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ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-3 : Power Systems [All topics] + Systems and Signal Processing-1 + Microprocessor-1 Electrical Circuits-2 + Control Systems-2 [Part Syllabus]

Name: Satyam Khairnar.

Roll No :

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	

Signature of Evaluator ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

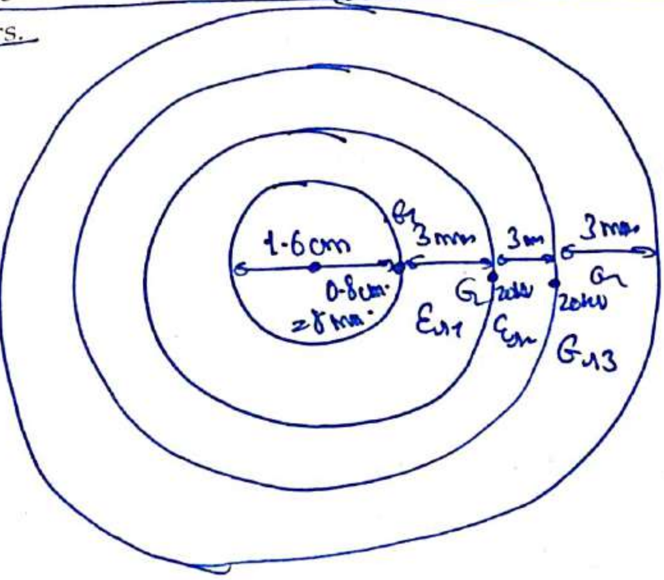
Do not write in this margin

Section A : Power Systems

Q.1 (a) A 66 kV concentric cable with intersheath has a core diameter of 1.6 cm. 3 mm thick dielectric materials constitute the three zones of insulation. Determine the maximum stress in each of the three layers if 20 kV is maintained across each of the inner two layers.

[12 marks]

(Ans.)



$$E_1 = \frac{\lambda}{2\pi\epsilon_0\epsilon_{r1} r_1} = \frac{k}{\epsilon_{r1}(8\,mm)}$$ (let us assume $k = \frac{\lambda}{2\pi\epsilon_0}$)

$$E_2 = \frac{\lambda}{2\pi\epsilon_0\epsilon_{r2} r_2} = \frac{k}{\epsilon_{r2}(11\,mm)}$$

$$E_3 = \frac{\lambda}{2\pi\epsilon_0\epsilon_{r3} r_3} = \frac{k}{\epsilon_{r3}(14\,mm)}$$

Now, $V_1 - V_2 = \frac{\lambda}{2\pi\epsilon_0\epsilon_{r1}} \ln\left(\frac{r_2}{r_1}\right)$

$$\Delta V_1 = \frac{k}{\epsilon_{r1}} \ln\left(\frac{11}{8}\right) = 20\,kV$$

$$\Delta V_2 = V_2 - V_3 = \frac{k}{\epsilon_{r2}} \ln\left(\frac{14}{11}\right) = 20\,kV$$

$$\Delta V_3 = \frac{k}{\epsilon_{r3}} \ln\left(\frac{17}{14}\right)$$

$$\frac{K r_1}{r_1 G_{r1}} \ln\left(\frac{11}{8}\right) = 20\times10^3$$

$$\Rightarrow g_{max1}\, r_1 \ln\left(\frac{11}{8}\right) = 20\times10^3$$

$$\Rightarrow g_{max1}\,(8mm) \ln\left(\frac{11}{8}\right) = 20\times10^3$$

$$g_{max1} = 7.85\ kV/mm.$$

Similarly,

$$g_{max2}\, r_2 \ln\left(\frac{14}{11}\right) = 20\times10^3$$

$$g_{max2}\times(11mm) \ln\left(\frac{14}{11}\right) = 20\times10^3$$

$$g_{max2} = 7.5393\ kV/mm.$$

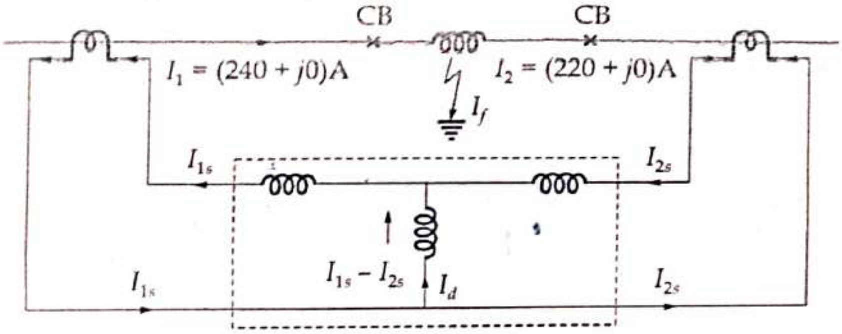
$$\Delta V_3 = 66\ kV - 20V - 20V$$

$$\Delta V_3 = 26\ kV$$

$$26\ kV = g_{max3}\,(14mm) \ln\left(\frac{17}{14}\right)$$

$$g_{max3} = 9.5652\ kV/mm.$$

1 (b) Figure below shows percentage differential relay is applied for the protection of a generator winding. The relay has 10% slope of its operating characteristic on $\frac{(I_{1s} + I_{2s})}{2}$ versus $(I_{1s} - I_{2s})$ diagram. A high resistance ground fault occurred near the grounded neutral end of the generator winding while generator is carrying load. As a consequence, the currents flowing at each end of the winding are shown in the figure below. Assuming CT ratio of 400/5 ampere, will the relay operate to trip the breaker?



[12 marks]

(Ans.) As per the figure, we know that.

$$(I_1)(N_p) = (I_{1s})(N_s)$$

$$\therefore (240)(5) = (I_{1s})(400)$$

$$I_{1s} = 3A$$

Similarly,

$$(I_2)(N_p) = (I_{2s})(N_s)$$

$$\Rightarrow (220)\times(5) = (I_{2s})(400)$$

$$I_{2s} = 2.75A$$

$$\therefore I_d = I_{1s} - I_{2s}$$

$$I_d = 0.25A$$

Also we have, $\left(\frac{I_{1s}+I_{2s}}{2}\right)$

$$= \frac{3+2.75}{2}$$

$$\Rightarrow \frac{I_{1s}+I_{2s}}{2} = 2.875A$$

Now for $N_r(I_{1s}-I_{2s})$, $N_o\left(\frac{I_{1s}+I_{2s}}{2}\right)$ $\times 5$

we have slope = 10%

$\therefore$ $\Rightarrow \frac{N_o}{N_r}\left(\frac{I_{1s}+I_{2s}}{2}\right) - (I_{1s}-I_{2s})$

$$= (0.1)(2.875) - 0.25$$

$$= 0.2875 - 0.25$$

$$= 0.0375A$$

$\therefore$ the relay will operate.

(c) A 50 Hz, 4 pole, turbo-generator rated 100 MVA, 11 kV has an inertia constant of 8 MJ/MVA.

(i) Determine the stored energy in the rotor at synchronous speed.

(ii) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, determine acceleration in elec-degree/sec^2, neglecting mechanical and electrical losses.

(iii) If the acceleration calculated in part (ii) is maintained for 10 cycle, determine the change in torque angle and rotor speed in revolutions per minute at the end of the period.

[2 + 5 + 5 marks]

(Ans): We have, $S = 100$ MVA; $V = 11$ kV; $H = 8$ MJ/MVA; $P = 4$;
$\Rightarrow G = 100$ MVA
$f = 50$ Hz.

(i). Stored energy $= GH$

$= 100 \times 8$

$\Rightarrow \boxed{\text{Stored energy} = 800\,J}$.

(ii). We have,

$$\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\Rightarrow \frac{(100)(8)}{\pi \times 50}\frac{d^2\delta}{dt^2} = 80 - 50$$

$$\Rightarrow \frac{d^2\delta}{dt^2} = 5.8904 \text{ rad/sec}^2$$

$$\Rightarrow \frac{d^2\delta}{dt^2} = 5.8904 \times \frac{180}{\pi} \text{ /sec}^2$$

~~$\frac{d^2\delta}{dt^2} = 337.5$ elec°/sec²~~

$$\frac{d^2\delta}{dt^2} = 337.5 \text{ elect degree/sec}^2$$

(iii) We have, accelerator maintain for 10 cycle.

$\therefore$ $t = \frac{10}{50} = 0.2$ sec.

$\therefore$ $\Delta\delta = \frac{1}{2} \times \left(\frac{d^2\delta}{dt^2}\right) t^2$

$= \frac{1}{2} \times (337.5) (0.2)^2$

$\Delta\delta = 6.75$ degree electrical.

Now, $\Delta\omega = \frac{d^2\delta}{dt^2}(t)$

$= (337.5) \times \frac{\pi}{180} \times 0.2$

$\Delta\omega = 1.178$ rad/sec (electrical.)

$\therefore$ $\Delta\omega = \frac{2}{P}(1.178) = 0.589$ mech-rad/sec.

$\Rightarrow \Delta N = \frac{60}{2\pi} \times \Delta\omega = \frac{60}{2\pi} \times 0.589$

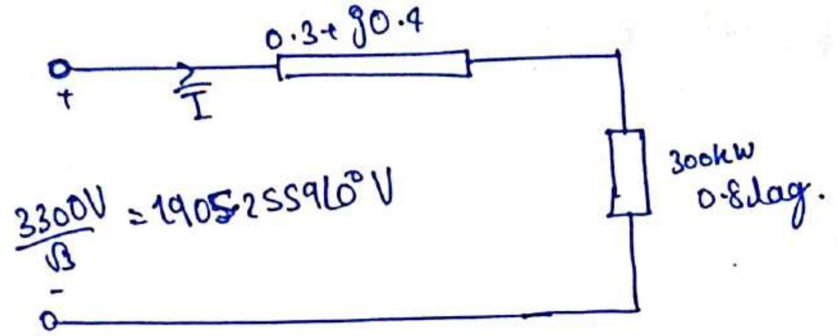
$\Delta N = 5.6245$ rpm.

$\Rightarrow N_f = 1505.6245$ rpm.

(d) The per phase impedance of 3-ϕ short transmission line is $(0.3 + j0.4)\Omega$. The sending-end line to line voltage is 3300 V and the load at the receiving end is 300 kW per phase at 0.8 pf lagging. Calculate receiving end voltage and line current.

[12 marks]

Ans.)



$P_L = 300\,kW$ $\quad\quad$ $V_s = 1905.2559\angle 0\ V$

$\Rightarrow S_L = \frac{300}{0.8} = 375\,kVA$

$\therefore Q_L = 225\,kVAR$

$\therefore |\bar{I}|^2 R = 300\,kW$

$|\bar{I}|^2 X = 225\,kVAR$

$\Rightarrow \frac{R}{X} = \frac{4}{3}$

& $\frac{\bar{V_s}}{\bar{I}} = R + jX + 0.3 + j0.4$

$1905.2559 = \left(\left(R + j\frac{3R}{4}\right) + (0.3 + j0.4)\right)\bar{I}$

$1905.2559 = \left|(R+0.3) + j\left(0.4 + \frac{3R}{4}\right)\right| |\bar{I}|$

$\Rightarrow 1905.2559 = |\bar{I}| \sqrt{(R+0.3)^2 + \left(0.4 + \frac{3R}{4}\right)^2}$

$1905.2559 = \sqrt{\frac{300 \times 10^3}{R}} \sqrt{R^2 + 0.09 + 0.6R + \frac{9}{16}R^2 + 0.16 + 0.6R}$

$(1905.2559)^2 = \left(\frac{300 \times 10^3}{R}\right)\left(1.5625R^2 + 1.2R + 0.25\right)$

$12.1R = 1.5625R^2 + 1.2R + 0.25$

$1.5625R^2 - 10.9R + 0.25 = 0$

$R = 6.9529\ \Omega,\ 0.023011\ \Omega$

$R = 6.9529\ \Omega$

$\Rightarrow X = \frac{3R}{4} = 5.2147\ \Omega$

$\therefore \bar{I}_L = \frac{1905.2559\angle 0^\circ}{(0.3 + j0.4 + 6.9529 + j5.2147)}$

$\bar{I}_L = \frac{1905.2559}{7.2529 + j5.6147}$

$\bar{I}_L = 207.7208\angle -37.7449^\circ\ A$

$\therefore \bar{V}_R = (207.7208\angle -37.7449^\circ)(6.9529 + j5.2147)$

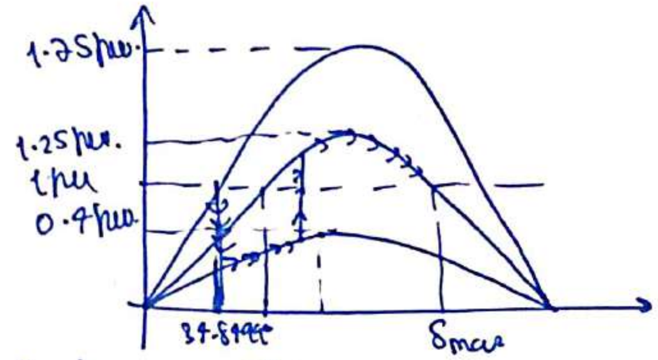
$\bar{V}_R = 1805.33105\angle -0.8749^\circ\ V$

$\bar{V}_{R(L-L)} = 3126.925\ V$

(e) A three phase generator delivers 1.0 p.u. power to an infinite bus through a transmission network when a fault occurs. The maximum power which can be transferred in pre-fault, during fault and post fault conditions are 1.75 p.u., 0.4 p.u and 1.25 p.u. respectively. Find the critical angle.

[12 marks]

(Ans.):



Initial power = 1 pu.

$\therefore \delta_0 = \sin^{-1}\left(\frac{1}{1.25}\right)$

$\delta_0 = 34.8499°$

$\delta_{max} = \pi - \sin^{-1}\left(\frac{1}{1.25}\right)$

$\delta_{max} = 2.21429 \text{ rad} = 126.8698°$

$$\therefore \int_{34.8499°}^{\delta_c} (1 - 0.4\sin\delta)\, d\delta = \int_{\delta_c}^{126.8698°} (1.25\sin\delta - 1)\, d\delta$$

$$\Rightarrow \delta_c - 0.6082 + \underline{0.4\cos\delta_c} - 0.4\cos(34.8499°)$$
$$= \underline{1.25\cos\delta_c - 1.25\cos(126.8698°) - 2.21429 + \delta_c}$$

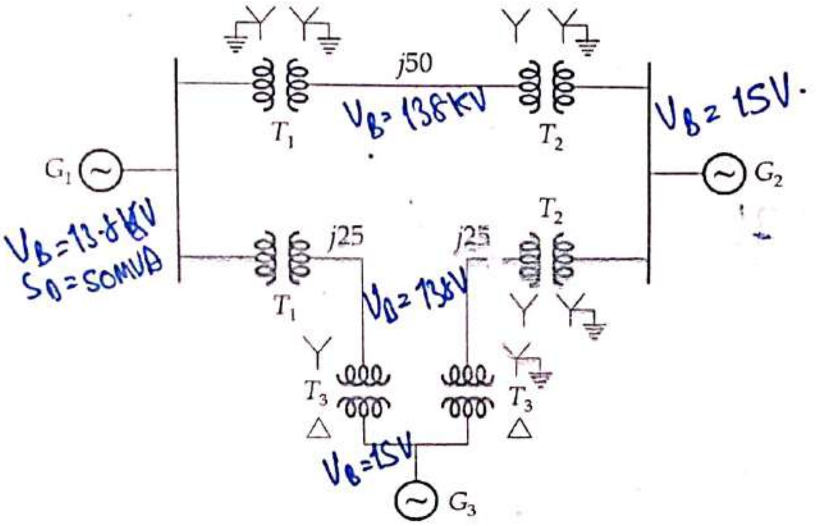
$0.85\cos\delta_c = 0.45583$

$$\Rightarrow \delta_c = 57.5698°$$

(a) A 3-bus system is given in figure below. The ratings of the various components are listed below :

Generator 1 = 50 MVA;	13.8 kV;	$X'' = 0.15$ pu
Generator 2 = 40 MVA;	13.2 kV;	$X'' = 0.20$ pu
Generator 3 = 30 MVA;	11 kV;	$X'' = 0.25$ pu
Transformer 1 = 45 MVA,	11 kV Δ/110 kV Y,	$X = 0.1$ pu
Transformer 2 = 25 MVA,	12.5 kV Δ/115 kV Y,	$X = 0.15$ pu
Transformer 3 = 40 MVA,	12.5 kV Δ/115 kV Y,	$X = 0.1$ pu

The line impedances are shown in figure below. Determine the reactance diagram based on 50 MVA and 13.8 kV as base quantities in Generator 1.



[20 marks]

(Ans.) We have,

$V_B = 13.8$ kV

$S_B = 50$ MVA

$\therefore$ For generator (1), we have

$X'' = 0.15$ pu.

$\Rightarrow V_B = 15$ kV & $S_B = 50$ MVA

For generator (2), we have, $X_{new} = X_{old} \times \frac{V_{old}^2}{S_{old}} \times \frac{S_{new}}{V_{new}^2}$

$$X'' = 0.2 \times \frac{(13.2)^2}{(40)} \times \frac{(50)}{(15)^2}$$

~~$X'' = 0.2287$ pu.~~ $\Rightarrow X'' = 0.1936$ pu.

For generator 3, we have

$$X'' = 0.25 \times \frac{(11)^2}{(30)} \times \frac{50}{(13.8)^2}$$

$\cancel{X'' = 0.2640\,pu}$ $X'' = 0.22407\,pu.$

For Transformer 3, we have $\left(\text{for } \Delta,\ Z_B = \frac{3(V)^2}{S}\right)$

$$X'' = (0.1) \times 3 \times \frac{(12.5)^2}{(40)} \times \frac{50)}{3\times(13.8)^2} \Rightarrow X'' = 0.0868\,pu.$$

$\cancel{X'' = 0.10756\,pu}$ $\cancel{X'' = 0.10256\,pu}$

For Transformer 2, we have

$$X'' = (0.15)\frac{(12.5)^2}{25} \times \frac{50}{3\times(13.8)^2}$$

$\cancel{X'' = 0.2801\,pu}$ $X'' = 0.2083333\,pu.$

For Transformer 1, we have

$$X'' = (0.1) \times 3\,\frac{(11)^2}{45} \times \frac{50}{(13.8)^2}$$

$$\Rightarrow X'' = 0.0706\,pu.$$

For line between T_1 & T_2,

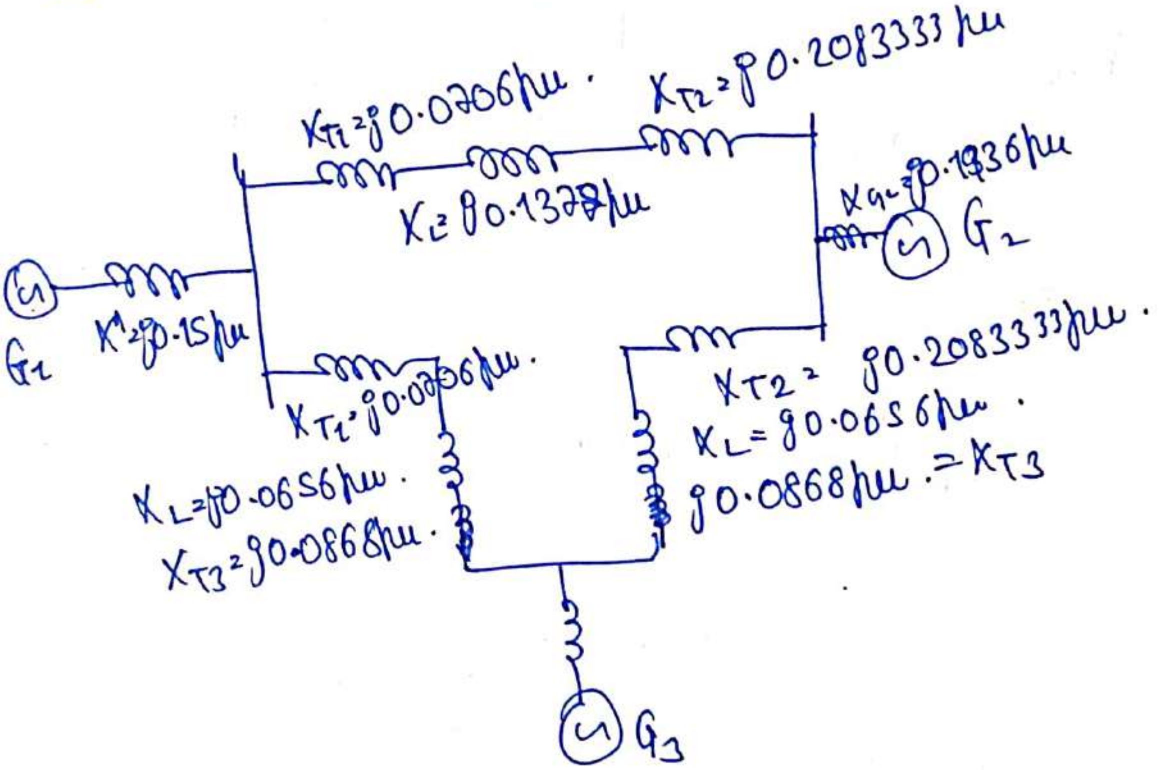
$Z = 50\,\Omega$ $Z_{pu} = \frac{50\times 50}{(138)^2} \Rightarrow Z_{pu} = 0.1327\,pu.$

Similarly, for line between (T_1 & T_3) & (T_2 & T_3), we have

$$Z = \frac{25 \times 50}{(138)^2}$$

$X_{pu} = 0.0656$ pu.

∴ we have the following reactance diagram.



Q.2 (b) Explain briefly what is swing equation and use dynamics of angular motion with time to formulate the equation for a synchronous generator of inertia constant H in seconds run by a mechanical turbine with input power P_m in p.u. to deliver electrical power P_e in p.u. to the electrical network at f Hz in terms of power angle δ in radians measured from rotating reference of generator axis.

[20 marks]

(Ans) Swing equation: Whenever there is variation in the input or output power, a transient is observed in the synchronous machine and it is as per the dynamics of angular motion. This variation is ~~known~~ governed by an equation which is called as swing equation.

As per swing equation,

$$\boxed{\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = \underset{\text{input}}{P_m} - \underset{\text{output}}{P_e}}$$

$$\Rightarrow \boxed{\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = \underset{(pu)}{P_m} - \underset{(pu)}{P_e}}$$

G = Base power (rating of generator)

H = Inertia constant

f = frequency

Do not write in this margin

$\frac{d^2\delta}{dt^2}$ ⇒ Angular acceleration of rotor in ~~elect radian/sec~~. ~~electrical radian~~ electrical radian/sec².

$\delta T = J\alpha$ (J = Moment of inertia, α = Angular acceleration)

$\Rightarrow \delta T\omega = J\alpha\omega$.

$\Rightarrow T\omega_s = J\alpha\omega_s$

$\Delta P = J\omega\alpha$

$\Delta P = \frac{J\omega^2}{\omega}\alpha$

$\Delta P = \frac{J\omega^2}{2\pi f}\alpha$

$\Delta P = \frac{\frac{1}{2}J\omega^2}{\pi f}\alpha$

$\Rightarrow \frac{K.E.}{\pi f}\alpha = \Delta P = P_m - P_e$

$$\Rightarrow \boxed{\frac{GH}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e}$$

Q.2 (c) A 3-ϕ, 400 km, 50 Hz long transmission line with series impedance of $(0.15 + j0.78)\ \Omega/\text{km}$ and shunt admittance of $j5.0 \times 10^{-6}\ ℧/\text{km}$. Determine A, B, C, D parameter of line assuming :

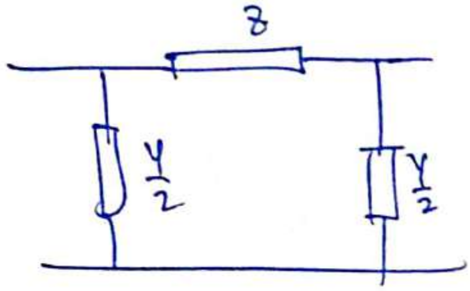
(i) The line could be represented by nominal-T.

(ii) The line could be represented by nominal-π.

(iii) The exact representation.

[20 marks]

(Ans.) (ii).



For nominal π, we have

$$= \begin{bmatrix} 1 & 0 \\ \frac{Y}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{Y}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & Z \\ \frac{Y}{2} & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{Y}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y+\frac{Y^2Z}{4} & 1+\frac{YZ}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y\left(1+\frac{YZ}{4}\right) & 1+\frac{YZ}{2} \end{bmatrix}$$

we have, $Z = (0.15 + j0.78) \times 400$

$Z = 60 + 312j$

$Z = 317.7168 \angle 79.1144^\circ\ \Omega$:

$Y = (j5 \times 10^{-6}) \times 400$

$Y = 0.002 \angle 90^\circ\ ℧$

$\therefore$ ~~$A = 0.002\angle 90^\circ$~~ $1+\frac{YZ}{2}$.

$$1+\frac{YZ}{2} = 1+\frac{(0.002\angle 90^\circ)(317.7168\angle 79.1149^\circ)}{2}$$

$$1+\frac{YZ}{2} = 2.203\angle 164.1974^\circ$$

$$Y\left(1+\frac{YZ}{4}\right) = (0.002\angle 90^\circ)\left(1+\frac{(0.002\angle 90^\circ)(317.7168\angle 79.1149^\circ)}{4}\right)$$

$$= 1.689\times 10^{-3}\angle 92.0357^\circ$$

$$\text{Nominal } \pi \Rightarrow \begin{bmatrix} 2.203\angle 164.1974^\circ & 317.7168\angle 79.1149^\circ \\ 1.689\times 10^{-3}\angle 92.0357^\circ & 2.203\angle 164.1974^\circ \end{bmatrix}$$

(ii) · Nominal T ⇒

$$\begin{bmatrix} 1+\frac{YZ}{2} & Z\left(1+\frac{YZ}{4}\right) \\ Y & 1+\frac{YZ}{2} \end{bmatrix}$$

$$Z\left(1+\frac{YZ}{4}\right) = (317.7168\angle 79.1149^\circ)\left(1+\frac{(0.002\angle 90^\circ)(317.72\angle 79.11^\circ)}{4}\right)$$

$$= 268.3227\angle 81.15^\circ$$

$$\begin{bmatrix} 2.203\angle 164.1974^\circ & 268.3227\angle 81.15^\circ \\ 0.002\angle 90^\circ & 2.203\angle 164.1974^\circ \end{bmatrix}$$

(iii) $\gamma l = \sqrt{YZ}$

$$= \sqrt{(317.7168\angle 79.1149^\circ)(0.002\angle 90^\circ)} = \sqrt{0.6354\angle 169.1149^\circ}$$

$$= 0.7971\angle 84.5572^\circ = 0.0756 + j0.7935$$

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{158858.4\angle -10.8856^\circ} = 398.57\angle -5.4428^\circ$$

$\sinh\gamma l = \sinh(0.0256 + j0.7935)$

$= \sinh(0.0256)\cos(0.7935) + j\cosh(0.0256)\sin(0.7935)$

$= 0.053 + j(1.002859)(0.8794)$

$= 0.8835\angle 86.56^\circ \Rightarrow$

$\cosh\gamma l = \cosh(0.0256 + j0.7935)$

$= \cosh(0.0256)\cos(0.7935) + j\sinh(0.0256)\sin(0.7935)$

$= 0.70385 + j(0.0256)(0.0539)$

$= 0.70355 + j\,4.02786\times10^{-3}$

$= 0.70356\angle 0.332^\circ$

$\therefore$ $\boxed{[ABCD] = \begin{bmatrix} 0.70356\angle 0.332^\circ & 352.1366\angle 81.1122^\circ \\ 2.2166\times10^{-3}\angle 92.0028^\circ & 0.70356\angle 0.332^\circ \end{bmatrix}}$

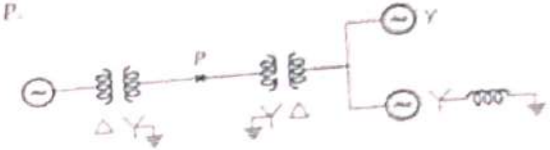
$Z_c \sinh\gamma l = 352.1366\angle 81.1122^\circ$

$\frac{1}{Z_c}\sinh\gamma l = 2.2166\times10^{-3}\angle 92.0028^\circ$

(a) A 50 Hz generator is delivering 50% of the power that it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactance between the generator and infinite bus to 400% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 80% of the original maximum value. Determine critical clearing angle for the condition described.

[20 marks]

A 30 MVA, 13.8 kV, 3-phase alternator has a subtransient reactance of 15% and negative and zero sequence reactance of 15% and 5% respectively. The alternator supplies two motors over a transmission line having tensiometers of both-ends as shown on one line diagram. The motors having rated input of 20 MVA and 10 MVA both with 12.5 kV with 20% subtransient reactance and negative and zero sequence reactances are 20% and 5% respectively. Current limiting reactor of 2 Ω each are in the alternator and larger motor. The 3-phase transformers are both rated 35 MVA, 13.2 Δ - 115 Y kV with leakage reactance of 10%. Series reactance of the line is 80 Ω. The zero sequence reactance of the line is 200 Ω. Determine the fault current when (i) L-G, (ii) L-L, (iii) LLG and fault takes place at point P.



(Assume, V_f = 120 kV)

[20 marks]

Q.3 (c) (i) Give the methods of improving string efficiency for an insulator.

(ii) A transmission line has a span of 375 m between level supports. The conductor has an effective diameters of 1.96 c.m. and weight 0.865 kg/m. Its ultimate strength is 9060 kg. If the conductor has ice coating of radial thickness 1.27 c.m. and subjected to a wind pressure of 3.9 gm/cm^2 of projected area. Calculate sag for a safety factor of 2. (Weight of 1 c.c. of ice is 0.91 gm).

[8 + 12 marks]

Q.4 (a) A star connected 3-phase, 10 MVA, 6.6 kV alternator has a per phase reactance of 20%. It is protected by Merz-price circulating current principle not less than 170 A. Calculate of the value of earthing resistance to be provided in order to ensure that only 20% of the alternator winding remains unprotected.

[20 marks]

(Ans.) We have, Y-connect genⁿ.

$V_{L-L} = 6.6 kV$; $X = 0.2$ pu.

$I \geq 170 A$; $S_B = 10 MVA$.

$x = 20\%$ (only 20% of wdg. remains protected.)

We have, $V_{ph} = \frac{6.6}{\sqrt{3}} kV$

$\Rightarrow V_{ph} = \frac{6600}{\sqrt{3}}$

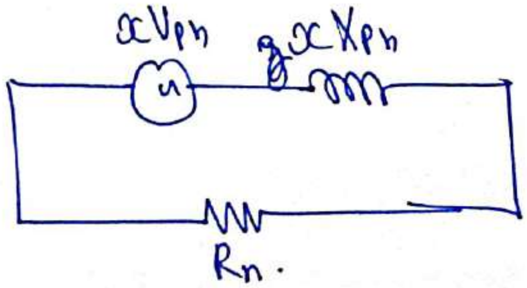
$\Rightarrow V_{ph} = 3810.5178 V$.

& $X = X_{pu} \times \frac{V_L^2}{S_B}$

$= \frac{0.2 \times (6.6 \times 10^3)^2}{10 \times 10^6}$

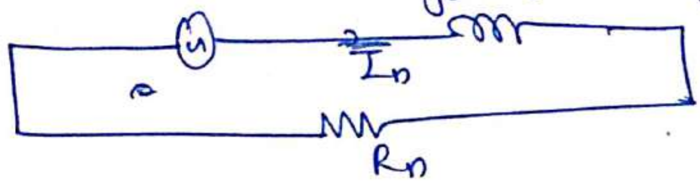
$X = 0.8712 \Omega$

∴ We have the following circuit per phase

⇒ 

$\Rightarrow$ $0.2 \times 3810.5178 = 762.1036\,V$

$j0.2 \times 0.8712 = j0.17424$



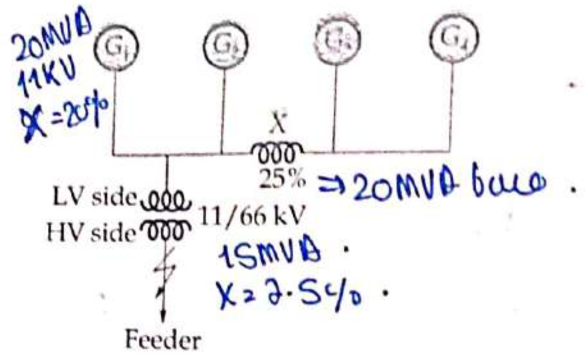
$\therefore \bar{I}_n = \dfrac{(762.1036)}{\sqrt{(R_n + jX_n)}}$

$\Rightarrow |\bar{I}_n| = \dfrac{762.1036}{\sqrt{R_n^2 + (0.17424)^2}} = 170\,A$ (given).

$\Rightarrow R_n^2 + (0.17424)^2 = 20.0969$.

$\Rightarrow R_n = 4.4796\,\Omega$

Q.4 (b) A generating station has four identical generators, G_1, G_2, G_3 and G_4 each of 20 MVA, 11 kV having 20% reactance. They are connected to a busbar which has a busbar reactor of 25% reactance on 20 MVA base, inserted between G_2 and G_3 as shown below. A 66 kV feeder is taken off from the bus bars through a 15 MVA, 11/66 kV transformer having 7.5% reactance. A symmetrical 3-phase fault occurs at the high voltage terminals of the transformers. Calculate the current fed into the fault.



[20 marks]

(Ans.): Let us assume,

$V_B = 11\text{kV}$

& $S_B = 20\text{MVA}$.

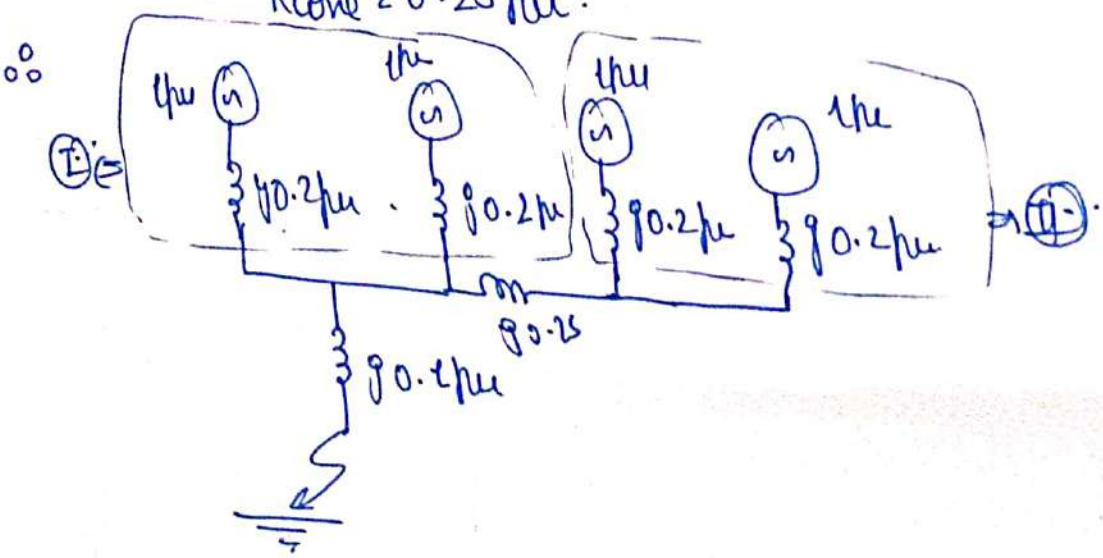
∴ $X_{alternator} = 0.2$ pu.

$$X_{Transf} = \left(\frac{7.5}{100}\right) \times \frac{(11)^2}{15} \times \frac{(20)}{(11)^2}$$

$\Rightarrow X_{Transf} = 0.1$ pu.

& $X_{core} = 0.25$ pu.

∴



For (I), we have

$$V_{eq} = \frac{V_1X_2+V_2X_1}{X_1+X_2} = V_1 \quad (\because X_1 = X_2 \ \& \ V_1 = V_2)$$

$$\frac{1}{X_{eq}} = \frac{1}{X_1}+\frac{1}{X_2} \Rightarrow X_{eq} = \frac{X_1}{2}$$

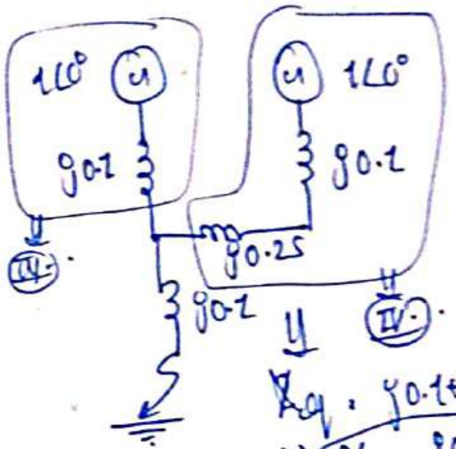
$\therefore V_{eq} = 1\angle 0°$ pu.

$X_{eq} = \frac{j0.2}{2} = j0.1$ pu

Similarly for (II), we have.

$V_{eq} = 1\angle 0°$ pu.

$X_{eq} = j0.1$ pu.



⇓ (IV)

$X_q = j0.1 + j0.25$

$\Rightarrow \boxed{X_q = j0.35 \text{ pu}}$

For (III) & (IV)

$$V_{eq} = \frac{(1)(j0.35)+(1)(j0.1)}{j(0.45)} \Rightarrow \boxed{V_{eq} = 1\angle 0° \text{ pu}}$$

$$\& \ X_{eq} = \frac{(j0.35)(j0.1)}{j0.35+j0.1} \Rightarrow \boxed{X_{eq} = j0.07777 \text{ pu}}$$

$\Rightarrow X_{eq} = j0.1 + j0.07777$

$X_{eq} = j0.17777$ pu.

$$\therefore \bar{I} = \frac{1\angle 0°}{j0.17777}$$

$$\Rightarrow \boxed{\bar{I} = 5.625\angle -90° \text{ pu}}$$

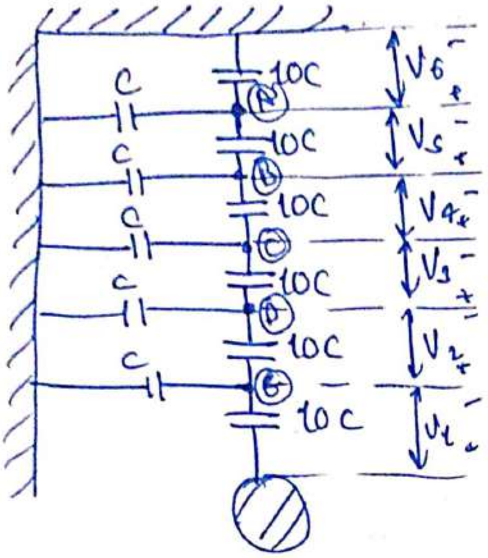
$\Rightarrow |\bar{I}| = 5.625$ pu

$$|\bar{I}| = \frac{5.625 \times 20\times 10^6}{\sqrt{3}\times 11\times 10^3} \Rightarrow \boxed{|\bar{I}| = 5904.718662 \text{ A}}$$

Q.4 (c) A string of six insulation unit has mutual capacitance 10 times the capacitance to ground. Determine the voltage across each unit as a fraction of the operating voltage. Also, determine string efficiency.

[20 marks]

(Ans.):



$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6.$

Now at (A), we have

$\cancel{j}(10\cancel{C})\cancel{\omega} V_6 + \cancel{j}(\cancel{C})\cancel{\omega} V_6 = \cancel{j}(10\cancel{C})\cancel{\omega} V_5$

$11V_6 = 10V_5$

$\Rightarrow V_5 = 1.1V_6$ — (i)

Now at (B), we have

$j\omega(10C)V_4 = j\omega(10C)V_5 + j\omega(C)(V_5 + V_6)$

$\Rightarrow 10V_4 = 10(1.1V_6) + (2.1V_6)$

$10V_4 = 13.1V_6$

$\Rightarrow V_4 = 1.31V_6$ — (ii)

Now at (C), we have

$j\omega(10C)V_3 = j\omega(10C)V_4 + j\omega(C)(V_4 + V_5 + V_6)$

$10V_3 = 10(1.31)V_6 + (1.31V_6 + 1.1V_6 + V_6)$

$10V_3 = 13.1V_6 + 3.41V_6$

$10V_3 = 16.51V_6$

$\Rightarrow V_3 = 1.651V_6$.

Now, at (D), we have

$j\omega(10C)V_2 = j\omega(10C)V_3 + j\omega C(V_3+V_4+V_5+V_6)$

$10V_2 = 10V_3 + (1.651+1.31+1.1+1)V_6$

$10V_2 = 10(1.651)V_6 + 5.061V_6$

$10V_2 = 21.571V_6$

$\Rightarrow V_2 = 2.1571V_6$.

Now at (E), we have

$j\omega(10C)V_1 = j\omega(10C)V_2 + j\omega C(V_2+V_3+V_4+V_5+V_6)$

$10V_1 = 21.571V_6 + (2.1571+1.651+1.31+1.1+1)V_6$

$V_1 = 2.8791V_6$.

Now, we have $V = V_1 + V_2 + \text{---} + V_6$

$\Rightarrow V = (2.8791 + 2.1571 + \text{---} + 1)V_6$

$\Rightarrow V = 10.0972V_6$

$\Rightarrow V_6 = 0.099V_6 = 9.903\%$ Voltage.

$V_5 = 10.89\%$ Voltage.

$V_4 = 12.97\%$ voltage.

$V_3 = 16.351\%$ voltage.

$V_2 = 21.3633\%$ voltages.

$V_1 = 28.5138\%$ voltage.

$\because V_1$ is maximum,

$\therefore \eta = \frac{V}{6 \times V_1} \times 100\%$

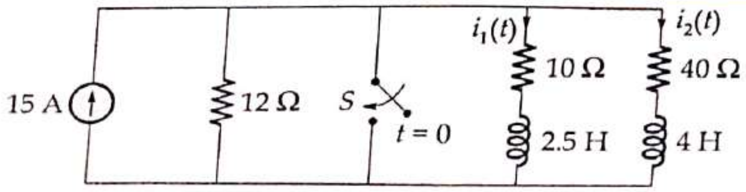
$$\Rightarrow \eta = \frac{V_1 + V_2 + \text{---} V_6}{6 \times V_1}$$

$$= \frac{(2.8791 + 2.1824 + \text{---} + 1.1 + 1) V_6}{6 \times 2.8791 \times V_1}$$

$$\boxed{\eta = 58.45\%}$$

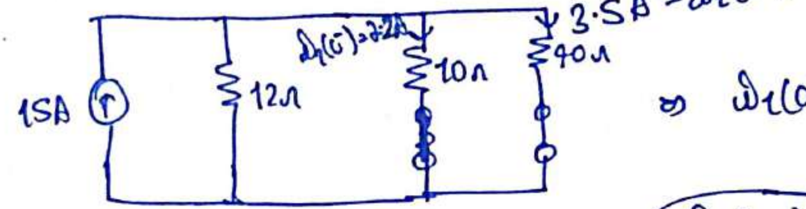
Section B : Systems and Signal Processing-1 + Microprocessor-1 + Electrical Circuits-2 + Control Systems-2

Q.5 (a) The switch 'S' in the circuit shown below is opened for a long time and closed at $t = 0$. Find the time domain expressions for currents $i_1(t)$ and $i_2(t)$ for $t > 0$.



[12 marks]

(Ans.) At $t = 0^-$, we have

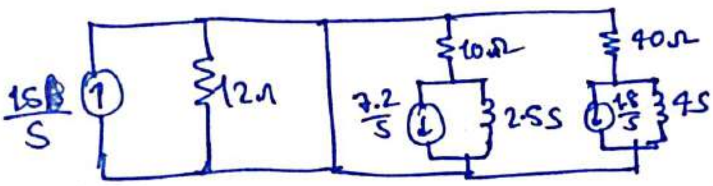


$$\Rightarrow i_1(0^-) = \frac{15 \times \left(\frac{1}{10}\right)}{\left(\frac{1}{12}\right) + \left(\frac{1}{10}\right) + \left(\frac{1}{40}\right)}$$

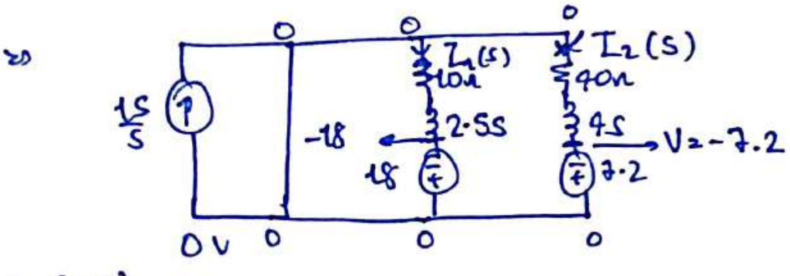
$$i_1(0^-) = 7.2\,A$$

$$i_2(0^-) = \frac{15\left(\frac{1}{40}\right)}{\left(\frac{1}{12}\right) + \left(\frac{1}{10}\right) + \left(\frac{1}{40}\right)} \Rightarrow i_2(0^-) = 1.8\,A$$

Now, at $t = 0^+$, we have



$\Rightarrow$



$$I_1(S) = \frac{0 - (-18)}{10 + 2.5S}$$

$$I_1(S) = \frac{\frac{18}{2.5}}{S + 4} = \frac{7.2}{S + 4}$$

$$\Rightarrow i_1(t) = 7.2e^{-4t}\,u(t)\,A$$

Similarly, we have

$$I_2(s) = \frac{0-(-7.2)}{40+4s}$$

$$I_2(s) = \frac{\frac{7.2}{4}}{(s+10)}$$

$$I_2(s) = \frac{1.8}{s+10}$$

$$\Rightarrow i_2(t) = 1.8e^{-10t}\,u(t)\,A$$

The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(s+2)}$$

The system is to have 25% maximum overshoot and peak time 1.0 second. Determine the value of K and tachometer feedback constant K_t.

[12 marks]

$$G(s) = \frac{K}{s(s+2)}$$

Characteristic equation is

$$s^2 + 2s + K = 0$$

Peak time $= \frac{\pi}{\omega_d} = 1$ sec.

Maximum peak overshoot $= 25\%$

$\Rightarrow \omega_d = \pi$ rad/sec.

& $e^{-\xi\pi/\sqrt{1-\xi^2}} = 25\%$

$$\Rightarrow \frac{-\xi\pi}{\sqrt{1-\xi^2}} = -1.3863$$

$$\frac{\xi^2}{1-\xi^2} = 0.19472$$

$$6.1356\,\xi^2 = 1$$

$$\xi = 0.4037$$

& $\omega_d = \pi$ rad/sec.

$$\Rightarrow \pi = \omega_n\sqrt{1-\xi^2}$$

$$\pi = \omega_n\sqrt{1-(0.4037)^2}$$

$$\Rightarrow \omega_n = 3.43386 \text{ rad/sec.}$$

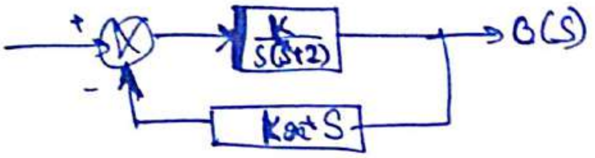
∴ we have characteristic equation as

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

$$\Rightarrow s^2+2\times 0.4037\times 3.43386s+(3.43386)^2=0$$

$$s^2+2.7725s+11.7914=0 \quad \text{—(i)}$$

Now, since feedback type of tachometer is not given in the question ∴ let us assume it to be as $\underline{s+k_t}$



$$\Rightarrow T(s)=\frac{\frac{K}{s(s+2)}}{1+\frac{(k_t+s)K}{s(s+2)}}=\frac{K}{s^2+2s+(k_t+s)K}$$

∴ Characteristic eqn $\Rightarrow s^2+(2+K)s+K(k_t+1)=0$ —(ii)

$$2+K=2.7725$$

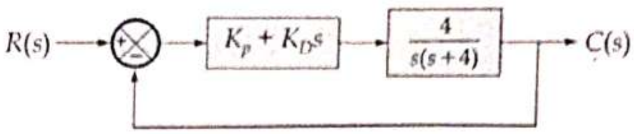
$$\Rightarrow K=0.7725$$

$$K(k_t+1)=11.7919$$

$$\Rightarrow 0.7725(1+k_t)=11.7919$$

$$\Rightarrow k_t=14.2646$$

(c) A control system with PD controller is shown below :



Determine the value of K_P and K_D such that the damping ratio of the system will be 0.75 and the steady state error for unit ramp input will be 0.20.

[12 marks]

(Ans.)

$e_{ss} = 0.2$

$$\text{OLTF} = \frac{4(K_p + K_D s)}{s(s+4)}$$

$$\lim_{s\to 0} \cancel{s}\frac{4(K_p + K_D s)}{\cancel{s}(s+4)} = \frac{4K_p}{4} = K_p = K_v$$

$$e_{ss} = \frac{1}{K_v} = 0.2 \Rightarrow K_v = 5$$

$$\therefore K_p = K_v = 5 \Rightarrow K_p = 5$$

Now, we have characteristic equation as

$$1 + \frac{(K_p + K_D s)4}{s(s+4)} = 0$$

$$s^2 + 4s + 4sK_D + 4K_p = 0$$

$$\Rightarrow s^2 + s(4 + 4K_D) + 4K_p = 0$$

$$\Rightarrow \omega_n^2 = 4K_p = 4 \times 5$$

$$\omega_n = \sqrt{20} \text{ rad/sec.}$$

$$2\zeta\omega_n = 4 + 4K_D$$

$$2 \times 0.75 \times \sqrt{20} = 4 + 4K_D$$

$$\Rightarrow K_D = 0.677$$

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5 (d) The Fourier transform $X(\omega)$ of a continuous time periodic signal $x(t)$ is given by

$$X(\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right)$$

Determine :

(i) The fundamental frequency of the signal $x(t)$.

(ii) The complex Fourier series coefficients of the signal $x(t)$.

(iii) The time domain expression of $x(t)$.

[12 marks]

(Ans.) (iii) $X(\omega) = j\,\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(j\,\delta\left(\omega - \frac{\pi}{3}\right) e^{j\omega t} + 2\delta\left(\omega - \frac{\pi}{7}\right) e^{j\omega t}\right) d\omega$$

$$x(t) = \frac{1}{2\pi}\left(\int_{-\infty}^{\infty} j\, e^{j\frac{\pi}{3}t}\, \delta\left(\omega - \frac{\pi}{3}\right) d\omega + \int 2\delta\left(\omega - \frac{\pi}{7}\right) e^{j\frac{\pi}{7}t}\, d\omega\right)$$

$$x(t) = \frac{1}{2\pi}\left(j\, e^{j\frac{\pi}{3}t} + 2\, e^{j\frac{\pi}{7}t}\right)$$

$$\boxed{x(t) = \frac{j}{2\pi} e^{j\frac{\pi}{3}t} + 2\, e^{j\frac{\pi}{7}t}}$$

(i) $\therefore\ \omega_1 = \frac{\pi}{3},\ \omega_2 = \frac{\pi}{7}$.

$T_1 = \frac{2\pi}{\pi/3} = 6\,\text{sec};\ T_2 = \frac{2\pi}{\pi/7} = 14\,\text{sec}$

$\therefore$ LCM $(6, 14) = 42$

$\therefore\ \left(\omega_0 = \frac{2\pi}{42} = \frac{\pi}{21}\,\text{rad/sec}\right) \Rightarrow \left(f_0 = 0.0238\,\text{rad/s}\right)$

$\therefore\ \omega_1 = 7\omega_0\ \ \&\ \ \omega_2 = 3\omega_0$

(ii). We have,

$$x(t) = \frac{j}{2\pi} e^{j\frac{7\pi}{3}t} + 2e^{j\frac{\pi}{3}t}$$

$$x(t) = 2e^{j(3\omega_0)t} + \frac{j}{2\pi} e^{j(7\omega_0)t}.$$

$\therefore$ We have, $C_1 = C_2 = 0$

$C_3 = 2 \Rightarrow |C_3| = 2$ & $\angle C_3 = 0°$.

$C_4 = C_5 = C_6 = 0$

$C_7 = \frac{j}{2\pi} \Rightarrow |C_7| = 0.1592$ & $\angle C_7 = 90°$.

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(e) Explain the following instructions:
(i) XCHG (ii) IN (iii) OUT (iv) DAA

[12 marks]

(i) XCHG ⇒

- It is an instruction of 8085 microprocessor
- It is used to exchange the content of register pair DE to Register pair HL.

(ii) IN

- It is used to access the externally connected input device as per I/O mapped I/O.
- It is used as IN XX H where XX represents the 8 bit address
- It transfers data from 8 bit address I/O to accumulator.
- Ex: IN 80H

(iii) OUT

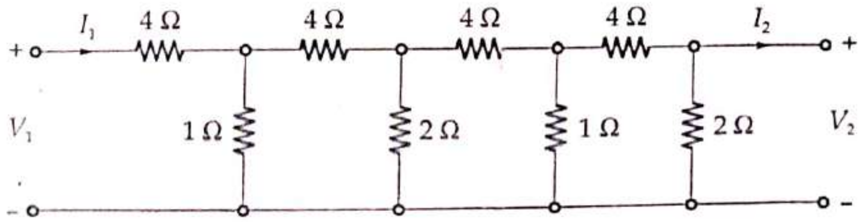
- It is used to access the externally connected output device as per I/O mapped I/O.
- It is used as OUT XX H where XX represent the 8 bit hexadecimal address.
- It transfers data from accumulator to 8 bit address of I/O output port.
- Ex: OUT 80H

(iv) DAA : ~~Decimal~~

- It is an instruction used in 8085 microprocessor.
- The fullform of this instruction is decimal adjust accumulator.
- It is used when the results of calculation are BCD and ~~...~~ hence ~~...~~ are differed values in ~~...~~ hexadecimal. It convert the same to a correct range.

6(a) **(i)** Two 2-port network are connected in cascade. Prove that the overall transmission parameter matrix equals to the multiplication of individual transmission parameter matrices.

(ii) Determine the transmission parameters of the 2-port network shown in the figure below:



[20 marks]

Q.6 (b) (i) Explain the similarities and differences between :

1. JUMP and CALL instructions.
2. STA and STAX instructions.

[10 marks]

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6(b) **(ii)** Write an assembly language program for an 8085 microprocessor, to find 2's complement of a 16-bit number. Write comments for selected instructions.

[10 marks]

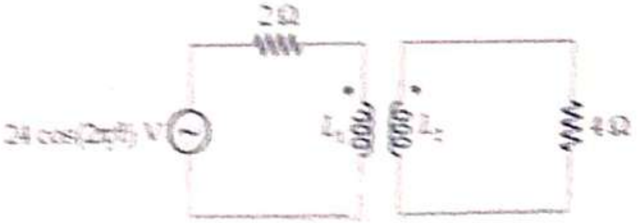
Do not write in this margin

6 (c) Check whether given signal $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}u(2t-n)$ is periodic. If yes, compute its average power.

[20 marks]

The coupled circuit shown below has a coefficient of coupling $K = 1$. Determine the energy stored in the mutually coupled inductor at $t = 5$ msec.

$L_1 = 3.185$ mH; $L_2 = 12.74$ mH; $f = 50$ Hz



[20 marks]

Obtain eigen values, eigen vectors and the state model in canonical form for a system described by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} u(t)$$

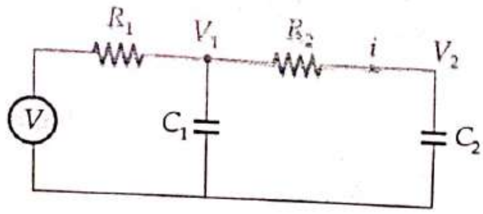
$$y(t) = [1 \ 0 \ 0]\, x(t)$$

[20 marks]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \left(\frac{1}{C_1 R_2}\right) \\ \left(\frac{1}{R_2 C_2}\right) & \left(\frac{-1}{R_2 C_2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C_1} \\ 0 \end{bmatrix} V$$

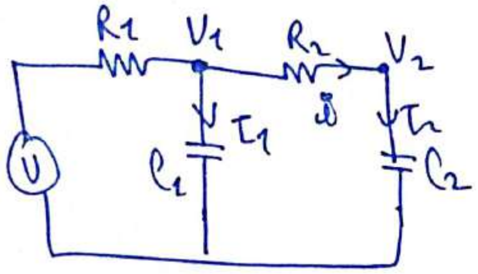
$$y = \begin{bmatrix} \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) Determine the state model for the network shown below considering $V_1 = x_1$; $V_2 = x_2$ and $y = i$.



[10 marks]

(Ans.):



$$i = \frac{V_1 - V_2}{R_2}$$

$$y = \left(\frac{1}{R_2}\right)x_1 + \left(\frac{-1}{R_2}\right)x_2$$

$$I_1 = C_1\frac{dV_1}{dt} = C_1\frac{dx_1}{dt} = C_1\dot{x}_1$$

$$I_1 = \frac{V - V_1}{R_1} + \frac{V_2 - V_1}{R_2}$$

$$I_1 = \frac{V - x_1}{R_1} + \frac{x_2 - x_1}{R_2}$$

$$C_1\dot{x}_1 = \left(\frac{-1}{R_1} - \frac{1}{R_2}\right)x_1 + \left(\frac{1}{R_2}\right)x_2 + \frac{V}{R_1}$$

$$\dot{x}_1 = -\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)x_1 + \left(\frac{1}{C_1R_2}\right)x_2 + \frac{V}{C_1R_1}.$$

$$\frac{V_1 - V_2}{R} = I_2 \Rightarrow C_2\dot{x}_2 = \left(\frac{1}{R_2}\right)x_1 + \left(\frac{-1}{R_2}\right)x_2$$

$$\dot{x}_2 = \left(\frac{1}{C_2R_2}\right)x_1 + \left(\frac{-1}{C_2R_2}\right)x_2$$

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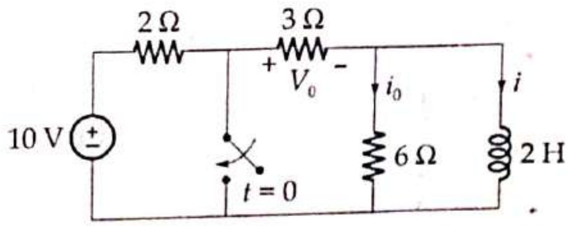
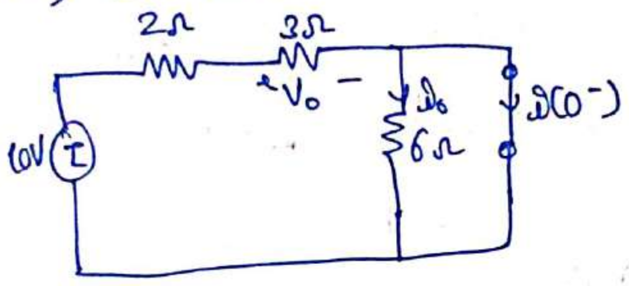
(a) (ii) In the circuit shown below:



Find i_0, V_0 and i for all time, assuming that the switch was open for a long time.

[10 marks]

Ans.): At $t < 0^-$, we have

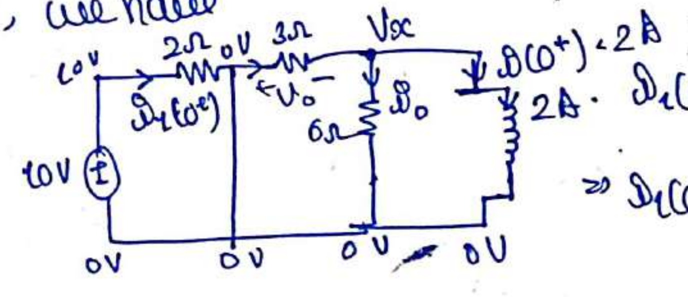


$\Rightarrow \quad i(0^-) = \frac{10}{2+3} = 2A.$

$i_0(0^-) = 0A.$

$V_0(0^-) = \frac{3}{2+3} \times 10 \Rightarrow V_0(0^-) = 6V.$

At $t = 0^+$, we have



$i(0^+) = 2A.$ $\quad i_1(0^+) = \frac{10}{2}$

$\Rightarrow i_1(0^+) = 5A.$

Now, $\frac{V_x}{6} + \frac{V_x}{3} + 2 = 0$

$\Rightarrow \frac{V_x}{2} + 2 = 0 \Rightarrow V_x = -4V.$

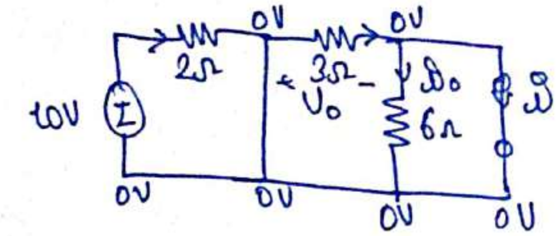
$\therefore \quad V_0(0^+) = 0 - (-4)$

$\Rightarrow V_0(0^+) = 4V.$

$i(0^+) = 2A$

$i_0(0^+) = \frac{V_x}{6} = -\frac{2}{3}\ A.$

At $t = \infty$, we have



$V_0(\infty) = 0 - 0 = 0V.$

$$i_0(\infty) = \frac{0-0}{6} = 0A$$

$$\therefore i(\infty) = \left(\frac{0-0}{3}\right) + \left(\frac{0-0}{6}\right)$$

$$i(\infty) = 0A$$

Now, we know that

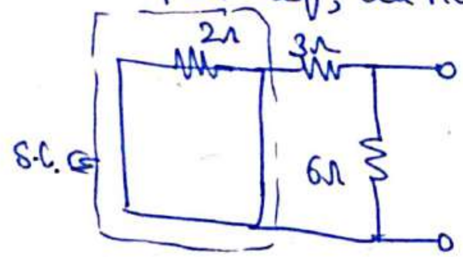
$$f(t) = f(\infty) + (f(0) - f(\infty))e^{-t/\tau}$$

$$\Rightarrow V_0(t) = V_0(\infty) + (V_0(t) - V_0(\infty))e^{-t/\tau}$$

$$= 0 + (4)e^{-t/1}$$

$$V_0(t) = 4e^{-t} \text{ V}.$$

For Req, we have



$$\Rightarrow R_{eq} = \frac{3 \times 6}{3+6}$$

$$\Rightarrow R_{eq} = 2\Omega.$$

$$\therefore \tau = \frac{L}{R} = \frac{2}{2} = 1 \text{ sec}$$

Similarly, $i_0(t) = 0 + \left(\frac{-2}{3} - 0\right)e^{-t}$

$$i_0(t) = -\frac{2}{3}e^{-t} \text{ A}$$

$$i(t) = 0 + (2)e^{-t}$$

$$i(t) = 2e^{-t} \text{ A}.$$

b) Consider a discrete time system with input $x(n)$ and output $y(n)$ related by

$$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$$

where n_0 is a finite positive integer

(i) Is this system linear?

(ii) Is this system time-invariant?

(iii) If $x(n)$ is known to be bounded by a finite integer B_x [i.e. $|x(n)| < B_x$ for all n], it can be shown that $y(n)$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B_x and n_0.

[20 marks]

(Ans.) (i) We have,

$$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$$

for $x_1(n) \rightarrow \sum_{k=n-n_0}^{n+n_0} x_1(n)$ — (i)

$$a x_1(n) \longrightarrow \sum_{k=n-n_0}^{n+n_0} a x_1(n) = a \sum_{k=n-n_0}^{n+n_0} x_1(n)$$

∴ homogeneous.

for $x_2(n) \longrightarrow \sum_{k=n-n_0}^{n+n_0} x_2(n)$ — (ii)

(i) + (ii), we have

$$x_1(n) + x_2(n) \longrightarrow \sum_{k=n-n_0}^{n+n_0} x_1(n) + \sum_{k=n-n_0}^{n+n_0} x_2(n)$$

$$\Rightarrow (x_1(n) + x_2(n)) \longrightarrow \sum_{k=n-n_0}^{n+n_0} (x_1(n) + x_2(n))$$

∴ the function is additive.

Hence the given function is <u>linear</u>.

(ii) we have

$$x(n) \longrightarrow \sum_{k=n-n_0}^{n+n_0} x(k)$$

$$x(n-n_0) \longrightarrow \sum_{k=n-n_0}^{n+n_0} x(k) \quad \text{(i)}.$$

while,

$$y(n-n_0) \longrightarrow \sum_{k=n-n_0-n_0}^{n-n_0+n_0} x(k)$$

$$= \sum_{k=n-2n_0}^{n} x(k) \quad \text{(ii)}.$$

$\because$ (i) $\neq$ (ii).

$\therefore$ the given system is time variant.

(iii). we have, $|x(n)| < B_x$

$\therefore$ for $y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$

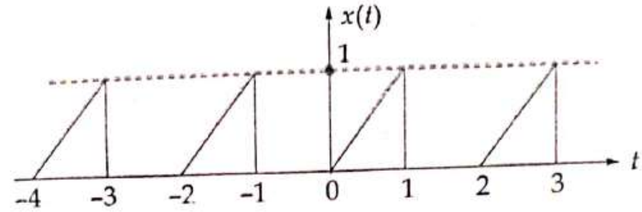
we have, $|y(n)| = \left|\sum_{k=n-n_0}^{n+n_0} x(k)\right|$

$$\Rightarrow \left|\sum_{k=n-n_0}^{n+n_0} x(k)\right| \leq \sum_{k=n-n_0}^{n+n_0} |x_k| \leq \sum_{k=n-n_0}^{n+n_0} B_x$$

$$\therefore y(n) \leq \sum_{k=n-n_0}^{n+n_0} B_x$$

$\therefore y_n \leq B_x(n+n_0-n+n_0) \Rightarrow \boxed{y_n \leq 2B_x n_0}$

Q.8 (c) Find the trigonometric Fourier series for the waveform shown in figure and sketch the line spectrum.



[20 marks]

(Ans.): We have,

$$x(t) = \begin{cases} t; & 0 \leq t < 1 \\ 0; & 1 < t < 2 \end{cases}$$

$T = 2$.

$$\therefore \quad a_0 = \frac{1}{T}\int_0^{T_0} f(t)\,dt$$

$$= \left(\frac{1}{2}\right)\left(\int_0^1 t\,dt + \int_1^2 0\,dt\right)$$

$$= \frac{1}{2}\left(\frac{t^2}{2}\right)_0^1 = \frac{1}{4} \Rightarrow a_0 = 1/4$$

Now, we have $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$.

$$a_n = \frac{2}{T}\int_0^{T_0} f(t)\cos n\omega_0 t\,dt.$$

$\omega_0 = \frac{2\pi}{2} = \pi$

$$\therefore a_n = \frac{2}{2}\left(\int_0^1 t\cdot\cos n\pi t\,dt + \int_1^2 0\,dt\right)$$

$$a_n = \int_0^1 t\cos n\pi t\,dt$$

$$a_n = \int_0^1 t \cos n\pi t \, dt$$

$n\pi t = y \Rightarrow dy = n\pi dt$

$$\int_0^{n\pi} \left(\frac{y}{n\pi}\right) \cos y \left(\frac{dy}{n\pi}\right) = \frac{1}{(n\pi)^2} \int_0^{n\pi} y \cos y \, dy$$

$$= \frac{1}{(n\pi)^2} \left(y \int \cos y - \int \int \cos y \, dy \right)$$

$$a_n = \frac{1}{(n\pi)^2} \left(y \sin y + \cos y \right)_0^{n\pi}$$

$$a_n = \frac{1}{(n\pi)^2} \left(n\pi \sin(n\pi) + \cos(n\pi) - 0 - 1 \right)$$

$$a_n = \frac{1}{(n\pi)^2} \left(\cos(n\pi) - 1 \right)$$

$$a_n = \begin{cases} \frac{-2}{(n\pi)^2} ; n = \text{odd}. \\ 0 \; ; n = \text{even}. \end{cases}$$

Now, $b_n = \frac{2}{T} \int_0^{T_0} f(t) \sin n\omega_0 t \, dt$

$$b_n = \int_0^1 t \sin n\pi t \, dt$$

$n\pi t = y \Rightarrow dy = n\pi dt$

$$b_n = \int_0^{n\pi} \frac{y}{n\pi} \sin y \left(\frac{dy}{n\pi}\right)$$

$$b_n = \frac{1}{(n\pi)^2} \int_0^{n\pi} y \sin y \, dy$$

$$b_n = \frac{1}{(n\pi)^2}\int_0^{n\pi} y \sin y \, dy$$

$$b_n = \frac{1}{(n\pi)^2}\left(y\int \sin y - \int\int \sin y \, dy\right)$$

$$b_n = \frac{1}{(n\pi)^2}\left(-y\cos y + \sin y\right)_0^{n\pi}$$

$$b_n = \frac{1}{(n\pi)^2}\left(-(n\pi)\cos(n\pi) + 0 + \sin(n\pi) - 0\right)$$

$$b_n = \frac{-1}{n\pi}\cos(n\pi)$$

$$b_n = \begin{cases} \frac{1}{n\pi} & ; n = \text{odd} \\ \frac{-1}{n\pi} & ; n = \text{even} \end{cases} \Rightarrow b_n = \frac{(-1)^{n+1}}{n\pi}$$

$$\therefore f(t) = \frac{1}{4} + \sum_{n=1,3,5\ldots} \frac{(-2)}{(n\pi)^2}\cos n\pi t + \sum_{n=1,2,3\ldots}^{\infty} \frac{(-1)^{n+1}}{n\pi}\sin n\pi t$$

$$f(t) = \frac{1}{4} + \left(\frac{-2}{\pi^2}\right)\cos \pi t + \frac{1}{\pi}\sin \pi t$$
$$+ \left(\frac{-1}{\pi}\right)\sin 2\pi t$$
$$+ \frac{-2}{9\pi^2}\cos 3\pi t + \left(\frac{1}{\pi}\right)\sin 3\pi t$$

$$\Rightarrow |C_1| = \sqrt{\left(\frac{2}{\pi^2}\right)^2 + \left(\frac{1}{\pi}\right)^2} \Rightarrow C_1 = \sqrt{\frac{4}{\pi^4} + \frac{1}{\pi^2}} \Rightarrow |C_1| = 0.5513$$

$$\angle C_1 = 122.481^\circ$$

$$|C_2| = \frac{1}{\pi} \qquad \angle C_2 = 180^\circ$$

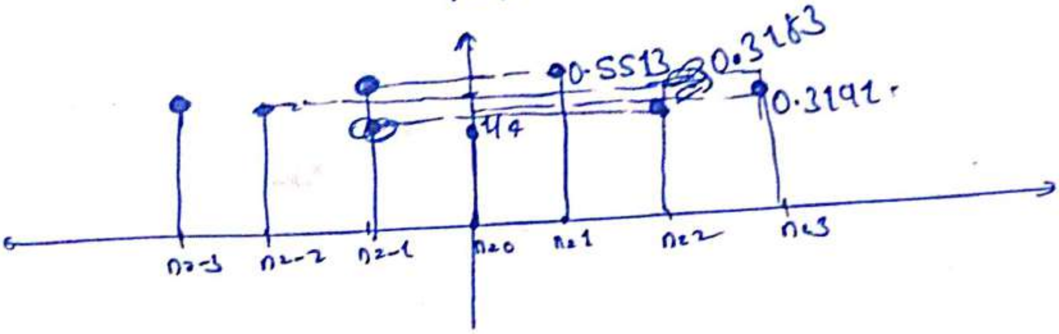
$$|C_3| = \sqrt{\left(\frac{2}{9\pi^2}\right)^2 + \left(\frac{1}{\pi}\right)^2} \Rightarrow |C_3| = 0.3191 \qquad \angle C_3 = \tan^{-1}\left(\frac{\frac{1}{\pi}}{\frac{-2}{9\pi^2}}\right)$$

oooo

Space for Rough Work

$\angle C_3 = 94.0461°$

$|C_n|$



$\angle C_n$