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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 25/01/2024

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (d) | 13. (d) | 19. (d) | 25. (b) |
| 2. (c) | 8. (a) | 14. (a) | 20. (a) | 26. (c) |
| 3. (c) | 9. (a) | 15. (a) | 21. (c) | 27. (a) |
| 4. (a) | 10. (b) | 16. (d) | 22. (d) | 28. (b) |
| 5. (a) | 11. (c) | 17. (d) | 23. (c) | 29. (d) |
| 6. (d) | 12. (d) | 18. (d) | 24. (b) | 30. (a) |

DETAILED EXPLANATIONS

1. (b)

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x-1) = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$

$\Rightarrow f$ is continuous at $x = 1$

And $Lf'(1) = 2, Rf'(1) = 1$

$\Rightarrow f$ is not differentiable at $x = 1$

2. (c)

$$\begin{aligned} \int_0^5 f(x) \cdot dx &= \frac{h}{2} [(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)] \\ &= \frac{1}{2} [(2 + 0.077) + 2(1 + 0.4 + 0.2 + 0.1176)] \\ &\simeq 2.75 \end{aligned}$$

3. (c)

Using Crout's method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$l_{11} = 2 \qquad l_{11} u_{12} = 4$$

$$u_{12} = \frac{4}{2} = 2$$

$$l_{21} = 6 \qquad l_{21} u_{12} + l_{22} = 3$$

$$6 \times 2 + l_{22} = 3$$

$$l_{22} = 3 - 12$$

$$l_{22} = -9$$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Note: Candidates can use options to solve such questions.

4. (a)

$$\begin{aligned} P(-1 \leq x \leq 1) &= \int_{-1}^1 (0.1) dx \\ &= 2 \times \frac{1}{10} = \frac{1}{5} \end{aligned}$$

5. (a)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\log x}{\cot x} &= \lim_{x \rightarrow 0} \frac{1}{x} = -\lim_{x \rightarrow 0} \frac{\sin^2 x}{x} \quad \left(\text{from } \frac{\infty}{\infty} \right) \\ &= -\lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{1} = 0 \quad \left(\text{from } \frac{0}{0} \right) \end{aligned}$$

6. (d)

$$\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \\ x = \frac{\pi}{2} &\Rightarrow u = \sin \frac{\pi}{2} = 1 \\ x = -\pi &\Rightarrow u = \sin(-\pi) = 0 \\ \int_{-\pi}^{\pi/2} \cos(x) \cos(\sin(x)) \, dx &= \int_0^1 \cos u \, du \\ &= \left[\sin u \right]_0^1 \\ &= (\sin 1) - \sin(0) = \sin 1 \end{aligned}$$

7. (d)

$$\begin{aligned} \frac{e^x}{(1-e^x)} dx + \frac{\sec^2 y}{\tan y} dy &= 0 \\ \text{Integrating on both sides, we get,} \\ -\ln(1-e^x) + \ln(\tan y) &= C_1 \\ \ln \left(\frac{\tan y}{(1-e^x)} \right) &= C_1 \\ \frac{\tan y}{(1-e^x)} &= e^{C_1} = C \\ \tan y &= C(1-e^x) \end{aligned}$$

8. (a)

$$(D^2 + D)y = x^2 + 2x + 8$$

The particular integral is,

$$\begin{aligned} P I &= \frac{x^2 + 2x + 8}{D(1+D)} \\ &= \frac{1}{D} (1+D)^{-1} (x^2 + 2x + 8) = \frac{1}{D} (1 - D + D^2 - D^3 + \dots)(x^2 + 2x + 8) \\ &= \frac{1}{D} (x^2 + 2x + 8 - 2x - 2 + 2) = \frac{1}{D} (x^2 + 8) = \frac{x^3}{3} + 8x \end{aligned}$$

9. (a)

$$\begin{aligned} \ln y &= \sin^{-1}x, & \ln z &= -\cos^{-1}x \\ \ln y - \ln z &= \sin^{-1}x + \cos^{-1}x \\ \ln\left(\frac{y}{z}\right) &= \frac{\pi}{2} \\ y &= ze^{\pi/2} \\ \frac{dy}{dz} &= e^{\pi/2} \\ \frac{d^2y}{dz^2} &= 0 \end{aligned}$$

10. (b)

We have

$$\begin{aligned} y &= e^x (A \cos x + B \sin x) \\ y' &= e^x (A \cos x + B \sin x) + e^x (-A \sin x + B \cos x) \\ &= y + e^x [-A \sin x + B \cos x] \\ y'' &= y' + e^x (-A \sin x + B \cos x) + e^x (-A \cos x - B \sin x) \\ &= y' + y' - y - y = 2y' - 2y \end{aligned}$$

 \Rightarrow

$$\begin{aligned} \text{Order} &= 2 \\ \text{Degree} &= 1 \end{aligned}$$

11. (c)

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^x - x}{x - 1 - \log x} & \quad \left(\frac{0}{0} \text{ form} \right) \\ &= \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(x^x) - 1}{1 - 0 - \frac{1}{x}} \end{aligned}$$

Let,

$$\begin{aligned} y &= x^x \\ \log y &= x \log x \end{aligned}$$

$$\therefore \frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \log x$$

or

$$\begin{aligned} \frac{d}{dx}(x^x) &= x^x(1 + \log x) \\ &= \lim_{x \rightarrow 1} \frac{x^x(1 + \log x) - 1}{1 - \frac{1}{x}} \quad \left(\frac{0}{0} \text{ form} \right) \\ &= \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(x^x) \cdot (1 + \log x) + x^x \left(\frac{1}{x} \right) - 0}{\frac{1}{x^2}} \\ &= \lim_{x \rightarrow 1} \frac{x^x(1 + \log x)^2 + x^x \left(\frac{1}{x} \right)}{x^{-2}} = \frac{1(1+0)^2 + 1 \cdot 1}{1} = 2 \end{aligned}$$

12. (d)

Let $u = \sqrt{x}$

Then $du = \frac{1}{2\sqrt{x}} dx$

$\therefore dx = du \cdot 2\sqrt{x}$

$$\begin{aligned} \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx &= \int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x} du = 2 \int \sin u du \\ &= -2 \cos \sqrt{x} + c \end{aligned}$$

13. (d)

$$\begin{aligned} I &= \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right) dx \\ &= \int_0^{\pi/2} [\log(\sin x) dx - \log(\cos x) dx] \\ &= \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx - \int_0^{\pi/2} \log(\cos x) dx \\ I &= 0 \end{aligned}$$

14. (a)

$$\begin{aligned} |\lambda - AI| &= (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda = -\lambda^3 + \lambda^2 \\ \Rightarrow -\lambda^3 + \lambda^2 &= 0 \\ \Rightarrow -\lambda^2(\lambda - 1) &= 0 \\ \lambda &= 0, \lambda = 1 \end{aligned}$$

The largest eigen value is 1

$$\begin{aligned} A - I &= \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2} \\ \Rightarrow &\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_3 + R_1} \\ \Rightarrow &\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2} \\ \Rightarrow &\begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2} \\ \Rightarrow &\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$[A - I]\vec{x} = 0$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$$

$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$

$$\therefore x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

15. (a)

$$\text{Parabola given : } x^2 = 4y \quad \dots(i)$$

$$\text{Straight line is } x - 2y + 4 = 0$$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$

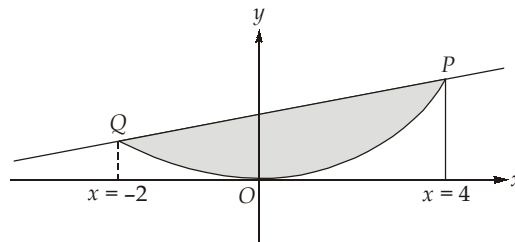
$$\Rightarrow x^2 = 2(x+4)$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x-4) + 2(x-4) = 0$$

$$\Rightarrow x = 4, -2$$



$$\text{Required area} = POQ$$

$$= \int_{-2}^4 y \, dx \text{ from straight line} - \int_{-2}^4 y \, dx \text{ from parabola}$$

$$= \int_{-2}^4 \left(\frac{x+4}{2} \right) - \int_{-2}^4 \frac{x^2}{4} \, dx$$

$$= \frac{1}{2} \left[\frac{x^2}{2} + 4x \right]_{-2}^4 - \frac{1}{4} \left[\frac{x^3}{3} \right]_{-2}^4$$

$$= \frac{1}{2} \{8 + 16 - (-6)\} - \frac{1}{12} (64 + 8)$$

$$= \frac{1}{2} \times 30 - \frac{1}{12} \times 72 = 15 - 6 = 9$$

16. (d)

$$AX = B$$

$$\text{Augmented matrix, } [A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_2 + R_1:$$

$$|A : B| = \left| \begin{array}{ccc|c} -2 & 1 & 1 & l \\ 1 & -2 & 1 & m \\ 0 & 0 & 0 & l+m+n \end{array} \right|$$

Since, $l + m + n = 0$
 Rank of $[A : B] = 2$
 Rank of $[A] = \text{Rank of } [A : B] = 2 < 3$ (Number of variables)
 \Rightarrow Infinitely many solutions are possible.

17. (d)

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$X_1 = c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{aligned} x_1 + x_3 &= 0 \\ 2x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

$$X_2 = c_2 \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 = -x_2$$

$$x_1 = \frac{-1}{2}x_3$$

$$X_3 = c_3 \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Since, $X_1^T X_2 \neq 0$
 $X_2^T X_3 \neq 0$
 $X_3^T X_1 \neq 0$

None of the above is correct.

18. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

$$x = -1, 2$$

$$f''(x) = 12x - 6$$

$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$

$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$

The function has maxima at $x = -1$ and minima at $x = 2$.Critical point $(-1, 2)$ draw plot on line graph:Since $0 \in (-1, 2)$ and $f'(0) = 6 \times 0^2 - 6 \times 0 - 12 = -12 < 0$ The function is decreasing between -1 and 2 .

19. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

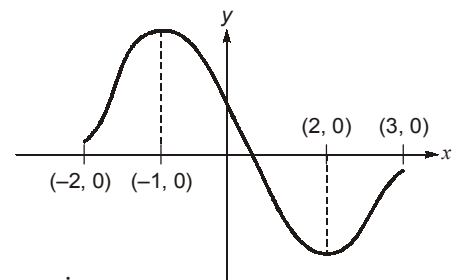
$$(x + 1)(x - 2) = 0$$

$$x = -1, 2$$

$$f''(x) = 12x - 6$$

$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$

$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$

The function has maxima at $x = -1$ and minima at $x = 2$.The function is decreasing between -1 and 2 .

20. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$

$$\log \frac{1}{\cos x \cos y} = C_1$$

$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

⇒ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

21. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, the given equation is not exact.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2 + 1)} = \frac{1}{y}$$

$$IF = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

22. (d)

$$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 6x$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = -6xy - 6y$$

$$\begin{aligned} dv &= \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy \\ &= (6xy + 6y)dx + (3x^2 - 3y^2 + 6x)dy \\ v &= 3x^2y + 6xy - y^3 + C \end{aligned}$$

23. (c)

If the two curves intersect, then at point of intersection,

$$3x^3 + 2x^2 + 8x - 5 = 2x^3 + 3x + 2$$

$$x^3 + 2x^2 + 5x - 7 = 0$$

$$f(x) = x^3 + 2x^2 + 5x - 7$$

$$f(0) = 0 + 0 + 0 - 7 = -7 < 0$$

$$f(1) = 1 + 2 + 5 - 7 = 1 > 0$$

⇒ One root lies between 0 and 1. Let us assume 1 as initial value.

$$f'(x) = 3x^2 + 4x + 5$$

$$x_1 = 1 - \frac{f(x)}{f'(x)} \Big|_{x=1} = 1 - \frac{1^3 + 2 \times 1^2 + 5 \times 1 - 7}{3 \times 1^2 + 4 \times 1 + 5} = 0.9167$$

$$x_2 = x_1 - \frac{f(x)}{f'(x)} \Big|_{x=0.9167} = 0.9136$$

24. (b)

Number of ways of throwing 6 is five $\Rightarrow (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)$ Number of ways of throwing 7 is six $\Rightarrow (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)$

$$\text{Probability of throwing 6, } p_1 = \frac{5}{36}$$

$$\text{Probability of failing to throw 6, } p_2 = 1 - \frac{5}{36} = \frac{31}{36}$$

$$\text{Probability of throwing 7, } q_1 = \frac{6}{36}$$

$$\text{Probability of failing to throw 7, } q_2 = 1 - \frac{6}{36} = \frac{30}{36}$$

$$\begin{aligned} \text{Probability of } B \text{ winning} &= p_2 q_1 + p_2 q_2 p_2 q_1 + p_2 q_2 p_2 q_2 p_2 q_1 + \dots \\ &= p_2 q_1 [1 + p_2 q_2 + (p_2 q_2)^2 + (p_2 q_2)^3 + \dots] \\ &= \frac{p_2 q_1}{(1 - p_2 q_2)} = \frac{\frac{31}{36} \times \frac{6}{36}}{1 - \frac{31}{36} \times \frac{30}{36}} = \frac{31 \times 6}{366} = \frac{31}{61} \end{aligned}$$

25. (b)

$$\phi_1 = ax^2 - byz - (a + 2)x$$

$$\nabla \phi_1 = [2ax - (a + 2)]\hat{i} - bz\hat{j} - by\hat{k}$$

$$\nabla \phi_1(1, -1, 2) = (a - 2)\hat{i} - 2b\hat{j} + b\hat{k}$$

$$\phi_2 = 4x^2y + z^3 - 4$$

$$\nabla \phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla \phi_2(1, -1, 2) = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

Since surfaces are orthogonal to each other at $(1, -1, 2)$

$$\nabla \phi_1 \cdot \nabla \phi_2 = 0$$

$$[(a - 2)\hat{i} - 2b\hat{j} + b\hat{k}] \cdot [-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$$

$$-8(a - 2) - 8b + 12b = 0$$

... (i)

Also point $(1, -1, 2)$ lies on the surface.

$$\Rightarrow a \times 1 + 2b = (a + 2)1$$

$$b = 1$$

Putting this in equation 1, we get,

$$-8(a - 2) - 8 + 12 = 0$$

$$a - 2 = -\frac{1}{8} \times (-4) = 0.5$$

$$a = 2.5$$

26. (c)

Case-I: White ball is transferred from urn A to urn B

$$\text{Probability of drawing white ball from } B = \frac{2}{2+4} \times \frac{6}{13} = \frac{2}{13}$$

Case-II: Black ball is transferred from A to B

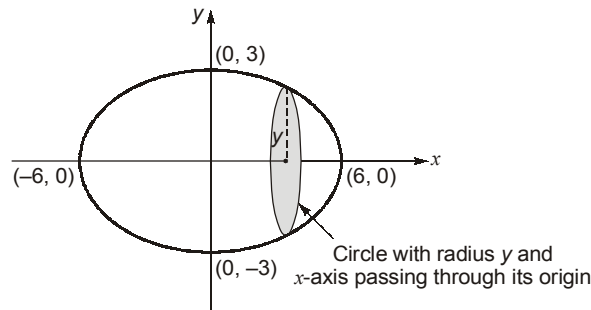
$$\text{Probability of drawing black ball from B} = \frac{4}{2+4} \times \frac{5}{13} = \frac{10}{39}$$

$$\text{Required probability} = \frac{2}{13} + \frac{10}{39} = \frac{16}{39}$$

27. (a)

$$\begin{aligned} \text{Mean} &= \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x^2 dx + \int_1^2 (2-x)x dx \\ &= \left. \frac{x^3}{3} \right|_0^1 + \left(x^2 - \frac{x^3}{3} \right) \Big|_1^2 = \frac{1}{3} + 4 - 1 - \frac{8-1}{3} = 1 \end{aligned}$$

28. (b)



$$\begin{aligned} \text{Volume generated} &= \int_{-6}^6 \pi y^2 dx = \int_{-6}^6 \pi \left(\frac{36-x^2}{4} \right) dx \\ &= \frac{\pi \times 2}{4} \int_0^6 (36-x^2) dx = \frac{\pi}{2} \left[36x - \frac{x^3}{3} \right]_0^6 \\ &= 72\pi \end{aligned}$$

29. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF \cdot f(x) \cdot f'(x) dx$$

$$y \times e^{f(x)} = \int e^{f(x)} \cdot f(x) \cdot f'(x) dx$$

Let

$$f(x) = t$$

$$f'(x) dx = dt$$

$$y \times e^t = \int e^t \cdot t dt$$

$$y \cdot e^t = t \cdot e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log [y + 1 - f(x)] + f(x) = c'$$

30. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2 + 4)} = 96 \frac{1}{4D^2 \left(1 + \frac{D^2}{4}\right)} x^2 = \frac{96}{4} \left[\frac{\left(1 - \frac{D^2}{4}\right) x^2}{D^2} \right]$$

$$= 24 \frac{\left(x^2 - \frac{1}{2}\right)}{D^2}$$

$$PI = 24 \left[\frac{x^4}{4 \times 3} - \frac{x^2}{4} \right] = 2x^2(x^2 - 3)$$

$$PI|_{x=2} = 2 \times 2^2(4 - 3) = 8$$

