \quad \text{...(v)}$$

Drift velocity of charge carrier $= v_d = \dfrac{E}{B} = \dfrac{J}{\rho_v}$

Now from equation (v),

$$V_H = \frac{BdJ}{\rho_v} = \frac{BId}{A\rho_v} = \frac{BId}{w \times d \times \rho_v}$$

$\Rightarrow$ $$V_H = \frac{BI}{\rho_v w} \quad \text{...(vi)}$$

This equation (vi) represents the Hall voltage (V_H) developed in a semiconductor bar.

Solution: 4

$$n_i = 2.29 \times 10^{21}/\text{m}^3$$

For electron, $v_d = 3.9$ km/sec

For hole, $v_d = 1.9$ km/sec

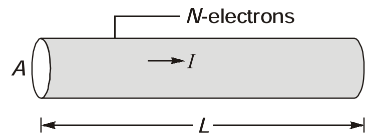
Solution: 5

$$n_i = 2.26 \times 10^{12}\ /\text{m}^3$$
$$\sigma = 3.22 \times 10^{-7}\ \text{S}$$

Solution: 6

Current density:

- If N electrons are contained in a length L of a conductor as shown in figure below. If T is time taken to traverse distance L, the total number of electrons passing through any cross section of wire in T per unit time is N/T.



(A = cross-sectional area)

Therefore, $$I = \frac{Nq}{T} = \frac{Nqv}{L} \quad \text{...(i)}$$

∴ Current density $$(J) = \frac{I}{A} = \frac{Nqv}{LA} \quad \text{...(ii)}$$

(unit of J = amp/m^2)

Since, $$\frac{N}{LA} = n \text{ (electron concentration in electrons per cubic meter)}$$

∴ $$J = nqv = nev = \rho v \quad \text{...(iii)}$$

where $\rho \equiv ne$ is the **charge density** in Coulombs per cubic meter and v in meters per second.

- The conductivity of a material can be related to the number of charge carriers present in the materials. Now, combining equations (i) and (iii) we get,

$$J = nqv = nq\mu E$$

⇒ $$J = \sigma E \quad \text{...(iv)}$$

where, $$\sigma = nq\mu \quad \text{...(v)}$$

where σ is conductivity in (ohm-meter)$^{-1}$

Solution: 7

$$(E_F' - E_F) = 0.083 \text{ eV}$$

Solution: 8

If an electric field is present in addition to the carrier gradient, the current densities will each have a drift component and a diffusion component.

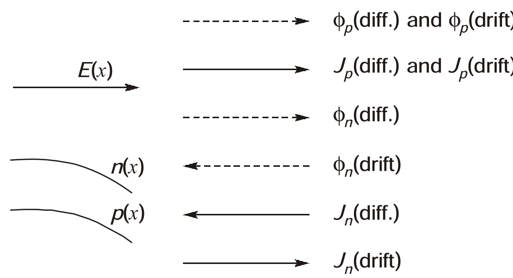
$$J_n(x) = \underbrace{q\mu_n n(x)E(x)}_{\text{drift}} + \underbrace{qD_n\frac{dn(x)}{dx}}_{\text{diffusion}} \quad \text{...(i)}$$

$$J_p(x) = q\mu_p p(x)E(x) - qD_p\frac{dp(x)}{dx} \quad \text{...(ii)}$$

and the total current density is the sum of the contributions due to electrons and holes :

$$J(x) = J_n(x) + J_p(x)$$

The relation between the particle flow and the current of equation (i) by considering a diagram such as shown in figure. In this figure an electric field is assumed to be in the x-direction, along with carrier distributions $n(x)$ and $p(x)$ which decrease with increasing x. Thus the derivatives in equation (i) & (ii) are negative, and diffusion takes place in the x-direction. The resulting electron and hole diffusion currents [J_n(diff.) and J_p(diff.)] are in opposite directions. Holes drift in the direction of the electric field [J_p (drift)], whereas electrons drift in the opposite direction because of their negative charge. The resulting drift current is in the x-direction in each case. Note that the drift and diffusion components of the current are additive for holes when the field is in the direction of decreasing hole concentration, whereas the two components are subtractive for electrons under similar conditions. The total current may be due primarily to the flow of electrons or holes, depending on the relative concentrations and the relative magnitudes and directions of electric field and carrier gradients.



An important result is that minority carriers can contribute significantly to the current through diffusion. Since the drift terms are proportional to carrier concentration, minority carriers seldom provide much drift current. On the other hand, diffusion current is proportional to the gradient of concentration. For example, in n-type material the minority hole concentration 'p' may be many orders of magnitude smaller than the electron concentration 'n', but the gradient (dp/dx) may be significant. As a result, minority carrier currents through diffusion can sometimes be as large as majority carrier currents.

Solution : 9

$$E_g = 1.25 \text{ eV}$$
$$N_{CO} N_{VO} = 1.15 \times 10^{29}$$

Solution : 10

$$N_D = 9.26 \times 10^{14}/\text{cm}^3$$
$$\rho(200°\text{K}) = 2.7\ \Omega\text{-cm}$$
$$\rho(300°\text{K}) = 9.64\ \Omega\text{-cm}$$

■■■■

2 PN Junction Diode and Special Devices

LEVEL 1 Objective Solutions

1.	(a)
2.	(b)
3.	(12.5)
4.	(a)
5.	(b)
6.	(c)
7.	(9.33)
8.	(0.45)
9.	(0.337)
10.	(d)
11.	(b)
12.	(4)
13.	(c)
14.	(1.33)
15.	(a)

LEVEL 2 Objective Solutions

16	(b)
17.	(a)
18.	(b)
19.	(c)
20.	(21)
21.	(d)
22.	(0.526)
23.	(d)
24.	(240.333)
25.	(b)
26.	(d)
27.	(c)
28.	(1.52)
29.	(0.952)
30.	(1.822)
31.	(0.3)
32.	(19.23)
33.	(b)
34.	(d)

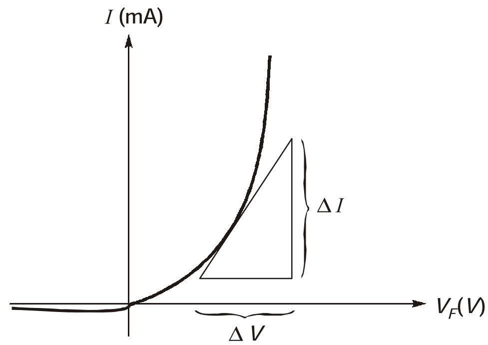
LEVEL 3 Conventional Solutions

Solution: 1

A.C or dynamic resistance (r_f) of a *p-n* junction diode is defined as the reciprocal of the slope of the volt-ampere characteristics.

$$r_f = \frac{\text{Change in voltage}}{\text{Resulting change in current}} = \frac{\Delta V}{\Delta I}$$

A specific change in the voltage and current which may be used to determine the A.C. (or) dynamic resistance for the region of diode characteristics.



Consider the current equation of a *p - n* junction diode

i.e. $$I = I_0\left(e^{\frac{V}{\eta V_T}} - 1\right) \quad \text{...(i)}$$

where, I_0 = reverse current of PN-junction diode

V = applied voltage of PN-junction diode

V_T = thermal voltage

By taking the derivative of the equation (i) w.r.t. the applied voltage, we get,

$$\frac{dI}{dV} = \frac{d}{dV}\left[I_0\left(e^{\frac{V}{\eta V_T}} - 1\right)\right] = I_0\left[\frac{1}{\eta V_T}\cdot e^{V/\eta V_T}\right] = \frac{I_0}{\eta V_T}e^{V/\eta V_T}$$

$$\frac{dI}{dV} = \frac{I + I_0}{\eta V_T}$$

Generally $I >> I_0$ in the vertical slope section of the characteristics.

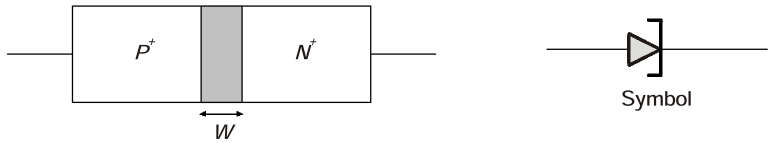
So, $$\frac{dI}{dV} \approx \frac{I}{\eta V_T}$$

$$\Delta I = \frac{dI}{dV}\Delta V$$

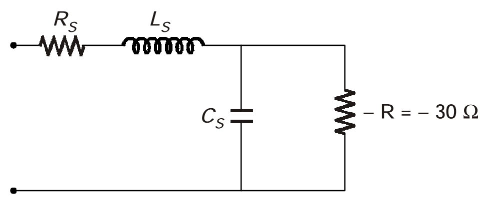
Therefore, $$r_f = \frac{\Delta V}{\Delta I} = \frac{1}{\left(\frac{dI}{dV}\right)} = \frac{\eta V_T}{I}$$

$\therefore$ The dynamic resistance varies inversely with the forward current of diode.

Solution : 2

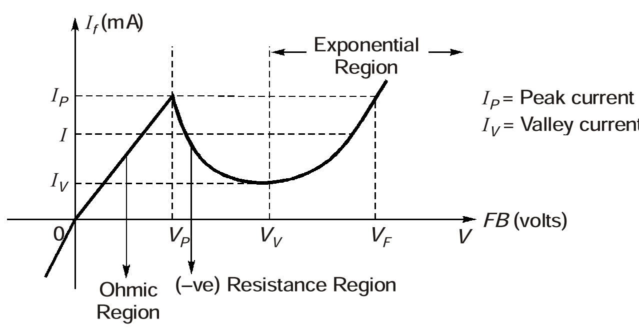


Equivalent circuit:



Tunnel diodes are basically P^+N^+ diode with doping concentration of 1:10^3. Due to high doping depletion layer is very narrow. Hence charge carrier will be penetrating the depletion layer almost at the speed of light. This behaviour of charge carriers is called as tunneling effect. Tunnel diodes are negative resistance device and are fastest switches. GaAS is popularly used for manufacturing of Tunnel Diodes.

VI characteristics of Tunnel Diode : →



Solution: 3

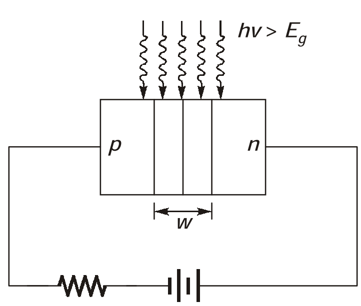
$$V_1 = 0.036$$
$$V_2 = 4.964 \text{ V}$$

Solution: 4

$$V_0 = 0.85 \text{ V}$$

Solution: 5

Let we have a *p-n* junction which is reverse biased and is illuminated by photon energy $hv > E_g$.



Due to this illumination an added generation rate gap (EHP/cm^3) is participated in current.

- The number of holes created within a diffusion length of transition region on the n-side will be $AL_p \cdot g_{op}$.

Where L_p → diffusion length of holes.

$A \rightarrow$ Area of cross-section.

- Similarly $AL_n g_{op}$ electrons are generated per second within L_h of x_{po} and AWg_{op} carriers are generated within W.
- The resulting current due to collection of these optically generated carriers by the junction is

$$I_{op} = qAg_{op}(L_p + L_n + W) \quad \text{...(i)}$$

If $I_0 \rightarrow$ is the reverse saturation current of diode so net current flowing through diode

$$I = I_0[e^{qV/kT} - 1] - I_{op} \quad \text{...(ii)}$$

$\because$ I_{op} is from n to p

When the diode is short circuited

i.e. $V = 0$

i.e short circuit current

$\therefore$ $I_{SC} = -I_{op}$ – sign because this current flow from n to p.

$$I_{SC} = -qAg_{op}(L_p + L_n + W) \quad \text{...(iii)}$$

From equation (ii) $I = I_0[e^{qV/kT} - 1] - I_{op}$

or
$$I = qA\left[\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}\cdot n_p\right](e^{qV/kT} - 1) - qAg_{op}(L_n + L_p + W)$$

When there is an open circuit across the device

i.e. $I = 0$

$$V = V_{OC}$$

$$\left(e^{qV_{OC}/kT} - 1\right) = \frac{qAg_{op}(L_n + L_p + W)}{qA\left[\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right]}$$

$$e^{qV_{OC}/kT} = \left[1 + \frac{L_p + L_n + W}{(L_p/\tau_p)\,p_n + (L_n/\tau_n)\,n_p}g_{op}\right]$$

$$\Rightarrow \quad V_{OC} = \frac{kT}{q}ln\left[1 + \frac{L_p + L_n + W}{(L_p/\tau_p)\,p_n + (L_n/\tau_n)n_p}g_{op}\right] \quad \text{...(iv)}$$

- When an illuminated junction is operated in 3rd quadrant i.e. voltage is reverse direction as well as current is also.
- Current becomes independent of applied voltage but is proportional to the optical generation rate.
- Such a device provides a useful means of measuring illumination level or of converting time varying optical signal into-electrical signal, so can be used as photo detector.
- From equation (iv)

$$V_{OC} = \frac{kT}{q}ln\left[\frac{L_p + L_n + W}{(L_p/\tau_p)p_n + (L_n/\tau_n)\,n_p}g_{op} + 1\right] \quad \text{...(v)}$$

For a special case of symmetrical junction

$$p_n = n_p, \quad \text{and} \quad \tau_p = \tau_n$$

We have
$$\frac{p_n}{\tau_n} = g_{th}$$

$g_{th} \rightarrow$ thermal generation rate

By neglecting the generation within W.

We can rewrite equation (v) as

$$V_{OC} = \frac{kT}{q} ln\left[\frac{g_{op}}{g_{th}} + 1\right]$$

$$V_{OC} \simeq \frac{kT}{q} ln\frac{g_{op}}{g_{th}} \quad \text{for } g_{op} >> g_{th}$$

$g_{th} = p_n/\tau_n$ represents the equilibrium thermal generation – recombination rates.

- As minority carriers concentration is increased by optical generation EHP's, the life time becomes shorter and $p_n/\tau_n = g_{th}$ becomes larger making $\frac{g_{op}}{g_{th}}$ constant.

  So V_{OC} can not increased indefinitely with increased generation rate infact limit on V_{OC} is equilibrium contact potential V_0.
- This effect of illuminated junction can be used in photo cell and this is also known photo-voltaic effect.

Solution: 6

$$V_0 = 697.32 \text{ mV}$$
$$W = 0.958836 \ \mu\text{m}$$

Solution: 7

$$\frac{\Delta V_z}{V_z} \times 100\% \Big/ {}^\circ\text{C} = 0.0113/{}^\circ\text{C}$$

Solution: 8

The term varactor is a shortened form of variable reactor, referring to the voltage variable capacitance of a reverse-biased *p-n* junction. The junction capacitance depends on the applied voltage and the design of the junction. In some cases a junction with fixed reverse bias may be used as a capacitance of a set value. More commonly the varactor diode is designed to exploit the voltage-variable properties of the junction capacitance. For example, a varactor (or a set of varactors) may be used in the tuning stage of a radio receiver to replace the bulky variable plate capacitor. The size of the resulting circuit can be greatly reduced, and its dependability is improved. Other applications of varactors include use in harmonic generation, microwave frequency multiplication and active filters.

If the *p-n* junction is abrupt, the capacitance varies as the square root of the reverse bias V_r in a graded junction, however, the capacitance can usually be written in the form.

$$C_j \propto V_r^{-n}$$

Where, $n = \frac{1}{2}$ for step graded or abrupt p-n junction diode

$n = \frac{1}{3}$ for linear graded diode

$n = \frac{1}{2.5}$ for diffused p-n junction diode

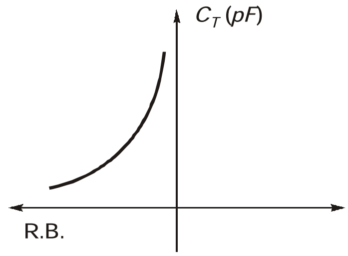
$n =$ grading coefficient

For varactor diode, $n = \frac{1}{3}$

So, for varactor diode, $C_T \propto \dfrac{1}{\sqrt[3]{R.B.\text{ voltage}}}$

$$C_T \propto V^{-1/3}$$

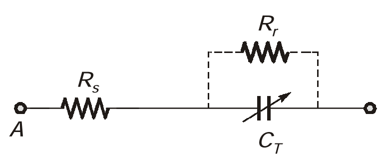
- Varactor diode operates on the principle of transition capacitance (C_T).
- Varactor diode is always operated under reverse bias.
- GaAs material is popularly used for designing of varactor diode.



Characteristic curve of varactor diode

Symbol of varactor diode:

Equivalent circuit:



Where R_s = Ohmic resistance or contact resistance

R_r = Reverse resistance of varactor diode

The tuning frequency of a varactor diode.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\text{ Hz}$$

Application of varactor diodes:

1. In designing of LC resonant tuning circuits.
2. For direct generation of frequency modulation (F.M) by using varactor diode modulator circuit.
3. For self balancing of AC bridges.
4. As a parametric amplifiers (Para-Amp). Para-amp is a microwave power amplifier and it is used in satellite communication.

Solution : 9

$$\eta = 1.94$$

Solution : 10

The hole concentration inside n-region is obtained by solving the continuity equation as

$$P_n'(x) = P_n'(0)e^{-x/L_p}$$

The hole diffusion current $I_p(x) = -AqD_p \dfrac{dp(x)}{dx}$

$$\Rightarrow \qquad I_{p_n}(x) = -AqD_p\left(\frac{-P_n(0)}{L_p}\right)e^{-x/L_p}$$

$$I_{p_n}(x) = \frac{AqD_p}{L_p}P_{n_o}(e^{-V/V_T} - 1)e^{-x/L_p}$$

At $x = 0$,

$$I_{p_n} = \frac{AqD_pP_{n_o}}{L_p}(e^{V/V_T} - 1)$$

Similarly,

$$I_{n_p}(0) = \frac{AqD_n n_{p_o}}{L_n}(e^{V/V_T} - 1) \qquad \text{...(i)}$$

$$I_{p_n}(0) = \frac{AqD_n P_{n_o}}{L_p}(e^{V/V_T} - 1) \qquad \text{...(ii)}$$

Dividing equation (i) by equation (ii) gives

$$\frac{I_{p_n}(0)}{I_{n_p}(0)} = \frac{D_p P_{n_o}}{L_p} \times \frac{L_n}{D_p n_{p_o}} = \frac{V_T \mu_p \dfrac{n_i^2}{N_D}}{V_T \mu_n \dfrac{n_i^2}{N_A}} \times \frac{L_n}{L_p}$$

$$\Rightarrow \qquad \frac{I_{p_n}(0)}{I_{n_p}(0)} = \frac{q\mu_p N_A L_n}{q\mu_n N_D L_p} = \frac{\sigma_p L_n}{\sigma_n L_p}$$

■■■■

3 Bipolar Junction Transistor

LEVEL 1 Objective Solutions

1.	(6.656)
2.	(b)
3.	(b)
4.	(c)
5.	(b)
6.	(b)
7.	(c)
8.	(c)
9.	(d)

LEVEL 2 Objective Solutions

10.	(a)
11.	(a)
12.	(a)
13.	(a)
14.	(d)
15.	(b)
16.	(380.28)
17.	(2.718)

LEVEL 3 Conventional Solutions

Solution: 1

(i) $t_1 = \frac{t_{ON}}{2} = 1\mu$ sec.

$t_2 = 97.5\ \mu$ sec.

(ii) $P_{ON} = 333.3$ Watts

$P_{OFF} = 333.3$ watts

(iii) $P_{av} = 333.3$ watts

Solution: 2

(i) With increase in base width, the recombination in base region will increase and therefore base current increases and collector current decreases so $\beta = I_C/I_B$ decreases.

(ii) As carrier (minority) life time in base region increase the number of recombinations in base region decreases and more and more carries reach to collector, therefore I_B decreases and I_C increases and as a result $\beta = I_C/I_B$ increases.

(iii) With increase in temperature, minority carrier life time increases in base region and this tend to increase β while on the otherhand due to increase in temperature, transit time τ_t increases which tend to decrease β, but the effect of increasing lifetime with temperature dominate so β increases with temperature.

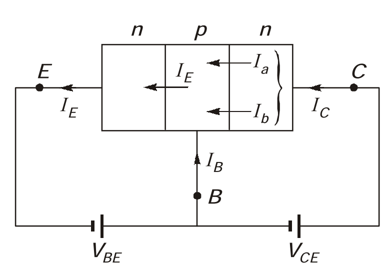
(iv) With increase in collector current, β will remain same, as collector current will increase when base current increases, as base current is controlling current on which collector current depends.

(v) As collector voltage increases, β effective base width decreases, so recombination in base region decreases due to this I_B decreases and I_C increases and therefore $\beta = I_C/I_B$ increases.

Solution: 3

The solution for the given problem can be obtained from the basic transport mechanism of the BJT.

Let us consider the internal structure of the BJT as shown below.



I_a = Current due to injected minority carriers crossing the CB junction

I_b = Current due to thermally generated minority carriers crossing the CB junction

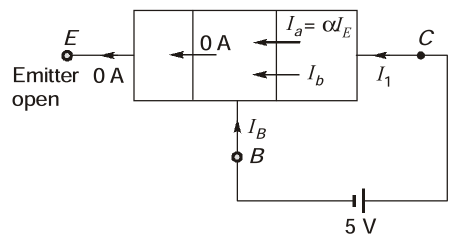
$I_a = \alpha I_E$

where, α = large signal current gain of CB configuration

β = large signal current gain of CE configuration

$I_b = I_{CO}$ = reverse saturation current of the CB junction

For the circuit given in the figure (a):



As the emitter is open circuited, $I_E = 0$ A

So, $$I_a = \alpha I_E = 0 \text{ A}$$

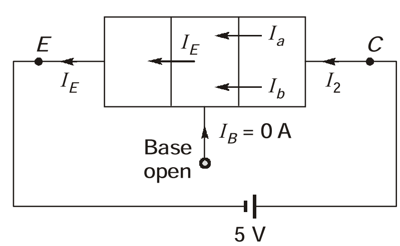
$$I_b = I_{CO} \quad \because$$

CB junction is reverse biased

and $$I_1 = I_C = I_a + I_b = 0 \text{ A} + I_{CO}$$

$$I_1 = I_{CO} \quad \text{...(i)}$$

For the circuit given in the figure (b) :



$$I_a = \alpha I_E$$

$$I_b = I_{CO}$$

$$I_2 = I_C = I_E \quad \because I_B = 0 \text{ A}$$

So, $$I_2 = I_E = \alpha I_E + I_{CO}$$

$$I_E(1-\alpha) = I_{CO}$$

$$I_E = \frac{1}{(1-\alpha)} I_{CO} = \left[1 + \frac{\alpha}{(1-\alpha)}\right] I_{CO}$$

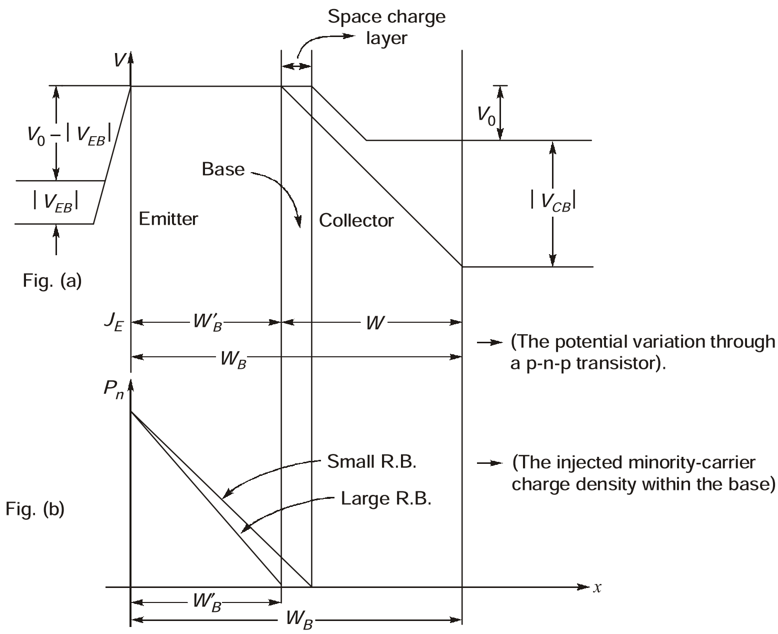
$$I_E = (1+\beta) I_{CO} \quad \because \beta = \frac{\alpha}{(1-\alpha)}$$

$$I_2 = I_E = (1+\beta) I_{CO} \quad \text{...(ii)}$$

From the equations (i) and (ii), it is clear that, the current I_2 is $(1 + \beta)$ times of current I_1.

Solution: 4

- As we know that the width (W) of the depletion region of a diode or a p-n junction increases with the magnitude of the reverse biasing voltage. In the active region, emitter junction is F.B. but the collector junction is R.B. then in the figure below we can see that the barrier width at J_E is negligible compared with the space-charge region width (W) at J_C. As the R.B. voltage increases, the transition region penetrates deeper into the collector and base. Since the doping in the base is ordinarily larger than that of the collector, so the penetration of the depletion region into the base is much smaller than into the collector. If the metallurgical base width is 'W_B' then the effective electrical base width is $W_{eB} = W_B - W$.
- This modulation (small change) of the effective base-width by altering the collector voltage is popularity known as **"Early effect".** This effect is also known as the **"Base width modulation".**



- The lowering the value of W'_B with increasing reverse bias collector voltage has the following main consequences:

⇒ There is less chances for recombination with in the base region. Hence 'α' increases with increasing the value of $|V_{CB}|$.

⇒ The concentration gradient of minority carriers p_n is increased within the base as shown in figure (b) above. Note that, $p_n = 0$ at the edge of the base-collector depletion layer.

⇒ For extremely large collector-junction R.B. voltage the effective base width of the transistor i.e. W'_B may be reduced to zero, which results a voltage breakdown in the transistor and this phenomena is called the Punch-through or Reach-through.

⇒ Due to early-effect the transit time is reduced and so the switching-time is reduced and hence transistor becomes faster.

Solution: 5

We know the relation,

[Power transmitted] × [Thermal resistance] = [Temperature difference]

Let,
P_c = Power radiated by collector
T_j = Junction temperature
G_a = Ambient temperature
θ_{JA} = Thermal resistance between the junction and the air

At steady state

⇒ $$P_c \times \theta_{JA} = [T_j - T_A]$$

To prevent thermal runaway, the right hand side of the equation must be greater than the left hand side during any variation of junction temperature.

$$P_c \theta_{JA} < T_j - T_A$$

differentiate with respect to T_j

$$\theta_{JA} \frac{\partial P_c}{\partial T_j} < 1; \qquad \frac{\partial P_c}{\partial T_j} < \frac{1}{\theta_{JA}}$$

■■■■

4 FET and MOSFET Devices

LEVEL 1 Objective Solutions

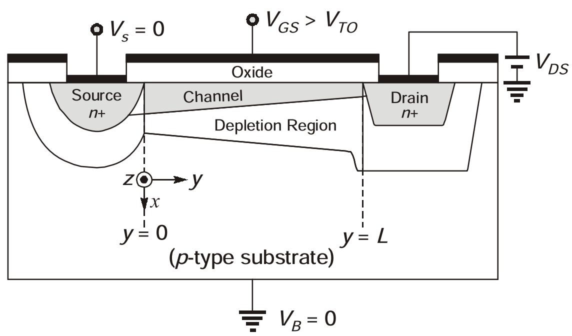
1.	(a)
2.	(a)
3.	(a)
4.	(5.07)
5.	(b)
6.	(b)
7.	(c)
8.	(1.42)
9.	(b)
10.	(b)
11.	(a)
12.	(0.67)
13.	(b)
14.	(b)

LEVEL 2 Objective Solutions

15.	(28.35)
16.	(1.2)
17.	(2.25)
18.	(1178.31)
19.	(a)
20.	(c)
21.	(–1.32)
22.	(b)
23.	(b)
24.	(18)
25.	(b)
26.	(a)

LEVEL 3 Conventional Solutions

Solution: 1



⇒ Cross-sectional view of an n-channel transistor, operating in linear region.

Gradual Channel Approximation: In the above diagram, I_D (current flowing in the channel from drain to source) under the applied drain voltage V_{DS}. We assume channel is formed already by applying $V_{GS} > V_{TO}$ (threshold voltage) we first define a co-ordinate system with x direction being perpendicular to the substrate and y is along the channel

⇒ Let $V_c(y)$ be channel voltage with respect to source (which is grounded). We assume V_{TO} independent of y and thus constant (this is assumption not strictly true) we also assume E_y is dominant an E_x, this will allow us to write

$$V_c(y = 0) = V_s = 0 \quad \text{...(i)}$$

$$V_c(y = L) = V_{DS}(V_D) \quad \text{...(ii)}$$

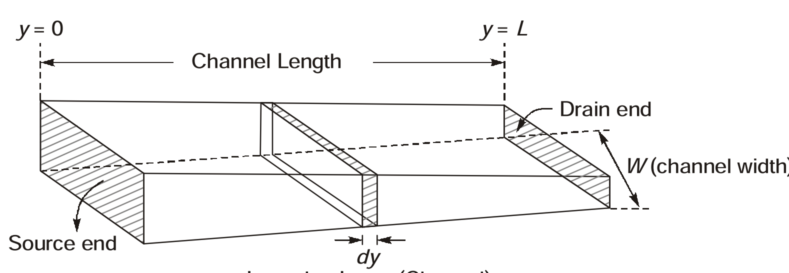
$$[V_{DS} = V_D - V_S = V_D - 0 = V_D]$$

The channel is completed inverted from $y = 0$ to $y = L$ as $V_{GS} \geq V_{TO}$ and so electron flow from source to drain under electric field $E_y(>> E_n)$.

Let $Q_I(y) = -C_{ox}(V_{GS} - V_c(y) - V_{TO})$

$Q_I(y)$ the amount of inversion charge stored in an elemental length dy of channel at a voltage $V_c(y)$ above source.

Next diagram show exclusively the channel or inversion charge layer of the previous diagram.



Inversion Layer (Channel)

Let dR (incremental resistance) in the channel for constant mobility (μ_n) of inversion layer

$$dR = -\frac{dy}{W\mu_n Q_I(y)} \quad \text{[negative sign is to adjst negative } Q_I(y)]$$

μ_n actually depends on the concentration of carrier and so the above assumption is for simplifying the solution. We further assume that current density is uniform across the length of the channel from $y = 0$ to $y = L$ i.e. I_D is independent of y.

Applying ohm's law for the element dy of the channel

$$dV_c = I_D \cdot d_R = -\frac{I_D}{W\mu_n Q_I(y)} \cdot dy \quad \text{...(iii)}$$

(i) Can be integrated from $y = 0$ to $y = L$ using boundary condition of equation (i) and equation (ii)

$$-W\mu_n \int_0^{V_{bs}} Q_I(y)V_c = \int_0^L I_D \cdot dy$$

$\because$ I_D is independent of y so $\int_0^L I_D \cdot dy = I_D \cdot L$

$$\Rightarrow \quad I_D \cdot L = -W\mu_n \int_0^{V_{DS}} Q_I(y)dV_c$$

$$Q_I(y) = -C_{ox}(V_{GS} - V_c(y) - V_{To})$$

$$\Rightarrow \quad I_D \cdot L = -W\mu_n(-C_{ox}) \int_0^{V_{DS}} (V_{GS} - V_c(y) - V_{To})dV_c$$

$$= W\mu_n C_{ox}\left[\int_0^{V_{DS}} (V_{GS} - V_{To})dV_c \int_0^{V_{DS}} -V_c(y)dy_c\right]$$

$$I_D = W\mu_n C_{ox}\left[(V_{GS} - V_{To}) - V_{DS} - \frac{V_{DS}^2}{2}\right]$$

at saturation $(V_{GS} - V_T) = V_{DS}$

$\Rightarrow$ I_{DS} = Saturation drain current

$$I_{DS} = \frac{W\mu_n C_{ox}}{L}\left[\frac{(V_{GS} - V_{To})^2}{2}\right]$$

$$\Rightarrow \quad g_{ms} = \text{Saturation transconductance} \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{W\mu_n C_{ox}}{L}(V_{GS} - V_{To})$$

Solution: 2

$\because$ We know the expression for threshold voltage is

$$V_T = Q_{ms} - \frac{Q_j}{C_{ox}} - \frac{Q_d}{C_{ox}} + 2Q_F \quad \text{...(i)}$$

(a) $\Delta Q_{ms} = Q_m - Q_s$ = Metal semiconductor work function difference.

Q_m and Q_s are difference of Fermi-level and vacuum level in metal and semiconductor respectively. Since Fermi-level is a function of doping. So Q_{ms} depends on doping of the substrate.

(b) Q_j = Sum total of oxide trapped charges and the charge at oxide-semiconductor interface.

(c) Q_d = Total depletion charge

Q_d for p-substrate. n-channel device is negative = $-2\sqrt{e\,\epsilon_o E_s N_a Q_{Fp}}$

Q_{Fp} – Fermi-potential of p-type substrate = $kTln\left(\frac{N_a}{n_i}\right)$

N_a = Acceptor doping concentration

$\in_s$ = Relative permittivity of substrate

Q_d for n is a similar manner = $2\sqrt{e \in_o \in_s N_d Q_{Fn}}$

Q_{Fn} Fermi-potential on *n*-substrate = $-kTln\left(\frac{N_d}{n_i}\right)$

N_d = Doron concentration in substrate

$\because$ Q_{Fp}, Q_{Fn} are function of doping level *SU*.

$\Rightarrow$ Q_d is a strong function of doping.

(d) Q_F again the Fermi-potential, a function of doping concentration.

(e) C_{ox} = Oxide capacitance = $\frac{\in_o \in_{ox}}{t_{ox}}$

$\in_{ox}$ = Relative permittivity of oxide

t_{ox} = Oxide thickness

So examining all right hand side of equation (i) we observe that V_T can be controlled by suitably controlling the doping concentration of the substrate and also by varying the oxide thickness. In many application interface charge is deliberately introduced to alter the threshold voltage of a MOS-device (specially depletion mode device).

PART-2

A short channel device is one in which the length of the channel (L) is of the same order of the depletion width at source/drain junction

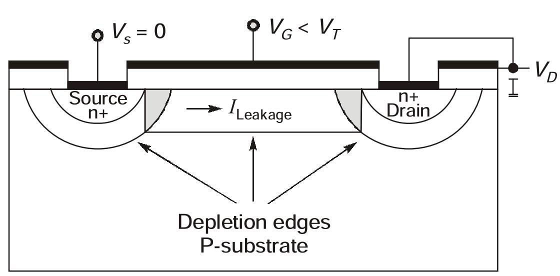


Figure-1
Sub-threshold leakage current in MOS due to DIBL

In long channel MOSFET, when $V_G < V_T$, source is unable to inject electron in the channel due to high source-channel depletion barrier. This sub-threshold source-channel barrier is high enough to stop any leakage current. But in short channel device. The influence of drain electric field is very strong at source junction. This influence leads to reduction in the barrier voltage at source junction and thus electron enter the channel and drain at drain end.

The effect is a subthreshold leakage current and the phenomena is called (DIBL) Drain Induced Barrier Lowering.

Then, one more phenomena which is most important in effecting the device performance in Short Channel Devices is Velocity Saturation.

The velocity of carriers in a semiconductor (*SC*) depends on electric field and doping level and it becomes constant beyond a certain electric field called critical electric field (E_c) as shown in the diagram below, whereas it increase as

$$v_n(\text{velocity}) = \frac{\mu_n E}{\left(1+\frac{E}{E_c}\right)}$$

[Variation of carrier velocity with applied electric field (E)]

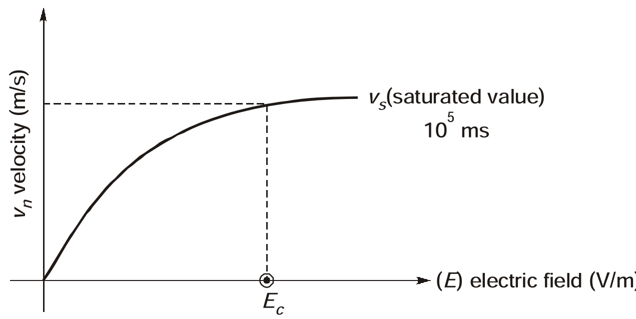


Figure-2: Velocity-Saturaton Effect

Let

μ_n = Mobility of electron

E = Applied electric field

V_{DS} = Drain to source voltage

E_c is typically between $\frac{1\,\text{V}}{\mu\text{m}}$ to $\frac{5\,\text{V}}{\mu\text{m}}$.

In long channel *n*-MOS say $L = 0.25\ \mu\text{m}$

$\Rightarrow$ We need $V_{DS} = 2$ Volts to get $E = \frac{V_{DS}}{L} = \frac{2\,\text{V}}{0.25\,\mu\text{m}} = \frac{4\,\text{V}}{\mu\text{m}} > E_c$

In short channel L is much smaller than 0.25 μm and hence $E = \frac{V_{DS}}{L} >> E_c$ for very small voltage and hence velocity saturation is easily seen to occur.

The velocity saturation has pronounced effect on the operation of MOS transistor having short channel.

Drain Current: From the continuity requirement of the velocity saturation diagram in Figure-2.

$$v_s = \left.\frac{\mu_n E}{1+E/E_c}\right|_{E=E_c}$$

$$\Rightarrow \quad v_s = \frac{\mu_n E_c}{2} \Rightarrow E_c = \frac{2v_s}{\mu_n}$$

Drain current expression in presence of velocity saturation becomes

$$I_D = \frac{\mu_n}{\left(1+\frac{V_{DS}}{L\,E_c}\right)} C_{ox} \frac{W}{L}\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

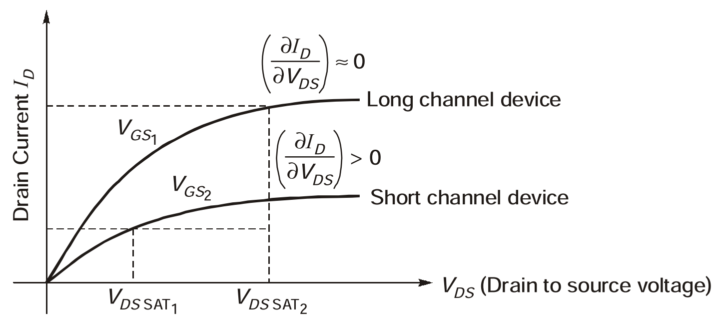
Where symbols have their usual meaning, $\frac{V_{DS}}{L}$ is average field in the channel

$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{gs} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]\left[\frac{1}{1+\left(\frac{V_{DS}}{LE_C}\right)}\right]$$

Let $$\frac{1}{1+\frac{V}{LE_c}} = K(V) \quad \Rightarrow \quad \text{If } K(V) \text{ is } 1$$

Velocity saturation is minimum and as V_{DS} increases beyond $(E_C \cdot L)$, velocity saturation effect take over.
For short channel devices $K(V)$ is less than 1 and hence current I_{DS} decreases due to velocity saturation.
Also the V_{DS} required for a device = $(E_c \cdot 1)$, for saturation to occur.

So $V_{DS\,SAT_1}$ for long channel > $V_{DS\,SAT_2}$ for short channel effect of velocity saturation of device characteristic.



$V_{GS_1} = V_{GS_2}$

Here we observe that short channel saturation current is linearly dependent of V_{DS}

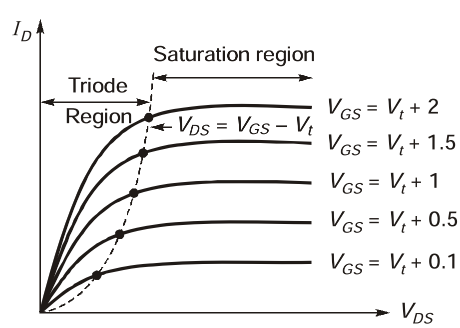
$$\Rightarrow \quad \frac{\partial I_D}{\partial V_{DS}} > 0 \qquad \left(\text{unlike long channel where } \frac{\partial I_D}{\partial V_{GS}} \approx 0\right)$$

⇒ Output impedance of short channel MOS device reduce due to velocity saturation.

Solution: 3

The n-channel enhancement MOSFET operates in the triode region when V_{GS} is greater than V_t and the drain voltage is lower than the gate by at least V_t volts. In the triode region,

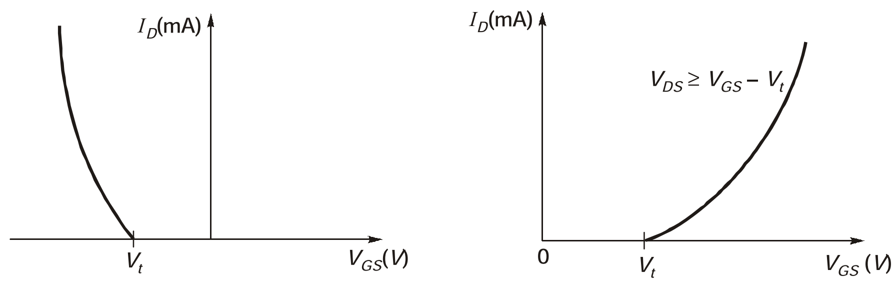
$$i_D = k_n' \frac{\omega}{L}\left[(V_{GS} - V_t)\, V_{DS} - \frac{1}{2}V_{DS}^2\right]$$



Output characteristics of n-channel MOSFET

The n-channel enhancement MOSFET operates in the saturation region when V_{GS} is greater than 'V_t' and the drain voltage does not, fall below the gate voltage by more than 'V_t' volts.

$$i_D = \frac{1}{2}k'_n\frac{\omega}{L}(V_{GS} - V_t)^2$$



Transfer characteristic of p-channel MOSFET

Transfer characteristics of n-channel MOSFET

Transfer characteristic shows that there will be no current until and unless

$V_{GS} \geq V_t$ for (n-MOS) and $V_{GS} \leq V_t$ for (p-MOS)

Solution: 4

(iii) $$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = 4 \text{ mS}$$

$$\mu = 840$$

Solution: 5

(i) $$\left(\frac{W}{L}\right) = 24$$

(ii) $$I_D = 1.152 \text{ mA}$$

(iii) $$L = 1.02\ \mu\text{m}$$

$$W = 24.48\ \mu\text{m}$$

■■■■

5 Diode Circuits

LEVEL 1 Objective Solutions

1. (b)
2. (b)
3. (d)
4. (a)
5. (2)
6. (1.27)
7. (27)
8. (b)
9. (b)
10. (b)

LEVEL 2 Objective Solutions

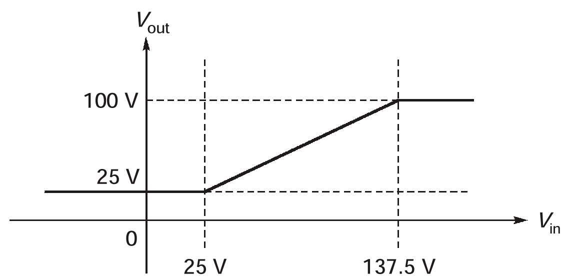
11. (0.01)
12. (b)
13. (d)
14. (c)
15. (a)
16. (d)
17. (b)
18. (b)
19. (c)
20. (b)
21. (c)
22. (c)

LEVEL 3 Conventional Solutions

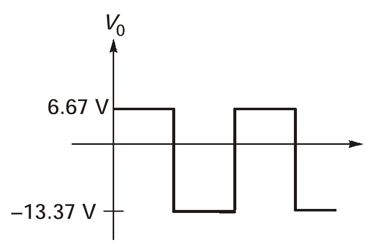
Solution: 1

$$C = 200\ \mu F$$

Solution: 2



Solution: 3



Solution: 4

(i) $i_{b(max)} = I = 100$ mA,

$i_{b(avg)} = 35.14$ mA

(ii) $i_{b(max)} = I = 100$ mA

(iii) $i_{b(avg)} = 33.3$ mA

Solution: 5

$$R = 13.65$$

■■■■

6 BJT and FET Biasing

LEVEL 1 Objective Solutions

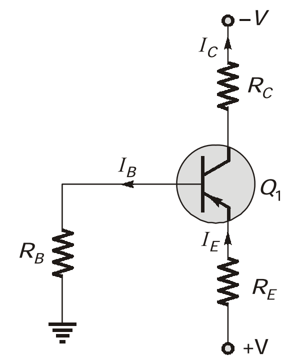
1.	(d)
2.	(3.87)
3.	(10)
4.	(c)
5.	(b)
6.	(c)
7.	(4)
8.	(c)
9.	(c)

LEVEL 2 Objective Solutions

10.	(0.396)
11.	(a)
12.	(1.407)
13.	(c)
14.	(b)
15.	(1.42)
16.	(25)
17.	(2.19)
18.	(a)
19.	(2)
20.	(2.5)
21.	(c)

LEVEL 3 Conventional Solutions

Solution: 1



Since, stability factor (s) = $\boxed{S = \left.\dfrac{\partial I_C}{\partial I_{CO}}\right|_{V_{be},\beta} = \dfrac{1+\beta}{1-\dfrac{\beta\cdot\partial I_b}{\partial I_C}}}$

Apply KVL,

$$\Rightarrow \quad V - V_{EB} = I_E R_E + I_B R_B \quad \text{...(i)}$$

also, $$I_E = I_C + I_B \quad \text{...(ii)}$$

and $$I_C = \beta I_B + (\beta+1) I_{CO} \quad \text{...(iii)}$$

$$\Rightarrow \quad V - V_{EB} = (I_C + I_B) R_E + I_B R_B$$

$$\Rightarrow \quad V - V_{EB} = I_C R_E + I_B (R_B + R_E)$$

Putting $I_B = \dfrac{I_C - (\beta+1) I_{CO}}{\beta}$ in the above equation,

$$V - V_{EB} = I_C R_E + (R_B + R_E)\left\{\frac{I_C - (\beta+1) I_{CO}}{\beta}\right\}$$

Differentiating the above equation with respect to I_C,

$$0 = R_E + \frac{(R_B + R_E)}{\beta}\left[1 - (\beta+1)\frac{\partial I_{CO}}{\partial I_C}\right]$$

$\because$ Stability factor = $S = \dfrac{\partial I_C}{\partial I_{C0}}$

$$\therefore \quad 0 = R_E + \frac{(R_B + R_E)}{\beta}\left[1 - \frac{(\beta+1)}{S}\right]$$

$$\Rightarrow \quad 1 - \frac{(\beta+1)}{S} = \frac{-\beta R_E}{R_B + R_E} \quad \Rightarrow \quad \frac{(\beta+1)}{S} = 1 + \frac{\beta R_E}{R_B + R_E} = \frac{R_B + R_E + \beta R_E}{R_B + R_E}$$

$$\Rightarrow \quad S = \frac{(\beta+1)(R_B + R_E)}{R_B + (\beta+1) R_E} = \frac{(\beta+1)\left(1 + \dfrac{R_B}{R_E}\right)}{(\beta+1) + \dfrac{R_B}{R_E}}$$

Solution: 2

$$i_C = 0.91 \text{ mA}$$
$$V_{CE} = 9.793 \text{ V}$$
$$S = 4.8$$

Solution: 3

$$I_{CQ} = 0.894 \text{ mA}$$
$$I_{EQ} = 0.90 \text{ mA}$$
$$V_{ECQ} = 6.53 \text{ V}$$

Solution: 4

$$V_1 = 0.9125 \text{ V}$$
$$V_2 = 1.613 \text{ V}$$
$$V_3 = -1.69 \text{ V}$$
$$V_4 = -2.432 \text{ V}$$
$$V_5 = 1.116 \text{ V}$$

Solution: 5

$$(I_{out})_{max} = 2.32 \text{ mA}$$

Solution: 6

$$V_{E_1} = 0.22 \text{ volts}$$

■■■■

7 Midfrequency Analysis of Amplifiers

LEVEL 1 Objective Solutions

1. (a)
2. (b)
3. (c)
4. (500)
5. (a)
6. (–214.8)
7. (102.38)
8. (b)
9. (6)

LEVEL 2 Objective Solutions

10. (b)
11. (b)
12. (b)
13. (707.35)
14. (6.8)
15. (48.27)
16. (1000)
17. (6.11)
18. (b)
19. (7.95)
20. (a)
21. (4.44)

LEVEL 3 Conventional Solutions

Solution : 1

(i) $g_m = 1.6176 \text{ m℧}$

(ii) $r_d = 50 \text{ k}\Omega$

(iii) Case-I:

$$A_v = -3.2352$$
$$Z_i = 2.36 \text{ M}\Omega$$
$$Z_o = 2 \text{ k}\Omega$$

Case-II:

$$A_v = -3.11$$
$$Z_i = 2.433 \text{ M}\Omega$$
$$Z_o = 1.923 \text{ k}\Omega$$

Solution: 2

$$\frac{\text{Area of } D_E}{\text{Area of } D_B} = (\beta + 1) = 101$$

Solution: 3

$$V_{CE} = -5.81 \text{ V}$$
$$Z_{\text{in}} = 6.78 \text{ k}\Omega$$

Solution: 4

$$\frac{V_0}{V_i} = -291.61$$

Solution: 5

$$I_C = 1.92 \text{ mA}$$
$$g_m = 73.9 \text{ mA/V}$$
$$r_\pi = 1.35 \text{ k}\Omega$$
$$r_0 = 52.1 \text{ k}\Omega$$

■■■■

8 High Frequency Analysis and Multistage Amplifier

LEVEL 1 Objective Solutions

1.	(b)
2.	(d)
3.	(132.70)
4.	(a)
5.	(a)
6.	(a)
7.	(c)
8.	(b)
9.	(b)

LEVEL 2 Objective Solutions

10.	(1.8)
11.	(98.06)
12.	(b)
13.	(d)
14.	(b)
15.	(18.3)

LEVEL 3 Conventional Solutions

Solution: 1

(i)
$$f_u = 19.615 \text{ kHz}$$
$$f_l = 5.098 \text{ Hz}$$

(ii) Let A_{vh} be the voltage gain at high frequencies and A_{vm} be the voltage gain at middle frequencies. So for single stage,

$$\frac{A_{vh}}{A_{vm}} = \frac{1}{1 + jf/f_u}$$

Considering magnitude only:

$$\left|\frac{A_{vh}}{A_{vm}}\right| = \frac{1}{\sqrt{1+(f/f_u)^2}}$$

For n stages:

$$\left|\frac{A_{vh}}{A_{vm}}\right|_{\text{overall}} = \left[\frac{1}{\sqrt{1+(f/f_u)^2}}\right]^n$$

At $f = f_{u\,\text{overall}}$, i.e., the upper cut-off frequency of overall amplifier

$$\left|\frac{A_{vh}}{A_{vm}}\right|_{\text{overall}} = \frac{1}{\sqrt{2}}$$

Putting values: we get,

$$\left[\frac{1}{\sqrt{1+\left(\frac{f_{u\,\text{overall}}}{f_u}\right)^2}}\right]^n = \frac{1}{\sqrt{2}} = \left(\frac{1}{2}\right)^{1/2}$$

$$\Rightarrow \quad \frac{1}{1+\left(\frac{f_{u\,\text{overall}}}{f_u}\right)^2} = \frac{1}{2^{1/n}} = 2^{1/n}$$

$$\Rightarrow \quad \frac{f_{u\,\text{overall}}}{f_u} = \sqrt{2^{1/n} - 1}$$

$$\Rightarrow \quad \boxed{f_{u\,\text{overall}} = f_u \cdot \sqrt{2^{1/n} - 1}} \quad \text{...(i)}$$

Let A_{vl} be the voltage gain at low frequencies

$$\frac{A_{vl}}{A_{vm}} = \frac{1}{1 + jf_l/f}$$

For n-stages:

$$\left|\frac{A_{vl}}{A_{vm}}\right|_{\text{overall}} = \left[\frac{1}{\sqrt{1+(f_l/f)^2}}\right]^n$$

At $f = f_{l\,\text{overall}}$, i.e., the lower cut-off frequency of overall amplifier

$$\left|\frac{A_{v_l}}{A_{v_m}}\right|_{\text{overall}} = \frac{1}{\sqrt{2}}$$

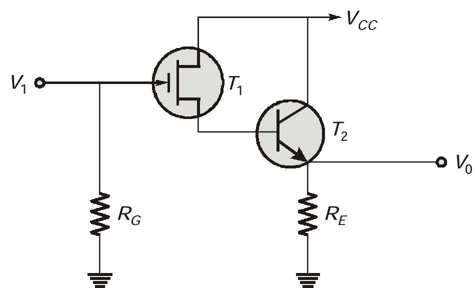
Putting values:

$$\left[\frac{1}{1+(f_l/f_{l\,\text{overall}})^2}\right]^{n/2} = \left[\frac{1}{2}\right]^{1/2}$$

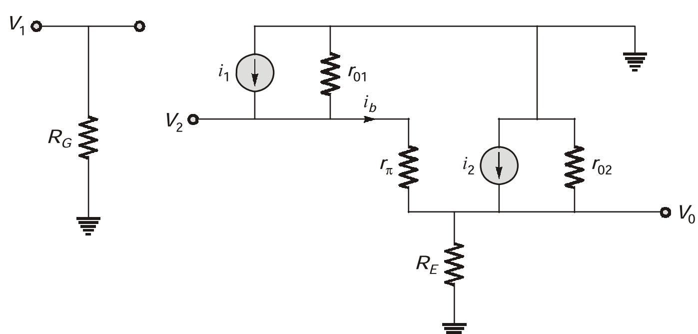
$$\Rightarrow \qquad \frac{1}{1+(f_l/f_{l\,\text{overall}})^2} = \frac{1}{2^{1/n}}$$

$$\Rightarrow \qquad \boxed{f_{l\,\text{overall}} = \frac{f_l}{\sqrt{2^{1/n}-1}}}$$

Solution: 2



Small signal equivalent circuit:



$$i_1 = g_{m_1}(V_1 - V_2) \qquad \text{...(1)}$$

$$i_2 = \beta i_b \qquad \text{...(2)}$$

Now,

$$i_b = i_1 - \frac{V_2}{r_{01}}$$

$$= g_{m_1}(V_1 - V_2) - \frac{V_2}{r_{01}}$$

$$\Rightarrow \qquad i_b = g_{m_1}V_1 + V_2\left(-g_{m_1} - \frac{1}{r_{01}}\right) \qquad \text{...(3)}$$

Now, current through $R_E = \dfrac{V_0}{R_E}$

$$\Rightarrow \qquad i_b + i_2 - \frac{V_0}{r_{02}} = \frac{V_0}{R_E}$$

$$\Rightarrow \qquad i_b + i_2 = V_0\left(\frac{1}{R_E} + \frac{1}{r_{02}}\right)$$

$$\Rightarrow \qquad (\beta + 1)i_b = V_0\left(\frac{1}{R_E} + \frac{1}{r_{02}}\right)$$

$$\Rightarrow \qquad (\beta + 1)\left[g_{m1}V_1 - V_2\left(g_{m1} + \frac{1}{r_{01}}\right)\right] = V_0\left(\frac{1}{R_E} + \frac{1}{r_{02}}\right) \qquad \text{...(4)}$$

Now, from (3)

$$\Rightarrow \qquad \frac{V_2 - V_0}{r_\pi} = g_{m1}V_1 - g_{m1}V_2 - \frac{V_2}{r_{01}}$$

$$\Rightarrow \qquad V_2\left(\frac{1}{r_\pi} + g_{m1} + \frac{1}{r_{01}}\right) = g_{m1}V_1 + \frac{V_0}{r_\pi}$$

$$\Rightarrow \qquad V_2 = \frac{g_{m1}V_1 + \dfrac{V_0}{r_\pi}}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}} \qquad \text{...(5)}$$

From (4) and (5)

$$\Rightarrow \qquad (\beta + 1)\left[g_{m1}V_1 - \frac{\left(g_{m1}V_1 + \dfrac{V_0}{r_\pi}\right)\left(g_{m1} + \dfrac{1}{r_{01}}\right)}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}}\right] = V_0\left(\frac{1}{R_E} + \frac{1}{r_{02}}\right)$$

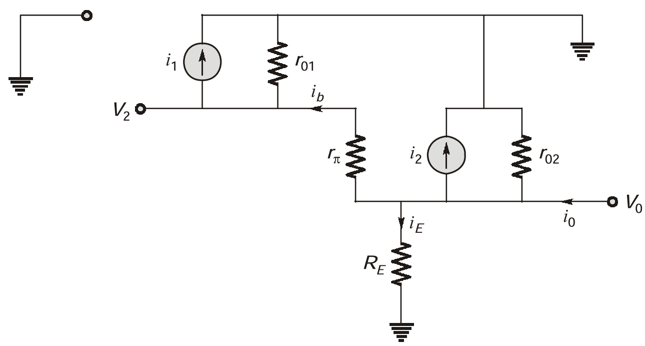
$$\Rightarrow \qquad g_{m1}V_1 - \frac{g_{m1}V_1\left(g_{m1} + \dfrac{1}{r_{01}}\right)}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}} - \frac{\dfrac{V_0}{r_\pi}\left(g_{m1} + \dfrac{1}{r_{01}}\right)}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}} = \frac{V_0\left(\dfrac{1}{R_E} + \dfrac{1}{r_{02}}\right)}{(\beta + 1)}$$

$$\Rightarrow \qquad V_1\left[g_{m1} - \frac{g_{m1}\left(g_{m1} + \dfrac{1}{r_{01}}\right)}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}}\right] = V_0\left[\frac{\dfrac{1}{R_E} + \dfrac{1}{r_{02}}}{(\beta + 1)} + \frac{\dfrac{1}{r_\pi}\left(g_{m1} + \dfrac{1}{r_{01}}\right)}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}}\right]$$

$$\Rightarrow \qquad \frac{V_0}{V_1} = \frac{g_{m1} - \dfrac{g_{m1}\left(g_{m1} + \dfrac{1}{r_{01}}\right)}{\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}}}{\dfrac{1}{(\beta + 1)}\left(\dfrac{1}{R_E} + \dfrac{1}{r_{02}}\right) + \dfrac{g_{m1} + \dfrac{1}{r_{01}}}{r_\pi\left(\dfrac{1}{r_\pi} + g_{m1} + \dfrac{1}{r_{01}}\right)}}$$

This is the gain of the amplifier.

Output resistance:



$$i_0 = i_E + i_2 + \frac{V_0}{r_{02}} + i_b$$

$$i_0 = \frac{V_0}{R_E} + \frac{V_0}{r_{02}} + (\beta+1)\,i_b$$

$$i_0 = V_0\left(\frac{1}{R_E} + \frac{1}{r_{02}}\right) + (\beta+1)\frac{(V_0 - V_2)}{r_\pi}$$

$$i_0 = V_0\left(\frac{1}{R_E} + \frac{1}{r_{02}} + \frac{\beta+1}{r_\pi}\right) - \frac{(\beta+1)V_2}{r_\pi} \qquad \text{...(1)}$$

Now, $$i_b = i_1 + \frac{V_2}{r_{01}}$$

$\Rightarrow$ $$\frac{V_0 - V_2}{r_\pi} = i_1 + \frac{V_2}{r_{01}}$$

$\Rightarrow$ $$\frac{V_0 - V_2}{r_\pi} = \left(g_{m1} + \frac{1}{r_{01}}\right)V_2$$

$\Rightarrow$ $$\frac{V_0}{r_\pi} = \left(g_{m1} + \frac{1}{r_{01}} + \frac{1}{r_\pi}\right)V_2$$

$\Rightarrow$ $$V_2 = \frac{V_0}{r_\pi\left(g_{m1} + \frac{1}{r_{01}} + \frac{1}{r_\pi}\right)}$$

$\Rightarrow$ $$V_2 = \frac{V_0}{g_{m1}r_\pi + \frac{r_\pi}{r_{01}} + 1} \qquad \text{...(2)}$$

From (1) and (2)

$\Rightarrow$ $$i_0 = V_0\left[\frac{1}{R_E} + \frac{1}{r_{02}} + \frac{\beta+1}{r_\pi} - \frac{(\beta+1)}{r_\pi\left(g_{m1}r_\pi + \frac{r_\pi}{r_{01}} + 1\right)}\right]$$

$\Rightarrow$ $$\frac{V_0}{i_0} = \frac{1}{\frac{1}{R_E} + \frac{1}{r_{02}} + \frac{\beta+1}{r_\pi} - \frac{(\beta+1)}{r_\pi\left(g_{m1}r_\pi + \frac{r_\pi}{r_{01}} + 1\right)}}$$

This is the output resistance.

Solution: 3

$$\frac{I_{out}}{I_{in}} = 2601$$

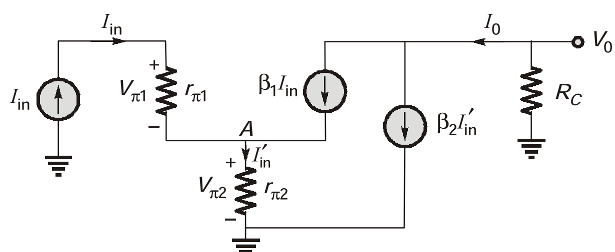
$$\frac{V_{in}}{i_2} = 2.601\ \text{M}\Omega$$

Solution: 4

$$A_v = -42$$

Solution: 5

(i) Since, we have to consider the input current of the transistors, thus the small signal equivalent of the transistor can be drawn as.



(ii) Now, $$I_0 = \beta_2 I'_{in} + \beta_1 I_1$$

Now, applying KCL at node A, we get

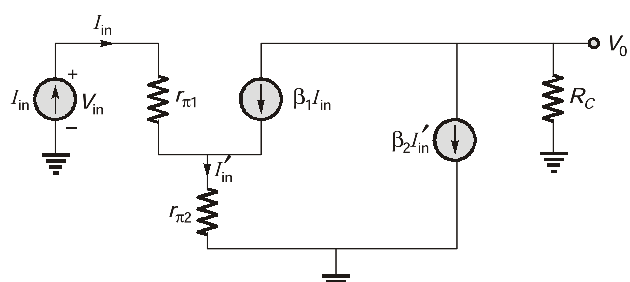
$$I'_{in} = I_{in} + \beta_1 I_{in}$$
$$I'_{in} = (1+\beta_1)I_{in}$$

Substituting this value in the above equation, we get,

$$I_0 = \beta_2(1+\beta_1)I_{in} + \beta_1 I_{in}$$

$$A_I = \frac{I_0}{I_{in}} = \beta_1 + \beta_2(1+\beta_1)$$

(iii) Assume the circuit as shown below



Now, $$V_{in} = r_{\pi1} I_{in} + r_{\pi2} I'_{in}$$

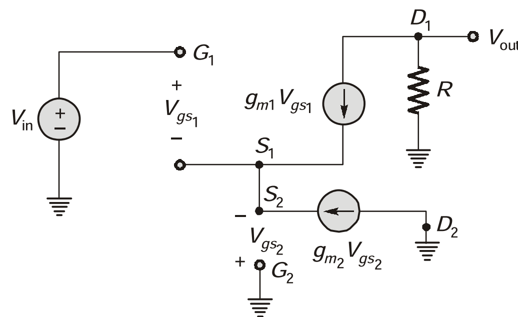
$$V_{in} = \left[r_{\pi1} + (\beta_1+1)r_{\pi2}\right]I_{in}$$

$$\frac{V_{in}}{I_{in}} = r_{\pi1} + (\beta_1+1)r_{\pi2}$$

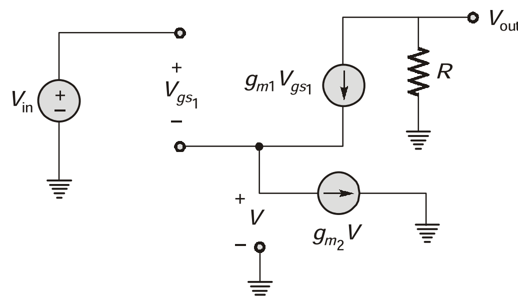
$\therefore$ $$R_{in} = r_{\pi1} + (\beta_1+1)r_{\pi2}$$

Solution: 6

By drawing the small signal equivalent circuit, we get



the above circuit can be redrawn as



Substituting $V = -V_{gs_2}$

Now, $$V_{in} = V_{gs_1} + V \quad \text{...(i)}$$

and $$g_{m_1} V_{gs_1} = g_{m_2} \cdot V \quad (\because \text{from KCL at node } S_1) \quad \text{...(ii)}$$

thus $$V_{out} = -\left[g_{m_1} V_{gs_1} R\right] \quad \text{...(iii)}$$

$$V_{out} = -g_{m_1} R(V_{in} - V) \quad \text{(from (i))}$$

$$= -g_{m_1} R V_{in} + g_{m_1} V R$$

now, $$V = \frac{g_{m_1} V_{gs_1}}{g_{m_2}} \quad \text{(from equation (ii))}$$

$$V_{out} = -g_{m_1} R V_{in} + \frac{g_{m_1} R V_{gs_1}}{g_{m_2}} \cdot g_{m_1}$$

now from (3), we get, $$V_{out} = -g_{m_1} R V_{in} - \frac{g_{m_1}}{g_{m_2}} V_{out}$$

$$\left(1 + \frac{g_{m_1}}{g_{m_2}}\right) V_{out} = -g_{m_1} R V_{in}$$

$$V_{out} = \frac{-g_{m_1} R}{1 + \dfrac{g_{m_1}}{g_{m_2}}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{-R}{\dfrac{1}{g_{m_1}} + \dfrac{1}{g_{m_2}}}$$

■■■■

9 Operational Amplifiers

LEVEL 1 Objective Solutions

1.	(b)
2.	(8.8)
3.	(c)
4.	(d)
5.	(0.1)
6.	(b)
7.	(d)

LEVEL 2 Objective Solutions

8.	(d)
9.	(d)
10.	(–8)
11.	(2.67)
12.	(b)
13.	(5.66)
14.	(d)
15.	(a)
16.	(b)
17.	(8)
18.	(d)
19.	(a)

LEVEL 3 Conventional Solutions

Solution: 1

Input offset voltage: It is defined as the opposite of the voltage that must be applied to the input terminals of an Op-amp to get zero voltage at the output terminals.

Bias current: In order for the Op-amp to operate, its two input terminals have to be supplied with dc currents, turned the input bias currents.

Slew rate: It is the maximum possible rate at which the output can change. Its unit is volts/second.

Output impedance: It is the impedance of the Op-amp seen from output terminals when the input is zero.

Common mode rejection ratio: It is the ratio of differential gain to common mode gain.

Solution: 2

$$\frac{V_0}{V_i} = 11$$

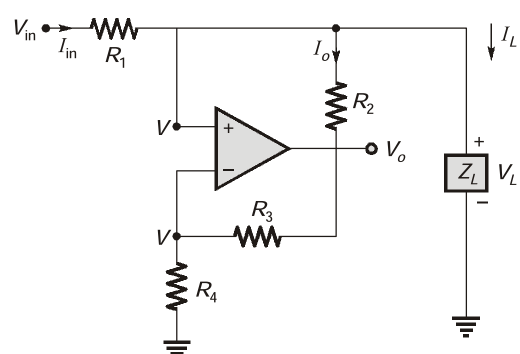
$$R_{iF} = 3611.738 \text{ M}\Omega$$

$$R_{OF} = 74.5\ \Omega$$

Solution: 3

$$I_{in} = I_o + I_L$$

$$\frac{V_{in} - V_L}{R_1} = \frac{V_L - V_o}{R_2} + I_L \quad \text{...(i)}$$



$$\frac{V_L - 0}{R_4} + \frac{V_L - V_o}{R_3} = 0$$

$$\frac{V_L}{R_4} + \frac{V_L}{R_3} = \frac{V_o}{R_3}$$

$$V_o = R_3 V_L \left[\frac{1}{R_4} + \frac{1}{R_3}\right]$$

$$V_o = V_L \left[\frac{R_3 + R_4}{R_4}\right] \quad \text{...(ii)}$$

By using equations (i) and (ii), we get

$$\frac{V_{in}}{R_1} - \frac{V_L}{R_1} = \frac{V_L}{R_2} - \frac{V_L}{R_2}\left[\frac{R_3 + R_4}{R_4}\right] + I_L$$

$$\frac{V_{in}}{R_1} - \frac{V_L}{R_1} - \frac{V_L}{R_2} + \frac{V_L}{R_2}\left[\frac{R_3 + R_4}{R_4}\right] = I_L$$

So, $$I_L = \frac{V_{in}}{R_1} - V_L\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{R_3 + R_4}{R_2 R_4}\right]$$

So, to make I_L independent of V_L, then term

$$\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{R_3 + R_4}{R_2 R_4}\right] = 0$$

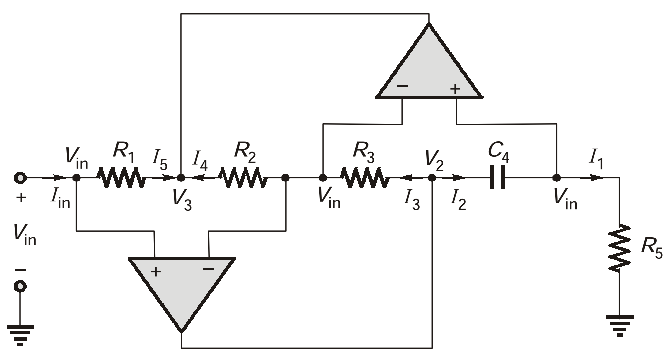
$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{R_3 + R_4}{R_2 R_4}$$

$$\frac{R_1 + R_2}{R_1 R_2} = \frac{R_3 + R_4}{R_2 R_4}$$

$$R_1 R_4 + R_2 R_4 = R_1 R_3 + R_1 R_4$$

$$\boxed{\frac{R_4}{R_3} = \frac{R_1}{R_2}}$$

Solution: 4



Now, $$I_1 = \frac{V_{in}}{R_5}$$

Thus, $$I_2 = I_1 = \frac{V_{in}}{R_5}$$

$\Rightarrow$ $$V_2 = V_{in} + \frac{V_{in}}{sC_4 R_5}$$

$\Rightarrow$ $$I_3 = \frac{V_{in}}{sR_5 R_3 C_4}$$

Now, $$I_3 = I_4 = \frac{V_{in}}{sC_4 R_5 R_3}$$

$$V_3 = V_{in} - \frac{V_{in} R_2}{sC_4 R_5 R_3}$$

Thus, $$I_5 = \frac{V_{in} R_2}{sC_4 R_5 R_3 R_1}$$

$\Rightarrow$ $$I_5 = I_{in}$$

$$I_{in} = \frac{V_{in}R_2}{sC_4R_5R_3R_1}$$

$$\Rightarrow \qquad Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = s\left[\frac{C_4R_5R_3R_1}{R_2}\right]$$

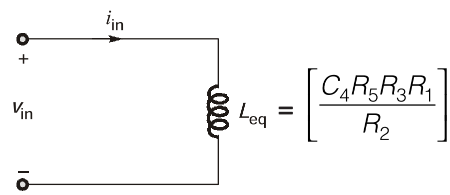
Comment:

The above equation is equivalent to

$$Z_{in}(s) = sL_{eq}$$

where $$L_{eq} = \left[\frac{C_4R_5R_3R_1}{R_2}\right]$$

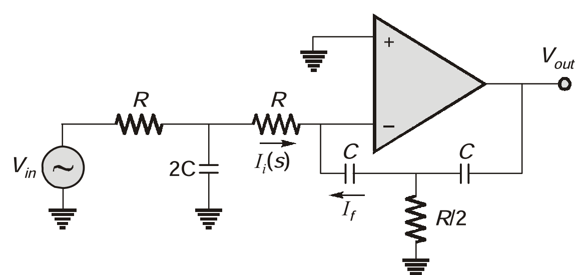
thus the circuit can be used for simulating an inductor. Thus an equivalent model, of the circuit can be represented as.



Solution: 5

$$V_{out}(t) = 50\sin t\,u(t) \text{ V}$$

Solution: 6



Now, taking the Laplace transform and computing

$$I_i(s) = \frac{\frac{V_{in}(s)}{2R}}{RsC+1}$$

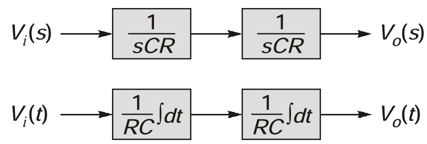
$$I_f(s) = \frac{RC^2S^2}{2(RCs+1)}\cdot V_{out}(s)$$

Now, $$I_i(s) = -I_f(s)$$

$$\therefore \qquad \frac{V_{in}(s)}{2R(RCs+1)} = \frac{-RC^2s^2}{2(RCs+1)}\cdot V_{out}(s)$$

$$\therefore \qquad \frac{V_{out}(s)}{V_{in}(s)} = -\frac{1}{(RCs)^2} = \left(\frac{-1}{R^2C^2}\right)\cdot\frac{1}{s^2}$$

Thus from the equation we can say that the circuit will behave as two integrators connected in cascaded as shown below:



Solution: 7

$$\frac{V_0}{V_{in}} = \frac{-R_2}{R_1}\left(\frac{1}{1+\frac{1}{A_{OL}}\left(1+\frac{R_2}{R_1}\right)}\right)$$

Solution: 8

$$V_0 = -2\text{ V}$$

■■■■

10 Feedback, Tuned Amplifiers and Timers

LEVEL 1 Objective Solutions

1.	(c)
2.	(a)
3.	(a)
4.	(d)
5.	(11)
6.	(b)
7.	(a)
8.	(b)
9.	(–50)
10.	(c)

LEVEL 2 Objective Solutions

11.	(b)
12.	(b)
13.	(b)
14.	(7.9)
15.	(c)
16.	(0.008)
17.	(a)
18.	(8)
19.	(b)
20.	(48)
21.	(a)
22.	(c)

LEVEL 3 Conventional Solutions

Solution: 1

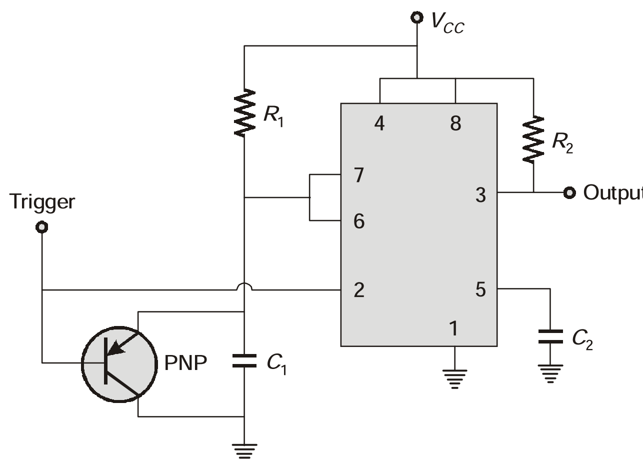
Method-I: $A_{vf} = 45.4$

Method-II: $R_{if} = 112.155 \text{ k}\Omega$

$A_{vs} = 45.4$

$R'_{of} = 129.03\ \Omega$

Solution: 2



The 555 timer is connected as a monostable multivibrator. A PNP transistor is connected across the capacitor and the input trigger pulse train is given to the base terminal of the transistor as well as the pin 2 trigger input of the IC 555. The train of the trigger pulses will continuously reset the timing cycle. Hence the output is always high. If any trigger pulse is missing, the device detects this missing pulse and the output goes low.

When the input is 0, the PNP transistor is turned on and the voltage across the capacitor is clamped to 0.7 V and the output is high. When the input trigger voltage is high, the transistor is cut off and the capacitor will start charging. If the input signal goes low again before the completion of the timing cycle, the voltage across the capacitor fall to 0.7 V before reaching the threshold voltage $\left(\frac{2}{3}V_{CC}\right)$ and the output continues to remain high. If the input trigger signal does not go low before the completion of the timing cycle due to a missing pulse, it allows the capacitor to charge to the threshold voltage and the output will become low. For this to work, the time period of the input trigger signal should be slightly lesser than the timing interval.

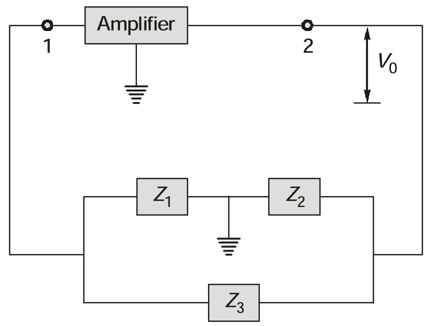
Solution: 3

$$\frac{V_f}{V_0} = \frac{1}{20}$$

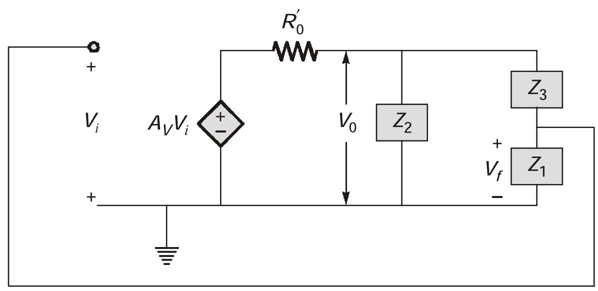
$$A_{vf} = -\frac{400}{19}$$

Solution: 4

Simplified circuit of LC oscillator (radio oscillator)



If amplifier is replaced with equivalent circuit then LC oscillator can be redrawn as



Net load impedance in LC oscillator is

$$Z_L = (Z_1 + Z_3) \parallel Z_2$$

$$Z_L = \frac{(Z_1 + Z_3)Z_2}{Z_1 + Z_2 + Z_3}$$

Using Voltage divider rule
$$V_0 = \frac{A_v \cdot V_i \times Z_L}{R_0' + Z_L} \qquad (R_0' = r_{ds} \parallel R_D)$$

$$\frac{V_0}{V_i} = A = \frac{A_v Z_L}{R_0' + Z_L}$$

Also
$$V_f = \frac{V_0 Z_1}{Z_1 + Z_3}$$

$$\frac{V_f}{V_0} = \beta = \frac{Z_1}{Z_1 + Z_3}$$

$$\text{Loop gain} = A \cdot \beta = \frac{A_v \cdot Z_1 \cdot Z_L}{(Z_1 + Z_3)(Z_L + R_0')}$$

Putting values of R_0', Z_L and

$$Z_1 = jX_1,\ Z_2 = jX_2,\ Z_3 = jX_3 \text{ we get}$$

$$\text{Loop gain} = \frac{-A_V X_1 X_2}{R_0' j(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

at $\omega = \omega_0$ phase of loop gain should be 360°

∴ Imaginary part should be zero.

∴ $X_1 + X_2 + X_3 = 0$ Condition for oscillations

For Hartley oscillator

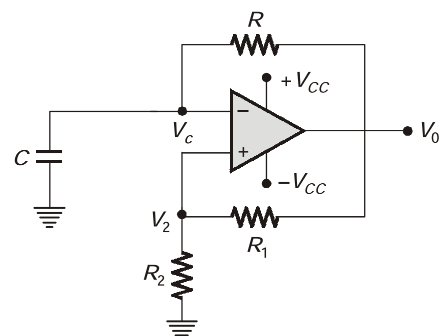
$$X_1 = \omega L_1,\ X_2 = \omega L_2,\ X_3 = \frac{1}{\omega C}$$

$$X_1 + X_2 + X_3 = 0$$

Gives $$f_0 = \frac{1}{2\pi\sqrt{(L_1 + L_2)C}}$$

Solution: 5

Astable multivibrator is constructed as shown in the figure below



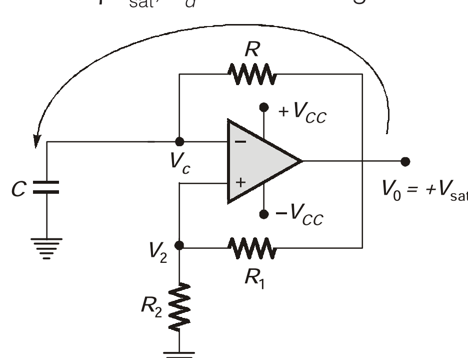
Output has two quasi stable states i.e. $+V_{sat}$ and $-V_{sat}$. Output waveform resembles square wave. It is also called square wave generator or free running oscillator.
Because of positive feedback the output is at two levels that is $+V_{sat}$ or $-V_{sat}$

$$V_2 = \frac{R_2}{R_1 + R_2} V_0 = \beta V_0$$

If $$V_0 = +V_{sat},$$
$$V_2 = +\beta V_{sat}$$

If $$V_0 = -V_{sat},$$
$$V_2 = -\beta V_{sat}$$

Circuit compares V_C with V_2
case (i) If $V_0 = +V_{sat}$, $V_2 = +\beta V_{sat}$, $V_d = V_2 - V_c$
capacitor charges through resistance R and V_C continuously increases
V_C continues to increase till $+\beta V_{sat}$
If V_C becomes slightly greater than $+\beta V_{sat}$, V_d becomes negative and then V_0 changes to $-V_{sat}$.

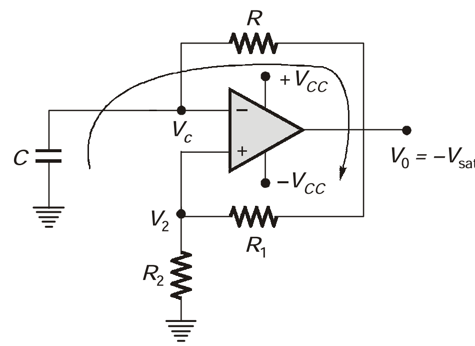


case (i)

case (ii) If $V_0 = -V_{sat} \Rightarrow$ $V_2 = -\beta V_{sat}$

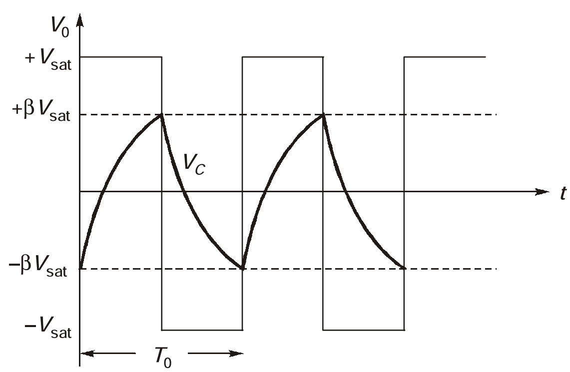
$$V_d = V_2 - V_c$$

Capacitor stops charging and start discharging through R.
V_c gradually decreases and V_c continues to decrease till $-\beta V_{sat}$



case (ii)

If V_C becomes slightly less than $-\beta V_{sat}$, V_d becomes positive and V_0 changes to $+V_{sat}$.
As capacitor charges and discharges through same resistance. The charging and discharging interval will be same and output waveform has 50% duty cycle.
Calculation of T_0:



$$V_c(t) = (V_i - V_f)e^{-t/RC} + V_f$$
$$V_c(0) = V_i = -\beta V_{sat}$$
$$V_c(\infty) = V_f = +V_{sat}$$
$$V_c(t) = [-\beta V_{sat} - V_{sat}]e^{-t/RC} + V_{sat}$$
$$V_c(t) = [1 - (1+\beta)e^{-t/RC}]\,V_{sat}$$

at $t = \dfrac{T_0}{2}$, $V_c(t) = +\beta V_{sat}$

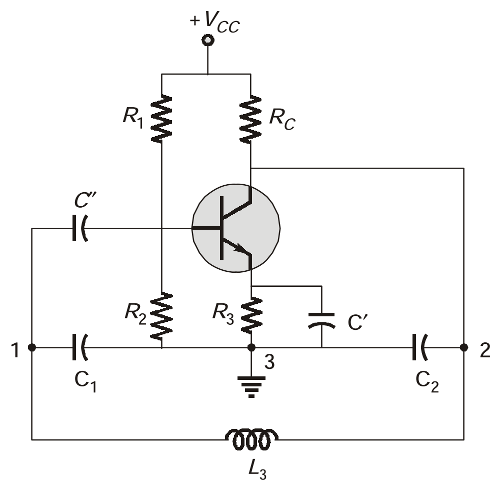
$$+\beta V_{sat} = [1 - (1+\beta)e^{-T_0/2RC}]\,V_{sat}$$
$$(1+\beta)e^{-T_0/2RC} = 1 - \beta$$
$$e^{-T_0/2RC} = \frac{1-\beta}{1+\beta}$$
$$\frac{-T_0}{2RC} = In\left(\frac{1-\beta}{1+\beta}\right)$$

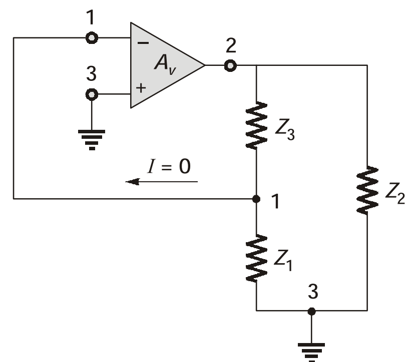
where β = feedback factor = $\dfrac{R_2}{R_1 + R_2}$

$$T_0 = -2RCln\left(\frac{1-\beta}{1+\beta}\right)$$

Solution: 6

(i)





(ii)

$$Z_1 = jX_1,\ Z_2 = jX_2,\ Z_3 = jX_3$$

$\Rightarrow$ $$X_1 = \frac{-1}{\omega C_1},\ X_2 = \frac{-1}{\omega C_2},\ X_3 = \omega L_3$$

Load impedance, $$Z_L = Z_2 \parallel (Z_1 + Z_3)$$

The gain without feedback is

$$A = \frac{-A_v Z_L}{Z_L + R_0}$$

Feedback factor, $$\beta = \frac{-Z_1}{Z_1 + Z_3}$$

Loop gain, $$-A\beta = \frac{-A_v Z_1 Z_2}{R_0(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

For loop gain to be real, $X_1 + X_2 + X_3 = 0$ at $\omega = \omega_0$ (zero phase shift)

Putting values $\dfrac{-1}{\omega_0 C_1} - \dfrac{1}{\omega_0 C_2} + \omega_0 L_3 = 0$

$\Rightarrow$ $$\omega_0^2 = \frac{1}{L_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$\Rightarrow$ $$\omega_0 = \sqrt{\frac{1}{L_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

Frequency of oscillation is,

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)}$$

When, $X_1 + X_2 + X_3 = 0$ then,

$$-A\beta = \frac{A_v X_1 X_2}{-X_2(X_1+X_3)} = \frac{-A_V X_1}{X_1+X_3} = \frac{A_V X_1}{X_2}$$

For oscillations to start, $-A\beta > 1$

$$\Rightarrow \qquad \frac{A_V X_1}{X_2} \geq 1$$

$$\Rightarrow \qquad \frac{X_2}{X_1} \leq A_V$$

$$\Rightarrow \qquad \frac{-1/\omega_0 C_2}{-1/\omega_0 C_1} \leq A_V$$

$$\Rightarrow \qquad \frac{C_1}{C_2} \leq A_V$$

Solution: 7

$$f_0 = 250.75\text{ kHz}$$

Solution: 8

$$f_0 = 7.96\text{ Hz}$$

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