UPPSC-AE 2020

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Mechanical Engineering

Mechanism and Machines

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

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Mechanism and Machines

Contents

CHAPTER 7

Balancing

7.1 Introduction

Balancing is the process of modifying machinery so that the unbalance force is reduced to an acceptable level and if possible eliminated entirely. Commonly balancing is done by redistributing the mass which must be done by addition or removal of mass from various machine members.

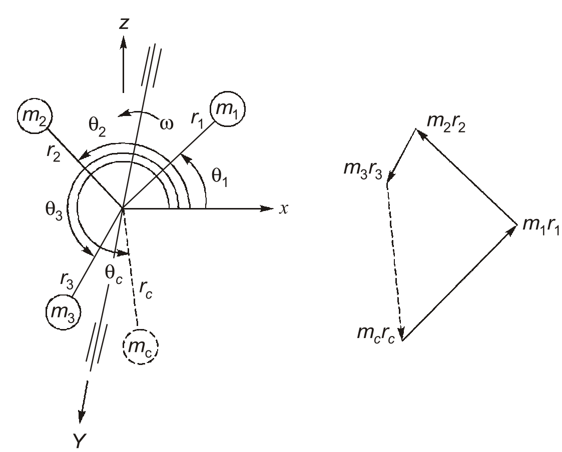
Cause of Unbalance

An unbalance of forces is produced in rotary or reciprocating machinery due to the inertia force associated with the moving masses.

7.2 Type of Balancing

7.2.1 Static Balancing

- It is required when masses are in same transverse plane.
- A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation, so that there is **no resultant centrifugal force**. Moments are automatically balanced.



Let 'F' is vector sum of centrifugal force due to different rotating masses.

$$F = m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2$$

Here, ω = Constant angular velocity;

m_1, m_2 and m_3 are Rotating masses.

r_1, r_2 and r_3 are Radii of masses.

In case of statically balanced condition the vector sum of forces will become zero. Thus

$$m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 = 0$$

Method for Balancing if Unbalance is Present

Introduce a counter weight of mass m_c at radius r_c to balance the rotor, thus,

$$m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 + m_cr_c\omega^2 = 0$$

$$m_1r_1 + m_2r_2 + m_3r_3 + m_cr_c = 0$$

$$\Sigma m_ir_i + m_cr_c = 0$$

Mathematical Solution

- To solve mathematically, divide each force into 'x' and 'z' components.

$$\Sigma m_ir_i \cos\theta + m_cr_c \cos\theta_c = 0 \quad \text{...(i)}$$

$$\Sigma m_ir_i \sin\theta + m_cr_c \sin\theta_c = 0 \quad \text{...(ii)}$$

By solving above equation

$$m_c r_c = \sqrt{(-\Sigma m_i r_i \cos\theta)^2 + (-\Sigma m_i r_i \sin\theta)^2}$$

Angle of introduction of counter weight (θ_c)

$$\tan\theta_c = \frac{-\Sigma m_i r_i \sin\theta}{-\Sigma m_i r_i \cos\theta}$$

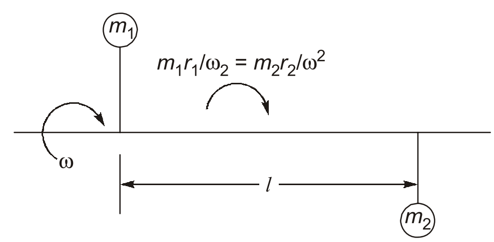
Here, negative sign identify the quadrant of angle.

Graphical Solution

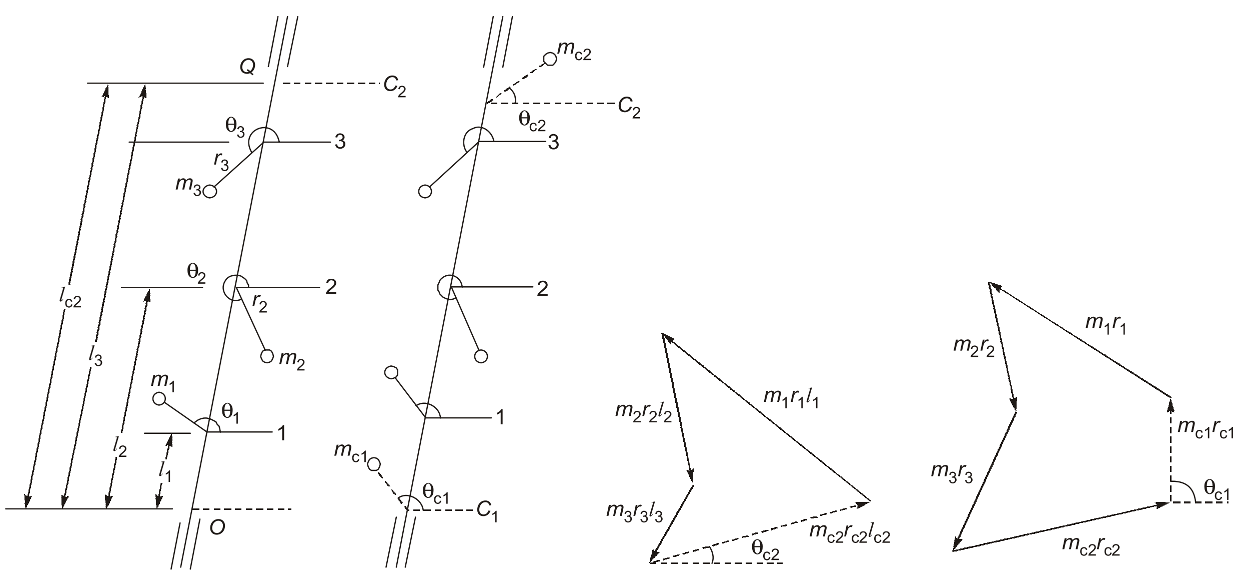
- Vectors m_1r_1, m_2r_2, m_3r_3 etc. are added. If they close in a loop, the system is balanced. Otherwise the closing vector will be given m_cr_c.
- Its direction identifies the angular position of the counter mass.

7.2.2 Dynamic Balancing

- It is required when several masses rotate in different planes. These masses result in centrifugal forces along with couples.
- A system of rotating masses is in dynamic balance when there **does not exist any resultant centrifugal force as well as resultant couple.**



Method of Balancing if Unbalance is Present



(a) (b) (c) (d)

Here, ω = Uniform angular velocity; m_1, m_2, m_3 = Mass attached to rotor at radii r_1, r_2 and r_3 in plane 1, 2, 3 respectively; c_1 = Reference plane; l = Distance of different masses from reference plane

For complete balance

$$m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^3 = 0 \quad \text{and} \quad m_1r_1l_1\omega^2 + m_2r_2l_2\omega^2 + m_3r_3l_3\omega^3 = 0$$

If there are unbalanced force and couples, a mass placed in reference plane may satisfy force equation but the couple equation is satisfied only by two equal force in difference planes. Thus **two planes are needed to balance a system of rotating masses for dynamic balancing (several masses in different plane)**.

In order to balance, introduce two counter-masses m_{c1} and m_{c2} at radii r_{c1} and r_{c2} respectively. Thus,

To Balance Force

$$m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 + m_{c1}r_{c1}\omega^2 + m_{c2}r_{c2}\omega^2 = 0$$

$$\Rightarrow \quad \Sigma m_ir_i + m_{c1}r_{c1} + m_{c2}r_{c2} = 0$$

To Balance Couple, (take moment about 'O')

$$m_1r_1l_1\omega^2 + m_2r_2l_2\omega^2 + m_3r_3l_3\omega^2 + m_{c2}r_{c2}l_{c2}\omega^2 = 0$$

$$\Rightarrow \quad \Sigma m_ir_il_i + m_{c2}r_{c2}l_{c2} = 0$$

Mathematical Solution

$$m_{c2}r_{c2}l_{c2} = \sqrt{(\Sigma m_ir_il_i\cos\theta)^2 + (\Sigma m_ir_il_i\sin\theta)^2}$$

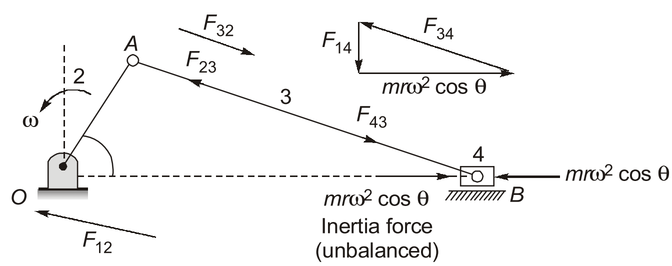
and

$$\tan\theta_{c2} = \frac{-\Sigma m_ir_il_i\sin\theta}{-\Sigma m_ir_il_i\cos\theta}$$

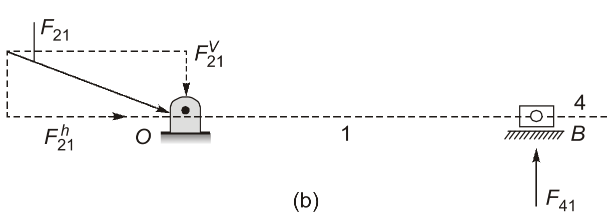
$$m_{c1}r_{c1} = \sqrt{(\Sigma m_ir_i\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})^2 + (\Sigma m_ir_i\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})^2}$$

$$\tan\theta_{c1} = \frac{-(\Sigma m_ir_i\sin\theta + m_{c2}r_{c2}\sin\theta_{c2})}{-(\Sigma m_ir_i\cos\theta + m_{c2}r_{c2}\cos\theta_{c2})}$$

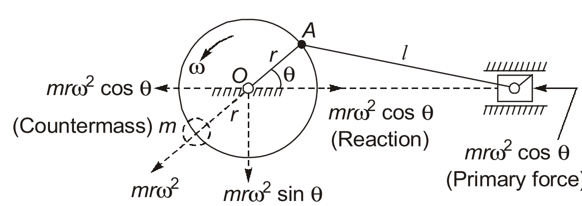
7.3 Balancing of Reciprocating Masses



(a)



(b)



(c)

∴ Force required to accelerate mass m

Force, $$F = mr\omega^2\cos\theta + mr\omega^2\frac{\cos 2\theta}{n}$$

Here, Primary accelerating force $= mr\omega^2\cos\theta$

Secondary accelerating force $= mr\omega^2\dfrac{\cos 2\theta}{n}$

$$n = \frac{\text{Length of connecting rod}}{\text{Crank radius}}$$

Generally 'n' is very high thus secondary force can be neglected for lower speed engine.

Maximum primary force $= mr\omega^2$

Maximum secondary force $= \dfrac{mr\omega^2}{n}$

F_{21}^{h} = Only horizontal unbalanced shaking force. It produces linear vibration.

F_{21}^{v} and F_{41}^{v} = Vertical balanced force but forms couple, which produces oscillating vibration.

Method of Balancing

Balancing the shaking force is done by addition of a rotating countermass at radius '*r*' directly opposite to the crank. This countermass is an addition to the mass used to balance the rotating unbalance due to the mass at the crank pin.

$mr\omega^2 \cos\theta$ = Horizontal component of balancing mass which balances reciprocating mass

$mr\omega^2 \sin\theta$ = Vertical component of balancing mass which remains unbalanced.

Vertical component ($mr\omega^2 \sin\theta$) become zero at the end of stroke when $\theta = 0°$ or $180°$ and maximum at the middle when $\theta = 90°$.

Thus maximum unbalanced force remains same but it tends to jump up and down. So, if we balance the whose reciprocating mass, an unbalanced vertical force ($mr\omega^2\sin\theta$) remains in the system. To minimize this partial balancing is done.

7.4 Minimization of Effect of Unbalanced Force

- If '*c*' is the fraction of the reciprocating mass, thus

  $$\text{Primary balanced force} = cmr\omega^2 \cos\theta$$

  $$\text{Primary unbalanced force} = (1 - c)mr\omega^2 \cos\theta$$

  Unbalanced vertical component of centrifugal force = $cmr\omega^2 \sin\theta$

  Resultant unbalanced force at any instant

  $$= \sqrt{\left[(1-c)mr\omega^2 \cos\theta\right]^2 + \left[cmr\omega^2 \sin\theta\right]^2}$$

- Resultant unbalanced force is minimum when $c = 1/2$
- In reciprocating engines, unbalanced forces in the direction of line of stroke are more dangerous than perpendicular to line of stroke.

Secondary Balancing

$$\text{Secondary force} = mr\omega^2 \frac{\cos 2\theta}{n} = mr(2\omega)^2 \frac{\cos 2\theta}{4n}$$

- Its frequency is twice that of the primary force and the magnitude $1/n$ times the magnitude of the primary force.
- The Effect of secondary force is equivalent to an imaginary crank of length $r/4n$ rotating at double the angular velocity i.e. twice of the engine speed.
- The secondary couple about a reference plane is given by the multiplication of the secondary force with the distance l of the plane from reference plane.

Condition for Complete Balancing of Reciprocating Mass

- Primary forces must balance i.e. Primary force polygon is enclosed.
- Primary couples must balance i.e. Primary couple polygon is enclosed.
- Secondary forces must balance i.e. secondary force polygon is enclosed.
- Secondary couples must balance i.e. secondary couple polygon is enclosed.

NOTE

(i) It is not possible to satisfy all the above condition fully for reciprocating masses.

(ii) Hence, we can say that rotating masses are always completely balanced but reciprocating masses are not completely balanced.

7.5 Effect of Partial Balancing in Locomotives

(i) Hammer Blow

- Hammer blow is the maximum vertical unbalanced force caused by the mass provided to balance the reciprocating masses. Its value is = $m r \omega^2$.
  m = Balance mass
- It varies as square of the speed.
- At high speeds, force of the hammer blow could exceed the static load on the wheels and the wheels can be lifted off the rail when the direction of the hammer blow will be vertically upwards.

(ii) Variation of Tractive Force

Unbalanced primary force for cylinder '1'

$$= (1 - c) m r \omega^2 \cos\theta$$

Unbalanced primary force for cylinder '2'

$$= (1 - c) m r \omega^2 \cos(90^\circ + \theta) = -(1 - c) m r \omega^2 \sin\theta$$

Thus, Total unbalanced primary force or the variation in the tractive force

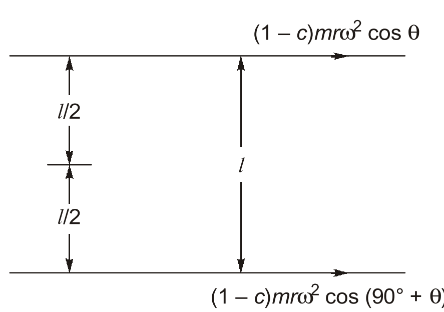
$$= (1 - c)\, m r \omega^2 (\cos\theta - \sin\theta)$$

NOTE: This is maximum when $\theta = 135^\circ$ or 315°.

Maximum variation in tractive force = $\pm\sqrt{2}(1 - c) m r \omega^2$

(iii) Swaying Couple

Unbalanced primary forces along the line of stroke are separated by a distance l apart and thus constitute a couple. This tends to make the leading wheels away from side to side.



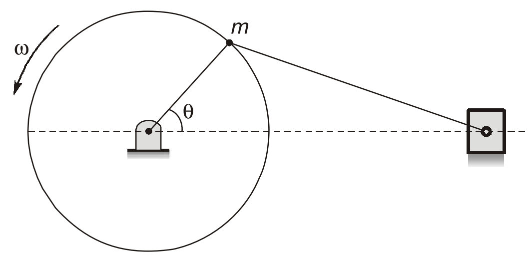
Swaying couple = moment of forces about the engine centre line

$$= (1 - c) m r \omega^2 (\cos\theta + \sin\theta) \cdot \frac{l}{2}$$

This is maximum when $\theta = 45^\circ$ or 225°

Maximum swaying couple = $\pm\frac{1}{\sqrt{2}}(1 - c) m r \omega^2 l$

7.6 Balancing of Radial Engines

- A radial engine is a multicylinder engine in which all the connecting rods are connected by a common crank.
- These are balanced by direct and reverse crank methods.
- As all the forces are in the same plane, no unbalance couple exists.
- 

- **Primary Force Balance :**

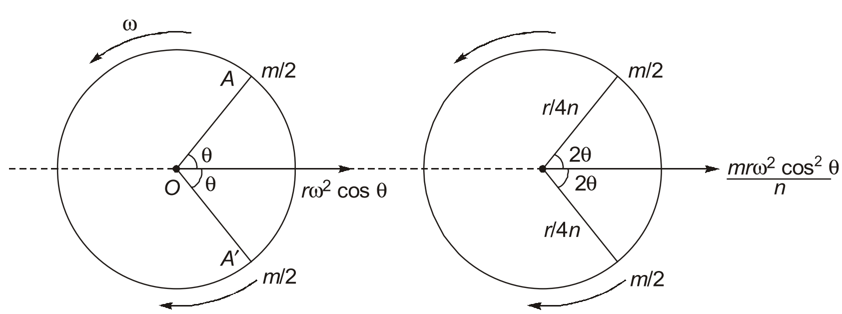
  Direct crank : $\frac{m}{2}$ at θ from line of stroke in same direction.

  Reverse crank : $\frac{m}{2}$ at θ from line of stroke in opposite direction.

- **Secondary Force Balance :**

  Direct Crank : $\frac{m}{2}$ at end of crank of length $\frac{r}{4n}$ at angle 2θ and rotating with '2ω' in the same direction.

  Secondary Crank : $\frac{m}{2}$ at end of crank of length $\frac{r}{4n}$ at angle 2θ and rotating with '2ω' in the opposite direction.



OA : Direct crank
OA′ : Reverse crank

Primary Balance

Secondary Balance

- This method is also used for *V* engines.

Example - 7.1 A system of masses rotating in different planes is in dynamic balance if

(a) resultant force is zero
(b) resultant couple is zero
(c) resultant couple is numerically equal to resultant force
(d) resultant force and resultant couple, both are zero.

Solution: (d)

$$\text{Different planes} \rightarrow \text{Dynamic balancing}$$

So, both couple and force should be zero.

Example - 7.2 In order to balance the reciprocating masses

(a) primary and secondary forces must be balanced
(b) primary couple must be balanced
(c) secondary couple must be balanced
(d) All options are correct

Solution: (d)

Theoretically all options are correct. But only partial balancing is done for reciprocating masses.

Example - 7.3 If the ratio of the length of the connecting rod to crank radius increases, then

(a) Primary unbalanced forces will increase
(b) Primary unbalanced forces will decrease
(c) Secondary unbalanced forces will increase
(d) Secondary unbalanced forces will decrease

Solution: (d)

$$F = mr\omega^2\cos\theta + \frac{mr\omega^2\cos 2\theta}{n}$$

if n i.e., $\frac{L}{r}$ increases, $\frac{mr\omega^2\cos 2\theta}{n}$ decreases.

Student's Assignment

Q.1 Balancing of rigid-rotors can be achieved by appropriately placing balancing weights in
(a) single plane (b) two plane
(c) three plane (d) four plane

Q.2 Shaking force in unbalanced slider-crank mechanism produces
(a) linear vibration (b) oscillating vibration
(c) both (a) and (b) (d) None of the above

Q.3 In reciprocating engine, unbalanced force in the direction of line of stroke as compared to perpendicular to the line of stroke is
(a) less dangerous (b) more dangerous
(c) equally dangerous (d) None of these

Q.4 In reciprocating engine, primary forces are
(a) completely balanced
(b) partially balanced
(c) can not be balanced
(d) None of these

Q.5 Hammer blow
(a) is the maximum horizontal unbalanced force caused by the mass provided to balance the reciprocating masses
(b) is the maximum vertical unbalance force caused by the mass added to balance the reciprocating masses
(c) varies as the square root of speed
(d) varies inversely with the square of the speed

Q.6 Consider the following statements :
Effect of unbalanced primary force along the line of stroke produces
1. Swaying couple
2. Variation in tractive forces
3. Hammer blow

Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

Q.7 An unbalanced couple of magnitude 300 Nm is noticed on a shaft of 200 cm length. The dynamic reactions at the bearings are
(a) 300 N and –300 N
(b) 300 N and 300 N
(c) 150 N and 150 N
(d) 150 N and –150 N

Q.8 Two rotors are mounted on a shaft. If the unbalanced force due to one rotor is equal in magnitude to the unbalanced force due to the other rotor, but positioned exactly 180° apart, then the system will be balanced
(a) statically
(b) dynamically
(c) both (a) and (b)
(d) None of these

Q.9 Resultant unbalanced force in reciprocating engine will be minimum at any instant when balanced mass is
(a) $\frac{1}{2} \times$ reciprocating mass
(b) $\frac{2}{3} \times$ reciprocating mass
(c) $\frac{1}{3} \times$ reciprocating mass
(d) $\frac{3}{4} \times$ reciprocating mass

Q.10 In locomotive unbalanced portion of the primary force, which act along the line of stroke of a locomotive engine, will be maximum at
(a) 180°
(b) 270°
(c) 135°
(d) 90°

ANSWER KEY // STUDENT'S ASSIGNMENT

1. (b)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (b)
6. (b)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (c)

HINTS & SOLUTIONS // STUDENT'S ASSIGNMENT

1. (b)

An unbalancing rigid rotor will behaves if several masses in different plane and it is balanced by placing weights in two plane.

2. (b)

Shaking couple produce oscillating vibration.

3. (b)

In later case, mechanism tend to jump up and down thus more dangerous.

4. (b)

In reciprocating engine, primary force are only partially balanced.

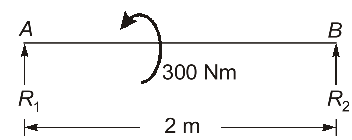
5. (b)

Hammer blow is the maximum vertical unbalance forced caused by the mass added to balance the reciprocating masses.

6. (b)

Hammer blow is perpendicular to line of stroke.

7. (d)



$$R_1 + R_2 = 0 \Rightarrow R_1 = -R_2$$

Now, $\Sigma M_A = 0$

$$R_2 \times 2 + 300 = 0$$

$$\Rightarrow \quad R_2 = -150 \text{ Nm}$$

$$R_1 = 150 \text{ Nm}$$

8. (a)

Because unbalanced couple will be present.

9. (a)

Resultant unbalanced force

$$= \sqrt{\left[(1-c)mr\omega^2 \cos\theta\right]^2 + \left[cmr\omega^2 \sin\theta\right]^2}$$

10. (c)

Maximum variation $= \pm\sqrt{2}(1-c)mr\omega^2$

at θ = 135°, 315°.