2019

RANK IMPROVEMENT WORKBOOK

Mechanical Engineering

Thermodynamics

Answer Key of Objective & Conventional Questions

MADE EASY
Publications

1 Basic Concepts and Zeroth Law of Thermodynamics

LEVEL 1 — Objective Questions

1. (c)
2. (b)
3. (a)
4. (b)
5. (a, b & c)
6. (c)
7. (a)
8. (d)
9. (a)
10. (d)
11. (b)
12. (a)

LEVEL 2 — Objective Questions

13. (b)
14. (d)
15. (b)
16. (b)
17. (d)
18. (a)
19. (c)
20. (c)
21. (4)
22. (c)
23. (d)

■■■■

LEVEL 3 — Conventional Questions

Solution: 24

$$p_{abs} = 82.168 \text{ kPa}$$

Solution : 25 (0.816)

$$\Delta x = 0.816 \text{ m}$$

Solution : 26 (0.2869)

$$R = 0.2869 \text{ kJ/kgK}$$

Solution : 27 (0.2296)

$$m = 0.2296 \text{ kg}$$

Solution : 28 (419.67)

$$T_{°C} = -40°C \text{ and } T_F = -40°F$$

On absolute scales,

$$T_K = 233.15 \text{ K}$$
$$T_R = 419.67 \text{ R}$$

Solution: 29

$$\text{Pressure rise in tire} = 26 \text{ kPa}$$
$$\Delta m = 0.1823 \text{ kg}$$

Solution : 30

$$\text{Power output of shaft} = 6.96 \text{ watt}$$
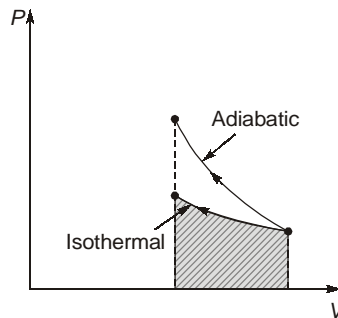$$T = 7.91 \times 10^{-2} \text{ Nm}$$

Solution : 31

For process 2-3, Isothermal compression

$$\text{Final temperature, } T_3 = T_2 = 208.73 \text{ K}$$
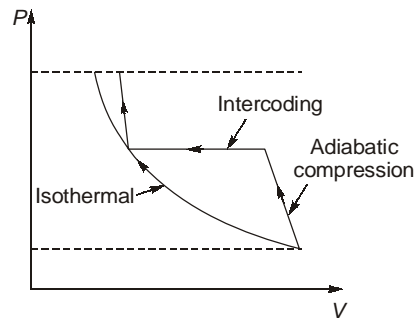$$\Rightarrow \quad \text{Final pressure, } P_3 = 5.18 \text{ kPa}$$

As can be seen in figure, slope of an isothermal process is less than that of an isentropic/adiabatic process.

Hence area under the curve, which is equal to work done is minimum in case of an isothermal process.

∴ Isothermal process should be used in compression

In Practise, for compression involving high compression ratios, adiabatic process with intercooling is employed. This method closely approaches an isothermal process.

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2 First law of Thermodynamics (Non-Flow Processes)

LEVEL 1 Objective Questions

1. (a)
2. (d)
3. (c)
4. (b)
5. (c)
6. (b)
7. (b)
8. (a)
9. (d)
10. (c)
11. (c)
12. (c)

LEVEL 2 Objective Questions

13. (c)
14. (a)
15. (c)
16. (15 kJ)
17. (−55.45)(−55.55 to − 55.446)
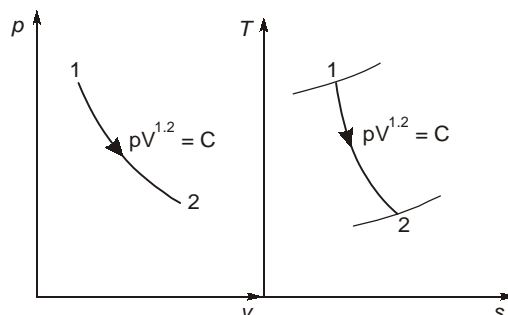18. (d)
19. (1.69)(1.61 to 1.71)
20. (b)
21. (c)
22. (a)
23. (a)

LEVEL 3 Conventional Questions

Solution : 24

$$Q = 2.75 \text{ kJ}$$

Solution : 25

$$W = 40 \text{ kJ}$$

Solution : 26

Change in internal energy,

$$dU = mc_v(T_2 - T_1)$$

$$dU = \frac{mR}{\gamma - 1}(T_2 - T_1) \qquad \qquad \dots (i)$$

$$\left(\because c_v = \frac{R}{\gamma - 1} \right)$$

Work done, $W = \int_{1}^{2} p\, dV$

where

$$pV^n = C$$

$$p = \frac{C}{V^n} = CV^{-n}$$

∴

$$W = \int_{1}^{2} V^{-n} dV = C\frac{[V^{-n+1}]_1^2}{-n+1}$$

$$= \frac{p_2 V_2^n V_2^{-n+1} - p_1 V_1^n V_1^{-n+1}}{1-n} = \frac{p_2 V_2 - p_1 V_1}{1-n}$$

$$W = \frac{mR(T_2 - T_1)}{1-n} \qquad \qquad \dots(ii)$$

Heat transfer, $Q = dU + W = \dfrac{mR(T_2 - T_1)}{\gamma - 1} + \dfrac{mR(T_2 - T_1)}{1-n}$

$$= \left(\frac{1-n}{\gamma-1}+1\right)\frac{mR(T_2-T_1)}{1-n} = \left(\frac{1-n+\gamma-1}{\gamma-1}\right)\frac{mR(T_2-T_1)}{1-n}$$

$$Q = \left(\frac{\gamma-n}{\gamma-1}\right)\frac{mR(T_2-T_1)}{1-n} \qquad \text{...(iii)}$$

$$Q \;:\; dU \;:\; W$$

$$\left(\frac{\gamma-n}{\gamma-1}\right)\frac{mR(T_2-T_1)}{1-n} \;:\; \frac{mR(T_2-T_1)}{\gamma-1} \;:\; \frac{mR(T_2-T_1)}{1-n}$$

$$\left(\frac{\gamma-n}{\gamma-1}\right)\times\frac{1}{1-n} \;:\; \frac{1}{\gamma-1} \;:\; \frac{1}{1-n}$$

$$(\gamma-n) \;:\; (1-n) \;:\; (\gamma-1)$$

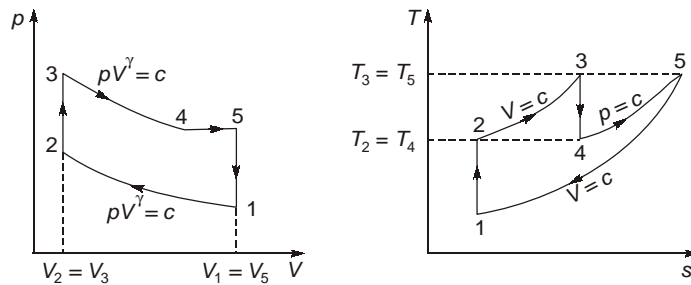$$\text{Heat transfer, } Q = mc_n(T_2-T_1) \qquad \text{...(iv)}$$

Comparing Eq. (iv) with Eq. (iii), we get

$$c_n = \left(\frac{\gamma-n}{\gamma-1}\right)\frac{R}{1-n} = \left(\frac{\gamma-n}{1-n}\right)c_v$$

where

$$c_v = \frac{R}{\gamma-1}$$
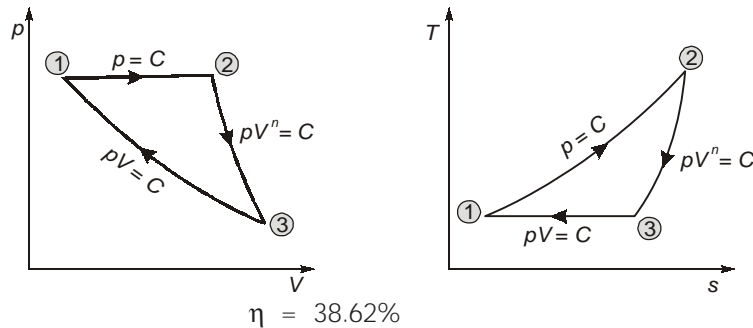
Solution : 27

or

$$T_1 = 313.28 \text{ K}$$

$$\text{Net work done: } w_{net} = 107.83 \text{ kJ/kg}$$

Solution : 28

$$\text{Time} = 8 \text{ hour } 23 \text{ min } 42.38 \text{ sec}$$

Solution : 29

$$\eta = 38.62\%$$

■■■■

3 First law of Thermodynamics (Flow Processes)

LEVEL 1 Objective Questions

1. (a)
2. (b)
3. (b)
4. (c)
5. (a)
6. (c)
7. (c)
8. (b)
9. (d)
10. (c)
11. (b)
12. (b)

LEVEL 2 Objective Questions

13. (a)
14. (70)(no range)
15. 523.02(523 to 524)
16. (b)
17. (c)
18. (d)
19. (2717)
20. (c)
21. (a)
22. (b)

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LEVEL 3 — Conventional Questions

Solution : 23

Assumption:

(i) Neglecting ΔK.E. and ΔP.E.

(ii) Assume flow is steady.

(iii) There is no work transfer in after cooler.

$$Q_1 = -0.163 \text{ kW}$$
$$Q_2 = -3.76 \text{ kW}$$

Solution : 24

$$\Delta P_{max} = 307.49 \text{ kPa}$$

Solution : 25 (212)

$$T_2 = 255°C$$
$$m = 0.442 \text{ kg/s}$$
$$d_2 = 212 \text{ mm}$$

Solution : 26

$$Q = -248.211 \text{ kJ}$$
$$V_P = 0.145 \text{ m}^3$$

■■■■

4 Second law of Thermodynamics

1. (d)
2. (4)(no range)
3. (d)
4. (c)
5. (b)
6. (d)
7. (a)
8. (4)
9. (b)
10. (b)
11. (d)
12. (d)

13. (17.32)(17 to 18)
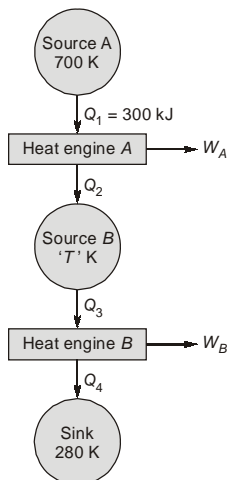14. (c)
15. (b)
16. (b)
17. (b)
18. (d)
19. (50)
20. (c)

■■■■

LEVEL **3** Conventional Questions

Solution : 21

T - Temperature of intermediate between A and B.

$$T = 420\ K$$

Efficiency of each engine,

$$\eta_A = 40\%$$

$$\eta_B = 33.33\%$$

Heat rejected by engine A and received by engine B

$$Q_3 = 180\ kJ$$

Heat rejected to the sink $= 120\ kJ$

Solution : 22

Total heat transferred to 30°C thermal reservoir $= Q_2 = 7764.56\ kJ$

Solution : 23

Work done $= 8.12\ kW$

Solution : 24

$$\eta = 33.33\%$$

Heat rejected to sink at $T_2 = (400\ K) = 240J$

and heat rejected to sink at $T_3 = 300\ K = 360\ kJ$
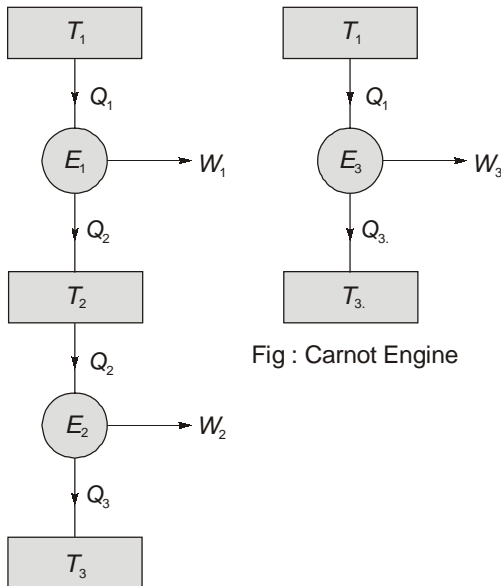
Solution : 25

Fig : Carnot Engine

Fig : Carrnot Engine
in series

$$\eta_1 \ = \ \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

or $$1 - \eta_1 \ = \ \frac{T_2}{T_1}$$

Similarly, $$1 - \eta_2 \ = \ \frac{T_3}{T_2}$$

$\therefore$ $$(1 - \eta_1)(1 - \eta_2) \ = \ \frac{T_3}{T_1}$$ . . . (i)

For the Carnot engine operating directly between the temperature limits T_1 and T_3

$$\eta_3 \ = \ \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

or $$1 - \eta_3 \ = \ \frac{T_3}{T_1}$$ . . . (ii)

Equating equation (i) and (ii), we get

$$1 - \eta_3 \ = \ (1 - \eta_1)(1 - \eta_2)$$
$$= \ 1 - \eta_1 - \eta_2 + \eta_1\eta_2$$

or $$\eta_3 \ = \ \eta_1 + \eta_2 - \eta_1\eta_2$$

Solution : 26
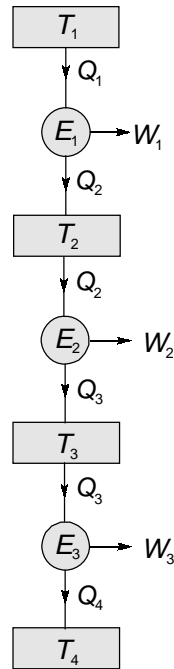
T_2 = 1100 K
T_3 = 500 K
T_4 = 250 K

■■■■

5 Entropy, Availability and Irreversibility

LEVEL 1 — Objective Questions

1. (a)
2. (a)
3. (a)
4. (400)(no range)
5. (c)
6. (600)
7. (b)
8. (b)
9. (a)
10. (b)
11. (b)
12. (c)

LEVEL 2 — Objective Questions

13. (d)
14. (c)
15. (b)
16. (a)
17. (b)
18. (c)
19. (c)

■■■■

LEVEL **3** Conventional Questions

Solution : 20

$$\text{Final pressure:} \quad p = 2 \text{ bar}$$

Now, partial pressure of O_2, $\qquad p_A = 0.5714 \text{ bar}$

Partial pressure of N_2, $\qquad p_B = 1.1428 \text{ bar}$

Partial pressure of CO_2, $\qquad p_c = 0.2857 \text{ bar}$

Change in entropy of O_2, $\qquad (\Delta S)_A = 0.0875 \text{ kJ/K}$

Change in entropy of N_2, $\qquad (\Delta S)_B = 0.07826 \text{ kJ/K}$

Change in entropy of CO_2, $\qquad (\Delta S)_C = 0.0680 \text{ kJ/K}$

Solution : 21
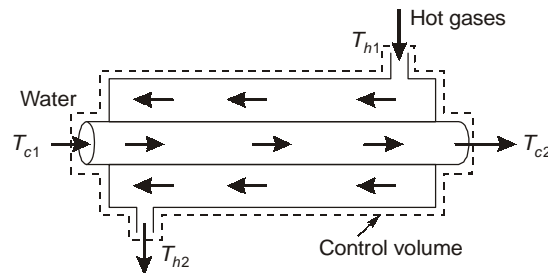
$$p_0 = 100 \text{ kPa}, \quad T_0 = 25°C = 298 \text{ K}$$
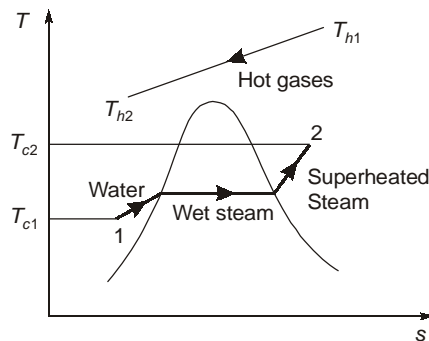
Change in availability of water on unit mass of water basis,

$$\psi_2 - \psi_1 = 769.07 \text{ kJ/kg}$$

Heat exchanged between combustion products (hot gases) and water in boiler,

Process irreversibility per unit mass of water flow,

$$I = 906.06 \text{ kJ}$$

Second law efficiency, $\eta_{II} = 45.90\%$

Entropy generated per kg of water,

$$(\Delta S)_{uni} = 3.04 \text{ kJ/K per kg of water}$$

Solution : 22

$$\text{Availability: } a = 271.75 \text{ kJ/kg}$$

Solution : 23

Availability of a system is the maximum useful work potential of a system at the given state. Availability is equal to the maximum work obtainable from the system when it moves from the current state to a state of complete equilibrium with the surroundings (also known as dead state) - temperature, pressure and chemical equilibrium.
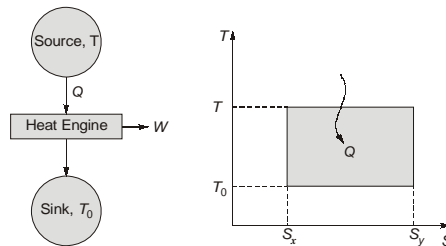
Consider a Carnot cycle as shown in figure which extracts heat Q from source at temperature T and rejects it to sink at temperature T_0.

Work 'W' is produced by the Carnot engine during the process. This is the maximum work which can be obtained by any cycle (reversible work) and hence, as per the definition of availability.

$$\text{Availability } = A = W_{max} = W_{Carnot}$$

$$\text{Efficiency of Carnot engine } = 1 - \frac{T_0}{T}$$

$$\therefore \quad \text{Work obtained, } W = \eta_C \times Q$$

$$\Rightarrow \quad W = \left(1 - \frac{T_0}{T}\right)Q$$

$$\therefore \quad \text{Availability } = W = \left(1 - \frac{T_0}{T}\right)Q$$

Solution : 24

$$\text{Decrease in availability of exhaust gas } = 65.75 \text{ kJ/kg}$$
$$\text{Total entropy production per kg gas } = 0.0738 \text{ kJ/kgK}$$
$$T_{2a} = 503.41 \text{ K}$$

Solution : 25

$$\text{Net change in entropy of gas } = m\Delta S_{net} = 0 \text{ kJ/K}$$

Solution : 26

$$\text{Availability of the system at the given state } = 560.04 \text{ kJ}$$
$$\text{Availability of air in this process } = 555.1 \text{ kJ}$$
$$\text{Effectiveness } \eta_{II} = 30.68\%$$

Solution : 27

$$m_0 = 137.14 \text{ kg}$$
$$\text{Entropy generated } = 3.18 \text{ kJ/K}$$

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6 Properties of Pure Substance

LEVEL 1 — Objective Questions

1. (a)
2. (c)
3. (c)
4. (c)
5. (b)
6. (b)
7. (b)
8. (b)
9. (c)
10. (a)
11. (d)

LEVEL 2 — Objective Questions

12. (b)
13. (d)
14. (b)
15. (a)
16. (a)
17. (c)
18. (b)
19. (3138.56)
20. (4158.97)
21. (a)

■■■■

LEVEL 3 Conventional Questions

Solution : 22
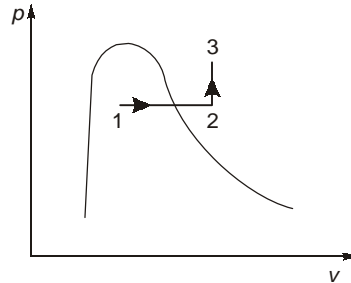
Total heat transfer $=$ 0.7085 kJ

Solution : 23
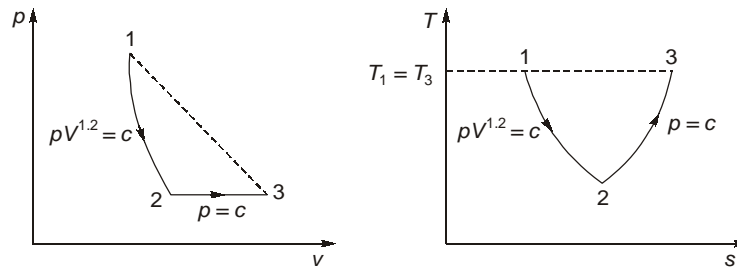
Work done for whole process, $=$ 32.11 kJ

Heat transfer for whole process, $=$ 32.114 kJ

Change in entropy of whole process, $\Delta S =$ 0.10511 kJ/K

Solution : 24

$$c_{ps} = 1.978 \text{ kJ/kgK}$$

Solution : 25

Mass of vapour, $m_g =$ 2.6094 kg

Mass of water, $m_f =$ 40.274 kg

Solution : 26

Mass of steam, $m =$ 0.78392 kg

Total heat content of steam, $H =$ 7625.63 kJ

Solution : 27

Supply rate of cooling water, $m_w =$ 138815.39 kg/hr

Solution : 28

$$Q = 129.82 \text{ kJ}$$

Total time elapsed $=$ 43.27 minutes

■■■■

7 Thermodynamic Relations and Clapeyron Equation

LEVEL 1 — Objective Questions

1. (a)
2. (b)
3. (c)
4. (c)
5. (b)
6. (a)
7. (c)
8. (b)
9. (c)
10. (c)

LEVEL 2 — Objective Questions

11. (b)
12. (c)
13. (d)
14. (c)
15. (d)
16. (2443.248)(2443.10 to 2443.45)
17. (1000)(999.5 to 1001)
18. (d)
19. (c)

■■■■

LEVEL 3 Conventional Questions

Solution : 20

For incompressible substance like liquids or solids, the specific volume (or density) is constant and the specific internal energy assumed to vary only with temperature.

Since the specific internal energy of an incompressible substance depends only on temperature, the specific heat c_v is also a function of temperature alone.

$$c_v(T) = \frac{du}{dT} \qquad ..(i)$$

Although the specific volume is constant and internal energy depends on temperature only, enthalpy varies with both pressure and temperature according to

$$h(T, p) = u(T) + pv \qquad ...(ii)$$

Differentiating equation (ii) with respect to temperature at constant pressure,

$$\left(\frac{dh}{dT}\right)_p = \left(\frac{du}{dT}\right) + 0$$

$$c_p = c_v$$

where
$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \text{ and } c_v = \frac{\partial u}{\partial T}$$

Thus, for an incompressible substance there is no difference between c_p and c_v, and both can be represent by the same symbol, c.

Solution : 21

By definition of Joule-Thomson coefficient,

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h \qquad ...(i)$$

Consider
$$s = f(T, p)$$

Then
$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \qquad ...(ii)$$

From Tds relations,

$$dh = Tds + vdp$$

Substituting the value of ds from Eq. (ii) in above equation.

$$dh = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp + vdp$$

$$= T\left(\frac{\partial s}{\partial T}\right)_p dT + \left[v + T\left(\frac{\partial s}{\partial p}\right)_T\right]dp$$

where
$$T\left(\frac{\partial s}{\partial T}\right)_p = c_p$$

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{from Maxwell's relation}$$

∴
$$dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

For throttling process,
$$dh = 0$$

∴
$$0 = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

$$-c_p dT = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

or
$$\left(\frac{\partial T}{\partial p}\right)_h = \frac{1}{c_p}\left[-v + T\left(\frac{\partial v}{\partial T}\right)_p\right] \qquad \ldots(iii)$$

and
$$\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p = v \times \left(\frac{-1}{T^2}\right) + \frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_p$$

$$\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p = \frac{1}{T^2}\left[-v + T\left(\frac{\partial v}{\partial T}\right)_p\right]$$

or
$$-v + T\left(\frac{\partial v}{\partial T}\right)_p = T^2\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right] \qquad \ldots(iv)$$

From Eqs. (iii) and (iv), we get

$$\left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$$

or
$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$$

Solution : 22

For liquid water, $c_p - c_v = 0.0248$ kJ/kgK

Solution : 23

$$C_P - C_V = R$$

Solution : 24

We know that third Maxwell relation,

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \qquad \text{. . .(i)}$$

When a phase change occurs, the saturated pressure p_s depends on saturated temperature T_s only.

That is, $\qquad p_s = f(T_s)$

The partial derivative, $\left(\frac{\partial p}{\partial T}\right)_v$ can be written as a total

derivative $\left(\frac{dp}{dT}\right)_{sat}$. The total derivative $\left(\frac{dp}{dT}\right)_{sat}$ is the

slope on saturated curve in p-T diagram at a saturated state, as shown in figure, and it is independent of specific

volume. Thus, the slope $\left(\frac{dp}{dT}\right)_{sat}$ is considered as a

constant during the integration of Eq. (i) between two saturated liquid state f and saturated vapour state g.

Equation (i) is written as

$$\left(\frac{dp}{dT}\right)_{sat} dv = ds$$

Integration between saturated liquid state f and saturated vapour state g, we get

$$\left(\frac{dp}{dT}\right)_{sat} \int_{v_f}^{v_g} dv = \int_{s_f}^{s_g} ds$$

$$\left(\frac{dp}{dT}\right)_{sat} (v_g - v_f) = (s_g - s_f)$$

$$\left(\frac{dp}{dT}\right)_{sat} v_{fg} = s_{fg}$$

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{s_{fg}}{v_{fg}}$$

...(ii)

We know that $\qquad dh = Tds + vdp$

During the phase change, both temperature and pressure are constant.
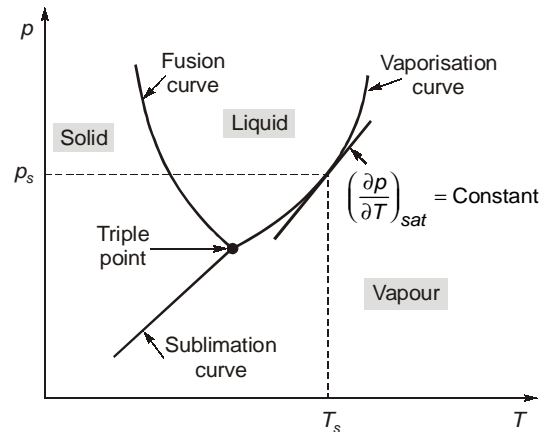
i.e., $\qquad T = C \quad \text{and} \quad p = C$

$\therefore \qquad dh = Tds$

Integration between two saturated states, we get

$$\int_{h_f}^{h_g} dh = T \int_{s_f}^{s_g} ds$$

$$h_g - h_f = T(s_g - s_f)$$

Phase diagram on *p-T* planes for water

$$h_{fg} = Ts_{fg}$$

where $h_{fg} = h_g - h_f$, specific enthalpy of vaporization

$s_{fg} = s_g - s_f$, change in specific entropy during a phase change process.

∴ $$s_{fg} = \frac{h_{fg}}{T}$$

Substituting the value of $s_{fg} = \dfrac{h_{fg}}{T}$ in Eq. (ii), we get

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}}$$

...(iii)

Equation (iii) is called the **Clapeyron equation**. The Clapeyron equation is used to determine the change in enthalpy when phase change takes place during a process *i.e.*, enthalpy of vaporization h_{fg}. This equation is valid only for any phase change at constant T and p.

For liquid-vapour and solid-vapour phase change process, the Clapeyron equation can be simplified by using some approximations.

At low pressure, $v_g \gg v_f$ for liquid and solid

∴ $$v_{fg} = v_g$$

∴ v_f is neglected

If vapour is considered as an ideal gas,

∴ $$pv_g = RT$$

from equation of state

or $$v_g = \frac{RT}{p}$$

∴ $$v_{fg} = \frac{RT}{p}$$

∵ $v_g = v_{fg}$

Substituting the values of $v_{fg} = \dfrac{RT}{p}$ in Eq. (iii), we get

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{ph_{fg}}{RT^2}$$

$$\frac{dp}{p} = \left[\frac{h_{fg}}{RT^2}\right]dT \qquad \qquad \text{. . . (iv)}$$

Equation (iv) is called the **Clapeyron-Clausius equation**. This equation is used to calculate the variation of saturated pressure with temperature.

Solution : 25

Consider variation of saturation pressure with saturation temperature.

During a phase change process, saturation pressure which varies only with saturation temperature.
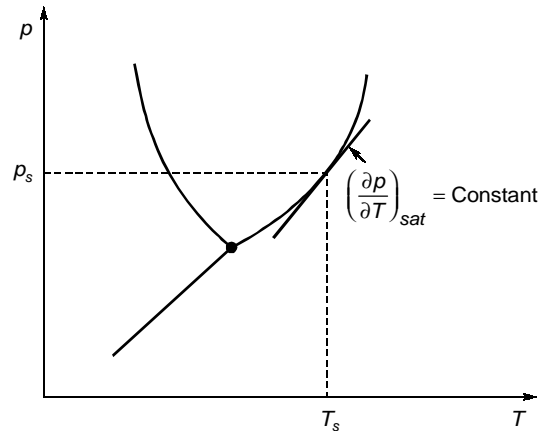
$$\therefore \qquad P_{sat} = \int(T_{sat})$$

$$\Rightarrow \qquad \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{dS}{dV}\right)_{sat}$$

From Maxwell's equation, we have:

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\therefore \qquad \left(\frac{dP}{dT}\right)_{sat} = \frac{S_g - S_f}{V_g - V_f} \qquad \qquad \dots \text{(i)}$$

Also, $\qquad TdS = dh - vdp$

During phase change process, $dp = 0$ (pressure remains constant at P_{sat}),

$$\therefore \qquad TdS = dh \Rightarrow S_g - S_f = \frac{h_{fg}}{T} \qquad \qquad \dots \text{(ii)}$$

$\therefore$ From (i) and (ii), we get: $\qquad \left(\frac{\partial P}{\partial T}\right)_{sat} = \frac{h_{fg}}{TV_{fg}}$

This equation is known as Clapeyron equation. Calculating h_{fg} using Claperon equation

$$\text{Error \%} = 0.78\%$$

Solution : 26

For an ideal gas, internal energy and enthalpy are functions of temperature alone.

$$\therefore \qquad dh = C_p dT \quad \text{and} \quad du = C_v dT$$

Also, ideal gas equation holds, $PV = RT$

From 1st law of thermodynamics,

$$Q = dU + W = dU + PdV$$

also, $\qquad Q = TdS = dU + PdV \qquad \qquad \dots \text{(i)}$

$$H = U + PV$$

$$\Rightarrow \qquad dH = dU + PdV + VdP$$

$$\therefore \qquad TdS = dH - VdP \qquad \qquad \dots \text{(ii)}$$

From (i) $\qquad TdS = C_V dT + PdV \qquad \qquad \dots \text{(iii)}$

From (ii)
$$TdS = C_p dT - VdP \qquad \qquad \dots \text{(iv)}$$

Combining (iii) and (iv),

$$dT = \frac{TdS - PdV}{C_V} = \frac{TdS + VdP}{C_P}$$

$\Rightarrow \quad T C_p \, dS - P C_p \, dV = T C_v \, dS + V C_v \, dP$

$\Rightarrow \quad T \, dS \, (C_p - C_v) = P C_p \, dV + V C_v \, dP$

We know, $\quad C_p - C_v = R$

$\Rightarrow \quad RTdS = PC_p \, dV + V C_v \, dP$

$\Rightarrow \quad PVdS = PC_p dV + V C_v dP$

$\Rightarrow \quad dS = C_p \dfrac{dV}{V} + C_v \dfrac{dP}{P} \quad$ Hence, proved

For isentropic process of an ideal gas,

$$dS = 0$$

$\therefore \quad dS = C_p \dfrac{dV}{V} + C_v \dfrac{dP}{P} = 0$

$\Rightarrow \quad \dfrac{dP}{dV} = -\dfrac{C_p}{C_v} \cdot \dfrac{P}{V} = -\gamma \cdot \dfrac{P}{V}$

where γ-ratio of specific heats of ideal gas.

$\therefore \quad \dfrac{dP}{P} = -\gamma \dfrac{dV}{V}$

Integrating both sides, we get

$\ln P = -\gamma \ln V + \ln C, \quad$ where C is a constant $= \ln (V^{-\gamma} \cdot C)$

$\Rightarrow \quad P = V^{-\gamma} \cdot C$

$\Rightarrow \quad PV^\gamma = C$

where C is a constant.

■■■■