

GATE

MADE EASY **WORKBOOK** 2024



**Detailed Explanations of
Try Yourself Questions**

Instrumentation Engineering Sensors and Industrial Instrumentation



MADE EASY
Publications

1

Introduction to Transducers



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\theta_F = 100 - 30 = 70^\circ\text{C}$$

We can write

$$66.5 = 70(1 - e^{-30/\tau})$$

$$\Rightarrow \tau = 10 \text{ sec.}$$

For second condition

$$68 = 70(1 - e^{-t/10})$$

by solving $t = 36.5 \text{ sec.}$

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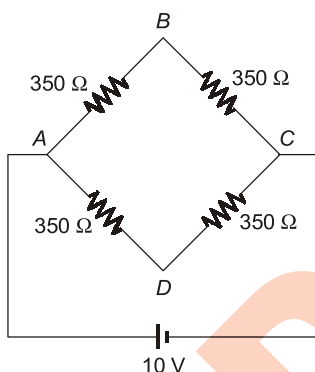
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Resistive Transducers



Detailed Explanation of Try Yourself Questions

T1. Sol.



Given, $G_f = 2.03$ Vs = 10 V
 $\epsilon = 1430 \times 10^{-6}$

Strain gauge in BC subjected to strain of 1450 $\mu\text{m/m}$.

$$V_{\text{bridge}} = \frac{V_o}{4} \times \frac{\Delta R}{R + 2\Delta R}$$

$$\Delta R = G_f \times \epsilon \times R$$

$$= 2.03 \times 1450 \times 10^{-6} \times 350$$

$$V_{\text{bridge}} = 7.314 \times 10^{-3} = 7.31 \text{ mV}$$

T2. Sol.

$$\frac{R_o}{R_t} = \frac{d_i}{d_t}$$

$$\frac{6000}{20 \text{ k}\Omega} = \frac{d_i}{100 \text{ mm}}$$

$$d_i = 30 \text{ mm},$$

$$\frac{R_D}{R_t} = \frac{d_i}{d_t}$$

$$\frac{15000}{20 \text{ k}\Omega} = \frac{d_i}{100 \text{ mm}}$$

$$d_i = 75 \text{ mm}$$

T3. Sol.

$$\therefore \epsilon_o = \frac{E G_F \epsilon}{4}$$

$$= \frac{2}{4} \times 10(10 + 20 \sin 314t) \times 10^{-6}$$

$$= [50 + 100 \sin 314t] \mu\text{V}$$

T4. Sol.

$$\therefore \Delta R = R_o G_F \epsilon$$

$$\epsilon = \frac{\Delta R}{R_o G_F} = \frac{0.52}{100 \times 2} = 0.0026$$

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Capacitive Transducers



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$C_1 = \frac{\epsilon_0 \epsilon_r \times l \times \omega}{d}$$

$$C_2 = \frac{\epsilon_0 \epsilon_r \times X \times \omega}{d} + \frac{\epsilon_0 \times \epsilon_r \times (l - X) \omega}{d}$$

Given $2C_1 = C_2$

$$\frac{2 \epsilon_0 \cdot \epsilon_r \times l \times \omega}{d} = \frac{\epsilon_0 \times \epsilon_r \times X \times \omega}{d} + \frac{\epsilon_0 \epsilon_r (l - X) \omega}{d}$$

$$X = \frac{l}{2}$$

T2. Sol.

$$V_o = -\frac{C_i d}{\epsilon_A} V_e$$

$$\frac{\partial V_o}{\partial d} = -\frac{C_i V_e}{\epsilon A}$$

$$= \frac{-10 \times 10^{-12} \times 10}{8.854 \times 10^{-12} \times 800 \times 10^{-6}}$$

$$= 1.1412 \times 10^4 \text{ V/m}$$

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4

Inductive Transducers



Detailed Explanation of Try Yourself Questions

T1. Sol.

Voltage produced in the stator of the transmitter

$$V_r \cos \theta = 5 \cos 60^\circ = 5 \times \frac{1}{2} = 2.5 \text{ Volts}$$

$$L = 6.10 \text{ mH}$$

Away from coil

$$\frac{6 \times 10^{-3}}{L} = \frac{2 + 0.035}{2}$$

$$L = 5.896 \text{ mH}$$

T2. Sol.

$$L = 6 \text{ mH} ; d = 2 \text{ mm}$$

Towards the coil

$$\frac{6 \text{ mH}}{L} = \frac{2 - 0.035}{2 \text{ mm}}$$

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Piezo-electric Transducers



Detailed Explanation of Try Yourself Questions

T1. Sol.

0.0193

T2. Sol.

$$V_o = gpt$$

$$300 = 0.65 \times \frac{f}{36 \times 10^{-6}} \times 2 \times 10^{-3}$$

$$F = 8.3 \text{ N}$$

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Measurement of Flow



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\frac{Q_1}{Q_2} = \sqrt{\frac{P_1}{P_2}}$$

$$\frac{80}{Q_2} = \sqrt{\frac{15}{25}}$$

$$Q_2 = 103.27 \text{ l/min}$$

T2. Sol.

$$Q = \frac{C_d A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\Delta p}{\rho}}$$

$$\rho = \frac{M}{Q}$$

$$1000 = \frac{4}{Q}$$

$$A_2 = \left(\frac{3.5}{2}\right)^2 = 9.62 \times 10^{-4}$$

$$Q = 4 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$4 \times 10^{-3} = 0.95 \times \frac{9.62 \times 10^{-4}}{\sqrt{1 - \frac{1}{4}}} \times \sqrt{\frac{2\Delta p}{1000}}$$

$$\Delta p = 8.9 \text{ kN/m}^2$$

T3. Sol.

$$\text{Given } \omega = 4.5 \times 10^4 \text{ Q}$$

$$\beta = 0.88, n = 4, Q_{\max} = 3.15 \times 10^{-3} \text{ m}^3\text{s}^{-1}$$

Now maximum amplitude

$$\begin{aligned} &= \omega_{\max} \times \beta \times n \\ &= 4.5 \times 10^4 \times 3.15 \times 10^{-3} \times 0.88 \times 4 \\ &= 498.96 \end{aligned}$$

$$\text{Maximum frequency, } f_{\max} = \frac{\omega_{\max} \times n}{2\pi}$$

$$\begin{aligned} &= \frac{4.5 \times 10^5 \times 3.15 \times 10^{-3} \times 4}{2\pi} \\ &= 90.2 \text{ kHz} \end{aligned}$$

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Measurement of Pressure



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\begin{aligned}\Delta p &= 9.81 \text{ Pa}, \quad d_t = 2 \text{ mm} = 0.002 \text{ m} \\ d_w &= 0.02 \text{ m} \\ d &= 4 \text{ mm}, \quad \text{specific gravity} = 0.864 \\ \rho &= 0.864 \times 1000\end{aligned}$$

We know,

$$\Delta p = \left(\sin \theta + \frac{\alpha_L}{\alpha_w} \right) \rho g$$

$$\sin \theta = \frac{\Delta p}{\rho g} - \left(\frac{d_t}{d_w} \right)^2$$

$$= \frac{9.81}{(0.0104) \times (0.864 \times 1000) (9.81)} - (0.1)^2$$

$$\theta = \sin^{-1} (0.2793^\circ) = 16.22^\circ$$

T2. Sol.

$$\begin{aligned}\Delta p &= \rho g h \\ 1 &= 13600 \times 9.8 \times h \\ h &= 7.5 \times 10^{-3} \text{ mm}\end{aligned}$$

$$\% \text{ error} = \frac{-a}{a+A} \times 100$$

$$= -\frac{(1)^2}{(1)^2 + (15)^2} \times 100 = -0.44$$

$$= -0.5\% \text{ approx.}$$

T3. (10)



$$\text{Total count} = 2000 \text{ counts}$$

$$\text{Max voltage} = 200 \text{ mV}$$

$$\text{Sensitivity} = \frac{1 \text{ mV/V}}{1 \text{ k pascal}}$$

$$\text{Input voltage} = 10 \text{ V}$$

$$\text{Full scale output} = 200 \text{ mV}$$

$$\text{Sensitivity} = \frac{200 \text{ mV}/10 \text{ V}}{P_{in}}$$

$$\Rightarrow \frac{1 \text{ mV}}{\text{V}} = \frac{200 \text{ mV}}{10 \text{ V}} \times \frac{1 \text{ k pascal}}{P_{in}}$$

$$\Rightarrow P_{in} = 20 \text{ k pascal}$$

$$\text{Resolution} = \frac{20 \text{ k pascal}}{2000 \text{ counts}} = 10 P_a$$

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Measurement of Temperature



Detailed Explanation of Try Yourself Questions

T1. Sol.

Sensitivity of each thermopile $60 \mu\text{V}/^\circ\text{C}$
 10 junction sensitivity = $10 \times 60 \mu\text{V}/^\circ\text{C}$

Resolution of AC is given as $\frac{V_{\text{ref}}}{2^n - 1}$

$$= \frac{6 \text{ V}}{2^{10} - 1} = 5.86 \text{ mV}$$

Resolution of system = $\frac{5.86 \text{ mV}}{6 \text{ mV}/^\circ\text{C}} = 0.977^\circ\text{C}$

T2. Sol.

Resistance of RTD at 1°C

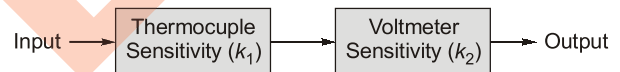
$$\begin{aligned} R_1^\circ\text{C} &= R_0 (1 + \alpha \Delta t) \\ &= 100 [1 + 0.00392 (1)] \\ &= 100.392 \end{aligned}$$

$$\begin{aligned} V_o &= \frac{10(100.392)}{1010.392} - \frac{10}{1010} \times 100 \\ &= 0.384 \text{ mV}/^\circ\text{C} \end{aligned}$$

$$g = \frac{10 \text{ mV}}{0.384 \text{ mV}} = 26/^\circ\text{C}$$

T3. Sol.

Block diagram representation of the given problem shown below:



Here, $k_1 = 4.8 \text{ mV}/^\circ\text{C}$

$$k_2 = 1^\circ/\text{mV} = \left(1^\circ \times \frac{\pi}{180} \times 30 \right) \frac{\text{mm}}{\text{mV}}$$

Overall sensitivity

$$\begin{aligned} k_1 \times k_2 &= 4.8 \frac{\text{mV}}{^\circ\text{C}} \times \left(1^\circ \times \frac{\pi}{180} \times 30 \right) \frac{\text{mm}}{\text{mV}} \\ &= 2.5 \text{ mm}/^\circ\text{C} \end{aligned}$$

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9

Measurement of Force



Detailed Explanation of Try Yourself Questions

T1. Sol.

Consider 1 kN is applied on load cell stress

$$= \frac{1 \times 10^3}{\frac{\pi}{4} \times (10^{-3} \times 50)^2}$$

$$= 0.5095 \times 10^6 \text{ N/m}^2$$

$$\text{Strain} = \frac{\text{Stress}}{E} = \frac{0.5095 \times 10^6}{200 \times 10^9}$$

$$= 2.5475 \times 10^{-6}$$

Fractional change in resistance

$$\frac{dR}{R} = F\epsilon = 2 \times 2.5475 \times 10^{-6}$$

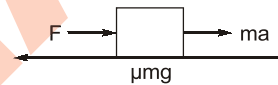
$$= 5.095 \times 10^{-6}$$

$$\text{Output voltage} = 2(1 + \mu) \left(5.095 \times 10^{-6} \times \frac{6}{4} \right)$$

$$= 19.87 \times 10^{-8}$$

Gauge sensitivity = 19.87 $\mu\text{V/kN}$

T2. Sol.



$$F = \mu mg + ma$$

$$4.9 = mg$$

$$m = 0.5 \text{ kg}$$

$$F = 0.5 \times 0.5 \times 9.8 + 0.5 \times 4$$

$$= 4.5 \text{ N.}$$

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10

Measurement of Torque, Vibration & Shock



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\frac{y}{x} = \frac{r^2}{\sqrt{(-r^2)^2 + (5\xi r)^2}}$$

At 30 kHz

$$r = \frac{\omega}{\omega_n} = \frac{30}{20} = 1.5 \quad \xi = 0.7$$

$$\frac{y}{x} = \frac{1.5 \times 1.5}{\sqrt{(1 - 2.25) + (2 \times 0.7 \times 1.5)^2}} = 0.92$$

For 1 kHz

$$r = \frac{1000}{20} = 50$$

$$\frac{y}{x} = \frac{50 \times 50}{\sqrt{(1 - 50 \times 50)^2 + (2 \times 0.7 \times 1.5)^2}} = 1$$

T2. Sol.

$$\begin{aligned} a_{\text{peak}} &= x_m \omega^2 \\ &= 10 \times 10^{-3} \times (2\pi \times 100)^2 \\ &= 10^{-2} \times 10^4 \times 39.4384 \\ &= 3943.84 \end{aligned}$$

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Measurement of Velocity



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\text{Speed} = \frac{6000}{10} = 600$$

Since points appear to be revolving once in 12 seconds or 5 times in 1 minute, we have

$$\text{Slip} = 5 \text{ rpm}$$

$$\begin{aligned} \text{Possible shaft speed} &= 600 \pm 5 \\ &= 605, 595 \text{ rpm} \end{aligned}$$

T2. Sol.

$$\begin{aligned} \text{(i)} \quad n &= \frac{f}{m} = \frac{4000}{10} \\ &= 400 \text{ rpm} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 400 + 20 &= 420 \text{ rpm} \\ \text{Shaft speed} &+ \text{disc speed.} \end{aligned}$$

T3. Sol.

Frequency of AC voltage, $f = m \times f_r$

m = Number of teeth

f_r = Rotation speed (rps)

$$f = 25 \times 100 = 2.5 \text{ kHz}$$

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Measurement of Humidity, Viscosity & pH Value



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\frac{\mu_A}{\mu_B} = \frac{V_B}{V_A}$$

$$\frac{12 \times 10^{-3}}{\mu_B} = \frac{0.003}{0.06}$$

$$\mu_B = 0.24 \text{ m}^2/\text{sec.}$$

$$= 59.1 \times 1.3 \times 100$$

$$= 7.683 \text{ V}$$

Loading condition

$$V_L = \frac{V_o}{R_o + R_L} \times R_L$$

$$= \frac{7.683}{10^{10} + 10^{11}} \times 10^{11} = 6.9 \text{ V}$$

T2. Sol.

pH change $7.8 - 6.5 = 1.3$
at 25°C slope factor = 59.1 mV/pH unit

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