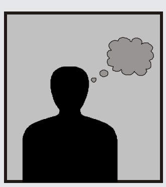
GATE

MADE EASY Workbook 2027



Detailed Explanations of Try Yourself *Questions*

Instrumentation Engineering

Communication

MADE EASY PUBLICATIONS

1 Amplitude Modulation

Detailed Explanation of Try Yourself Questions

T1. Sol.

The signal $$s(t) = A_C[1 + \mu \cos(\omega_m t)]\cos(\omega_c t)$$

The signal can be represented as $$s(t) = \text{Re}\left[A_C e^{j\omega_c t} + \frac{A_C\mu}{2}\left(e^{j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t}\right)\right]$$

$$s(t)|_{\text{complex}} = \left[A_C e^{j\omega_c t} + \frac{A_C\mu}{2}\left(e^{j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t}\right)\right]$$

$$s(t)|_c = \left[s(t)_{ce}\, e^{-j\omega_c t}\right]$$

(where, $s(t)|_c$ = the complex signal $s(t)$ and $s(t)|_{ce}$ = the complex low pass equal of the signal $s(t)$)

$$\therefore \quad s(t)|_{ce} = A_C + \frac{A_C\mu}{2}[\cos\omega_m + j\sin\omega_m t] + \frac{A_C\mu}{2}[\cos\omega_m - j\sin\omega_m t]$$

Putting the conditions given in the questions we get:

$$s(t)|_{ce} = 1 + \frac{1}{8}[\cos\omega_m + j\sin\omega_m t] + \frac{1}{4}[\cos\omega_m - j\sin\omega_m t]$$

$$s(t)|_{ce} = 1 + \frac{3}{8}\cos\omega_m t - j\frac{1}{8}\sin(\omega_m t)$$

$$\therefore \quad \text{A envelop} = \left[\left(1 + \frac{3}{8}\cos(\omega_m t)\right)^2 + \left(\frac{1}{8}\sin(\omega_m t)\right)^2\right]^{1/2}$$

T2. Sol.

Expression for AM signal

$$V_{AM}(t) = A_C\cos\omega_c t + A_C m_a \cos(\omega_c + \omega_m)t + A_C m_a \cos(\omega_c - \omega_m)t$$

$$\because \quad P_C = 100 = \frac{A_C^2}{2}$$

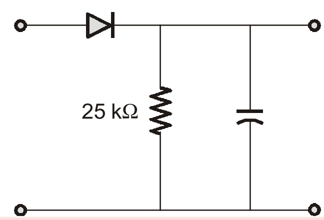
$$\therefore \quad A_C = 14.14 \text{ V}$$

Also $$\eta = \frac{m_a^2}{2+m_a^2} = 40\%$$

or $$0.8 + 0.4\, m_a^2 = m_a^2$$
$$m_a = 1.154$$

∴ $$B = A_C m_a/2 = 8.16$$

T3. Sol.



$$RC \le \frac{1}{\omega_n}\frac{\sqrt{1-\mu^2}}{\mu}$$
$$C \le \frac{1}{R\omega_n}\frac{\sqrt{1-\mu^2}}{\mu}$$
$$C \le \frac{1}{10^4 \times 2\pi \times 25 \times 10^3}\cdot\frac{\sqrt{1-(0.5)^2}}{0.5}$$
$$C \le 1.1 \text{ nF}$$

T4. (c)

$$x(t) = m(t) + \cos\omega_c t$$
$$y(t) = 4(m(t) + \cos\omega_c t) + 10\left[m^2(t) + \cos^2\omega_c t + 2m(t)\cos\omega_c t\right]$$
$$= 4m(t) + 10m^2(t) + 4\cos\omega_c t + \frac{10}{2} + \frac{10}{2}\cos 2\omega_c t + 20\,m(t)\cos\omega_c t$$

after passing through filter

$$y(t) = 4\cos\omega_c t + 20\,m(t)\cos\omega_c t$$
$$= 4[1 + 5\,m(t)]\cos\omega_c t$$
$$\mu = 5 \times M$$
$$0.8 = 5 \times M$$
$$M = \frac{0.8}{5} = 0.16$$

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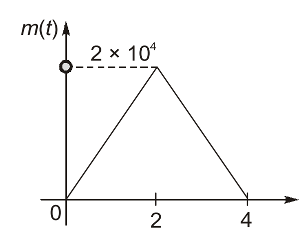
2 Angle Modulation

Detailed Explanation of Try Yourself Questions

T1. Sol.



Maximum instantenious frequency

$$f_i = f_c + \frac{K_p}{2\pi}\dot{m}(t)$$

$$115.95 \times 10^3 = \frac{10^5}{2\pi} + \left(\frac{K_p}{2\pi}\right) \times 10^4$$

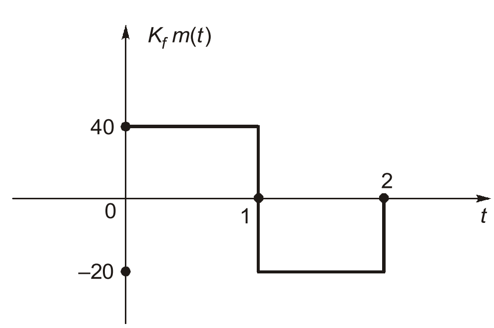
$$10^5 = \left(\frac{K_p}{2\pi}\right) \times 10^4$$

$$10 = \left(\frac{K_p}{2\pi}\right)$$

$$K_p = 2\pi \times 10 \text{ Hrtz/Volt}$$

$$K_p = 10 \text{ rad/volt}$$

T2. Sol.



$$s(t) = 10 \cos [2\pi \times 10^6 t + 20\pi[4r(t) - 6r(t-1) + 2r(t-2)]]$$

Standard FM expression is given by:

$$s(t) = A_c \cos\left[2\pi fct + 2\pi k_f \int m(t)dt\right]$$

$$2\pi k_f \int m(t)dt = 20\pi\,(4r(t) - 6r(t-1) + 2r(t-2))$$

$$k_f\, m(t) = 10[4\,u(t) - 6u\,(t-1) + 2r\,(t-2)]$$

$$\Delta f = \max|\,k_f m(t)\,| = 40 \text{ Hz}$$

T3. Sol.

Maximum frequency deviation

$$\Delta f_{max} = \frac{K_p}{2\pi}\left|\frac{d}{dt}m(\text{t})\right|_{max} = \frac{K_p}{2\pi}2t\,e^{-t^2}$$

$$= \frac{8000}{2\pi}.2.\frac{1}{\sqrt{2}}.e^{-1/2} \qquad \left(\because \max 2 + e^{-t^2} \text{ is at } t = \frac{1}{\sqrt{2}}\right)$$

$$= 3.43 \text{ kHz}$$

T4. Sol.

Compairing the equation with the standard equation.

$$s(t) = A\cos[\omega_c t + k_p\, m(t)]$$

$$\therefore \quad k_p\, m(t) = 0.1\sin(10^3\pi t)$$

$$m(t) = \frac{0.1}{k_p}\sin(10^3\pi t)$$

$$= 0.01\sin(10^3\pi t)$$

Similarly

$$s(t) = A\cos\left[\omega_c t + K_f \int m(t)dt\right]$$

$$K_f \int m(t)\,dt = 0.1\sin(10^3\pi t)$$

$$\int m(t)dt = \frac{0.1}{10\pi}\sin(10^3\pi t) = \frac{0.1\times 10^3\pi}{10\pi}\cos(10^3\pi t) = 10\cos(10^3\pi t)$$

T5. Sol.

$A_m = 5$ V, $f_m = 100$ Hz } $\Delta f = k_f A_m = 1$ kHz
$A_m = 10$ V, $f_m = 50$ Hz } $\Delta f = 2$ kHz
To get $\Delta f = 30$ kHz
frequency multiplication factor should be 15.

T6. (a)

$$BW = 2[\beta + 1]f_m$$

$$\beta = k_p A_m = 5$$

A_m is doubled $\Rightarrow \beta = 10\,; f_m = \frac{1}{2}$ kHz

$$BW = 2[10+1]\cdot\frac{1}{2} = 11 \text{ kHz}$$

T7. Sol.

$$\beta_f = \frac{k_f \max\{m(t)\}}{f_m} = \frac{100\text{ k} \times 1}{1\text{ k}} = 100$$

$$BW_f = 2\,(100 + 1)\;1\text{ k} = 202\text{ kHz.}$$

$$\beta_p = k_p \max\{m(t)\}$$

$$= 10 \times 1 = 10$$

$$BW_p = 2\,(10 + 1)\;1\text{k} = 22\text{ kHz}$$

Bandwidth required for channel

$$= 202 + 22 = 224\text{ kHz}$$

T8. Sol.

The phase modulated signal can be given by,

$$s(t) = A_c\cos[2\pi f_c t + k_p m(t)] = A_c\cos[\theta(t)]$$

The instantaneous frequency of the modulated signal,

$$f_i = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

Given that,

$$m(t) = 100\,\text{sinc}(1000t)\text{ V} = 100\frac{\sin(1000\pi t)}{1000\pi t}$$

$$\frac{dm(t)}{dt} = 100\left[\frac{1000\pi\cos(1000\pi t)}{1000\pi t} - \frac{\sin(1000\pi t)}{1000\pi t^2}\right]$$

At $t = 1$ ms,

$$\frac{dm(t)}{dt} = \frac{100\cos(\pi)}{10^{-3}} = -10^5\text{ V/s}$$

So,

$$f_i = f_c + \frac{1}{2\pi}(-10^5 k_p) = 100 - \frac{100 \times 2}{2\pi}\text{ kHz}$$

$$= 100 - \frac{100}{\pi}\text{ kHz} = 68.17\text{ kHz}$$

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3 Sampling and Pulse Code Modulation

Detailed Explanation of Try Yourself Questions

T1. (d)

$$f_m = 100 \text{ Hz}$$

$$Q_e = \pm\frac{\Delta}{2}$$

$$f_s = 1.5 \times f_m \times 2 = 300 \text{ Hz}$$

$$\frac{\Delta}{2} \le \frac{0.1}{100} \times A_m$$

$$\frac{2A_m}{2^n \times 2} \le \frac{0.1}{100} \times A_m$$

$$\Rightarrow \quad n = 10$$

$$r_b = N\,n\,f_s = 8 \times 10 \times 300 = 24000 \text{ Hz} = 24 \text{ kbits/sec}$$

T2. Sol.

$$\text{Sampling frequency } (f_s) = 1.5 \times 2 \times 4 = 12 \text{ kHz}$$

$$\text{step size } (\Delta) = 10 \text{ mV}$$

To avoid slope overload distortion in Delta modulation;

$$\frac{\Delta}{T_s} \ge \left|\frac{dm(t)}{dt}\right|_{max}$$

i.e., $$\frac{\Delta}{T_s} \ge 2\pi f_m \cdot A_m$$ ...for sinusoidal message signal

$$A_m \le \frac{\Delta}{T_s(2\pi f_m)}$$

$$(A_m)_{max} = \frac{\Delta}{T_s(2\pi f_m)} = \frac{\Delta \cdot f_s}{2\pi f_m}$$

$$= \frac{10\times10^{-3}\times12\times10^{3}}{2\pi\times10^{3}} = 19.09\times10^{-3} \approx 19.1\text{mV}$$

T3. (c)

To prevent slope overload

$$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

$$\delta\times200\times10^3 \geq 2\pi A_m f_m$$

$$\delta \geq \frac{2\times\pi\times(10\times10^3)\times\frac{1}{2}}{200\times10^3}$$

$$\delta \geq 0.157 \text{ Volts}$$

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4 Digital Carrier Modulation Schemes

Detailed Explanation *of* Try Yourself Questions

T1. Sol.

Average energy $= \frac{1}{16}\left[4(\sqrt{2}a)^2 + 8(\sqrt{10}a)^2 + 4(\sqrt{18}a)^2\right]$

$= \frac{1}{4}[2a^2 + 20a^2 + 18a^2]$

$= 10\,a^2$.

T2. Sol.

Let signal *I* be represented as

$$S_1(t) = \begin{cases} A_1 \sin\frac{\pi t}{T}\ ; & 0 \le t \le T \\ 0\ ; & 0 \le t \le T \end{cases}$$

and signal II be represented as

$$S_2(t) = \begin{cases} A_2 \sin\frac{\pi t}{T}\ ; & 0 \le t \le T \\ -A_2 \sin\frac{\pi t}{T}\ ; & 0 \le t \le T \end{cases}$$

The average energy of signal will be

$$P_{avg_1} = \frac{1}{2}\left(\frac{A_1^2}{2}\right) + \frac{1}{2}(0) = \frac{A_1^2}{4}$$

Average energy of signal (ii)

$$P_{avg_2} = \frac{1}{2}\left(\frac{A_2^2}{2}\right) + \frac{1}{2}\left(\frac{A_2^2}{2}\right) = \frac{A_2^2}{2}$$

$\therefore \quad \frac{A_1^2}{4} = \frac{A_2^2}{2}$

$\Rightarrow \quad \frac{A_1}{\sqrt{2}} = A_2$

T3. (b)

Sampling frequency $(f_s) = 1.25 \times (2f_m) + \text{Guard band}$
$= 1.25 \times 2 \times 10 + 1$
$= 26 \text{ kHz}$

Bit rate $(R_b) = n.f_s = 4 \times 26$ ...$[L \le 2^n]$
$= 104 \text{ kHz}$

$\because$ Bandwidth of channel is 100 kHz i.e.,

$\text{B.W} \le 100 \text{ kHz}$

$R_s(1+\alpha) \le 100$ ...[For M-ary PSK $(\text{BW})_{min} = R_s(1+\alpha)$]

$\because \quad R_s = \frac{R_b}{\log_2 M}$ ...$[R_s$ = symbol rate]

$\therefore \quad \frac{R_b}{\log_2 M}(1+\alpha) \le 100$

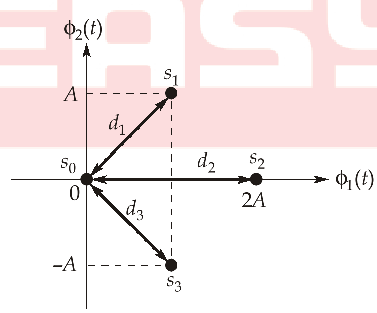
$\frac{104}{\log_2 M}(1+0.3) \le 100$

$\log_2 M \ge 1.352$

$M \ge 2^{1.352}$

$M_{min} = 4$ $(\because M = 2^n)$

T4. (d)



Let the energy associated with the symbols s_0, s_1, s_2 and s_3 are E_0, E_1, E_2 and E_3 respectively.

$E_i = (d_i)^2$; $i = 0, 1, 2, 3$

From the above diagram,

$d_0 = 0$

$d_1 = d_3 = \sqrt{A^2 + A^2} = \sqrt{2A^2}$

$d_2 = 2A$

So, $E_0 = 0$

$E_1 = E_3 = 2A^2$

$E_2 = 4A^2$

The average symbol energy of the modulation scheme can be given as,

$$E_s = \sum_{i=0}^{3} E_i P(s_i) \; ; \qquad P(s_i) = \text{probability of occurrence of the symbol } s_i$$

$$= 0(0.3) + 2A^2(0.2) + 4A^2(0.4) + 2A^2(0.1)$$

$$= (0.4 + 1.6 + 0.2)A^2$$

$$= 2.2\,A^2$$

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