GATE

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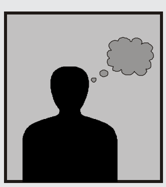
Workbook 2025



Detailed Explanations of Try Yourself *Questions*

Instrumentation Engineering
Signals and Systems

MADE EASY PUBLICATIONS

1 Introduction

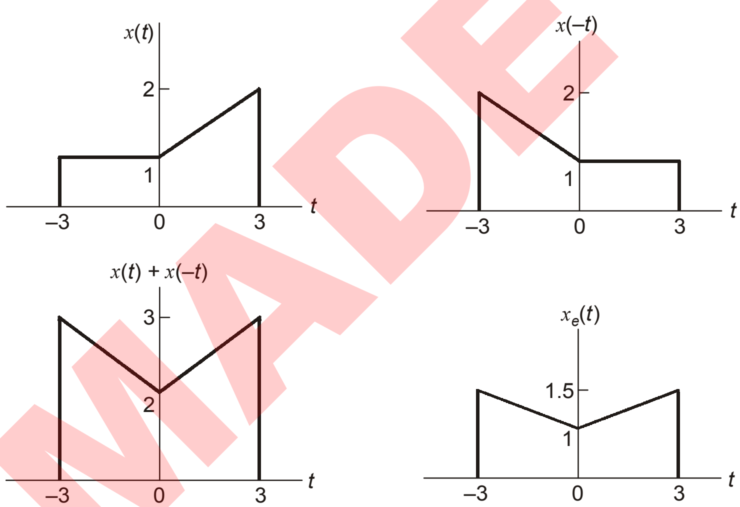
Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

Even part of $x(t)$, $\quad x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Signal $x_e(t)$ is obtained as follows:



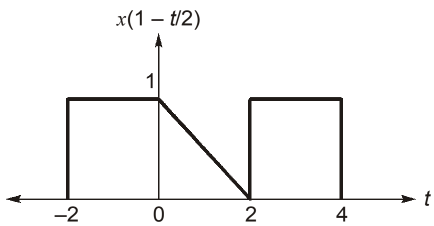
T2 : Solution

(c)

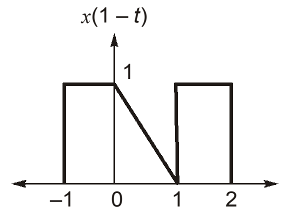
We can perform following sequence of transformation.

$$x\left(1-\frac{t}{2}\right) \xrightarrow[\text{time compression}]{t-2t} x(1-t) \xrightarrow[\text{folding}]{t\to -t} x(t+1) \xrightarrow[\text{time shifting}]{t\to t-1} x(t)$$

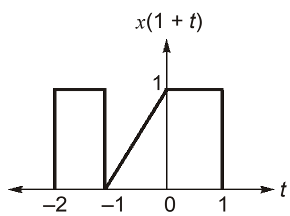
Graphically it is obtained as



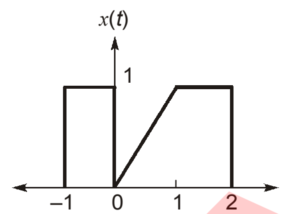
Original signal



Time Compression



Time Reflection



Time Shifting

T3 : Solution

(a)

The expression of $x(t)$ is $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-4k) - \delta(t-4k-1)$.

So $x(t)$ is a subtraction of two signals each periodic with period 4. So $x(t)$ is periodic with period 4.

T4 : Solution

The signal is, $x(t) = 3e^{-t}u(t)$

Now, energy of signal will be $E_x = \int_0^{\infty} [3e^{-t}]^2 dt = 4.5$

T5 : Solution

(d)

$$y(t) = 4^2\cos^2\left(200t + \frac{\pi}{6}\right)$$

$$= 4^2 \frac{\left(1 + \cos 2\left(200t + \frac{\pi}{6}\right)\right)}{2}$$

$$= 8 + 8\cos\left(400t + \frac{\pi}{3}\right)$$

Thus the DC component is 8.

T6 : Solution

(b)
Cosine function is a periodic signal. As all periodic signals are power signals, therefore the given signal is power signal.

T7 : Solution

(a)

$$\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt = f(0) = \cos\left(\frac{3\times 0}{2}\right) = \cos 0 = 1$$

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2 Fourier Series

Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

Given that Fourier series coefficient of $x(t)$ is a_k

So, $$x(t) \xrightarrow{\text{F.S.}} a_k$$

Now, real part of $x(t)$ is $\dfrac{x(t)+x^*(t)}{2}$

and if $$x(t) \xrightarrow{\text{F.S.}} a_k$$

then $$x^*(t) \xrightarrow{\text{F.S.}} a^*_{-k}$$

So real part of $x(t)$, $$\frac{x(t)+x^*(t)}{2} \xrightarrow{\text{F.S.}} \frac{a_k + a^*_{-k}}{2}$$

T2 : Solution

(d)

T3 : Solution

(b)

T4 : Solution

Power of signals is $$\sum_{-\infty}^{\infty} |C_n|^2 \Rightarrow \sum_{-2}^{2} |C_n|^2$$

So power is $$= \sum_{-2}^{2} |C_n|^2 = (2)^2 + (8)^2 + (8)^2 + (2)^2 = 136$$

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3 Fourier Transform

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

The Fourier transform is $X(\omega) = u(\omega) - u(\omega - 2)$, we know that

- If signal is real then $X(\omega)$ is conjugate symmetric.
- If signal is imaginary then $X(\omega)$ is conjugate anti-symmetric

The given $X(\omega)$ is neither conjugate symmetric nor conjugate anti-symmetric.

So $x(t)$ is complex signal.

T2 : Solution

(c)

Fourier transform of $x(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} x(t)\cos(\omega t)dt - j\int_{-\infty}^{\infty} x(t)\sin(\omega t)dt$$

If $x(t)$ is odd, then $x(t)$ sin ωt is an even function and $x(t)$ cos ωt is an odd function.

So, $$\int_{-\infty}^{\infty} x(t)\cos(\omega t)dt = 0$$

and, $$X(j\omega) = j\int_{-\infty}^{\infty} x(t)\sin(\omega t)dt$$

or, $$X(j\omega) = -2j\int_{0}^{\infty} x(t)\sin(\omega t)dt$$

T3 : Solution

(a)

Given $X(j\omega)$ is real and odd, so $x(t)$ is imaginary and odd.

T4 : Solution

(a)

Fourier transform is $G(\omega) = \dfrac{\omega^2 + 21}{\omega^2 + 9}$

So,
$$G(\omega) = \frac{\omega^2}{\omega^2+9} + \frac{21}{\omega^2+9} = 1 + \frac{12}{\omega^2+9}$$

As we know that Fourier transform of $e^{-a|t|}$ is $\dfrac{2a}{a^2+\omega^2}$

So
$$g(t) = \delta(t) + 2\exp(-3|t|)$$

T5 : Solution

(d)

If,
$$x(t) \xleftrightarrow{F} X(j\omega)$$

then,
$$\frac{dx(t)}{dt} \xleftrightarrow{F} (j\omega)X(j\omega) \qquad \text{(Time differentiation property)}$$

and,
$$\frac{d^2x(t)}{dt^2} \xleftrightarrow{F} -\omega^2 X(j\omega)$$

$$\frac{d^2[x(t-2)]}{dt^2} \xleftrightarrow{F} -\omega^2 e^{-j2\omega} X(j\omega) \qquad \text{(Time-shifting property)}$$

T6 : Solution

(a)

We have,
$$y(t) = \int_{-\infty}^{\tau} x(\tau)d\tau$$

$$\int_{-\infty}^{\tau} x(\tau)d\tau \xleftrightarrow{F} \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega) \qquad \text{(Time integration property)}$$

So,
$$Y(j\omega) = \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

$$= \frac{1}{j\omega}\left(\frac{j\omega}{5+\frac{j\omega}{10}}\right) + 0 = \frac{1}{\left(5+\frac{j\omega}{10}\right)} \qquad X(0) = 0$$

Now, area under $y(t)$,
$$\int_{-\infty}^{\infty} y(t)dt = Y(0)$$

Thus,
$$Y(0) = \frac{1}{5+0} = \frac{1}{5}$$

T7 : Solution

(c)

Properties of distortionless system are:

- Magnitude should be constant w.r.t. frequency.
- Phase should depend linearly on frequency.

Only function given in option (c) follow the given conditions.

T8 : Solution

(a)

The signal $x(t) = (2 + e^{-3t})\, u(t)$ then final value i.e. $x(\infty)$ will be 2.

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4 Laplace Transform

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

Convolution in time domain is multiplication in s-domain.

$$\therefore \quad L[h(t)] = L[f(t)] \times L[g(t)] = \frac{1}{s+3}$$

T2 : Solution

(c)

$$r(t) \xleftrightarrow{\text{L.T.}} \frac{1}{s^2}$$

$$r(t-a) \xleftrightarrow{\text{L.T.}} e^{-as} \times \frac{1}{s^2} = \frac{e^{-as}}{s^2}$$

T3 : Solution

(c)

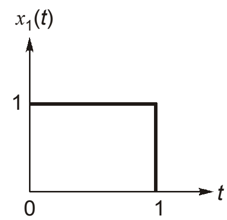
$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} s\,I(s)$$

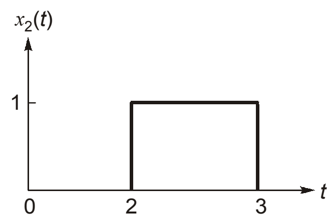
$$= \lim_{s\to 0} s.\frac{2}{s(1+s)} = 2$$

T4 : Solution

(b)

We can express the given function in terms of unit step function as follows:





$x_1(t) = u(t) - u(t-1)$ $\qquad$ $x_2(t) = u(t-2) - u(t-3)$

Thus,
$$x(t) = x_1(t) + x_2(t) + x_3(t) + \ldots$$
$$= u(t) - u(t-1) + u(t-2) - u(t-3) + \ldots$$

We know that
$$u(t) \xleftrightarrow{L} \frac{1}{s}$$
$$u(t-t_0) \xleftrightarrow{L} \frac{1}{s}e^{-st_0} \qquad \text{(time-shifting)}$$

The Laplace transform of $x(t)$ is
$$X(s) = \frac{1}{s} - \frac{1}{s}e^{-s} + \frac{1}{s}e^{-2s} - \frac{1}{s}e^{-3s} + \frac{1}{s}e^{-4s} - \frac{1}{s}e^{-5s} + \ldots.$$
$$= \frac{1}{s}\left[1 + e^{-2s} + e^{-4s} + \ldots.\right] - \frac{1}{s}\left[e^{-s} + e^{-3s} + e^{-5s} + \ldots.\right]$$
$$= \frac{1}{s}\left[\frac{1}{1-e^{-2s}}\right] - \frac{1}{s}\left[\frac{e^{-s}}{1-e^{-2s}}\right]$$
$$= \frac{1}{s}\left[\frac{1-e^{-s}}{1-e^{-2s}}\right]$$
$$= \frac{1}{s}\left[\frac{1}{1+e^{-s}}\right]$$

T5 : Solution

(d)

From the time integration property of Laplace transform
$$x(t) \xleftrightarrow{L} X(s)$$
$$\int_0^t x(\tau)d\tau \xleftrightarrow{L} \frac{1}{s}X(s) \qquad \text{Time integration Property}$$
$$\int_0^t x(\tau)d\tau \xleftrightarrow{L} \frac{(s+1)}{s(s^2+4s+5)}$$

T6 : Solution

(d)

Given,
$$H(s) = \frac{k(s^2+\omega_0^2)}{s^2 + \left(\dfrac{\omega_0}{Q}\right)s + \omega_0^2}$$

So value of $H(s)$ at $s \to \infty$ is k
and value of $H(s)$ at $s \to 0$ is k.
So the filter is a band stop filter or notch filter.

T7 : Solution

(c)

$$X(s) = L[x(t)] = \frac{s}{s^2+1}$$

$$H(s) = L[h(t)] = \frac{1}{s^2+1}$$

$$y(t) = x(t) * h(t)$$

$$Y(s) = L[x(t) * h(t)] = X(s)H(s) = \frac{s}{(s^2+1)^2}$$

Using partial fractional, $$Y(s) = \frac{-j/4}{(s-j)^2} + \frac{j/4}{(s+j)^2}$$

We know that $te^{-at}u(t) \xleftrightarrow{L} \dfrac{1}{(s+a)^2}$

so, $\dfrac{1}{(s-j)^2} \xleftrightarrow{L^{-1}} te^{jt}$

$\dfrac{1}{(s+j)^2} \xleftrightarrow{L^{-1}} te^{-jt}$

so, $$y(t) = \frac{j}{4}\left[-te^{jt} + te^{-jt}\right] = \frac{j}{4}t\left[e^{-jt} - e^{jt}\right] = \frac{t}{2}\sin t, \qquad t \geq 0$$

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5 Sampling Theorem and Discrete Time System

Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} 1 = \infty$$

T2 : Solution

(c)

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]$$

$$y[n] = \sum_{n=-\infty}^{\infty} n^2\delta[n+2]$$

$$= n^2\Big|_{n=-2}$$

$$= (-2)^2 = 4$$

T3 : Solution

(d)

(A)
$$y[n] = x[n^2]$$
$$x_1[n] \rightarrow y_1[n] = x_1[n^2]$$
$$x_2[n] \rightarrow y_2[n] = x_2[n^2]$$
$$ax_1[n] + bx_2[n] \rightarrow ax_1[n^2] + bx_2[n^2]$$
$$= ay_1[n] + by_2[n]$$

Hence the system is linear.

(B)
$$y[n] = x^2[n-1]$$

For a delayed input $x[n-n_0]$, output is

$$y[n, n_0] = x^2[n-n_0-1]$$

The delayed output

$$y[n-n_0] = x^2[n-n_0-1]$$

Since $y[n, n_0] = y[n - n_0]$ Hence the system is time-invariant.

(C) $y[n] = x[n] + n$

$y[n]$ depends on present value of $x[n]$, so the system is causal.

(D)

$$y[n] = x[3n]$$
$$y[-1] = x[-3]$$
$$y[1] = x[3]$$

System has memory, therefore it is a dynamic system.

T4 : Solution

(c)

Since $x[n]$ is even symmetric about mid point ($n = 1$) and $h[n]$ is odd symmetric about mid point ($n = 1$) so $y[n]$ will be odd symmetric about its mid point $n = 2$.

$x[n]$ \ $h[n]$	$\underset{\downarrow}{1}$	0	–1
$\rightarrow$ 2	2	0	–2
1	1	0	–1
2	2	0	–2

$$y[n] = x[n] * h[n] = \{\underset{\uparrow}{2}, 1, 0, -1, -2\}$$

$y[n]$ is odd symmetric about $n = 2$.

T5 : Solution

(a)

Causality:

$$h[n] = 0,\ n < 0$$

The system is causal.

Stability:

$$\sum_{n=-\infty}^{\infty} |h[n]| = 2\sum_{n=0}^{\infty}(0.4)^n - \sum_{n=0}^{\infty}(0.2)^n$$

$$= 2\left[\frac{1}{1-0.4}\right] - \frac{1}{(1-0.2)} < \infty$$

The sytem is stable.

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6 Z-Transform

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

Given that,
$$x(n) = \sum_{k=0}^{\infty} \delta(n-k)$$
$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \cdots$$
$$x(n) = u(n)$$
$$X(z) = \text{Z.T.}[u(n)]$$
$$X(z) = \frac{z}{z-1}$$

T2 : Solution

(c)

z-transform of $x[n]$,
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} \alpha^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty} \alpha^{-n} z^{-n} u[n]$$
$$= \sum_{n=0}^{\infty} (\alpha z^{-1}) + \sum_{n=0}^{\infty} (\alpha z)^{-n} = \underbrace{\frac{1}{1-\alpha z^{-1}}}_{\text{I}} + \underbrace{\frac{1}{1-(\alpha z)^{-1}}}_{\text{II}}$$

Series I converges, if $\alpha z^{-1} < 1$ or $|z| > |\alpha|$

Series II converges, if $(\alpha z)^{-1} < 1$ or $\alpha z > 1$ or $|z| > \frac{1}{|\alpha|}$

So, ROC is interaction of both

$$\text{ROC} : |z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$$

T3 : Solution

(b)

$$X(z) = \frac{z+1}{z(z-1)}$$

$$= -\frac{1}{z} + \frac{2}{z-1} = -\frac{1}{z} + 2z^{-1}\left(\frac{z}{z-1}\right) \qquad \text{By partial fraction}$$

Taking inverse z-transform

$$x[n] = -\delta[n-1] + 2u[n-1]$$
$$x[0] = -0 + 0 = 0$$
$$x[1] = -1 + 2 = 1$$
$$x[2] = -0 + 2 = 2$$

T4 : Solution

(c)

By taking z-transform of $x[n]$ and $h[n]$

$$H(z) = 1 + 2z^{-1} - z^{-3} + z^{-4}$$
$$X(z) = 1 + 3z^{-1} - z^{-2} - 2z^{-3}$$

From the convolution property of z-transform

$$Y(z) = H(z)\,X(z)$$
$$Y(z) = 1 + 5z^{-1} + 5z^{-2} - 5z^{-3} - 6z^{-4} + 4z^{-5} + z^{-6} - 2z^{-7}$$

Sequence is $y[n] = \{1, 5, 5, -5, -6, 4, 1, -2\}$

$$y[4] = -6$$

T5 : Solution

(d)

Given that $x(n)$ is right sided and real, $X(z)$ has two poles, two zeros at origin and one pole at $e^{j\pi/2}$, $X(1) = 1$. Since $x(n)$ is real so poles of $X(z)$ should be in conjugate pairs so other pole will be at $e^{-j\pi/2}$.

So, $$X(z) = \frac{k\,z^2}{(z-e^{-j\pi/2})(z-e^{+j\pi/2})} = \frac{k\,z^2}{z^2+1}$$

Since, $X(1) = 1$ so, $k = 2$

So, $$X(z) = \frac{2z^2}{z^2+1} \text{ and } |z| > 1$$

T6 : Solution

(a)

We know that, $$\alpha^n u[n] \xleftrightarrow{Z} \frac{z}{z-\alpha}$$

$$\alpha^{n-10}u[n-10] \xleftrightarrow{Z} \frac{z^{-10}z}{z-\alpha} \qquad \text{(time shifting property)}$$

T7 : Solution

(b)

$$Y(z) - \frac{1}{3}z^{-1}Y(z) = X(z)$$

$$Y(z)\left[1 - \frac{1}{3}z^{-1}\right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

where, $$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$\therefore$ $$Y(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)} = \frac{3}{1 - \frac{1}{2}z^{-1}} - \frac{2}{1 - \frac{1}{3}z^{-1}}$$

$$y[n] = \left[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right]u[n]$$

T8 : Solution

(a)

We know that convolution of $x[n]$ with unit step function $u[n]$ is given by

$$x[n] * u[n] = \sum_{k=-\infty}^{\infty} x[k]$$

so, $$y[n] = x[n] * u[n]$$

Taking z-transform on both sides

$$Y(z) = X(z)\frac{z}{(z-1)} = X(z)\frac{1}{(1-z^{-1})}$$

Transfer function, $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1-z^{-1})}$$

Now, consider the inverse system of $H(z)$, let impulse response of the inverse system is given by $H_1(z)$, then we can write

$$H(z)H_1(z) = 1$$

$$H_1(z) = \frac{X(z)}{Y(z)} = 1 - z^{-1}$$

$$(1 - z^{-1})Y(z) = X(z)$$

$$Y(z) - z^{-1}Y(z) = X(z)$$

Taking inverse z-transform

$$y[n] - y[n-1] = x[n]$$

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7 DTFT, DTFS & DFT

Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

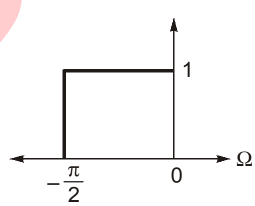
Since $$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$$

Thus $$e^{j\Omega_0 n} x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega-\Omega_0)})$$ (Frequency shifting property)

$\Omega_0 = -\pi/4$

$$e^{-j\frac{\pi}{4}n} x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega+\pi/4)})$$

The graph of $X(e^{j\Omega})$ is shifting to left by $\frac{\pi}{4}$ units. So, DTFT of $e^{-j\frac{\pi n}{4}} x[n]$ is



T2 : Solution

(a)

N-point DfT is given as

$$X_{DFT}[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}}, \; k = 0, 1, \ldots N-1$$

$$X_{DFT}[k] = \sum_{n=0}^{3} x[n] e^{-j\frac{\pi nk}{2}} \qquad \because N = 4$$

For $k = 0$,
$$X_{DFT}[0] = \sum_{n=0}^{3} x[n]$$
$$= x[0] + x[1] + x[2] + x[3]$$
$$= \cos 0 + \cos \pi + \cos 2\pi + \cos 3\pi$$
$$= 1 - 1 + 1 - 1 = 0$$

For $k = 1$,
$$X_{DFT}[1] = \sum_{n=0}^{3} x[n]e^{-j\frac{\pi n}{2}}$$
$$= x[0]e^{0} + x[1]e^{-j\frac{\pi}{2}} + x[2]e^{-j\pi} + x[3]e^{-j\frac{3\pi}{2}}$$
$$= \cos 0 + \cos \pi(-j) + \cos 2\pi(-1) + \cos 3\pi(j)$$
$$= 1 + (-1)(-j) + 1(-1) + (-1)(j)$$
$$= 1 + j - 1 - j$$
$$= 0$$

Similarly we can obtain $X_{DFT}[2]$ and $X_{DFT}[3]$ for $k = 2$ and $k = 3$ respectively,
$$X_{DFT}[2] = 1 + 1 + 1 + 1 = 4$$
$$X_{DFT}[3] = 1 - j - 1 + j = 0$$
$$X_{DFT}[k] = \{0, 0, 4, 0\}$$

T3 : Solution

(c)

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = \sum_{n=-2}^{2} e^{-j\Omega n}$$
$$= e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}$$
$$= e^{-j2\Omega}(1 + e^{j\Omega} + e^{j2\Omega} + e^{j3\Omega} + e^{j4\Omega})$$
$$= e^{-j2\Omega}\frac{(1 - e^{j5\Omega})}{1 - e^{j\Omega}} \qquad \text{(Summation of finite GP)}$$
$$= \frac{e^{-j5\pi/2} - e^{j5\Omega/2}}{e^{-j\pi/2} - e^{j\Omega/2}} = \frac{\sin 2.5\Omega}{\sin 0.5\Omega}$$

T4 : Solution

(b)

$$X(e^{j\Omega}) = j4 \sin 4\Omega - 1$$
$$= 2(e^{j4\Omega} - e^{-j4\Omega}) - 1$$

Taking inverse Fourier transform, we have
$$x[n] = 2\delta[n+4] - 2\delta[n-4] - \delta[n]$$

Since,
$$\delta[n - n_0] \xleftrightarrow{DTFT} e^{-j\Omega n_0}$$

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