

OPSC

2019

Odisha Public Service Commission Assistant Engineer Examination

Civil Engineering

Structural Analysis

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**



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Structural Analysis

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Stability and Indeterminacy

1.1 Support System

1.1.1 2-D Supports

(a) Fixed Support

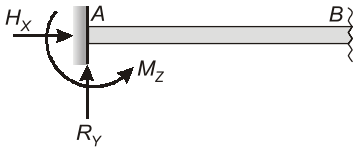


Fig. 1.1 (i) Number of reaction = 3

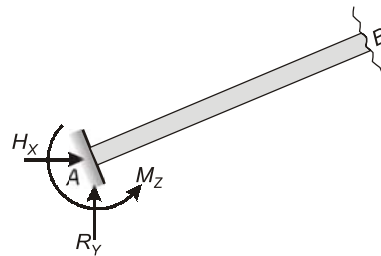


Fig. 1.1 (ii) Number of reactions = 3

At 2-D fixed support, there can be three reactions:

- (i) one vertical reaction (R_y)
- (ii) one horizontal reaction (H_x)
- (iii) one moment reaction (M_z)

(b) Hinge Support

Hinge support is represented by the symbol .

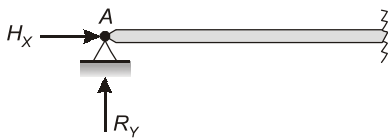


Fig. 1.2 (i) Number of reactions = 2

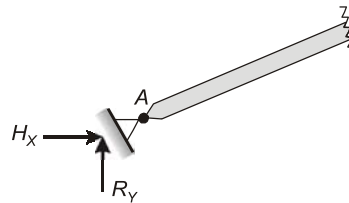



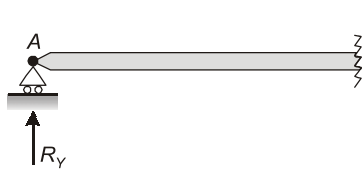
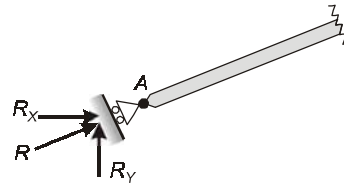
Fig. 1.2 (ii) Number of reactions = 2

At hinged support, there can be two reactions:

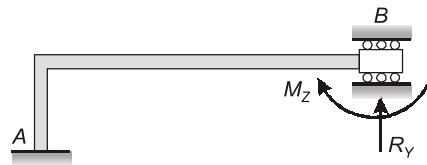
- (i) one horizontal reaction (H_x)
- (ii) one vertical reaction (R_y)

(c) Roller Support

Roller support is represented by the symbol  or .

**Fig. 1.3 (i)** Number of reactions = 1**Fig. 1.3 (ii)** Number of reactions = 1

At roller support there can be only one externally independent reaction which is normal to the contact surface.

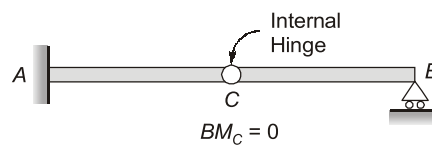
(d) Guided Roller Support**Fig. 1.4** Number of reactions = 2

At guided roller supports there can be two reactions:

- (i) one vertical reaction (R_y)
- (ii) one moment reaction (M_z)

1.1.2 2-D Internal Joints**(a) Internal Hinge**

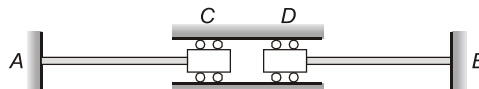
At internal hinge bending moment will be zero.

**Fig. 1.5**

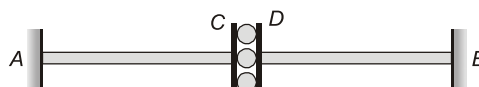
NOTE: An internal hinge provides one additional equilibrium equation for structures.

(b) Internal Roller

At internal roller either axially force or shear force will be zero.

**Fig. 1.6**

In fig. 1.6, axially force at C and D is zero.

**Fig. 1.7**

In fig. 1.7, shear force at C and D will be zero i.e., $S_C = S_D = 0$

(c) Internal Link

If any member is connected by hinges at its end and subjected to no external loading in between then it can be termed as internal link and carry axial force only.

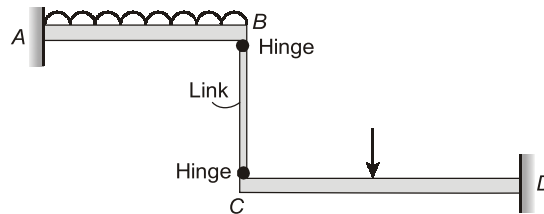


Fig. 1.8

Here BC is a link, link BC carry only axial force

Also $BM_B = 0$ and $BM_C = 0$

NOTE: Internal release also provides additional equation for analysis of structure.

1.1.3 3-D Supports

(a) Fixed Support

At 3-D fixed support there can be six reactions:

(i) three reactions R_x , R_y and R_z

(ii) three moment reactions M_x , M_y and M_z

The fixed support are also called **Built-in support**.

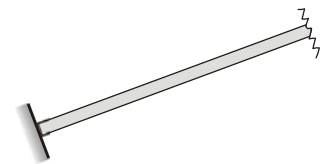


Fig. 1.9: Number of reactions = 6

(b) 3-D Hinged Support

At 3-D hinged support there can be three reactions

(i) R_x

(ii) R_y

(ii) R_z

The 3-D hinged support is also called 'ball and socket joint'.

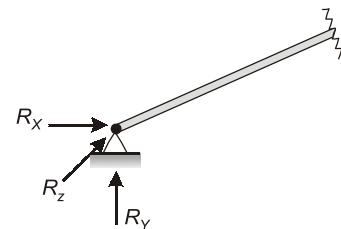


Fig. 1.10 Number of reactions = 3

(c) Roller Support

At 3-D roller support there can be only one externally independent reaction which is perpendicular to the contact surface

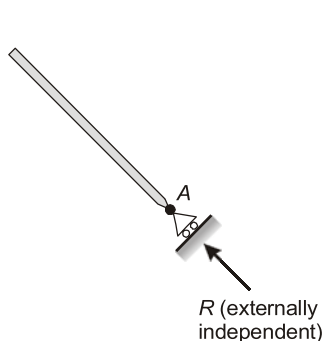


Fig. 1.12 (i)

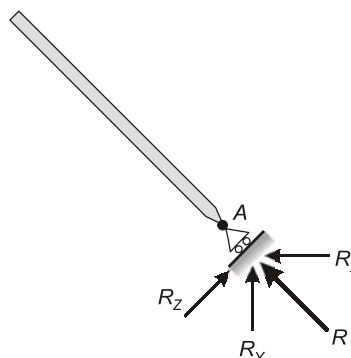


Fig. 1.12 (ii)

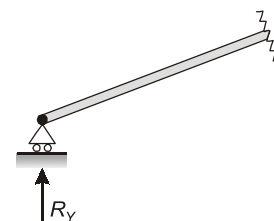


Fig. 1.11 Number of reactions = 1

in figure 1.12.(ii), reactions at roller support A , R_x , R_y and R_z are externally dependent reactions which depends on reaction R .

1.2 Structure

1.2.1 Elements of Structure

Some of the major elements of structure by which structures are fabricated are as follows:

(a) Beams: Beams are structural members which is predominantly subjected to bending. On the basis of support system beams can be classified as:

(i) Simply supported beam

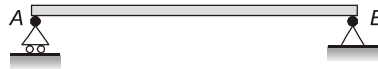


Fig. 1.13

(ii) Cantilever beam



Fig. 1.14

(iii) Propped cantilever

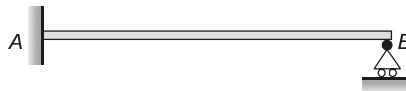


Fig. 1.15

(iv) Fixed beam



Fig. 1.16

(v) Continuous beam

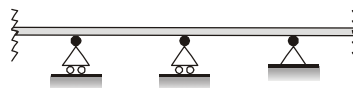


Fig. 1.17

(b) Columns: A column is a vertical compression member which is slender and straight. Generally columns are subjected to axial compression and bending moment as shown in figure.

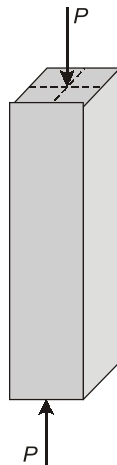


Fig. 1.18 (i)

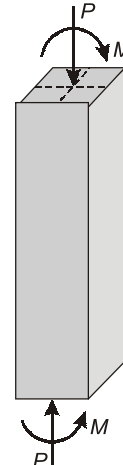


Fig. 1.18 (ii)

- (c) **Tie Members:** Tie members are tension members of trusses and frame, which are subjected to axial tensile force. (Figure : 1.19)



Fig. 1.19 Tie Rod

1.2.2 Types of Structures

- (a) **Trusses:** A truss is constructed from pin jointed slender members, usually arranged in triangular manner. In trusses, loads are applied on joints due to which each member of truss subjected to only axial forces i.e., either axial compression or axial tension. Generally trusses are used when span of structure is large.

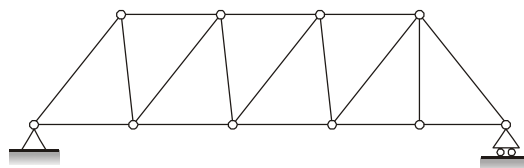


Fig. 1.20 Truss

- (b) **Frames:** A frame is constructed from either pin jointed or fixed jointed beam and columns. Generally loads are applied on beams and this loading causes axial force, shear force and bending to the members of frame.

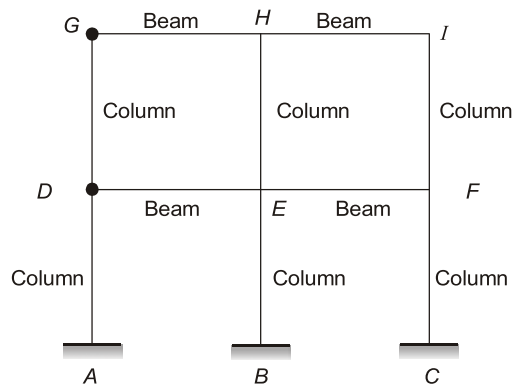


Fig. 1.21 Frames

- (c) **Arches:** Arches are used in bridges, dome roof, auditorium, where span of structures are relatively more due to external loading, Arch can be subjected to axial compression, shear force or bending moment.

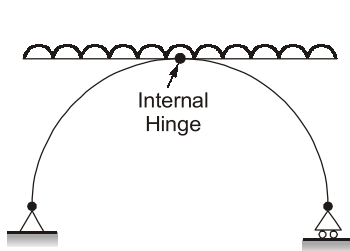


Fig. 1.22 (i) Three Hinge Arch

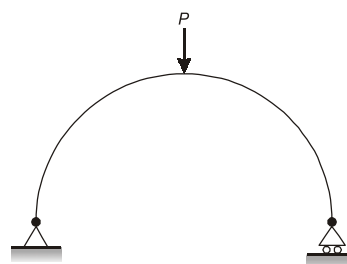


Fig. 1.22 (ii) Two Hinge Arch

- (d) **Cables:** Cables are used to support long span bridges. Cables are flexible members and due to external loading it is subjected to axial tension only.

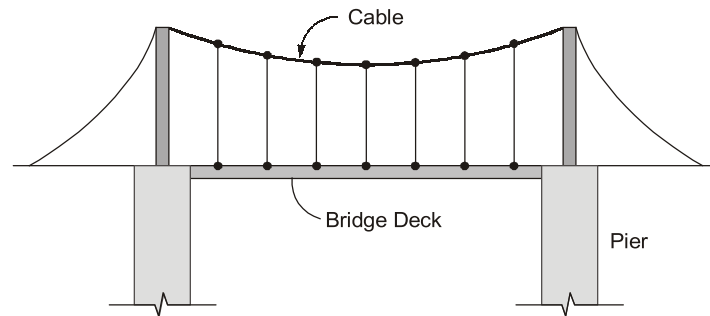


Fig. 1.23 Cable and Bridge

1.3 Types of Loading

- (a) **Point load:** A point load is considered to be acting at a point. It is also called concentrated load. In actual practice point loads are distributed load which are distributed over very small area.

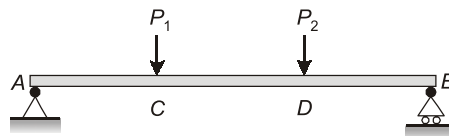


Fig. 1.24 Point Load

- (b) **Distributed loads:** Distributed loads are those loads, which acts over some measurable area. Distributed loads are measured by the intensity of loading per unit length along the beam.

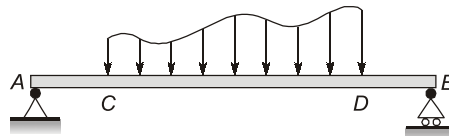


Fig. 1.25 Distributed Loads

- (c) **Uniformly distributed loads:** Uniformly distributed loads are those distributed loads which have uniform intensity of loading over the area.

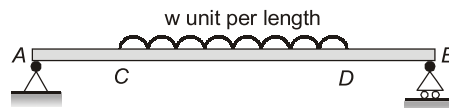


Fig. 1.26 Uniformly Distributed Loads

- (d) **Uniformly varying loads:** A uniformly varying load, commonly abbreviated as UVL, is the one in which the intensity of loading varies from one end to other. For example, intensity is zero at one end and w at other end.

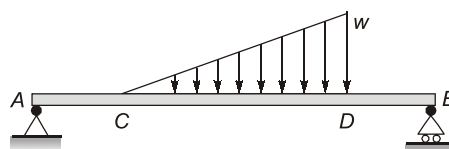


Fig. 1.27 Uniformly Varying Loads

- (e) **Couple :** A system of forces with resultant moment, but no resultant force is called couple. It is statically equivalent to force times the offset distance.

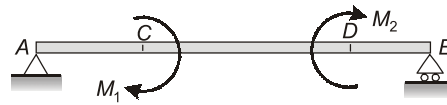


Fig. 1.28 Couple

1.4 Stability of Structures

Structural stability is the major concern of the structural designer. To ensure the stability, a structure must have enough support reaction along with proper arrangement of members. The overall stability of structures can be divided into

- (i) External stability
- (ii) Internal stability

1.4.1 External Stability

- (a) **2-D Structures:** For stability of 2-D structures there should be no rigid body movement of structure due to loading so, it should have support in x -direction, y -direction and no rotation in x - y plane. So there should be enough reactions to restrain the rigid body motion.

For stability of 2-D structures, following three conditions of static equilibrium should be satisfied.

- (i) $\Sigma F_x = 0$ (To prevent Δ_x)
- (ii) $\Sigma F_y = 0$ (To prevent Δ_y)
- (iii) $\Sigma M_z = 0$ (To prevent θ_z)

For stability in 2-D structures following conditions also be satisfied:

- (i) There should be minimum three number of externally independent support reaction.
- (ii) All reactions should not be parallel, otherwise linearly instability will set up.

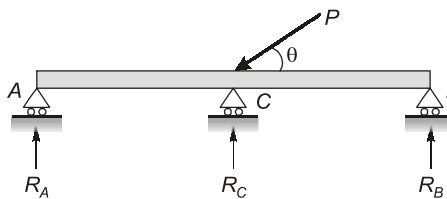


Fig. 1.29 Unstable

- (iii) All reactions should not be linearly concurrent otherwise rotational instability will setup.

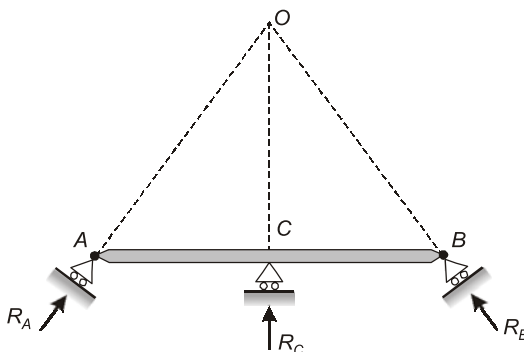


Fig. 1.30 (i) Unstable

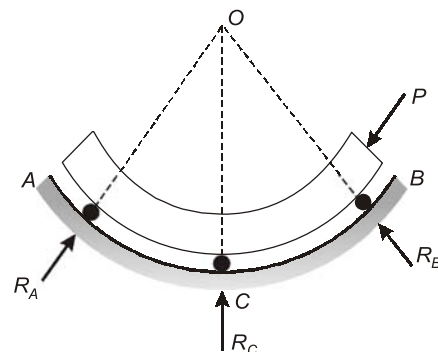


Fig. 1.30 (ii) Unstable

(iv) Reactions should be non-trivial i.e. there should be enough magnitude and enough difference between them.

(b) 3-D Structures: In case of 3-D structures, there should be a minimum of six independent external reactions to prevent rigid body displacement of structure. The displacement to be prevented are: $\Delta_x, \Delta_y, \Delta_z, \theta_x, \theta_y$ and θ_z . Therefore, there will be six equation of static equilibrium.

(i) $\Sigma F_x = 0$

(ii) $\Sigma F_y = 0$

(iii) $\Sigma F_z = 0$

(iv) $\Sigma M_x = 0$

(v) $\Sigma M_y = 0$

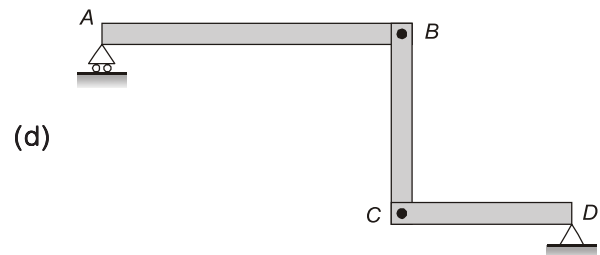
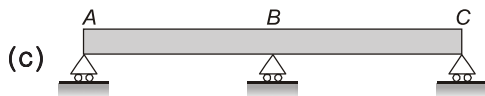
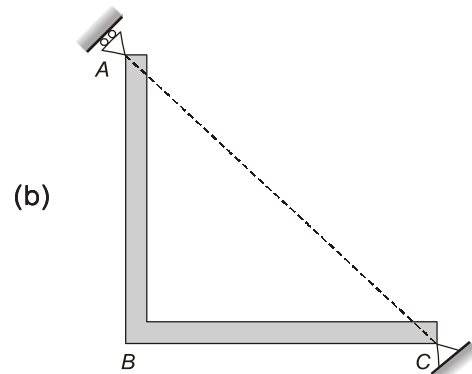
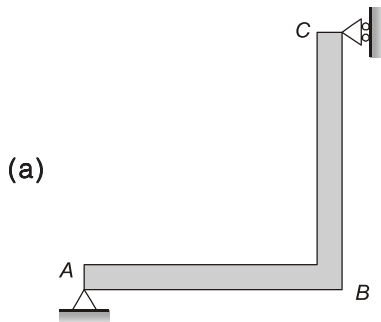
(vi) $\Sigma M_z = 0$

For stability in 3-D structures, all the reactions should be non-coplanar, non-concurrent and non-parallel.

NOTE: If a structure is constructed from elastic members then small elastic displacement may be permitted but small rigid body displacement will not be permitted.

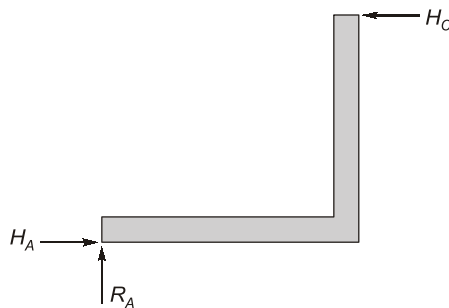
Example -1.1

Which one of the following structures is stable?

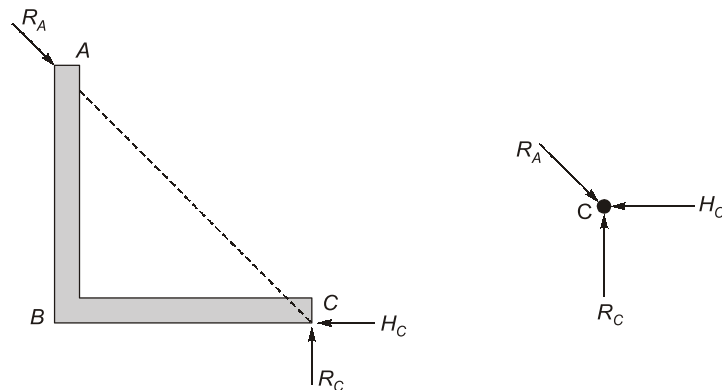


Solution:(a)

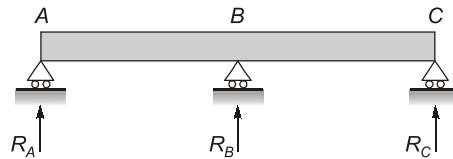
Member (a) is stable, since reactions are non-parallel and non-concurrent.



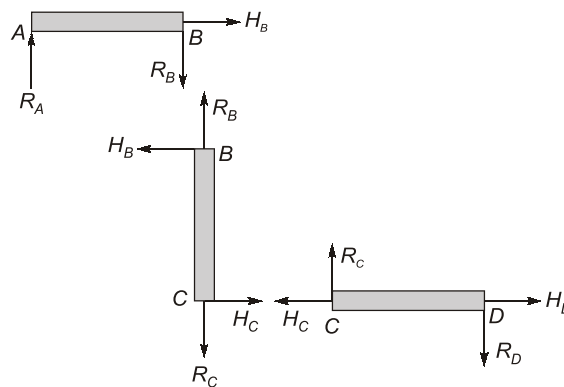
Member (b) is unstable since all the reactions are concurrent at C.



Beam (c) is unstable, since all three reactions are parallel.



Structure (d) is unstable, since the member AB can move horizontally without any restraint. i.e. $\sum F_x \neq 0$



1.4.2 Internal Stability

For the internal stability, no part of the structure can move rigidly relative to the other part so that geometry of the structure is preserved, however small elastic deformations are permitted. To preserve geometry, enough number of members and their adequate arrangement is required. For the geometric stability, there should not be any condition of mechanism. Mechanism is formed when there are three collinear hinges, hence to preserve geometric stability there should not be three collinear hinges. For 2-D truss the minimum number of members needed for geometric stability are:

$$m = 2j - 3$$

and for 3-D truss,

$$m = 3j - 6$$

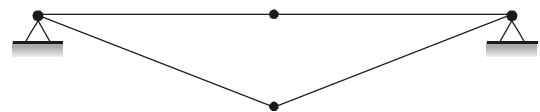


Fig. 1.31

where,

j = Number of joint in truss

m = Member required for geometrical stability.

All the members should be arranged in such a way that truss can be divided into triangular blocks. i.e. no rectangular or polygonal blocks.

Hence, for overall geometrical stability of truss:

(i) Minimum number of member should be present

$$m = 2j - 3 \quad (2\text{-D truss})$$

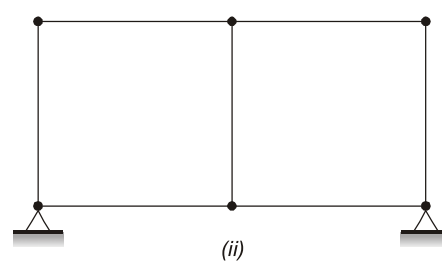
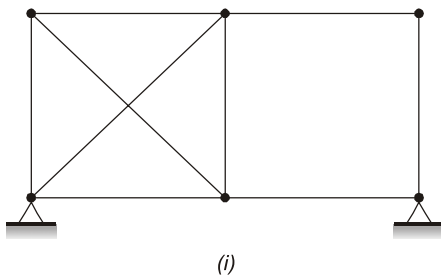
and

$$m = 3j - 6 \quad (3\text{-D truss})$$

(ii) There should be no condition of mechanism i.e. no three collinear hinges.

Example-1.2

Check geometrical stability for given trusses.



Solution:

(i) In case (i), arrangement of members is not adequate, hence right panel is unstable and left panel is over stiff. For geometric stability, all panels of truss should be stable so given truss is geometrical unstable.

For right panel:

$$j = 4$$

Number of member present,

$$m = 4$$

$$\text{But minimum number of member needed} = 2j - 3 = 2 \times 4 - 3 = 5$$

Hence Right panel is deficient.

For left panel:

$$j = 4$$

Number of member present,

$$m = 6$$

$$\text{But minimum number of member needed} = 2j - 3 = 2 \times 4 - 3 = 5$$

Hence left panel is over stiff.

(ii)

$$j = 6$$

Number of members present,

$$m = 7$$

$$\text{But minimum number of member needed} = 2j - 3 = 2 \times 6 - 3 = 9$$

Hence, above truss is geometrically unstable and it can be called 'deficient structure'.

$$\therefore \text{Number of deficiency} = 2$$

1.5 Statically Determinate and Indeterminate Structures

1.5.1 Statically Determinate Structures

A structure is said to be determinate if conditions of static equilibrium are sufficient to analyse the structure.

- In determinate structures, bending moment and shear force are independent of properties of material and cross-sectional area.
- No stresses are induced due to temperature changes.
- No stresses are induced due to lack of fit and support settlement.

1.5.2 Statically Indeterminate Structures

A structure is said to be statically indeterminate if conditions of static equilibrium are not sufficient to analyse the structure. To analyse these structures, additional compatibility conditions are required.

- In indeterminate structures, bending moment and shear force depends upon the properties of material and cross-sectional area.
- Stresses are induced due to temperature variation.
- Stresses are induced due to lack of fit and support settlement.

1.6 Degree of Indeterminacy

The degree of indeterminacy can be divided into:

1. Static indeterminacy, which can be classified as
 - (a) external indeterminacy
 - (b) internal indeterminacy
2. Kinematic indeterminacy

1.6.1 Static Indeterminacy

Those structures which can not be analyse using equations of static equilibrium alone are called indeterminate structures or hyper static structures. To analyse these structures extra equation are required which is called compatibility equation.

(a) External Static Indeterminacy (D_{Se}):

It is related to support system of the structure. External static indeterminacy is equal to number of independent external reactions in excess to available equilibrium condition for static equilibrium.

$$D_{Se} = r_e - r$$

where,

r_e = Total number of independent support reaction

r = Total number of available equations of static equilibrium

$$= 3 \text{ [2-D]}$$

... [2-D]

$$= 6 \text{ [3-D]}$$

... [3-D]

Case-1: (2-D beam subjected to general loading)

Here,

$$r_e = 6$$

$$r = 3 \quad \dots (2\text{-D})$$

Therefore,

$$D_{Se} = r_e - r$$

$$D_{Se} = 6 - 3 = 3$$

For general loading system, a fixed beam is statically indeterminate to 3rd degree.

However for vertical loading system.

Case-2: (2-D beam vertical loading)

$$r_e = 4$$

and equations of static equilibrium available,

$$r = 2$$

therefore,

$$D_{Se} = r_e - r$$

$$= 4 - 2 = 2$$

Here beam indeterminate to 2nd degree.

Hence, for general loading, the external indeterminacy is given by

$$D_{Se} = r_e - 3 \quad [\text{For 2-D}]$$

and

$$D_{Se} = r_e - 6 \quad [\text{For 3-D}]$$



Fig. 1.34

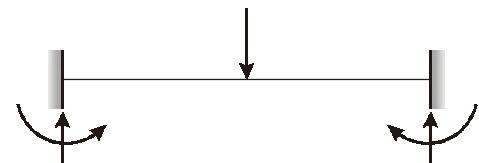


Fig. 1.35