Civil Engineering

Soil Mechanics and Foundation Engineering

Comprehensive Theory

with Solved Examples and Practice Questions

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Soil Mechanics and Foundation Engineering

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CHAPTER 10

Shear Strength of Soils

10.1 Introduction

One of the most important considerations in the design and construction on earth and earth supporting structures is the stability of soil mass.

The bearing capacity of soil, stability of soils and earth pressure against retaining structure directly depends upon the 'shear strength' of soil.

Due to external loadings or internal stress changes may fail by shear. So it is very important to examine the mechanism of shear failure and the factors affecting the same.

10.2 Concept of Stress

- Stress is an internal force acting per unit area of a surface. It is a tensor quantity.
- It has two components; one that acts normal to the sectional plane and other that acts along the plane.
- The normal component is called direct stress and the component which acts along the plane is called tangential stress. This tangential stress is responsible for the shearing of the material hence referred as 'shearing stress'.

10.3 Shear Strength of Soil

'Shear strength' is the resistance offered by soil against shear deformation, its value is equal to the shear stress on critical plane (plane A-A). The critical plane is that plane on which Resultant stress has maximum angle of obliquity with the normal of that plane.

where, σ_1 = Major principal stress
σ_2 = Minor principal stress
θ_c = Angle of critical plane or failure plane with the major principal plane.

- On the plane of θ_c (critical plane), σ_R is most inclined i.e., $\beta = \beta_{max}$ for frictional soils $\beta_{max} \simeq$ internal frictional angle of soil (ϕ)
- When angle β is maximum, then the shear stress on plane *A*-*A* will be equal to the shear strength of soil.

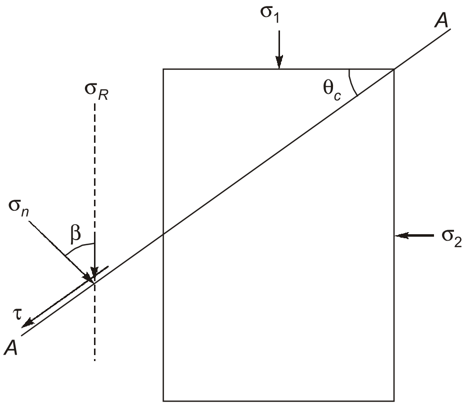


Fig. 10.1

10.3.1 Mechanism of Shear Resistance

Shearing resistance of a soil is the property of the soil that enables the soil mass to keep its equilibrium when its surface is not level or under any loading situation that is producing shearing stresses.

A soil may derived its shearing strength from the following parameters

(i) Frictional Resistance
(ii) Interlocking of particles
(iii) Cohesion and adhesion of molecules.

The granular soils (like sand) derived their shearing strength from friction (Both sliding and rolling) and interlocking.

Fine grained soils derived their strength from friction and cohesion.

Highly plastic clay i.e., pure clays, however only have 'cohesion' as their source of shear strength.

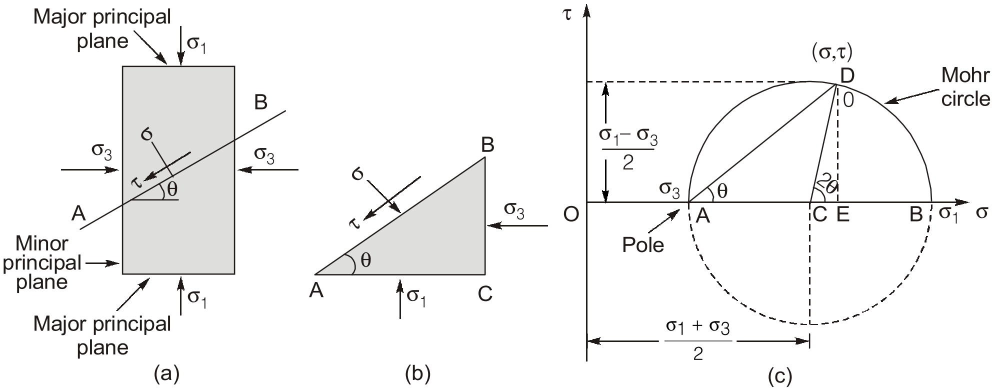
10.3.2 Stress at a Point—Mohr Circle of Stress

- In a stressed soil mass, shear failure can occur along any plane.
- At any stressed point, there exists three mutually perpendicular planes on which there are no shearing stresses acting. These are known as principal planes. The normal stresses that act on these planes are called principal stresses; the largest of these is called the major principal stress (σ_1), the smallest is called the minor principal stress (σ_3) and the third one is called the intermediate principal stress (σ_2). The corresponding planes are respectively designated as the major, minor and intermediate principal planes. However, the critical stress conditions occurs only at σ_1 and σ_3.
- In a two dimensional stress system, the major and minor principal planes occur on horizontal and vertical directions as shown in figure.
- If σ_1 and σ_3 are known it can be shown that on any plane *AB* inclined at angle θ to the direction of major principal plane, the normal stress σ and the shear stress τ are given by:

$$\sigma = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta \qquad \text{...(i)}$$

$$\tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cdot \sin 2\theta \qquad \text{...(ii)}$$

- Mohr demonstrated that these equations tend themselves to graphical representation. It can be shown that "the locus of stress coordinates (σ, τ) for all planes through a point is a circle, called, the "Mohr circle of Stress".



(a) (b) (c)

Fig. 10.2

Method of Drawing Mohr's Circle:

1. Normal stress σ is plotted on *X*-axis.
2. Shear stress τ is plotted on *y*-axis.
3. Compressive normal stresses are taken as positive.
4. Shear stresses that produce counter-clockwise couples of the element are considered positive.
5. The centre of circle is at $C\left(\frac{\sigma_1+\sigma_3}{2}, 0\right)$ and radius is equal to $\frac{\sigma_1-\sigma_3}{2}$; and the circle cuts the *X* axis at two points.
   - Now from fig. (*C*).
     $\angle BCD = 2\theta$, where θ is the angle made by the line joining point $(\sigma_3, 0)$ and parallel to the plane AB of fig. (b)
     and,
     $$\sigma = OE = OC + CE$$
     $$= \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cdot\cos 2\theta$$
     $$\tau = DE = \frac{\sigma_1-\sigma_3}{2}\cdot\sin 2\theta$$
   - Point *A* on the Mohr Circle is a unique point called the '**pole**' or "**Origin of planes**".
   - The property of pole is : "a line drawn through the pole intersects the Mohr Circle at a point which represents the state of stress on a plane which has the same inclination in space as the line itself".
   - This property can be utilized in locating the pole in a situation where the state of stress (σ, τ) on a certain plane is known.

Important Relationship Obtained from the Mohr Circle:

1. Maximum shearing stress occurs on planes inclined at 45° to principal planes.
$$\tau_{max} = \frac{\sigma_1-\sigma_3}{2} \quad (\text{at}\,\theta = 45°)$$
2. The normal stresses on plane of maximum shear are equal to each other and they are given by
$$\sigma_{1,2} = \frac{\sigma_1+\sigma_3}{2}$$
3. The sum of normal stresses on mutually perpendicular planes is a constant. i.e,
$$\sigma_1 + \sigma_3 = \sigma_{n1} + \sigma_{n2} = \text{constant}$$
4. When the principal stresses are equal to each other, the radius of the Mohr's circle becomes zero, which means that shear stresses vanish on all planes. Such a point is called **ISOTROPIC point**.
5. The resultant stress at any point is $\sqrt{\sigma^2+\tau^2}$ and the obliquity, β, equal to $\tan^{-1}(\tau/\sigma)$.
6. The maximum angle of obliquity (β_{max}) is obtained from failure envelope and is given by:
$$\theta_{cr} = 45° + \frac{\beta_{max}}{2}$$
7. The plane of maximum obliquity is most liable to failure and ***not*** the plane of maximum shear.
8. Failure becomes incipient when β_{max} approaches and equals the angle of internal friction ϕ

- For granular soil $\beta_{max} = \phi$
- For cohesive soils : ϕ = o, therefore, β_{max} = o therefore, $\theta = \frac{\pi}{4} + \frac{\beta}{2} = \frac{\pi}{4}$

9. In failure plane, $\sigma_n = \sigma_1(1 - \sin\beta_{max})$

$$\sigma_n = \sigma_3(1 + \sin\beta_{max})$$

$$\therefore \quad \frac{\sigma_1}{\sigma_3} = \frac{1 + \sin\beta_{max}}{1 - \sin\beta_{max}}$$

$$\Rightarrow \quad \sin\beta_{max} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

- On critical plane at limiting conditions, τ is called shear strength for cohesionless soil.

$$\tau = S = \sigma_n \tan\phi$$

- Normal stress on the plane of τ_{max} is given by *OC*,

$$\sigma_n = OC = \frac{\sigma_1 + \sigma_3}{2}$$

angle of plane of τ_{max}, $\theta = 45°$ with the major principal plane.

- Resultant stress on the plane of τ_{max} *OA*,

$$\sigma_R = OA = \sqrt{\sigma_n^2 + \tau_{max}^2} = \sqrt{\left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 + \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2}$$

$$= \sqrt{\frac{\sigma_1^2 + \sigma_3^2}{2}}$$

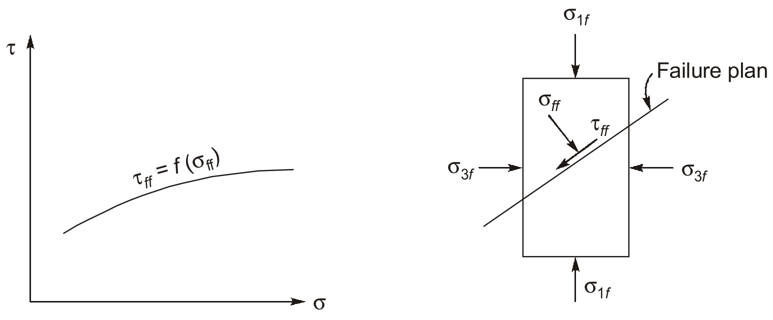
Mohr failure criterion:

- Mohr failure theory is based on the hypothesis that materials fails when the shear stress on the failure plane at failure reaches a value which is a unique function of the normal stress on the plane. i.e.,

$$\tau_{ff} = f(\sigma_{ff})$$

where, τ = shear stress and σ = normal stress.

The first subscript refers to 'failure plane' and the second subscript denotes 'at failure'.



(a) Mohr failure criterion (b) Soil element at failure

Fig. 10.3

- From data of different tests we can have a series of Mohr's circles. A line tangential to the Mohr circle gives a curve called Mohr failure envelope. It expresses the Functional Form of relationship between τ and σ_{ff} as given by above equation.
- If Mohr circle lies below the failure envelope (circle-I), every plane passing through this point has a shearing stress which is smaller than the shearing strength; this is known as stable condition.
- Mohr circle lying above the envelope, (circle-II) cannot exist because it is not possible for the shear stress to exceed the shearing strength.
- Any Mohr circle whose tangent is the Mohr failure envelope, represents a condition where in the point of tangency gives the stress conditions on the failure plane at failure.

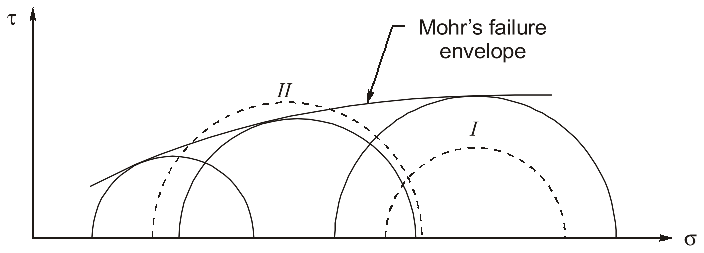


Fig. 10.4

Note: Mohr envelope is unique for a given material and independent of stress induced in material.

Coulomb's Equation and Mohr–Coulomb's Criterion:

- Coulomb observed that one component of the shearing strength called intrinsic cohesion (or apparent cohesion) is constant for a given soil and is independent of applied stress. The other component, namely the frictional resistance, varies directly as the magnitude of the normal stress on the plane of rupture.

  Coulomb equation is written as,

$$\tau_f = c + \sigma \tan \phi$$

where τ_f = shear strength of soil
c = apparent cohesion
s = Normal stress on plane of rupture
ϕ = Angle of internal friction

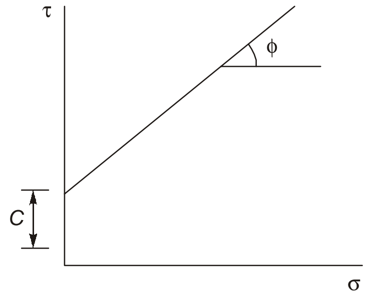


Fig. 10.5 Graphical Representation of Coulomb equation

- Mohr-coulomb failure criteria can be expressed in the form of

$$\tau_{ff} = c + \sigma_{ff} \tan \phi$$

- Also, angle of failure can be expressed in the term of angle of shearing resistance ϕ.

$$\theta_f = 45° + \frac{\phi}{2}$$

Do You Know

- According to Mohr-coulomb theory, failure will take place at a plane which has $\tau_f < \tau_{max}$.
- 'C' and ϕ are referred as the shear strength parameters of the soil. But C and ϕ are not inherent properties of the soil. There are, infect, related to the type of test and the condition under which these are measured.

Terzaghi Modification:

- The original form of Coulomb's equation was in terms of total normal stress.
- After terzaghi establishment of effective stress principle, it was found that the shear strength of soil is depends on effective parameters not on total parameters.

The shear strength of the soil is accordingly expressed as

$$\tau_f = c' + \bar{\sigma}\tan\phi'$$

where $\bar{\sigma} = \sigma - u$

c' = effective cohesion

ϕ' = angle of shearing resistance referred to effective stress.

u = pore pressure on the plane of rupture.

Relationship between C, ϕ and principal stresses at failure:

- Mohr coulomb failure criteria can be expressed in terms of the relationship between the principal stresses σ_{1f} and σ_{3f}.

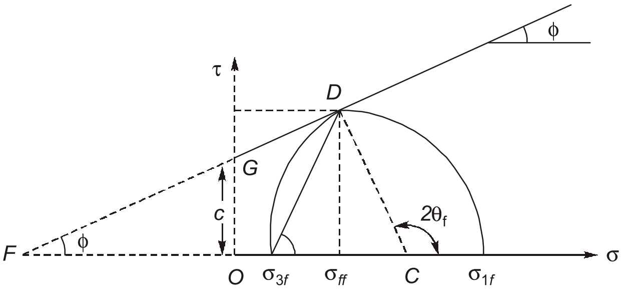


Fig. 10.6

$$\sin\phi = \frac{ED}{FC} = \frac{ED}{CO+OF} = \frac{(\sigma_{1f}-\sigma_{3f})/2}{(\sigma_{1f}+\sigma_{3f})/2 + c\cot\phi}$$

or $$(\sigma_{1f} - \sigma_{3f}) = (\sigma_{1f} + \sigma_{3f})\sin\phi + 2c\cos\phi$$

or Rearranging,

$$\sigma_{1f} = \sigma_{3f}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\left(\frac{\cos\phi}{1-\sin\phi}\right)$$

or, $$\sigma_{1f} = \sigma_{3f}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

or, $$\boxed{\sigma_{1f} = \sigma_{3f}\tan^2\left(45° + \frac{\phi}{2}\right) + 2c\tan\left(45° + \frac{\phi}{2}\right)}$$

Limitation of Mohr–Coulomb theory:

- It neglects the effect of the intermediate principal stress (σ_2).
- This theory, approx the failure envelope into straight line which may be a little curve for over consolidated soil.
- For some clays, there is no fixed relationship between the normal and shear stresses on the plane of failure. The theory cannot be used for such soils.
- In case of pure clays, according to this theory, shear strength is constant with the depth. However in practice a little increase is observed.

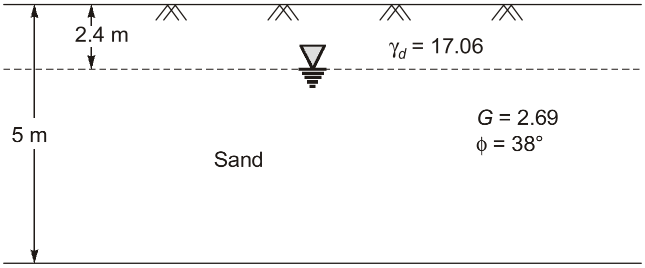
NOTE

- c' and ϕ' are effective stress parameters, c' and ϕ' would take constant values for a soil provided the void ratio, density and pore pressure all remain same for different normal stress value.
- The angle of repose of a granular soil can be determined by pouring the material on a level surface from a small height and measuring the angle between the sloping surface and the horizontal. When sand or gravel is dumped, the material is in a loose state near its surface. Hence, the angle of repose is approximately equal to the angle of internal friction of the soil in its loose state, that is, the angle of internal friction corresponding to the ultimate deviator stress.

Example 10.1 Compute the shearing strength of a soil along a horizontal plane at a depth of 5 m in a deposit of sand having the following properties:

Angle of internal friction, $\phi = 38°$
Dry unit weight, $\gamma_d = 17.06$ kN/m^3
Specific gravity, $G = 2.69$
Assume the ground W.T at a depth of 2.4 m from the ground level.
Also determine the change, in the shear strength, if the water-table rises upto the ground level.

Solution:



We know the shear strength for sand deposit is given by,

$$\tau = c + \bar{\sigma}\tan\phi$$

For sand, $c = 0$

$\therefore$ $\tau = \bar{\sigma}\tan\phi$

where $\bar{\sigma}$ = effective stress at 5 m below sand

$$\therefore \quad \bar{\sigma} = \gamma_d \times 2.4 + \gamma' \times 2.6 \qquad \text{...(i)}$$

Using,

$$\gamma_d = \frac{G \cdot \gamma_w}{1+e}$$

$$17.06 = \frac{2.69 \times 9.81}{1+e}$$

$$e = 0.546$$

Now,

$$\gamma' = \left(\frac{G-1}{1+e}\right)\gamma_w = \left(\frac{2.69-1}{1+0.546}\right) \times 9.81 = 10.723 \text{ kN/m}^2$$

from equation (i),

$$\bar{\sigma} = 17.06 \times 2.4 + 10.723 \times 2.6 = 68.82 \text{ kN/m}^2$$

Thus, the shear strength of the soil,

$$\tau = \bar{\sigma} \tan \phi = 68.82 \times \tan 38° = 53.76 \text{ kN/m}^2$$

When water table rises to the ground level, the entire soil below ground level become submerged, then $\bar{\sigma}$ is given by,

$$\bar{\sigma} = \gamma' \times 5 \text{ m} = 10.723 \times 5 = 53.615 \text{ kN/m}^2$$

Now shear strength becomes,

$$\tau = \bar{\sigma} \tan \phi = 53.615 \times \tan 38° = 41.89 \text{ kN/m}^2$$

$\therefore$ Change in shear strength, $\Delta\tau = 53.76 - 41.89 = 11.87 \text{ kN/m}^2$

Example 10.2 A given saturated clay is known to have effective strength parameters of $c' = 10$ kPa and $\phi' = 28°$. A sample of this clay was brought to failure quickly so that no dissipation of the pore water could occur at failure it was known that $\bar{\sigma}_1' = 60$ kPa, $\bar{\sigma}_3' = 10$ kPa and $u_f = 20$ kPa.

(a) Estimate the values of σ_1 and σ_3 at failure

(b) What was the effective normal stress on the failure plane?

Solution:

Given,

$$c' = 10 \text{ kPa}, \ \phi' = 28°$$

$$\bar{\sigma}_1 = 60 \text{ kPa}, \ \bar{\sigma}_3 = 10 \text{ kPa}$$

$$U_f = 20 \text{ kPa}$$

(a) We know,

$$\bar{\sigma} = \sigma - u$$

$$\therefore \quad \bar{\sigma}_1 = \sigma_1 - u_f$$

$$60 = \sigma_1 - 20$$

$$\sigma_1 = 80 \text{ kPa}$$

Similarly,

$$\bar{\sigma}_3 = \sigma_3 - u_f$$

$$10 = \sigma_3 - 20$$

$$\therefore \quad \sigma_3 = 30 \text{ kPa}$$

(b) Inclination of failure plane with the major principal plane, $\theta_c = 45° + \frac{\phi'}{2} = 45 + \frac{28°}{2} = 59°$

The effective normal stress at the failure plane is given by the relation,

$$\bar{\sigma} = \frac{\bar{\sigma}_1 + \bar{\sigma}_3}{2} + \frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2} \cos 2\theta_c$$

$$= \frac{60+10}{2} + \frac{60-10}{2} \cos(2 \times 59°)$$

$$= 35 - 11.74 = 23.26 \text{ kPa}$$

10.4 Factors Affecting Shear Strength

(a) Confining Stress:

(b) Drainage Conditions:

- It has been point out earlier, that effective stress which governs the shearing strength of soil.

$$\tau = c + \bar{\sigma} \tan \phi'$$

- Drained condition occurs when the excess pore water pressure developed during loading of a soil dissipates i.e. $\Delta u = 0$.
- Undrained conditions occurs when the excess pore water pressure cannot drain from the soil. i.e., $\Delta u \neq 0$.
- The existence of either condition–drained or undrained depends on the soil type, geological formation (fissures, sand layer in clays etc.), and the rate of loading.
- The values of c and ϕ depends on the drainage conditions in saturated soils.

$$\phi = \phi' \text{ in case of drain test}$$

$$\phi = 0 \text{ in case of an undrained test}$$

(c) Density Index:

- The most important factor affecting the shear strength of granular soil is density index.
- For the same composition of the soil, higher the density, higher the angle of friction. Hence higher will be shear strength.

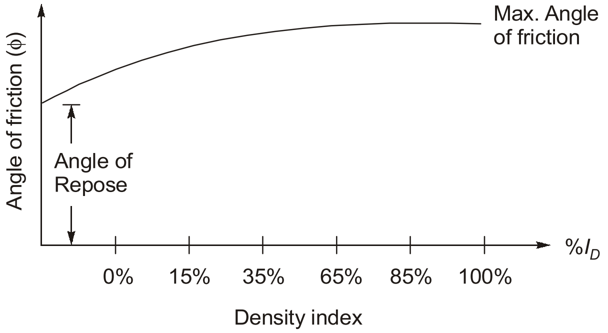


Fig. 10.7

(d) Water content and saturation:

- In case of fine grained soils, cohesion between soil particles is inversely proportional to water content. The relationship between cohesion and the water content is given in figure.
- The degree of saturation also affects the cohesion and the cohesion increases upto an optimum value above which it decreases with increasing for a given void ratio.
- Other hand, in unsaturated soils negative excess pore water pressure increases the effective stress ($\bar{\sigma} = \sigma - u$). Thus, if the pore water pressure is negative, the effective stress increases.

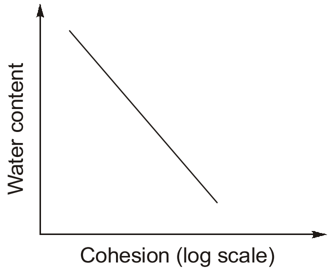


Fig. 10.8

(e) Composition and Particle Characteristics:

- Angle of internal friction depends on the grading of the soil. A well graded soil with high uniformity coefficient has a higher angle of friction as compared to poorly graded (i.e. uniform) soil.
- Similarly, sharp angular grains which can interlock well with adjacent grain will show higher friction angles. Hence minerals such as mica and flaky particles will show low angles of internal friction.

10.5 Measurement of Shear Strength

- Determination of shearing strength of a soil involves the plotting of failure envelopes and evaluation of the shear strength parameters for the necessary condition. Following test are carried out for this purpose:

  Field Tests:

  1. Direct shear test
  2. Triaxial Test
  3. Unconfined Compression Test
  4. Vane shear Test (Laboratory)
  5. Torsion Test
  6. Ring shear Test

  Field Test:

  1. Vane Shear Test
  2. Penetration Test