GATE

MADE EASY
Workbook 2027

Detailed Explanations of Try Yourself *Questions*

Instrumentation Engineering

Electrical Circuits

MADE EASY PUBLICATIONS

1 Basics

Detailed Explanation of Try Yourself Questions

T1. (b)

Writing node equation at the top center node

$$\frac{V_1 - 0}{2+3} + \frac{(V_1 - 1)}{1} + \frac{V_1 - \alpha V_x}{5} = 0$$

$$\frac{V_1}{5} + \frac{V_1 - 1}{1} + \frac{V_1 - \alpha V_x}{5} = 0 \quad \text{...(i)}$$

Since $V_x = \left(\frac{2}{2+3}\right) V_1 = \frac{2}{5} V_1$ (Voltage Division)

Now, by substituting $V_1 = (5/2)\, V_x$ into equation (1), we get

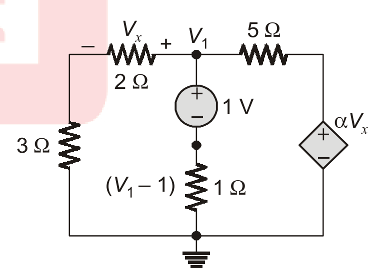
$$\frac{1}{5}\left(\frac{5}{2}V_x\right) + \left(\frac{5}{2}V_x - 1\right) + \frac{1}{5}\left(\frac{5}{2}V_x - V_x\right) = 0$$

$$\frac{V_x}{2} + \frac{5}{2}V_x + \frac{V_x}{2} = 1$$

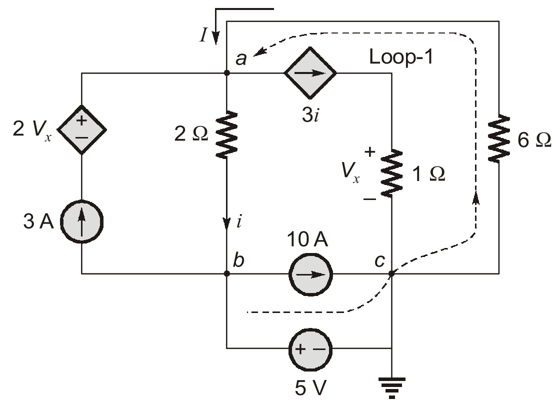
$$\frac{7}{2}V_x - \alpha + \frac{V_x}{5} = 1$$

$$35\, V_x - 2\,\alpha V_x = 10$$

$$V_x = \frac{10}{(35 - 2\alpha)}$$



T2. Sol.



Taking node 'C' as reference,
KCL at node 'a':

$$I + 3 = 3i + i$$

$$I = 4i - 3 \quad \text{...(i)}$$

KVL in loop-1

$$2i + 5 + 6I = 0$$

$$6I = -2i - 5 \quad \text{...(ii)}$$

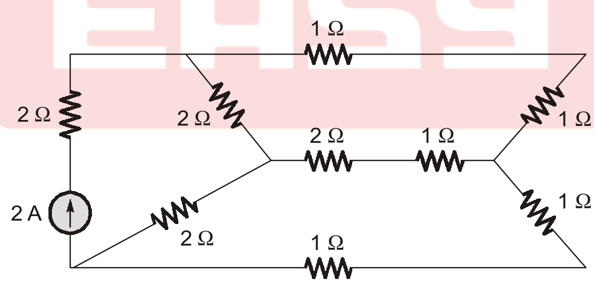
Solving equation (i) and (ii), we get

$$I = -1 \text{ A}, \quad i = \frac{1}{2} \text{ A}$$

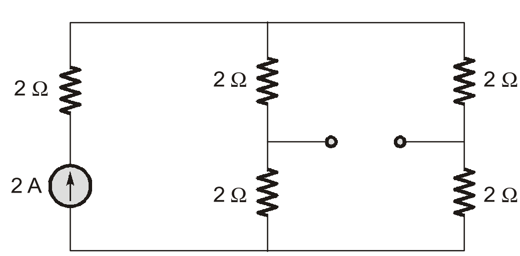
$$i = 0.5 \text{ A}$$

T3. Sol.

Transform Δ to Y the circuit can be reduced as below,



From balanced bridge
Network can be reduced to

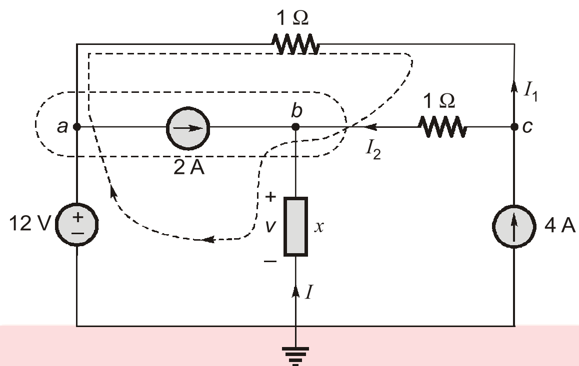


Total resistance across current source,

$$R_{eq} = 2 + \frac{4 \times 4}{4 + 4} = 4\ \Omega$$

Power delivered by current source, $P = I^2 R_{eq} = 2^2 \times 4 = 16$ W

T4. (b)



$$V = AI + B \quad \text{...(i)}$$

At node C;

$$4 = I_1 + I_2 \quad \text{...(ii)}$$

KVL in loop-1,

$$-12 - I_1 + I_2 + V = 0$$
$$V - I_1 + I_2 = 12 \quad \text{...(iii)}$$

KCL at node (b),

$$2 + I + I_2 = 0 \quad \text{...(iv)}$$

From equation (ii) and (iii),

$$I_1 = 4 - I_2$$

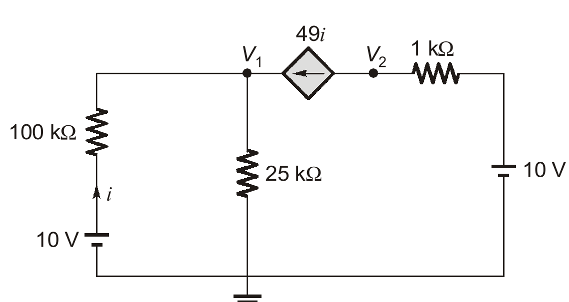
and

$$V + I_2 - 4 + I_2 = 12$$
$$2I_2 = 12 - V + 4 \quad \text{...(v)}$$

From equation (iv) and (v),

$$2 + I + 6 - \frac{V}{2} + 2 = 0$$
$$10 + I - \frac{V}{2} = 0$$
$$V = 2I + 20$$
$$A = 2, \quad B = 20$$

T5. (c)



KCL at node V_1,

$$i + 49i = \frac{V_1}{25}$$

$$50i = \frac{V_1}{25} \quad \text{...(i)}$$

$$i = \frac{10 - V_1}{100K} \quad \text{...(ii)}$$

From equation (i) and (ii),

$$50 \times \frac{10 - V_1}{100K} = \frac{V_1}{25}$$

$$\Rightarrow \quad \frac{1}{2K}(10 - V_1) = \frac{V_1}{25}$$

$$\Rightarrow \quad 10 - V_1 = \frac{(2K) \cdot V_1}{25}$$

$$10 = 81\ V_1$$

$$\Rightarrow \quad V_1 = \frac{10}{81} \text{ volts}$$

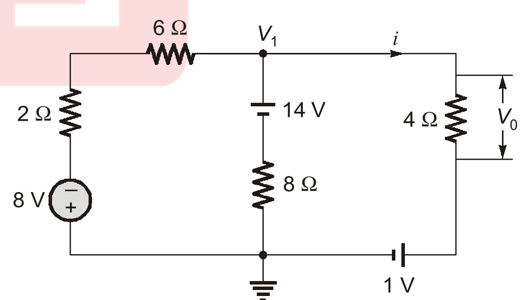
and

$$i = \frac{10 - \frac{10}{81}}{(100\text{ K})}$$

$$V_2 = 10 - 1\text{K} \times \frac{\left(10 - \frac{10}{81}\right)}{100\text{ K}} \times 49$$

$$= 5.16 \text{ volts}$$

T6. (a)



Transform current source to voltage source,
Applying KCL, at node V_1,

$$\frac{V_1 + 8}{8} + \frac{V_1 - 14}{8} + \frac{V_1 - 1}{4} = 0$$

$$V_1 = 2\text{ V}$$

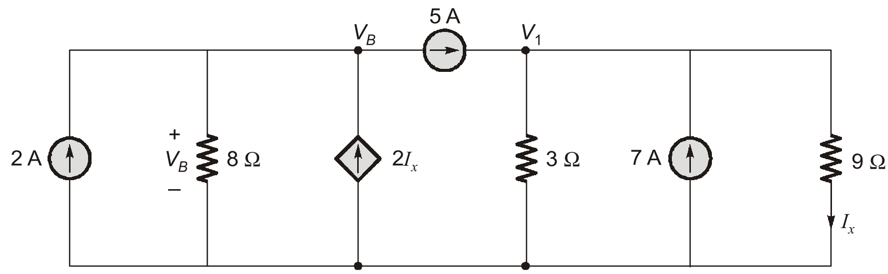
Current,

$$i = \frac{V_1 - 1}{4} = \frac{1}{4}\text{ A}$$

then,

$$V_0 = \frac{1}{4} \times 4 = 1\text{ V}$$

T7. (a)



By KCL at node 1,

$$\frac{V_1}{3} + \frac{V_1}{9} = 5 + 7$$

$$V_1 = 27\text{ V}$$

$$I_x = \frac{V_1}{9} = \frac{27}{9} = 3\text{ A}$$

By KCL at node B,

$$2 + 2I_x = \frac{V_B}{8} + 5$$

$$2 + 2(3) = \frac{V_B}{8} + 5$$

$\Rightarrow$ $$V_B = 24\text{ V}$$

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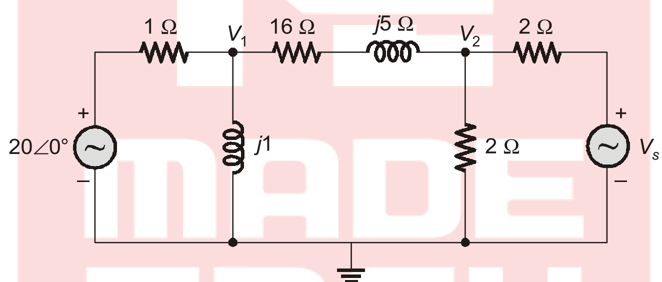
2 Steady State AC Analysis

Detailed Explanation of Try Yourself Questions

T1. Sol.



From given data current in 16 Ω is equal to zero, hence

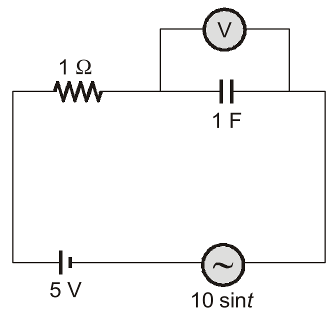
$$\boxed{V_1 = V_2}$$

$$V_1 = 20\angle 0 \times \frac{j1}{1+j1} = 20\frac{1\angle 90}{\sqrt{2}\angle 45} = \frac{20}{\sqrt{2}}\angle 45°$$

$$V_1 = V_2 = \frac{20}{\sqrt{2}}\angle 45°$$

$$V_s = 2\,V_2 = 2\frac{20}{\sqrt{2}}\angle 45° = 20\sqrt{2}\angle 45°\text{ V}$$

T2. (b)



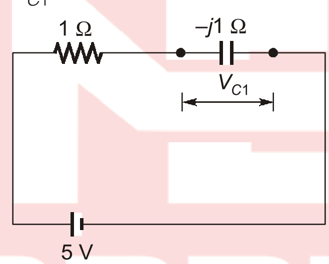
$$\because \quad X_C = \frac{1}{\omega C} = \frac{1}{1 \times 1}$$

$$X_C = 1\ \Omega$$

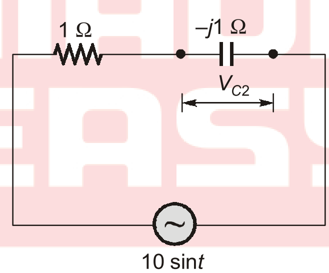
using superposition principle,

(i) For 5 V source

In steady-state, $V_{C1} = 5\text{ V}$



(ii) For 10 sin*t* source:



$$V_{C2} = \frac{10}{\sqrt{2} \times \sqrt{2}} \times 1 = 5\text{ V}$$

Now, $V_C = \sqrt{V_{C1}^2 + V_{C2}^2} = \sqrt{5^2 + (5)^2} = \sqrt{50} = 7.07\text{ V}$

T3. (c)

$$I = 4.24\sin(500t + 45°)$$

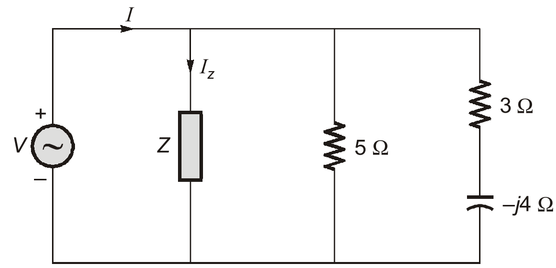
$$P = 180\text{ W}, \quad \text{p.f.} = 0.8\text{ lag}$$

$\because$ Power dissipated in resistor $= P = I_{or}^2 \times R$

$$180 = \left(\frac{4.24}{\sqrt{2}}\right)^2 \times R$$

$$R = 20.02 \simeq 20\ \Omega$$

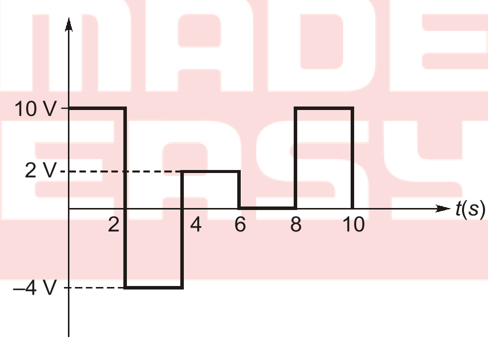
T4. (a)



$$V = 50\angle 30^\circ$$
$$I = 27.9\angle 57.8^\circ$$
$$Z_{eq} = \frac{V}{I} = \frac{50\angle 30^\circ}{27.9\angle 57.8^\circ} = 1.8\angle -27.8\ \Omega$$
$$= 1.8\angle -27.8^\circ\ \Omega$$

$\because$
$$\frac{1}{Z_{eq}} = \frac{1}{Z} + \frac{1}{5} + \frac{1}{3-j4}$$
$$\frac{1}{1.8\angle 27.8} = \frac{1}{Z} + \frac{1}{5} + \frac{3+j4}{25}$$
$$\frac{1}{Z} = \frac{1}{1.8\angle -27.8} - \frac{1}{5} - \frac{3+j4}{25}$$
$$Z = 5\angle -30^\circ\ \Omega$$

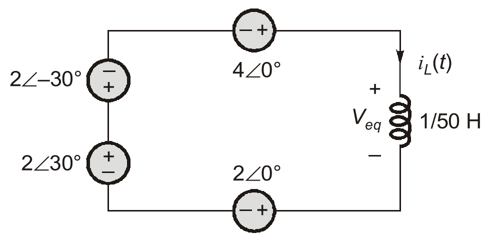
T5. Sol.



$$\text{Rms value} = \left[\frac{1}{T}\int_0^T f^2(t)\,d(t)\right]^{1/2} = \left[\frac{1}{10}\int_0^{10} f^2(t)\cdot d(t)\right]^{1/2}$$
$$= \left[\frac{1}{10}\left\{\int_0^2 100\,dt + \int_2^4 16\,dt + \int_4^6 4\,dt + \int_6^8 0 + \int_8^{10} 100\,dt\right\}\right]^{1/2}$$
$$= \left[\frac{1}{10}\{100\times 2 + 16\times 2 + 4\times 2 + 100\times 2\}\right]^{1/2}$$
$$= \left[\frac{1}{10}(440)\right]^{1/2} = \sqrt{44} = 6.633 \text{ unit}$$

T6. (c)

Given redundant network can be reduced as,



$$X_L = \omega L = 100\left(\frac{1}{50}\right) = 2$$

$$i_L(t) = \frac{V_{eq}}{jX_L}$$

$$i_L(t) = \frac{2+j2}{j2} = \sqrt{2}\angle{-45}$$

$$i_L(t) = 1.414\cos(100t - 45°)\text{ A}$$

T7. (a)

Current, $i(t) = C\dfrac{dv(t)}{dt}$

For $0 < t < 0.5\text{ s}$, $v(t) = 30t^2\text{ V}$

$i(t) = 20 \times 10^{-6}\,(60t) = 1.2\,t\text{ mA}$

For $0.5\text{ s} < t < 1\text{ s}$, $v(t) = 30\,(t-1)^2$

$i(t) = (20 \times 10^{-6})\,[60\,(t-1)] = 1.2(t-1)\text{ mA}$

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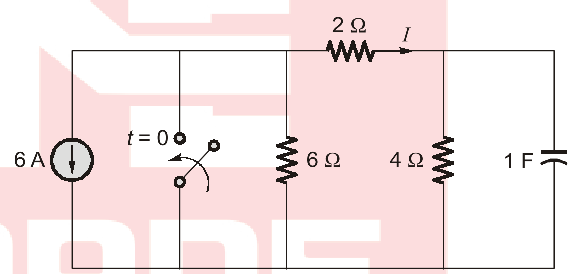
3 Transient Response

Detailed Explanation of Try Yourself Questions

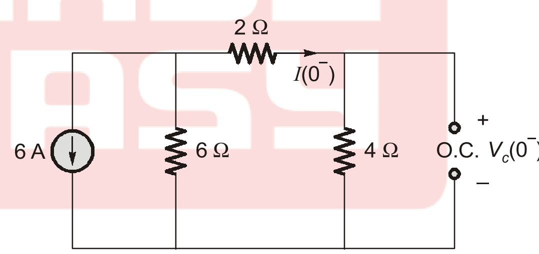
T1. (c)

For the given circuit,



For $t < 0$; at

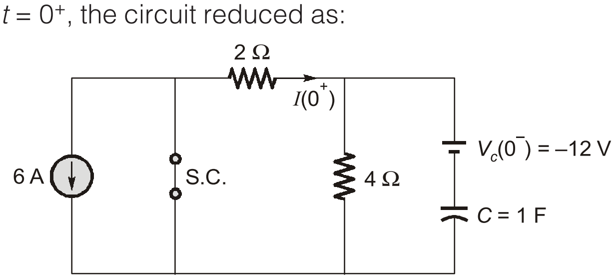
Steady-state:



$$I(0^-) = -\frac{6}{12} \times 6 = -3 \text{ A}$$

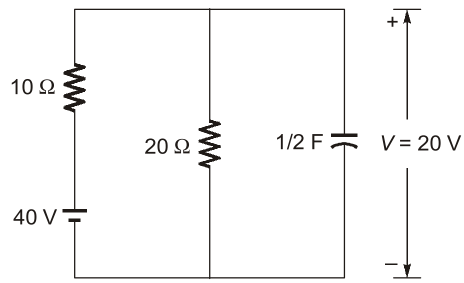
and $\quad V_C(0^-) = -12 \text{ V}$

after closing switch at $t = 0^+$, the circuit reduced as:



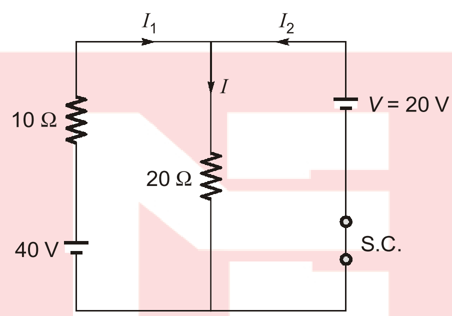
$$I(0^+) = \frac{12}{2} = 6 \text{ A}$$

T2. (c)



Suppose at time $t = 0$, the voltage 'V' = 20 V

The circuit can be reduced as



at $t = 0^+$;

$$\because \quad I = \frac{20}{20} = 1\text{ A}$$

and
$$I_1 = \frac{40-20}{10} = 2\text{ A}$$

$$I_2 = -1\text{ A}$$

$\because$ Current flowing across capacitor at $t = 0^+$;

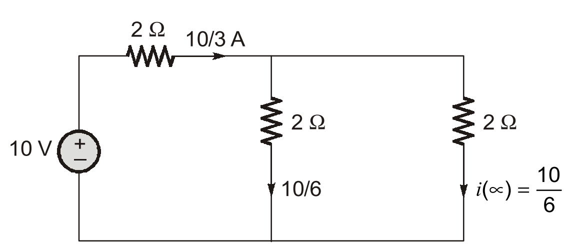
$$C\left.\frac{dV}{dt}\right|_{t=0^+} = -I_2 \text{ or } \left|\frac{dV}{dt}\right|_{\text{at } t=0^+} = 2\text{ V/s}$$

T3. Sol.

From given data,

$$i(0^+) = \frac{\Psi(0^+)}{L} = \frac{10}{1} = 10\text{ A}$$

at $t = \infty$;

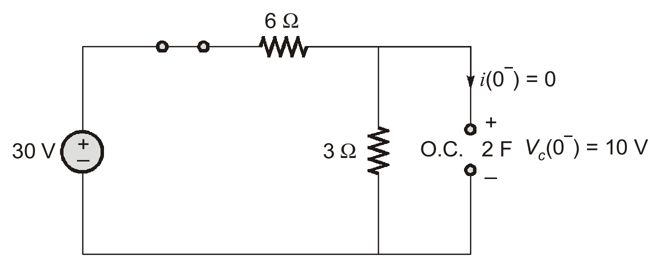


$$i(t) = [i(0^+) - i(\infty)]\, e^{-Rt/L} + i(\infty)$$

$$i(t) = \left[10 - \frac{10}{6}\right] e^{-\frac{3t}{1}} + \frac{10}{6} = [1.67 + (8.333)\, e^{-3t}] \text{A.}$$

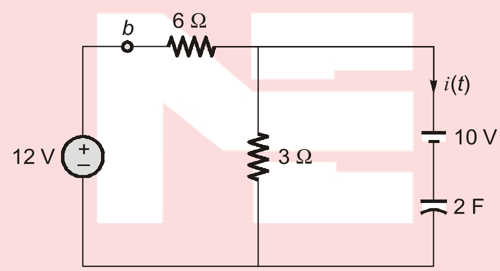
T4. (c)

∵ At at $t < 0$; the circuit is behaving as shown in figure,



$$i(0^-) = 0; \quad V_c(0^-) = 10 \text{ V}$$

At $t > 0$; now here we can find,



$$i(\infty) = 0$$

and

$$V_c(\infty) = 4 \text{ V}$$

$$V_c(t) = V_c(\infty) + [V(0^+) - V(\infty)]\, e^{-t/\tau}$$

Here,

$$\tau = R_{eq} \cdot C = (6 || 3) \times 2$$

$$= 2 \times 2 = 4 \text{ sec}$$

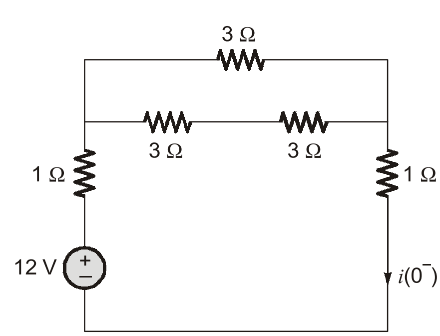
$$V_c(t) = 4 + [10 - 4]\, e^{-t/4} = 4 + 6e^{-t/4}$$

$$i_c(t) = -C\frac{dV}{dt} = -C \times \left[\frac{d[4 + 6e^{-t/4}]}{dt}\right]$$

$$= -2 \times \frac{6}{4} \cdot e^{-t/4} = -3e^{-t/4} \text{ A}$$

T5. (b)

At $t = 0^-$

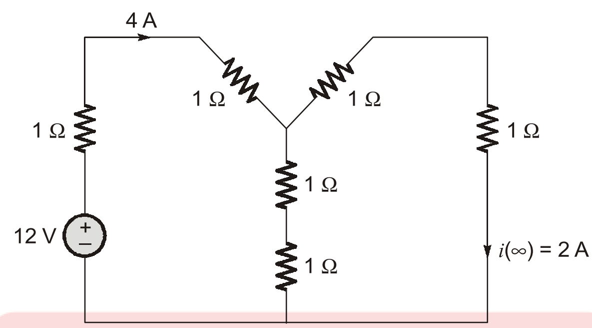


$$i(0^-) = \frac{12}{R_{eq}} = \frac{12}{4} = 3\text{ A}$$

$$i(0^+) = i(0^-) = 3\text{ A}$$

At $t = \infty$;

Transform Δ to Y;



$$i(\infty) = 2\text{ A}$$

$$i(t) = [(i(0^+) - i(\infty)]e^{-\frac{Rt}{L}} + i(\infty)$$

$$i(t) = (3 - 2)\, e^{-3t} + 2$$

$$i(t) = (2 + e^{-3t})\text{A}$$

T6. Sol.

$$F(s) = \frac{4e^{-2s}(s+2)}{s}$$

$$\text{Initial value} = \lim_{S\to\infty} s \cdot F(s)$$

$$f(0) = \lim_{S\to\infty} \frac{s \times 4e^{-2s}(s+2)}{s} = \lim_{S\to\infty} 4e^{-2s}(s+2)$$

$$= \lim_{S\to\infty} \frac{4(s+2)}{\left[1 + 2s + \frac{(2s)^2}{2!} + \frac{(2s)^3}{3!} + \ldots\right]}$$

$$= \lim_{S\to\infty} \frac{4s\left[1 + \frac{2}{s}\right]}{s\left[\frac{1}{s} + 2 + \frac{2^2 s}{2!} + \frac{2^3 s^2}{3!} + \ldots\right]}$$

$$= \frac{4[1+0]}{[0 + 2 + \infty + \infty + \ldots\ldots\infty]} = \frac{4}{\infty} = 0$$

Final value,

$$f(\infty) = \lim_{s\to 0} s \cdot F(s) = \lim_{s\to 0} \frac{s \times 4e^{-2s} \cdot (s+2)}{s}$$

$$= 4e^{-2\times 0}(0 + 2) = 8e^{-0} = 8(1)$$

$$= 8$$

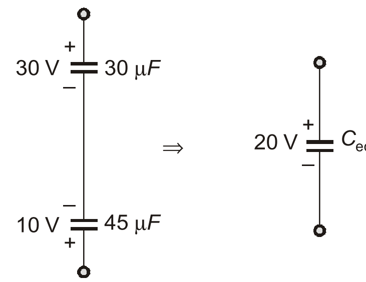
T7. (d)

The series connected capacitors can be replaced with an equivalent capacitor as shown

$$v_o(0) = 20 \text{ V}$$

$$C_{eq} = \frac{(30)(45)}{30+45} = 18\mu F$$

$$v_o(t) = v_o(0) + \frac{1}{C_{eq}}\int_0^t i(t)dt$$



–ve sign is taken because current flows from –ve to +ve polarity.

$$v_o(t) = 20 - \frac{1}{C_{eq}}\int_0^t i(t)dt$$

$$= 20 - \frac{1}{18\times10^{-6}}\int_0^t i(900\times10^{-6})e^{-2.5t}dt$$

$$= 20 - \frac{900\times10^{-6}}{18\times10^{-6}}\left[\frac{e^{-2.5t}}{-2.5}\right]_0^t$$

$$= 20 + 20\,[e^{-2.5t} - 1] = 20\,e^{-2.5t} \text{ V}$$

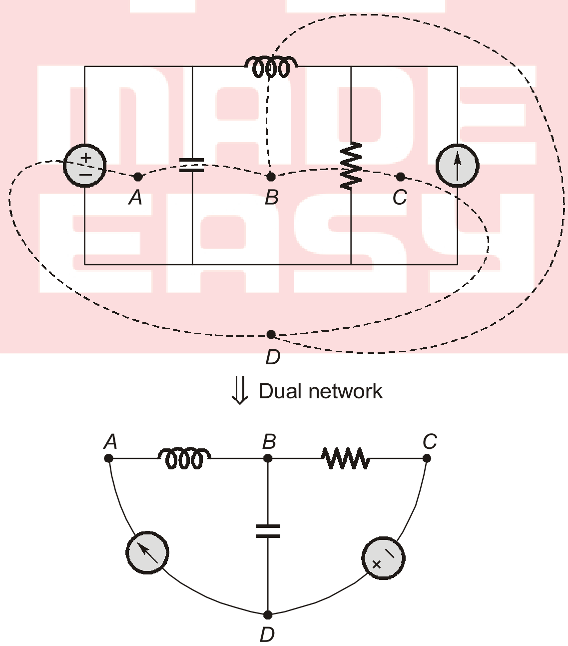
4 Graph Theory

Detailed Explanation *of* Try Yourself Questions

T1. (c)

T2. (d)

Dual of the given network is



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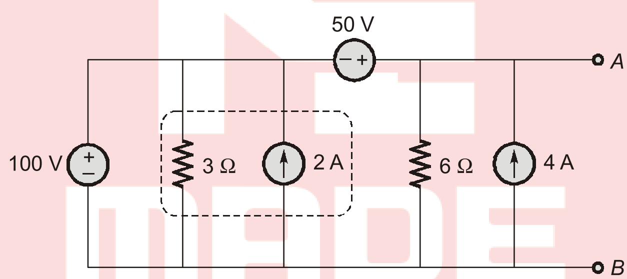
5 Network Theorems

Detailed Explanation of Try Yourself Questions

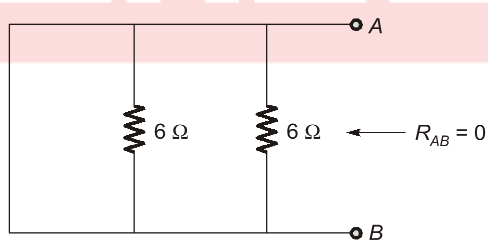
T1. (d)



$$V_{AB} = 50 + 100 = 150 \text{ V}$$

For R_{AB};

$$R_{AB} = 0$$



then here, $$I_N = \frac{V_{AB}}{R_{AB}} = \infty$$

Therefore, norton's equivalent circuit between terminals A and B does not exist.

T2. (a)

$$Z_S = \frac{R(j\omega L)}{R + j\omega L}$$

To seperate peal and imaginary,

$$Z_S = \frac{R(j\omega L)}{R+j\omega L}\times\frac{R-j\omega L}{R-j\omega L} = \frac{R\omega^2L^2}{R^2+\omega^2L^2}+j\frac{R^2\omega L}{R^2+\omega^2L^2}$$

From maximum power theorems,

$$Z_L = Z_S^*$$

$$R_1 - j\frac{1}{\omega C} = \frac{R\omega^2L^2}{R^2+\omega^2L^2}-j\frac{R^2\omega L}{R^2+\omega^2L^2}$$

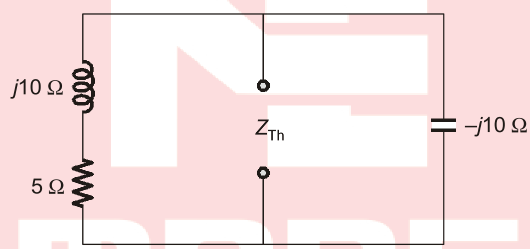
Compare real and imaginary part on both sides

$$R_1 = \frac{R\omega^2L^2}{R^2+\omega^2L^2}$$

$$C = \frac{R^2+\omega^2L^2}{R^2\omega^2L}$$

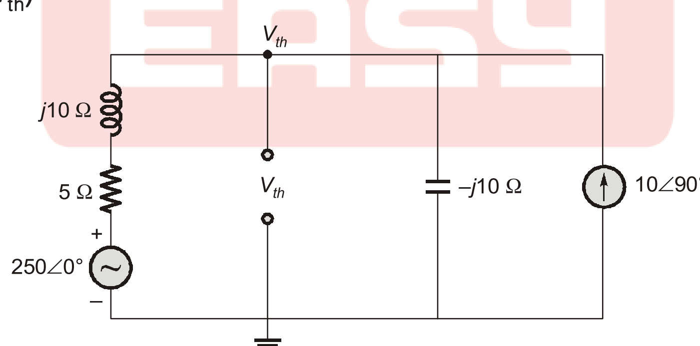
T3. Sol.

Case-1: To find (Z_{th})



$$Z_{th} = \frac{(5+j10)(-j10)}{5+10-j10} = 20-j10\,\Omega$$

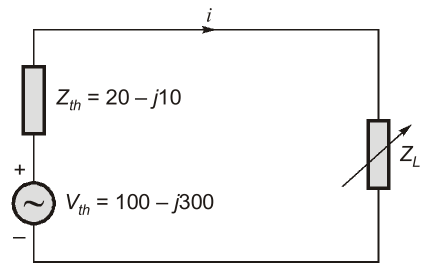
Case-2: To find (V_{th})



$$\frac{V_{th}-250}{5+j10}+\frac{V_{th}}{-j10} = 10\angle 90°$$

$$V_{th} = (100-j300)\text{ V}$$

From maximum power theorem,



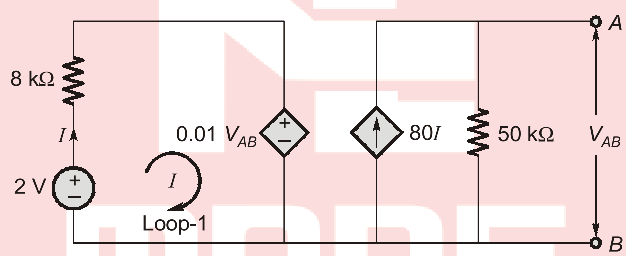
$$Z_L = Z_{th}^*$$

$$Z_L = 20 + j10$$

Current,
$$i = \frac{\sqrt{100^2 + 300^2}}{40}$$

$$P_L = i^2 R_L = i^2 \times 20 = 1250 \text{ W}$$

T4. (c)



$$V_{AB} = V_{Th} = 50 \text{ K} \times 80I \quad \text{...(i)}$$

KVL in loop-1,

$$-2 + (8 \times I) \times 10^3 + 0.01 \times V_{AB} = 0$$

$$2 = 10^3 [8I + 0.01 \times 50 \times 80I]$$

$$2 = 10^3 [8I + 40I]$$

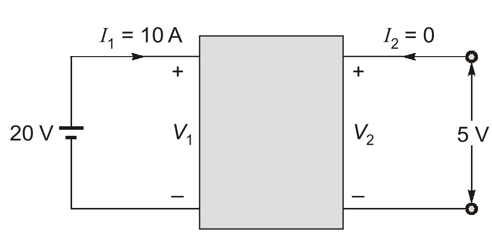
⇒
$$2 = 48I \times 10^3$$

$$I = \frac{2}{48\text{K}} \quad \text{...(ii)}$$

Now,
$$V_{Th} = 50\text{K} \times 80 \times \frac{2}{48\text{ K}} = 166.67 \text{ V}$$

T5. (b)

From the given network,



$\because \quad I_2 = 0$

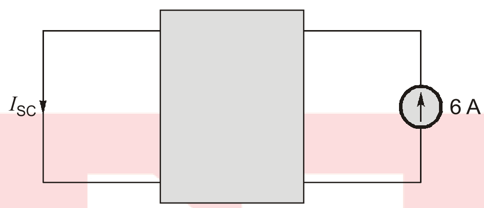
$$Z_{11} = \frac{V_1}{I_1} = \frac{20}{10} = 2\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{5}{10} = 0.5\ \Omega$$

For a reciprocal network,

$$Z_{12} = Z_{21} = 0.5$$

$\because$ For the given second network,



$\because \quad I_{SC} = I_1, \quad I_2 = 6\text{ A}$

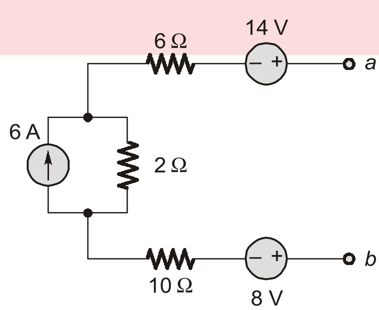
$\because \quad \frac{I_1}{I_2} = \frac{Z_{12}}{Z_{11}} = \frac{V_1/I_2}{V_1/I_1} = \frac{0.5}{2} = \frac{1}{4}$

$\because \quad I_{SC} = I_1 = \frac{6}{4} = 1.5\text{ A}$

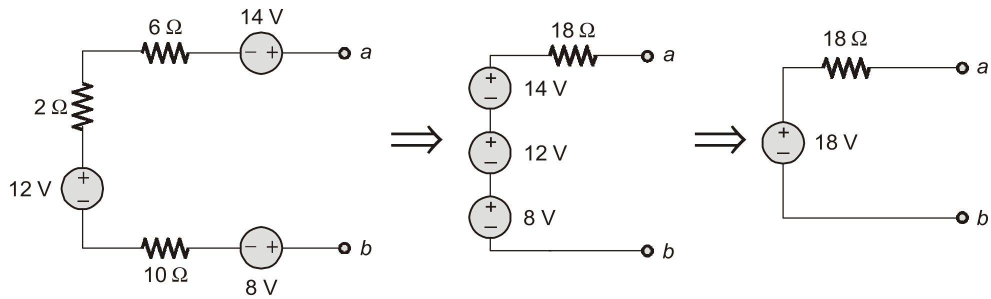
T6. (c)

Combining the parallel resistance and adding the parallel connected current sources.

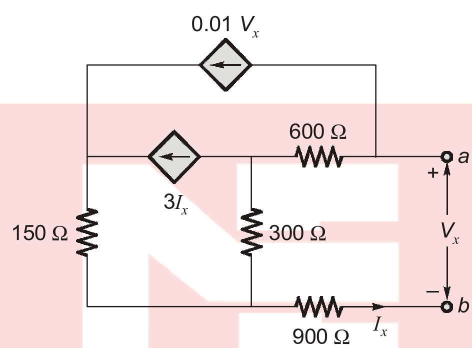
$$9\ A - 3\ A = 6\ A\text{ (upward)}$$
$$3\ \Omega || 6\ \Omega = 2\ \Omega$$



Source transformation of 6 *A* source



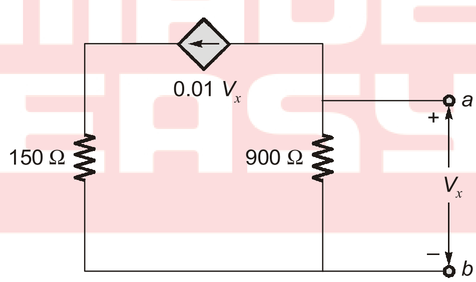
T7. (d)



For V_{OC} across *a* and *b*,

$\because$ $I_x = 0$; as O.C.

Now, circuit is reduced as below,



$$V_x = -900 \times 0.01\ V_x$$

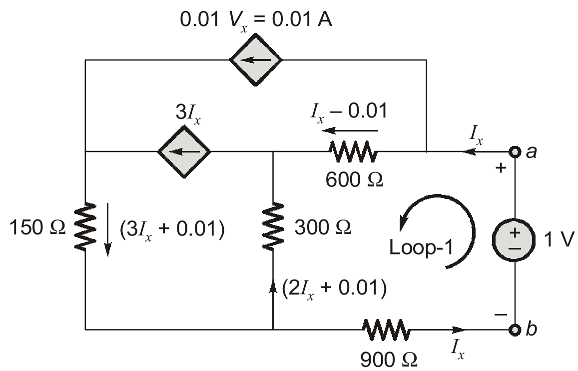
Only one case is possible,

i.e. $$V_{OC} = 0\text{ V}$$

For R_{Th}:

$$R_{\text{Th}} = \frac{V_x}{I_x}$$

$$V_x = 1\text{ V}$$



KVL in loop-1,

$$1 = 600(I_x - 0.01) - 300(2I_x + 0.01) + 900I_x$$
$$1 = 600\,I_x - 6 - 600\,I_x - 3 + 900\,I_x$$
$$10 = 900\,I_x$$

$\Rightarrow$
$$I_x = \frac{1}{90}$$
$$R_{Th} = 90\ \Omega$$

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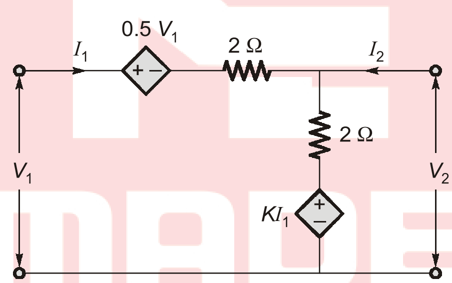
6 Two-Port Networks

Detailed Explanation of Try Yourself Questions

T1. (a)



For $I_2 = 0$;

$$V_2 = (2 \times I_1 + KI_1)$$
$$V_2 = (2 + K)\, I_1$$
$$\frac{V_2}{I_1} = Z_{21} = (2 + K) \quad \text{...(i)}$$

For $I_1 = 0$;

$$Z_{12} = \frac{V_1}{I_2};$$
$$V_1 = 0.5\, V_1 + 2I_2$$
$$\Rightarrow \quad 0.5\, V_1 = 2I_2$$
$$Z_{12} = \frac{V_1}{I_2} = 4 \quad \text{...(ii)}$$

From equation (i) and (ii),

From reciprocal network,

$$Z_{12} = Z_{21}$$
$$\because \quad 2 + K = 4$$
$$K = 2$$

T2. Sol.

To find Z_{22};

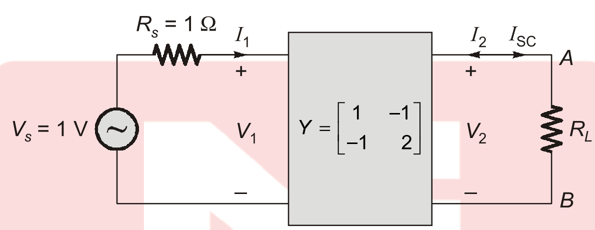
$$\because \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

For $I_1 = 0$;
R_{Th} from port port-2.

$$R_{Th} = 1.732\ \Omega$$
$$Z_{22} = 1.732\ \Omega$$

T3. Sol.

I_{SC} = for $V_2 = 0$;



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = V_1 - V_2$$
$$I_2 = V_1 - V_2$$

For $V_2 = 0$;

$$I_{SC} = -I_2 = -V_1$$
$$= -(V_s - I_1 R_s)$$
$$-V_1 = V_s + I_1 R_s$$

$\because$ For $V_2 = 0$;

$$I_1 = V_1$$

then,

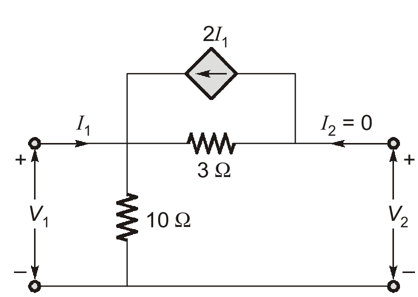
$$-V_1 = -V_s + V_1 \times R_s$$
$$V_s = V_1(1 + R_s)$$
$$V_1 = \frac{V_s}{(1+R_s)} = \frac{1}{2}$$

Now,

$$I_{SC} = -V_1 = -0.5\ \text{A}$$

T4. Sol.

To find Z-parameters:



we need to get;

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}; \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

then the circuit becomes,

$$V_1 = 10\,I_1$$

$$Z_{11} = \frac{V_1}{I_1} = 10$$

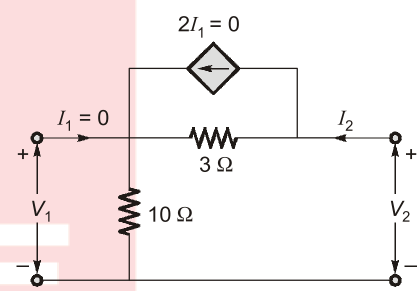
and $$V_2 = V_1 - 3\times 2I_1$$

$$V_2 = 10I_1 - 6I_1; \quad Z_{21} = \frac{V_2}{I_1} = 4$$

For $$Z_{12} = \left.\frac{V_2}{I_1}\right|_{I_1=0}$$

and $$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

then the circuit becomes,



$$V_1 = 10I_2;$$

$$Z_{12} = \frac{V_1}{I_2} = 10;$$

and $$V_2 = 13I_2$$

$$Z_{22} = \frac{V_2}{I_2} = 13$$

then, $$[Z] = \begin{bmatrix} 10 & 10 \\ 4 & 13 \end{bmatrix}$$

T5. Sol.

Given, $$[h] = \begin{bmatrix} 16\,\Omega & 3 \\ -2 & 0.015 \end{bmatrix}$$

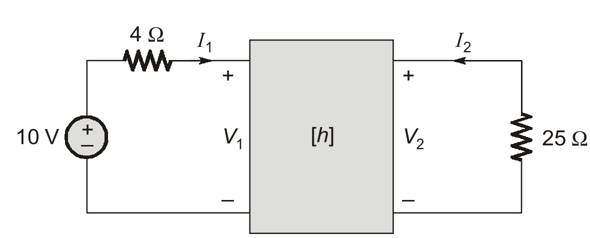
$\therefore$ $$V_1 = 16\,I_1 + 3\,V_2 \quad \text{...(i)}$$

$$I_2 = -2I_1 + 0.01\,V_2 \quad \text{...(ii)}$$

From the given circuit, $$I_2 = -\frac{V_2}{25}$$

KVL in the loop: $$-10 + 4I_1 + V_1 = 0$$

$$I_1 = \frac{10 - V_1}{4}$$



Substituting I_1 and I_2 in the equation (i) and (ii),

$$V_1 = 16\left(\frac{10 - V_1}{4}\right) + 3V_2$$

$$5V_1 - 3V_2 = 40$$

$$\left(-\frac{V_2}{25}\right) = -2\left(\frac{10 - V_1}{4}\right) + 0.01V_2$$

$$0.5V_1 + 0.05V_2 = 5$$

$$V_1 = 9.71\text{ V}; \quad V_2 = 2.8571\text{ V}$$

The ratio,
$$\frac{V_2}{V_1} = \frac{2.8571}{9.71} = 0.294$$

$$I_1 = \frac{10 - V_1}{4} = \frac{10 - 9.71}{4} = 0.072\text{ A}$$

$$\frac{I_1}{V_1} = \frac{0.072}{9.71} = 0.0074\ \mho$$

$$\frac{V_2}{V_1} = \frac{2.8571}{0.072} = 39.682\ \Omega$$

$$\frac{I_2}{I_1} = -0.11$$

T6. Sol.

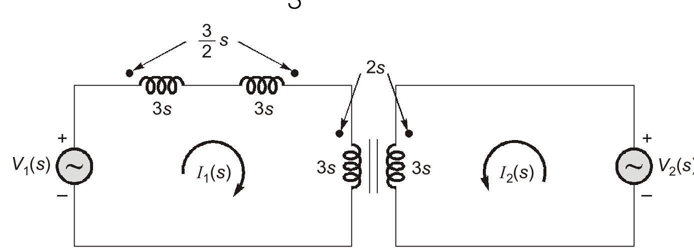
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{11}{34} & -10 \\ \frac{1}{34} & -4 \end{bmatrix}$$

T7. (c)

$$M = K\sqrt{L_1 \cdot L_2}$$

For first transformer,
$$M_1 = \frac{1}{2}\sqrt{3 \times 3} = \frac{3}{2}\text{ H}$$

For second transformer,
$$M_2 = \frac{2}{3}\sqrt{3 \times 3} = 2\text{ H}$$



By KVL for Loop 1,

$$V_1(s) = 9s\,I_1(s) - 3s\,I_1(s) + 2s\,I_2(s)$$

$$V_1(s) = 6s\,I_1(s) + 2s\,I_2(s)$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

We find, $Z_{11} = 6s$

$Z_{12} = 2s$

By KVL for Loop 2, $V_2(s) = 2s\,I_1(s) + 3s\,I_2(s)$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

We find, $Z_{21} = 2s$

$Z_{22} = 3s$

$\therefore$ $[Z] = \begin{bmatrix} 6s & 2s \\ 2s & 3s \end{bmatrix}$

T8. (c)

From circuit,

$V_1 = 100\angle 0°$

and $V_2 = -10\,I_2$

$\because$ $V_1 = 40\,I_1 + j20\,I_2$ ...(i)

$V_2 = j30\,I_1 + 50\,I_2$ ...(ii)

From equation (ii), $-10I_2 = j30\,I_1 + 50\,I_2$

$-60I_2 = j30\,I_1;\quad I_2 = -\frac{j}{2} \times I_1$

From equation (i), $100 = 40I_1 + j20 \times \left(-\frac{j}{2}\right) I_1$

$100 = 50\,I_1$

$\Rightarrow$ $I_1 = 2\angle 0°\text{ A}$

then, $I_2 = -\frac{j}{2} \times 2\text{ A} = 1\angle -90°\text{ A}$

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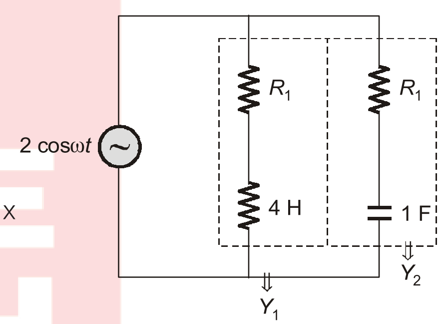
7 Resonance

Detailed Explanation of Try Yourself Questions

T1. Sol.

For reactive power from source to zero.

The network is behaving as purely resistive.



$\because$
$$Y_{eq} = Y_1 + Y_2$$

$$Y_{eq} = \frac{1}{R + X_L} + \frac{1}{R + X_C}$$

$\because$
$$X_L = j\omega L, \quad X_C = -j\frac{1}{\omega C}$$

$$Y_{eq} = \frac{1}{R + j\omega L} + \frac{1}{R - \dfrac{j}{\omega C}}$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2} + \frac{R + \dfrac{j}{\omega C}}{R^2 + \dfrac{1}{\omega^2 C^2}}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + \frac{R}{R^2 + \left(\dfrac{1}{\omega C}\right)^2} + j\left[\frac{\dfrac{1}{\omega C}}{R^2 + \dfrac{1}{\omega^2 C^2}} - \frac{\omega L}{R^2 + \omega^2 L^2}\right]$$

$\because$ $I_m(Y_{eq}) = 0$; for resistive network

$$\frac{\dfrac{1}{\omega C}}{R^2 + \dfrac{1}{\omega^2 C^2}} = \frac{\omega L}{R^2 + \omega^2 L^2}$$

$$\frac{1}{\omega C}\times(R^2+\omega^2L^2) = \omega L\left(R^2+\frac{1}{\omega^2C^2}\right)$$

$$\frac{1}{\omega^2LC}[R^2+\omega^2L^2] = R^2+\frac{1}{\omega^2C^2}$$

$$\frac{R^2}{\omega^2LC}+\frac{L}{C} = R^2+\frac{1}{\omega^2C^2}$$

$$R^2\left[1-\frac{1}{\omega^2LC}\right] = \frac{L}{C}-\frac{1}{\omega^2C^2}$$

$$R^2 = \frac{\left(\frac{L}{C}-\frac{1}{\omega^2C^2}\right)}{\left(1-\frac{1}{\omega^2LC}\right)}$$

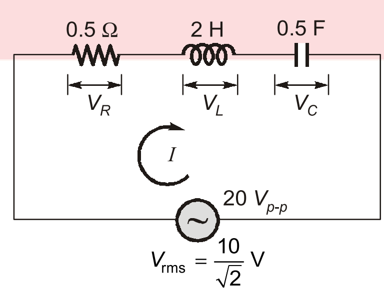
$$R = \sqrt{\frac{\frac{L}{C}-\frac{1}{\omega^2C^2}}{1-\frac{1}{\omega^2LC}}}$$

or, $L = 4\text{ H},\ C = 1\text{ F}$

$$R = \sqrt{\frac{4-\frac{1}{\omega^2}}{1-\frac{1}{4\omega^2}}} = 2\ \Omega$$

T2. (a)

Voltage across 'R' is maximum.



When V_C and V_L are in phase opposition i.e. at resonance.

∵ At resonance:

Total impedance, $Z = R = 0.5\ \Omega$

Current, $I = \frac{10/\sqrt{2}}{0.5} = \frac{20}{\sqrt{2}} = 10\sqrt{2} = 14.142\text{ A}$

Voltage across capacitor, $$V_C = \frac{I}{\omega_0 C}$$

$\because$ $$\omega_0 = \frac{1}{\sqrt{LC}}$$

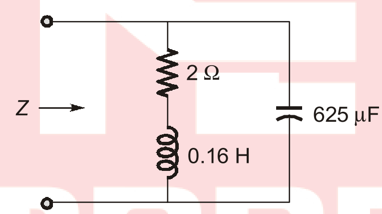
$$V_C = \frac{I}{\frac{1}{\sqrt{LC}} \times C} = I\sqrt{\frac{L}{C}}$$

$$= 10\sqrt{2}\sqrt{\frac{2}{0.5}} = 10\sqrt{2} \times 2 = 20\sqrt{2}$$

$$= \frac{40}{\sqrt{2}} \text{ V}$$

T3. (c)

T4. (d)



$\because$ At resonance,

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \sqrt{\frac{10^6}{0.16 \times 625} - \frac{4}{0.16 \times 0.16}} = \sqrt{10^4 - 156.25}$$

$$= 99.216 \text{ rad/sec.}$$

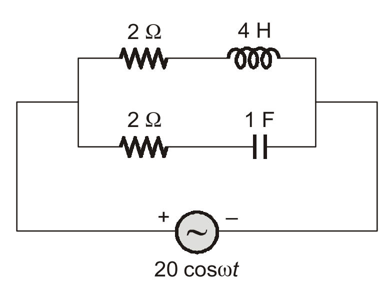
$$Z \text{ at } \omega = \omega_0 = \frac{\omega_0^2 L^2}{R} + R = 2 + \frac{(99.216)^2 \times (0.16)^2}{2}$$

$$= 2 + 126 = 128\ \Omega$$

T5. Sol.

(i) At resonance;

$$Y_{eq} = G_{eq} = \frac{R}{R^2 + \frac{1}{(\omega C)^2}} + \frac{R}{R^2 + (\omega L)^2}$$



Average power consumed,

$$P = V_{rms}^2 \times G_{eq} = (20/\sqrt{2})^2 \times G_{eq}$$

$$\because \omega_0 \text{ (resonant frequency)} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times 1}} = 0.5 \text{ rad/sec.}$$

$$G_{eq} = \frac{2}{4+\dfrac{1}{(0.5\times 1)^2}} + \frac{2}{4+(0.5\times 4)^2}$$

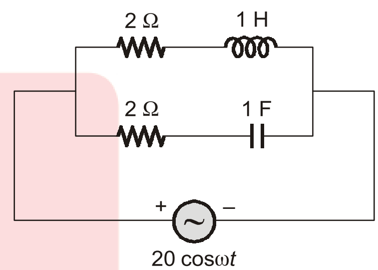
$$= \frac{2}{4+4} + \frac{2}{4+4} = 0.5\ \mho$$

$$P = \frac{20^2}{2}\times 0.5 = 100 \text{ W}$$

(ii)

$$R_1 = R_2 = R = 2\ \Omega$$

$$L = 1 \text{ H};\quad C = 1 \text{ F}$$



$$\omega_0 = \frac{1}{\sqrt{1\times 1}} = 1 \text{ rad/sec.}$$

$$G_{eq} = \frac{2}{(2)^2+(1\times 1)^2} + \frac{2}{(2)^2+\left(\dfrac{1}{1\times 1}\right)^2}$$

$$= \frac{2+2}{4+1} = \frac{4}{5}\ \mho$$

$$P = \frac{(20)^2\times 4}{2\times 5} = 160 \text{ W}$$

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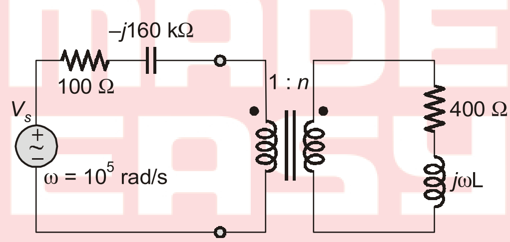
8 Filters and Magnetic Coupled Circuits

Detailed Explanation of Try Yourself Questions

T1. (c)

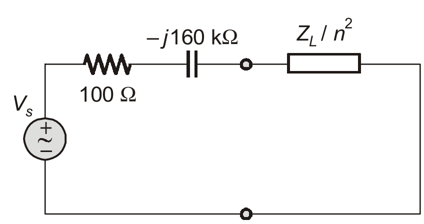
T2. (c)

In phaser domain,
$$L \Rightarrow j\omega L$$
$$C \Rightarrow \frac{1}{j\omega C} = \frac{1}{j(10^5)(62.5\times10^{-12})} = -j160\ \text{k}\Omega$$



Load impedance, $Z_L = 400 + j\omega L$

Reflecting the secondary impedance to the primary side



For maximum power transfer $Z_L/n^2 = Z_s^*$

$$\frac{Z_L}{n^2} = (100 - j160\times10^3) \qquad Z_s = (100 - j160\times10^3)\ \Omega$$

$$\frac{400 + j\omega L}{n^2} = 100 + j160\times10^3$$

Comparing real parts on both sides of the equation,

$$\frac{400}{n^2} = 100 \Rightarrow n = 2$$

Comparing imaginary parts,

$$\frac{\omega L}{n^2} = 160 \times 10^3$$

$$\Rightarrow \quad L = \frac{160 \times 10^3}{10^5} \times 4 = 6.4 \text{ H}$$

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