UPPSC-AE 2020

Uttar Pradesh Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Mechanical Engineering

Mechatronics and Robotics

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Contents

CHAPTER 4

Control Systems

4.1 Introduction

Control systems in an interdisciplinary field covering many areas of engineering and sciences. It is a device that controls the behavior of other devices. It is working as an interconnection between connected components or related in such a manner as to give commands, or regulate other component or itself. It is found in abundance in all sectors of industry, such as quality control of manufactured products, automatic assembly lines, machine-tool control, space technology, computer control, transportation systems, power systems, robotics, Micro-Electro-Mechanical Systems (MEMS), nanotechnology, and many others.

The study and design of automatic control system is also a field known as control engineering. In the modern era, it has become a part of everyday life. From simple device such as a toaster to complex systems like space shuttle control engineering plays a crucial role. The control system is a component that is added to other components to increase system functionality and also to get the desired output from the system.

The basic ingredients of a control system can be described by:

1. Objectives of control.
2. Control-system components.
3. Results or outputs.

Basic components of Control System

The basic relationship among these three components is illustrated in Fig above. In more technical terms, the objectives can be identified with inputs, or actuating signals, and the results are also called outputs, or controlled variables. In general, the objective of the control system is to control the outputs in some prescribed manner by the inputs through the elements of the control system.

4.2 Characteristics of Control Systems

A good control system must have the following characteristics:

- **Accuracy:** It is the measurement tolerance of any instrument and it defines the limit of error that instrument can make in working condition. It can be improved by feedback element and the detector must be present in the control system.
- **Sensitivity:** Control system parameter always varies with the surrounding conditions, internal disturbance or because of any other reason. Any good control system should be insensitive to such parameter and sensitive to input signals only. This change can be called as sensitivity.
- **Noise:** An undesirable input signal is known as noise, a good control system should ignore these noise effects for better performance.
- **Stability:** It is one of the most desirable characteristics of control system, for any bounded input the output must also be bounded. If the input is zero the output must also be zero and it is known as the stable system.

- **Bandwidth:** Bandwidth should be as large as possible for better frequency response of a control system.
- **Speed:** It is the time which system takes to reach stable condition. For a good control system, speed should be higher and have minimum transient period.
- **Oscillation:** For stability of the control system, it must have a minimum or constant oscillation.

4.3 Examples of Control Systems

Control systems find numerous and widespread applications from everyday life to extraordinary applications in science, industry, and society. Here are a few examples:

(a) Residential heating and air-conditioning systems controlled by a thermostat.

(b) The speed control of an automobile.

(c) Automatic traffic signal system at roadway intersections.

(d) Control system which automatically turns on a room lamp at dusk, and turns it off in daylight.

(e) Automatic hot water heater.

(f) Environmental test chamber temperature control system.

(g) An automatic positioning system for a missile launcher.

(h) An automatic speed control for a field controlled DC motor.

(i) The altitude control system of a typical space vehicle.

(j) Automatic position control system of a high speed automated train system.

(k) Human heart using a pacemaker.

(l) An elevator position control system used in high rise multi-level buildings.

4.4 Controller

It is a system which can control the behavior of other devices or systems. There are a number of different standard types of control systems such as P, PD, PI, and PID controllers. They are very common in the production of physical systems, however they have several drawbacks.

4.5 Compensator

It is a control system that regulates another system, this is done by conditioning the input or output to that system. Compensators are typically employed to correct a single design flaw, with the intention of applying effect on other aspects of the design in a minimal manner. There are a number of different compensator units that can be employed to help fix certain system that are outside of a proper operating range.

The phase characteristics are necessary for compensation system, especially if the magnitude response is to remain constant. Occasionally, it is necessary to alter the phase characteristics of a given system, without altering the magnitude characteristics. To do this, we need to alter the frequency response in such a way that the phase response is altered, but the magnitude response is not altered. To do this, we implement a special variety of controllers known as phase compensators. They are called compensators because they help to improve the phase response of the system. There are two general types of compensators: Lead Compensators and Lag Compensators. If we combine the two types, we can get a special Lead-Lag Compensator system.

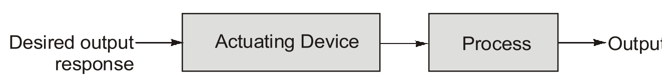
4.6 Control System Configuration

There are two types of control system configurations:
1. Open loop control systems or non-feedback control systems.
2. Closed loop control systems or feedback control systems.

4.6.1 Open loop Systems

An open loop control system utilizes an actuating device to control the process directly without using feedback channel. A component or process to be controlled can be represented by a block diagram as shown in figure. The block can be investigated one at a time, the output of one block will be input to another block and we can use cause and effect reasoning. This will become complicated when it is connected in a closed loop system because of interaction between different blocks.

(i) Process to be controlled



(ii) Open loop control system (Non feedback)

Example: Electronic fan switch.

Reference input: When the fan switches on, a 230 V is applied. So reference input is 230 V signal.

Controller: The electronic voltage controller when turning the knob to the desired position, the voltage will increase or decrease to an appropriate value. It can have a value like 230 V which is at maximum speed and 115 V at half speed and so on. Once we set the speed nothing need to be done, let there are three fans and you turn the knob at same amount but the speed is slightly different for every fan. This happens because of inappropriate settings, inconsistency in ball bearing performance, imperfect blade design which causes different amount of drag forces. So, an open loop system is one where we cannot correct the error of desired output and the actual output.

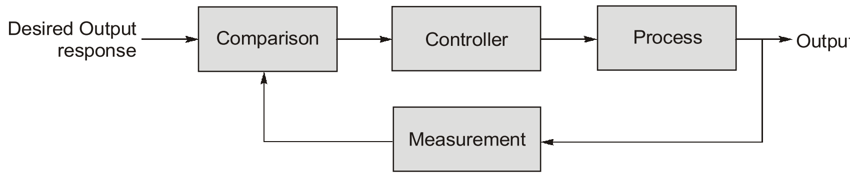
Advantages:
1. Simple construction
2. Low cost
3. Maintenance is easy
4. Stability is good

Disadvantages
1 Less Accuracy
2. Reliability is less
3. The change in output cannot be modified

Other Examples: Electric hand dryer, Automatic washing machine, Bread toaster, Automatic tea or coffee maker, Light switches etc.

4.6.2 Closed Loop Systems

A closed loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output. The system can be easily spotted by seeing its block relation, it does not always maintain only relation from one block head to another block tail, it also has relation from different blocks as shown in Fig. given below of closed loop feedback control system. Since the increasing complexity of the system is under control and the optimum performance of feedback system has grown it in past few decades.



Closed loop control system (feedback system)

Advantages:

1. Closed loop control is more accurate, even in the case of non-linearity.
2. Highly accurate because the error of output can be modified by feedback signal.
3. Range of Bandwidth is large.
4. Less sensitive to disturbance and noise.
5. Less sensitive to the characteristics and parameter variations.

Disadvantages:

1. The cost is high.
2. Design is complicated.
3. More maintenance required, which further increases the cost.
4. Overall gain reduces due to feedback.

4.7 Comparison between Open Loop and Close Loop Control Systems

Table

Open Loop System	Close Loop System
1. So long as the calibration is good, open-loop system will be accurate	1. Due to feedback, the close-loop system is more accurate
2. Organization is simple and easy to construct	2. Complicated and difficult
3. Generally stable in operation	3. Stability depends on system components
4. If non-linearity is present, system operation degenerates	4. Comparatively, the performance is better than open-loop system if non-linearity is present

Example - 4.1 Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List-I

A. Human respiration system
B. Pointing of an object with a finger
C. A man driving a car
D. A thermostatically controlled room heater

List II

1. Man-made control system
2. Natural including biological control system
3. Control system whose components are both man-made and natural

Codes:

	A	B	C	D
(a)	2	2	3	1
(b)	3	1	2	1
(c)	3	2	2	3
(d)	2	1	3	3

Solution: (a)

Example - 4.2 If the initial conditions for a system are inherently zero, what does it physically mean?

(a) The system is at rest but stores energy
(b) The system is working but does not store energy
(c) The system is at rest or no energy is stored in any of its part
(d) The system is working with zero reference input

Solution: (c)

A system with zero initial conditions is said to be at rest since there is no stored energy.

Example - 4.3 What are the properties of linear systems which are not valid for non-linear systems? Explain each briefly?

Solution:

- Linear systems satisfy properties of superposition and homogeneity. Any system that does not satisfy these properties is non-linear.

  Property of superposition: When the output corresponding to v_{in_1} is v_{out_1} and the output corresponding to v_{in_2} is v_{out_2} then the output corresponding to $av_{in_1} + bv_{in_2}$ is $av_{out_1} + bv_{out_2}$.
  Property of homogeneity: It states that for a given input x in the domain of the function f and for any real number k

  $$f(kx) = kf(x)$$

- Linear systems have one equilibrium point at the origin. Non-linear systems may have many equilibrium points.

4.8 Mathematical Modelling of Physical Systems

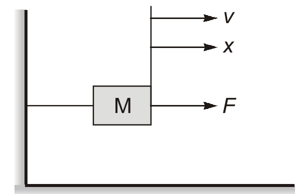
4.8.1 Mechanical Systems

All mechanical systems are divided into two parts:

1. Mechanical Translational System:

Input = Force (F),
Output = Linear displacement (x) or Linear velocity (v)
The three ideal elements are

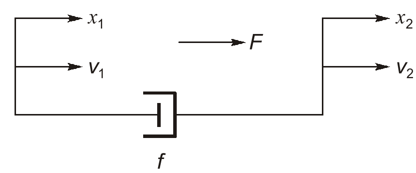
(a) Mass Element:



$$F = M\frac{d^2x}{dt^2}$$

or $$F = M\frac{dv}{dt}$$

(b) Damper Element:



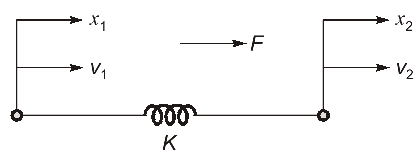
$$F = f\frac{d}{dt}(x_1 - x_2) = f\frac{dx}{dt}$$

where $$x_1 - x_2 = x$$

or $$F = f(v_1 - v_2) = fv$$

where $$v = v_1 - v_2$$

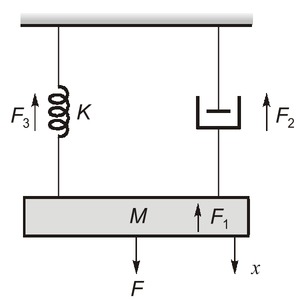
(c) Spring Element:



$$F = K(x_1 - x_2) = Kx$$

where $$x_1 - x_2 = x$$

Consider,



$$F = F_1 + F_2 + F_3$$

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx \quad \text{... (i)}$$

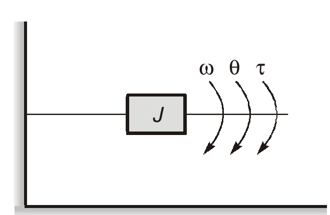
2. Mechanical Rotational System

Input = Torque (τ),

Output = Angular displacement (θ) or Angular velocity (ω)

The three ideal elements are

(a) Inertial Element:

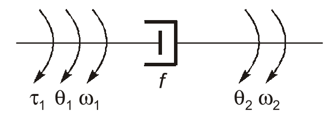


$$\tau = J\frac{d^2\theta}{dt^2}$$

or $$\tau = J\frac{d\omega}{dt}$$

(b) Torsional Damper Element:

$$\tau = f\frac{d}{dt}(\theta_1 - \theta_2) = f\frac{d\theta}{dt},$$



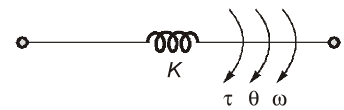
where, $\theta = \theta_1 - \theta_2$

or $\tau = f(\omega_1 - \omega_2) = f\omega$

where, $\omega = \omega_1 - \omega_2$

(c) Torsional Spring Element:

$$\tau = K\theta$$

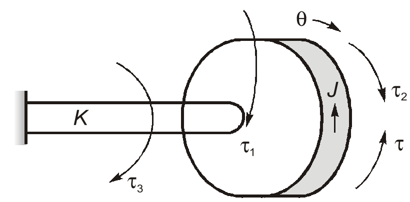


or $\tau = K\int \omega dt$

Consider,

$$\tau = \tau_1 + \tau_2 + \tau_3$$

$$\tau = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta \quad \text{... (ii)}$$

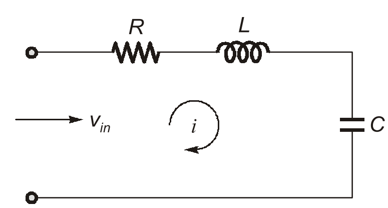


4.8.2 Electrical Systems

1. Series RLC Circuit:

Applying KVL

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_{in}$$



but $i = \dfrac{dq}{dt}$

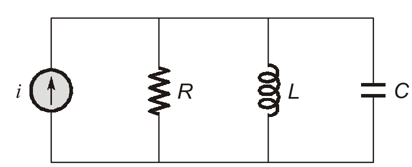
or $q = \int i\,dt$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_{in} \quad \text{... (iii)}$$

2. Parallel RLC Circuit:

Applying KCL

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = i,$$



where V = node voltage

$$V = \frac{d\phi}{dt}$$

where, ϕ = Magnetic flux

$$C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} = i \quad \text{... (iv)}$$

Duality: $C \to L,$

$R \to \dfrac{1}{R},$

$\phi \to q$

4.8.3 Analogy

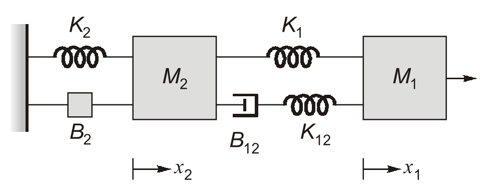
On comparing equations (i), (ii), (iii) and (iv), Force, Torque, Voltage, Current Analogy can be developed as follows:

Mechanical		Electrical	
Translation	Rotation	Force-voltage analogy	Force-current analogy
f	T	V	i
M	J	L	C
D	D	R	$1/R$
k	k	$1/C$	$1/L$
x	θ	q	ϕ

Here, $f \rightarrow$ Force, $M \rightarrow$ Mass, $D \rightarrow$ Damping coefficient, $k \rightarrow$ Spring coefficient, $x \rightarrow$ Displacement, $T \rightarrow$ Torque, $J \rightarrow$ Moment of inertia, $\theta \rightarrow$ Angular displacement.

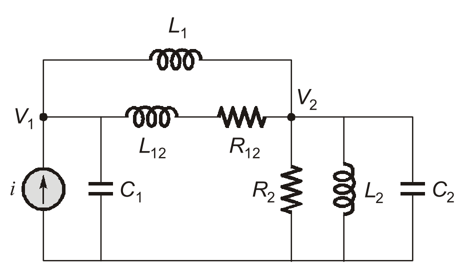
NOTE: From the two analogies it is clear that mass and spring elements are energy storing elements.

Example - 4.4 Consider the following mechanical system shown in the diagram:



Which one of the following circuits shows the correct force-current analogous electrical circuit for the mechanical diagram shown above?

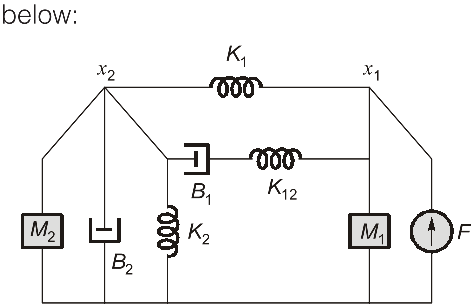
Solution:



Nodal diagram

Mechanical system $\longrightarrow$	Force current analog
Mass (M)	Capacitance (C)
Damper (B)	Conductance ($1/R$)
Spring coefficient (K)	Inductance current ($1/L$)

Mechanical network is shown below:

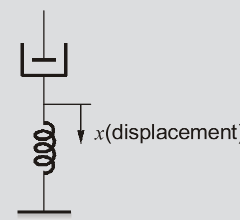


4.9 Nodal Method for Writing Differential Equation of Complex Mechanical System

- Number of nodes = Number of displacements.
- Take an additional node which is a reference node.
- Connect the mass and inertial mass elements always between the principle node and reference node.
- Connect the spring and damping elements either between the principle nodes or between principle nodes and reference depending on their position.
- Obtain the nodal diagram and write the describing differential equations at each node.

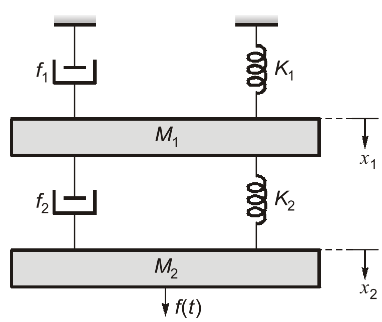
NOTE

- One mass element ⇒ order of differential equation = 2
  Two mass element ⇒ order of differential equation = 4
  n mass element ⇒ order of differential equation = $2n$
- A point or node between damper and spring must be taken as a displacement.



- Each mass must be taken as a displacement.
- Mass or inertial mass elements and spring elements are known as consecutive elements because they are analogous to inductor/capacitor in electrical analogy.

Example - 4.5 Obtain the mathematical system equations and its corresponding mechanical circuit diagram of the following mechanical system.

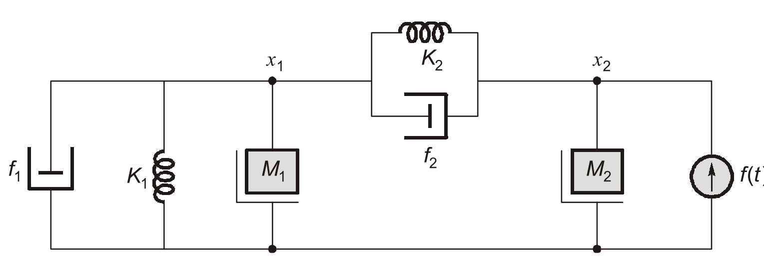


Solution:

At node x_1: $$f_1\frac{dx_1}{dt} + K_1x_1 + M_1\frac{d^2x_1}{dt^2} + K_2(x_1 - x_2) + f_2\frac{d(x_1 - x_2)}{dt} = 0$$

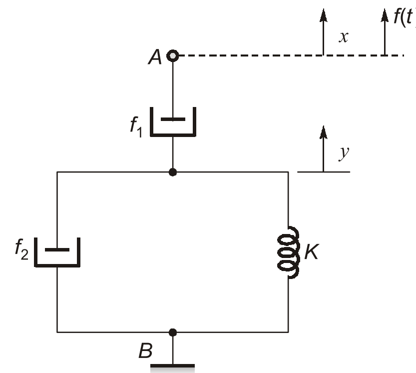
At node x_2: $$M_2\frac{d^2x_2}{dt^2} + K_2(x_2 - x_1) + f_2\frac{d(x_2 - x_1)}{dt} = f(t)$$

Now, mechanical circuit diagram can be drawn either from given system diagram or from these system equations as well.



Mechanical circuit diagram

Example - 4.6 Find the transfer function relating displacements y and x for the mechanical system of given figure.



Solution:

From the mechanical circuit diagram the equations for the system are

$$f_2\frac{dy}{dt}+Ky+f_1\frac{d}{dt}(y-x) = 0 \quad \text{...(i)}$$

$$f_1\frac{d}{dt}(x-y) = f(t) \quad \text{...(ii)}$$

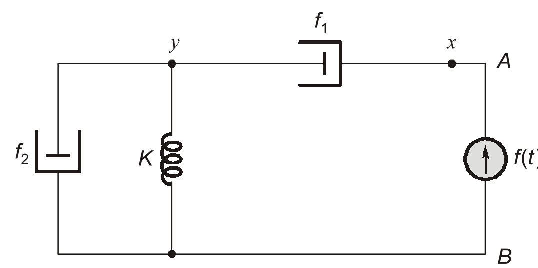
Assuming initial conditions as zero and taking Laplace transform on both sides of equation (i),

$$f_2sY(s)+KY(s)+f_1s[Y(s)-X(s)] = 0 \quad \text{...(iii)}$$

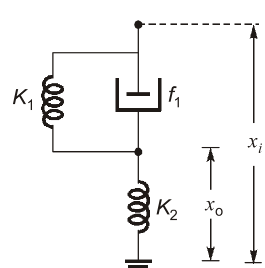
From equation (iii), the required transfer function is

$$\frac{Y(s)}{X(s)} = \frac{f_1 s}{[(f_1+f_2)s+K]} = \frac{T_1 s}{(T_2 s+1)}$$

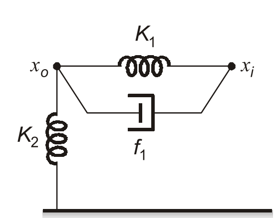
where, $T_1 = \frac{f_1}{K}$ and $T_2 = \frac{(f_1+f_2)}{K}$.



Example - 4.7 Derive an expression for the transfer function $X_o(s)/X_i(s)$ for the mechanical system given below:



Solution:



At node x_o, $$K_2x_o+K_1(x_o-x_i)+f_1\frac{d}{dt}(x_o-x_i) = 0$$

Using Laplace transform,

$$K_2X_o(s)+K_1\left(X_o(s)-X_i(s)\right)+f_1s(X_o(s)-X_i(s)) = 0$$

On solving, $$\frac{X_o(s)}{X_i(s)} = \frac{f_1s+K_1}{f_1s+K_1+K_2}$$

4.10 Control Signal

The control signal or manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. The controlled variable is the quantity or condition that is measured or controlled. Normally the controlled variable is the output of the system. Whereas control means measuring the value of controlled variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from the desired value.

In some automatic control systems, control signals are altered to make the controlled variable correspond to the desired value (as in servomechanism, stabilization systems, and programmed control) or achieve some optimum or extreme value (as in optimal control systems and self-aligning systems). In other automatic control systems, the control signal depends on the control law and is determined by the characteristics of the controlled object and the nature of commands and disturbances acting on the automatic control system.

Control signals may be applied in single-capacity or multiple-capacity systems. In the latter, each control signal may influence one or several controlled variables, which makes controlling the object more difficult. Thus, one of the most important problems to be solved in the design of automatic control systems is the lessening or elimination of the effect of the control signal on all but the desired controlled variable.

4.11 Controllability

If the internal states of the system is changing from one value to another value in a finite time by a finite input then we can say that the system is controllable. Otherwise it is uncontrollable.

For IInd Order System:

$$[\phi_C] = [B \quad AB]_{2\times 2}$$

If $|\phi_C| = 0 \Rightarrow$ Then system is called uncontrollable system

If $|\phi_C| \neq 0 \Rightarrow$ Then system is controllable.

Where,
$$\dot{X}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

where, A is system matrix, B is input matrix and C is output matrix.

For IIIrd Order System:

$$[\phi_C] = [A^0B \quad AB \quad A^2B]_{3\times 3}$$

If $|\phi_C| = 0 \Rightarrow$ System is uncontrollable.

If $|\phi_C| \neq 0 \Rightarrow$ System is controllable.

4.12 Observability

If the internal states of the system are able to evaluate from the output of the system at anytime then we can say that the system is observable.

For IInd Order System:

$$[\phi_0] = [C^T \quad AC^T]_{2\times 2}$$

For IIIrd Order System:

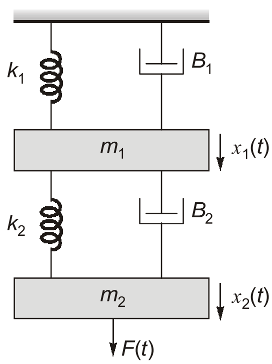
$$[\phi_0] = [C^T \quad AC^T \quad A^2C^T]_{3\times 3}$$

If $|\phi_0| = 0$, then the system is non-observable.

If $|\phi_0| \neq 0$, then the system is observable.

Student's Assignment

Q.1 A mechanical system consists of two mass-spring-friction systems as shown in the given figure. The order of the transfer function $\frac{X_1(s)}{F(s)}$ is



(a) 1
(b) 2
(c) 3
(d) 4

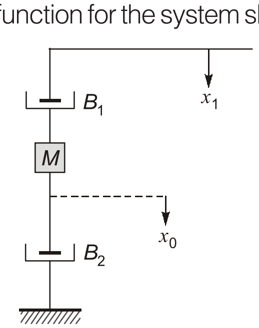
Q.2 The describing equation of a mass-damper spring system is given by

$$\frac{2d^2x}{dt^2} + \frac{dx}{dt} + 0.5x = f(t)$$

where, $f(t)$ is the external force acting on the system and x is the displacement of the mass. The steady state displacement corresponding to a force of $2N$ is given by

(a) 0.25 m
(b) 0.5 m
(c) 2 m
(d) 4 m

Q.3 The transfer function for the system shown below is

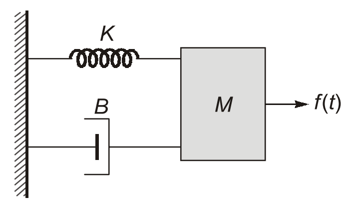


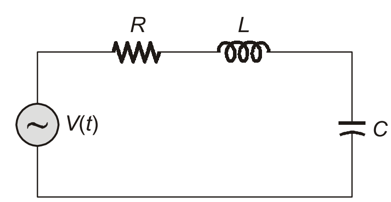
(a) $\frac{1}{Ms^2 + B_1s + B_2s}$
(b) $\frac{(B_1 + B_2)}{Ms^2 + B_1s + B_2s}$
(c) $\frac{B_2s}{Ms + (B_1 + B_2)}$
(d) $\frac{B_1s}{Ms^2 + (B_1 + B_2)s}$

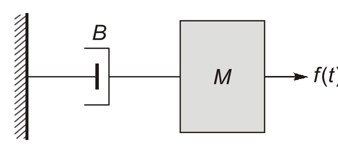
Q.4 Which of the following is an example of an open-loop system?

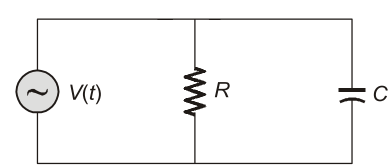
(a) Household refrigerator
(b) Respiratory system of an animal
(c) Stabilization of air pressure entering into a mask
(d) Execution of a program by a computers

Q.5 Consider the following systems:

1.


2.


3.


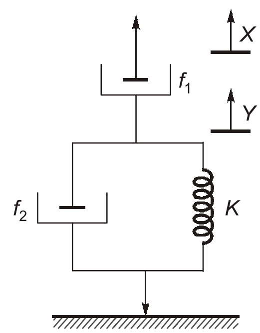
4.


Which of these systems can be modelled by differential equation

$$a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = x(t)$$

(a) 1 and 2
(b) 1 and 3
(c) 2 and 4
(d) 1, 2 and 4

Q.6 Find the transfer function relating displacements Y and X for the mechanical system of the given figure.



(a) $\dfrac{Y(s)}{X(s)} = \dfrac{f_1(s)}{(f_1+f_2)(s)+K}$

(b) $\dfrac{Y(s)}{X(s)} = \dfrac{K}{(f_1+f_2)(s)+K}$

(c) $\dfrac{Y(s)}{X(s)} = \dfrac{f_2(s)}{(f_1+f_2)(s)+K}$

(d) $\dfrac{Y(s)}{X(s)} = \dfrac{f_1(s)}{f_2(s)+K}$

Q.7 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

A. Hydraulic actuator
B. Flapper valve
C. Potentiometer error detector
D. Dumb-bell rotor

List-II

1. Linear device
2. AC servo systems
3. Large power to weight ratio
4. Pneumatic systems

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	3	4	2	1
(c)	3	4	1	2
(d)	4	3	1	2

Q.8 Which of the following are the characteristics of closed loop systems?

1. It does not compensate for disturbances
2. It reduces the sensitivity of plant-parameter variations
3. It does not involve output measurements
4. It has the ability to control the system transient response

Select the correct answer using the codes given below:

(a) 1 and 4 (b) 2 and 4
(c) 1 and 3 (d) 2 and 3

Q.9 Consider the following statements regarding advantages of closed loop negative feedback control systems over open loop systems:

1. The overall reliability of the closed loop system is more than that of open loop system.
2. The transient response in a closed loop system decays more quickly than in open loop system.
3. In an open loop system, closing of the loop increases the overall gain of the system.
4. In the closed loop system, the effect of variation of component parameters on its performance is reduced.

Which of these statements are correct?

(a) 1 and 2 (b) 1 and 3
(c) 1, 2 and 4 (d) 3 and 4

Q.10 Check the controllability and observability for the following system

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \ ; \ Y = [1 \quad 1]X$$

(a) not controllable, not observable
(b) controllable but not observable
(c) controllable, observable
(d) not controllable but observable

Q.11 Check controllability and observability

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \ ; \ Y = [1 \quad 0]X$$

(a) not controllable, not observable
(b) controllable, observable
(c) controllable but not observable
(d) not controllable but observable

Q.12 Consider a second order system whose state space representation is of the form $\dot{X} = AX + BU$. If $x_1(t) = x_2(t)$, then system is

(a) controllable (b) uncontrollable
(c) observable (d) unstable

Q.13 For the system $X = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, which of the following statement is true?

(a) The system is controllable but unstable.
(b) The system is uncontrollable and unstable.
(c) The system is controllable and stable.
(d) The system is uncontrollable and stable.

Q.14 Consider the system

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t), \; c(t) = [d_1 \quad d_2]\, x(t)$$

The conditions for complete state controllability and complete observability are

(a) $d_1 \neq 0$, $b_2 \neq 0$, b_1 and d_2 can be anything
(b) $d_1 \neq 0$, $d_2 \neq 0$, b_1 and b_2 can be anything
(c) $b_1 \neq 0$, $b_2 \neq 0$, d_1 and d_2 can be anything
(d) $b_1 \neq 0$, $d_2 \neq 0$, b_1 and d_1 can be anything

Q.15 If $\dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} U$ and $Y = [1 \quad 1]\, X$, then

1. system is observable
2. system is controllable
3. system is not observable
4. system is not controllable

Which of the above statements are correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 4 (d) 3 and 4

Q.16 The state space representation of a system is given by

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad ; \quad Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T x$$

The system is

(a) both controllable as well as observable
(b) controllable but not observable
(c) Observable but not controllable
(d) Neither controllable nor observable

Q.17 A good control system has

(a) high accuracy
(b) high sensitivity
(c) large range bandwidth
(d) all of the above

Q.18 Advantage of open loop system

(a) high accuracy
(b) high reliability
(c) low cost and simple in construction
(d) all of the above

Q.19 The performance of controller depends upon

(a) Efficiency of controller algorithm
(b) Sensor performance
(c) Actuator performance
(d) All the above

Q.20 Which statements are true about loop system?

1. Regulatory mechanism is generally provided by closed loop control system.
2. Open loop control system can make error equal to zero at steady state in the absence of disturbance.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) None of these

Q.21 Which is correct for proportional controller?

(a) $k_d \dfrac{de(t)}{dt} + p_0$

(b) $k_d e(t) + p_0$

(c) $k_d e(t) + k_i \displaystyle\int_0^t e(t)\,dt + k_d \dfrac{de(t)}{dt} + p_0$

(d) $k_i \displaystyle\int_0^t e(t)\,dt + p_0$

Q.22 Which of the following controller can results zero steady state error?

(a) Proportional controller
(b) Integral controller
(c) Derivative controller
(d) None of these

Q.23 Which of the following controller can be called as "Anticipatory controller"?

(a) Proportional controller
(b) Integral controller
(c) Derivative controller
(d) None of these

Q.24 A system is said to be completely observable

(a) if every state $x(t_0)$ cannot completely identified by measurement of output $y(t)$ over a finite time interval.

(b) if every state $x(t_0)$ can be completely identified by measurement of output $y(t)$ over a finite time interval.

(c) Both (a) and (b) are correct

(d) None of the above

Q.25 A state variable description of a single-input single-output linear system is given by

$$\dot{x} = Ax(t) + Bu(t)$$
$$Y(t) = Cx(t)$$

where, $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = [1 \quad -1]$

The system is

(a) Controllable

(b) Observable

(c) Both controllable and absorbable

(d) None of these

Q.26 If $\dot{X} = \begin{bmatrix} 1 & 2 \\ 0 & b \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V$

where b is unknown constant. This system is

(a) uncontrollable for $b = 1$

(b) uncontrollable for $b = 0$

(c) uncontrollable for all value of b

(d) controllable for all value of b

Q.27 Negative feedback in a closed loop control system does not

(a) reduce the overall gain

(b) reduce bandwidth

(c) improve disturbance rejection

(d) reduce sensitivity to parameter variation

Q.28 The transfer function of a system refers to

(a) ratio of Laplace transform of input to Laplace transform of output

(b) ratio of Laplace transform of output to Laplace transform of input

(c) product of Laplace transform of output and Laplace transform of input

(d) summation of Laplace transform of output and Laplace transform of input

Q.29 $\dot{X} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V,\ y = [b \quad 0]x$

where b is unknown constant. This system is

(a) observable for all value of b

(b) unobservable for all value of b

(c) observable for all non-zero value of b

(d) unobservable for all non-zero value of b

Q.30 Consider the system with state equation

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

(a) Controllable (b) Uncontrollable

(c) Both (a) and (b) (d) None of above

Q.31 Which of the following are the basic building block elements for a mechanical system where forces and straight line displacements are involved without any rotation?

1. Spring
2. Dashpot
3. Mass
4. Moment of inertia

Select the correct answer using the code given below:

(a) 1, 2 and 4 (b) 1, 3 and 4

(c) 2, 3 and 4 (d) 1, 2 and 3

Q.32 For the control signal to change at a rate proportional to the error signal, the robotic controller must employ

(a) integral control

(b) proportional-plus-integral control

(c) proportional-plus-derivative control

(d) proportional-plus-integral-plus-derivative control

ANSWER KEY // **STUDENT'S ASSIGNMENT**

1. (d)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (a)
6. (a)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)
11. (b)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)
16. (c)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (c)
21. (b)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (a)
26. (b)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (a)
31. (d)	**32.** (a)			

HINTS & SOLUTIONS / STUDENT'S ASSIGNMENT

17. (d)

A good control system should have
(i) high accuracy
(ii) high sensitiveness
(iii) can ignore undesirable noise
(iv) stable
(v) minimum or constant oscillation

18. (c)

Open loop system

Advantage:
(i) Simple construction
(ii) Low cost
(iii) Easy maintenance
(iv) stability of good

Disadvantage:
(i) Less accuracy
(ii) Less reliable
(iii) Change in output cannot be modified

19. (a)

Performance of controller depends upon efficiency of controller algorithm. It does not depend upon actuator performance and sensor performance because these are not a part of controller but are separate.

20. (c)

- Regulatory mechanism i.e., feedback are generally provided in closed loop system.
- All steady state in absence of disturbance, open loop control system can make zero error.

21. (b)

Proportional controller : $k_d e(t) + p_0$

Integral controller : $k_i \int_0^t e(t)\,dt + p_0$

Derivative controller : $k_d \dfrac{de(t)}{dt} + p_0$

PID controller : $k_d e(t) + k_i \int_0^t e(t)\,dt + k_d \dfrac{de(t)}{dt} + p_0$

22. (b)

- Integral controller : $k_i \int_0^t e(t)\,dt + p_0$
- Integral controller can results zero steady state error.
- Derivative controller called anticipatory controller.

23. (c)

- Derivative controller are often called as anticipatory controller.
- Derivative controller: $k_d \dfrac{de(t)}{dt} + p_0$
- PID controller are called universal controller.

24. (b)

A system is said to be completely observable, if every state $x(t_0)$ can be completely identified by measurement of output $y(t)$ over a finite time interval.

25. (a)

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_C = [B \;:\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|Q_C| = 1$$

Hence, $|Q_C| \neq 0$ so (controllable)

$$C^T = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} C^T & A^T C^T \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$|Q_0| = 0$$

Hence, $|Q_0| = 0$ so unobservable

26. (b)

$$A = \begin{bmatrix} 1 & 2 \\ 0 & b \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Controllability matrix = $[B \quad AB]$

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & b \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ b \end{bmatrix}$$

$$|Q_C| = \begin{bmatrix} 0 & 2 \\ 1 & b \end{bmatrix} = -2 \neq 0$$

Controllable for any value of 'b'.

28. (b)

Transfer function of a system is defined as ratio of Laplace transform of output to Laplace transform of input.

29. (c)

Observability condition

$$Q_0 = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} b & 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix}$$

$$|Q_0| = 2b^2 - 0 = (2b^2)$$

Observable for all non-zero value of 'b'.

30. (a)

Condition for controllability

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$A^2 B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 25 \end{bmatrix}$$

$$|Q_C| \neq 0$$

So, system is completely controllable.

32. (a)

Given that the control signal to change at a rate proportional to the error signal it means

$$\frac{d}{dt}(\Delta I(t)) \propto E(t)$$

$$\Delta I(t) = K_I \int E(t)dt$$

$\Delta I(t)$ = change of control signal (controller output)

$E(t)$ = Error signal

As the change of control signal is proportional to the integral of error, it represents "Integral controller".

■■■■