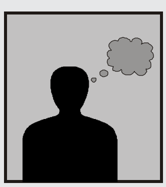
# GATE

MADE EASY

# Workbook 2027



## Detailed Explanations of Try Yourself *Questions*

# Instrumentation Engineering

## Measurements

MADE EASY PUBLICATIONS

# 1 Error Analysis

## Detailed Explanation *of* Try Yourself Questions

**T1 : Solution**

(d)

Probable error, $$\delta I = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 \delta I_1^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 \delta I_2^2}$$

Here, $$I = I_1 + I_2$$

So, $$\frac{\partial I}{\partial I_1} = \frac{\partial I}{\partial I_2} = 1$$

$$\delta I = \sqrt{(1)^2 (1)^2 + (1)^2 (2)^2} = 2.24 \text{ A}$$

therefore, $$I = 300 \pm 2.24 \text{ A}$$

# 2 Indicating Meters

## Detailed Explanation of Try Yourself Questions

**T1 : Solution**

$$S_{dc} = \frac{1}{I_{fs}} = \frac{1}{1\times 10^{-3}} = 1000\ \Omega/\text{v}$$

$$R_m = S_{dc}V = 1000 \times 1 = 1000\ \Omega$$

$\because$
$$R_s = 0.45 \times 1000 \times 10 - 1000$$

$$R_s = 3.5\ \text{k}\Omega$$

**T2 : Solution**

(d)

Average value of rectangular current wave $= \sqrt{\frac{1}{2T}\left[(12^2 \times T) + (5^2 \times T)\right]} \simeq 9.2$

$$\text{Average volts} = 9.2 \times 10 = 92\ \text{V}$$

The MI meter will read 92 V.

**T3 : Solution**

For the range extension of electrostatic voltmeter the capacitor is connected in series with meter and its value is given by

$$C_s = \frac{C_v}{m-1}$$

Where
$$m = \frac{V}{v} = \frac{20\ \text{kV}}{2\ \text{kV}} = 10$$

So,
$$C_s = \frac{0.5}{10-1} = 0.05\ \text{pF}$$

■■■■

# 3 Power & Energy Measurement

## Detailed Explanation *of* Try Yourself Questions

**T1 & T2 : Sol.**

Total power in the circuit, $P = W_1 + W_2 = 500\text{ W} + (-100)\text{ W} = 400\text{ W}$

Power factor of the circuit, $\cos\phi = \cos\tan^{-1}\left\{\left[\frac{W_1 - W_2}{W_1 + W_2}\right].\sqrt{3}\right\}$

$$= \cos\tan^{-1}\left\{\left[\frac{0.5-(-0.1)}{0.5+(-0.1)}\right].\sqrt{3}\right\}$$

$$= \cos\tan^{-1}(1.5\times\sqrt{3}) = 0.359$$

Load current per phase, $I_P = \dfrac{P}{\sqrt{3}\,V_L\cos\phi} = \dfrac{400}{\sqrt{3}\times 440\times 0.359} = 1.462\text{ A}$

Load impedance per phase, $Z_p = \dfrac{V_p}{I_p} = \dfrac{440/\sqrt{3}}{1.462} = 173.76\ \Omega$

Load resistance per phase, $R_p = Z_p\cos\phi = 62.38\ \Omega$

Load reactance per phase, $X_p = Z_p\sin\phi = 162.18\ \Omega$

Reading of wattmeter $B$ will be zero when p.f. $= \cos\phi' = 0.5$

or $\phi' = 60°$

Since there is no change in resistance,

Reactance in the circuit per phase,

$$X_p' = R_p\tan\phi'$$

$$X_p' = 62.38\times\sqrt{3} = 108.045\ \Omega$$

value of capacitive reactance to be introduced in each phase $= X_p - X_p'$

$$= 162.18\ \Omega - 108.045\ \Omega$$

$$= 54.135\ \Omega$$

**T3 : Solution**

Case-1:

$$\text{p.f.} = 1 \Rightarrow \phi = 0°$$
$$P_m = V_L I_L \sin(\Delta - \phi)$$
$$P_m = V_L I_L \sin(88° - 0°)$$
$$P_T = V_L I_L \sin(90° - 0°)$$
$$\%\text{ error} = \frac{P_m - P_T}{P_T} \times 100$$
$$= \frac{\sin(88°) - \sin(90°)}{\sin(90°)} \times 100 = -0.061\%$$

Case-2:

$$\text{p.f.} = 0.5 \Rightarrow \phi = 60°$$
$$P_m = V_L I_L \sin(88° - 60°)$$
$$P_m = V_L I_L \sin(28°)$$
$$P_T = V_L I_L \sin(90° - 60°)$$
$$P_T = V_L I_L \sin 30°$$
$$\%\text{ error} = \frac{\sin(28°) - \sin(30°)}{\sin(30°)} \times 100 = -6.1\%$$

■■■■

# 4 Instrument Transformers

## Detailed Explanation of Try Yourself Questions

**T1 : Solution**

(c)

$$\text{Phase angle error for CT is} = \frac{180}{\pi}\left(\frac{I_m \cos\delta - I_e \sin\delta}{K_t\, I_s}\right)\text{degree}$$

Here,

$$K_t = \frac{1000}{5} = 200,\ I_s = 5\text{ A}$$

$$I_m = 11\text{ A}$$

$$I_e = 6.5\text{ A}$$

$$\delta = 30°$$

$$\text{So, phase angle error} = \frac{180}{\pi}\left(\frac{11\cos 30° - 6.5\sin 30°}{200 \times 5}\right) = 0.359°$$

**T2 : Solution**

(b)

Secondary circuit phase angle, $\delta = \tan^{-1}\left(\frac{1}{1.5}\right) = 33.69°$

$$\cos\delta = \cos 33.39° = 0.835$$

or, $\sin\delta = \sin 33.69° = 0.555$

Turn ratio, $K_t = \frac{N_s}{N_p} = \frac{300}{1} = 300$

Magnetizing current, $I_m = \frac{\text{Magnetising mmf}}{N_p} = \frac{100}{1} = 90\text{ A}$

Secondary circuit burden impedance = $\sqrt{(1.5)^2 + (1.0)^2} = 1.8\ \Omega$

Secondary induced voltage, $E_s = 5 \times 1.8 = 9$ V

Primary induced voltage, $E_p = \dfrac{E_s}{300} = \dfrac{9\text{ V}}{300}$

Loss component, $I_w = \dfrac{\text{iron loss}}{E_p} = \dfrac{1.2}{(9/300)} = 40$ A

Phase angle, $\theta = \dfrac{180}{\pi}\left(\dfrac{I_m \cos\delta - I_w \sin\delta}{K_t I_s}\right)$

$$= \frac{180}{\pi}\left(\frac{100 \times 0.835 - 40 \times 0.555}{300 \times 5}\right) = 2.34°$$

■■■■

# 5 Measurement of R, L, C Bridges

## Detailed Explanation of Try Yourself Questions

**T1 : Solution**

Schering bridge D-factor = $\omega C_x R_x$ after substituting $C_x$ and $R_x$.

$$\text{D-factor} = \omega R_1 C_1$$
$$= 2\pi \times 10^3 \times 10^3 \times 0.5 \times 10^{-6}$$
$$= 3.14$$

**T2 : Solution**

(b)

At balance,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\frac{10 \times 10^3 \times X_C}{10 \times 10^3 + X_C} \times Z = 500 \times 10^3$$

as, $$X_C = \frac{1}{j\omega C} = \frac{1}{j \times 100\pi \times 100 \times 10^{-9}} = -j\frac{10^5}{\pi}$$

$\therefore$ $$\frac{-j10^4 \times 10^5}{\pi\left(10^4 - \frac{j \times 10^5}{\pi}\right)} \times 2 = 5 \times 10^5$$

$\Rightarrow$ $$\frac{-j10^3}{1000\pi - j10^4}(R + jX) = 5$$

$\Rightarrow$ $$-jR + X = 5\pi - j5 \times 10$$

$\Rightarrow$ $$R = 50\ \Omega$$

and $$L = \frac{5}{2 \times 50} = 50 \text{ mH}$$

**T3 : Solution**

$$R_3 = 5\ \Omega,$$
$$C = 1\ \text{mF},$$
$$R_1 = 160\ \Omega,$$
$$R_2 = 20\ \Omega$$

By using balance equation,
$$R = \frac{R_2 R_1}{R_3}$$
$$L = R_2 R_1 C$$

and
$$\text{quality factor} = Q = \frac{\omega L}{R}$$

So,
$$R = \frac{20 \times 160}{5} = 640\ \Omega$$
$$L = 20 \times 160 \times 1 \times 10^{-3} = 3.2\ \text{H}$$
$$Q = \frac{2\pi \times 50 \times 3.2}{640} = 1.57$$

# 6 CRO

## Detailed Explanation of Try Yourself Questions

**T1 : Solution**

Using the equation,

$$V_{p-p} = \left(\frac{\text{volts}}{\text{div}}\right) \times \left(\frac{\text{no. of div}}{1}\right)$$

$$V_{p-p} = 0.5 \text{ V} \times 3 = 1.5 \text{ V}$$

**T2 : Solution**

The period of the signal is calculate using the equation

$$T = \left(\frac{\text{time}}{\text{div}}\right) \times \left(\frac{\text{no. of div}}{\text{cycle}}\right)$$

$$T = 2\ \mu\text{s} \times 4 = 8\ \mu\text{s}$$

Hence, frequency is

$$f = \frac{1}{T} = \frac{1}{8\ \mu\text{s}} = 125 \text{ kHz}$$

■■■■