

GATE

MADE EASY **WORKBOOK 2024**



**Detailed Explanations of
Try Yourself Questions**

Instrumentation Engineering Signals and Systems



MADE EASY
Publications

1

Introduction



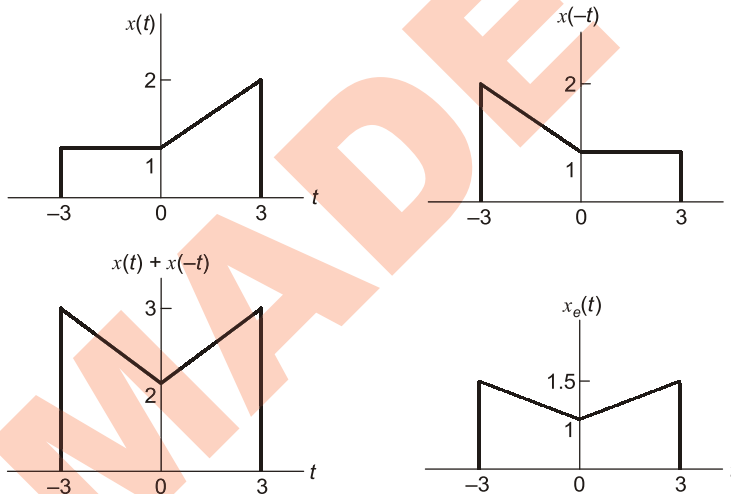
Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

Even part of $x(t)$, $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Signal $x_e(t)$ is obtained as follows:



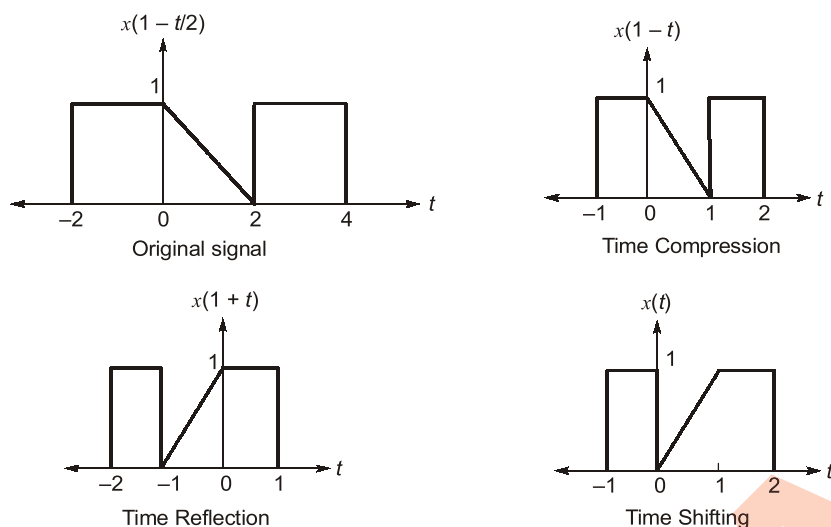
T2 : Solution

(c)

We can perform following sequence of transformation.

$$x\left(1 - \frac{t}{2}\right) \xrightarrow[\text{time compression}]{t \rightarrow 2t} x(1 - t) \xrightarrow[\text{folding}]{t \rightarrow -t} x(t + 1) \xrightarrow[\text{time shifting}]{t \rightarrow t - 1} x(t)$$

Graphically it is obtained as



T3 : Solution

(a)

The expression of $x(t)$ is $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 4k) - \delta(t - 4k - 1)$.

So $x(t)$ is a subtraction of two signals each periodic with period 4. So $x(t)$ is periodic with period 4.

T4 : Solution

The signal is,

$$x(t) = 3e^{-t}u(t)$$

Now, energy of signal will be

$$E_x = \int_0^{\infty} [3e^{-t}]^2 dt = 4.5$$

T5 : Solution

(d)

$$\begin{aligned} y(t) &= 4^2 \cos^2\left(200t + \frac{\pi}{6}\right) \\ &= 4^2 \frac{1 + \cos 2\left(200t + \frac{\pi}{6}\right)}{2} \\ &= 8 + 8 \cos\left(400t + \frac{\pi}{3}\right) \end{aligned}$$

Thus the DC component is 8.

T6 : Solution

(b)

Cosine function is a periodic signal. As all periodic signals are power signals, therefore the given signal is power signal.

T7 : Solution

(a)

$$\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt = f(0) = \cos\left(\frac{3 \times 0}{2}\right) = \cos 0 = 1$$

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2

Fourier Series



Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

Given that Fourier series coefficient of $x(t)$ is a_k

So, $x(t) \xrightarrow{\text{F.S.}} a_k$

Now, real part of $x(t)$ is $\frac{x(t) + x^*(t)}{2}$

and if

$x(t) \xrightarrow{\text{F.S.}} a_k$

then

$x^*(t) \xrightarrow{\text{F.S.}} a_{-k}^*$

So real part of $x(t)$, $\frac{x(t) + x^*(t)}{2} \xrightarrow{\text{F.S.}} \frac{a_k + a_{-k}^*}{2}$

T2 : Solution

(d)

T3 : Solution

(b)

T4 : Solution

Power of signals is

$$\sum_{-\infty}^{\infty} |C_n|^2 \Rightarrow \sum_{-2}^2 |C_n|^2$$

So power is

$$= \sum_{-2}^2 |C_n|^2 = (2)^2 + (8)^2 + (8)^2 + (2)^2 = 136$$

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3

Fourier Transform



Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

The Fourier transform is $X(\omega) = u(\omega) - u(\omega - 2)$, we know that

- If signal is real then $X(\omega)$ is conjugate symmetric.
- If signal is imaginary then $X(\omega)$ is conjugate anti-symmetric

The given $X(\omega)$ is neither conjugate symmetric nor conjugate anti-symmetric.

So $x(t)$ is complex signal.

T2 : Solution

(c)

Fourier transform of $x(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t)\cos(\omega t)dt - j \int_{-\infty}^{\infty} x(t)\sin(\omega t)dt$$

If $x(t)$ is odd, then $x(t)\sin \omega t$ is an even function and $x(t)\cos \omega t$ is an odd function.

So,
$$\int_{-\infty}^{\infty} x(t)\cos(\omega t)dt = 0$$

and,
$$X(j\omega) = j \int_{-\infty}^{\infty} x(t)\sin(\omega t)dt$$

or,
$$X(j\omega) = -2j \int_0^{\infty} x(t)\sin(\omega t)dt$$

T3 : Solution

(a)

Given $X(j\omega)$ is real and odd, so $x(t)$ is imaginary and odd.

T4 : Solution

(a)

Fourier transform is $G(\omega) = \frac{\omega^2 + 21}{\omega^2 + 9}$

So, $G(\omega) = \frac{\omega^2}{\omega^2 + 9} + \frac{21}{\omega^2 + 9} = 1 + \frac{12}{\omega^2 + 9}$

As we know that Fourier transform of $e^{-a|t|}$ is $\frac{2a}{a^2 + \omega^2}$

So $g(t) = \delta(t) + 2\exp(-3|t|)$

T5 : Solution

(d)

If, $x(t) \xrightarrow{F} X(j\omega)$

then, $\frac{dx(t)}{dt} \xrightarrow{F} (j\omega)X(j\omega)$ (Time differentiation property)

and, $\frac{d^2x(t)}{dt^2} \xrightarrow{F} -\omega^2 X(j\omega)$

$\frac{d^2[x(t-2)]}{dt^2} \xrightarrow{F} -\omega^2 e^{-j2\omega} X(j\omega)$ (Time-shifting property)

T6 : Solution

(a)

We have, $y(t) = \int_{-\infty}^t x(\tau) d\tau$

$\int_{-\infty}^t x(\tau) d\tau \xrightarrow{F} \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$ (Time integration property)

So, $Y(j\omega) = \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$

$= \frac{1}{j\omega} \left(\frac{j\omega}{5 + \frac{j\omega}{10}} \right) + 0 = \frac{1}{\left(5 + \frac{j\omega}{10} \right)}$ $X(0) = 0$

Now, area under $y(t)$, $\int_{-\infty}^{\infty} y(t) dt = Y(0)$

Thus, $Y(0) = \frac{1}{5+0} = \frac{1}{5}$

T7 : Solution

(c)

Properties of distortionless system are:

- Magnitude should be constant w.r.t. frequency.
- Phase should depend linearly on frequency.

Only function given in option (c) follow the given conditions.

T8 : Solution

(a)

The signal $x(t) = (2 + e^{-3t}) u(t)$ then final value i.e. $x(\infty)$ will be 2.

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4

Laplace Transform



Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

Convolution in time domain is multiplication in s -domain.

$$\therefore L[h(t)] = L[f(t)] \times L[g(t)] = \frac{1}{s+3}$$

T2 : Solution

(c)

$$r(t) \xleftrightarrow{\text{L.T.}} \frac{1}{s^2}$$

$$r(t-a) \xleftrightarrow{\text{L.T.}} e^{-as} \times \frac{1}{s^2} = \frac{e^{-as}}{s^2}$$

T3 : Solution

(c)

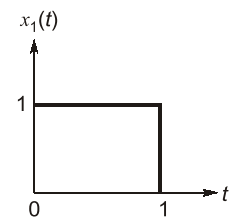
$$\lim_{t \rightarrow \infty} i(t) = \lim_{s \rightarrow 0} sI(s)$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{2}{s(1+s)} = 2$$

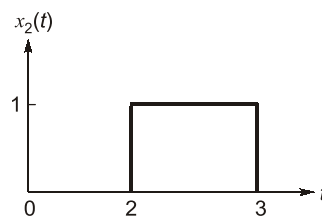
T4 : Solution

(b)

We can express the given function in terms of unit step function as follows:



$$x_1(t) = u(t) - u(t-1)$$



$$x_2(t) = u(t-2) - u(t-3)$$

Thus,

$$\begin{aligned} x(t) &= x_1(t) + x_2(t) + x_3(t) + \dots \\ &= u(t) - u(t-1) + u(t-2) - u(t-3) + \dots \end{aligned}$$

We know that

$$u(t) \xrightarrow{L} \frac{1}{s}$$

$$u(t-t_0) \xrightarrow{L} \frac{1}{s} e^{-st_0}$$

(time-shifting)

The Laplace transform of $x(t)$ is

$$\begin{aligned} X(s) &= \frac{1}{s} - \frac{1}{s} e^{-s} + \frac{1}{s} e^{-2s} - \frac{1}{s} e^{-3s} + \frac{1}{s} e^{-4s} - \frac{1}{s} e^{-5s} + \dots \\ &= \frac{1}{s} [1 + e^{-2s} + e^{-4s} + \dots] - \frac{1}{s} [e^{-s} + e^{-3s} + e^{-5s} + \dots] \\ &= \frac{1}{s} \left[\frac{1}{1 - e^{-2s}} \right] - \frac{1}{s} \left[\frac{e^{-s}}{1 - e^{-2s}} \right] \\ &= \frac{1}{s} \left[\frac{1 - e^{-s}}{1 - e^{-2s}} \right] \\ &= \frac{1}{s} \left[\frac{1}{1 + e^{-s}} \right] \end{aligned}$$

T5 : Solution

(d)

From the time integration property of Laplace transform

$$x(t) \xrightarrow{L} X(s)$$

$$\int_0^t x(\tau) d\tau \xrightarrow{L} \frac{1}{s} X(s)$$

Time integration Property

$$\int_0^t x(\tau) d\tau \xrightarrow{L} \frac{(s+1)}{s(s^2 + 4s + 5)}$$

T6 : Solution

(d)

Given,

$$H(s) = \frac{k(s^2 + \omega_0^2)}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

So value of $H(s)$ at $s \rightarrow \infty$ is k
and value of $H(s)$ at $s \rightarrow 0$ is k.
So the filter is a band stop filter or notch filter.

T7 : Solution

(c)

$$X(s) = L[x(t)] = \frac{s}{s^2 + 1}$$

$$H(s) = L[h(t)] = \frac{1}{s^2 + 1}$$

$$y(t) = x(t) * h(t)$$

$$Y(s) = L[x(t) * h(t)] = X(s)H(s) = \frac{s}{(s^2 + 1)^2}$$

Using partial fractional,

$$Y(s) = \frac{-j/4}{(s-j)^2} + \frac{j/4}{(s+j)^2}$$

$$\text{We know that } te^{-at}u(t) \xleftrightarrow{L} \frac{1}{(s+a)^2}$$

$$\text{so, } \frac{1}{(s-j)^2} \xleftrightarrow{L^{-1}} te^{jt}$$

$$\frac{1}{(s+j)^2} \xleftrightarrow{L^{-1}} te^{-jt}$$

$$\text{so, } y(t) = \frac{j}{4} [-te^{jt} + te^{-jt}] = \frac{j}{4} t [e^{-jt} - e^{jt}] = \frac{t}{2} \sin t, \quad t \geq 0$$

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5

Sampling Theorem and Discrete Time System



Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} 1 = \infty$$

T2 : Solution

(c)

$$\begin{aligned} y[n] &= \sum_{n=-\infty}^{\infty} n^2 \delta[n+2] \\ &= n^2 \Big|_{n=-2} \\ &= (-2)^2 = 4 \end{aligned}$$

$$\sum_{n=-\infty}^{\infty} x[n] \delta[n - n_0] = x[n_0]$$

T3 : Solution

(d)

(A)

$$\begin{aligned} y[n] &= x[n^2] \\ x_1[n] \rightarrow y_1[n] &= x_1[n^2] \\ x_2[n] \rightarrow y_2[n] &= x_2[n^2] \\ ax_1[n] + bx_2[n] &\rightarrow ax_1[n^2] + bx_2[n^2] \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Hence the system is linear.

(B)

$$y[n] = x^2[n - 1]$$

For a delayed input $x[n - n_0]$, output is

$$y[n, n_0] = x^2[n - n_0 - 1]$$

The delayed output

$$y[n - n_0] = x^2[n - n_0 - 1]$$

Since

$$y[n, n_0] = y[n - n_0]$$

Hence the system is time-invariant.

(C)

$$y[n] = x[n] + n$$

$y[n]$ depends on present value of $x[n]$, so the system is causal.

(D)

$$y[n] = x[3n]$$

$$y[-1] = x[-3]$$

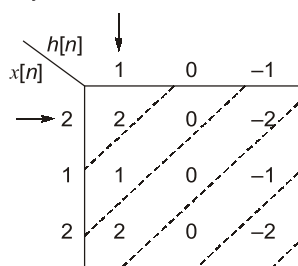
$$y[1] = x[3]$$

System has memory, therefore it is a dynamic system.

T4 : Solution

(c)

Since $x[n]$ is even symmetric about mid point ($n = 1$) and $h[n]$ is odd symmetric about mid point ($n = 1$) so $y[n]$ will be odd symmetric about its mid point $n = 2$.



$$y[n] = x[n] * h[n] = \{2, 1, 0, -1, -2\}$$

$y[n]$ is odd symmetric about $n = 2$.

T5 : Solution

(a)

Causality:

$$h[n] = 0, n < 0$$

The system is causal.

Stability:

$$\begin{aligned} \sum_{n=-\infty}^{\infty} |h[n]| &= 2 \sum_{n=0}^{\infty} (0.4)^n - \sum_{n=0}^{\infty} (0.2)^n \\ &= 2 \left[\frac{1}{1-0.4} \right] - \frac{1}{(1-0.2)} < \infty \end{aligned}$$

The system is stable.

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6

Z-Transform



Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

Given that,

$$x(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \dots$$

$$x(n) = u(n)$$

$$X(z) = \text{Z.T.}[u(n)]$$

$$X(z) = \frac{z}{z-1}$$

T2 : Solution

(c)

z-transform of $x[n]$,

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \alpha^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty} \alpha^{-n} z^{-n} u[n]$$

$$= \sum_{n=0}^{\infty} (\alpha z^{-1})^n + \sum_{n=0}^{\infty} (\alpha z)^{-n} = \underbrace{\frac{1}{1-\alpha z^{-1}}}_I + \underbrace{\frac{1}{1-(\alpha z)^{-1}}}_{II}$$

Series I converges, if $\alpha z^{-1} < 1$ or $|z| > |\alpha|$

Series II converges, if $(\alpha z)^{-1} < 1$ or $\alpha z > 1$ or $|z| > \frac{1}{|\alpha|}$

So, ROC is intersection of both

$$\text{ROC : } |z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$$

T3 : Solution

(b)

$$\begin{aligned} X(z) &= \frac{z+1}{z(z-1)} \\ &= -\frac{1}{z} + \frac{2}{z-1} = -\frac{1}{z} + 2z^{-1} \left(\frac{z}{z-1} \right) \end{aligned}$$

By partial fraction

Taking inverse z-transform

$$\begin{aligned} x[n] &= -\delta[n-1] + 2u[n-1] \\ x[0] &= -0 + 0 = 0 \\ x[1] &= -1 + 2 = 1 \\ x[2] &= -0 + 2 = 2 \end{aligned}$$

T4 : Solution

(c)

By taking z-transform of $x[n]$ and $h[n]$

$$\begin{aligned} H(z) &= 1 + 2z^{-1} - z^{-3} + z^{-4} \\ X(z) &= 1 + 3z^{-1} - z^{-2} - 2z^{-3} \end{aligned}$$

From the convolution property of z-transform

$$\begin{aligned} Y(z) &= H(z) X(z) \\ Y(z) &= 1 + 5z^{-1} + 5z^{-2} - 5z^{-3} - 6z^{-4} + 4z^{-5} + z^{-6} - 2z^{-7} \\ y[n] &= \{1, 5, 5, -5, -6, 4, 1, -2\} \\ y[4] &= -6 \end{aligned}$$

Sequence is

T5 : Solution

(d)

Given that $x(n)$ is right sided and real, $X(z)$ has two poles, two zeros at origin and one pole at $e^{j\pi/2}$, $X(1) = 1$. Since $x(n)$ is real so poles of $X(z)$ should be in conjugate pairs so other pole will be at $e^{-j\pi/2}$.

So,

$$X(z) = \frac{k z^2}{(z - e^{-j\pi/2})(z - e^{+j\pi/2})} = \frac{k z^2}{z^2 + 1}$$

Since,

$$X(1) = 1 \quad \text{so, } k = 2$$

So,

$$X(z) = \frac{2z^2}{z^2 + 1} \quad \text{and } |z| > 1$$

T6 : Solution

(a)

We know that,

$$\begin{aligned} \alpha^n u[n] &\xleftrightarrow{z} \frac{z}{z - \alpha} \\ \alpha^{n-10} u[n-10] &\xleftrightarrow{z} \frac{z^{-10} z}{z - \alpha} \end{aligned}$$

(time shifting property)

T7 : Solution

(b)

$$Y(z) - \frac{1}{3}z^{-1}Y(z) = X(z)$$

$$Y(z) \left[1 - \frac{1}{3}z^{-1} \right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

where,

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

 $\therefore$

$$Y(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)} = \frac{3}{1 - \frac{1}{2}z^{-1}} - \frac{2}{1 - \frac{1}{3}z^{-1}}$$

$$y[n] = \left[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n \right] u[n]$$

T8 : Solution

(a)

We know that convolution of $x[n]$ with unit step function $u[n]$ is given by

$$x[n] * u[n] = \sum_{k=-\infty}^{\infty} x[k]$$

so,

$$y[n] = x[n] * u[n]$$

Taking z-transform on both sides

$$Y(z) = X(z) \frac{z}{(z-1)} = X(z) \frac{1}{(1-z^{-1})}$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1-z^{-1})}$$

Now, consider the inverse system of $H(z)$, let impulse response of the inverse system is given by $H_1(z)$, then we can write

$$H(z)H_1(z) = 1$$

$$H_1(z) = \frac{X(z)}{Y(z)} = 1 - z^{-1}$$

$$(1 - z^{-1})Y(z) = X(z)$$

$$Y(z) - z^{-1}Y(z) = X(z)$$

Taking inverse z-transform

$$y[n] - y[n-1] = x[n]$$

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Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

Since

$$x[n] \xrightarrow{\text{DTFT}} X(e^{j\Omega})$$

Thus

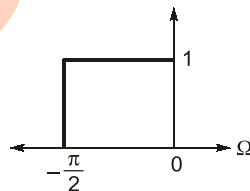
$$e^{j\Omega_0 n} x[n] \xrightarrow{\text{DTFT}} X(e^{j(\Omega - \Omega_0)})$$

(Frequency shifting property)

$$\Omega_0 = -\pi/4$$

$$e^{-j\frac{\pi}{4}n} x[n] \xrightarrow{\text{DTFT}} X(e^{j(\Omega + \pi/4)})$$

The graph of $X(e^{j\Omega})$ is shifting to left by $\frac{\pi}{4}$ units. So, DTFT of $e^{-j\frac{\pi}{4}n} x[n]$ is



T2 : Solution

(a)

N-point DFT is given as

$$X_{DFT}[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}}, k = 0, 1, \dots, N-1$$

$$X_{DFT}[k] = \sum_{n=0}^3 x[n] e^{-j\frac{\pi nk}{2}}$$

$$\because N = 4$$

For $k = 0$,

$$\begin{aligned} X_{DFT}[0] &= \sum_{n=0}^3 x[n] \\ &= x[0] + x[1] + x[2] + x[3] \\ &= \cos 0 + \cos \pi + \cos 2\pi + \cos 3\pi \\ &= 1 - 1 + 1 - 1 = 0 \end{aligned}$$

For $k = 1$,

$$\begin{aligned} X_{DFT}[1] &= \sum_{n=0}^3 x[n] e^{-j\frac{\pi n}{2}} \\ &= x[0]e^0 + x[1]e^{-j\frac{\pi}{2}} + x[2]e^{-j\pi} + x[3]e^{-j\frac{3\pi}{2}} \\ &= \cos 0 + \cos \pi(-j) + \cos 2\pi(-1) + \cos 3\pi(j) \\ &= 1 + (-1)(-j) + 1(-1) + (-1)(j) \\ &= 1 + j - 1 - j \\ &= 0 \end{aligned}$$

Similarly we can obtain $X_{DFT}[2]$ and $X_{DFT}[3]$ for $k = 2$ and $k = 3$ respectively,

$$\begin{aligned} X_{DFT}[2] &= 1 + 1 + 1 + 1 = 4 \\ X_{DFT}[3] &= 1 - j - 1 + j = 0 \\ X_{DFT}[k] &= \{0, 0, 4, 0\} \end{aligned}$$

T3 : Solution

(c)

$$\begin{aligned} X(e^{j\Omega}) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-2}^2 e^{-j\Omega n} \\ &= e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega} \\ &= e^{-j2\Omega} (1 + e^{j\Omega} + e^{j2\Omega} + e^{j3\Omega} + e^{j4\Omega}) \\ &= e^{-j2\Omega} \frac{(1 - e^{j5\Omega})}{1 - e^{j\Omega}} \quad \text{(Summation of finite GP)} \\ &= \frac{e^{-j5\pi/2} - e^{j5\Omega/2}}{e^{-j\pi/2} - e^{j\Omega/2}} = \frac{\sin 2.5\Omega}{\sin 0.5\Omega} \end{aligned}$$

T4 : Solution

(b)

$$\begin{aligned} X(e^{j\Omega}) &= j4 \sin 4\Omega - 1 \\ &= 2(e^{j4\Omega} - e^{-j4\Omega}) - 1 \end{aligned}$$

Taking inverse Fourier transform, we have

$$x[n] = 2\delta[n+4] - 2\delta[n-4] - \delta[n]$$

Since, $\delta[n - n_0] \xrightarrow{DTFT} e^{-j\Omega n_0}$

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