

GATE

MADE EASY **WORKBOOK 2024**



**Detailed Explanations of
Try Yourself Questions**

**Instrumentation Engineering
Communication**



MADE EASY
Publications

1

Amplitude Modulation



Detailed Explanation of Try Yourself Questions

T1. Sol.

The signal

$$s(t) = A_C [1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$$

The signal can be represented as

$$s(t) = \operatorname{Re} \left[A_C e^{j\omega_c t} + \frac{A_C \mu}{2} (e^{j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m)t}) \right]$$

$$s(t)|_{\text{complex}} = \left[A_C e^{j\omega_c t} + \frac{A_C \mu}{2} (e^{j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m)t}) \right]$$

$$s(t)|_c = [s(t)|_{ce} e^{-j\omega_c t}]$$

(where, $s(t)|_c$ = the complex signal $s(t)$ and $s(t)|_{ce}$ = the complex low pass equal of the signal $s(t)$)

$\therefore$

$$s(t)|_{ce} = A_C + \frac{A_C \mu}{2} [\cos \omega_m + j \sin \omega_m t] + \frac{A_C \mu}{2} [\cos \omega_m - j \sin \omega_m t]$$

Putting the conditions given in the questions we get:

$$s(t)|_{ce} = 1 + \frac{1}{8} [\cos \omega_m + j \sin \omega_m t] + \frac{1}{4} [\cos \omega_m - j \sin \omega_m t]$$

$$s(t)|_{ce} = 1 + \frac{3}{8} \cos \omega_m t - j \frac{1}{8} \sin(\omega_m t)$$

$\therefore$

$$\text{A envelop} = \left[\left(1 + \frac{3}{8} \cos(\omega_m t) \right)^2 + \left(\frac{1}{8} \sin(\omega_m t) \right)^2 \right]^{1/2}$$

T2. Sol.

Expression for AM signal

$$V_{AM}(t) = A_C \cos \omega_c t + A_C m_a \cos(\omega_c + \omega_m)t + A_C m_a \cos(\omega_c - \omega_m)t$$

$\therefore$

$$P_C = 100 = \frac{A_C^2}{2}$$

$\therefore$

$$A_C = 14.14 \text{ V}$$

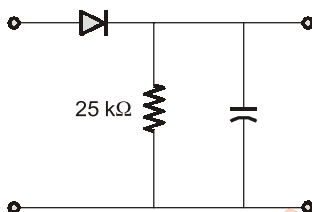
Also

$$\eta = \frac{m_a^2}{2 + m_a^2} = 40\%$$

or

$$\begin{aligned} 0.8 + 0.4 m_a^2 &= m_a^2 \\ m_a &= 1.154 \\ \therefore B &= A_C m_a / 2 = 8.16 \end{aligned}$$

T3. Sol.



$$RC \leq \frac{1}{\omega_n} \frac{\sqrt{1-\mu^2}}{\mu}$$

$$C \leq \frac{1}{R\omega_n} \frac{\sqrt{1-\mu^2}}{\mu}$$

$$C \leq \frac{1}{10^4 \times 2\pi \times 25 \times 10^3} \cdot \frac{\sqrt{1-(0.5)^2}}{0.5}$$

$$C \leq 1.1 \text{ nF}$$

T4. (c)

$$x(t) = m(t) + \cos \omega_c t$$

$$y(t) = 4(m(t) + \cos \omega_c t) + 10[m^2(t) + \cos^2 \omega_c t + 2m(t)\cos \omega_c t]$$

$$= 4m(t) + 10m^2(t) + 4\cos \omega_c t + \frac{10}{2} + \frac{10}{2} \cos 2\omega_c t + 20m(t)\cos \omega_c t$$

after passing through filter

$$y(t) = 4\cos \omega_c t + 20m(t)\cos \omega_c t$$

$$= 4[1 + 5m(t)] \cos \omega_c t$$

$$\mu = 5 \times M$$

$$0.8 = 5 \times M$$

$$M = \frac{0.8}{5} = 0.16$$

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Angle Modulation



Detailed Explanation of Try Yourself Questions

T1. Sol.

Maximum instantaneous frequency

$$f_i = f_c + \frac{K_p}{2\pi} \dot{m}(t)$$

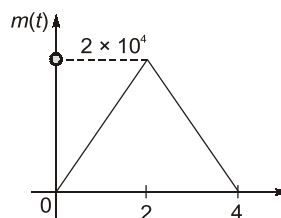
$$115.95 \times 10^3 = \frac{10^5}{2\pi} + \left(\frac{K_p}{2\pi} \right) \times 10^4$$

$$10^5 = \left(\frac{K_p}{2\pi} \right) \times 10^4$$

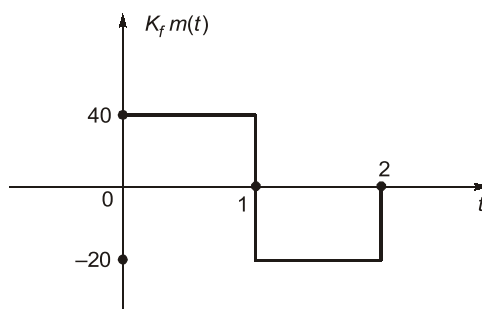
$$10 = \left(\frac{K_p}{2\pi} \right)$$

$$K_p = 2\pi \times 10 \text{ Hertz/Volt}$$

$$K_p = 10 \text{ rad/volt}$$



T2. Sol.



$$s(t) = 10 \cos [2\pi \times 10^6 t + 20\pi [4r(t) - 6r(t-1) + 2r(t-2)]]$$

Standard FM expression is given by:

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int m(t) dt \right]$$

$$2\pi k_f \int m(t) dt = 20\pi (4r(t) - 6r(t-1) + 2r(t-2))$$

$$k_f m(t) = 10[4u(t) - 6u(t-1) + 2r(t-2)]$$

$$\Delta f = \max |k_f m(t)| = 40 \text{ Hz}$$

T3. Sol.

Maximum frequency deviation

$$\Delta f_{\max} = \frac{K_p}{2\pi} \left| \frac{d}{dt} m(t) \right|_{\max} = \frac{K_p}{2\pi} 2t e^{-t^2}$$

$$= \frac{8000}{2\pi} \cdot 2 \cdot \frac{1}{\sqrt{2}} \cdot e^{-1/2} \quad \left(\because \max 2t e^{-t^2} \text{ is at } t = \frac{1}{\sqrt{2}} \right)$$

$$= 3.43 \text{ kHz}$$

T4. Sol.

Comparing the equation with the standard equation.

$$s(t) = A \cos[\omega_c t + k_p m(t)]$$

$$\therefore k_p m(t) = 0.1 \sin(10^3 \pi t)$$

$$m(t) = \frac{0.1}{k_p} \sin(10^3 \pi t)$$

$$= 0.01 \sin(10^3 \pi t)$$

Similarly

$$s(t) = A \cos[\omega_c t + K_f \int m(t) dt]$$

$$K_f \int m(t) dt = 0.1 \sin(10^3 \pi t)$$

$$\int m(t) dt = \frac{0.1}{10\pi} \sin(10^3 \pi t) = \frac{0.1 \times 10^3 \pi}{10\pi} \cos(10^3 \pi t) = 10 \cos(10^3 \pi t)$$

T5. Sol.

$$A_m = 5 \text{ V}, f_m = 100 \text{ Hz} \} \Delta f = k_f A_m = 1 \text{ kHz}$$

$$A_m = 10 \text{ V}, f_m = 50 \text{ Hz} \} \Delta f = 2 \text{ kHz}$$

To get $\Delta f = 30 \text{ kHz}$

frequency multiplication factor should be 15.

T6. (a)

$$BW = 2[\beta + 1]f_m$$

$$\beta = k_p A_m = 5$$

$$A_m \text{ is doubled} \Rightarrow \beta = 10; f_m = \frac{1}{2} \text{ kHz}$$

$$BW = 2[10 + 1] \cdot \frac{1}{2} = 11 \text{ kHz}$$

T7. Sol.

$$\beta_f = \frac{k_f \max \{m(t)\}}{f_m} = \frac{100 \text{ k} \times 1}{1 \text{ k}} = 100$$

$$BW_f = 2(100 + 1) 1 \text{ k} = 202 \text{ kHz.}$$

$$\begin{aligned}\beta_p &= k_p \max \{m(t)\} \\ &= 10 \times 1 = 10\end{aligned}$$

$$BW_p = 2(10 + 1) 1 \text{ k} = 22 \text{ kHz}$$

Bandwidth required for channel

$$= 202 + 22 = 224 \text{ kHz}$$

T8. Sol.

The phase modulated signal can be given by,

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] = A_c \cos[\theta(t)]$$

The instantaneous frequency of the modulated signal,

$$f_i = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$$

Given that,

$$m(t) = 100 \sin(1000\pi t) \text{ V} = 100 \frac{\sin(1000\pi t)}{1000\pi t}$$

$$\frac{dm(t)}{dt} = 100 \left[\frac{1000\pi \cos(1000\pi t)}{1000\pi t} - \frac{\sin(1000\pi t)}{1000\pi t^2} \right]$$

At $t = 1 \text{ ms}$,

$$\frac{dm(t)}{dt} = \frac{100 \cos(\pi)}{10^{-3}} = -10^5 \text{ V/s}$$

So,

$$f_i = f_c + \frac{1}{2\pi} (-10^5 k_p) = 100 - \frac{100 \times 2}{2\pi} \text{ kHz}$$

$$= 100 - \frac{100}{\pi} \text{ kHz} = 68.17 \text{ kHz}$$

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3

Sampling and Pulse Code Modulation



Detailed Explanation of Try Yourself Questions

T1. (d)

$$f_m = 100 \text{ Hz}$$

$$Q_e = \pm \frac{\Delta}{2}$$

$$f_s = 1.5 \times f_m \times 2 = 300 \text{ Hz}$$

$$\frac{\Delta}{2} \leq \frac{0.1}{100} \times A_m$$

$$\frac{2A_m}{2^n \times 2} \leq \frac{0.1}{100} \times A_m$$

$\Rightarrow$

$$n = 10$$

$$r_b = N n f_s = 8 \times 10 \times 300 = 24000 \text{ Hz} = 24 \text{ kbits/sec}$$

T2. Sol.

$$\begin{aligned} \text{Sampling frequency } (f_s) &= 1.5 \times 2 \times 4 \\ &= 12 \text{ kHz} \end{aligned}$$

$$\text{step size } (\Delta) = 10 \text{ mV}$$

To avoid slope overload distortion in Delta modulation;

$$\frac{\Delta}{T_s} \geq \left| \frac{dm(t)}{dt} \right|_{\max}$$

i.e.,

$$\frac{\Delta}{T_s} \geq 2\pi f_m \cdot A_m$$

...for sinusoidal message signal

$$A_m \leq \frac{\Delta}{T_s(2\pi f_m)}$$

$$\begin{aligned}
 (A_m)_{\max} &= \frac{\Delta}{T_s(2\pi f_m)} = \frac{\Delta \cdot f_s}{2\pi f_m} \\
 &= \frac{10 \times 10^{-3} \times 12 \times 10^3}{2\pi \times 10^3} = 19.09 \times 10^{-3} \approx 19.1 \text{ mV}
 \end{aligned}$$

T3. (c)

To prevent slope overload

$$\begin{aligned}
 \delta f_s &\geq \max \left| \frac{dm(t)}{dt} \right| \\
 \delta \times 200 \times 10^3 &\geq 2\pi A_m f_m \\
 \delta &\geq \frac{2 \times \pi \times (10 \times 10^3) \times \frac{1}{2}}{200 \times 10^3} \\
 \delta &\geq 0.157 \text{ Volts}
 \end{aligned}$$

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4

Digital Carrier Modulation Schemes



Detailed Explanation of Try Yourself Questions

T1. Sol.

$$\begin{aligned}\text{Average energy} &= \frac{1}{16} [4(\sqrt{2}a)^2 + 8(\sqrt{10}a)^2 + 4(\sqrt{18}a)^2] \\ &= \frac{1}{4} [2a^2 + 20a^2 + 18a^2] \\ &= 10a^2.\end{aligned}$$

T2. Sol.

Let signal I be represented as

$$S_1(t) = \begin{cases} A_1 \sin \frac{\pi t}{T} ; & 0 \leq t \leq T \\ 0 & ; 0 \leq t \leq T \end{cases}$$

and signal II be represented as

$$S_2(t) = \begin{cases} A_2 \sin \frac{\pi t}{T} ; & 0 \leq t \leq T \\ -A_2 \sin \frac{\pi t}{T} ; & 0 \leq t \leq T \end{cases}$$

The average energy of signal will be

$$P_{\text{avg}_1} = \frac{1}{2} \left(\frac{A_1^2}{2} \right) + \frac{1}{2} (0) = \frac{A_1^2}{4}$$

Average energy of signal (ii)

$$P_{\text{avg}_2} = \frac{1}{2} \left(\frac{A_2^2}{2} \right) + \frac{1}{2} \left(\frac{A_2^2}{2} \right) = \frac{A_2^2}{2}$$

$$\therefore \frac{A_1^2}{4} = \frac{A_2^2}{2}$$

$$\Rightarrow \frac{A_1}{\sqrt{2}} = A_2$$

T3. (b)

$$\begin{aligned} \text{Sampling frequency } (f_s) &= 1.25 \times (2f_m) + \text{Guard band} \\ &= 1.25 \times 2 \times 10 + 1 \\ &= 26 \text{ kHz} \end{aligned}$$

$$\begin{aligned} \text{Bit rate } (R_b) &= n \cdot f_s = 4 \times 26 \quad \dots [L \leq 2^n] \\ &= 104 \text{ kHz} \end{aligned}$$

$\therefore$ Bandwidth of channel is 100 kHz i.e.,

$$\text{B.W} \leq 100 \text{ kHz}$$

$$R_s(1 + \alpha) \leq 100$$

$$\dots [\text{For M-ary PSK } (BW)_{\min} = R_s(1 + \alpha)]$$

$$\therefore R_s = \frac{R_b}{\log_2 M}$$

$$\dots [R_s = \text{symbol rate}]$$

$$\therefore \frac{R_b}{\log_2 M} (1 + \alpha) \leq 100$$

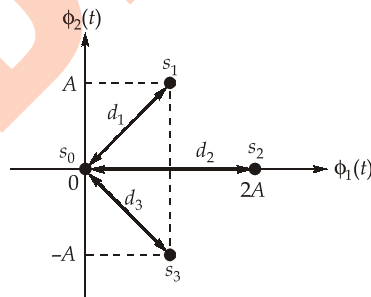
$$\frac{104}{\log_2 M} (1 + 0.3) \leq 100$$

$$\log_2 M \geq 1.352$$

$$M \geq 2^{1.352}$$

$$M_{\min} = 4$$

$$(\because M = 2^n)$$

T4. (d)

Let the energy associated with the symbols s_0, s_1, s_2 and s_3 are E_0, E_1, E_2 and E_3 respectively.

$$E_i = (d_i)^2; \quad i = 0, 1, 2, 3$$

From the above diagram,

$$d_0 = 0$$

$$d_1 = d_3 = \sqrt{A^2 + A^2} = \sqrt{2A^2}$$

$$d_2 = 2A$$

$$E_0 = 0$$

$$E_1 = E_3 = 2A^2$$

$$E_2 = 4A^2$$

So,

The average symbol energy of the modulation scheme can be given as,

$$\begin{aligned} E_s &= \sum_{i=0}^3 E_i P(s_i) ; \quad P(s_i) = \text{probability of occurrence of the symbol } s_i \\ &= 0(0.3) + 2A^2(0.2) + 4A^2(0.4) + 2A^2(0.1) \\ &= (0.4 + 1.6 + 0.2)A^2 \\ &= 2.2 A^2 \end{aligned}$$

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