## GATE

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Detailed Explanations of<br>Try Yourself Questions

## Instrumentation Engineering

Electrical Circuits

## Basics



## Detailed Explanation of <br> Try Yourself Questions

T1. (b)
Writing node equation at the top center node

$$
\begin{align*}
\frac{V_{1}-0}{2+3}+\frac{\left(V_{1}-1\right)}{1}+\frac{V_{1}-\alpha V_{x}}{5} & =0 \\
\frac{V_{1}}{5}+\frac{V_{1}-1}{1}+\frac{V_{1}-\alpha V_{x}}{5} & =0 \tag{i}
\end{align*}
$$

Since

$$
V_{x}=\left(\frac{2}{2+3}\right) V_{1}=\frac{2}{5} V_{1} \quad \text { (Voltage Division) }
$$

Now, by substituting

$$
V_{1}=(5 / 2) V_{x} \text { into equation (1), we get }
$$

$$
\begin{aligned}
\frac{1}{5}\left(\frac{5}{2} V_{x}\right)+\left(\frac{5}{2} V_{x}-1\right)+\frac{1}{5}\left(\frac{5}{2} V_{x}-V_{x}\right) & =0 \\
\frac{V_{x}}{2}+\frac{5}{2} V_{x}+\frac{V_{x}}{2} & =1 \\
\frac{7}{2} V_{x}-\alpha+\frac{V_{x}}{5} & =1 \\
35 V_{x}-2 \alpha V_{x} & =10 \\
V_{x} & =\frac{10}{(35-2 \alpha)}
\end{aligned}
$$



## T2. Sol.



Taking node 'C' as reference,
KCL at node 'a':

$$
\begin{align*}
I+3 & =3 i+i \\
I & =4 i-3 \tag{i}
\end{align*}
$$

KVL in loop-1

$$
\begin{align*}
2 i+5+6 I & =0 \\
6 I & =-2 i-5 \tag{ii}
\end{align*}
$$

Solving equation (i) and (ii), we get

$$
\begin{aligned}
I & =-1 \mathrm{~A}, \quad i=\frac{1}{2} \mathrm{~A} \\
i & =0.5 \mathrm{~A}
\end{aligned}
$$

## T3. Sol.

Transform $\Delta$ to $Y$ the circuit can be reduced as below,


From balanced bridge
Network can be reduced to


Total resistance across current source,

$$
R_{e q}=2+\frac{4 \times 4}{4+4}=4 \Omega
$$

Power delivered by current source, $\quad P=I^{2} R_{e q}=2^{2} \times 4=16 \mathrm{~W}$
T4. (b)


$$
\begin{equation*}
V=A I+B \tag{i}
\end{equation*}
$$

At node C;

$$
\begin{equation*}
4=I_{1}+I_{2} \tag{ii}
\end{equation*}
$$

KVL in loop-1,

$$
\begin{align*}
-12-I_{1}+I_{2}+V & =0 \\
V-I_{1}+I_{2} & =12 \tag{iii}
\end{align*}
$$

KCL at node (b),

$$
\begin{equation*}
2+I+I_{2}=0 \tag{iv}
\end{equation*}
$$

From equation (ii) and (iii),
and

$$
\begin{align*}
I_{1} & =4-I_{2} \\
V+I_{2}-4+I_{2} & =12 \\
2 I_{2} & =12-V+4 \tag{v}
\end{align*}
$$

From equation (iv) and (v),

$$
\begin{aligned}
2+I+6-\frac{V}{2}+2 & =0 \\
10+I-\frac{V}{2} & =0 \\
V & =2 I+20 \\
A & =2, \quad B=20
\end{aligned}
$$

T5. (c)


KCL at node $V_{1}$,

$$
\begin{align*}
i+49 i & =\frac{V_{1}}{25} \\
50 i & =\frac{V_{1}}{25}  \tag{i}\\
i & =\frac{10-V_{1}}{100 K} \tag{ii}
\end{align*}
$$

From equation (i) and (ii),

$$
\begin{array}{rlrl}
\Rightarrow & & 50 \times \frac{10-V_{1}}{100 \mathrm{~K}} & =\frac{V_{1}}{25} \\
\Rightarrow & \frac{1}{2 \mathrm{~K}}\left(10-V_{1}\right) & =\frac{V_{1}}{25} \\
\Rightarrow & 10-V_{1} & =\frac{(2 \mathrm{~K}) \cdot V_{1}}{25} \\
10 & =81 V_{1} \\
\text { and } & V_{1} & =\frac{10}{81} \text { volts } \\
& & i & =\frac{10-\frac{10}{81}}{(100 \mathrm{~K})}
\end{array}
$$

$$
V_{2}=10-1 \mathrm{~K} \times \frac{\left(10-\frac{10}{81}\right)}{100 \mathrm{~K}} \times 49
$$

$$
=5.16 \text { volts }
$$

T6. (a)
Transform current source to voltage source,
Applying KCL, at node $V_{1}$,

$$
\begin{aligned}
\frac{V_{1}+8}{8}+\frac{V_{1}-14}{8}+\frac{V_{1}-1}{4} & =0 \\
V_{1} & =2 \mathrm{~V} \\
i & =\frac{V_{1}-1}{4}=\frac{1}{4} \mathrm{~A}
\end{aligned}
$$


then,
$V_{0}=\frac{1}{4} \times 4=1 \mathrm{~V}$

T7. (a)


By KCL at node 1,

$$
\begin{aligned}
\frac{V_{1}}{3}+\frac{V_{1}}{9} & =5+7 \\
V_{1} & =27 \mathrm{~V} \\
I_{x} & =\frac{V_{1}}{9}=\frac{27}{9}=3 \mathrm{~A}
\end{aligned}
$$

By KCL at node B,

$$
2+2 I_{x}=\frac{V_{B}}{8}+5
$$

$$
\begin{aligned}
& 2+2(3) & =\frac{V_{B}}{8}+5 \\
\Rightarrow & V_{B} & =24 \mathrm{~V}
\end{aligned}
$$

## Steady State AC Analysis



T1. Sol.


From given data current in $16 \Omega$ is equal to zero, hence

$$
\begin{aligned}
& V_{1}=V_{2} \\
& V_{1}=20 \angle 0 \times \frac{j 1}{1+j 1}=20 \frac{1 \angle 90}{\sqrt{2} \angle 45}=\frac{20}{\sqrt{2}} \angle 45^{\circ} \\
& V_{1}=V_{2}=\frac{20}{\sqrt{2}} \angle 45^{\circ} \\
& V_{s}=2 V_{2}=2 \frac{20}{\sqrt{2}} \angle 45^{\circ}=20 \sqrt{2} \angle 45^{\circ} \mathrm{V}
\end{aligned}
$$

T2. (b)

using superposition principle,
(i) For 5 V source

In steady-state,

$$
V_{C 1}=5 \mathrm{~V}
$$


(ii) For 10 sint source:

$$
\begin{aligned}
& V_{C 2}=\frac{10}{\sqrt{2} \times \sqrt{2}} \times 1=5 \mathrm{~V} \\
& V_{C}=\sqrt{V_{C 1}^{2}+V_{C 2}^{2}}=\sqrt{5^{2}+(5)^{2}}=\sqrt{50}=7.07 \mathrm{~V}
\end{aligned}
$$

Now,
T3. (c)

$$
\begin{aligned}
I & =4.24 \sin \left(500 t+45^{\circ}\right) \\
P & =180 \mathrm{~W}, \quad \text { p.f. }=0.8 \mathrm{lag}
\end{aligned}
$$

$\because \quad$ Power dissipated in resistor $=P=I_{o r}^{2} \times R$

$$
\begin{aligned}
180 & =\left(\frac{4.24}{\sqrt{2}}\right)^{2} \times R \\
R & =20.02 \simeq 20 \Omega
\end{aligned}
$$

T4. (a)

$$
\begin{array}{rl}
I & z \\
V & =50 \angle 30^{\circ} \\
I & =27.9 \angle 57.8^{\circ} \\
Z_{e q} & =\frac{V}{I}=\frac{50 \angle 30^{\circ}}{27.9 \angle 57.8^{\circ}}=1.8 \angle-27.8 \Omega \\
& =1.8 \angle-27.8^{\circ} \Omega \\
\frac{1}{Z_{e q}} & =\frac{1}{Z}+\frac{1}{5}+\frac{1}{3-j 4} \\
\frac{1}{1.8 \angle 27.8} & =\frac{1}{Z}+\frac{1}{5}+\frac{3+j 4}{25} \\
\frac{1}{Z} & =\frac{1}{1.8 \angle-27.8}-\frac{1}{5}-\frac{3+j 4}{25} \\
Z & =5 \angle-30^{\circ} \Omega
\end{array}
$$

T5. Sol.


$$
\begin{aligned}
\text { Rms value } & =\left[\frac{1}{T} \int_{0}^{T} f^{2}(t) d(t)\right]^{1 / 2}=\left[\frac{1}{10} \int_{0}^{10} f^{2}(t) \cdot d(t)\right]^{1 / 2} \\
& =\left[\frac{1}{10}\left\{\int_{0}^{2} 100 d t+\int_{2}^{4} 16 d t+\int_{4}^{6} 4 d t+\int_{6}^{8} 0+\int_{8}^{10} 100 d t\right\}\right]^{1 / 2} \\
& =\left[\frac{1}{10}\{100 \times 2+16 \times 2+4 \times 2+100 \times 2\}\right]^{1 / 2} \\
& =\left[\frac{1}{10}(440)\right]^{1 / 2}=\sqrt{44}=6.633 \text { unit }
\end{aligned}
$$

T6. (c)
Given redundant network can be reduced as,


$$
\begin{aligned}
X_{L} & =\omega L=100\left(\frac{1}{50}\right)=2 \\
i_{L}(t) & =\frac{V_{e q}}{j X_{L}} \\
i_{L}(t) & =\frac{2+j 2}{j 2}=\sqrt{2} \angle-45 \\
i_{L}(t) & =1.414 \cos \left(100 t-45^{\circ}\right) \mathrm{A}
\end{aligned}
$$

T7. (a)
Current,

$$
i(t)=c \frac{d v(t)}{d t}
$$

For

$$
0<t<0.5 \mathrm{~s}, v(t)=30 t^{2} \mathrm{v}
$$

$i(t)=20 \times 10^{-6}(60 t)=1.2 \mathrm{tmA}$
For $\quad 0.5 \mathrm{~s}<t<1 \mathrm{~s}, v(t)=30(t-1)^{2}$

$$
i(t)=\left(20 \times 10^{-6}\right)[60(t-1)]=1.2(t-1) \mathrm{mA}
$$

## Transient Response

## Detailed Explanation of <br> Try Yourself Questions

T1. (c)
For the given circuit,


For $t<0$; at
Steady-state:
and

$I\left(0^{-}\right)=-\frac{6}{12} \times 6=-3 \mathrm{~A}$
$V_{C}\left(0^{-}\right)=-12 \mathrm{~V}$
after closing switch at $t=0^{+}$, the circuit reduced as:

$I\left(0^{+}\right)=\frac{12}{2}=6 \mathrm{~A}$

T2. (c)


Suppose at time $t=0$, the voltage ' V ' $=20 \mathrm{~V}$
The circuit can be reduced as

at $t=0^{+}$;
$\because \quad I=\frac{20}{20}=1 \mathrm{~A}$
and

$$
\begin{aligned}
& I_{1}=\frac{40-20}{10}=2 \mathrm{~A} \\
& I_{2}=-1 \mathrm{~A}
\end{aligned}
$$

$\because$ Current flowing across capacitor at $t=0^{+}$;

$$
\left.C \frac{d V}{d t}\right|_{t=0^{+}}=-I_{2} \text { or }\left|\frac{d V}{d t}\right|_{\mathrm{at} t=0^{+}}=2 \mathrm{~V} / \mathrm{s}
$$

## T3. Sol.

From given data,

$$
i\left(0^{+}\right)=\frac{\Psi\left(0^{+}\right)}{L}=\frac{10}{1}=10 \mathrm{~A}
$$

at $t=\infty$;


$$
\begin{aligned}
& i(t)=\left[i\left(0^{+}\right)-i(\infty)\right] e^{-R t / L}+i(\infty) \\
& i(t)=\left[10-\frac{10}{6}\right] e^{-\frac{3 t}{1}}+\frac{10}{6}=\left[1.67+(8.333) e^{-3 t}\right] A
\end{aligned}
$$

T4. (c)
$\because$ At at $t<0$; the circuit is behaving as shown in figure,


At $t>0$; now here we can find,


$$
i(\infty)=0
$$

and

Here,

$$
\begin{aligned}
V_{c}(\infty) & =4 V \\
V_{c}(t) & =V_{c}(\infty)+\left[V\left(0^{+}\right)-V(\infty)\right] e^{-t / \tau} \\
\tau & =R_{e q} \cdot C=(6 \| 3) \times 2 \\
& =2 \times 2=4 \mathrm{sec} \\
V_{c}(t) & =4+[10-4] e^{-t / 4}=4+6 e^{-t / 4} \\
i_{c}(t) & =-C \frac{d V}{d t}=-C \times\left[\frac{d\left[4+6 e^{-t / 4}\right]}{d t}\right] \\
& =-2 \times \frac{6}{4} \cdot e^{-t / 4}=-3 e^{-t / 4} \mathrm{~A}
\end{aligned}
$$

T5. (b)
At $t=0$


$$
\begin{aligned}
& i\left(0^{-}\right)=\frac{12}{R_{e q}}=\frac{12}{4}=3 \mathrm{~A} \\
& i\left(0^{+}\right)=i\left(0^{-}\right)=3 \mathrm{~A}
\end{aligned}
$$

At $t=\infty$;
Transform $\Delta$ to $Y$;


$$
\begin{aligned}
i(\infty) & =2 \mathrm{~A} \\
i(t) & =\left[\left(i\left(0^{+}\right)-i(\infty)\right] e^{-\frac{R t}{L}}+i(\infty)\right. \\
i(t) & =(3-2) \mathrm{e}^{-3 t}+2 \\
i(t) & =\left(2+e^{-3 t}\right) \mathrm{A}
\end{aligned}
$$

T6. Sol.

$$
\begin{aligned}
F(s) & =\frac{4 e^{-2 s(s+2)}}{s} \\
\text { Initial value } & =\operatorname{Lim}_{s \rightarrow \infty} s \cdot F(s)
\end{aligned}
$$

$$
f(0)=\operatorname{Lim}_{s \rightarrow \infty} \frac{s \times 4 e^{-2 s}(s+2)}{s}=\operatorname{Lim}_{s \rightarrow \infty} 4 e^{-2 s}(s+2)
$$

$$
=\operatorname{Lim}_{s \rightarrow \infty} \frac{4(s+2)}{\left[1+2 s+\frac{(2 s)^{2}}{2!}+\frac{(2 s)^{3}}{3!}+\ldots . \cdot\right]}
$$

$$
=\operatorname{Lim}_{s \rightarrow \infty} \frac{4 s\left[1+\frac{2}{s}\right]}{s\left[\frac{1}{s}+2+\frac{2^{2} s}{2!}+\frac{2^{3} s^{2}}{3!}+\ldots . .\right]}
$$

$$
=\frac{4[1+0]}{[0+2+\infty+\infty+\ldots \ldots . . \infty]}=\frac{4}{\infty}=0
$$

Final value,

$$
\begin{aligned}
f(\infty) & =\operatorname{Lim}_{s \rightarrow 0} s \cdot F(s)=\operatorname{Lim}_{s \rightarrow 0} \frac{s \times 4 e^{-2 s} \cdot(s+2)}{s} \\
& =4 e^{-2 \times 0}(0+2)=8 e^{-0}=8(1) \\
& =8
\end{aligned}
$$

T7. (d)
The series connected capacitors can be replaced with an equivalent capacitor as shown

$$
\begin{aligned}
v_{0}(0) & =20 V \\
C_{e q} & =\frac{(30)(45)}{30+45}=18 \mu F \\
v_{0}(t) & =v_{0}(0)+\frac{1}{C_{e q}} \int_{0}^{t} i(t) d t
\end{aligned}
$$

-ve sign is taken because current flows from -ve to +ve polarity.

$$
\begin{aligned}
v_{0}(t) & =20-\frac{1}{C_{e q}} \int_{0}^{t} i(t) d t \\
& =20-\frac{1}{18 \times 10^{-6}} \int_{0}^{t} i\left(900 \times 10^{-6}\right) e^{-2.5 t} d t \\
& =20-\frac{900 \times 10^{-6}}{18 \times 10^{-6}}\left[\frac{e^{-2.5 t}}{-2.5}\right]_{0}^{t} \\
& =20+20\left[e^{-2.5 t}-1\right]=20 e^{-2.5 t} \mathrm{~V}
\end{aligned}
$$



## Graph Theory

## Detailed Explanation of Try Yourself Questions

T1. (c)
T2. (d)
Dual of the given network is


■■■■

## Network Theorems



T1. (d)

$$
V_{A B}=50+100=150 \mathrm{~V}
$$

For $R_{A B}$;

$$
R_{A B}=0
$$


then here,

$$
I_{N}=\frac{V_{A B}}{R_{A B}}=\infty
$$

Therefore, norton's equivalent circuit between terminals $A$ and $B$ does not exist.
T2. (a)

$$
Z_{S}=\frac{R(j \omega L)}{R+j \omega L}
$$

To seperate peal and imaginary,

$$
z_{S}=\frac{R(j \omega L)}{R+j \omega L} \times \frac{R-j \omega L}{R-j \omega L}=\frac{R \omega^{2} L^{2}}{R^{2}+\omega^{2} L^{2}}+j \frac{R^{2} \omega L}{R^{2}+\omega^{2} L^{2}}
$$

From maximum power theorems,

$$
\begin{aligned}
Z_{L} & =Z_{S}^{*} \\
R_{1}-j \frac{1}{\omega C} & =\frac{R \omega^{2} L^{2}}{R^{2}+\omega^{2} L^{2}}-j \frac{R^{2} \omega L}{R^{2}+\omega^{2} L^{2}}
\end{aligned}
$$

Compare real and imaginary part on both sides

$$
\begin{aligned}
R_{1} & =\frac{R \omega^{2} L^{2}}{R^{2}+\omega^{2} L^{2}} \\
C & =\frac{R^{2}+\omega^{2} L^{2}}{R^{2} \omega^{2} L}
\end{aligned}
$$

T3. Sol.
Case-1: To find $\left(Z_{t h}\right)$


Case-2: To find ( $\mathrm{V}_{\mathrm{th}}$ )


$$
\begin{aligned}
\frac{V_{t h}-250}{5+j 10}+\frac{V_{\text {th }}}{-j 10} & =10 \angle 90^{\circ} \\
V_{\text {th }} & =(100-j 300) \mathrm{V}
\end{aligned}
$$

From maximum power theorem,

$Z_{L}=Z_{t h}^{*}$

$$
z_{L}=20+j 10
$$

Current,

$$
\begin{aligned}
i & =\frac{\sqrt{100^{2}+300^{2}}}{40} \\
P_{L} & =i^{2} R_{L}=i^{2} \times 20=1250 \mathrm{~W}
\end{aligned}
$$

T4. (c)


$$
\begin{equation*}
V_{A B}=V_{\mathrm{Th}}=50 \mathrm{~K} \times 80 I \tag{i}
\end{equation*}
$$

KVL in loop-1,

$$
\begin{align*}
-2+(8 \times I) \times 10^{3}+0.01 \times V_{A B} & =0 \\
2 & =10^{3}[8 I+0.01 \times 50 \times 80 I] \\
2 & =10^{3}[8 I+40 I] \\
2 & =48 I \times 10^{3} \\
\Rightarrow \quad I & =\frac{2}{48 \mathrm{~K}}  \tag{ii}\\
\text { Now, } \quad V_{\mathrm{Th}} & =50 \mathrm{~K} \times 80 \times \frac{2}{48 \mathrm{~K}}=166.67 \mathrm{~V}
\end{align*}
$$

## T5. (b)

From the given network,


$$
\because \quad \begin{aligned}
I_{2} & =0 \\
Z_{11} & =\frac{V_{1}}{I_{1}}=\frac{20}{10}=2 \Omega \\
Z_{21} & =\frac{V_{2}}{I_{1}}=\frac{5}{10}=0.5 \Omega
\end{aligned}
$$

For a reciprocal network,

$$
Z_{12}=Z_{21}=0.5
$$

$\because \quad$ For the given second network,


$$
\begin{array}{ll}
\because & I_{\mathrm{SC}}=I_{1}, \quad I_{2}=6 \mathrm{~A} \\
\because & \frac{I_{1}}{I_{2}}=\frac{Z_{12}}{Z_{11}}=\frac{V_{1} / I_{2}}{V_{1} / I_{1}}=\frac{0.5}{2}=\frac{1}{4}
\end{array}
$$

$$
\because \quad I_{\mathrm{SC}}=I_{1}=\frac{6}{4}=1.5 \mathrm{~A}
$$

T6. (c)
Combining the parallel resistance and adding the parallel connected current sources.

$$
\begin{aligned}
& 9 A-3 A=6 A \text { (upward) } \\
& 3 \Omega \| 6 \Omega=2 \Omega
\end{aligned}
$$



Source transformation of 6 A source


T7. (d)


For $V_{O C}$ across $a$ and $b$,
$\because \quad I_{x}=0$; as O.C.
Now, circuit is reduced as below,


Only one case is possible, i.e.

$$
\begin{aligned}
V_{O C} & =0 \mathrm{~V} \\
R_{\text {Th }} & =\frac{V_{x}}{I_{x}} \\
V_{x} & =1 \mathrm{~V}
\end{aligned}
$$



KVL in loop-1,

$$
\begin{aligned}
1 & =600\left(I_{x}-0.01\right)-300\left(2 I_{x}+0.01\right)+900 I_{x} \\
1 & =600 I_{x}-6-600 I_{x}-3+900 I_{x} \\
10 & =900 I_{x} \\
\Rightarrow \quad I_{x} & =\frac{1}{90} \\
R_{\mathrm{Th}} & =90 \Omega
\end{aligned}
$$

## Two-Port Networks

## Detailed Explanation <br> of <br> Try Yourself Questions

T1. (a)


For $I_{2}=0$;

$$
\begin{align*}
V_{2} & =\left(2 \times I_{1}+K I_{1}\right) \\
V_{2} & =(2+K) I_{1} \\
\frac{V_{2}}{I_{1}} & =Z_{21}=(2+K) \tag{i}
\end{align*}
$$

For $I_{1}=0$;

$$
\begin{align*}
Z_{12} & =\frac{V_{1}}{I_{2}} \\
V_{1} & =0.5 V_{1}+2 I_{2} \\
0.5 V_{1} & =2 I_{2} \\
Z_{12} & =\frac{V_{1}}{I_{2}}=4 \tag{ii}
\end{align*}
$$

From equation (i) and (ii),
From reciprocal network,
$\because$

$$
\begin{aligned}
Z_{12} & =Z_{21} \\
2+K & =4 \\
K & =2
\end{aligned}
$$

## T2. Sol.

To find $Z_{22}$;

$$
\because \quad z_{22}=\left.\frac{V_{2}}{I_{2}}\right|_{I_{1}=0}
$$

For $I_{1}=0$;
$R_{\text {Th }}$ from port port-2.

$$
\begin{aligned}
& R_{\text {Th }}=1.732 \Omega \\
& Z_{22}=1.732 \Omega
\end{aligned}
$$

## T3. Sol.

$I_{S C}=$ for $V_{2}=0 ;$


$$
\begin{aligned}
{\left[\begin{array}{l}
I_{1} \\
I_{2}
\end{array}\right] } & =\left[\begin{array}{cc}
1 & -1 \\
-1 & 2
\end{array}\right]\left[\begin{array}{l}
V_{1} \\
V_{2}
\end{array}\right] \\
I_{1} & =V_{1}-V_{2} \\
I_{2} & =V_{1}-V_{2}
\end{aligned}
$$

For $V_{2}=0$;
$\because$ For $V_{2}=0$;
then,

Now,

$$
\begin{aligned}
I_{\mathrm{SC}} & =-I_{2}=-V_{1} \\
& =-\left(V_{s}-I_{1} R_{s}\right) \\
-V_{1} & =V_{s}+I_{1} R_{s}
\end{aligned}
$$

T4. Sol.
To find Z-parameters:

we need to get;

$$
Z_{11}=\left.\frac{V_{1}}{I_{1}}\right|_{I_{2}=0} ; \quad Z_{21}=\left.\frac{V_{2}}{I_{1}}\right|_{I_{2}=0}
$$

then the circuit becomes,
and

$$
\begin{aligned}
V_{1} & =10 I_{1} \\
Z_{11} & =\frac{V_{1}}{I_{1}}=10 \\
V_{2} & =V_{1}-3 \times 2 I_{1} \\
V_{2} & =10 I_{1}-6 I_{1} ; \quad Z_{21}=\frac{V_{2}}{I_{1}}=4 \\
Z_{12} & =\left.\frac{V_{2}}{I_{1}}\right|_{I_{1}=0}
\end{aligned}
$$

For
and

$$
Z_{22}=\left.\frac{V_{2}}{I_{2}}\right|_{I_{1}=0}
$$

then the circuit becomes,

$$
Z_{12}=\frac{V_{1}}{I_{2}}=10 ;
$$

and

$$
V_{1}=10 I_{2}
$$

$$
\begin{aligned}
& V_{2}=13 I_{2} \\
& Z_{22}=\frac{V_{2}}{I_{2}}=13
\end{aligned}
$$


then,

$$
[Z]=\left[\begin{array}{cc}
10 & 10 \\
4 & 13
\end{array}\right]
$$

## T5. Sol.

Given,

$$
\begin{align*}
{[h] } & =\left[\begin{array}{cc}
16 \Omega & 3 \\
-2 & 0.015
\end{array}\right]  \tag{i}\\
V_{1} & =16 I_{1}+3 V_{2}  \tag{ii}\\
I_{2} & =-2 I_{1}+0.01 V_{2}
\end{align*}
$$

$$
\therefore \quad V_{1}=16 I_{1}+3 V_{2}
$$

From the given circuit,

$$
I_{2}=-\frac{V_{2}}{25}
$$

KVL in the loop: $\quad-10+4 I_{1}+V_{1}=0$

$$
I_{1}=\frac{10-V_{1}}{4}
$$

Substituting $I_{1}$ and $I_{2}$ in the equation (i) and (ii),


$$
V_{1}=16\left(\frac{10-V_{1}}{4}\right)+3 V_{2}
$$

$$
\begin{aligned}
5 V_{1}-3 V_{2} & =40 \\
\left(-\frac{V_{2}}{25}\right) & =-2\left(\frac{10-V_{1}}{4}\right)+0.01 V_{2} \\
0.5 V_{1}+0.05 V_{2} & =5 \\
V_{1} & =9.71 \mathrm{~V} ; \quad V_{2}=2.8571 \mathrm{~V}
\end{aligned}
$$

The ratio,

$$
\begin{aligned}
\frac{V_{2}}{V_{1}} & =\frac{2.8571}{9.71}=0.294 \\
I_{1} & =\frac{10-V_{1}}{4}=\frac{10-9.71}{4}=0.072 \mathrm{~A} \\
\frac{I_{1}}{V_{1}} & =\frac{0.072}{9.71}=0.0074 \mathrm{~V} \\
\frac{V_{2}}{V_{1}} & =\frac{2.8571}{0.072}=39.682 \Omega \\
\frac{I_{2}}{I_{1}} & =-0.11
\end{aligned}
$$

## T6. Sol.

$$
\left[\begin{array}{ll}
A & B \\
C & D
\end{array}\right]=\left[\begin{array}{cc}
\frac{11}{34} & -10 \\
\frac{1}{34} & -4
\end{array}\right]
$$

T7. (c)

For first transformer,

$$
\begin{aligned}
M & =K \sqrt{L_{1} \cdot L_{2}} \\
M_{1} & =\frac{1}{2} \sqrt{3 \times 3}=\frac{3}{2} H \\
M_{2} & =\frac{2}{3} \sqrt{3 \times 3}=2 \mathrm{H}
\end{aligned}
$$

For second transformer,


By KVL for Loop 1,

$$
\begin{aligned}
V_{1}(s) & =9 s I_{1}(s)-3 s I_{1}(s)+2 s I_{2}(s) \\
V_{1}(s) & =6 s I_{1}(s)+2 s I_{2}(s) \\
V_{1} & =Z_{11} I_{1}+Z_{12} I_{2}
\end{aligned}
$$

We find,
By KVL for Loop 2,

$$
\begin{aligned}
Z_{11} & =6 s \\
Z_{12} & =2 s \\
V_{2}(s) & =2 s I_{1}(s)+3 s I_{2}(s) \\
V_{2} & =Z_{21} I_{1}+Z_{22} I_{2} \\
Z_{21} & =2 s \\
Z_{22} & =3 s
\end{aligned}
$$

We find,

$$
\therefore \quad[Z]=\left[\begin{array}{cc}
6 s & 2 s \\
2 s & 3 s
\end{array}\right]
$$

T8. (c)
From circuit,
and
$\because$
From equation (ii),

From equation (i),
$V_{1}=100 \angle 0^{\circ}$
$V_{2}=-10 I_{2}$
$V_{1}=40 I_{1}+j 20 I_{2}$
$V_{2}=\beta 0 I_{1}+50 I_{2}$
$-10 I_{2}=j 30 I_{1}+50 I_{2}$
$-60 I_{2}=j 30 I_{1} ; \quad I_{2}=-\frac{j}{2} \times I_{1}$
$100=40 I_{1}+j 20 \times\left(-\frac{j}{2}\right) I_{1}$
$100=50 I_{1}$
$\Rightarrow \quad I_{1}=2 \angle 0^{\circ} \mathrm{A}$
then,

$$
I_{2}=-\frac{j}{2} \times 2 \mathrm{~A}=1 \angle-90^{\circ} \mathrm{A}
$$

## Resonance

## Detailed Explanation <br> of <br> Try Yourself Questions

## T1. Sol.

For reactive power from source to zero.
The network is behaving as purely resistive.
$\because$

$$
\begin{aligned}
Y_{e q} & =Y_{1}+Y_{2} \\
Y_{e q} & =\frac{1}{R+X_{L}}+\frac{1}{R+X_{C}} \\
X_{L} & =j \omega L, \quad X_{C}=-j \frac{1}{\omega C} \quad \times \\
Y_{e q} & =\frac{1}{R+j \omega L}+\frac{1}{R-\frac{j}{\omega C}} \\
& =\frac{R-j \omega L}{R^{2}+\omega^{2} L^{2}}+\frac{R+\frac{j}{\omega C}}{R^{2}+\frac{1}{\omega^{2} C^{2}}}
\end{aligned}
$$

$$
=\frac{R}{R^{2}+\omega^{2} L^{2}}+\frac{R}{R^{2}+\left(\frac{1}{\omega C}\right)^{2}}+j\left[\frac{\frac{1}{\omega C}}{R^{2}+\frac{1}{\omega^{2} C^{2}}}-\frac{\omega L}{R^{2}+\omega^{2} L^{2}}\right]
$$

$\because \quad I_{m}\left(Y_{e q}\right)=0$; for resistive network

$$
\frac{\frac{1}{\omega C}}{R^{2}+\frac{1}{\omega^{2} C^{2}}}=\frac{\omega L}{R^{2}+\omega^{2} L^{2}}
$$

$$
\begin{aligned}
\frac{1}{\omega C} \times\left(R^{2}+\omega^{2} L^{2}\right) & =\omega L\left(R^{2}+\frac{1}{\omega^{2} C^{2}}\right) \\
\frac{1}{\omega^{2} L C}\left[R^{2}+\omega^{2} L^{2}\right] & =R^{2}+\frac{1}{\omega^{2} C^{2}} \\
\frac{R^{2}}{\omega^{2} L C}+\frac{L}{C} & =R^{2}+\frac{1}{\omega^{2} C^{2}} \\
R^{2}\left[1-\frac{1}{\omega^{2} L C}\right] & =\frac{L}{C}-\frac{1}{\omega^{2} C^{2}}
\end{aligned}
$$

$$
R^{2}=\frac{\left(\frac{L}{C}-\frac{1}{\omega^{2} C^{2}}\right)}{\left(1-\frac{1}{\omega^{2} L C}\right)}
$$

$$
R=\sqrt{\frac{\frac{L}{C}-\frac{1}{\omega^{2} C^{2}}}{1-\frac{1}{\omega^{2} L C}}}
$$

or,

$$
L=4 \mathrm{H}, \mathrm{C}=1 \mathrm{~F}
$$

$$
R=\sqrt{\frac{4-\frac{1}{\omega^{2}}}{1-\frac{1}{4 \omega^{2}}}}=2 \Omega
$$

Voltage across ' $R$ ' is maximum.


When $V_{C}$ and $V_{L}$ are in phase opposition i.e. at resonance.
$\because$ At resonance:
Total impedance,

$$
Z=R=0.5 \Omega
$$

Current,

$$
I=\frac{10 / \sqrt{2}}{0.5}=\frac{20}{\sqrt{2}}=10 \sqrt{2}=14.142 \mathrm{~A}
$$

$$
\begin{array}{rlrl}
\text { Voltage across capacitor, } & V_{C} & =\frac{I}{\omega_{0} C} \\
\because & \omega_{0} & =\frac{1}{\sqrt{L C}} \\
V_{C} & =\frac{I}{\frac{1}{\sqrt{L C}} \times C}=I \sqrt{\frac{L}{C}} \\
& =10 \sqrt{2} \sqrt{\frac{2}{0.5}}=10 \sqrt{2} \times 2=20 \sqrt{2} \\
& =\frac{40}{\sqrt{2}} \mathrm{~V}
\end{array}
$$

T3. (c)
T4. (d)
$\because$ At resonance,

$$
\begin{aligned}
\omega_{0} & =\sqrt{\frac{1}{L C}-\frac{R^{2}}{L^{2}}} \\
& =\sqrt{\frac{10^{6}}{0.16 \times 625}-\frac{4}{0.16 \times 0.16}}=\sqrt{10^{4}-156.25} \\
& =99.216 \mathrm{rad} / \mathrm{sec} . \\
Z \text { at } \omega=\omega_{0} & =\frac{\omega_{0}^{2} L^{2}}{R}+R=2+\frac{(99.216)^{2} \times(0.16)^{2}}{2} \\
& =2+126=128 \Omega
\end{aligned}
$$

T5. Sol.
(i) At resonance;

$$
Y_{e q}=G_{e q}=\frac{R}{R^{2}+\frac{1}{(\omega C)^{2}}}+\frac{R}{R^{2}+(\omega L)^{2}}
$$

Average power consumed,

$$
P=V_{r m s}^{2} \times G_{e q}=(20 / \sqrt{2})^{2} \times G_{e q}
$$


$\because \omega_{0}($ resonant frequency $)=\frac{1}{\sqrt{L C}}=\frac{1}{\sqrt{4 \times 1}}=0.5 \mathrm{rad} / \mathrm{sec}$.

$$
\begin{aligned}
& G_{e q}=\frac{2}{4+\frac{1}{(0.5 \times 1)^{2}}}+\frac{2}{4+(0.5 \times 4)^{2}} \\
& =\frac{2}{4+4}+\frac{2}{4+4}=0.5 \mathrm{~J} \\
& P=\frac{20^{2}}{2} \times 0.5=100 \mathrm{~W}
\end{aligned}
$$

(ii)

$$
\begin{aligned}
R_{1}=R_{2} & =R=2 \Omega \\
L & =1 \mathrm{H} ; \quad C=1 \mathrm{~F} \\
\omega_{0} & =\frac{1}{\sqrt{1 \times 1}}=1 \mathrm{rad} / \mathrm{sec} . \\
G_{e q} & =\frac{2}{(2)^{2}+(1 \times 1)^{2}}+\frac{2}{(2)^{2}+\left(\frac{1}{1 \times 1}\right)^{2}} \\
& =\frac{2+2}{4+1}=\frac{4}{5} \mho \\
P & =\frac{(20)^{2} \times 4}{2 \times 5}=160 \mathrm{~W}
\end{aligned}
$$



Filters and Magnetic Coupled Circuits


T1. (c)
T2. (c)
In phaser domain,
$L \Rightarrow j \omega L$
$C \Rightarrow \frac{1}{j \omega C}=\frac{1}{j\left(10^{5}\right)\left(62.5 \times 10^{-12}\right)}=-j 160 \mathrm{k} \Omega$


Load impedance,

$$
Z_{L}=400+j \omega L
$$

Reflecting the secondary impedance to the primary side


For maximum power transfer

$$
\begin{aligned}
Z_{L} / n^{2} & =Z_{s}^{*} \\
\frac{Z_{L}}{n^{2}} & =\left(100-j 160 \times 10^{3}\right) \quad Z_{s}=\left(100-j 160 \times 10^{3}\right) \Omega \\
\frac{400+j \omega L}{n^{2}} & =100+j 160 \times 10^{3}
\end{aligned}
$$

Comparing real parts on both sides of the equation,

$$
\frac{400}{n^{2}}=100 \Rightarrow n=2
$$

Comparing imaginary parts,

$$
\begin{aligned}
\frac{\omega L}{n^{2}} & =160 \times 10^{3} \\
\Rightarrow \quad & L \\
= & =\frac{160 \times 10^{3}}{10^{5}} \times 4=6.4 \mathrm{H}
\end{aligned}
$$

