GATE

MADE EASY
Workbook 2027

Detailed Explanations of Try Yourself *Questions*

Instrumentation Engineering
Control Systems and Process Control

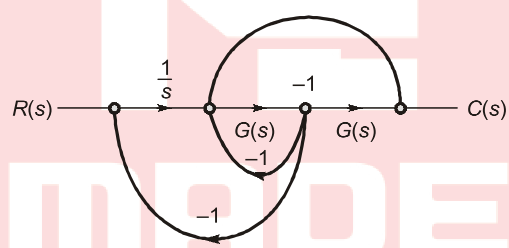
MADE EASY PUBLICATIONS

1 Mathematical Models and Block Diagram

Detailed Explanation of Try Yourself Questions

T1. (b)

Drawing SFG of the above



Here, $P_1 = \dfrac{G^2(s)}{s}$; $L_2 = -G(s)$; $L_1 = -G^2(s)$; $L_3 = -\dfrac{G(s)}{s}$

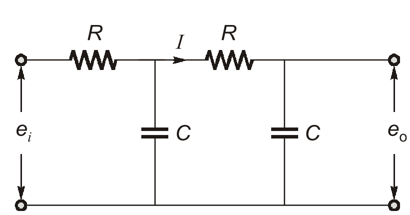
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G^2(s)}{s}}{1-\left(-G^2(s)-\dfrac{G(s)}{s}-G(s)\right)}$$

$$= \frac{G^2(s)}{s+sG^2(s)+G(s)+sG(s)}$$

Put $G(s) = s$,

$$\frac{C(s)}{R(s)} = \frac{s^2}{s+s^3+s+s^2} = \frac{s^2}{s^3+s^2+2s} = \frac{s}{s^2+s+2}$$

T2. Sol.



$$E_o(s) = \frac{1}{sC} I(s) \qquad \text{...(i)}$$

$$I(s) = \frac{E_i(s)}{\left[R + \dfrac{\left(R+\dfrac{1}{sC}\right)\times\dfrac{1}{sC}}{\left(R+\dfrac{1}{sC}+\dfrac{1}{sC}\right)}\right]} \times \frac{\dfrac{1}{sC}}{\left(R+\dfrac{1}{sC}+\dfrac{1}{sC}\right)}$$

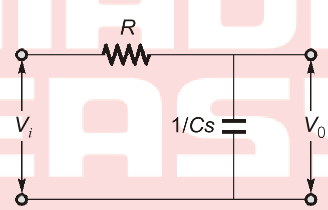
(Using current division rule)

$$= \frac{E_i(s)}{\dfrac{R+\dfrac{1}{sC}}{sCR+2}+R} \times \frac{1}{sCR+2} = \frac{E_i(s)}{\left(R+\dfrac{1}{sC}\right)+R(sCR+2)}$$

$$E_0(s) = \frac{\dfrac{1}{sC}\times E_i(s)}{\dfrac{(1+RSC)+SCR(SCR+2)}{SC}}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{S^2C^2R^2+3\,SCR+1} = \frac{1}{S^2T^2+3ST+1}$$

T3. (b)



$$\frac{V_0}{V_i} = \frac{1/Cs}{R+\dfrac{1}{Cs}} = \frac{1}{RCs+1}$$

■■■■

2 Time Response Analysis

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$G(s) = \frac{K}{s(s+p)}$$

Now, the closed loop system

$$T(s) = \frac{K}{s^2 + sp + K}$$

∴ Comparing it with standard equation

$$K = \omega_n^2$$

$$2\xi\omega_n = p$$

$$t_s = \frac{4}{\xi\omega_n}$$

⇒ $$\xi\omega_n = 1$$

∴ $$p = 2$$

now, $$e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.1$$

$$\xi = 0.537$$

∴ $$\omega_n = \frac{p}{2\xi} = 1.69$$

∴ $$K = \omega_n^2 = 2.85$$

T2. Sol.

Taking Laplace transform we get

$$X(s) = \frac{1}{(s^2 + 6s + 5)} . 12\left[\frac{1}{s} - \frac{1}{(s+2)}\right]$$

$$X(s) = \frac{12}{s(s+5)(s+1)} .12\left[\frac{1}{s} - \frac{1}{(s+2)}\right]$$

Now, using final value theorem

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$\therefore \quad X(s) = \lim_{s\to 0}\left[\frac{12}{(s+5)(s+1)} - \frac{12s}{(s+5)(s+1)(s+3)}\right]$$

$$= \frac{12}{5} = 2.4$$

T3. Sol.

To find the impulse response let us difference the response.

$$c'(t) = 12\,e^{-10t} - 12e^{-60t}$$

tanking inverse laplace transform we get

$$C'(s) = \frac{600}{(s+10)(s+60)}$$

$$C'(s) = \frac{600}{s^2 + 70s + 600}$$

$\therefore$ $c'(s)$ is the impulse response thus comparing it with the standard equation.

$$2\xi\omega_n = 70$$

$$\omega_n = \sqrt{600}$$

$$\therefore \quad \xi = 1.428 \approx 1.43$$

T4. Sol.

Since real port of the given second order equation is at –0.602 thus they can be considered as dominant poles.

Thus $$t_p = \frac{\pi}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_n = \sqrt{2.829} = 1.6819$$

and $$2\xi\omega_n = 1.204$$

$$\xi = \frac{1.204}{2\times 1.6819} = 0.3579$$

$$\therefore \quad \omega_d = 1.6819\sqrt{1-\xi^2}$$

$$\omega_d = 1.577$$

$$\therefore \quad t_p = 1.999 \approx 2$$

■■■■

3 Stability

Detailed Explanation of Try Yourself Questions

T1. (d)

T2. (d)

The correct sequence of steps needed to improve system stability is to use negative feedback, reduce gain and insert deviation action.

T3. Sol.

$$1 + G(s) = 0$$

$$\Rightarrow \quad s\tau_1\left[1 + s(\tau_1 + \tau_2) + s^2\tau_1\tau_2\right] + K = 0$$

$$s^3\tau_1^2\tau_2 + s^2\tau_1(\tau_1 + \tau_2) + s\tau_1 + K = 0$$

Using R-H criteria

s^3	$\tau_1^2\tau_2$	τ_1
s^2	$\tau_1(\tau_1+\tau_2)$	K
s^1	$\dfrac{\left[\tau_1^2(\tau_1+\tau_2) - K\tau_1^2\tau_2\right]}{\tau_1(\tau_1+\tau_2)}$	
s^0	K	

$$\Rightarrow \quad K > 0\ ;\ \tau_1 > 0\ ;\ \tau_2 > 0$$

Also,

$$\frac{\tau_1(\tau_1+\tau_2) - K\tau_2}{(\tau_1+\tau_2)} > 0$$

$$K\tau_2\tau_1 < \tau_1(\tau_1+\tau_2)$$

$$\Rightarrow \quad K < \left(1 + \frac{\tau_1}{\tau_2}\right)$$

$0 < K < \left(1 + \dfrac{\tau_1}{\tau_2}\right)$; [$\tau_1 > 0$ and $\tau_2 > 0$ and this is the only possible case.]

4 Root Locus Technique

Detailed Explanation of Try Yourself Questions

T1. Sol.

Characteristic equation is given as

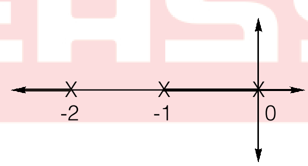
$$1 + G(s)\,H(s) = 0$$

On comparing this characteristic equation with the equation given in problem, we have

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)}$$

P = Number of open loop poles = 3 = number of branches on root locus

Z = 0 = Number of branches terminating at zeros.



Angle of Asymptotes: The $P - Z$ branches terminating at infinity will go along certain straight lines.

$$\text{Number of asymptotes} = P - Z = 3 - 0 = 3$$

$$\theta = \frac{180°(2q+1)}{P-Z} \qquad q = 0.\ 1,\ 2\ ...$$

$$\theta_1 = \frac{180 \times (2 \times 0 + 1)}{3} = 60°$$

$$\theta_2 = \frac{180°(2 \times 1 + 1)}{3} = 180°$$

$$\theta_3 = \frac{180°(2 \times 2 + 1)}{3} = 300°$$

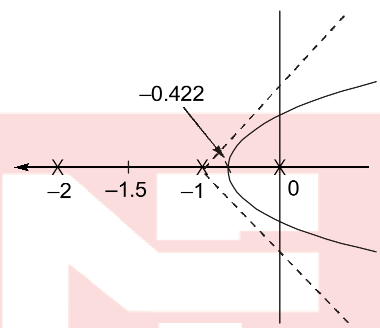
Centroid: It is the interpection point of the asymptotes on the real axis. It may or may not be a part of root locus.

$$\text{Centroid} = \frac{\Sigma \text{ Real part of open loop poles } - \Sigma \text{ Real part of open loop zeros}}{P - Z}$$

$$= \frac{0-1-2}{3} = -1$$

Centroid $\rightarrow (-1, 0)$

Break-away or break-in points: These are those points on whose multiple roots of the characteristic equation occur.



$$s(s^2 + 3s + 2) + K = 0$$

$$K = -(s^3 + 3s^2 + 2s)$$

$$\frac{dK}{ds} = -(3s^2 + 6s + 2) = 0$$

$$s = -0.422, -1.577$$

Now verify the valid break-away point

$K = 0.234$ (valid) at $s = -0.422$

$K =$ negative (not valid) at $s = -1.577$

T2. Sol.

$$OLTF = \frac{K}{s(s^2 + 4s + 8)}$$

Poles are at $s_1 = 0$

and $s_{2,\,3} = \dfrac{-4 \pm \sqrt{16 - 32}}{2} = -2 \pm j2$

There are 3 poles and no zero with root loci, all terminating at infinity.

$$\phi = \frac{(2q+1)180°}{P-Z} = 60°, 180°, 300° \quad \text{for } q = 0, 1, 2$$ (angle of asymptotes)

$$\text{Centroid} = \frac{0 - 2 + j2 - 2 - j2}{3} = \frac{-4}{3} = -1.33$$

$$1+\frac{K}{s(s^2+4s+8)} = 0$$

$$\Rightarrow \quad K = -s(s^2+4s+8) = -(s^3+4s^2+8s)$$

for break away points, $\frac{dK}{ds} = 0$

$$\Rightarrow \quad \frac{dK}{ds} = -(3s^2+8s+8)$$

$$s_{1,\,2} = -\frac{8\pm\sqrt{64-4\times8\times3}}{2\times3}$$

$$= -\frac{8\pm\sqrt{64-96}}{6} = -1.33\pm j0.943$$

As $\frac{dK}{ds}$ is imaginary, there is no breakaway point from the real axis.

Imaginary axis crossing:

$$\text{Characteristic equation} = s(s^2+4s+8)+K$$
$$= s^3+4s^2+8s+K = 0$$

s^3	1	8
s^2	4	K
s^1	$\frac{32-K}{4}$	
s^0	K	

From Routh-Hurwitz Criteria:

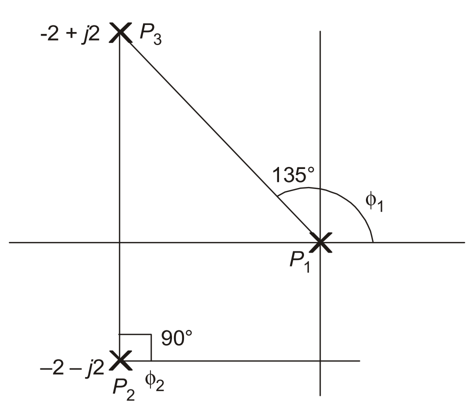
For $K = 32$, the system is marginally stable and beyond $K = 32$ the system becomes unstable.

Hence, $4s^2 + K = 4s^2 + 32$

$$s = j2\sqrt{2} = j\omega$$

$$\omega = 2\sqrt{2} = 2.83$$

The root locus cuts the imaginary axis at $\pm j2.83$.



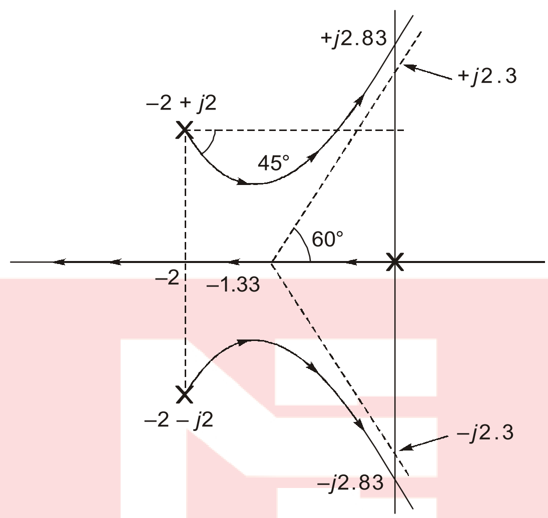
$$\phi_1 = 180° - \tan^{-1}\left(\frac{2}{2}\right) = 135°$$

$$\phi_2 = 90°$$
$$\Sigma\phi_p = \phi_1 + \phi_2 = 135° + 90° = 225°$$
$$\phi = \Sigma\phi_2 - \Sigma\phi_p = 0 - 225° = -225°$$

Angle of departure,

$$\phi_D = 180 + \phi = 180 - 225° = -45°$$

Root locus of the given system:



T3. Sol.

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+4)}$$

Step-1 Number of open loop poles ;

$$P = 3$$

Number of open loop zeros ; $Z = 0$

Number of branches terminating at infinity

$$= P - Z = 3$$

Step-2 Angle of asymptotes

$$\theta = \frac{(2q+1)180°}{P-Z} \quad \text{where } q = 0, 1, 2$$

$$\theta_1 = \frac{180°}{3} = 60°$$

$$\theta_2 = \frac{3 \times 180°}{3} = 180°$$

$$\theta_3 = \frac{5 \times 180°}{3} = 300°$$

Step-3

$$\text{Centroid} = \frac{\Sigma \text{ real part of open loop poles} - \Sigma \text{ real part of open loop zeros}}{P-Z}$$

$$= \frac{(-1-4)-(0)}{3-0} = -\frac{5}{3}$$

Step-4 Break away point

$$K + s(s^2 + 5s + 4) = 0$$

$$K = -s^3 - 5s^2 - 4s$$

$$\frac{dK}{ds} = -3s^2 - 10s - 4 = 0$$

$$3s^2 + 10s + 4 = 0$$

$$\Rightarrow \quad s_1, s_2 = -0.4648, -2.8685$$

Valid break-away point will be –0.4648

(i) Routh array table

s^3	1	4
s^2	5	K
s^1	$\frac{20-K}{5}$	0
s^0	1	

For system to be stable

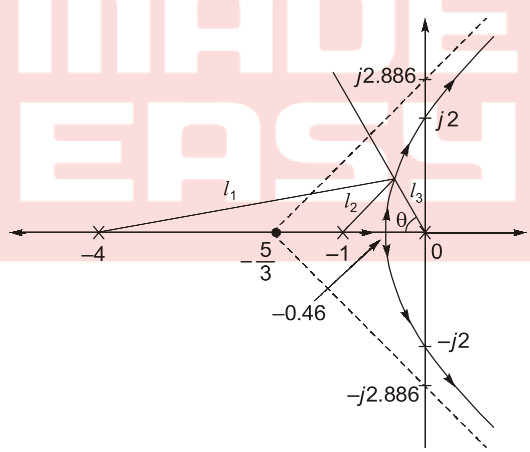
$$20 - K > 0 \quad \Rightarrow \quad K < 20$$

For system to be marginally stable.

$$K = 20$$

$$A(s) = 5s^2 + 20 = 0$$

$$\Rightarrow \quad s = \pm 2j$$



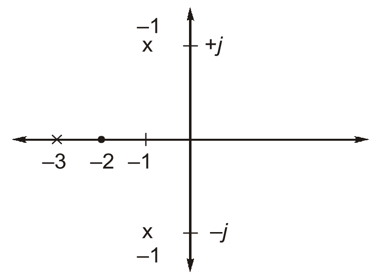
(ii) To find K,

$$\theta = \cos^{-1}\xi = \cos^{-1}(0.34) = 70.123°$$

$$K = l_1\, l_2\, l_3$$

$$\text{Gain margin } (GM) = \frac{\text{K(Marginal stability)}}{\text{K(desired)}}$$

T4. Sol.



$$\phi_d = 180 - (\phi_p - \phi_z)$$

$$= 180° - \left(180 + \tan^{-1}\left(\frac{1}{2}\right) + 90° - 225°\right)$$

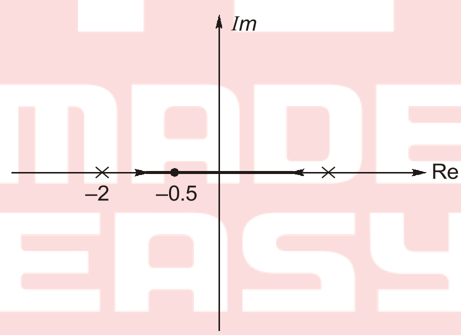
$$= 108.4°$$

T5. Sol.

Given that

$$G(s) = \frac{K}{(s+2)(s-1)}$$

Using root locus method, the break point can be



obtain as

$\Rightarrow$ $$1 + G(s) = 0$$

$$1 + \frac{K}{(s+2)(s-1)} = 0$$

or $$K = -(s+2)(s-1)$$

$$\frac{dK}{ds} = -2s - 1 = 0$$

or $$s = -0.5$$

To have, both the poles at the same directions

$$|G(s)|_{s=0.5} = 1$$

$$K = 2.25$$

■■■■

5 Frequency Response Analysis

Detailed Explanation of Try Yourself Questions

T1. Sol.

Given,
$$G(s) = \frac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$

$(1 + as)$ is addition of zero to the transfer function whose contribution in slope = +20 dB/decade or –6 dB/octave.

$(1 + bs)$ is addition of pole to the transfer function whose contribution in slope = –20 dB/decade or –6 dB/octave

Observing the change in the slope at different corner frequencies, we conclude that

$$a = \frac{1}{4}\text{ rad/s and } b = \frac{1}{24}\text{ rad/s}$$

From $\omega = 0.01$ rad/s to $\omega = 8$ rad/s,

slope = –20 dB/decade

Let the vertical length in dB be y

$$\therefore \quad -20 = \left(\frac{0-y}{\log 8 - \log 0.01}\right)$$

or, $$-20 = \frac{y}{\log 8 + 2}$$

or, $y = 58$ dB

Applying $y = mx + C$ at $\omega = 0.01$ rad/s,

we have: $58 = -20 \log 0.01 + C$

or, $C = 58 - 40 = 18$

Now, $C = 20 \log K$

or, $$\log K = \frac{18}{20} = 0.9$$

$$\therefore \quad K = \log^{-1}(0.9) = (10)^{0.9} = 7.94$$

$$\therefore \quad \frac{a}{bK} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{24}\right)\times 7.94}$$

$$= \frac{24}{4\times 7.94} = 0.755$$

$$\therefore \quad \frac{a}{bK} = 0.755$$

T2. Sol.

$$\text{OLTF} = G(s) = \frac{1}{(s+2)^2}$$

For unity feedback system,

$$H(s) = 1$$

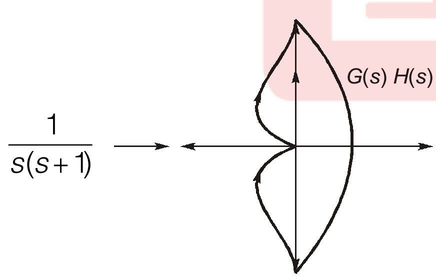
$$\therefore \quad \text{CLTF} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{(s+2)^2}}{1+\frac{1}{(s+2)^2}}$$

$$= \frac{1}{s^2+4s+5}$$

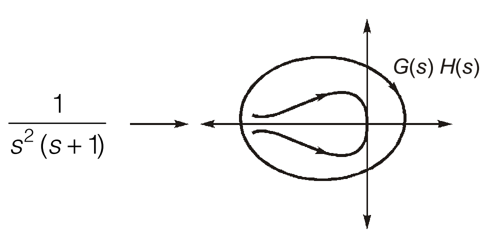
$\therefore$ Close loop poles will be the roots of $s^2 + 4s + 5 = 0$

i.e. $s = -2 + j$ and $-2 - j$

T3. (b)



After adding pole at origin



So, nyquist plot of a system will rotate by 90° in clockwise direction.

T4. Sol.

For gain margin we have to find

$$G(s)H(s) = \frac{0.75}{s(1+s)(1+0.5s)}$$

Phase over freqeuncy

$$-180° = -90° - \tan^{-1}(\omega) - \tan^{-1}(0.5 - \omega)$$
$$-90° = \tan^{-1}(\omega) + \tan^{-1}(0.5\,\omega)$$
$$\frac{1.5\omega}{1-0.5\omega^2} = \tan(90°)$$
$$0.5\,\omega^2 = 1$$
$$\omega = \sqrt{2}$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \frac{0.75}{\omega\sqrt{1+\omega^2}\sqrt{1+0.25\omega^2}} = \frac{0.75}{\sqrt{2}\sqrt{1+2}\sqrt{1+0.5}} = \frac{1}{4}$$

$$\therefore \quad \text{Gain margin} = 20\log\frac{1}{|G(j\omega)H(j\omega)|}$$
$$= 20\log 4 = 12 \text{ dB}$$

T5. Sol.

$$-90° - \tan^{-1}(2\omega) - \tan^{-1}(3\omega) = -180°$$
$$\tan^{-1}(2\omega) + \tan^{-1}(3\omega) = 90°$$
$$\frac{5\omega}{1-6\omega^2} = \tan(90°)$$
$$\therefore \quad 1 - 6\,\omega^2 = 0$$
$$\omega = \frac{1}{\sqrt{6}} = 0.41$$

T6. Sol.

The Bode plot is of type zero system
thus steady state error

$$e_{ss} = \frac{1}{1+K_p}$$

Where K_p = propational error constant

K_p = 40 db

or K_p = 100

$$\therefore \quad e_{ss} = \frac{1}{1+100} = \frac{1}{101} = 0.009$$

■■■■

6 Controllers and Compensators

Detailed Explanation of Try Yourself Questions

T1. Sol.

The given compensator represents phase lead compensator having maximum phase

$$\phi = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

Here,
$$\alpha = \frac{R_2}{R_1 + R_2} = \frac{(1/2)\,\Omega}{1 + (1/2)\,\Omega} = 0.333$$

∴
$$\phi = \sin^{-1}\left(\frac{1-0.333}{1+0.333}\right) = \sin^{-1}\left(\frac{0.667}{1.33}\right) = \sin^{-1}(0.5) = 30°$$

T2. (a)

The effect of addition of a zero to a transfer function is providing a phase lead.

T3. Sol.

$$G(s) = \frac{\left(1+\frac{s}{4}\right)}{\left(1+\frac{s}{25}\right)}$$

Comparing it with the standard transfer function of phase lead compensator

$$G(s) = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

$$T = \frac{1}{4}, \quad \alpha T = \frac{1}{25}$$

Now, frequency ω_m occurs at
$$= \sqrt{\frac{1}{\alpha T}\cdot\frac{1}{T}} = \sqrt{25\times 4} = 10 \text{ rad/sec.}$$

■■■■

7 State Space Analysis

Detailed Explanation of Try Yourself Questions

T1. (b)

T2. Sol.

Given
$$\dot{x}(t) = Ax(t), \quad x(0) = x_0$$

Taking the Laplace transform
$$sX(s) - x(0) = AX(s)$$
$$[sI - A]\,X(s) = x(0)$$
$$X(s) = [sI - A]^{-1}\,x(0)$$
$$x(t) = \mathcal{L}^{-1}[sI - A]^{-1}\,x(0) \qquad \text{...(i)}$$

Conditions given are

For
$$x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$$

For
$$x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Using the linearity property in equation (i)
$$K_1\,x_1(t) = \mathcal{L}^{-1}[sI - A]^{-1}\,x_1(0)\,K_1$$
$$K_2\,x_2(t) = \mathcal{L}^{-1}[sI - A]^{-1}\,x_2(0)\,K_2$$

Using the linearity property as
$$K_1\,x_1(t) + K_2\,x_2(t) = \mathcal{L}^{-1}[sI - A]^{-1}$$
$$[K_1x_1(0) + K_2x_2(0)] \qquad \text{...(ii)}$$

Also
$$X_3(s) = [sI - A]^{-1}\,x_3(0)$$

So,
$$K_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + K_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} K_1 + 0K_2 \\ -K_1 + K_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad K_1 = 3$$

$$K_2 = 8$$

So, from equation (ii), we get $x(t)$

$$x(t) = K_1 x_1(t) + K_2 x_2(t)$$

$$= 3\begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + 8\begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix}$$

T3. Sol.

Given

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$[sI - A] = s^2$$

$$\phi(t) = \mathcal{L}^{-1}[sI - A]^{-1}$$

$$= \frac{1}{s^2}\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

T4. (b)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}u$$

$$y = [1 \quad 1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \; C = [1 \;\; 1 \;\; 1]$$

Check for controllability:

$$Q_c = [B \; : \; AB \; : \; A^2B]$$

$$= \begin{bmatrix} 0 & 4 & -8 \\ 4 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

For controllable

$$|Q_c| \neq 0$$

Here, $|Q_c| = 4(0) = 0 \;\; \therefore$ Uncontrollable.

Check for observability:

$$Q_o = [C^T \; : \; A^T C^T \; : \; A^{2T} C^T]$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 4 \end{bmatrix}$$

For observable

$$|Q_o| \neq 0$$

Here $|Q_o| = 1 \;\; \therefore$ Observable.

T5. Sol.

$$\text{Characterstic equation} = \left|(sI - A)^{-1}\right|$$

$$= \begin{bmatrix} 2 & -1 \\ 3 & s+5 \end{bmatrix}^{-1}$$

$$= s(s+5) + 3$$

$$= s^2 + 5s + 3$$

■■■■

8 Process Control

Detailed Explanation of Try Yourself Questions

T1. Sol.

The open loop transfer function for the primary loop is given by

$$G(s)_{\text{primary}} = K_2 \frac{1}{(s+1)} \cdot \frac{8}{(s+2)(s+4)}$$

Phase cross-over frequency for primary loop is given by

$$-\tan^{-1}\omega_{pc} - \tan^{-1}\frac{\omega_{pc}}{2} - \tan^{-1}\frac{\omega_{pc}}{4} = -180^\circ$$

$$\tan^{-1}\omega_{pc} + \tan^{-1}\frac{\omega_{pc}}{2} + \tan^{-1}\frac{\omega_{pc}}{4} = 180^\circ$$

$$\tan^{-1}\omega_{pc} + \tan^{-1}\left[\frac{\frac{\omega_{pc}}{2}+\frac{\omega_{pc}}{4}}{1-\frac{\omega_{pc}^2}{8}}\right] = 180^\circ$$

$$\tan^{-1}\omega_{pc} + \tan^{-1}\left[\frac{6\omega_{pc}}{8-\omega_{pc}^2}\right] = 180^\circ$$

$$\tan^{-1}\left[\frac{\omega_{pc}+\frac{6\omega_{pc}}{8-\omega_{pc}^2}}{1-\frac{6\omega_{pc}^2}{8-\omega_{pc}^2}}\right] = 180^\circ$$

$$\omega_{pc} + \frac{6\omega_{pc}}{8-\omega_{pc}^2} = 0$$

$$1+\frac{6}{8-\omega_{pc}^2} = 0$$

$$\omega_{pc} = \sqrt{14}$$

We know that magnitude = 1, at ω_{pc} (By polar plot)

$$M = K_2 \frac{1}{\sqrt{1+\omega^2}} \frac{1}{\sqrt{\omega^2+4}} \frac{8}{\sqrt{\omega^2+16}}$$

$$1 = K_2 \frac{1}{\sqrt{1+\omega_{pc}^2}} \frac{1}{\sqrt{\omega_{pc}^2+4}} \frac{8}{\sqrt{\omega_{pc}^2+16}}$$

$$1 = K_2 \frac{1}{\sqrt{15}} \frac{1}{\sqrt{18}} \frac{8}{\sqrt{30}} = \frac{8K_2}{\sqrt{8100}}$$

$$K_2 = \frac{90}{8} = 11.25$$

The open loop transfer function for the secondary loop is given by

$$G_{secondary} = K_1 \frac{1}{s+1}$$

Cross-over frequency for secondary loop

$$-\tan^{-1}\omega_{pc} = -180°$$

$$\omega_{pc} = 0$$

Since there is no cross-over frequency for the secondary loop, so we can use any value of gain K_1 for secondary loop.

∴ Upper limit of $K_1 = \infty$

Upper limit of $K_2 = 11.25$

T2. Sol.

$$[D(s)\,G_{ff}(s) - Y(s)]\left[\frac{1}{s(s+1)}\right] + D(s) = Y(s)$$

$$D(s)\,G_{ff}(s) + (s^2+s)\,D(s) = (s^2+s+1)\,Y(s)$$

$$\frac{Y(s)}{D(s)} = H(s) = \frac{G_{FF}(s)+s^2+s}{s^2+s+1}$$

∵ $G_{ff}(s) = 1 + s$ [*P-D* controller]

∴ $$H(s) = \frac{(s^2+2s+1)}{(s^2+s+1)}$$

$$|H(j\omega)|_{\omega=2} = \frac{5}{\sqrt{9+4}} = \frac{5}{\sqrt{13}}$$

T3. Sol.

Using the value of K_u and P_u, Ziegler and Nichols recommended the following settings for feedback controllers

	K_C	$\tau_{I(\min)}$	$\tau_{D(\min)}$
P	$K_u/2$	–	–
P-1	$K_u/2.2$	$P_u/1.2$	–
P-1-*D*	$K_u/1.7$	$P_u/2$	$P_u/8$

Then, the Ziegeer-Nichols setting for the proportional controller is

$$\frac{K_u}{2} = \frac{10}{2} = 5$$

T4. Sol.

Open loop transfer function is given by

$$G(s) = \left[K_p + \frac{K_I}{s}\right]\frac{1}{(s+2)(s+10)} = \left[\frac{K_p s + K_I}{s}\right]\frac{1}{(s+2)(s+10)}$$

Ch: equation is given by

$$1 + G(s) = 0$$
$$s(s+2)(s+10) + K_p s + K_I = 0$$
$$s^3 + 12s^2 + (20 + K_p)\,s + K_I = 0$$

By R-H critieria

$$\begin{array}{c|cc} s^3 & 1 & 20 + K_p \\ s^2 & 12 & K_I \\ s^1 & \dfrac{12(20+K_p) - K_I}{12} & 0 \\ s^0 & K_I & \end{array}$$

For stable system, $\boxed{K_I > 0}$

$$12(20 + K_p) \geq 0$$
$$240 + 20K_p - K_I \geq 0$$
$$1240 + 20\,K_p \geq K_I$$
$$K_p \geq \frac{K_I - 240}{20}$$